IdylliQ matter:

Momentum shell in Quarkyonic matter from explicit duality

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Quarkyonic duality

Collins,Perry (1974)

Contrary to the common belief of free deconfied quarks at high μ…

Large-*Nc* QCD implies:

McLerran,Pisarski (2007)

Duality between quark matter and baryonic matter

$$r_{\rm Debye}^{-1} \sim \frac{1}{N_{\rm c}} \lambda_{\rm 't\ Hooft}\, \mu^2$$ … Never screened when $N_c \to \infty$

$$(\lambda_{\rm 't\ Hooft} = g^2 N_{\rm c})$$

Real QCD ($N_c = 3$):

Confinement when $r_{\rm Debye} > r_{\rm conf} \sim \Lambda_{\rm QCD}^{-1}$

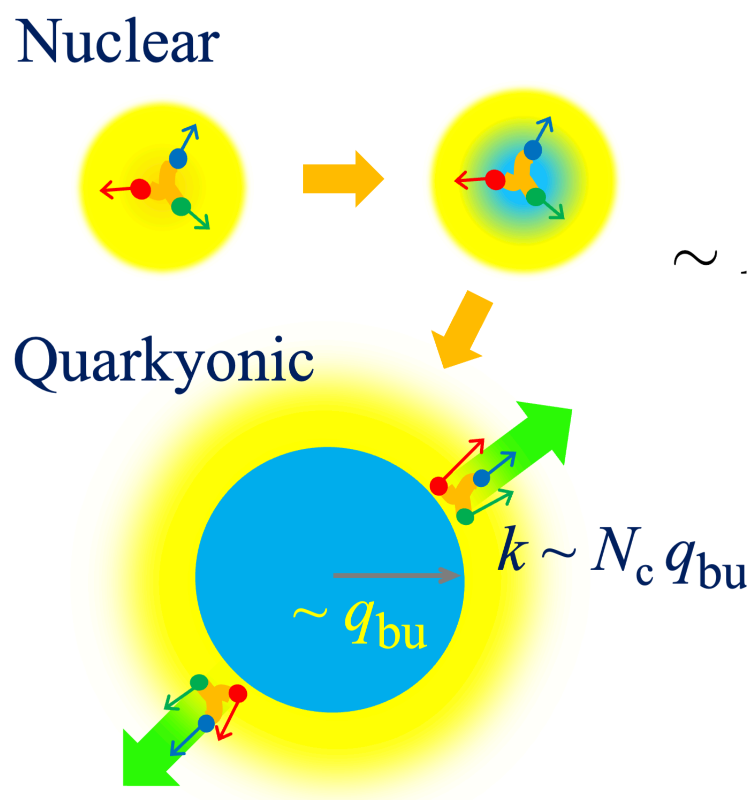
→ Quarkyonic regime: $\Lambda_{\rm QCD} \ll \mu \ll \sqrt{N_{\rm c}}\Lambda_{\rm QCD}$

…Quark Fermi sea formed but confined (baryonic)

Fermi “shell” picture

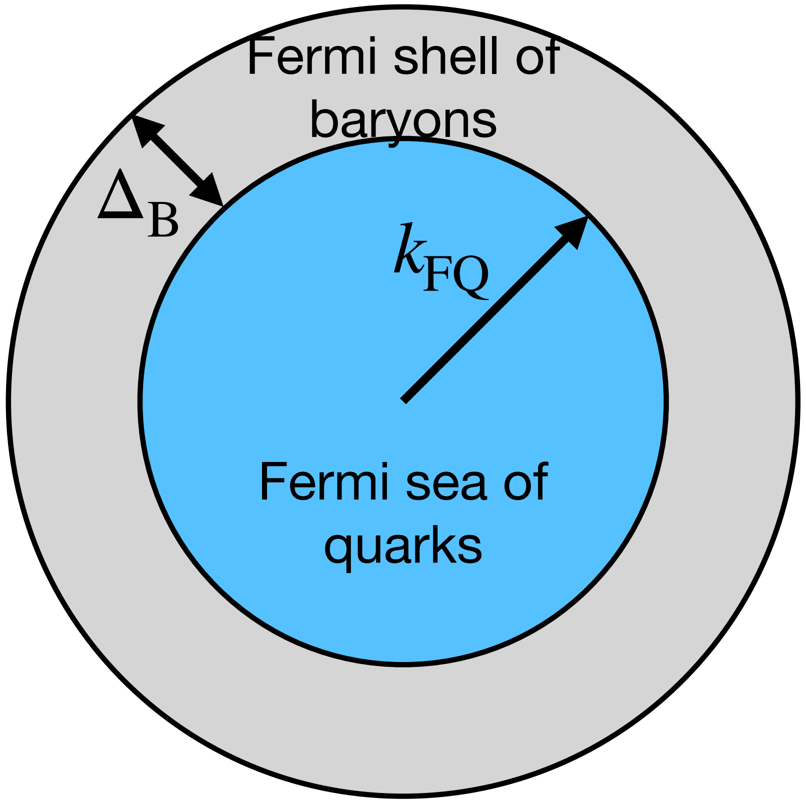
McLerran,Pisarski (2007);
see also: Jeong,McLerran,Sen (2019); Koch,Vovchenko (2022)

Resolution to the duality paradox is given by assuming the Fermi shell picture



Fermi sea: dominated by interaction that is less sensitive to IR → quarks

Fermi shell: interaction sensitive to IR d.o.f. → baryons, mesons, glues.



