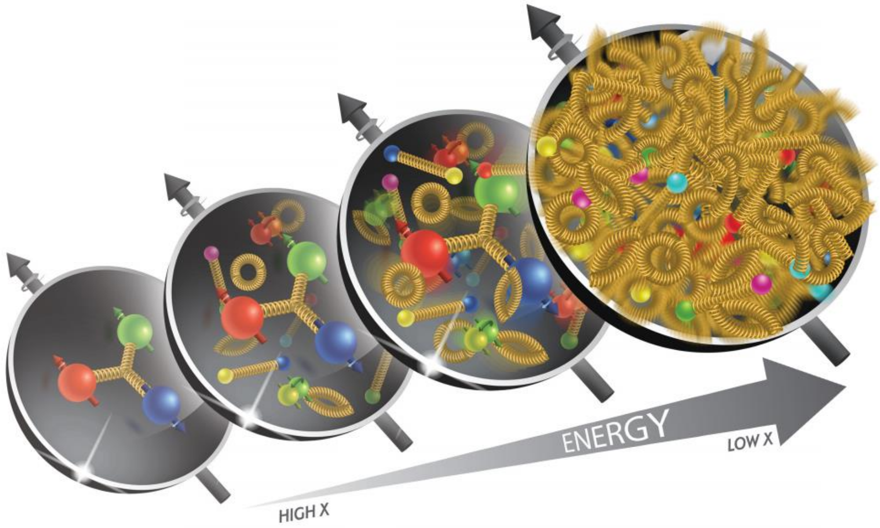
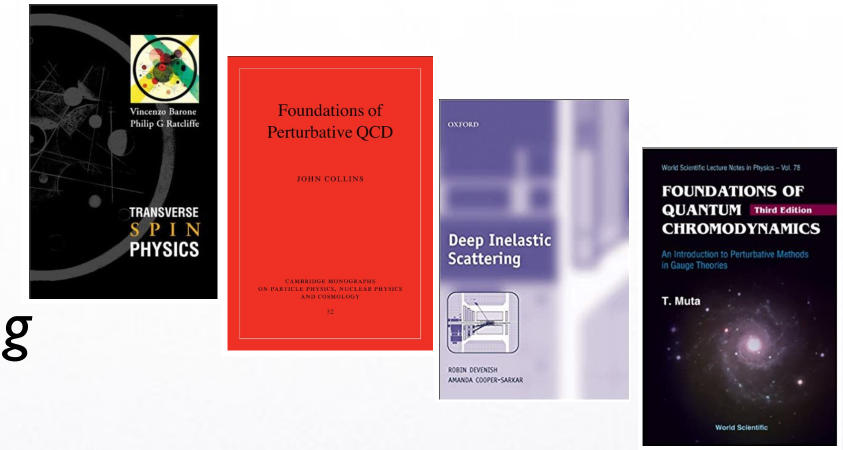


Phenomenology of hadron spin structure with (polarized) TMDs

Marco Radici
INFN - Pavia

Useful references

- Lecture notes
  - V. Barone - Cabeo School https://www.fe.infn.it/cabeo_school/2010/cabeo_school_2010.pdf
  - A. Bacchetta - Trento School https://www2.pv.infn.it/~bacchett/teaching/Bacchetta_Trento2012.pdf
  - R. Jaffe - Erice School https://arxiv.org/pdf/hep-ph/9602236.pdf
  - P. Mulders - GGI School http://www.nat.vu.nl/~mulders/tmdreview-vs3.pdf
- Books
  - V. Barone, P. Ratcliffe - *Transverse Spin Physics*
  - J. Collins - *Foundations of perturbative QCD*
  - R. Devenish, A. Cooper-Sarkar - *Deep Inelastic Scattering*
  - T. Muta - *Foundations of Quantum Chromodynamics*



- Papers
  - EPJ-A topical issue: The 3D structure of the nucleon https://link.springer.com/journal/10050/topicalCollection/AC_628286e999d9a60c9a780398df15f93d
  - M. Diehl - *Introduction to GPDs and TMDs* https://inspirehep.net/literature/1408303
  - A. Metz, A. Vossen - *Parton fragmentation functions* https://inspirehep.net/literature/1475000

Outline

- Why Transverse-Momentum Dependent (TMD) partonic functions ?

- The “TMD zoo”

- Where to find TMDs: observables in lepton-hadron, hadron-hadron, lepton-lepton collisions

Outline

- Why TMDs ?

First, a short recap of collinear factorization

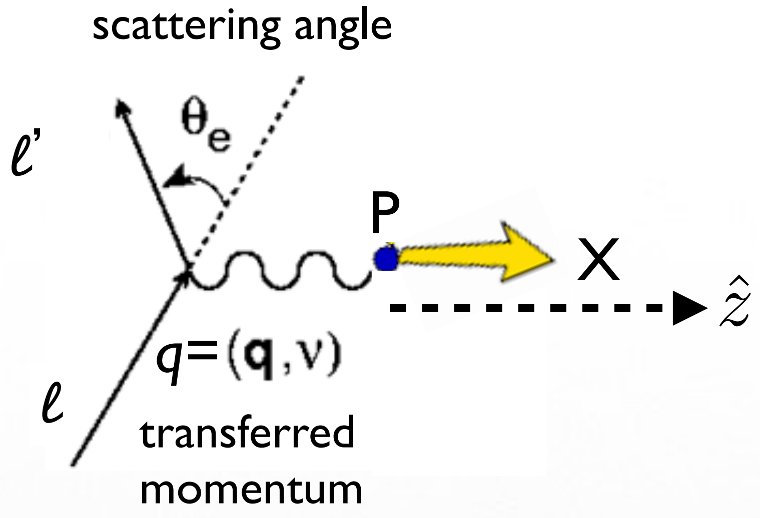
“Deep-Inelastic” kinematics

Internal hadron structure is best explored with a powerful “microscopic lense”
need a process with a hard scale; example: inclusive lepton-proton scattering

ℓ + N(P) → ℓ’ + X

Kinematic invariants

$$
\begin{aligned}
P^2 &= M^2 \\
Q^2 &= -q^2 \approx 2EE'(1-\cos\theta_e) = 4EE'\sin^2\theta_e/2 \\
\nu &= \frac{P\cdot q}{M} \overset{\mathrm{TRF}}{=} E - E' && \text{transferred energy} \\
y &= \frac{P\cdot q}{P\cdot \ell} \overset{\mathrm{TRF}}{=} \frac{E-E'}{E} && \text{fraction of ” ” } 0\leq y\leq 1 \\
x_B &= \frac{Q^2}{2P\cdot q} \overset{\mathrm{TRF}}{=} \frac{Q^2}{2M\nu} && \text{inelastic } 0<x\leq 1 \text{ elastic} \\
W^2 &= (P+q)^2 = M^2 + Q^2(1/x-1) \geq M^2 && \text{invariant mass}
\end{aligned}
$$



$P = (M,0,0,0)$ initially at rest

$$\downarrow \quad q = (\nu,0,0,|\mathbf{q}|)$$

$$P' = (\sqrt{M^2 + P_z'^2},0,0,P_z')$$

$$P_z' = |\mathbf{q}|$$

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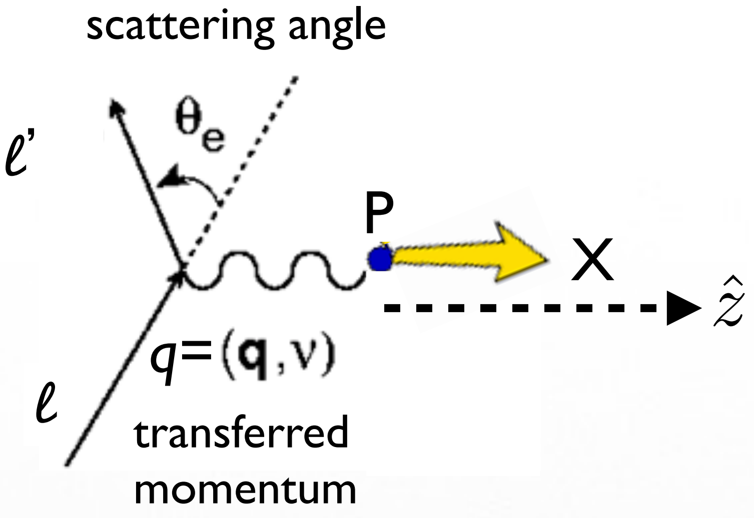

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$P_z' = |\mathbf{q}|$

$P' \approx (|\mathbf{q}|,0,0,|\mathbf{q}|)$

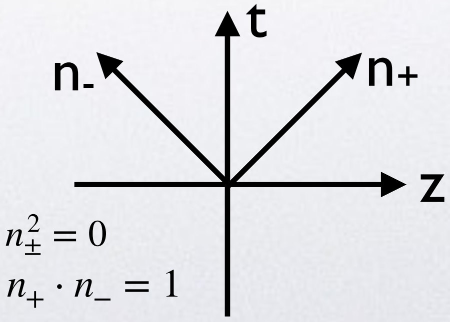
$P' \approx (P_+' = \sqrt{2}\,|\mathbf{q}|, P_-' \approx 0, \mathbf{0}_\perp)$

Deep-Inelastic regime: $\begin{cases} Q^2 \to \infty \\ x_B = \dfrac{Q^2}{2P\cdot q} \quad \text{finite} \end{cases}$

Light-Cone coordinates:

$$a^\mu = (a_0, a_1, a_2, a_3) = (a_+, a_-, \mathbf{a}_\perp)$$

$$a_\pm = \frac{a_0 \pm a_3}{\sqrt{2}} \qquad \mathbf{a}_\perp = (a_1, a_2)$$



$n_\pm^2 = 0$

$n_+ \cdot n_- = 1$

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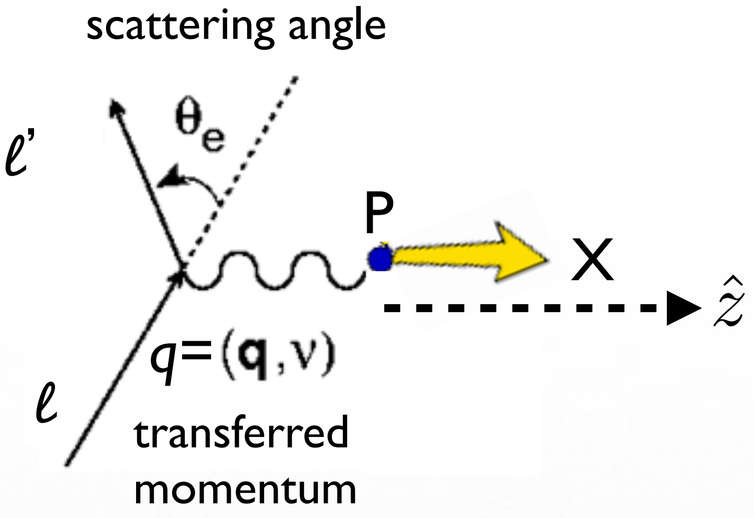

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$P' \approx (|\mathbf{q}|,0,0,|\mathbf{q}|)$

In this regime, only one single dominant component of proton momentum, P_+

Light-Cone coordinates:

$$a^\mu = (a_0, a_1, a_2, a_3) = (a_+, a_-, \mathbf{a}_\perp)$$

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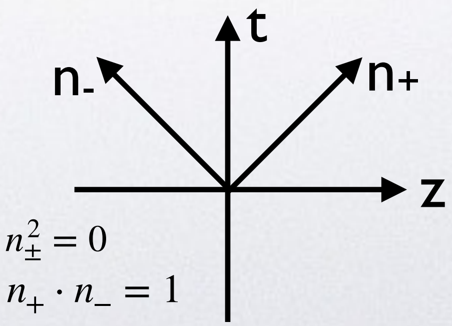


$n_\pm^2 = 0$
$n_+ \cdot n_- = 1$

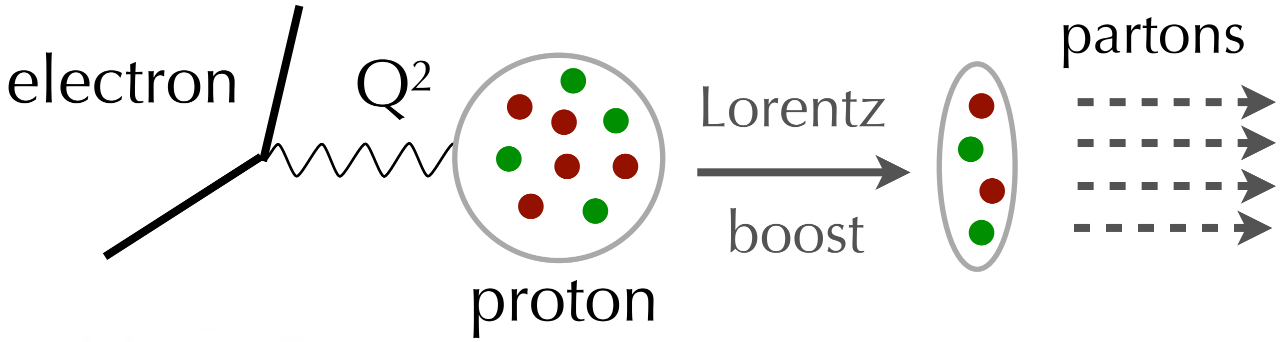
$P' \approx (P_+' = \sqrt{2}\,|\mathbf{q}|, P_-' \approx 0, \mathbf{0}_\perp)$

in general $P_+' \gg P_-'$

collinear framework

inclusive Deep-Inelastic Scattering (DIS): **1 dominant direction of momenta**

→ all partons collinear to proton



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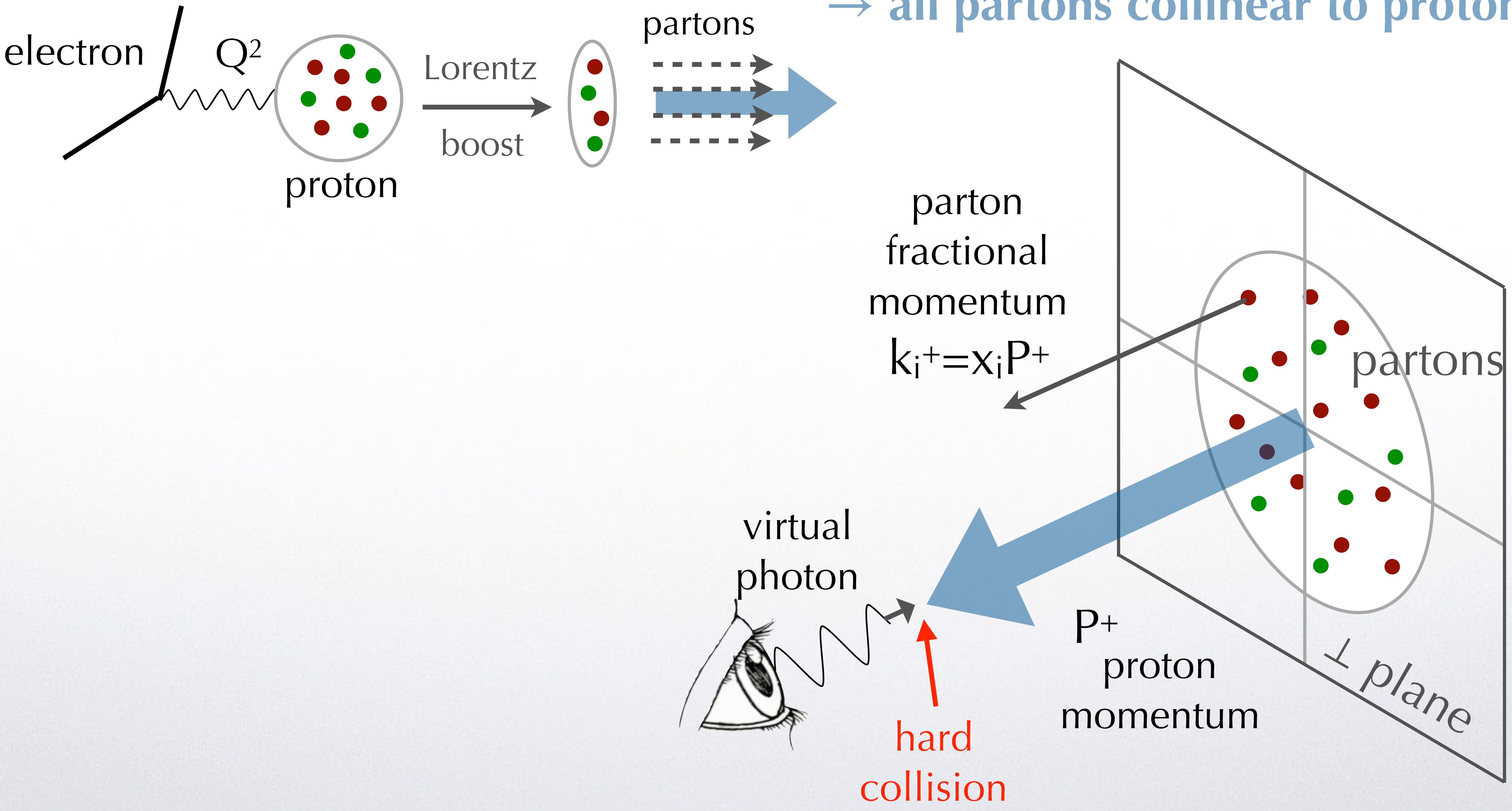

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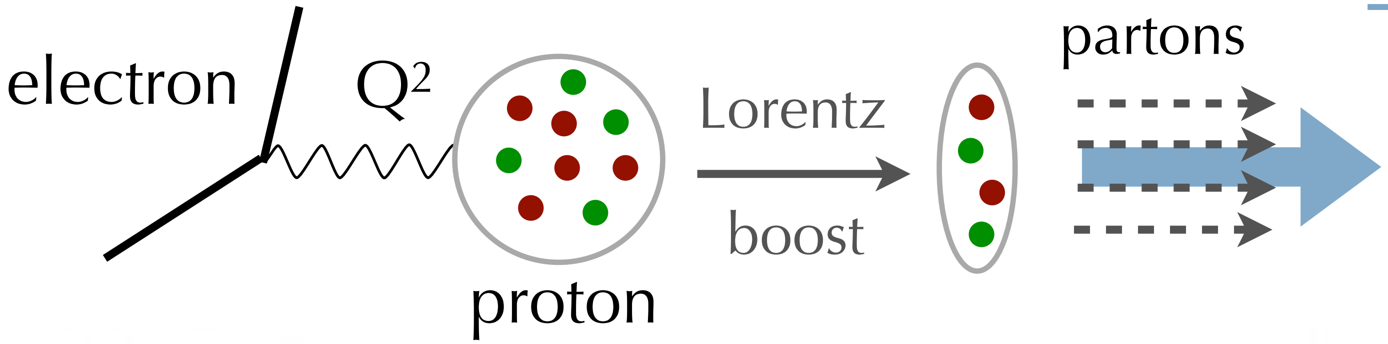


electron

Q^2

proton

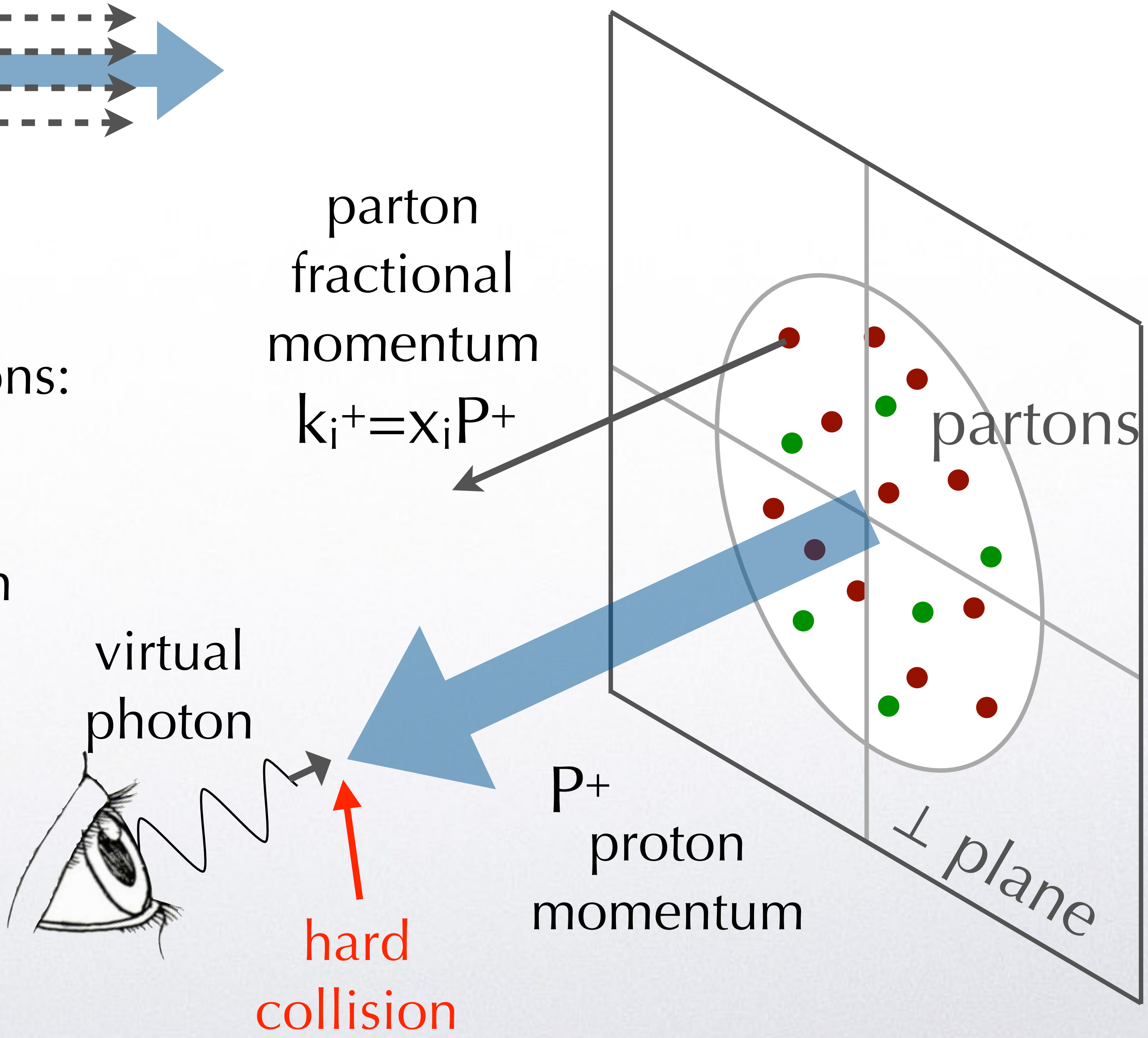
Lorentz

boost

partons

Basics of Feynman parton model:

- DIS regime and relativistic corrections: the virtual photon probes a frozen ensemble of partons
- **factorisation** between hard collision and proton structure
- **1D imaging of proton structure**, parametrised by collinear Parton Distribution Functions **PDF(x,Q²)**



parton

fractional

momentum

$k_i^+ = x_i P^+$

partons

virtual

photon

hard

collision

P^+

proton

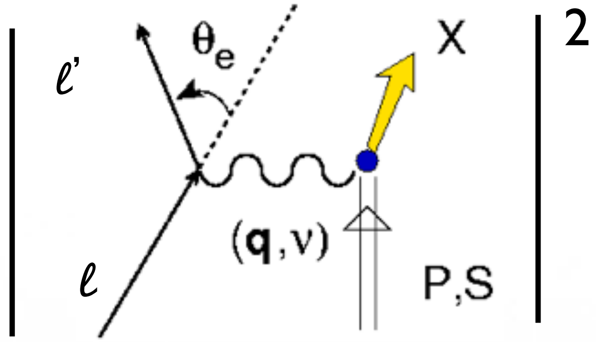
momentum

⊥ plane

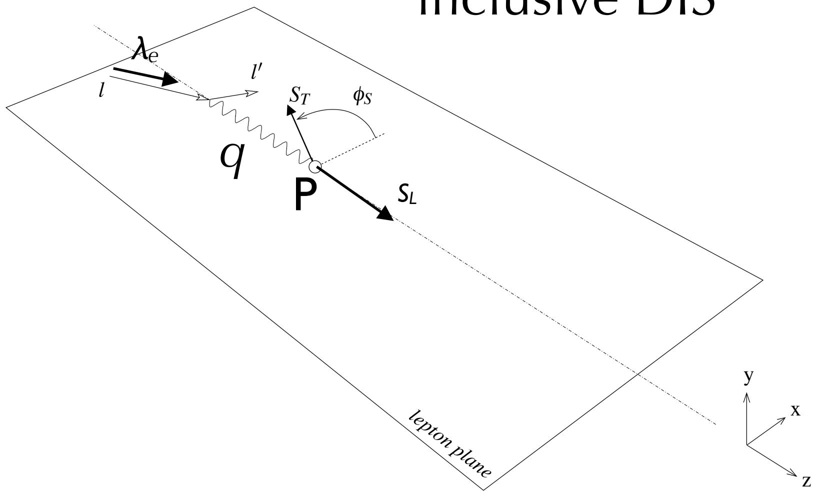
inclusive DIS

More rigorously:

one photon-exchange approximation

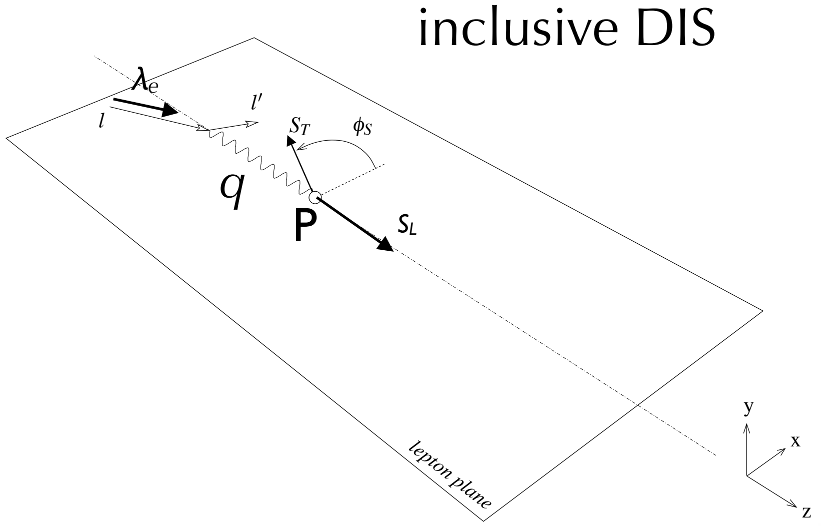


inclusive DIS



inclusive DIS

More rigorously:



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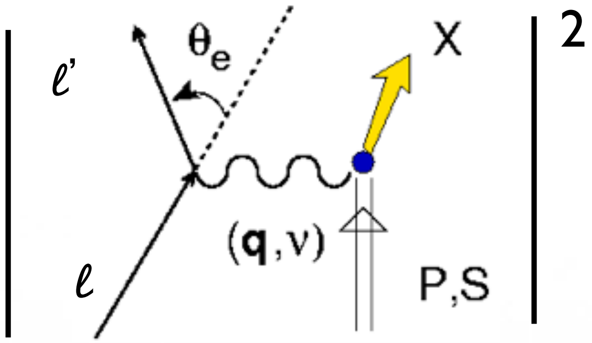
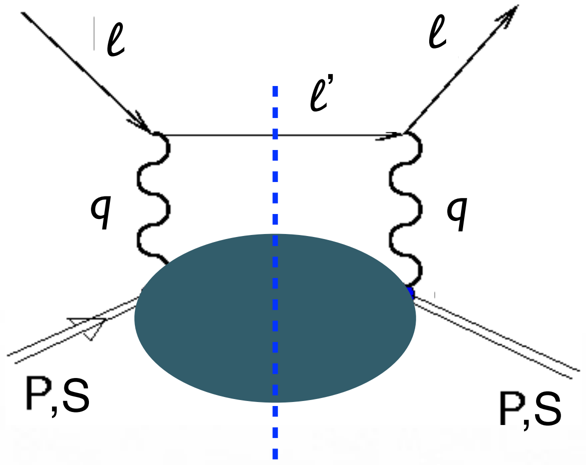


optical theorem

$\longrightarrow$

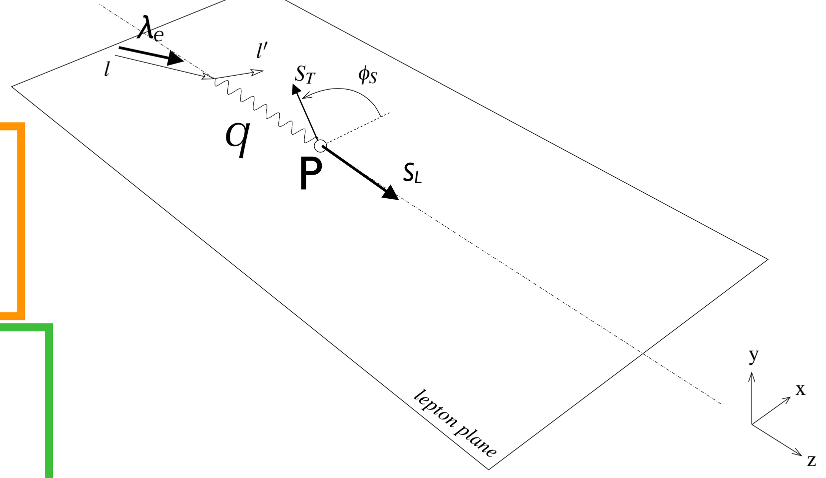


cut-diagram notation:
cross section = product of two amplitudes
particles entering cut are on-shell

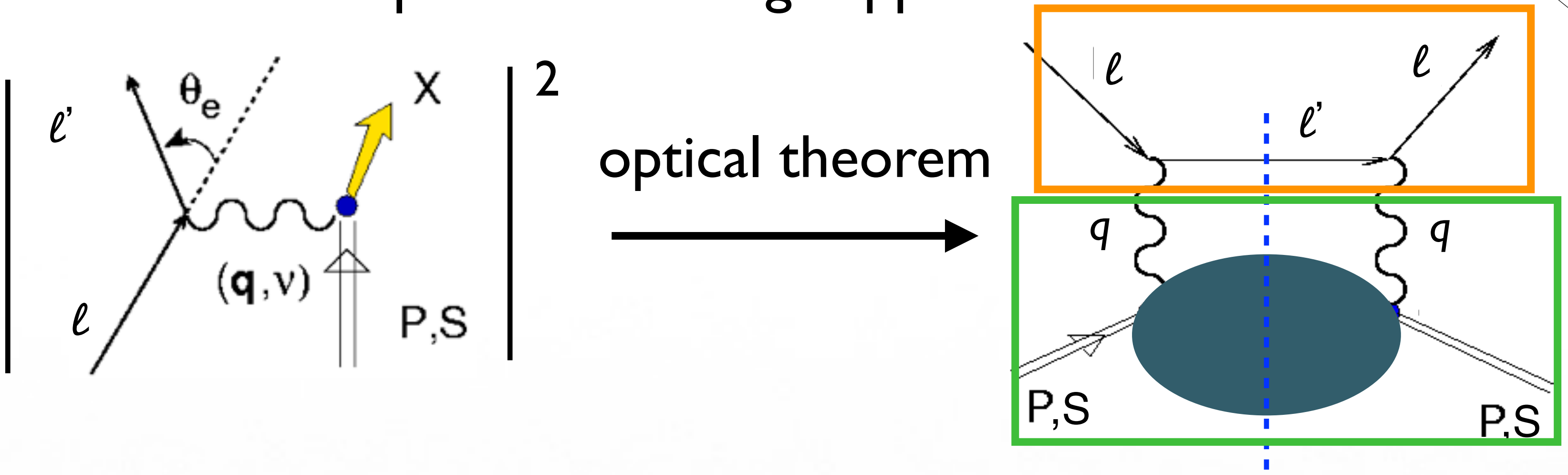
inclusive DIS

More rigorously:

inclusive DIS



one photon-exchange approximation



$$\frac{d\sigma}{dx_B\, dy\, d\phi_S} = \frac{\alpha^2 y}{2Q^4} \underbrace{L_{\mu\nu}(\ell, \ell', \lambda_e)}_{\text{leptonic tensor}} \underbrace{W^{\mu\nu}(q, P, S)}_{\text{hadronic tensor}}$$

calculable in QED

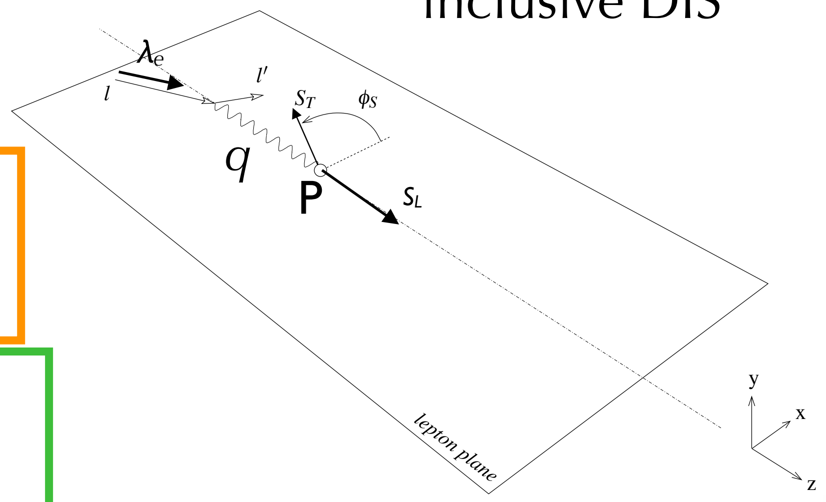
cut-diagram notation:
cross section = product of two amplitudes
particles entering cut are on-shell

linear combination of all tensor structures with q, P, S, subject to Hermiticity, gauge-, parity- and time reversal- invariance
→ parametrised with four structure functions

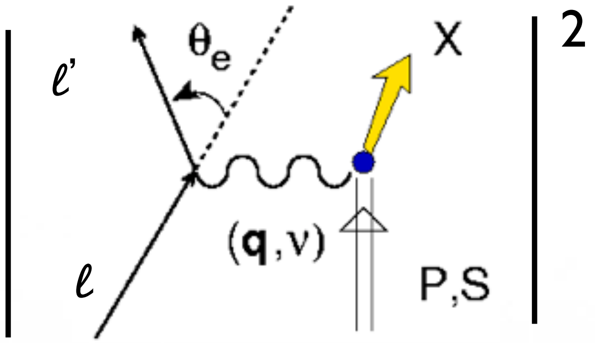
inclusive DIS

More rigorously:

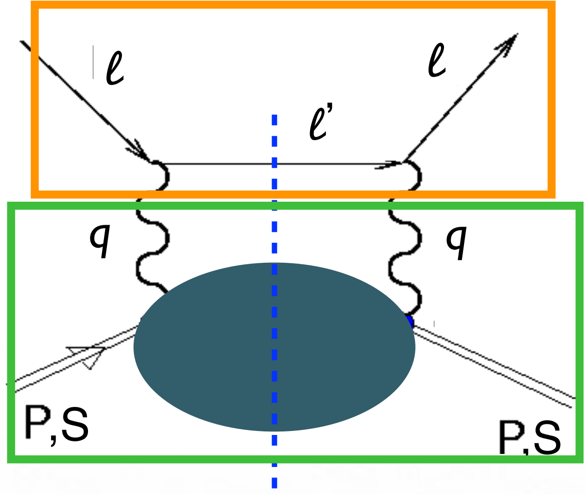
inclusive DIS



one photon-exchange approximation



optical theorem →



$$\frac{d\sigma}{dx_B\,dy\,d\phi_S} = \frac{\alpha^2 y}{2Q^4}\,\underbrace{L_{\mu\nu}(\ell,\ell',\lambda_e)}_{\text{leptonic tensor}}\,\underbrace{W^{\mu\nu}(q,P,S)}_{\text{hadronic tensor}}$$

leptonic tensor → calculable in QED

hadronic tensor → linear combination of all tensor structures with q, P, S, subject to Hermiticity, gauge-, parity- and time reversal- invariance
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$$\frac{d\sigma}{dx_B\,dy\,d\phi_S} = \frac{2\alpha^2}{x_B y Q^2}\left\{A(y)F_{UU,T} + B(y)F_{UU,L} + \lambda_e S_L C(y) F_{LL} + \lambda_e|\mathbf{S}_T|D(y)\cos\phi_S F_{LT}\right\}$$

each F.. (x,Q^2)

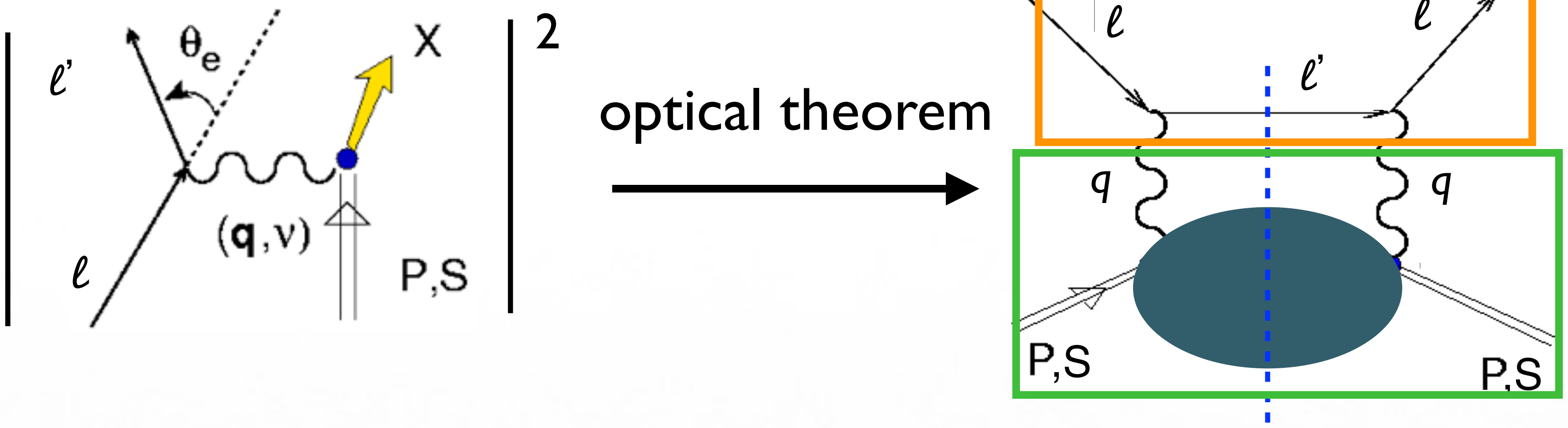
$$F_{\overset{\uparrow}{X}\overset{\uparrow}{Y},\overset{\uparrow}{Z}}$$

ℓ P γ^*

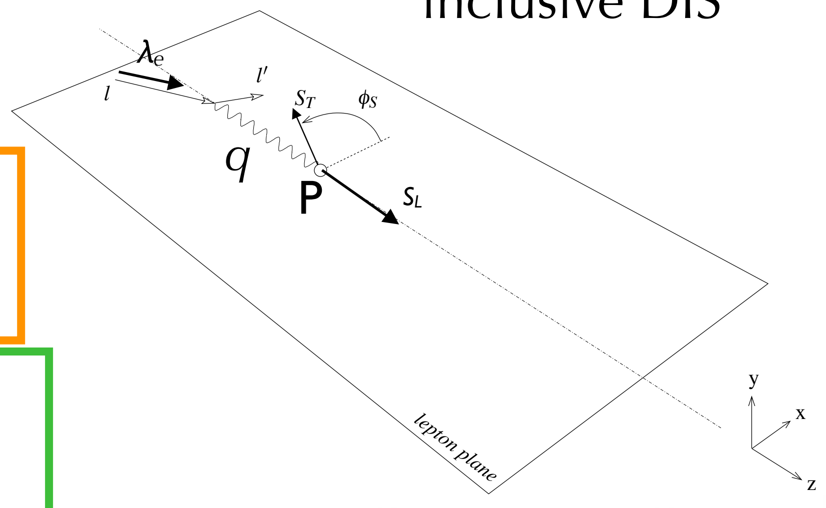
inclusive DIS

More rigorously:

one photon-exchange approximation



inclusive DIS



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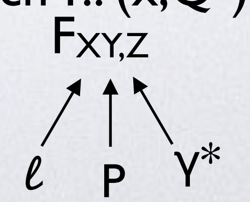


connection to standard notation

$F_{UU,T} = 2x_B F_1$

$F_{UU,L} = F_2 - 2x_B F_1 + \mathcal{O}(\gamma^2) F_2$

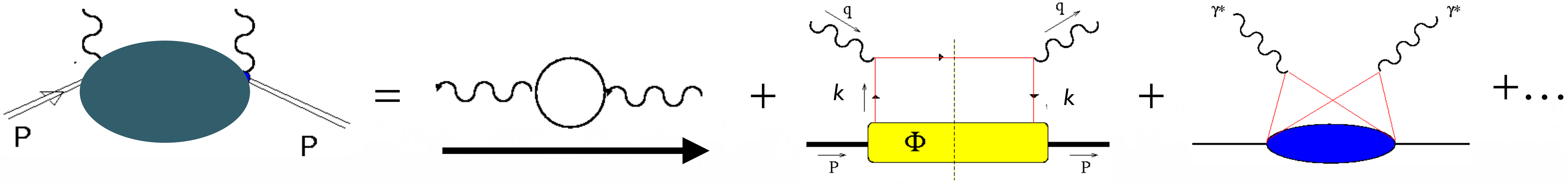
$F_{LL} = 2x_B g_1 + \mathcal{O}(\gamma^2) g_2$

$F_{LT} = \mathcal{O}(\gamma)(g_1 + g_2)$

$\gamma = \frac{2xM}{Q}$ target mass correction

OPE → factorisation

hadronic tensor $W^{\mu\nu}$: Operator Product Expansion (OPE)



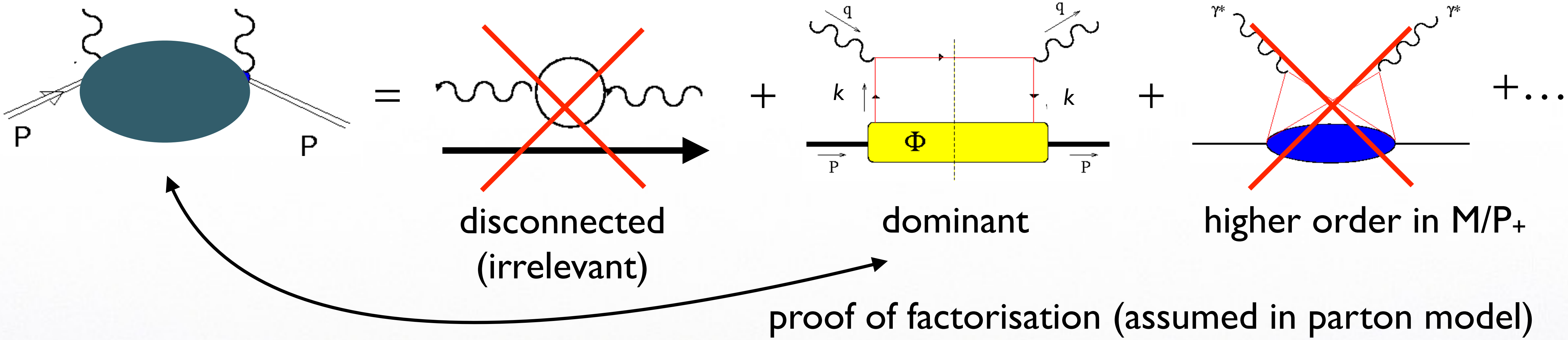
disconnected
(irrelevant)

dominant

higher order in M/P_+

OPE → factorisation

hadronic tensor $W^{\mu\nu}$: Operator Product Expansion (OPE)



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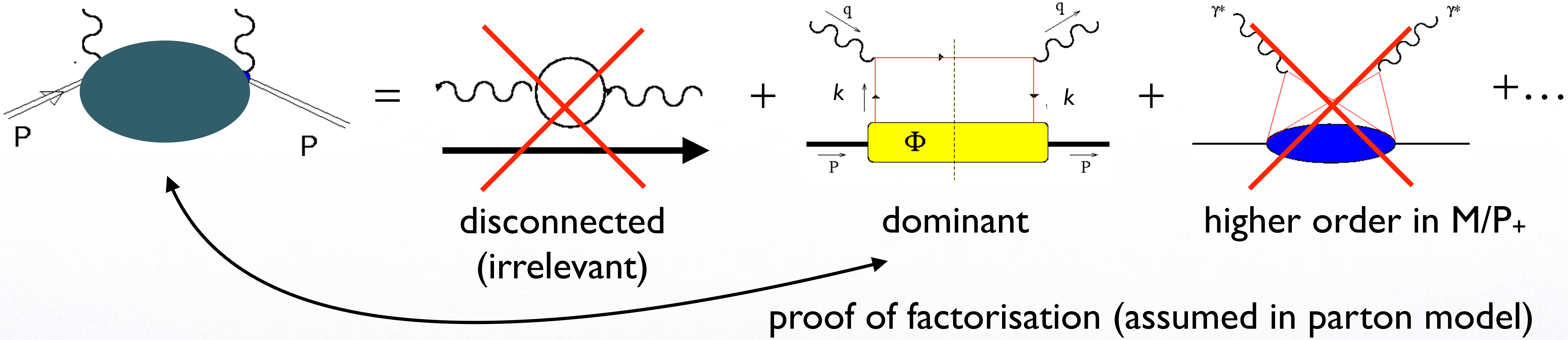
proof of factorisation (assumed in parton model)

OPE → factorisation

hadronic tensor $W^{\mu\nu}$: Operator Product Expansion (OPE)



disconnected (irrelevant)

dominant

higher order in M/P₊

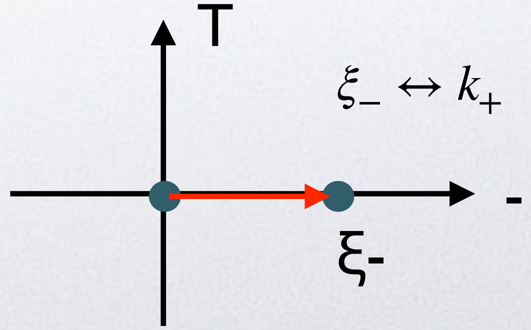
proof of factorisation (assumed in parton model)

$$2MW^{\mu\nu}(q,P,S) \approx \sum_q e_q^2 \frac{1}{2}\,\mathrm{Tr}\Big[\Phi(x_B,S)\,\gamma^\mu\,\gamma_+\,\gamma^\nu\Big] \qquad x_B \sim x = \frac{k_+}{P_+}$$

non-local correlator

$$\Phi_{ij}(x,S) = \int \frac{d\xi_-}{2\pi}\, e^{ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(0)\,\psi_i(\xi)\,|\,P,S\rangle_{\xi_+=\xi_T=0}$$

quark field with quantum numbers *i* on light-cone path



parton-parton correlation

non-local correlator

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$$= \int dk_+ dk\_ d\mathbf{k}_T\, \delta(k_+ - xP_+) \int \frac{d\xi}{(2\pi)^4}\, e^{ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(0)\,\psi_i(\xi)\,|\,P,S\rangle$$

parton-parton correlation

non-local correlator

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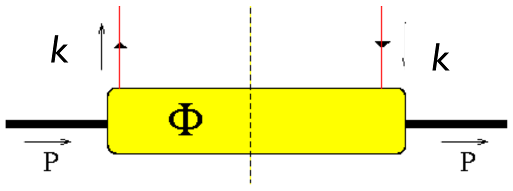


$\Phi(k, P, S)$ = linear combination of all tensor structures with k, P, S, subject to Hermiticity and parity-invariance (see later about time reversal)

OPE on $\Phi(k, P, S)$ → expansion in powers of M/P_+

Caveat

canonical OPE on local operators $\hat{\mathcal{O}}$

expansion in twist = dim($\hat{\mathcal{O}}$) - spin($\hat{\mathcal{O}}$)

Here, Φ is non-local, but can be expanded in local operators of same twist

“working” def. twist = 2 + powers of M/P_+

OPE → PDFs

OPE on $\Phi(k, P, S)$ → expansion in powers of M/P₊ → keeping only leading twist

$$\Phi(x,S) = \int dk_+ dk\_ d\mathbf{k}_T \, \delta(k_+ - xP_+) \, \Phi(k,P,S)$$

$$= \frac{1}{2}\Big[f_1(x)\,\gamma_- +$$

$$g_1(x)\, S_L \gamma_5 \gamma_- +$$

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right] \qquad h_1(x)\, i\sigma_{-\nu}\,\gamma_5 S_T^\nu \Big]$$

$$f_1(x) = \frac{1}{2}\text{Tr}\big[\Phi\,\gamma_+\big] \equiv \Phi^{[\gamma_+]}$$

$$S_L\, g_1(x) = \frac{1}{2}\text{Tr}\big[\Phi\,\gamma_+\gamma_5\big] \equiv \Phi^{[\gamma_+\gamma_5]}$$

$$(S_T)_i\, h_1(x) = \frac{1}{2}\text{Tr}\big[\Phi\, i\sigma_{+i}\,\gamma_5\big] \equiv \Phi^{[i\sigma_{+i}\,\gamma_5]}$$

