Entropy Production in Spin Hydrodynamics

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arXiv:2309.05789

(https://online.kitp.ucsb.edu/online/relfluids23/becattini/rm/jwvideo.html)

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Outline

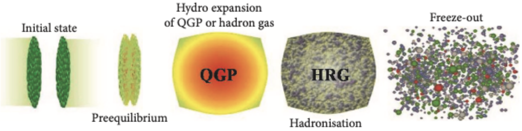
1. **Motivation: Why? What? Goal?**
2. **Local Equilibrium State**
3. **Entropy Current and Entropy-Gauge Transformation**
4. **Entropy Production Rate**
5. **Conclusion and Outlook**

Motivation: Why? What? Goal?

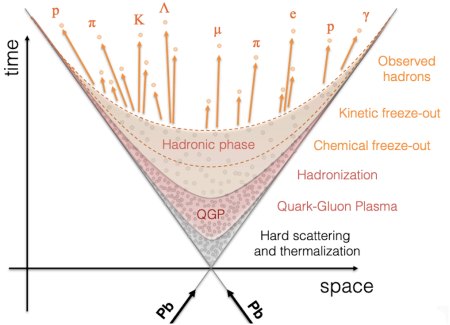
Why spin hydro?

Successes of relativistic hydrodynamics in heavy ion collisions

[W.Florkowski-Phenomenology of Ultra-relativistic Heavy-ion Collisions-World Scientific]



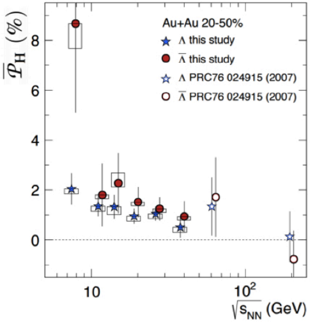
Various stages of relativistic heavy-ion collision [http://qgp.phy.duke.edu]



Spacetime diagram of relativistic heavy-ion collision [D.D. Chinellato]

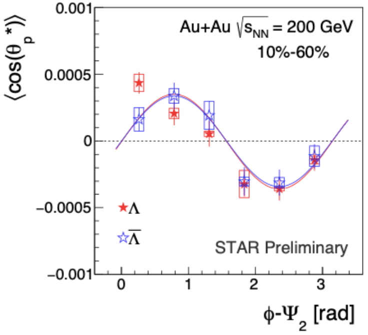
Why spin hydro?

First positive measurements of global spin polarization of Λ hyperons by STAR



Average Λ global polarization [STAR, L. Adamczyk et al., Nature 548, 62 (2017)]

Measurement of longitudinal polarization



[T.Niida, NPA 982 (2019) 511514]

What is spin hydro?

Spin hydrodynamics stipulates that the description of a relativistic fluid requires the addition of a *spin tensor* (that is the mean value of a rank 3 tensor operator $\widehat{\mathcal{S}}^{\lambda\mu\nu}$)

$$\widehat{\mathcal{J}}^{\lambda\mu\nu} = x^{\mu}\widehat{\mathcal{T}}^{\lambda\nu} - x^{\nu}\widehat{\mathcal{T}}^{\lambda\mu} + \widehat{\mathcal{S}}^{\lambda\mu\nu}\,, \tag{1}$$

where $\widehat{\mathcal{T}}^{\mu\nu}$ and $\widehat{\mathcal{J}}^{\lambda\mu\nu}$ are the energy-momentum and total angular momentum tensor operators,

$$\partial_{\mu}\widehat{\mathcal{T}}^{\mu\nu} = 0 \ \ , \ \ \partial_{\lambda}\widehat{\mathcal{J}}^{\lambda\mu\nu} = 0. \tag{2}$$

This implies that the spin tensor fulfills the continuity equation:

$$\partial_{\lambda}\widehat{\mathcal{S}}^{\lambda\mu\nu} = \widehat{\mathcal{T}}^{\nu\mu} - \widehat{\mathcal{T}}^{\mu\nu} = -2\widehat{\mathcal{T}}_{a}^{\mu\nu}. \tag{3}$$

What is spin hydro?

$$\boxed{\partial_\mu T^{\mu\nu} = 0} \quad \text{s.t} \quad \boxed{T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + T_s^{\mu\nu} + T_a^{\mu\nu}} \tag{4}$$

$$\begin{cases} T_s^{\mu\nu} = h^\mu u^\nu + h^\nu u^\mu + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu}, \\ T_a^{\mu\nu} = q^\mu u^\nu - q^\nu u^\mu + \Phi^{\mu\nu}. \end{cases}$$

$$\boxed{\partial_\lambda S^{\lambda\mu\nu} = -2T_a^{\mu\nu}} \quad \text{s.t} \quad \boxed{S^{\lambda\mu\nu} = u^\lambda S^{\mu\nu} + S_1^{\lambda\mu\nu}} \tag{5}$$

To solve the system $\longrightarrow$ determine $T_s^{\mu\nu}, T_a^{\mu\nu}, S_1^{\lambda\mu\nu}$.
(1^{st} step to reproduce the polarization data)

Goal

Several groups are working on spin hydrodynamics/spin polarization:

- Spin hydro with torsion
- Macroscopic approach
- Kinetic approach

[1] M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, and H.-U. Yee, "Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation," *JHEP* **11** (2021) 150, arXiv:2107.14231 [hep-th].