In this talk, we give an alternative explanation to this shell structure based on an explicit duality

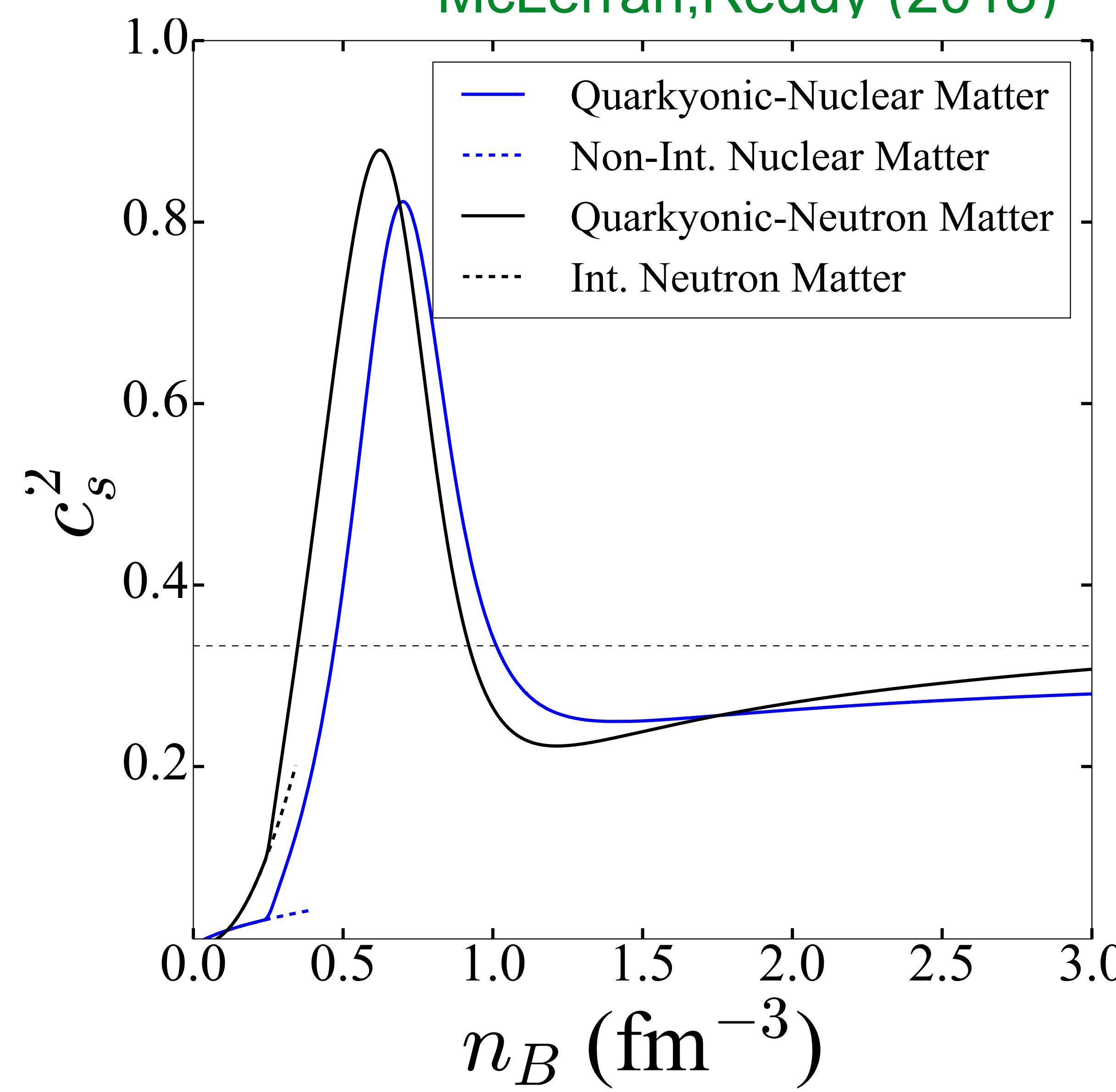
Implication to neutron-star EoS

McLerran,Pisarski (2007);
McLerran,Reddy (2018)

- Large sound speed at the onset of Quarkyonic matter
  → Transition is crossover.
  Different from the first-order phase transition.

- Rapid stiffening needed to support $2M_\odot$ neutron stars.

- Approaches to conformality at high density.



Theory with an explicit duality

Kojo (2021); Fujimoto,Kojo,McLerran (2023)

Quantum occupation of baryons and quarks in momentum space:

$$0 \le f_{\rm B}(k) \le 1, \quad 0 \le f_{\rm Q}(q) \le 1$$

- Free energy and density with an explicit duality

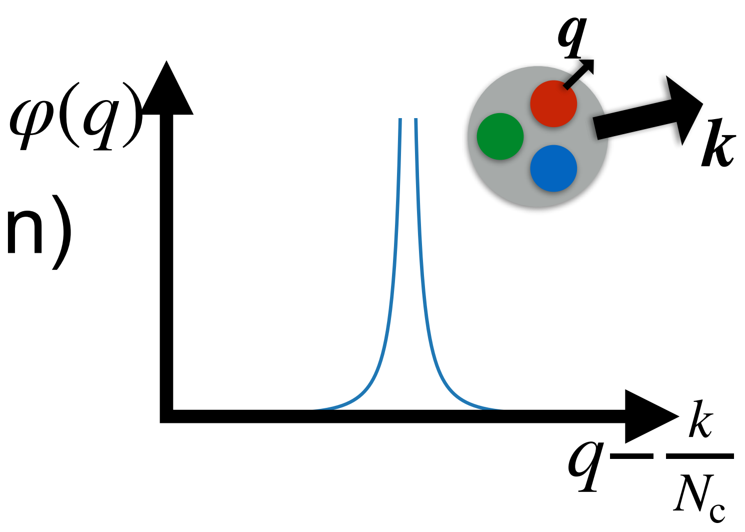
(= described in two ways, i.e., baryons and quarks)