OPE → PDFs

OPE on $\Phi(k, P, S)$ → expansion in powers of M/P_+ → keeping only leading twist

$$\Phi(x, S) = \int dk_+ dk_- d\mathbf{k}_T \, \delta(k_+ - xP_+) \, \Phi(k, P, S)$$

$$= \frac{1}{2}\Big[f_1(x)\,\gamma_- +$$ unpolarized Parton Distribution Function (PDF)

$$g_1(x)\, S_L \gamma_5 \gamma_- +$$ longitudinally polarized PDF (requires hadron long. pol. S_L)

$$\sigma^{\mu\nu} = \frac{i}{2}\,[\gamma^\mu, \gamma^\nu]$$

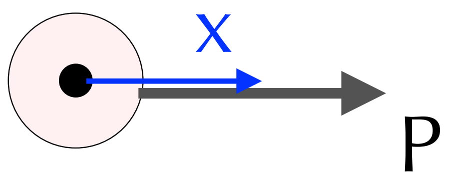
$$h_1(x)\, i\sigma_{-\nu}\,\gamma_5 S_T^\nu \Big]$$ transversely polarized PDF (requires hadron transv. pol. S_T)

$$f_1(x) = \frac{1}{2}\mathrm{Tr}\big[\Phi\,\gamma_+\big] \equiv \Phi^{[\gamma_+]}$$ (fractional) momentum distribution

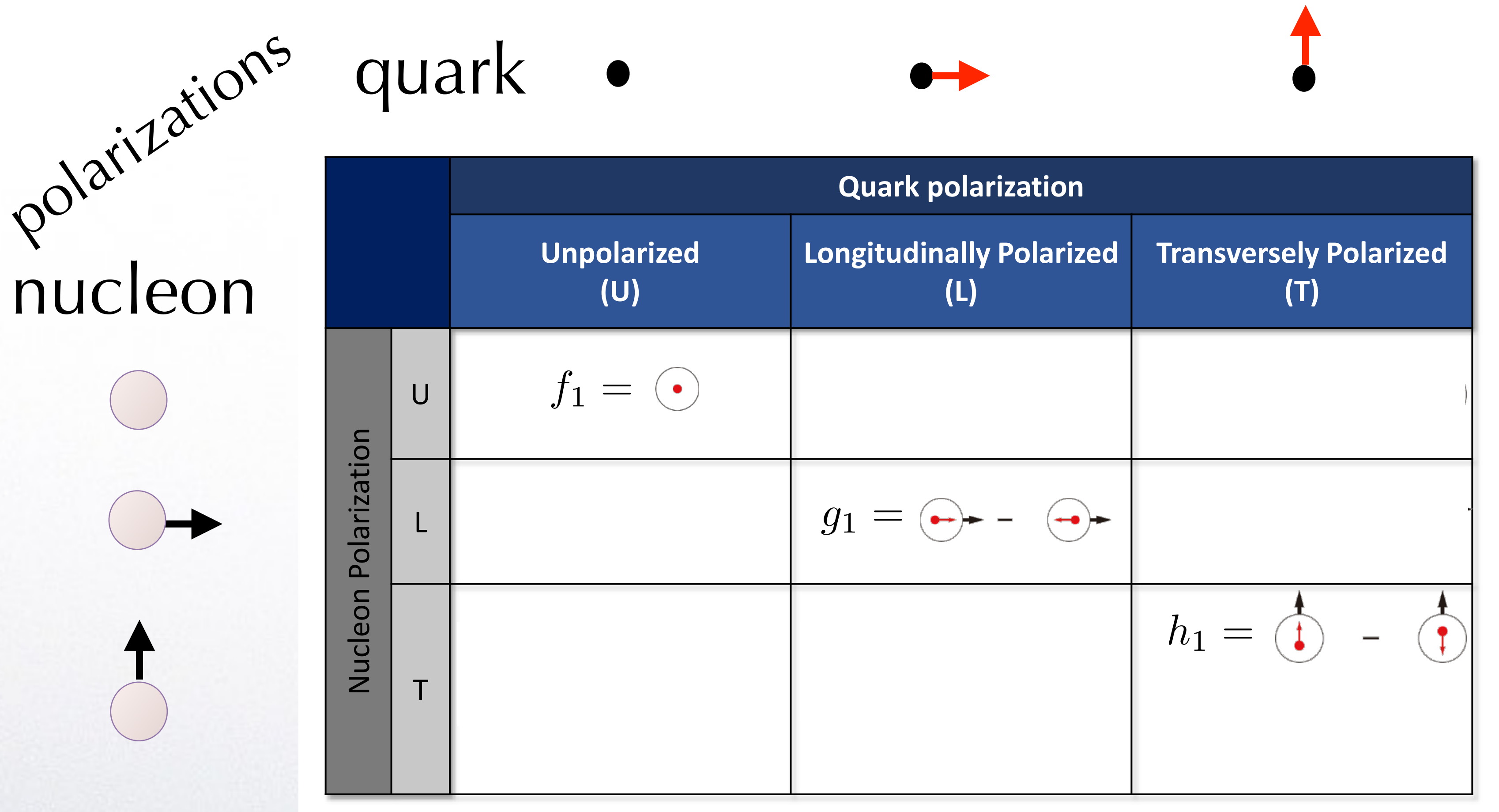
$$S_L\, g_1(x) = \frac{1}{2}\mathrm{Tr}\big[\Phi\,\gamma_+\gamma_5\big] \equiv \Phi^{[\gamma_+\gamma_5]}$$ helicity distribution

$$(S_T)_i\, h_1(x) = \frac{1}{2}\mathrm{Tr}\big[\Phi\, i\sigma_{+i}\,\gamma_5\big] \equiv \Phi^{[i\sigma_{+i}\gamma_5]}$$ transversity distribution

The PDF table



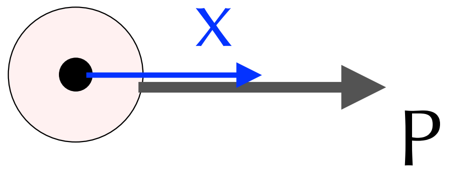
PDFs (x; Q^2) at leading twist for a spin-1/2 hadron (Nucleon)



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		
	L		$g_1 =$ ⊙ - ⊙	
	T			$h_1 =$ ⊙ - ⊙

N.B. Using appropriate projections $\Phi^{[\Gamma]}$ one can extract also subleading-twist PDFs, but no probabilistic interpretation

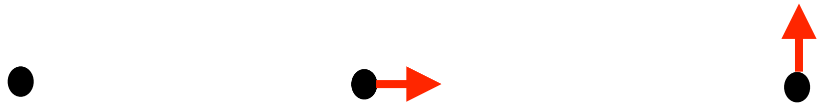
The PDF table



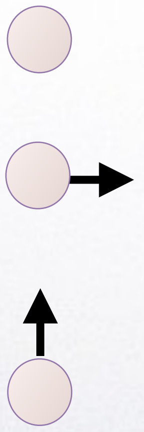
PDFs (x; Q^2) at leading twist
for a spin-1/2 hadron (Nucleon)

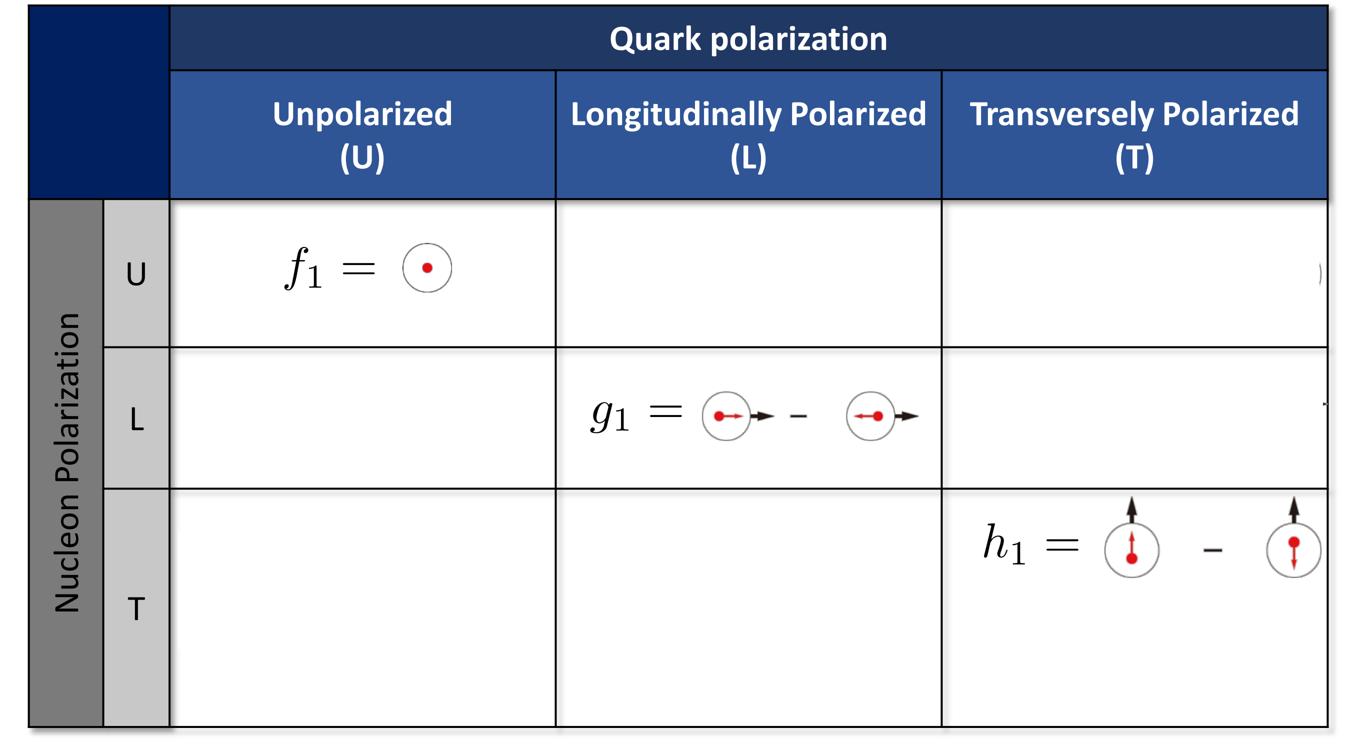
polarizations

quark



nucleon





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Nucleon Polarization	U	$f_1 =$		
	L		$g_1 =$ -	
	T			$h_1 =$ -

probabilistic interpretation

probability density of finding
an unpol. quark in an unpol. nucleon

probability density of finding a
long. pol. quark in a long. pol. nucleon

probability density of finding a
transv. pol. quark in a transv. pol. nucleon

N.B. Using appropriate projections $\Phi^{[\Gamma]}$ one can extract also subleading-twist PDFs, but no probabilistic interpretation

“observable” PDFs

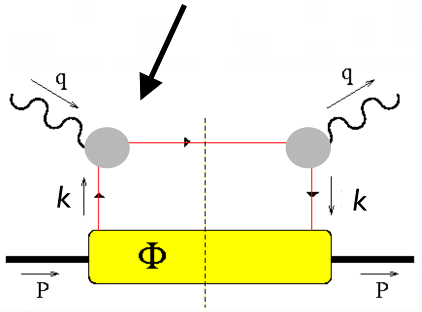
connection of PDFs with measurable structure functions

at leading order $\mathcal{O}(\alpha_s^0)$ and leading twist

$$F_{UU,T}(x_B, Q^2) = x_B \sum_q e_q^2 \, f_1^q(x_B, Q^2) \qquad F_{UU,L}(x_B, Q^2) \approx 0$$

$$F_{LL}(x_B, Q^2) = x_B \sum_q e_q^2 \, g_1^q(x_B, Q^2) \qquad F_{LT}(x_B, Q^2) \approx 0$$

hard cross section $d\hat{\sigma} = 1 + c_1 \alpha_s + \dots$
produce F_L , F_{LT} $\neq 0$



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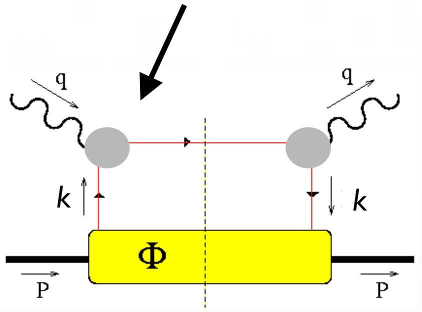

Transversity PDF does not appear in inclusive DIS cross section!

It happens because transverse polarization mixes quark helicities:

$$\langle \uparrow | .. | \uparrow \rangle \propto \langle + | .. | - \rangle , \langle - | .. | + \rangle$$

chirality = helicity for a spin-1/2 object; hence, h$_1$(x) is a chiral-odd PDF and can appear in the cross section only paired to another chiral-odd structure.

Transversity is not suppressed (as expected in perturbative QCD as m_q/Q),
it can be extracted in processes with at least two hadrons

The gauge link

$$\Phi(k,P,S) = \int \frac{d\xi}{(2\pi)^4}\, e^{ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(0)\,\psi_i(\xi)\,|\,P,S\rangle$$

this non-local operator is not color-gauge invariant under $\psi(x) \to e^{i\alpha^a(x)\,t^a}\,\psi(x) \equiv U(x)\,\psi(x)$

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$\downarrow$

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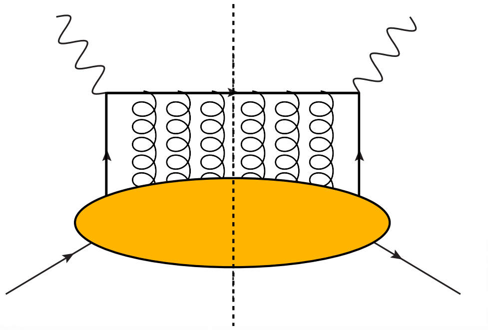

gauge-link operator $U_{[a,b]} = \mathscr{P}\exp\left[-ig\int_a^b d\eta_\mu\, A^\mu(\eta)\right]$

it transforms as $U_{[0,\xi]} \to U(0)\,U_{[0,\xi]}\,U^\dagger(\xi)$ so that $\Phi(k, P, S)$ is invariant

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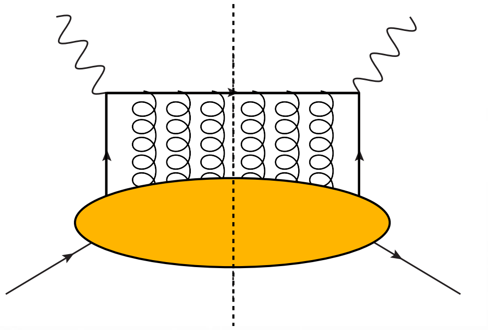

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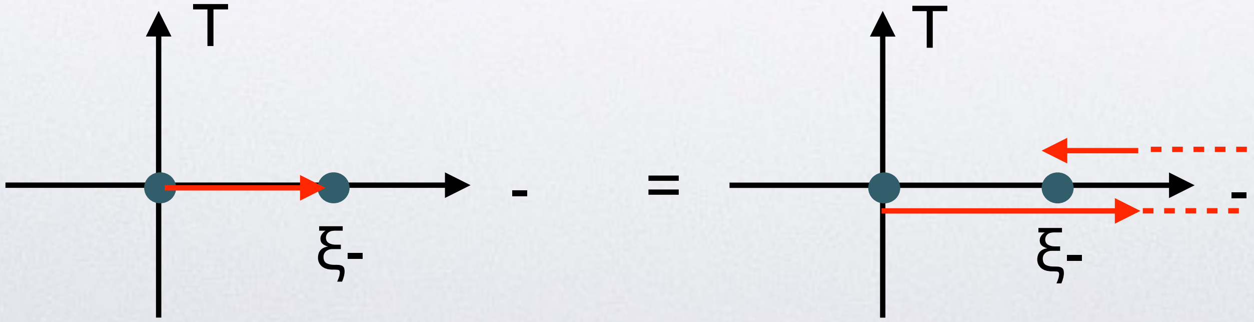
Φ(*x*, *S*) involves only the LC "-" direction: $\int d\xi_- \,\ldots\, |_{\xi_+,\xi_T=0}$

trick: $\Phi(x,S) \propto \langle P,S|\, \bar{\psi}_j(0)\, U_{[0,\xi_-]}\, \psi_i(\xi_-)\, |P,S\rangle = \langle P,S|\, \bar{\psi}_j(0)\, U_{[0,\infty_-]}\, U_{[\infty_-,\xi_-]}\, \psi_i(\xi_-)\, |P,S\rangle \equiv \langle P,S|\, \{\bar{\psi}_j(0)\}\, \{\psi_i(\xi_-)\}\, |P,S\rangle$



$\{\psi(x_-)\} = U_{[\infty_-,x_-]}\, \psi_i(x_-)$

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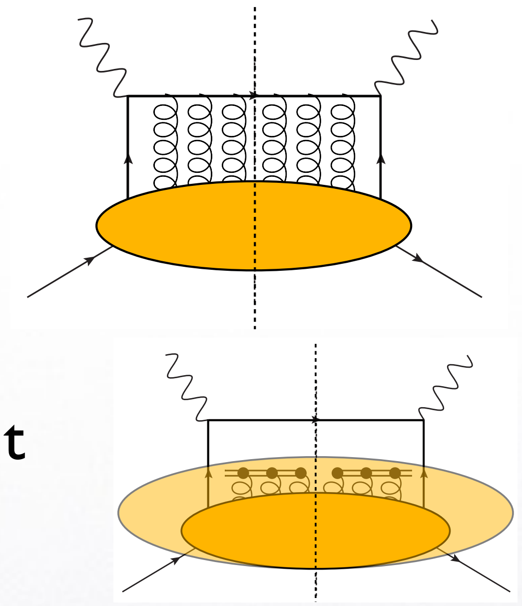

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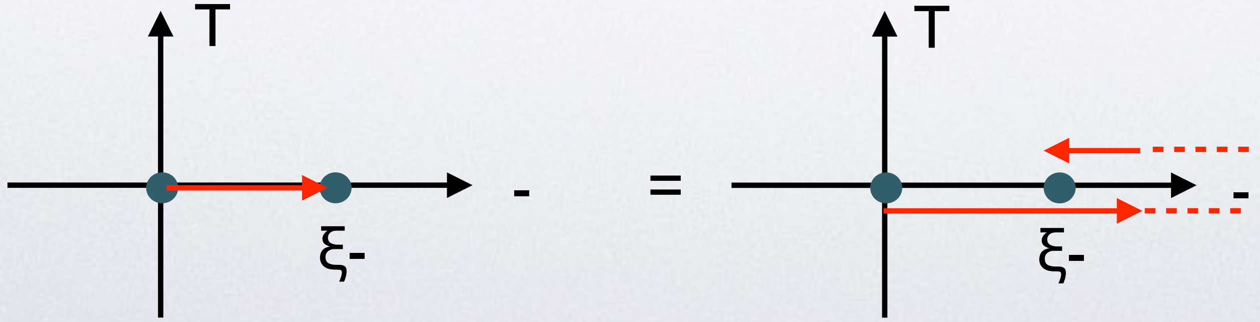
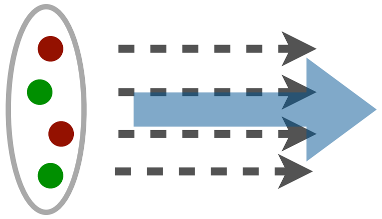


factorisation
is preserved

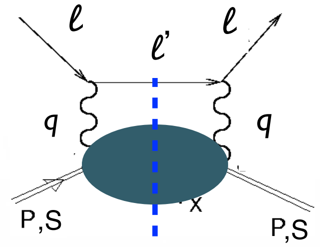
$\{\psi(x_-)\} = U_{[\infty_-,x_-]}\,\psi_i(x_-)$

Recap

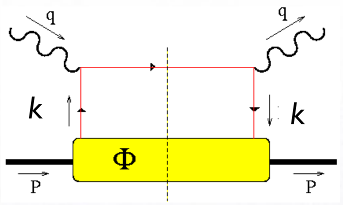
- hadron structure better explored in processes with a hard scale (much bigger than involved masses, $Q^2 \gg M^2$) ; on the Light-Cone, it implies one dominant direction → collinear framework natural choice



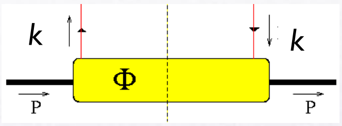
- Example: inclusive DIS, cross section $d\sigma \sim L_{\mu\nu} W^{\mu\nu}$ can be parametrised in terms of 4 structure functions (including polarization)



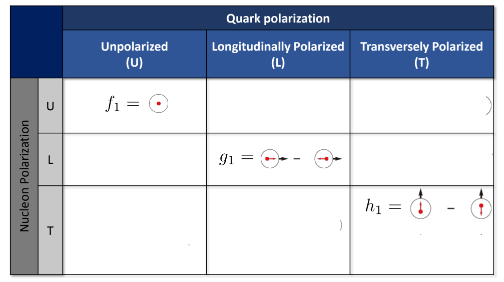
- OPE on $W^{\mu\nu}$ → factorisation of hadron structure in parton-parton non-local correlator Φ. It can be made color-gauge invariant by inserting proper gauge link



- Expansion of Φ in powers of *M*/*Q* (effective twist) contains operator-definition of collinear PDFs, that can be extracted by suitable projections



- Leading-twist PDFs have nice probabilistic interpretations, and can be connected to structure functions (except the chiral-odd transversity PDF)



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		
	L		$g_1 =$ ⊙ − ⊙	
	T			$h_1 =$ ⊙ − ⊙

Evidences to go beyond collinear

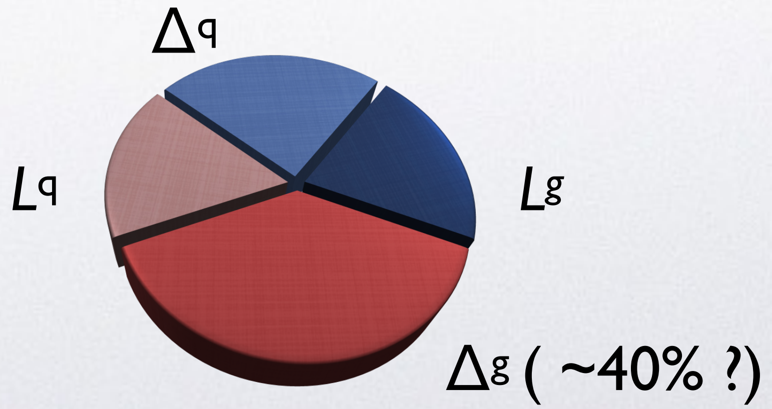
Evidences of going beyond the collinear framework

Example #1: the "Spin Crisis"

Ashmann et al. (EMC), P.L. ***B206*** *(88) 364*

- In 1988, the EMC Collaboration at CERN measures the F_{LL} structure function in the polarized inclusive DIS process $\vec{\mu} + \vec{p} \to \mu' + X$. Surprisingly, the sum of quark helicities Δq contributes at most 25% of spin 1/2 of the proton (depending on Q^2).
- There has been an intense activity to measure the gluon helicity Δg, which is currently known with a large error. But it's very unlikely that it amounts to the missing 75%…
- Missing contribution must come from the orbital angular momentum of partons L^q, L^g → need to be sensitive also to transverse components of parton momentum

$$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta q(Q^2) + L^q(Q^2)\right) + \Delta g(Q^2) + L^g(Q^2)$$



