Lambda hyperon spin polarization evolution using spin hydro

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Related papers: arXiv:2103.02592, arXiv:2011.14907, arXiv:1901.09655

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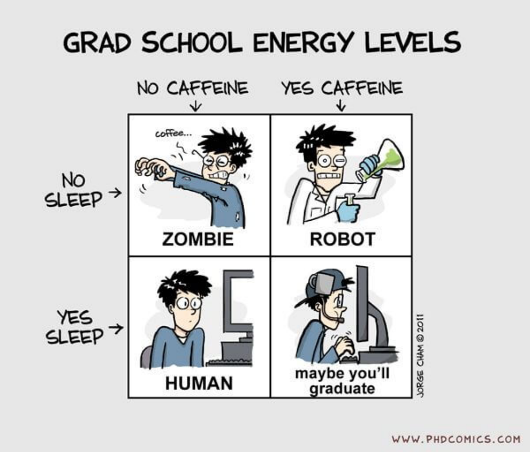
GRAD SCHOOL ENERGY LEVELS
NO CAFFEINE
YES CAFFEINE
NO SLEEP
YES SLEEP
coffee...
ZOMBIE
ROBOT
HUMAN
maybe you'll graduate
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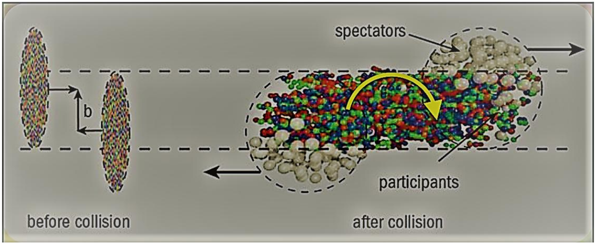
Heavy-ion collisions:

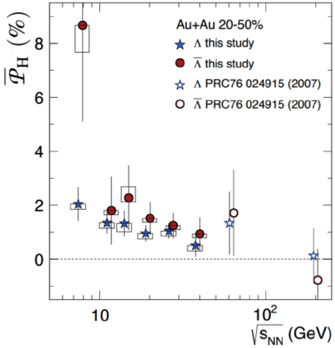
- Non-central relativistic heavy-ion collisions create global rotation of matter, which may induce spin polarization.
- Emerging particles are expected to be globally polarized with their spins on average pointing along the systems angular momentum.

  nucl-th/0410079, nucl-th/0410089, arXiv:0708.0035.



Source: CERN Courier

First positive measurements of global spin polarization of Λ hyperons by STAR



thermal approach $\longrightarrow$ $P_\Lambda \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T}$ $\quad P_{\bar{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$

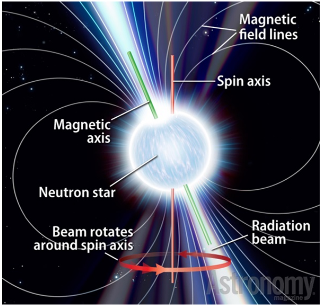
Becattini, F., Karpenko, I., Lisa, M., Upsal, I. , Voloshin, S. , PRC 95, 054902 (2017)

. . . the hottest, least viscous – and now, most vortical – fluid produced in the laboratory . . .

$$\omega = \left(P_\Lambda + P_{\bar{\Lambda}}\right) k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22} \mathrm{s}^{-1}$$

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

Even larger than...



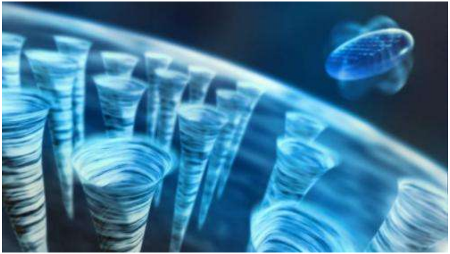


Figure: PSR J1748-2446ad ($716 s^{-1}$) & Nanodroplets of superfluid helium ($10^7 s^{-1}$).

10.1126/science.1123430, Science 345, 906–909 (2014)

MCU

Global polarization:

- Good agreement between experiment and models based on local thermodynamic equilibrium of spin degrees of freedom.

  0711.1253, 1304.4427, 1303.3431, 1501.04468, 1610.02506, 1610.04717, 1605.04024, 1703.03770, etc...

- But...

Longitudinal polarization:

- Good agreement between experiment and models based on local thermodynamic equilibrium of spin degrees of freedom.