$$\varepsilon = \varepsilon_{\rm B}[f_{\rm B}(k)] = \varepsilon_{\rm Q}[f_{\rm Q}(q)], \quad n_{\rm B} = \int_{\boldsymbol{k}} f_{\rm B}(k) = \int_{\boldsymbol{q}} f_{\rm Q}(q)$$

- The duality relation between $f_{\rm B}$ and $f_{\rm Q}$

(= probability to find quarks inside a single baryon)

$$f_{\rm Q}(q) = \int_{\boldsymbol{k}} \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\rm c}}\right) f_{\rm B}(k)$$



- Goal: Minimize ε w.r.t. $f_{\rm B}$ or $f_{\rm Q}$ → Variational problem

IdylliQ matter

Kojo (2021); Fujimoto,Kojo,McLerran (2023)

Free energy with an explicit duality

In this work, we use the **ideal** gas expression for baryons

$$\varepsilon = \varepsilon_{\rm B}[f_{\rm B}(k)] = \varepsilon_{\rm Q}[f_{\rm Q}(q)]$$

$$\varepsilon_{\rm B}[f_{\rm B}(k)] = \int_{\boldsymbol{k}} E_{\rm B}(k) f_{\rm B}(k), \qquad \left(E_{\rm B}(k) = \sqrt{k^2 + M_N^2}\right)$$

$$\varepsilon_{\rm Q}[f_{\rm Q}(q)] = \int_{\boldsymbol{q}} E_{\rm Q}(q) f_{\rm Q}(q)$$

We fix the baryon expression because we know this gives a suitable low-density description. Quark dispersion is fixed via the duality relation.

We name it as **IdylliQ** (Ideal dual Quarkyonic) **matter**

Explicitly solvable model

Fujimoto,Kojo,McLerran (2023)

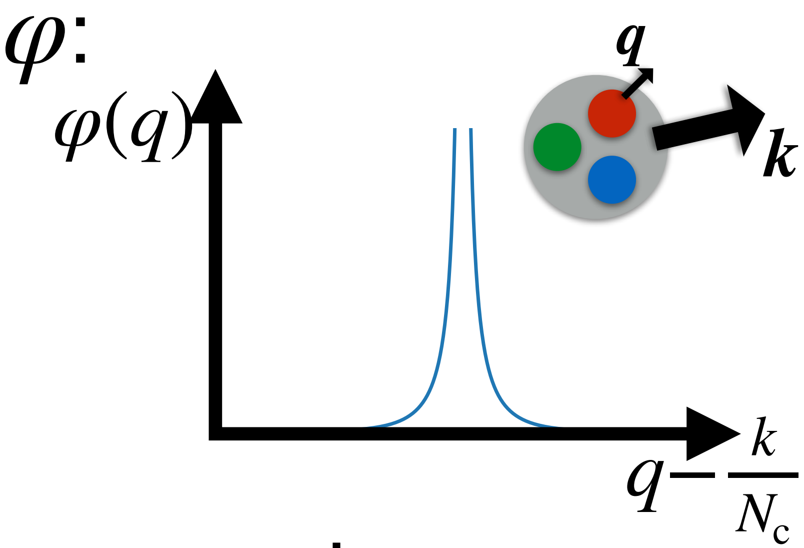
The duality relation between f_{B} and f_{Q} (quark model):

$$f_{\mathrm{Q}}(q) = \int \frac{d^d\boldsymbol{k}}{(2\pi)^d}\varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right) f_{\mathrm{B}}(k)$$

In this work, we assume the specific form for φ:

$$\varphi(q) = \frac{2\pi^2}{\Lambda^2}\frac{e^{-q/\Lambda}}{q}$$

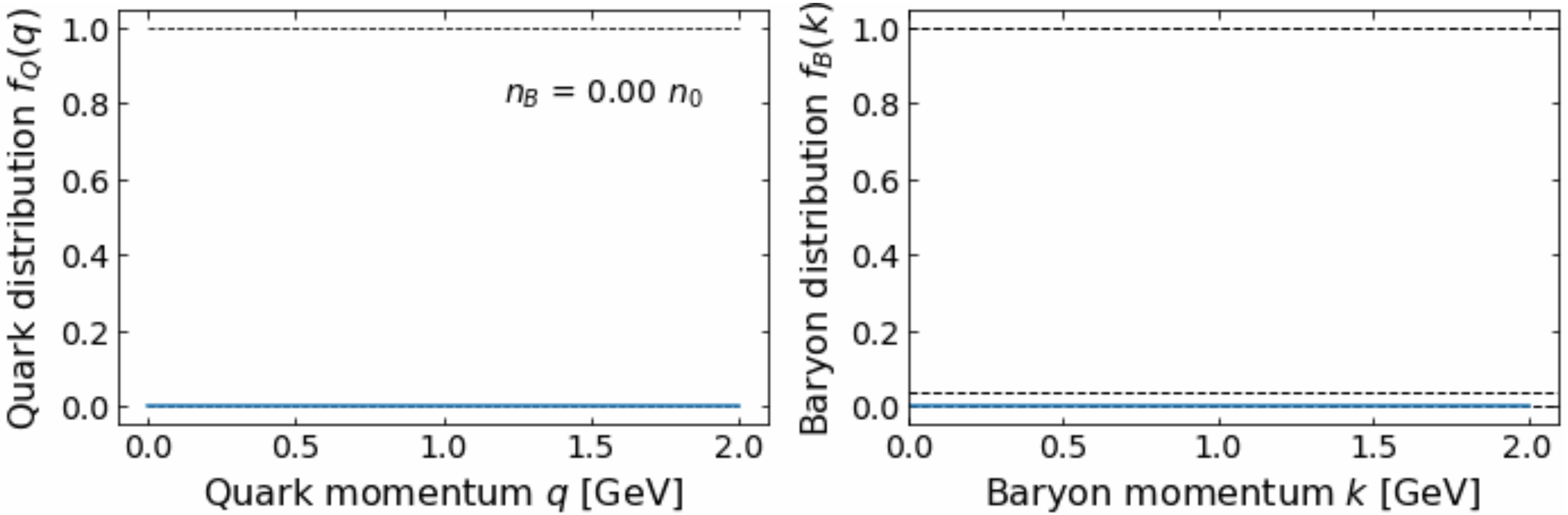
Λ: confining scale



This specific choice entails the one-to-one correspondence:

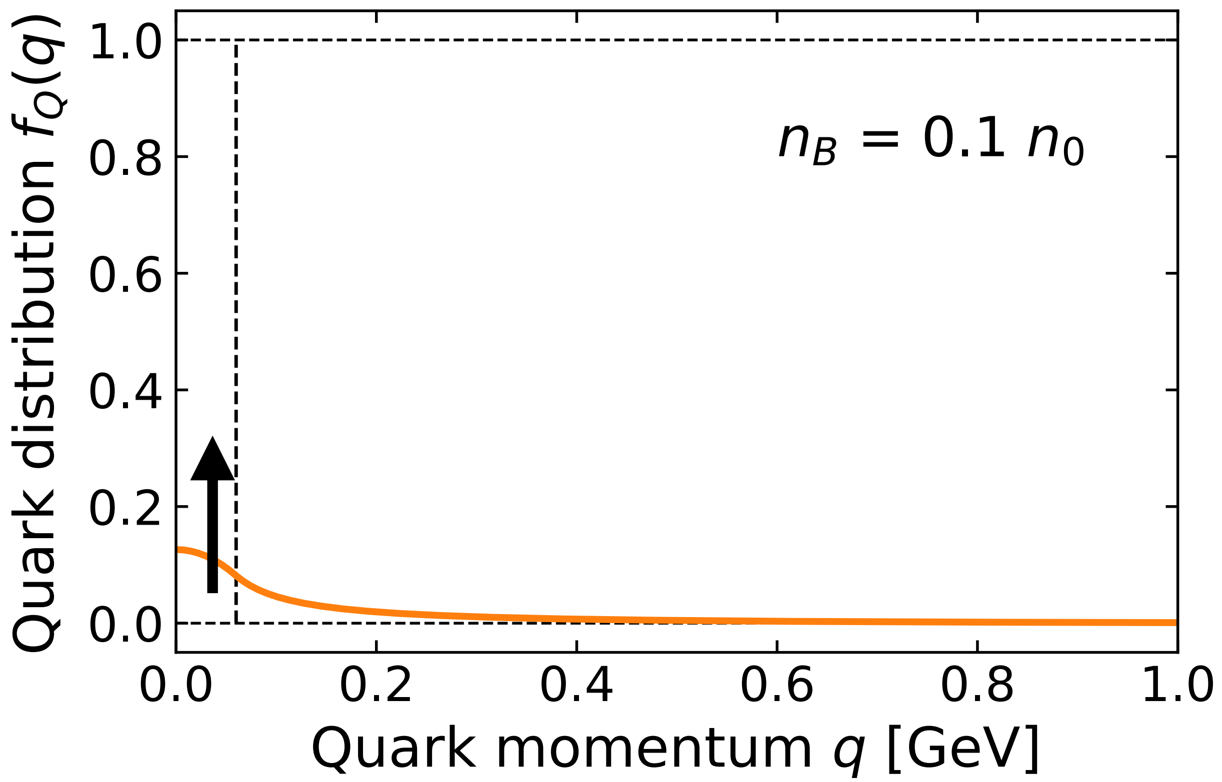
$$f_{\mathrm{B}}(N_{\mathrm{c}}q) = \frac{\Lambda^2}{N_{\mathrm{c}}^3}\left(-\nabla_q^2 + \frac{1}{\Lambda^2}\right) f_{\mathrm{Q}}(q)$$