Evidences to go beyond collinear

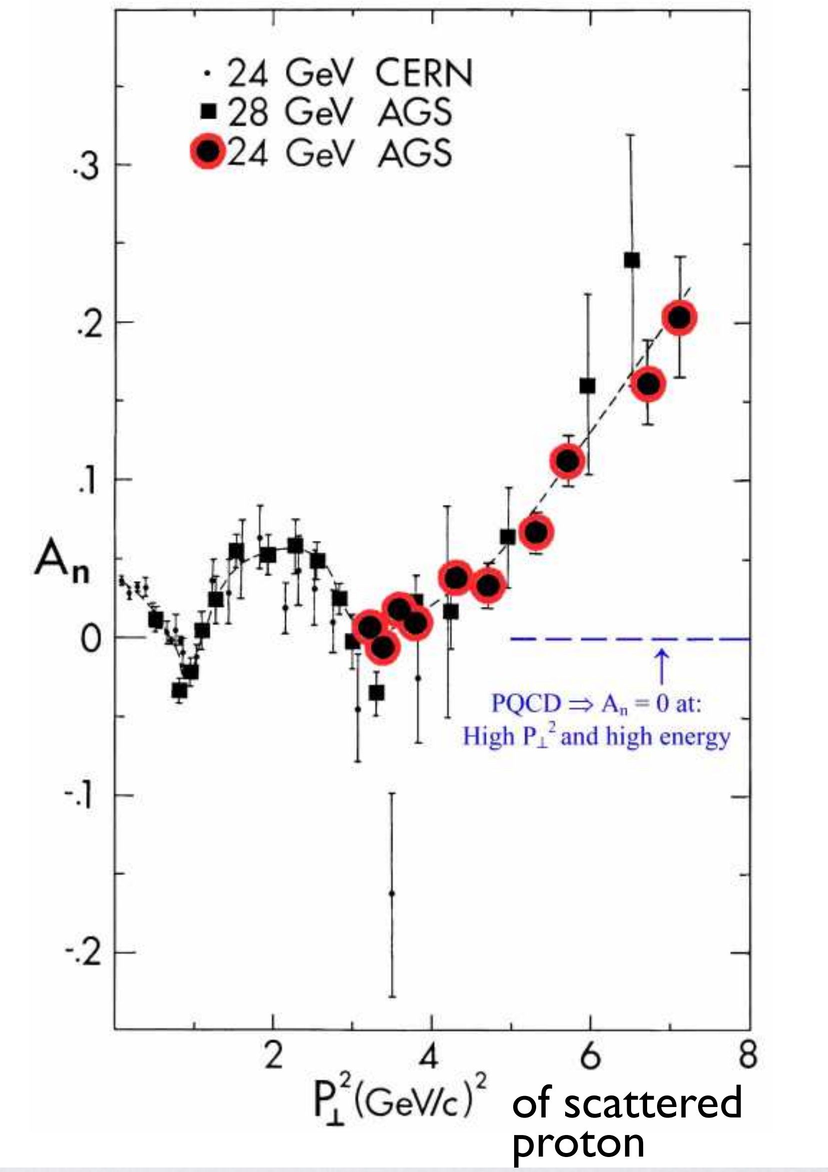
Evidences of going beyond the collinear framework

Example #2: elastic p-p scattering

$$A_N = \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}}$$

$p^{\uparrow} p \to p\, p$ versus $p^{\downarrow} p \to p\, p$

correlation between
spin of the proton and
k_T of partons



for a review, see
Krisch, E.P.J. **A31** (07) 417

Evidences to go beyond collinear

Evidences of going beyond the collinear framework

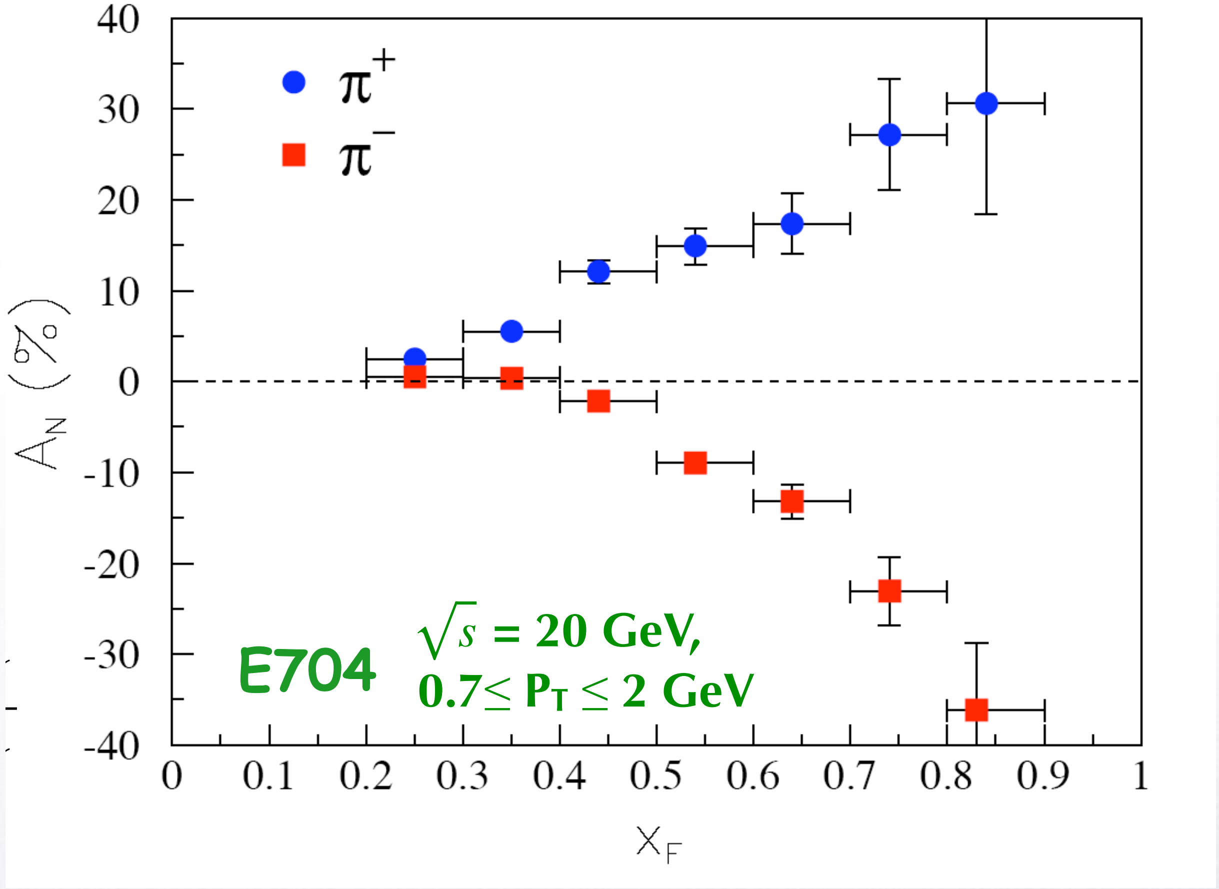
Example #3: semi-inclusive p-p collisions

$$p^\uparrow\, p \longrightarrow \pi\, X$$

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

single-spin asymmetry

correlation between
spin of the proton and
k_T and flavor of partons



Persisting also at higher energies up to $\sqrt{s} = 200$ GeV

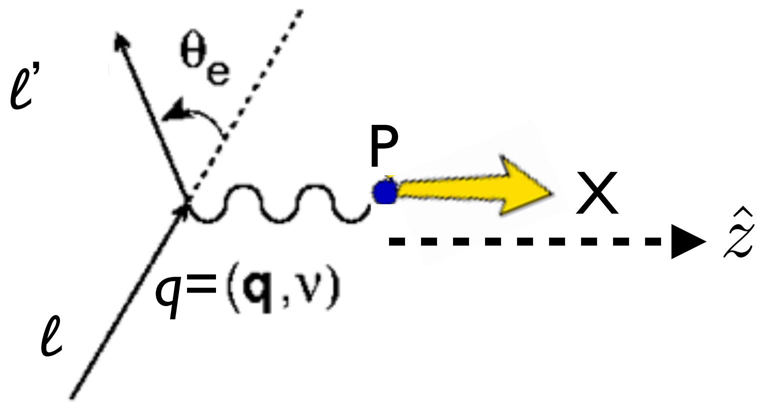
Adams *et al.* (STAR), PRL **92** (04) 171801

also in the $p + N \to \Lambda^\uparrow + X$ channel

Outline

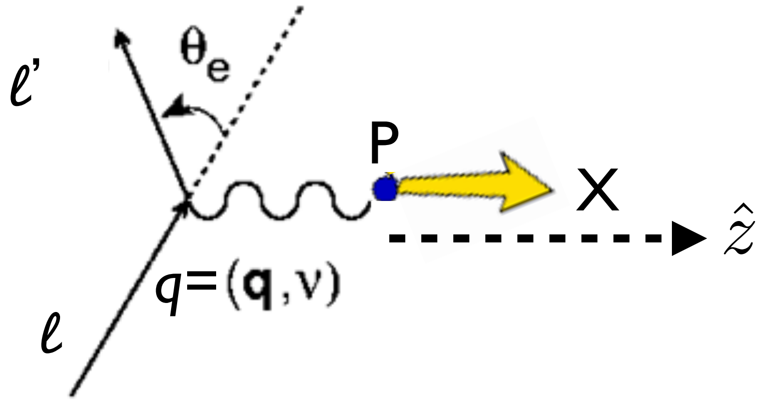
- The "TMD zoo"
  - factorisation th. and general properties (generalising same steps to get to PDFs)
  - specific properties

Need semi-inclusive process



inclusive DIS: - hard scale $Q^2 = -q^2 \gg M^2$ to "see" partons
- factorisation → isolate PDFs
- no further scale to probe proton interior

Need semi-inclusive process



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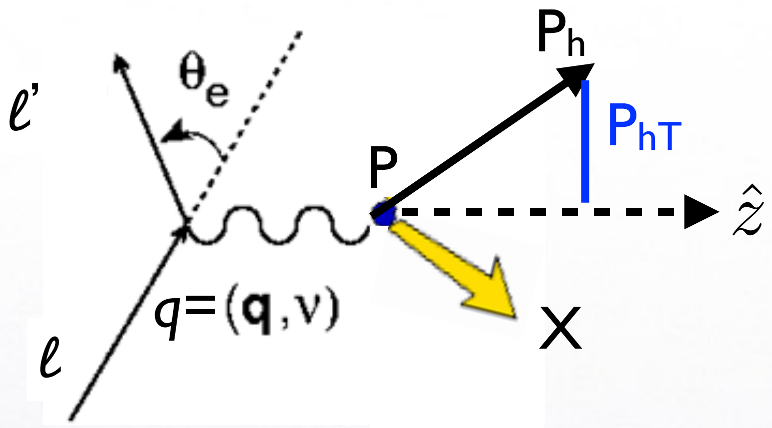
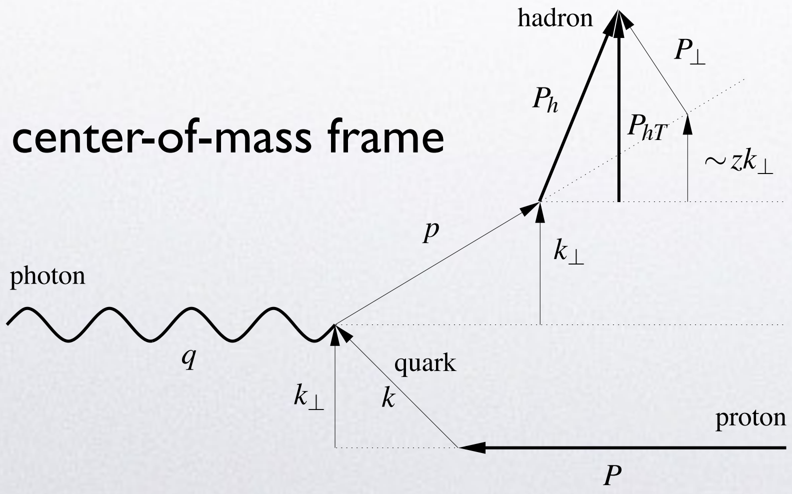


semi-inclusive DIS (SIDIS):

- hard scale $Q^2 = -q^2 \gg M^2$ to "see" partons
- soft scale: detect hadron *h* with $P_{hT}^2 \sim M^2 \ll Q^2$
- factorisation → isolate TMDs

center-of-mass frame



with these two scales, the process is factorizable into a hard photon-quark vertex and a quark→hadron fragmentation

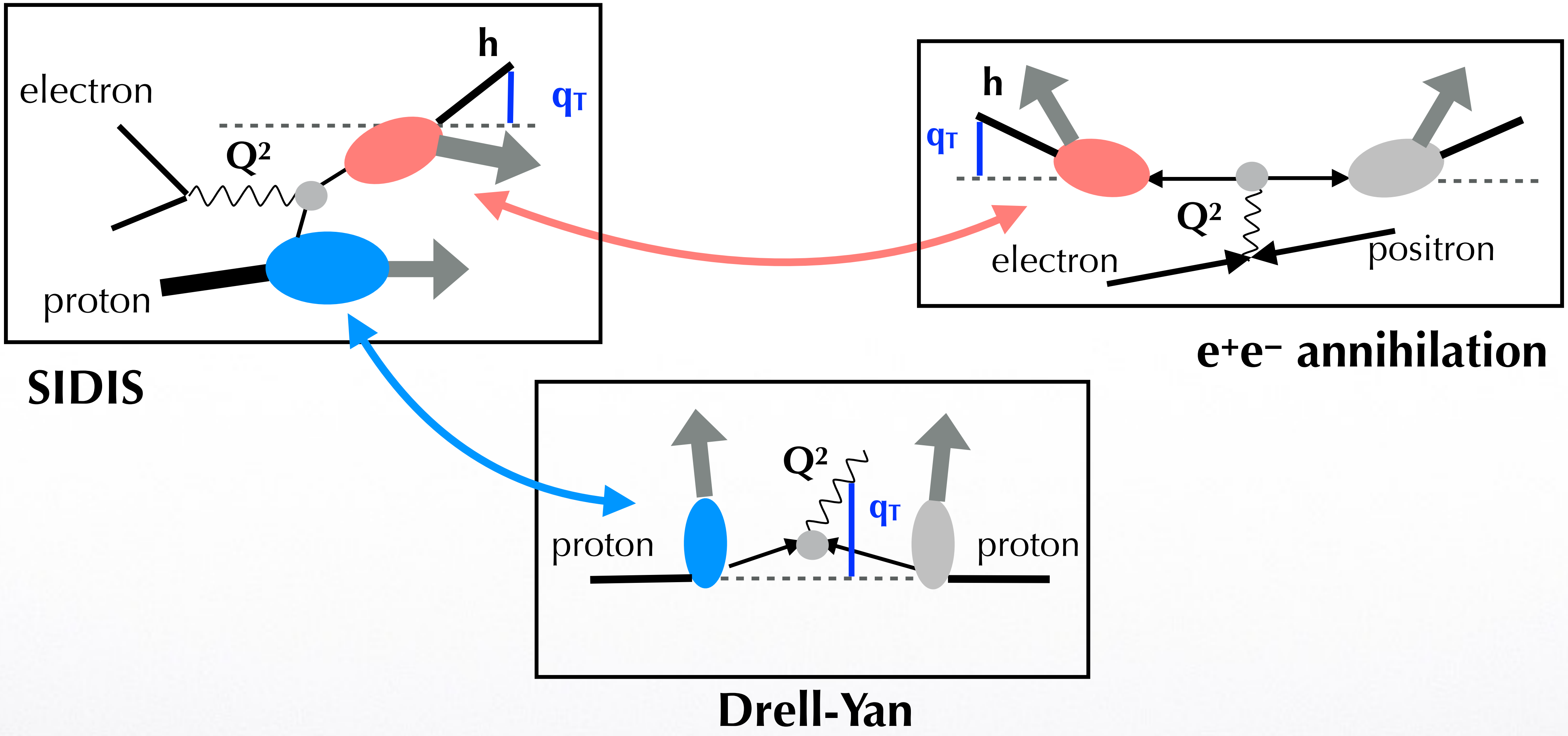
$$\mathbf{P}_{hT} = z\mathbf{k}_\perp + \mathbf{P}_\perp + \mathcal{O}(\mathbf{k}_\perp^2/Q^2)$$

z = fractional energy of *h* (analogous of *x*)

hadron P_{hT} arises from struck quark $k_\perp$ and transverse momentum $P_\perp$ generated during fragmentation

measure P_{hT} → get to $k_\perp$

TMDs : factorisation theorems



Factorization theorems well understood for $q_T \ll Q$
universality of TMD **PDFs** and **FFs** (but see later)

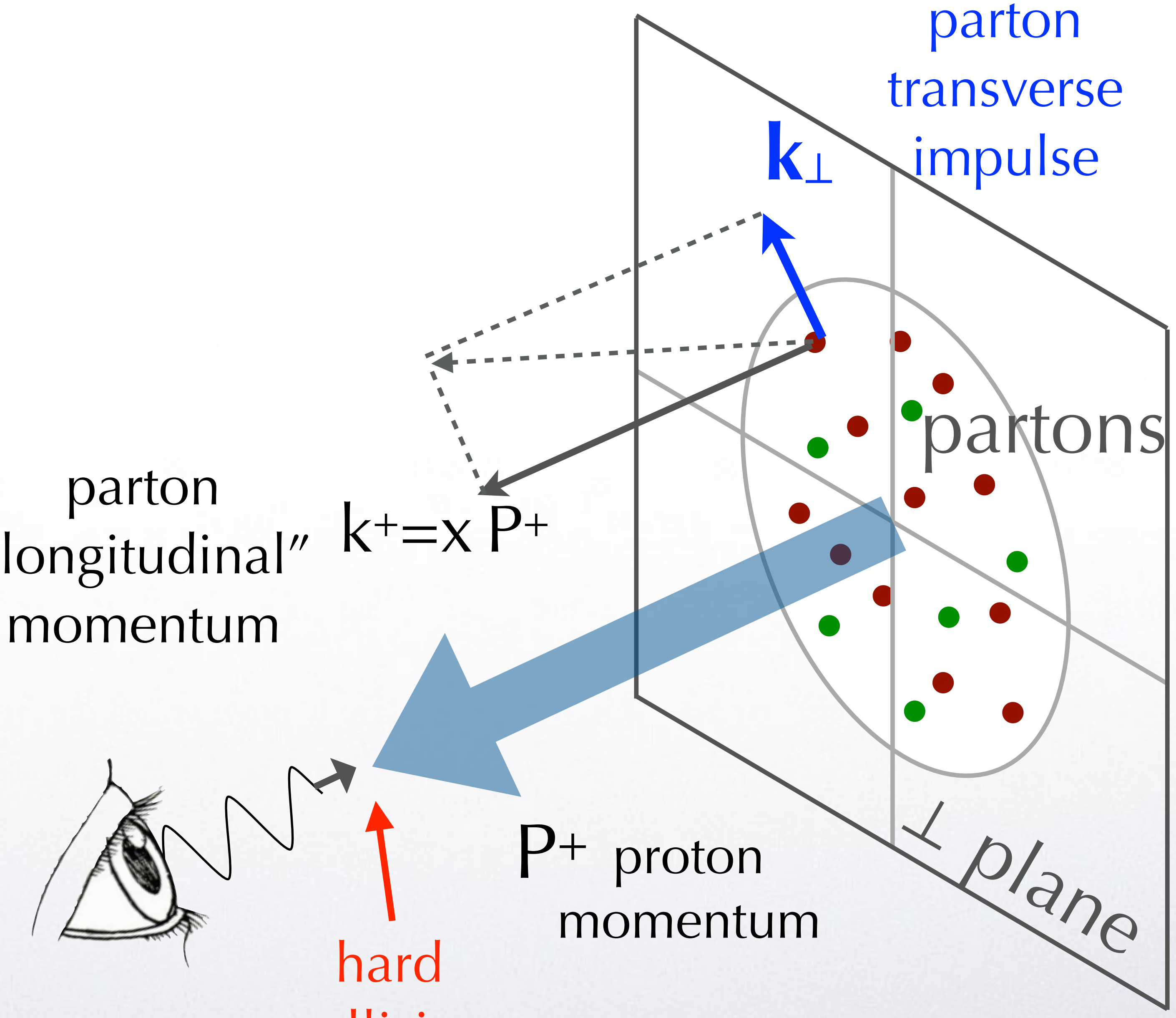
*Ji, Yuan, Ma, P.R. D**71** (05)*
*Rogers & Aybat, P.R. D**83** (11)*
Collins, "Foundations of Perturbative QCD" (11)
*Echevarria, Idilbi, Scimemi, JHEP **1207** (12)*

For $H_1+H_2 \rightarrow h+X$ no factor. th. but also no counterexample disproving it
Factorization broken for 2→2 processes

*Rogers & Mulders, P.R. D**81** (10)*
Buffing, Kang, Lee, Liu, arXiv:1812.07549

The TMD framework

parametrised by
Transverse-Momentum
Dependent PDFs
TMD PDF(x, k⊥; Q²)



parton
transverse
impulse

$k_\perp$

partons

parton
"longitudinal"
momentum

$k^+ = x\,P^+$

P^+ proton
momentum

hard
collision

⊥ plane

The TMD framework

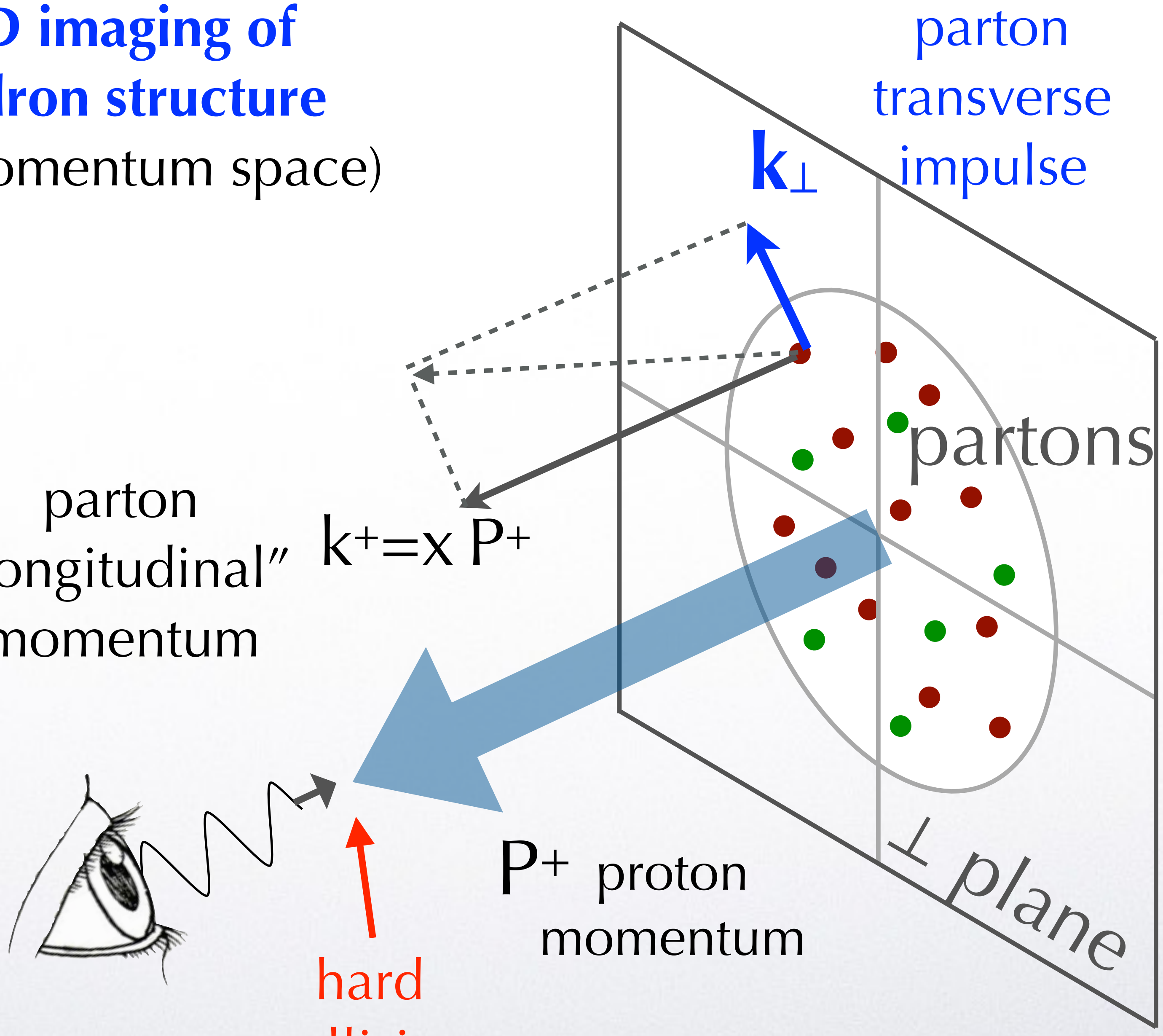
ECG

↓

cardio
MR

A new paradigm:
3D imaging of
hadron structure
(in momentum space)

parametrised by
Transverse-Momentum
Dependent PDFs
TMD PDF(x, $k_\perp$; Q^2)



parton
transverse
impulse

$k_\perp$

partons

parton
"longitudinal"
momentum

$k^+ = x\, P^+$

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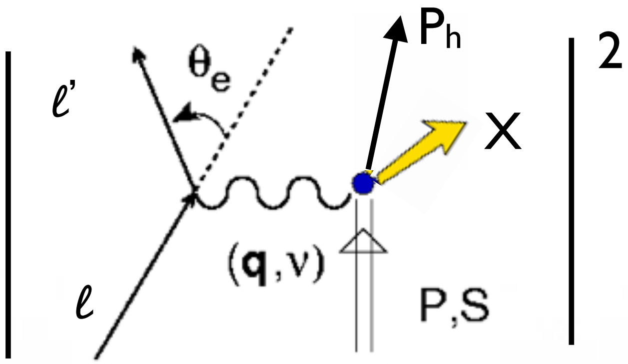
hard
collision