[2] K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya, "Fate of spin polarization in a relativistic fluid: An entropy-current analysis," *Phys. Lett. B* **795** (2019) 100–106, arXiv:1901.06615 [hep-th].

[3] K. Fukushima and S. Pu, "Spin hydrodynamics and symmetric energy-momentum tensors – A current induced by the spin vorticity –," *Phys. Lett. B* **817** (2021) 136346, arXiv:2010.01608 [hep-th].

[4] A. Daher, A. Das, W. Florkowski, and R. Ryblewski, "Canonical and phenomenological formulations of spin hydrodynamics," *Phys. Rev. C* **108** no. 2, (2023) 024902, arXiv:2202.12609 [nucl-th].

[5] D. She, A. Huang, D. Hou, and J. Liao, "Relativistic viscous hydrodynamics with angular momentum," *Sci. Bull.* **67** (2022) 2265–2268, arXiv:2105.04060 [nucl-th].

[6] A. D. Gallegos, U. Gürsoy, and A. Yarom, "Hydrodynamics of spin currents," *SciPost Phys.* **11** (2021) 041, arXiv:2101.04759 [hep-th].

[7] A. D. Gallegos, U. Gursoy, and A. Yarom, "Hydrodynamics, spin currents and torsion," arXiv:2203.05044 [hep-th].

[8] N. Weickgenannt, D. Wagner, E. Speranza, and D. H. Rischke, "Relativistic second-order dissipative spin hydrodynamics from the method of moments," *Phys. Rev. D* **106** no. 9, (2022) 096014, arXiv:2203.04766 [nucl-th].

Goal

In this approach, we will use first-principle quantum-density operator method, i.e, to formulate a "quantum relativistic approach to spin hydrodynamics" in order to re-derive and compare:

1. Entropy current,
2. Entropy production rate,
3. Dissipative currents.

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For experts: This will be achieved without assuming the local thermodynamic relations,

$$\epsilon + p = Ts + \frac{1}{2}\omega_{\mu\nu}S^{\mu\nu}. \tag{6}$$

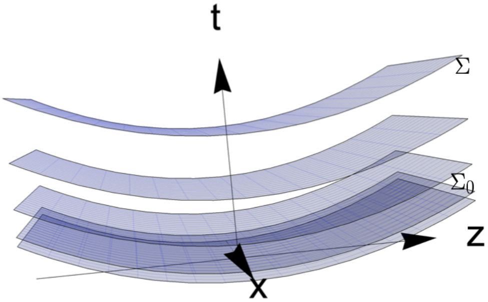
Local Equilibrium State

Typical Problem in QFT

Heisenberg Picture	$\widehat{\mathcal{T}}^{\mu\nu}(x,t) = e^{i\widehat{\mathcal{H}}t}\ \widehat{\mathcal{T}}^{\mu\nu}(x,0)\ e^{-i\widehat{\mathcal{H}}t}$	$\widehat{\rho}(0)$ fixed
Schrodinger Picture	$\widehat{\rho}_{(t)} = e^{i\widehat{\mathcal{H}}t}\ \widehat{\rho}_{(0)}\ e^{-i\widehat{\mathcal{H}}t}$	$\widehat{\mathcal{T}}^{\mu\nu}(x,0)$ fixed

As the system evolves, i.e, at later hypersurface, maybe we can find the form of the density operator (yet not the exact). This requires a physical assumption:

"At Σ_0 local thermodynamic equilibrium is achieved"

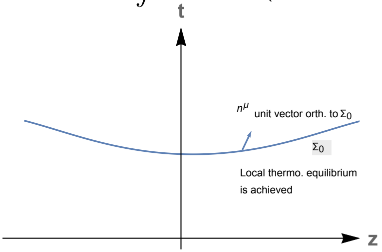


3D space-like hypersurfaces defining foliation

A LE state is obtained by looking for maximum entropy with specific constraints:

$$F[\widehat{\rho}] = -Tr[\widehat{\rho}\log\widehat{\rho}] - \int d\Sigma\ n_\mu\,(T^{\mu\nu}_{\rm LE} - T^{\mu\nu})\,\beta_\nu(x) - \int d\Sigma\ n_\mu\left(S^{\mu\lambda\nu}_{\rm LE} - S^{\mu\lambda\nu}\right)\Omega_{\lambda\nu}(x)$$

where : $\beta_\nu, \Omega_{\lambda\nu}$ are Lagrange multipliers



$$\frac{\delta F[\widehat{\rho}]}{\delta\widehat{\rho}} = 0$$

$$\Longrightarrow\ \widehat{\rho}_{(\Sigma_0)\rm LE} = \frac{1}{Z_{\rm LE}}\exp\left[-\int_{\Sigma_0} d\Sigma_\mu\left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu\lambda\nu}\right)\right] \qquad (7)$$