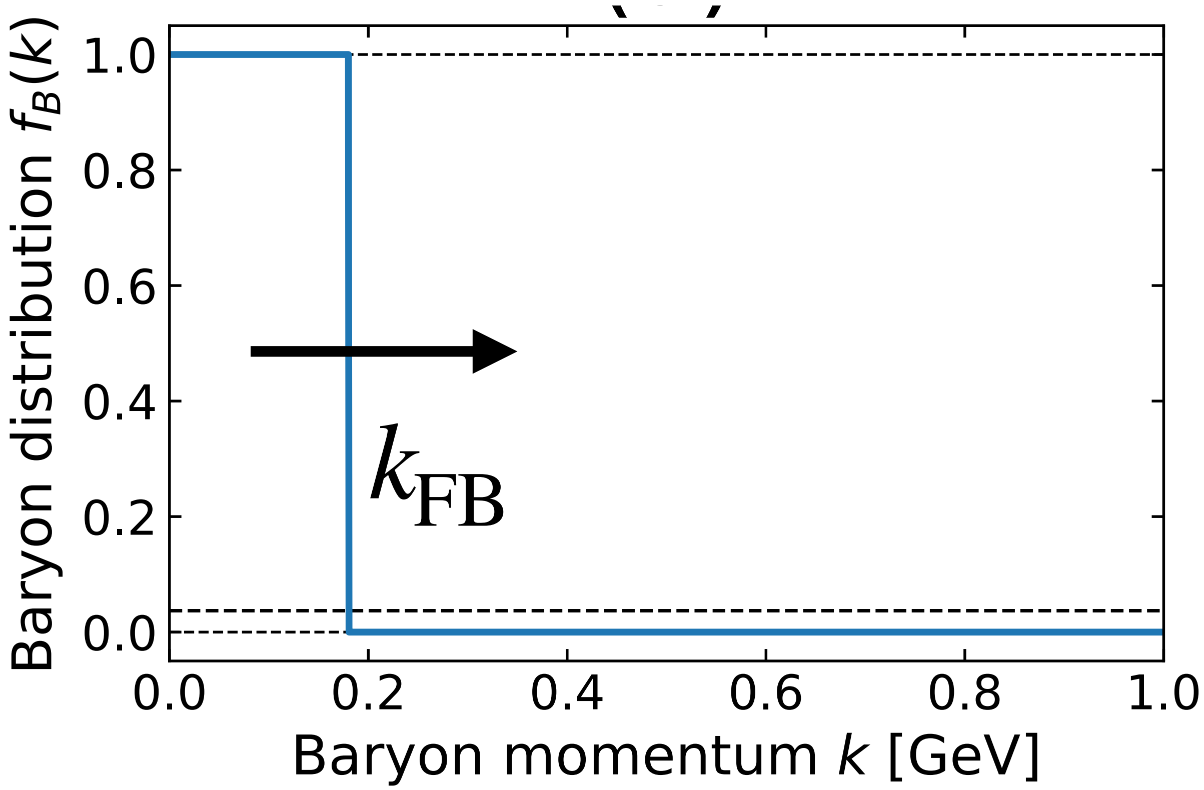
Overview on the analytic solution



Solution at low density

Kojo (2021), Fujimoto,Kojo,McLerran (2023)



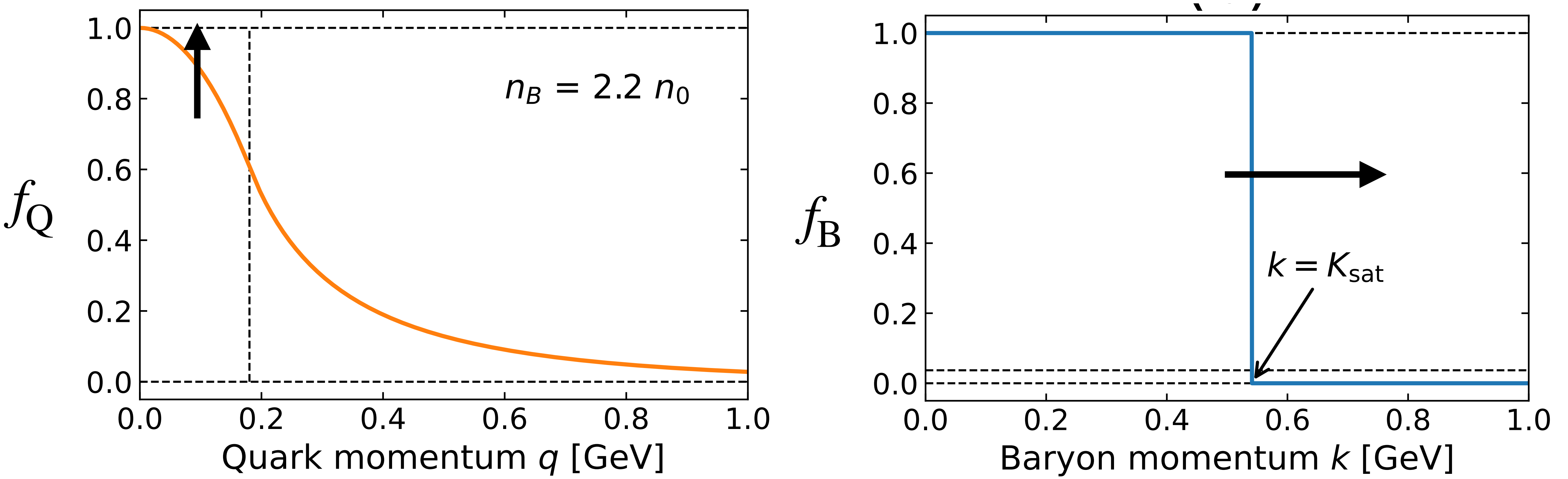


Fermi gas of baryons is formed.