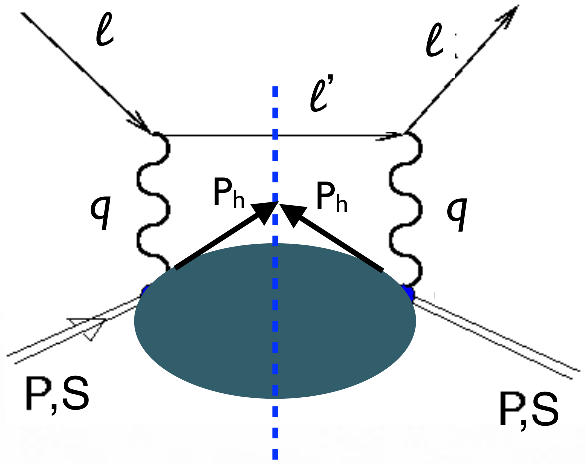
$\perp$ plane

Example : SIDIS

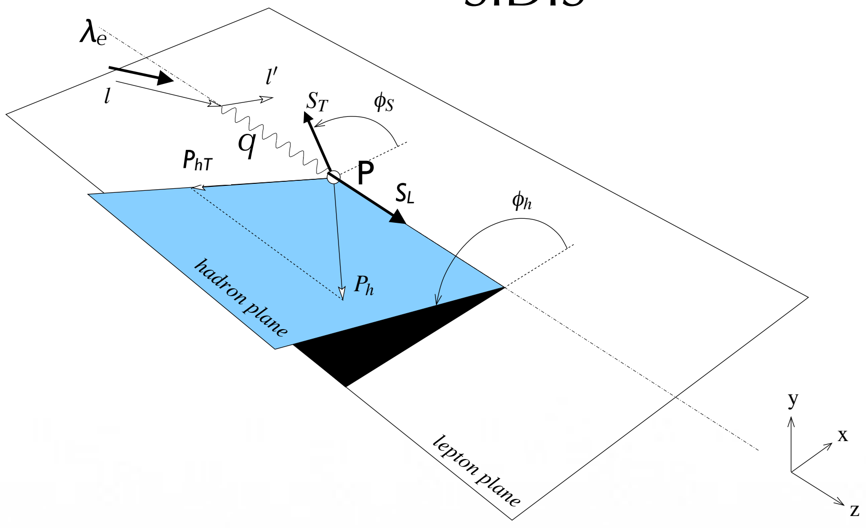
one photon-exchange approximation



optical theorem $\longrightarrow$



SIDIS



same invariants as inclusive DIS plus

$$z_h = \frac{P \cdot P_h}{P \cdot q}$$

"energy fraction" of fragmenting parton carried by final hadron

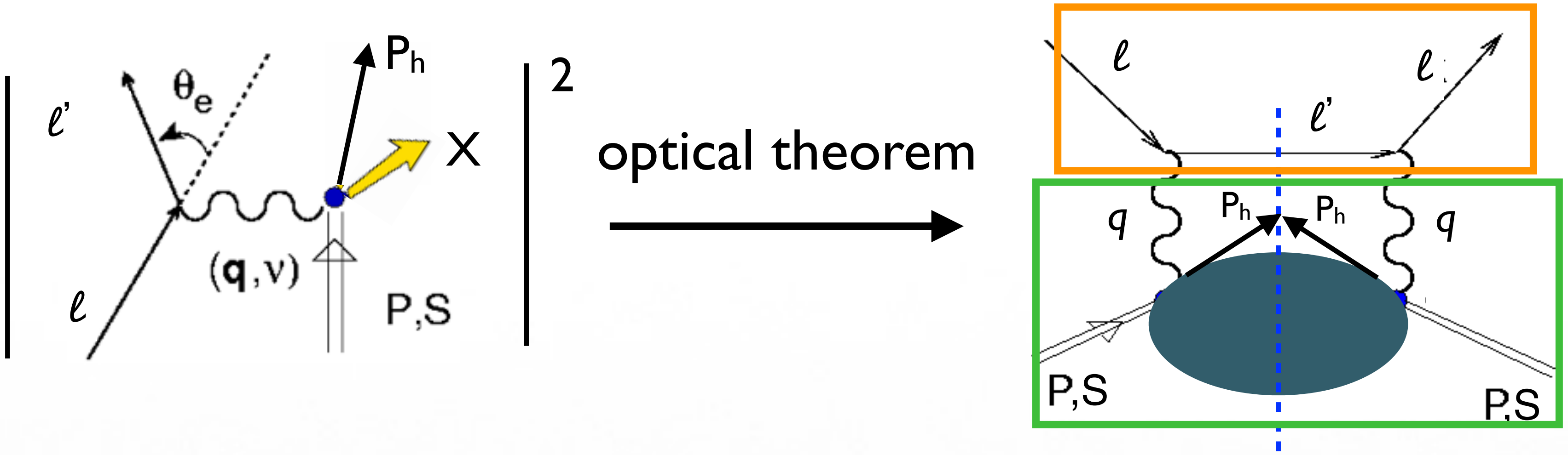
$$\frac{d\sigma}{dx_B dy d\phi_S \underline{dz_h d\phi_h dP_{hT}^2}} = \frac{\alpha^2 y}{2 z_h Q^4} L_{\mu\nu}(\ell, \ell', \lambda_e)\, W^{\mu\nu}(q, P, S, P_h)$$

new dependence

(for unpolarized hadron, S_h=0)

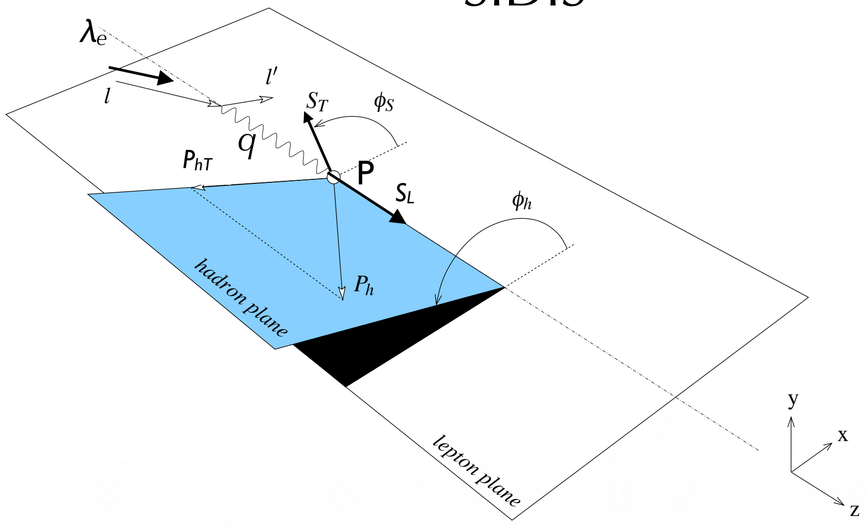
Example : SIDIS

one photon-exchange approximation



optical theorem

SIDIS



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new dependence
(for unpolarized hadron, S_h=0)

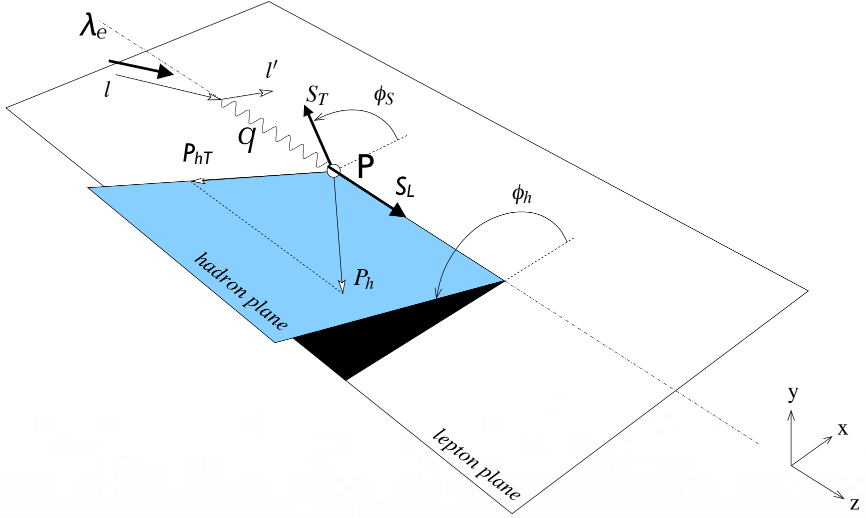
leptonic
tensor

hadronic
tensor

parametrised with
8 structure functions at leading twist
(18 including subleading twist)

SIDIS cross section

SIDIS



$$\frac{d\sigma}{dx_B dy d\phi_S dz_h d\phi_h dP_{hT}^2} =$$

$$= \frac{\alpha^2}{x_B y Q^2} \Big[A(y)\, F_{UU,T} + B(y)\, \cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$

$$+ S_L\, \sin 2\phi_h\, F_{UL}^{\sin 2\phi_h}$$

$$+ \lambda_e\, S_L\, C(y)\, F_{LL}$$

$$+ S_T \big[A(y)\, \sin(\phi_h - \phi_S)\, F_{UT,T}^{\sin(\phi_h - \phi_S)} + B(y)\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h + \phi_S)} + B(y)\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h - \phi_S)} \big]$$

$$+ \lambda_e\, S_T\, C(y)\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h - \phi_S)} \Big] \quad + \mathcal{O}\left(\frac{M}{Q}\right)$$

each

$F_{\dots}(x_B, z_h, P_{hT}^2, Q^2)$

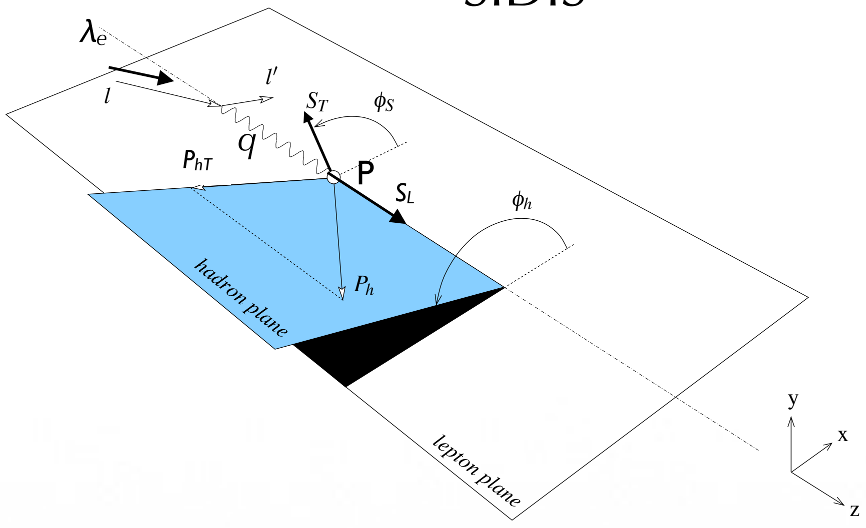
$F_{XY,Z}$

$\uparrow \; \uparrow \; \uparrow$

ℓ P γ^*

SIDIS cross section

SIDIS



$$\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, dP_{hT}^2} =$$

$$= \frac{\alpha^2}{x_B y Q^2} \Big[A(y)\, F_{UU,T} + B(y)\, \cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$

$$+ S_L\, \sin 2\phi_h\, F_{UL}^{\sin 2\phi_h}$$

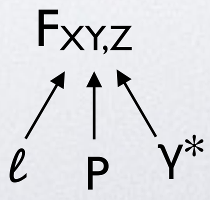
$$+ \lambda_e\, S_L\, C(y)\, F_{LL}$$

$$+ S_T \Big[A(y)\, \sin(\phi_h - \phi_S)\, F_{UT,T}^{\sin(\phi_h - \phi_S)} + B(y)\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h + \phi_S)} + B(y)\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h - \phi_S)} \Big]$$

$$+ \lambda_e\, S_T\, C(y)\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h - \phi_S)} \Big] \quad + \mathcal{O}\left(\frac{M}{Q}\right)$$

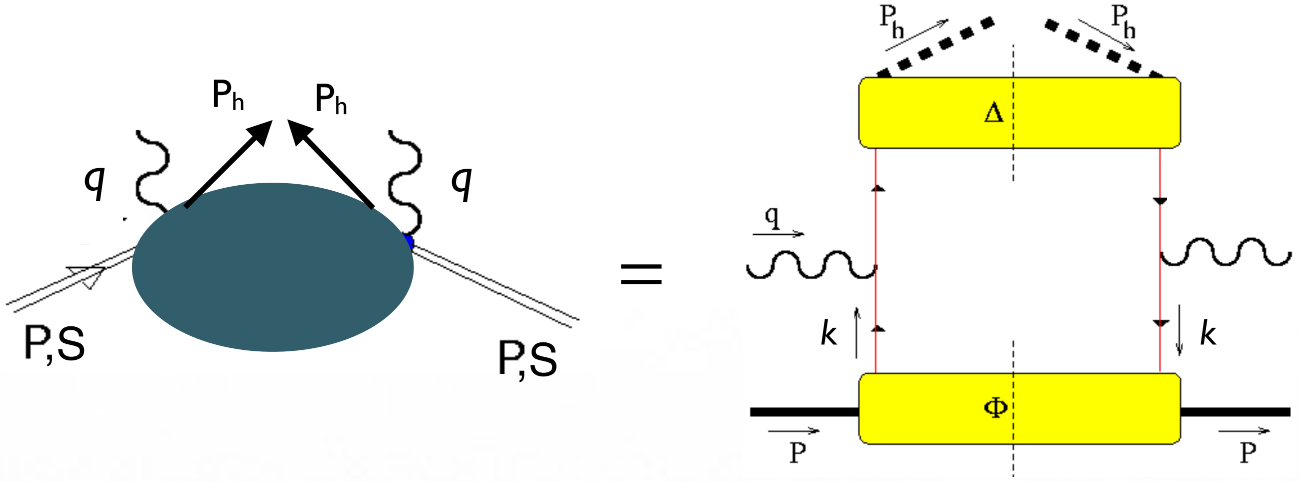
each

$F_{\dots}(x_B, z_h, P_{hT}^2, Q^2)$



SIDIS : factorisation

OPE not possible, use diagrammatic approach (select dominant diagram by counting powers of divergences)



+ higher twists (suppressed)

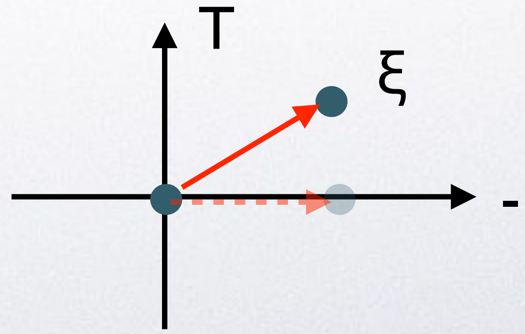
$$2MW^{\mu\nu}(q,P,S,P_h) = 2z_h\,\mathcal{C}\Big[\mathrm{Tr}\big[\Phi(x_B,\mathbf{k}_\perp,S)\,\gamma^\mu\,\Delta(z_h,\mathbf{P}_\perp)\,\gamma^\nu\big]\Big]$$

$$\mathcal{C}\big[\dots\big] = \int d\mathbf{P}_\perp d\mathbf{k}_\perp\,\delta^{(2)}(z\mathbf{k}_\perp + \mathbf{P}_\perp - \mathbf{P}_{hT})\big[\dots\big]$$

<u>non-local correlator</u>:

from collinear $\Phi_{ij}(x,S) = \int \frac{d\xi_-}{2\pi}\, e^{ik\cdot\xi}\,\langle P,S|\,\bar\psi_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,|P,S\rangle_{\xi_+=\xi_T=0}$

to $\Phi_{ij}(x,\mathbf{k}_\perp,S) = \int \frac{d\xi\_d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\,\langle P,S|\,\bar\psi_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,|P,S\rangle_{\xi_+=0}$



$x_B \sim x = \frac{k_+}{P_+}$

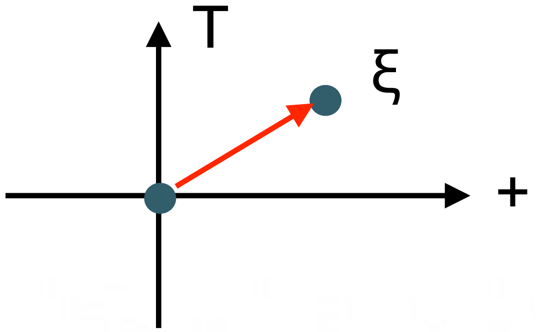
$\xi_- \leftrightarrow k_+$
$\xi_T \leftrightarrow k_\perp$

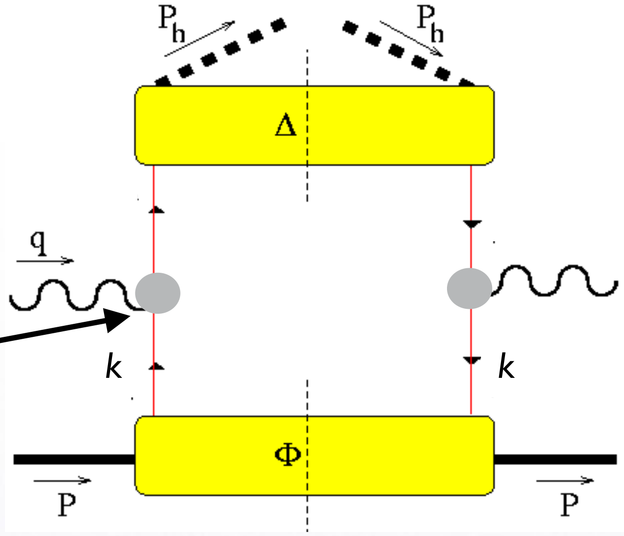
SIDIS : factorisation

non-local correlators

$$z_h \sim z = \frac{P_{h-}}{k_-} \qquad \begin{array}{l} \xi_+ \leftrightarrow k_- \\ \xi_T \leftrightarrow k_\perp \end{array}$$

$$\Delta_{ij}(z, \mathbf{k}_\perp) = \sum_X \int \frac{d\xi_+ d^2\xi_T}{(2\pi)^3} \, e^{ik\cdot\xi} \, \langle 0 \,|\, \psi_i(\xi) \,|\, X, P_h \rangle \langle X, P_h \,|\, \bar{\psi}_j(0) \,|\, 0 \rangle_{\xi_-=0}$$

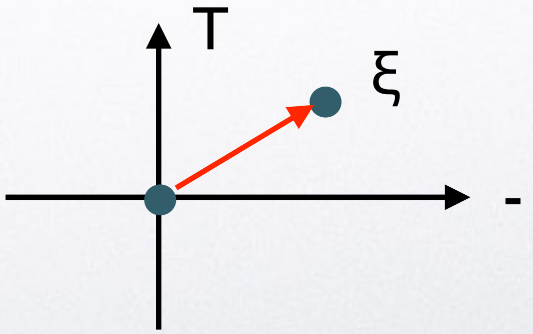




hard cross section

$d\hat{\sigma} = 1 + c_1\alpha_s + \ldots$

$$\Phi_{ij}(x, \mathbf{k}_\perp, S) = \int \frac{d\xi_- d^2\xi_T}{(2\pi)^3} \, e^{ik\cdot\xi} \, \langle P, S \,|\, \bar{\psi}_j(0) \, U_{[0,\xi]} \, \psi_i(\xi) \,|\, P, S \rangle_{\xi_+=0}$$



$$x_B \sim x = \frac{k_+}{P_+} \qquad \begin{array}{l} \xi_- \leftrightarrow k_+ \\ \xi_T \leftrightarrow k_\perp \end{array}$$

SIDIS : factorisation

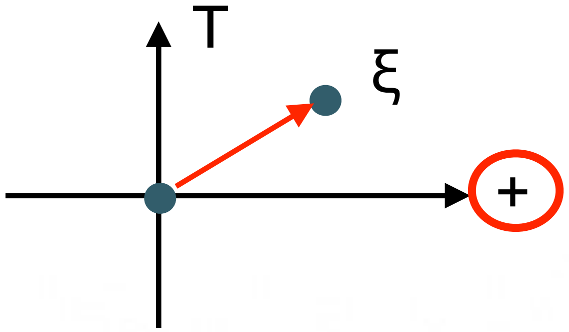
non-local correlators

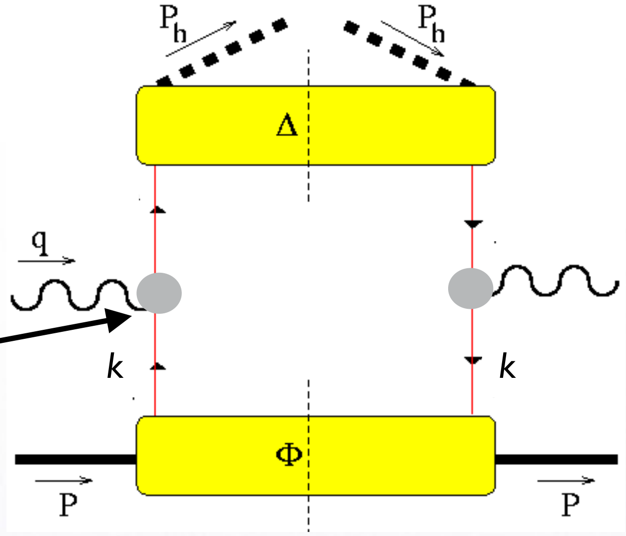
$z_h \sim z = \frac{P_{h-}}{k_-}$

$\xi_+ \leftrightarrow k_-$

$\xi_T \leftrightarrow k_\perp$

$$\Delta_{ij}(z,\mathbf{k}_\perp) = \sum_X \int \frac{d\xi_+ d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle 0|\psi_i(\xi)|X,P_h\rangle\langle X,P_h|\bar{\psi}_j(0)|0\rangle_{\xi_-=0}$$

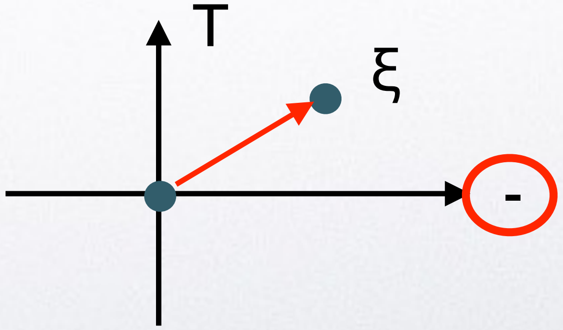




hard cross section

$d\hat{\sigma} = 1 + c_1\alpha_s + \ldots$

flipping LC-dominant direction



$$\Phi_{ij}(x,\mathbf{k}_\perp,S) = \int \frac{d\xi_- d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P,S|\bar{\psi}_j(0)\, U_{[0,\xi]}\, \psi_i(\xi)|P,S\rangle_{\xi_+=0}$$