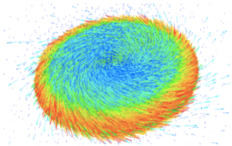
Entropy Current and Entropy-Gauge Transformation

Near local equilibrium at a given hypersurface Σ, the total entropy:

$$S = -Tr(\widehat{\rho}_{\rm LE} \log \widehat{\rho}_{\rm LE})$$
$$= \log Z_{\rm LE} + \int_\Sigma d\Sigma_\mu \left[Tr(\widehat{\rho}_{\rm LE} \widehat{\mathcal{T}}^{\mu\nu})\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu} Tr(\widehat{\rho}_{\rm LE} \widehat{\mathcal{S}}^{\mu\lambda\nu}) \right] \quad (8)$$

Is it possible to show that $logZ_{\rm LE}$ is an extensive quantity?

$$\log Z_{\rm LE} \sim \int_\Sigma d\Sigma_\mu \, \phi^\mu \quad (9)$$



Rotating disk [www.mr-cfd.com]

Therefore entropy current exists,

$$S = \int_{\Sigma} d\Sigma_{\mu} \left(\phi^{\mu} + T^{\mu\nu}\beta_{\nu} - \frac{1}{2}\Omega_{\lambda\nu}S^{\mu\lambda\nu} \right) \quad (10)$$

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$$s^{\mu} = \phi^{\mu} + T^{\mu\nu}\beta_{\nu} - \frac{1}{2}\Omega_{\lambda\nu}S^{\mu\lambda\nu} \tag{11}$$

$$\phi^{\mu} = \int_{0}^{T} \frac{dT'}{T'^{2}} \left[T^{\mu\nu}(T')u_{\nu} - \frac{1}{2}\omega_{\lambda\nu}S^{\mu\lambda\nu}(T') \right] \tag{12}$$

But what is the physics of ϕ^μ

$\phi^\mu :=$ Thermodynamic potential vector field.

For a fluid at global equilibrium with vanishing thermal vorticity $\varpi_{\mu\nu} = 0$:

$$\phi^\mu = p\,\beta^\mu = p\,\frac{u^\mu}{T}, \tag{13}$$

where "p" is the hydrostatic pressure that is the diagonal spatial component of the mean value of the energy-momentum operator.

Even though the forms s^μ and ϕ^μ are objective, they are not **unique**. It is quite clear that a transformation,

$$\phi^\mu \rightarrow \phi^{\mu'} = \phi^\mu + \partial_\lambda A^{\lambda\mu} \iff s^\mu \rightarrow s^{\mu'} = s^\mu + \partial_\lambda A^{\lambda\mu}$$

$$\implies S = \int_\Sigma d\Sigma_\mu \, s^\mu = \int_\Sigma d\Sigma_\mu \, s^{\mu'} \tag{14}$$

($A^{\lambda\mu}$ is arbitrary anti-symmetric tensor)

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Therefore, just like $T^{\mu\nu}$ and $S^{\mu\lambda\nu}$ are not invariant due to psuedo-gauge, the entropy current s^μ is not uniquely defined and can be changed, henceforth defined as entropy-gauge transformations.

Entropy Production Rate

- Entropy production is invariant under entropy-gauge transformation, i.e.,

$$\partial_\mu s^\mu = \partial_\mu s^{\mu'} . \tag{15}$$

- Using the entropy current $s^\mu = \phi^\mu + T^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}S^{\mu\lambda\nu}$, we obtain:

$$\begin{aligned} \partial_\mu s^\mu \;=\; & \partial_\mu \beta_\nu \left[T_s^{\mu\nu} - T_{s(\mathrm{LE})}^{\mu\nu} \right] + (\Omega_{\mu\nu} - \varpi_{\mu\nu}) \left[T_a^{\mu\nu} - T_{a(\mathrm{LE})}^{\mu\nu} \right] \\ & - \frac{1}{2} \partial_\mu \Omega_{\lambda\nu} \left[S^{\mu\lambda\nu} - S_{(\mathrm{LE})}^{\mu\lambda\nu} \right] \end{aligned} \tag{16}$$

$\varpi_{\mu\nu}$: *is the thermal vorticity*

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2. We stress that the formula is exact and not an approximation at some order of a gradient expansion.
3. A novel feature is apparently the simultaneous appearance of the last two terms of the right hand side.

Conclusion and Outlook

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- We expect to reproduce all the dissipative currents along side with what this quantum approach might add.
- Finally (later on...) to reproduce the spin polarization data.

[63. Cracow school of theoretical physics]

Thank You!