  0711.1253, 1304.4427, 1303.3431, 1501.04468, 1610.02506, 1610.04717, 1605.04024, 1703.03770, etc...

- But...

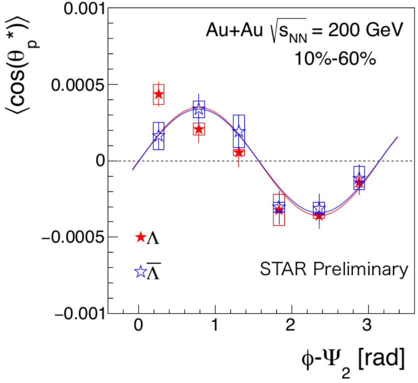


Figure: Longitudinal polarization of Λ-$\bar{\Lambda}$ (1905.11917)

Bigger picture:

- This study will help us to know the formation and characteristics of the QGP, a state of matter believed to exist at sufficiently high energy densities.

- Detecting and understanding the QGP allows us to understand better the universe in the moments after the Big Bang.

Our approach:

- Include spin degrees of freedom into the ideal standard hydrodynamics to form spin hydrodynamics formalism.
- $J^{\mu,\alpha\beta}(x) = x^\alpha T^{\mu\beta}(x) - x^\beta T^{\mu\alpha}(x) + S^{\mu,\alpha\beta}(x)$
- And, conservation of total angular momentum, $\partial_\lambda J^{\lambda,\mu\nu}(x) = 0$ gives $\partial_\lambda S^{\lambda,\mu\nu}(x) = T^{\nu\mu}(x) - T^{\mu\nu}(x)$
- For symmetric energy-momentum tensor, $T^{\nu\mu}_{\rm GLW}(x) = T^{\mu\nu}_{\rm GLW}(x)$, we have $\partial_\lambda S^{\lambda,\mu\nu}_{\rm GLW}(x) = 0$
- Hence conservation of the angular momentum implies the conservation of its spin part in the de Groot-van Leeuwen-van Weert (GLW) formulation.

1705.00587, 1712.07676, 1806.02616, 1811.04409, S. R. De Groot *et. al.*, Relativistic Kinetic Theory: Principles and Applications (1980).

Steps of spin hydrodynamic framework:

- Solving the standard perfect-fluid hydrodynamic equations without spin.

- Determination of the spin evolution in the hydrodynamic background.

- Determination of the Pauli-Lubański (PL) vector on the freeze-out hypersurface.

- Calculation of the spin polarization of particles in their rest frame. The spin polarization obtained is a function of the three-momenta of particles and can be directly compared with the experiment.
  1901.09655

Conservation laws:

- $d_\alpha N^\alpha \equiv d_\alpha(\mathcal{N} U^\alpha) = 0 \rightarrow$ **Conservation of net baryon number**
  where $\mathcal{N} = 4\ \sinh(\mu/T)\ \mathcal{N}_{(0)}$.
- $d_\alpha T^{\alpha\beta} \equiv d_\alpha \left[(\mathcal{E} + \mathcal{P})\, U^\alpha U^\beta - \mathcal{P} g^{\alpha\beta}\right] = 0 \rightarrow$ **Conservation of EMT**
  where $\mathcal{E} = 4\cosh(\mu/T)\mathcal{E}_{(0)}$ and $\mathcal{P} = 4\cosh(\mu/T)\mathcal{P}_{(0)}$.

These laws provide closed system of 5 eqns. for 5 funcs: μ, T, and three independent components of U^μ which need to be solved to get the perfect-fluid background evolution.

- $d_\alpha S^{\alpha,\beta\gamma} \equiv d_\alpha \left[\mathcal{A}_1 U^\alpha \omega^{\beta\gamma} + \mathcal{A}_2 U^\alpha U^{[\beta} \kappa^{\gamma]} + \mathcal{A}_3 (U^{[\beta} \omega^{\gamma]\alpha} + g^{\alpha[\beta} \kappa^{\gamma]})\right] = 0$
  $\downarrow$
  Conservation of spin

where $\mathcal{A}_1 = \cosh(\mu/T)\left(n_{(0)} - \mathcal{B}_{(0)}\right)$, $\mathcal{A}_2 = \cosh(\mu/T)\left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)}\right)$,
$\mathcal{A}_3 = \cosh(\mu/T)\,\mathcal{B}_{(0)}$
with, $\mathcal{B}_{(0)} = -\frac{2}{(m/T)^2}(\mathcal{E}_{(0)} + \mathcal{P}_{(0)})/T$ and $\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2\mathcal{N}_{(0)}$.
Here, $\omega^{\beta\gamma}$ is known as spin polarization tensor.