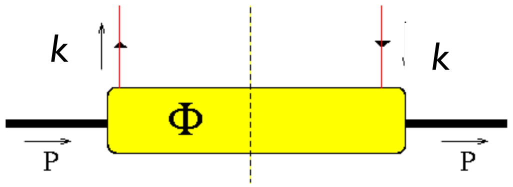
$x_B \sim x = \frac{k_+}{P_+}$

$\xi_- \leftrightarrow k_+$

$\xi_T \leftrightarrow k_\perp$

parton-parton correlator

linear combination of all tensor structures with *k*, *P*, *S*, subject to Hermiticity and parity-invariance (see later about time reversal)



expansion of Φ in powers of M/P₊ . At leading twist:

$$\begin{aligned}\Phi(x,\mathbf{k}_\perp,S) = \frac{1}{2}\Big[& f_1\,\gamma_- - f_{1T}^\perp\,\frac{(\mathbf{k}_\perp\times\mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}\,\gamma_- \\ & + g_{1L}\,S_L\,\gamma_5\gamma_- + g_{1T}\,\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\,\gamma_5\gamma_- \\ & + h_{1T}\,i\sigma_{-\nu}\,\gamma_5\,S_T^\nu + h_{1L}^\perp\,i\sigma_{-\nu}\,\gamma_5\,S_L\,\frac{k_\perp^\nu}{M} \\ & + h_{1T}^\perp\,\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\,i\sigma_{-\nu}\,\gamma_5\,\frac{k_\perp^\nu}{M} - h_1^\perp\,\sigma_{-\nu}\,\frac{k_\perp^\nu}{M}\Big]\end{aligned}$$

$\sigma^{\mu\nu} = \frac{i}{2}\,[\gamma^\mu,\gamma^\nu]$

Notations:

t = *f* unpolarized parton

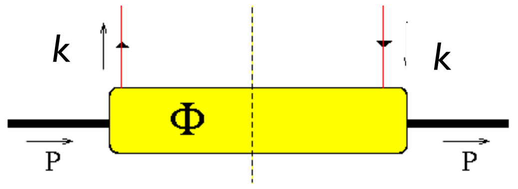
t = *g* longitudinally polarized parton

t = *h* transversely polarized parton

parton-parton correlator

linear combination of all tensor structures with k, P, S, subject to Hermiticity and parity-invariance (see later about time reversal)

expansion of Φ in powers of M/P+ . At leading twist:



$$\begin{aligned}\Phi(x,\mathbf{k}_\perp,S) = \frac{1}{2}\Big[& f_1 \gamma_- - f_{1T}^\perp \frac{(\mathbf{k}_\perp\times\mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}\gamma_- \\ & + g_{1L} S_L \gamma_5\gamma_- + g_{1T}\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\gamma_5\gamma_- \\ & + h_{1T}\, i\sigma_{-\nu}\gamma_5 S_T^\nu + h_{1L}^\perp\, i\sigma_{-\nu}\gamma_5 S_L \frac{k_\perp^\nu}{M} \\ & + h_{1T}^\perp \frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\, i\sigma_{-\nu}\gamma_5\frac{k_\perp^\nu}{M} - h_1^\perp\, \sigma_{-\nu}\frac{k_\perp^\nu}{M}\Big]\end{aligned}$$

$$\frac{1}{2}\mathrm{Tr}[\Phi\,\gamma_+] \equiv \Phi^{[\gamma_+]}$$

→ 2 TMDPDFs for unpol. parton

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu]$$

Notations: $t_{1X}^{(\perp)}(x,\mathbf{k}_\perp^2)$

waited by $k_\perp^i$

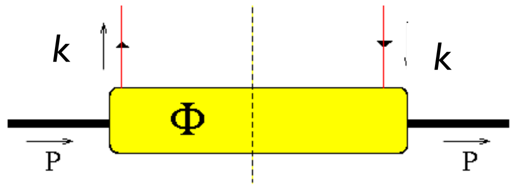
leading twist

$X = L$ longitudinally polarized hadron
$X = T$ transversely polarized hadron

$t = f$ unpolarized parton
$t = g$ longitudinally polarized parton
$t = h$ transversely polarized parton

parton-parton correlator

linear combination of all tensor structures with *k*, *P*, *S*, subject to Hermiticity and parity-invariance (see later about time reversal)



expansion of Φ in powers of M/P₊ . At leading twist:

$$\begin{aligned}\Phi(x,\mathbf{k}_\perp,S) = \frac{1}{2}\Big[& f_1\,\gamma_- - f_{1T}^\perp \frac{(\mathbf{k}_\perp\times\mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}\gamma_- \\ & + g_{1L}\,S_L\,\gamma_5\gamma_- + g_{1T}\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\gamma_5\gamma_- \\ & + h_{1T}\,i\sigma_{-\nu}\gamma_5\,S_T^\nu + h_{1L}^\perp\,i\sigma_{-\nu}\gamma_5\,S_L\frac{k_\perp^\nu}{M} \\ & + h_{1T}^\perp\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\,i\sigma_{-\nu}\gamma_5\frac{k_\perp^\nu}{M} - h_1^\perp\,\sigma_{-\nu}\frac{k_\perp^\nu}{M}\Big]\end{aligned}$$

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu,\gamma^\nu\right]$$

$\frac{1}{2}\mathrm{Tr}[\Phi\,\gamma_+] \equiv \Phi^{[\gamma_+]}$ → 2 TMDPDFs for unpol. parton

$\frac{1}{2}\mathrm{Tr}[\Phi\,\gamma_+\gamma_5] \equiv \Phi^{[\gamma_+\gamma_5]}$ → 2 TMDPDFs for long. pol. parton

$\frac{1}{2}\mathrm{Tr}[\Phi\,i\sigma_{+i}\gamma_5] \equiv \Phi^{[i\sigma_{+i}\gamma_5]}$

→ 4 TMDPDFs for transv. pol. parton along *i*

Notations: $t_{1X}^{(\perp)}(x,\mathbf{k}_\perp^2)$

waited by $k_\perp{}^i$

leading twist

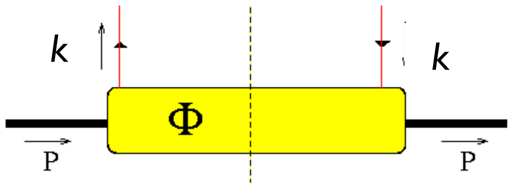
X = *L* longitudinally polarized hadron
X = *T* transversely polarized hadron

t = *f* unpolarized parton
t = *g* longitudinally polarized parton
t = *h* transversely polarized parton

parton-parton correlator

linear combination of all tensor structures with *k*, *P*, *S*, subject to Hermiticity and parity-invariance (see later about time reversal)

expansion of Φ in powers of M/P₊ . At leading twist:



$$\Phi(x,\mathbf{k}_\perp,S) = \frac{1}{2}\Big[\boxed{f_1}\gamma_- - f_{1T}^\perp\frac{(\mathbf{k}_\perp\times\mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}\gamma_-$$

$$+\boxed{g_{1L}}S_L\gamma_5\gamma_- + g_{1T}\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\gamma_5\gamma_-$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu] \qquad +\boxed{h_{1T}}i\sigma_{-\nu}\gamma_5 S_T^\nu + h_{1L}^\perp i\sigma_{-\nu}\gamma_5 S_L\frac{k_\perp^\nu}{M}$$

$$+h_{1T}^\perp\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}i\sigma_{-\nu}\gamma_5\frac{k_\perp^\nu}{M} - h_1^\perp\sigma_{-\nu}\frac{k_\perp^\nu}{M}\Big]$$

$\frac{1}{2}\mathrm{Tr}[\Phi\,\gamma_+] \equiv \Phi^{[\gamma_+]}$ → **2 TMDPDFs** for unpol. parton

$\frac{1}{2}\mathrm{Tr}[\Phi\,\gamma_+\gamma_5] \equiv \Phi^{[\gamma_+\gamma_5]}$ → **2 TMDPDFs** for long. pol. parton

$\frac{1}{2}\mathrm{Tr}[\Phi\,i\sigma_{+i}\gamma_5] \equiv \Phi^{[i\sigma_{+i}\gamma_5]}$ → **4 TMDPDFs** for transv. pol. parton along *i*

Notations: $t_{1X}^{(\perp)}(x,\mathbf{k}_\perp^2)$

$(\perp)$: waited by $k_\perp{}^i$

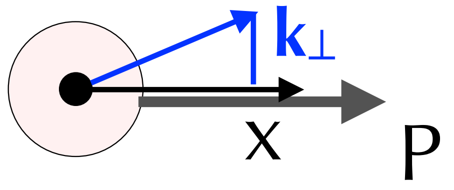
1: leading twist

X:
X = *L* longitudinally polarized hadron
X = *T* transversely polarized hadron

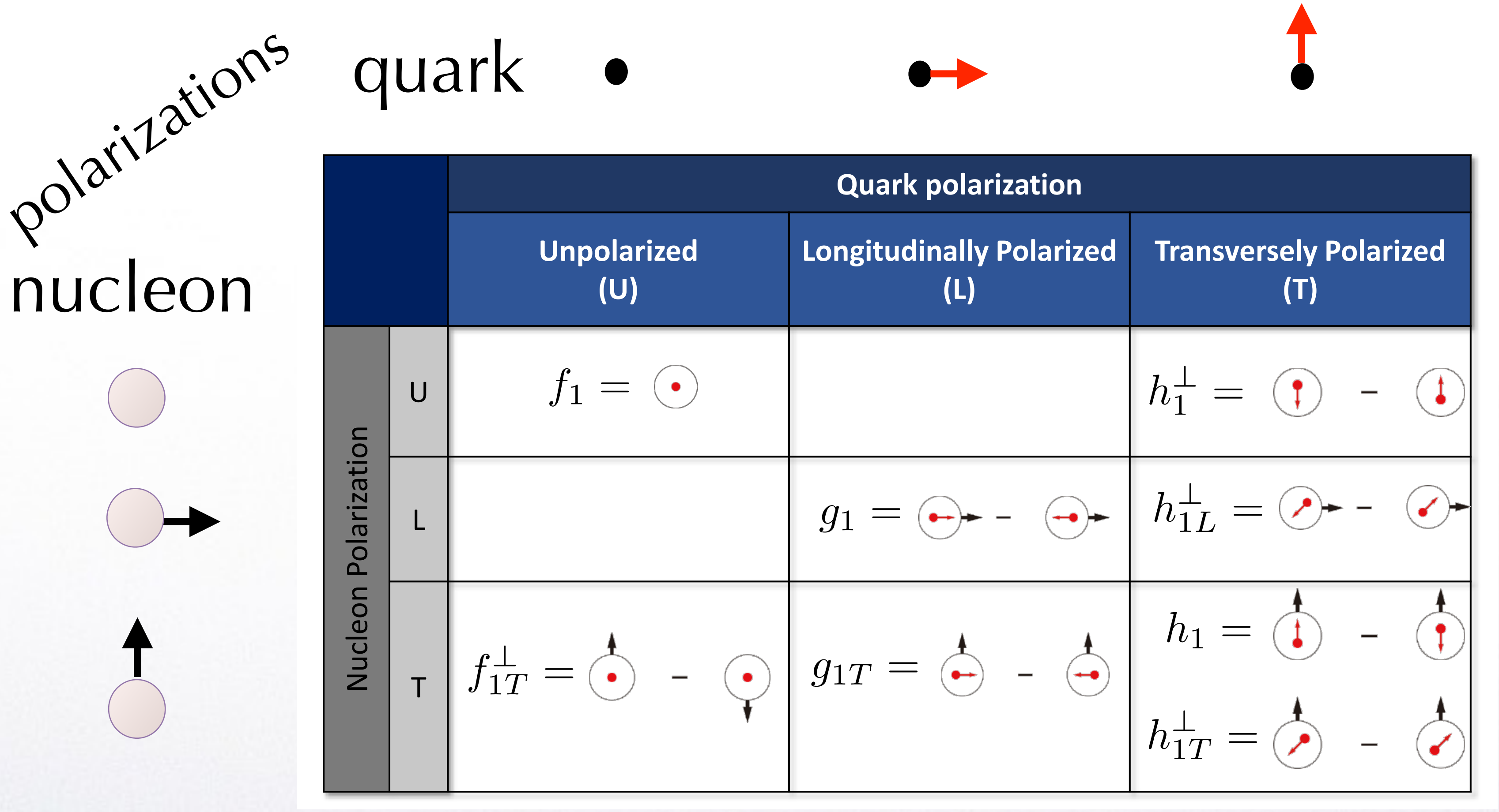
t = *f* unpolarized parton
t = *g* longitudinally polarized parton
t = *h* transversely polarized parton

$\boxed{\text{survive upon } \int d\mathbf{k}_\perp \to \text{collinear PDF}}$

The TMD PDF table



TMD PDFs (x, $k_\perp$; Q^2) at leading twist for a spin-1/2 hadron (Nucleon)

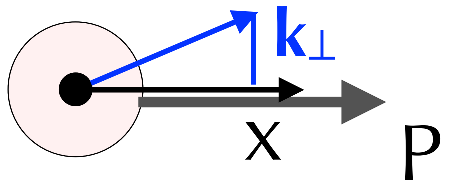


		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^{\perp} =$ -
	L		$g_1 =$ -	$h_{1L}^{\perp} =$ -
	T	$f_{1T}^{\perp} =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^{\perp} =$ -

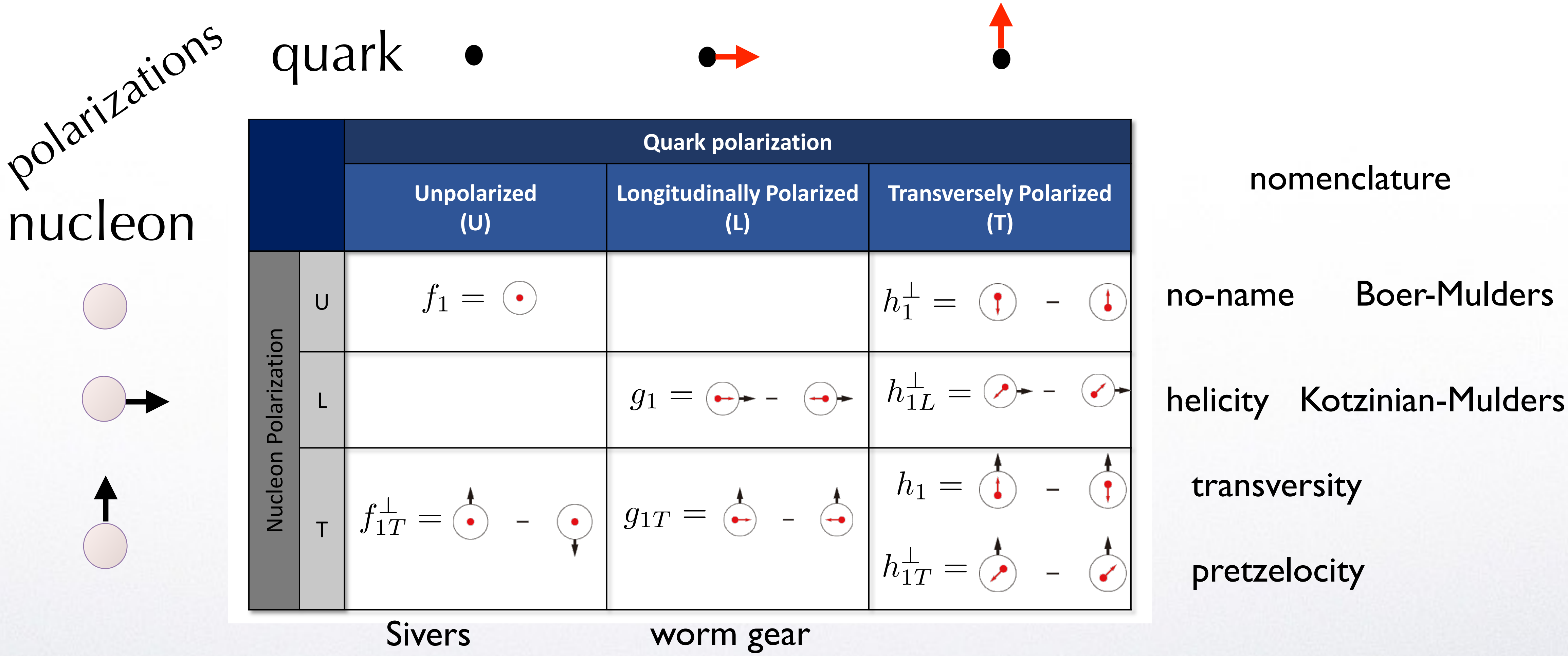
Each entry has a nice probabilistic interpretation

N.B. Using appropriate projections $\Phi^{[\Gamma]}$ one can extract also subleading-twist TMD PDFs, but no probabilistic interpretation

The TMD PDF table



TMD PDFs (x, k⊥; Q²) at leading twist for a spin-1/2 hadron (Nucleon)



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^{\perp} =$ -
	L		$g_1 =$ -	$h_{1L}^{\perp} =$ -
	T	$f_{1T}^{\perp} =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^{\perp} =$ -

nomenclature

no-name Boer-Mulders

helicity Kotzinian-Mulders

transversity

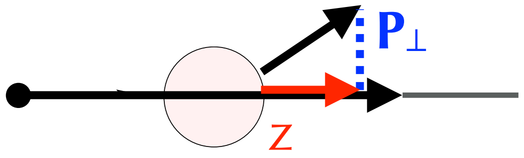
pretzelocity

Sivers worm gear

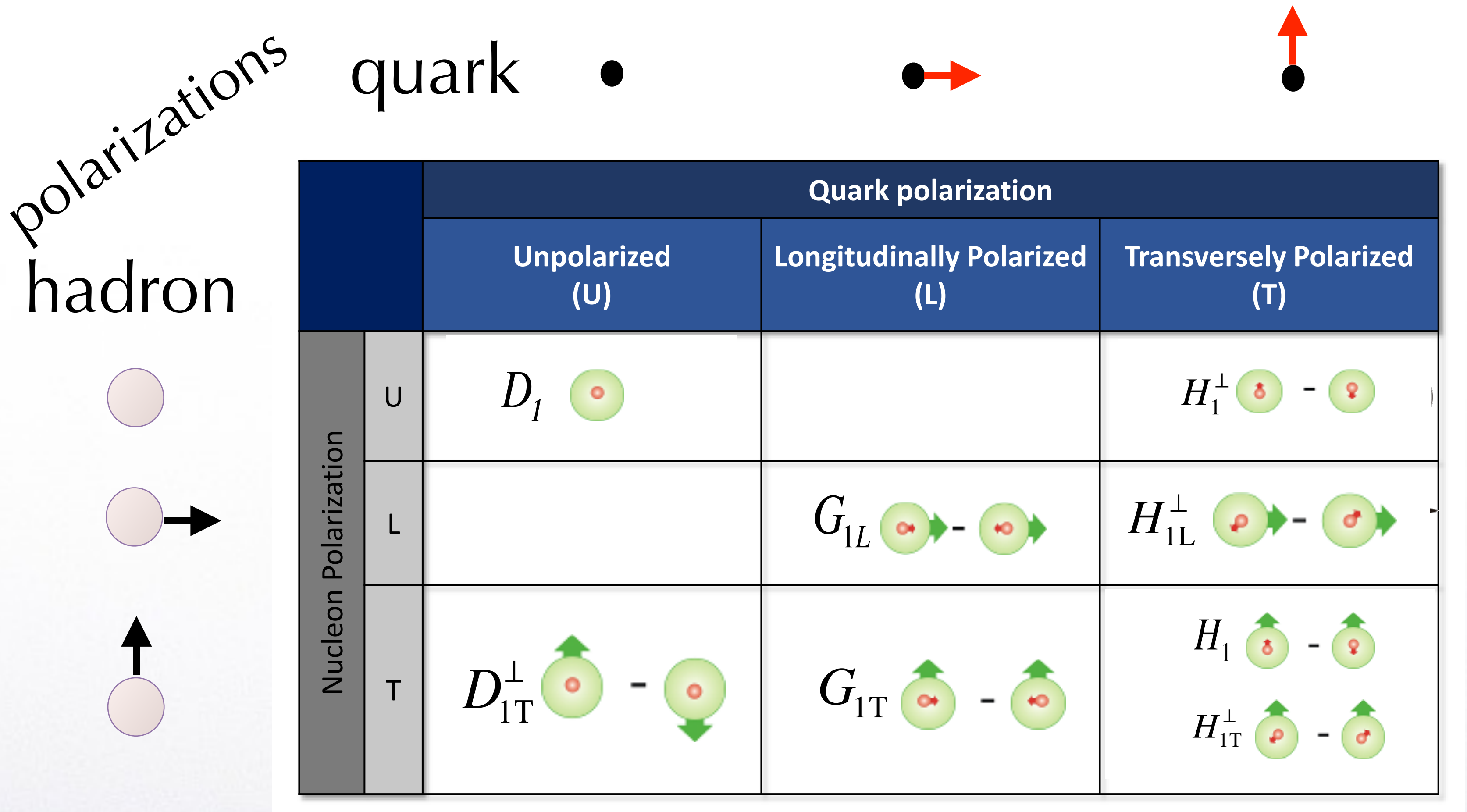
Each entry has a nice probabilistic interpretation

N.B. Using appropriate projections $\Phi^{[\Gamma]}$ one can extract also subleading-twist TMD PDFs, but no probabilistic interpretation

The TMD FF table



TMD FFs (z, $P_\perp$; Q^2) at leading twist (and $S_h \leq 1/2$)



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1		$H_1^\perp$ -
	L		G_{1L} -	$H_{1L}^\perp$ -
	T	$D_{1T}^\perp$ -	G_{1T} -	H_1 - $H_{1T}^\perp$ -

nomenclature

no-name Collins

… ….