Baryonic Fermi momentum k_{FB} grows until f_{Q} reaches 1

Saturation of the quark distribution

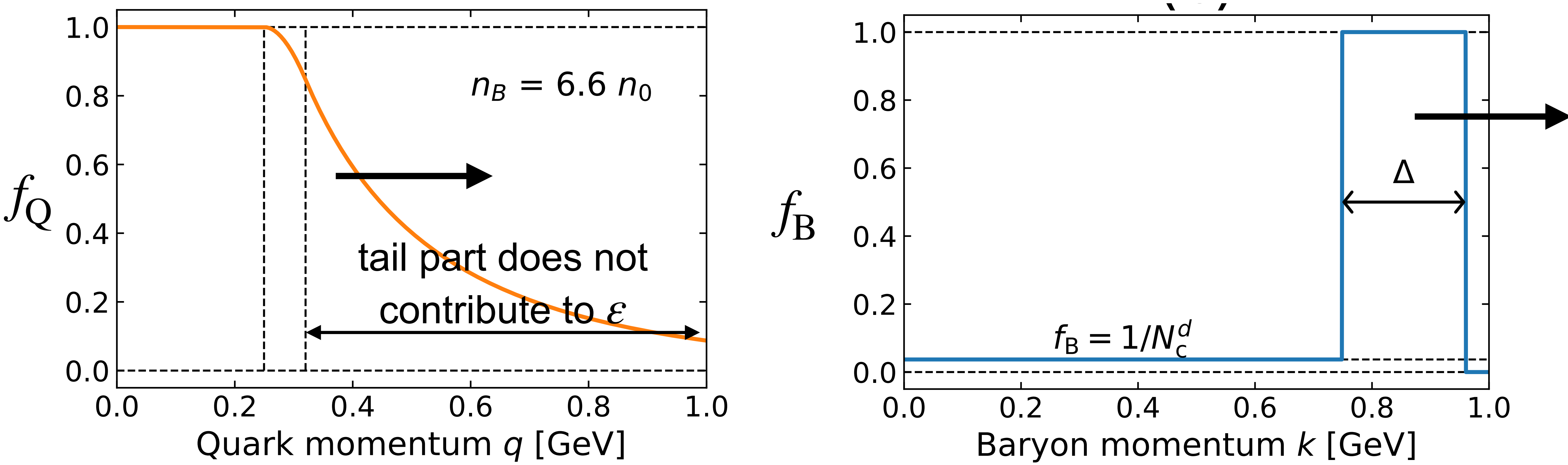
Kojo (2021), Fujimoto,Kojo,McLerran (2023)



At this point, f_Q “saturates” and Pauli blocking constraint becomes essential.

Solution at high density

Fujimoto,Kojo,McLerran (2023)



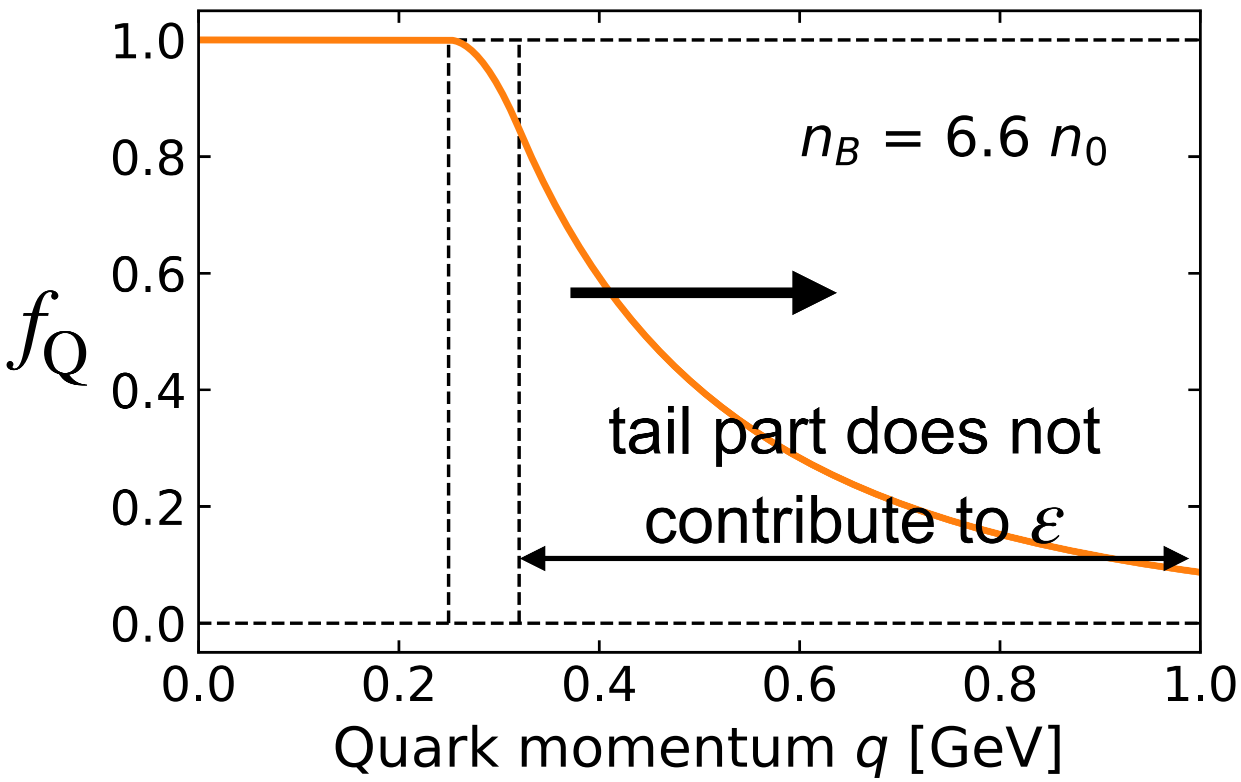
Saturation in low-momentum $f_Q \to$ Depletion in $f_B \sim 1/N_c^3$ (in 3d)