1811.04409

Spin polarization tensor:

$\omega_{\mu\nu}$ is an anti-symmetric tensor of rank 2 and can be parameterized by the four-vectors κ^μ and ω^μ,

$$\boxed{\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta},$$

where $\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \quad \omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha.$
U, X, Y and Z form a 4-vector basis satisfying the following normalization conditions: $U \cdot U = 1, \qquad X \cdot X = Y \cdot Y = Z \cdot Z = -1.$

$$\omega_{\alpha\beta} = \begin{bmatrix} 0 & e^1 & e^2 & e^3 \\ -e^1 & 0 & -b^3 & b^2 \\ -e^2 & b^3 & 0 & -b^1 \\ -e^3 & -b^2 & b^1 & 0 \end{bmatrix}$$

$$\begin{aligned} C_{\kappa X} &= e^1 \cosh(\vartheta + \eta) - b^2 \sinh(\vartheta + \eta), \\ C_{\omega Y} &= b^2 \cosh(\vartheta + \eta) - e^1 \sinh(\vartheta + \eta), \\ C_{\kappa Y} &= e^2 \cosh(\vartheta + \eta) + b^1 \sinh(\vartheta + \eta), \\ C_{\omega X} &= b^1 \cosh(\vartheta + \eta) + e^2 \sinh(\vartheta + \eta), \\ C_{\kappa Z} &= e^3, \quad C_{\omega Z} = b^3. \end{aligned}$$

Spin polarization (local and global):

$$\langle \pi_\mu \rangle_p(\boldsymbol{p}) = \frac{E_p \frac{d\overset{*}{\Pi}_\mu(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}} \quad \frac{\rightarrow Total\ \ Pauli\ \ Lubanski\ \ vector}{\rightarrow Momentum\ \ density\ \ of\ \ all\ \ particles} \sim \ \boldsymbol{p}\ dependent$$

$$\langle \pi_\mu \rangle = \frac{\int dP \langle \pi_\mu \rangle_p E_p \frac{d\mathcal{N}(p)}{d^3p}}{\int dP E_p \frac{d\mathcal{N}(p)}{d^3p}} = \frac{\int d^3p\ \ \frac{d\overset{*}{\Pi}_\mu(p)}{d^3p}}{\int d^3p\ \ \frac{d\mathcal{N}(p)}{d^3p}} \sim \ \boldsymbol{p}\ integrated$$

where, $$E_p \frac{d\overset{*}{\Pi}_\mu(p)}{d^3p} = \frac{1}{(2\pi)^3 m} \int \cosh\left(\frac{\mu}{T}\right) \Delta\Sigma_\lambda p^\lambda \, e^{-\beta \cdot p} \left(\tilde{\omega}_{\beta\mu}\ p^\beta\right)^*$$

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4}{(2\pi)^3} \int \cosh\left(\frac{\mu}{T}\right) \Delta\Sigma_\lambda p^\lambda \, e^{-\beta \cdot p}$$

* meaning quantities calculated in particle rest frame.

1901.09655

Spin angular momentum components at the freeze-out:

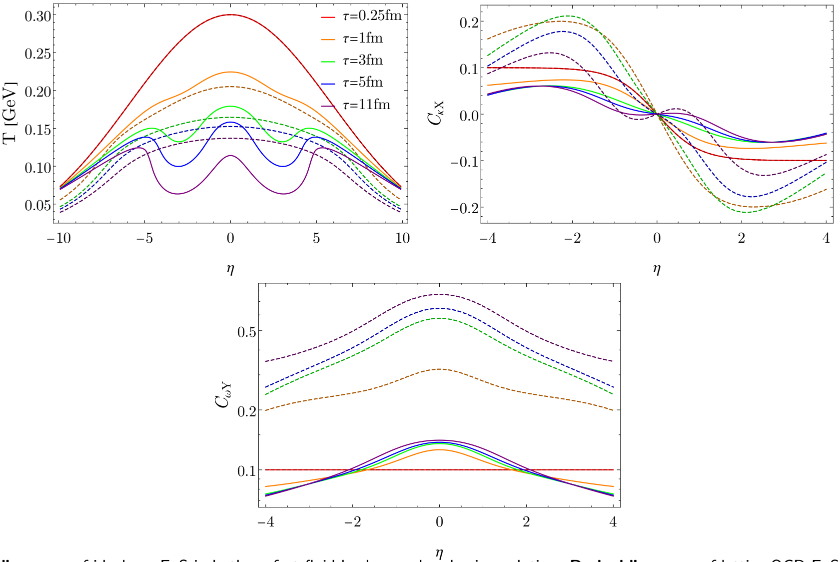
$$
\begin{aligned}
S^{\mu\nu}_{\rm FO} &= \int \Delta\Sigma_\lambda S^{\lambda,\mu\nu}_{\rm GLW} = \int dxdy\,\tau d\eta\, U^{\rm B}_\lambda S^{\lambda,\mu\nu}_{\rm GLW} \\
&= \pi R^2 \tau \int\limits_{-\eta_{\rm FO}/2}^{+\eta_{\rm FO}/2} d\eta\, U^{\rm B}_\lambda\; S^{\lambda,\mu\nu}_{\rm GLW}
\end{aligned}
$$

$$
S^{\rm FO}_{13} = \pi R^2 \tau \int\limits_{-\eta_{\rm FO}/2}^{+\eta_{\rm FO}/2} d\eta \Big[\mathcal{A}_1\, C_{\omega Y} \cosh(\eta) + \mathcal{A}_3\, C_{\kappa X} \sinh(\eta)\Big].
$$

$$
\boxed{C_{\kappa X} = -C_{\omega Y} \tanh(\vartheta + \eta)}.
$$

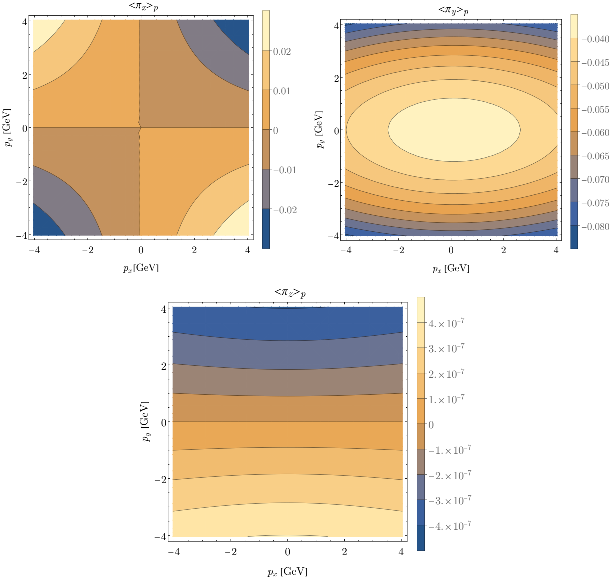
Non-Boost-invariant and transversely homogeneous flow

Perfect-fluid background and spin components evolution:

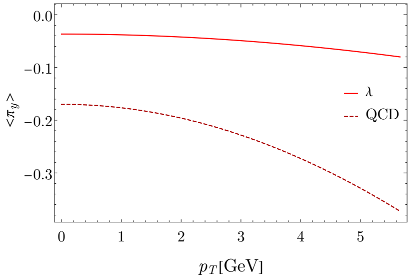


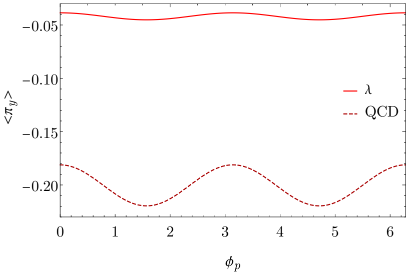
Solid lines: use of ideal gas EoS in both perfect fluid background and spin evolution. **Dashed lines**: use of lattice QCD EoS in background and ideal gas EoS for the spin evolution. 3+1 D simulation in progress.

Momentum dependence of polarization:



Global polarization:





Summary:

- Discussed relativistic hydrodynamics with spin based on the GLW formulation of energy-momentum and spin tensors.
- Showed how our formalism can be compared with the experiments.
- Obtained dynamics of spin polarization in the non-boost background.
- Incorporation of spin in full 3+1D hydro model required to address the problem of longitudinal polarization (which will be out pretty soon, stay tuned).

Thank you for your attention!