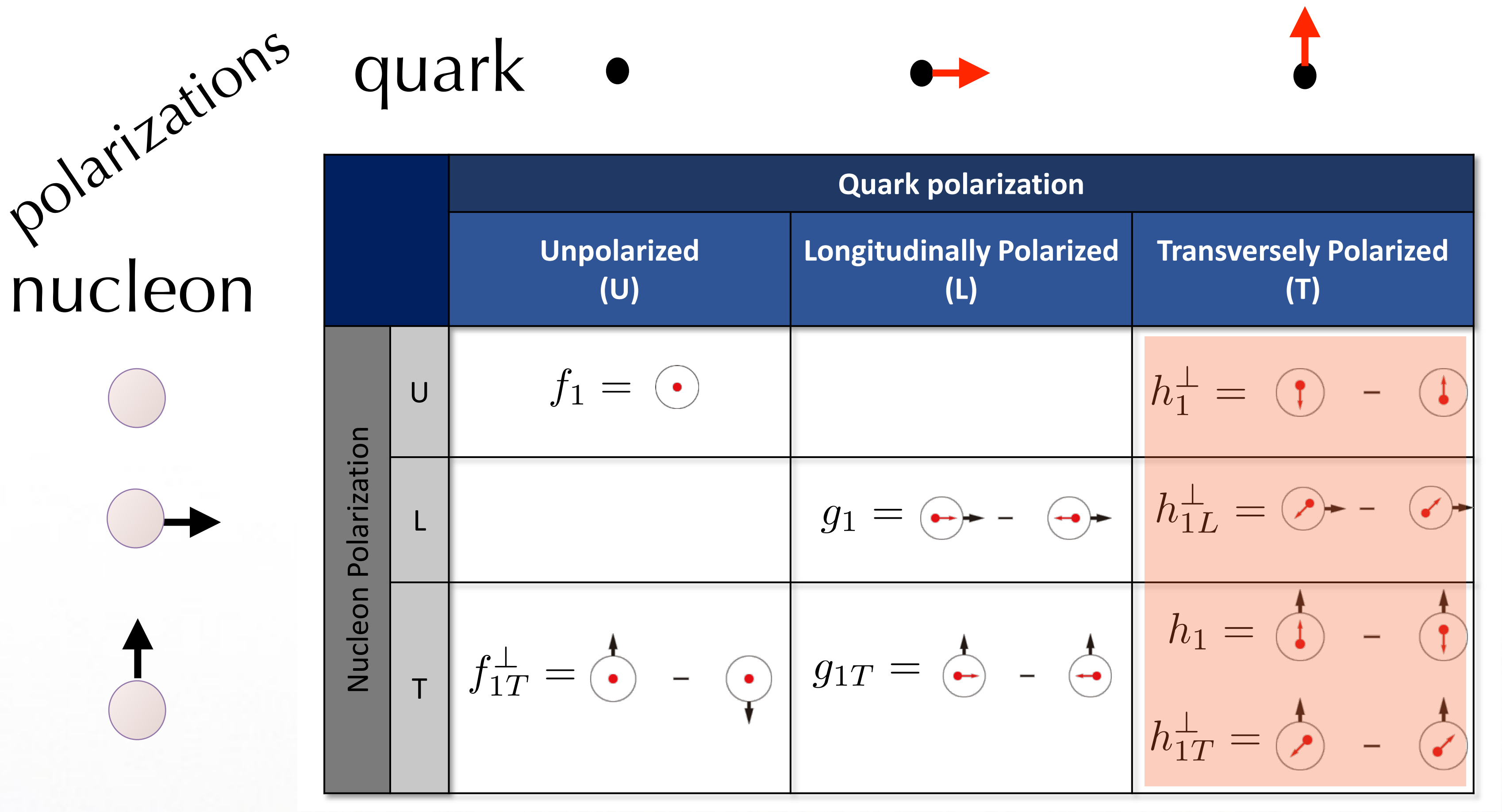
… ….

polarising FF …

Each entry has a nice probabilistic interpretation

N.B. Using appropriate projections $\Phi^{[\Gamma]}$ one can extract also subleading-twist TMD FFs, but no probabilistic interpretation

The chiral-odd TMD PDFs

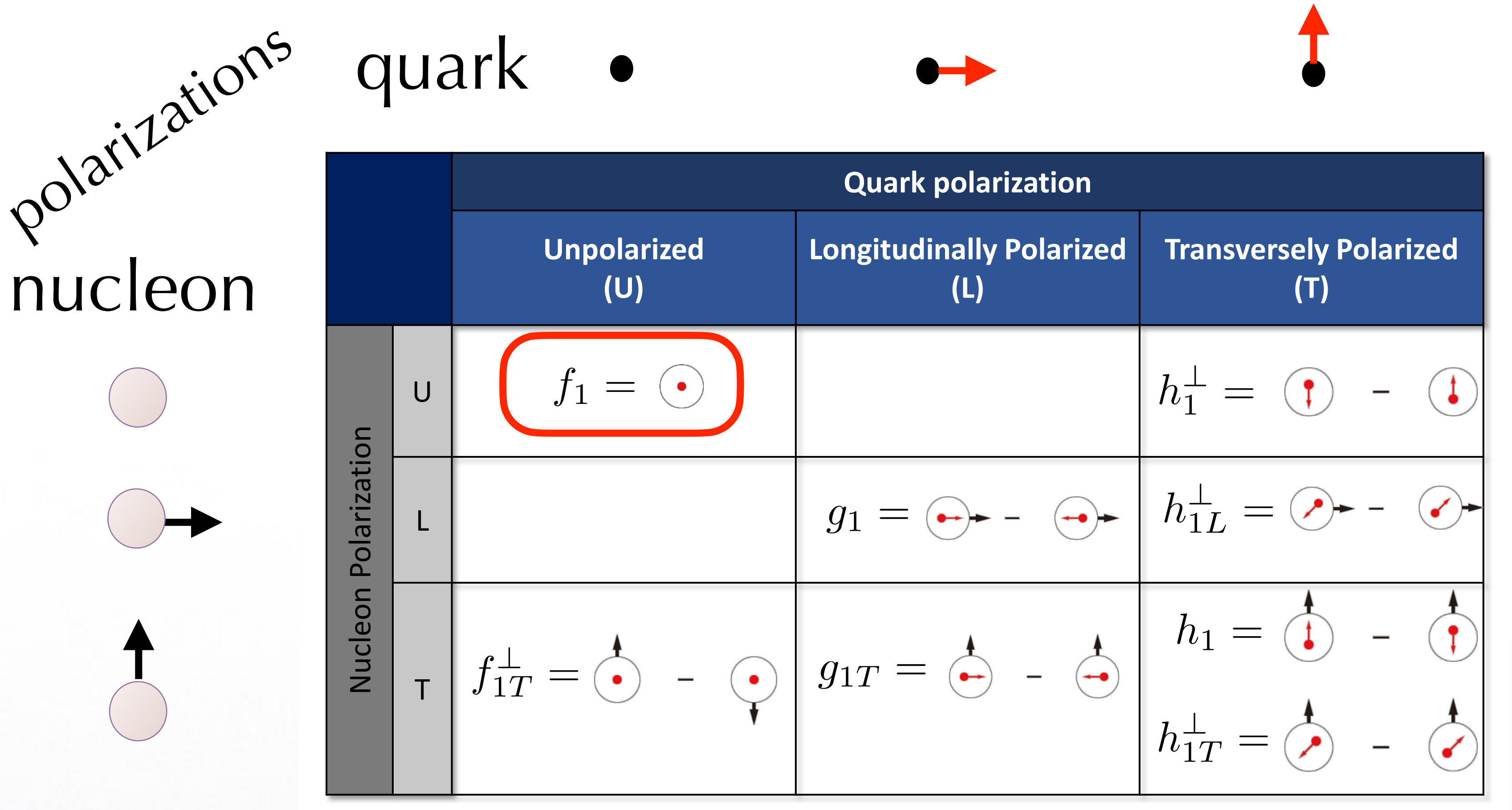


		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ -
	L		$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

all TMD PDFs belonging to right column involve transverse polarization of quarks, hence they are "chiral-odd" and are suppressed in perturbative QCD as m_q/Q.

Similarly to transversity h_1, they can appear in the cross section at leading twist if paired to another chiral-odd structure.

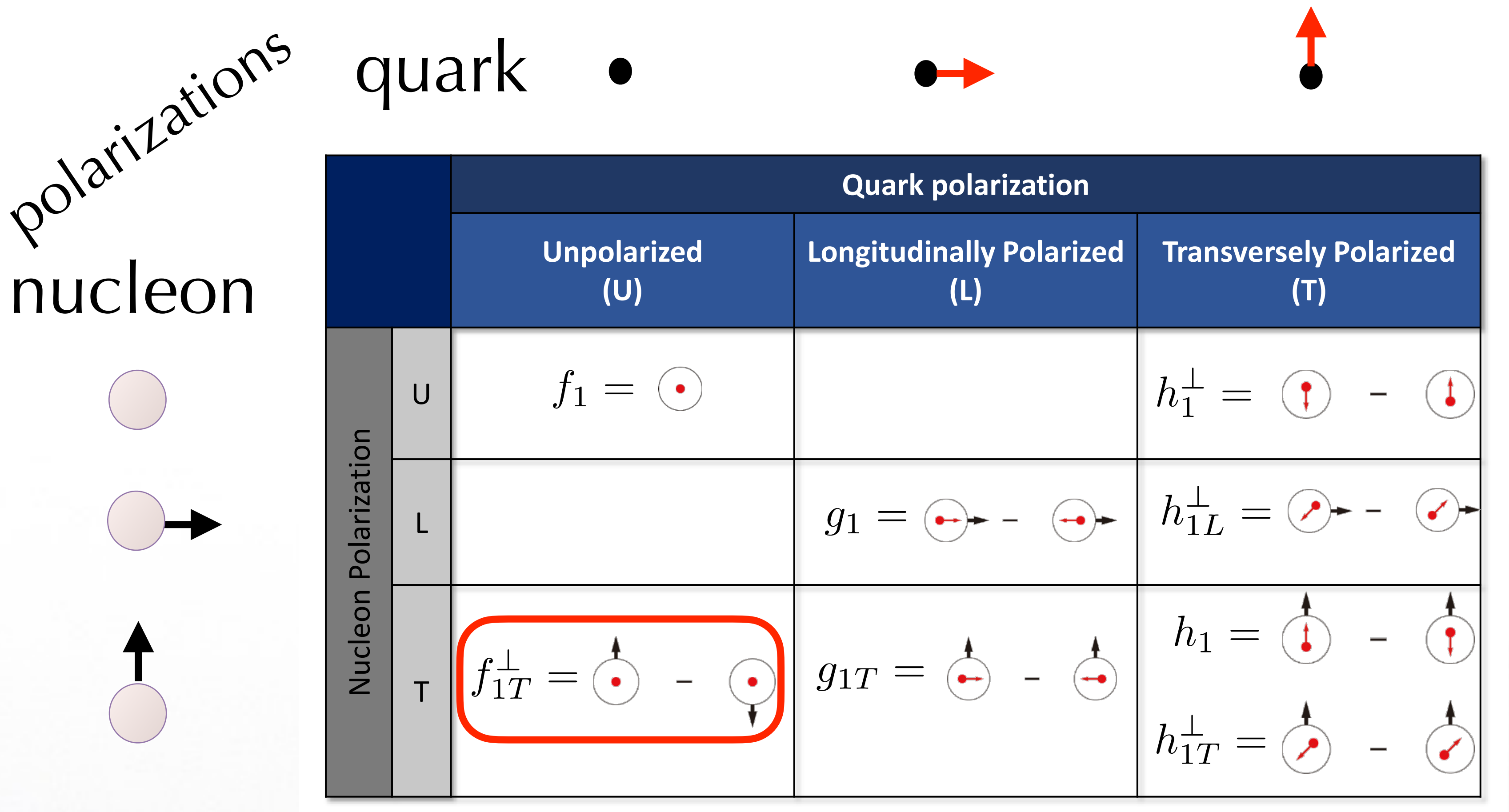
The unpolarized TMD PDF



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ -
	L		$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

$f_1^q(x, \mathbf{k}_\perp^2)$ probability density of finding a quark *q* with “longitudinal” (along “+” LC direction) fraction *x* of nucleon momentum, and transverse momentum $\mathbf{k}_\perp$

The Sivers TMD PDF



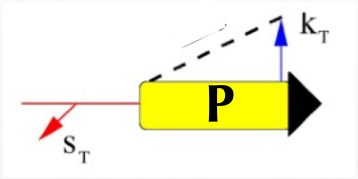
polarizations

quark

nucleon

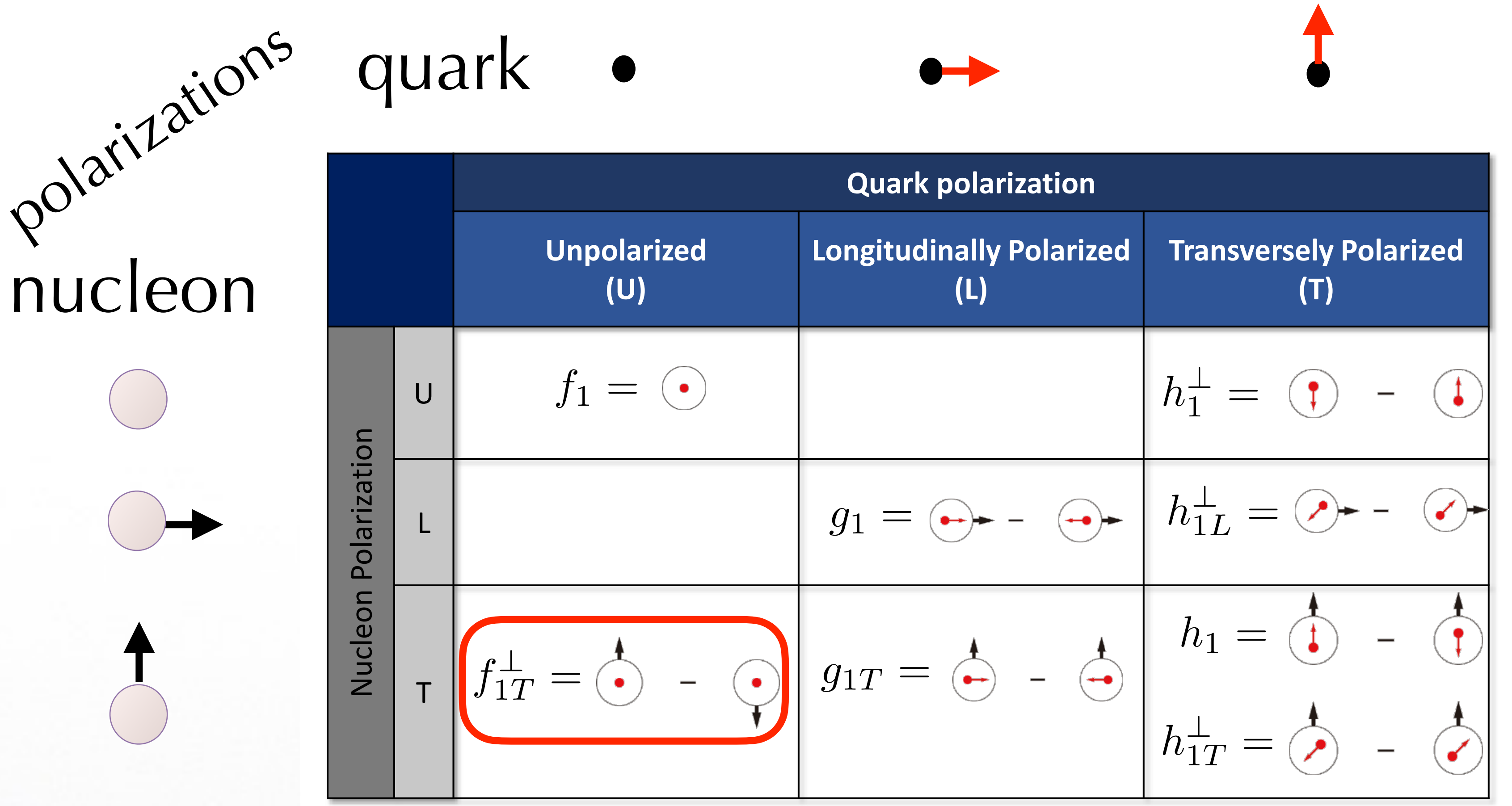
		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ -
	L		$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

$$\frac{1}{2}\mathrm{Tr}[\Phi\,\gamma_+] \to f_1 - f_{1T}^\perp \frac{(\mathbf{k}_\perp \times \mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}$$



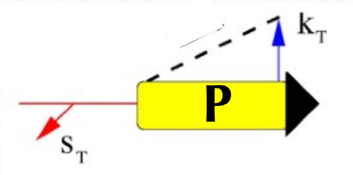
$\mathbf{S}_\mathrm{T} \cdot \mathbf{k}_\perp \times \mathbf{P}$

The Sivers TMD PDF

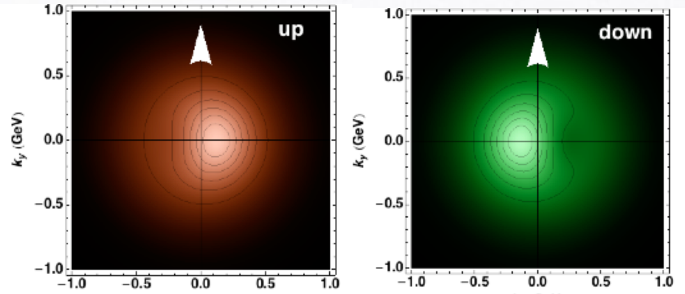


		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^{\perp} =$ -
	L		$g_1 =$ -	$h_{1L}^{\perp} =$ -
	T	$f_{1T}^{\perp} =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^{\perp} =$ -

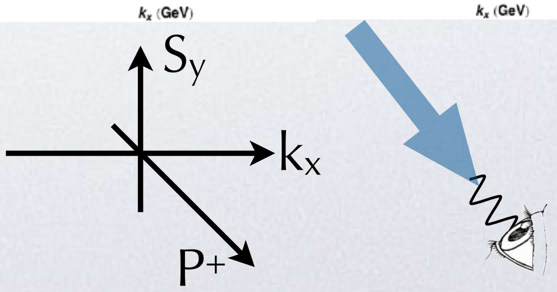
$$\frac{1}{2}\mathrm{Tr}[\Phi\,\gamma_+] \to f_1 - f_{1T}^{\perp}\,\frac{(\mathbf{k}_\perp \times \mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}$$



$\mathbf{S}_\mathrm{T} \cdot \mathbf{k}_\perp \times \mathbf{P}$

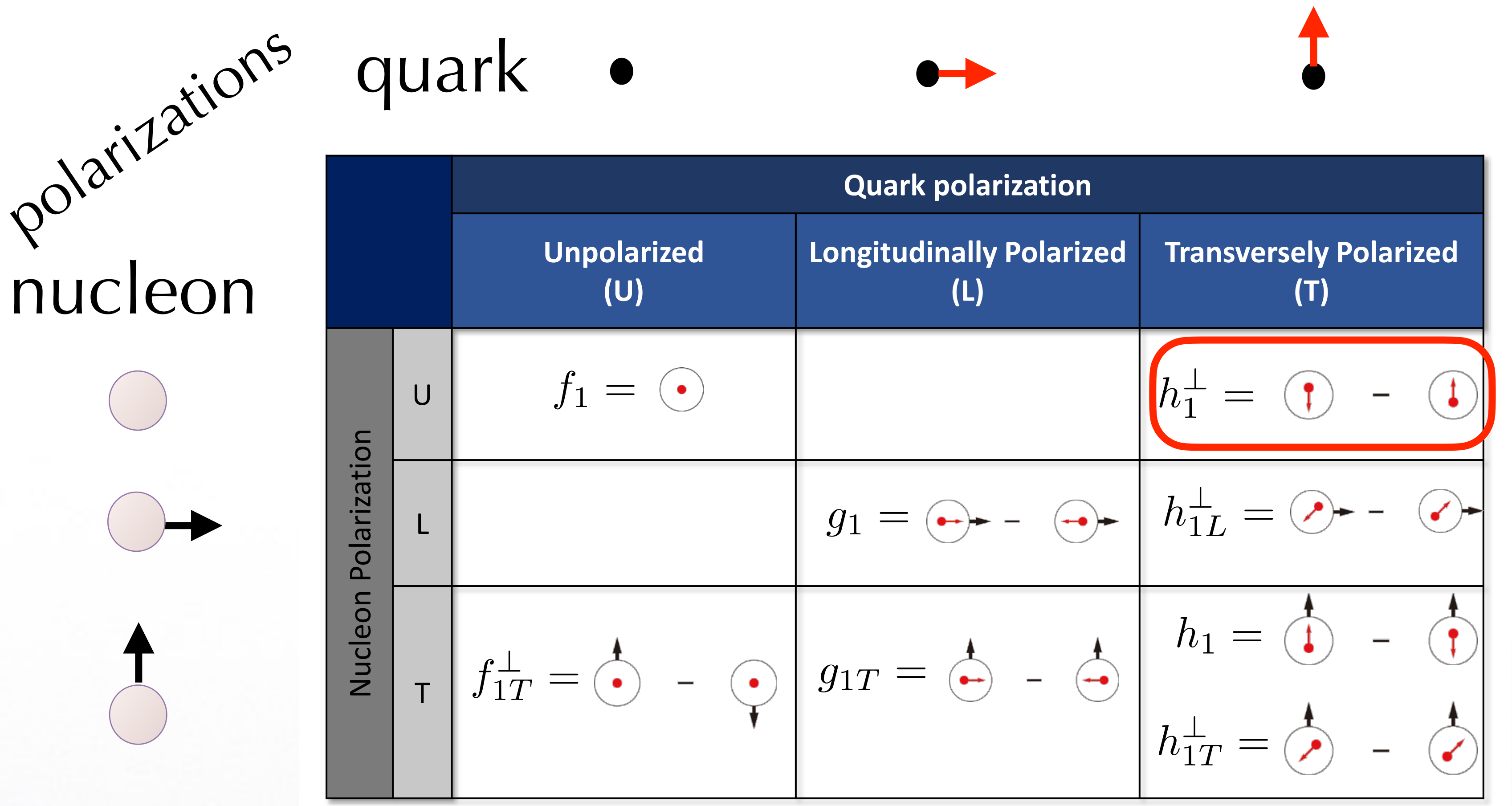


Sivers effect: how the momentum distribution of quarks is distorted by the transverse polarization of parent nucleon (“spin-orbit” correlation)



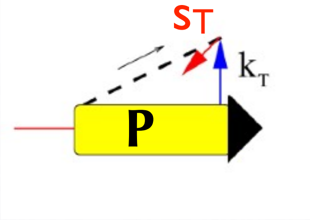
Sivers function $f_{1T}^{\perp}$ → access to quark orbital angular momentum

The Boer-Mulders TMD PDF



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		$h_1^\perp =$ ⊙ - ⊙
	L		$g_1 =$ ⊙ - ⊙	$h_{1L}^\perp =$ ⊙ - ⊙
	T	$f_{1T}^\perp =$ ⊙ - ⊙	$g_{1T} =$ ⊙ - ⊙	$h_1 =$ ⊙ - ⊙ $h_{1T}^\perp =$ ⊙ - ⊙

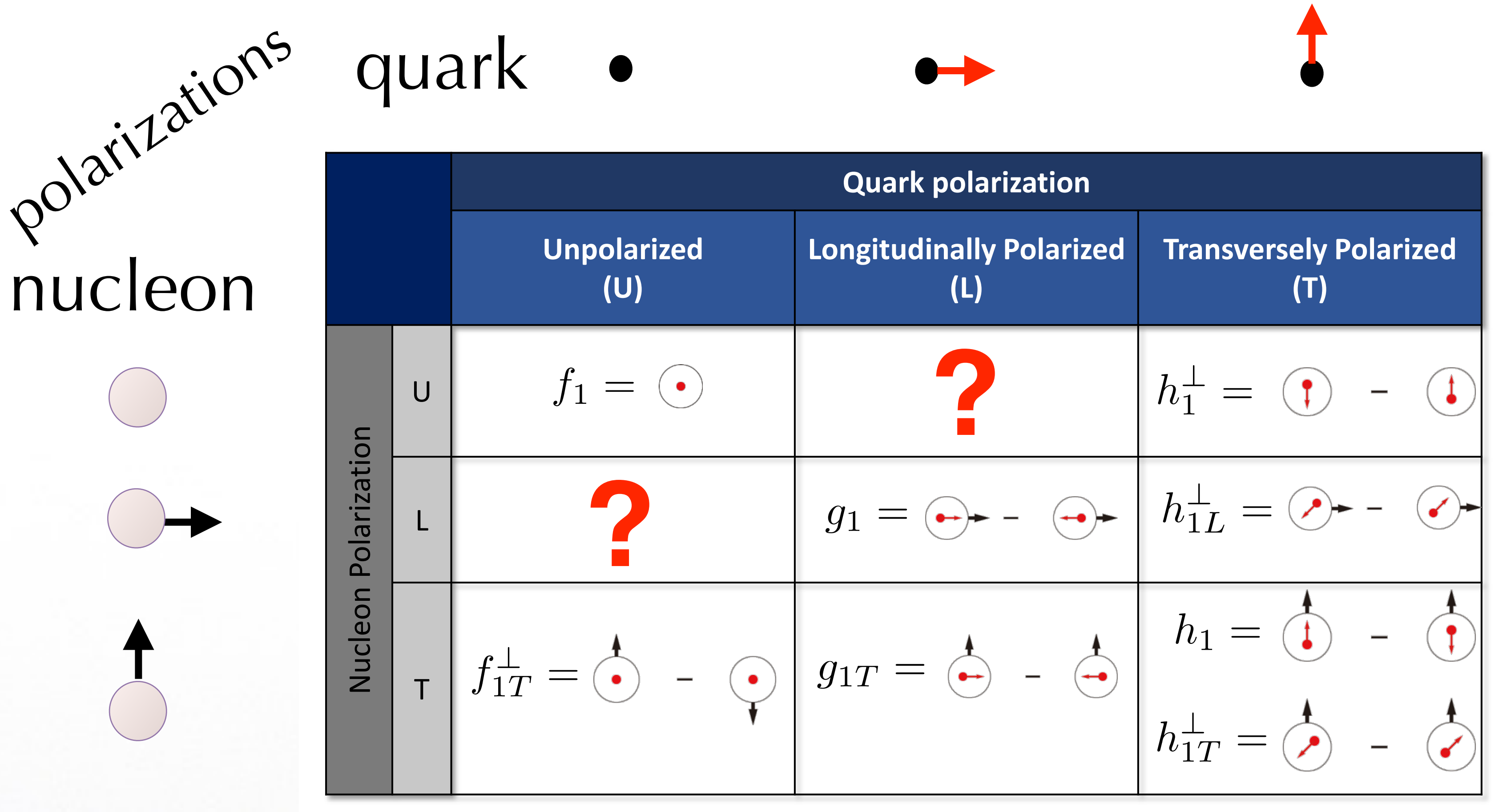
$$\frac{1}{2}\mathrm{Tr}[\Phi\, i\sigma_{+i}\gamma_5] \to \ \ldots + h_1^\perp \frac{(\mathbf{k}_\perp \times \mathbf{s}_T)\cdot\hat{\mathbf{P}}}{M}$$