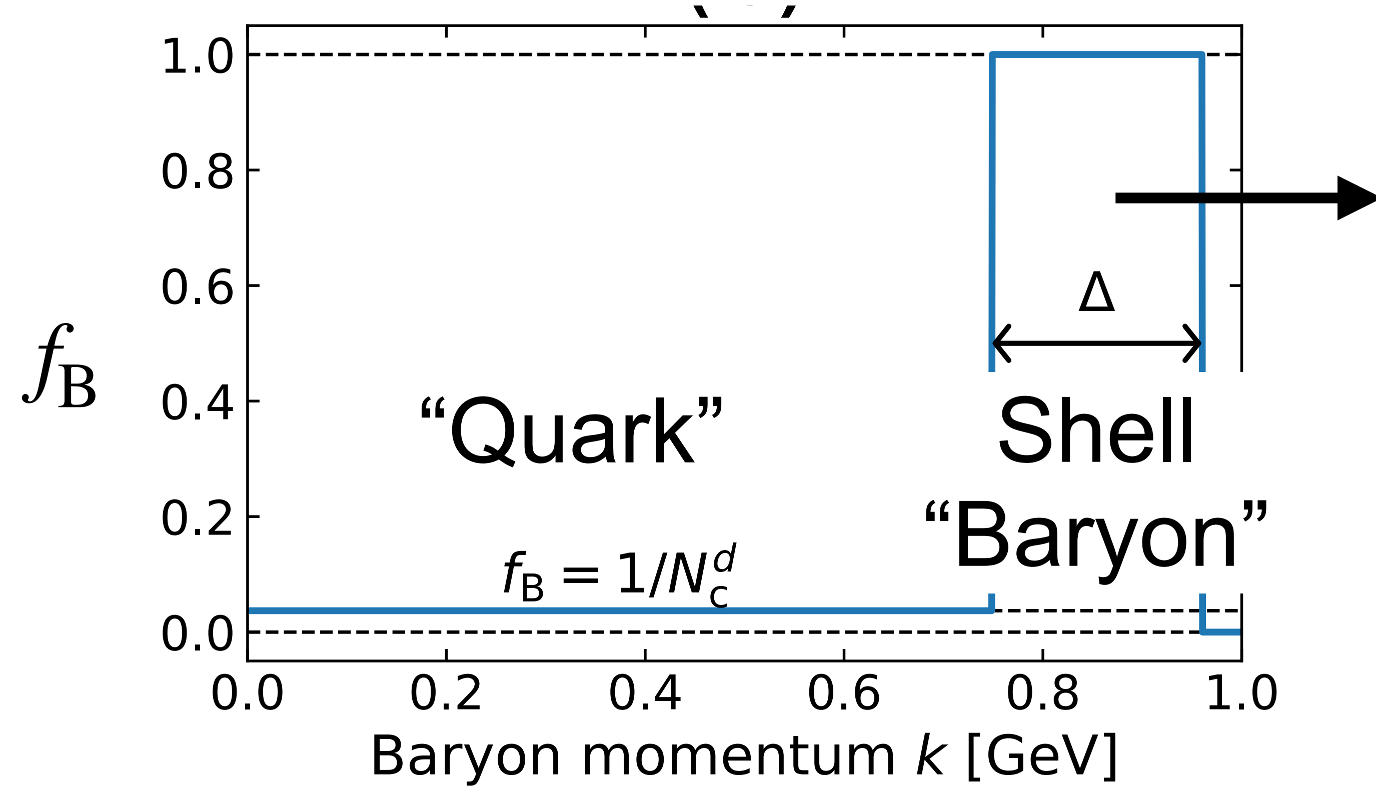
Tail in f_Q w/ a width $\sim \Lambda$

$\to$ Shell formation in high-momentum f_B w/ $\Delta_B \sim \dfrac{\Lambda}{N_c^2}$

Solution at high density

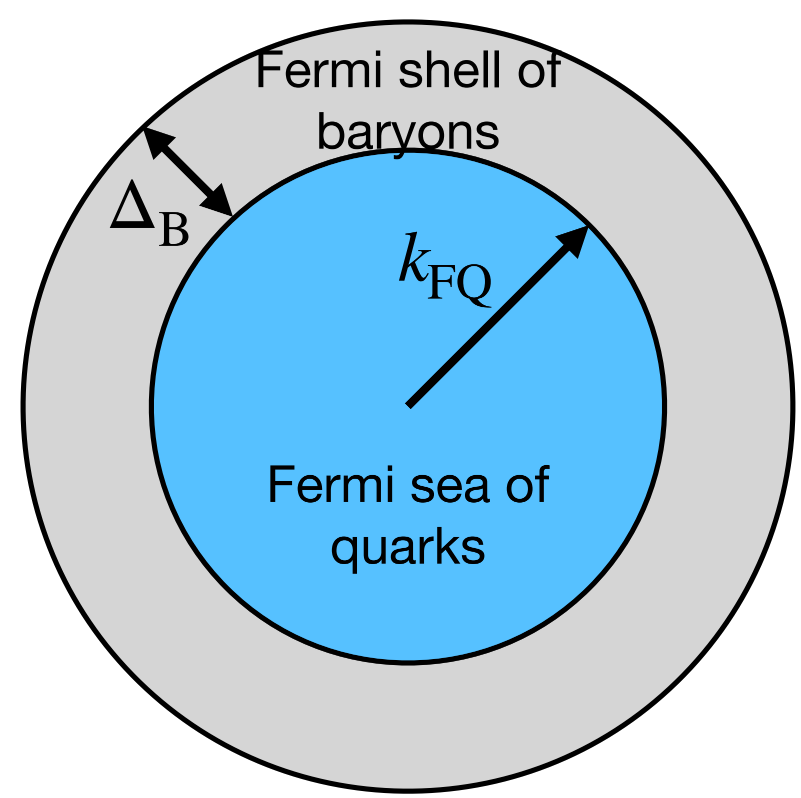
Fujimoto,Kojo,McLerran (2023)





Fermi shell structure arises in f_B

This picture is equivalent to the McLerran-Pisarski shell picture apart from the behavior of Δ_B



Underoccupied $f_{\rm B}$ and occupied $f_{\rm Q}$

Baryon number in the bulk "quark" region in the quark language:

$$n_{\rm B} = \int_0^{k_{\rm FQ}} \frac{d^3q}{(2\pi)^3} f_{\rm Q}(q) \sim k_{\rm FQ}^3 f_{\rm Q}$$

In the baryon language:

$$n_{\rm B} = \int_0^{k_{\rm FB}} \frac{d^3k}{(2\pi)^3} f_{\rm B}(k) \sim k_{\rm FB}^3 f_{\rm B} \sim N_{\rm c}^3 k_{\rm FQ}^3 f_{\rm B}$$

where the Fermi momenta are related as $k_{\rm FB} \sim N_{\rm c} k_{\rm FQ}$.

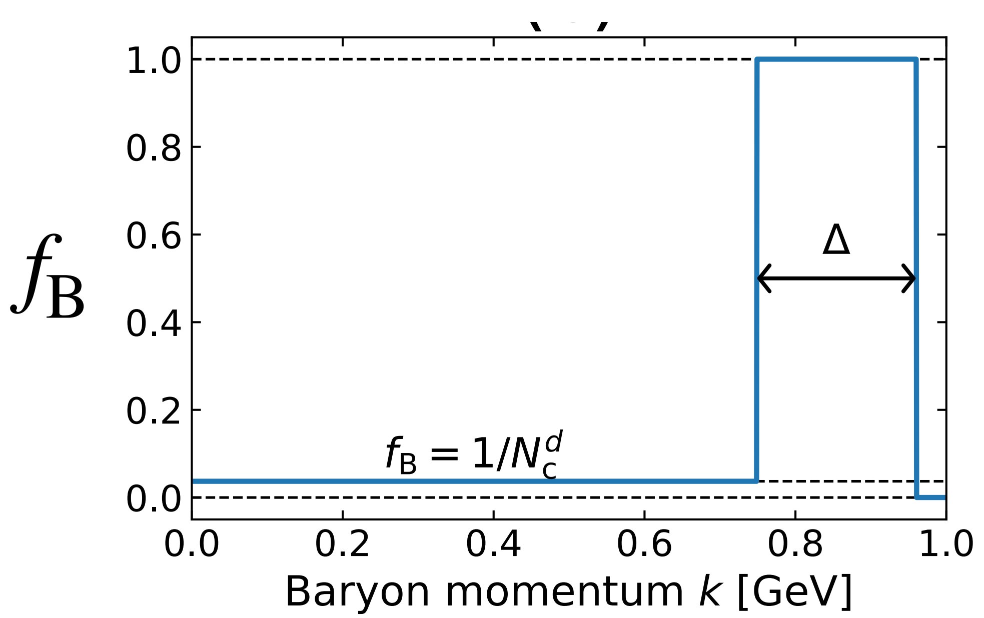
Because $f_{\rm Q} \le 1, \quad f_{\rm B} \sim 1/N_{\rm c}^3$ … composite baryon states are underoccupied

Rapid stiffening in the EoS

A partial occupation of available baryon phase space leads to the **large speed of sound**.

$$v_s^2 = \frac{n_B}{\mu_B dn_B/d\mu_B} \to \frac{\delta\mu_B}{\mu_B} \sim v_s^2 \frac{\delta n_B}{n_B}$$

If baryon is underoccupied, the density is does not vary a lot, with the variation of the Fermi energy



Summary

- We formulate the quantum-mechanical theory of **IdylliQ matter** — Quarkyonic matter with an explicit duality and ideal baryonic dispersion relation

- Previously proposed Fermi “shell” structure of Quarkyonic matter naturally arises in the baryon distribution

- Rapid rise in the sound speed is still there — the observational signature of the Quarkyonic matter