$\mathbf{s}_T \cdot \mathbf{k}_\perp \times \mathbf{P}$

Boer-Mulders effect: “spin-orbit” correlation at partonic level

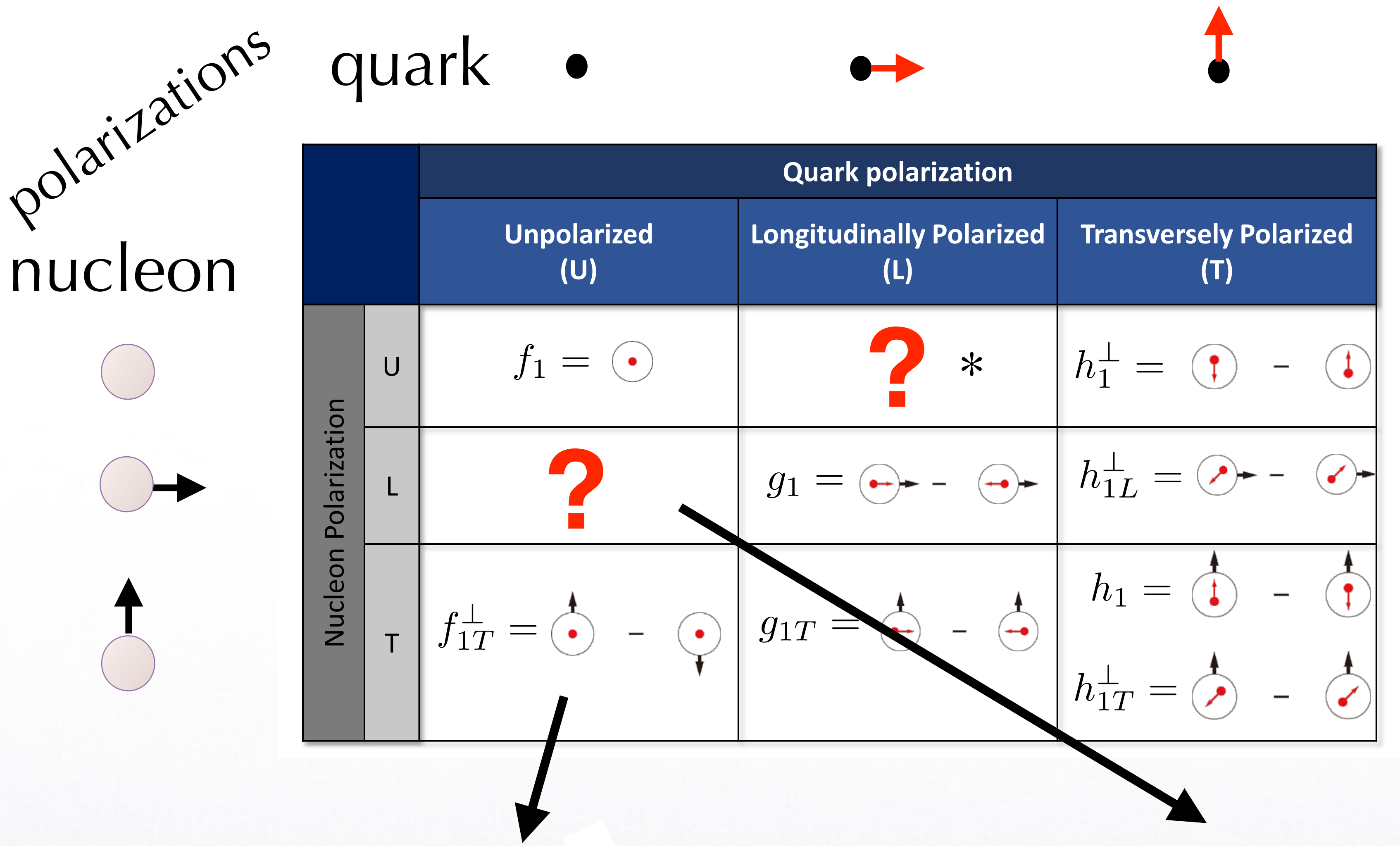
Forbidden combinations



		Quark polarization		
		Unpolarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
Nucleon Polarization	U	$f_1 =$	**?**	$h_1^{\perp} =$ -
	L	**?**	$g_1 =$ -	$h_{1L}^{\perp} =$ -
	T	$f_{1T}^{\perp} =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^{\perp} =$ -

Why?

Forbidden combinations



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$	**?** *	$h_1^\perp =$ -
	L	**?**	$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

Why?

prohibited by parity invariance

$\mathbf{S}_T \cdot \mathbf{k}_\perp \times \mathbf{P}$

$\mathbf{S}_L \cdot \mathbf{k}_\perp \times \mathbf{P} = 0$

not enough vectors for $f_{1L}^\perp$!

* similarly for "swapped" combination

T-odd TMD PDFs

Sivers and Boer-Mulders TMD PDFs vanish without gauge link *U*

$$\Phi_{ij}(x,\mathbf{k}_\perp,S) = \int \frac{d\xi_- d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(0)\, U_{[0,\xi]}\, \psi_i(\xi)\,|\,P,S\rangle_{\xi_+=0}$$

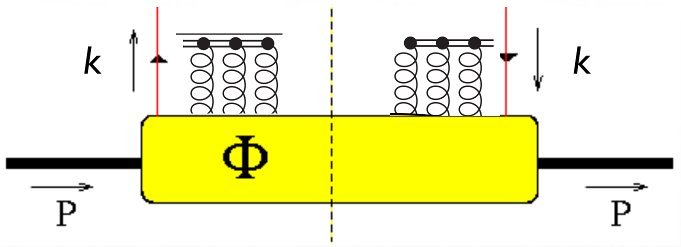
$$U_{[a,b]} = \mathscr{P}\,\exp\Big[-ig\int_a^b d\eta_\mu\, A^\mu(\eta)\Big]$$

They are generated by interference of different channels.

(for example, $f_{1T}^{\perp}$ can be reproduced by interference of model LC wave functions with different orbital angular momentum)

Gauge link *U* represents the residual color interactions that generate the necessary phase difference for the interference.

As such, time reversal puts no constraints on these structures.



Sivers and Boer-Mulders TMD PDFs are conventionally named "T-odd" TMD PDFs

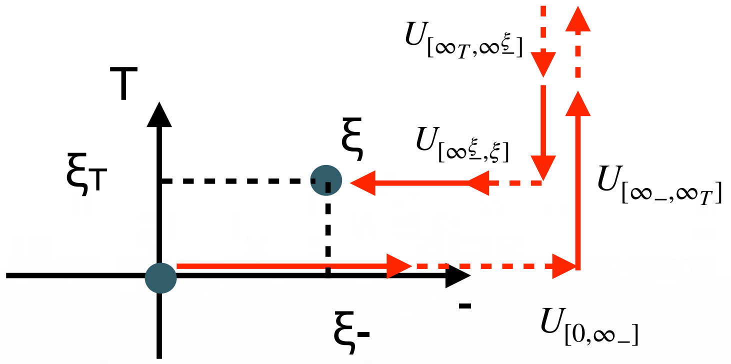
The gauge link

$$\Phi_{ij}(x,\mathbf{k}_\perp,S) = \int \frac{d\xi\_d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P,S|\bar\psi_j(0)\, U_{[0,\xi]}\, \psi_i(\xi)\,|P,S\rangle_{\xi_+=0}$$

TMD factorisation for SIDIS process suggests a trick similar to collinear framework case:

$$\langle P,S|\bar\psi(0)\,U_{[0,\xi]}\,\psi(\xi)\,|P,S\rangle = \langle P,S|\bar\psi(0)\,U_{[0,\infty_-]}\,U_{[\infty_-,\infty_T]}\,U_{[\infty_T,\infty^{\xi}_{-}]}\,U_{[\infty^{\xi}_{-},\xi]}\,\psi(\xi)\,|P,S\rangle$$

$$= \langle P,S|\,\{\bar\psi(0)\}\,\{\psi(\xi)\}\,|P,S\rangle$$



The gauge link

$$\Phi_{ij}(x,\mathbf{k}_\perp,S) = \int \frac{d\xi\_d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P,S\,|\,\bar\psi_j(0)\, U_{[0,\xi]}\, \psi_i(\xi)\,|\,P,S\rangle_{\xi_+=0}$$

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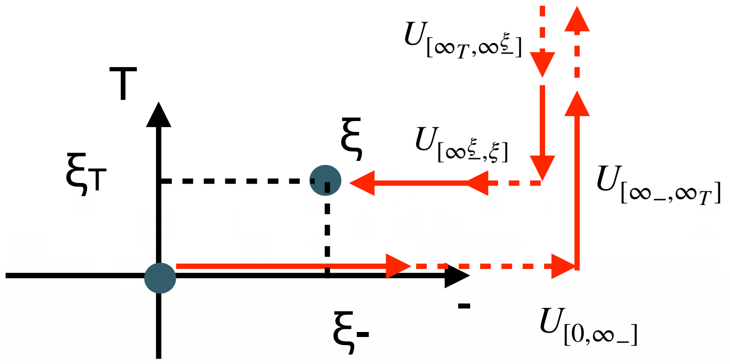
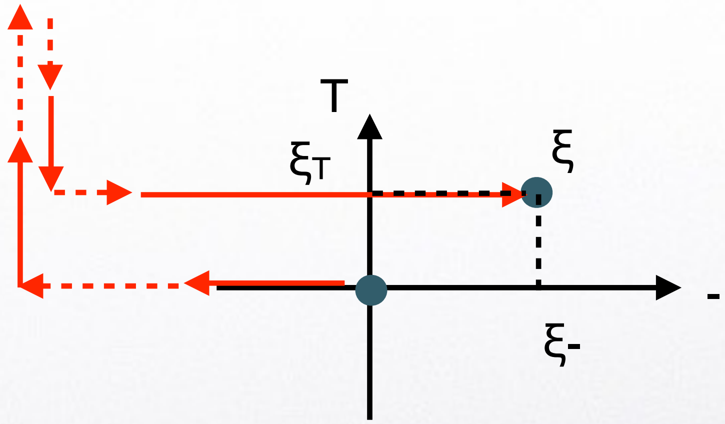
$$= \langle P,S\,|\,\{\bar\psi(0)\}\,\{\psi(\xi)\}\,|\,P,S\rangle$$



In Drell-Yan process, TMD factorisation gives the following path for gauge link:



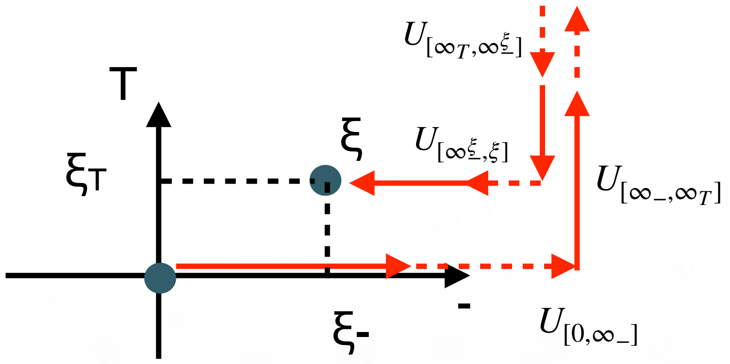
The gauge link

$$\Phi_{ij}(x,\mathbf{k}_\perp,S) = \int \frac{d\xi\_d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi} \langle P,S\,|\,\bar\psi_j(0)\, U_{[0,\xi]}\, \psi_i(\xi)\,|\,P,S\rangle_{\xi_+=0}$$

TMD factorisation for SIDIS process suggests a trick similar to collinear framework case:

$$\langle P,S\,|\,\bar\psi(0)\,U_{[0,\xi]}\,\psi(\xi)\,|\,P,S\rangle = \langle P,S\,|\,\bar\psi(0)\,U_{[0,\infty_-]}\,U_{[\infty_-,\infty_T]}\,U_{[\infty_T,\infty^{\xi}_{-}]}\,U_{[\infty^{\xi}_{-},\xi]}\,\psi(\xi)\,|\,P,S\rangle$$

$$= \langle P,S\,|\,\{\bar\psi(0)\}\,\{\psi(\xi)\}\,|\,P,S\rangle$$



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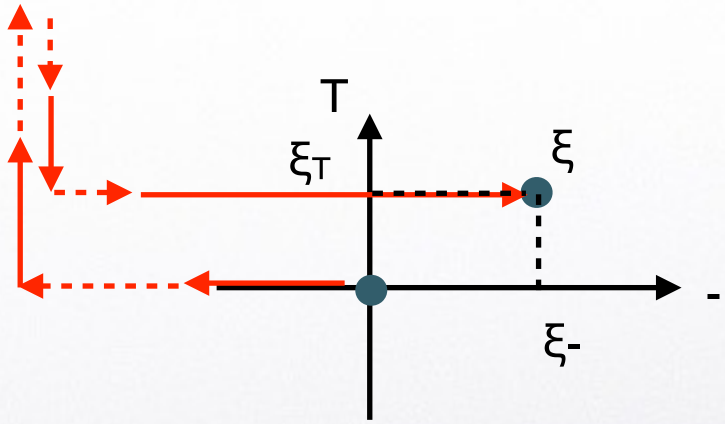


Notations: gauge link $U_{[+]}$ for SIDIS; $U_{[-]}$ for Drell-Yan

Important result: T-even TMD PDF$_{[+]}$ = TMD PDF$_{[-]}$
T-odd TMD PDF$_{[+]}$ = - TMD PDF$_{[-]}$

← breaking universality!
(but in a calculable way)

Process dependence

Sivers $f_{1T}^{\perp\,[+]} = -\,f_{1T}^{\perp\,[-]}$

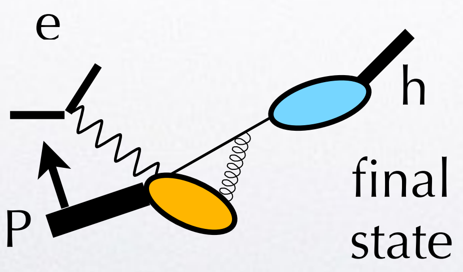
Boer-Mulders $h_1^{\perp\,[+]} = -\,h_1^{\perp\,[-]}$

SIDIS Drell-Yan

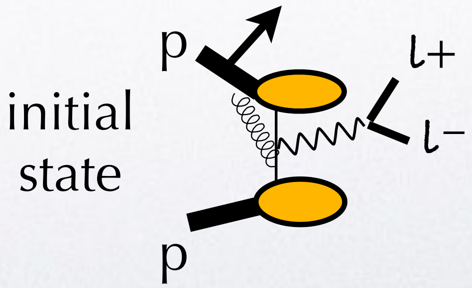
Prediction of QCD based on interplay between time-reversal and (color) gauge symmetry

Intense experimental work to test this prediction (see next lecture)

Intuition: in SIDIS, gauge link $U_{[+]}$ describes color final-state interactions
in Drell-Yan, gauge link $U_{[-]}$ describes color initial-state interactions

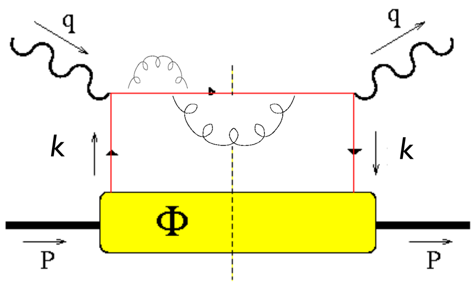


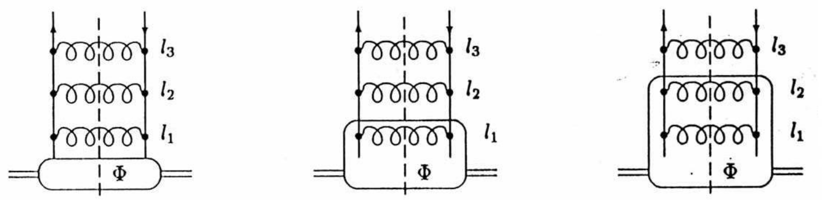
SIDIS



Drell-Yan

More on factorisation → evolution





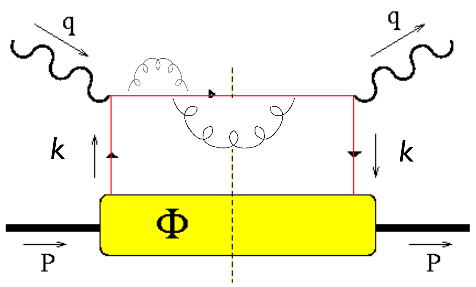
inclusive DIS: QCD corrections generate soft and collinear divergences

sum of real and virtual diagrams cancel soft divergences

collinear divergences reabsorbed in collinear PDFs

factorisation scale μ determines what is perturbative (calculable) from what is non perturbative (inside PDFs)
→ scale dependence given by DGLAP evolution eq.'s

More on factorisation → evolution



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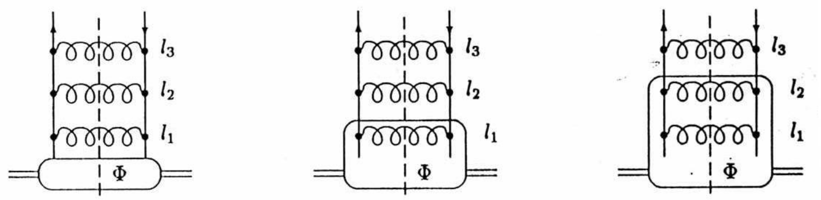

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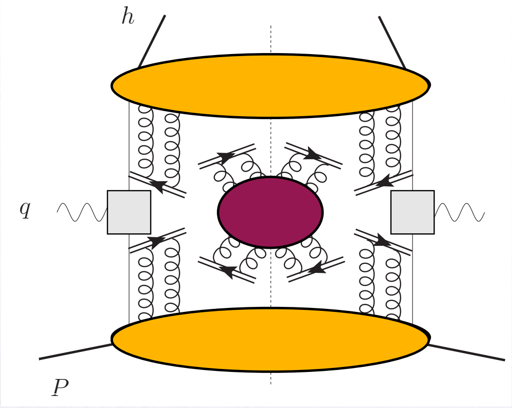
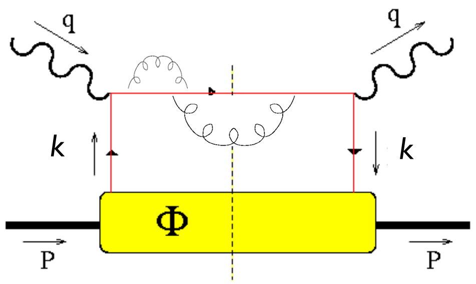
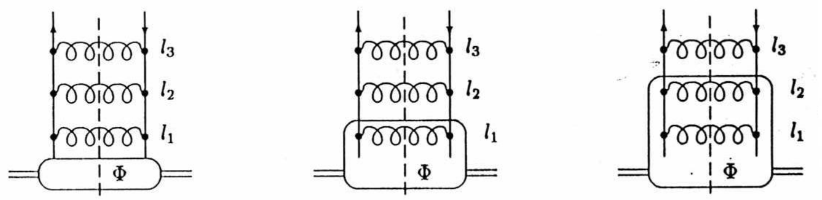


SIDIS: soft divergences do not cancel anymore
new class of light-cone (rapidity) divergences

need to introduce a **soft factor** convoluted with TMD PDFs and FFs

More on factorisation → evolution





inclusive DIS: QCD corrections generate soft and collinear divergences

sum of real and virtual diagrams cancel soft divergences

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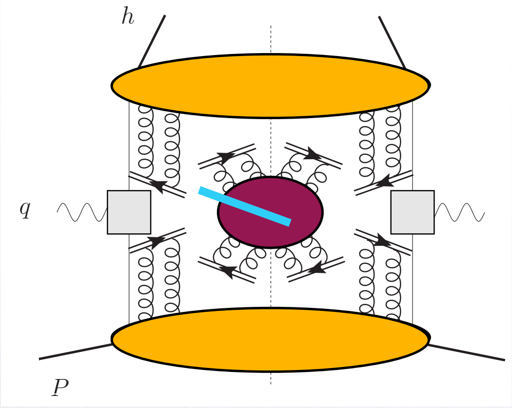


SIDIS: soft divergences do not cancel anymore
new class of light-cone (rapidity) divergences

need to introduce a **soft factor** convoluted with TMD PDFs and FFs

need to introduce a new **"rapidity scale"** ζ that regulates the rapidity divergences and splits soft factor content between TMD PDFs and FFs → new scale dependence

DGLAP eq.'s $$\frac{d\log \text{TMD}}{d\log\mu} = \gamma_D(\mu,\zeta)$$

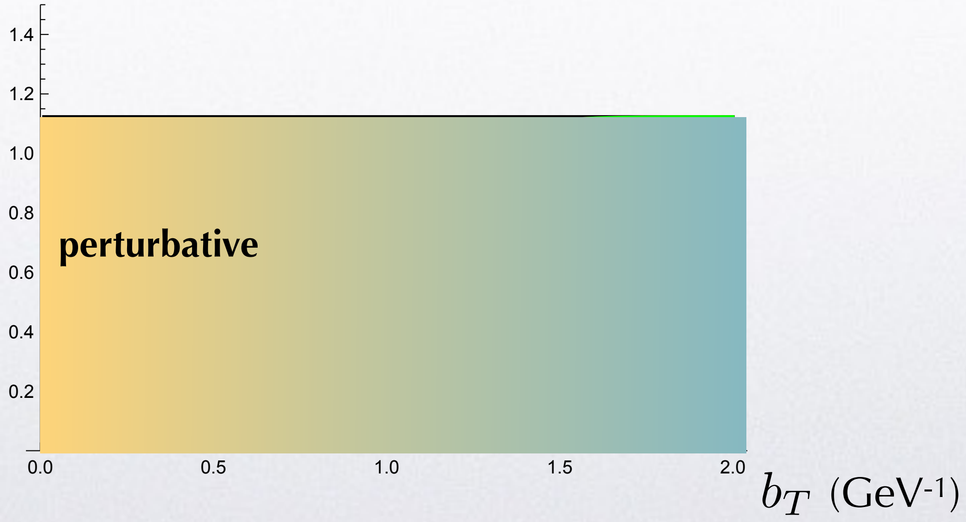
CSS eq.'s $$\frac{d\log \text{TMD}}{d\log\sqrt{\zeta}} = K(\mu)$$

More on factorisation → evolution

TMD evolution from initial (μ₀,ζ₀) scales is better studied in position space b_T (↔ k⊥)

For $b_T \ll 1/\Lambda_{QCD}$ perturbation theory is valid

$$f_1^q(x, b_T^2; \mu, \zeta) = \quad \text{Evo}\Big[(\mu, \zeta) \leftarrow (\mu_0, \zeta_0)\Big] \quad f_1^q(x, b_T^2; \mu_0, \zeta_0)$$



More on factorisation → evolution

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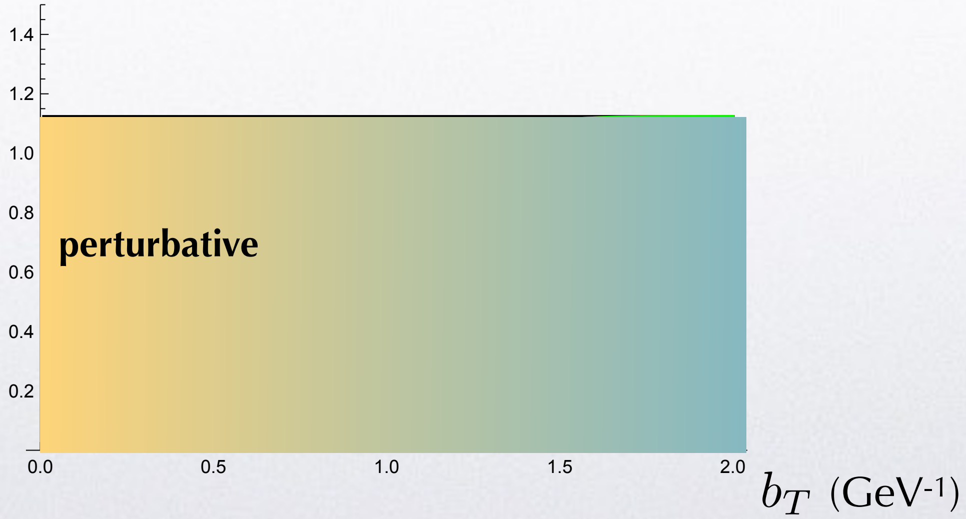
DGLAP+CSS eqs. ↓

$$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu,\zeta) + K(\mu_0) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\right]$$



More on factorisation → evolution

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For $b_T \ll 1/\Lambda_{QCD}$ perturbation theory is valid

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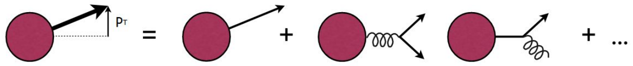
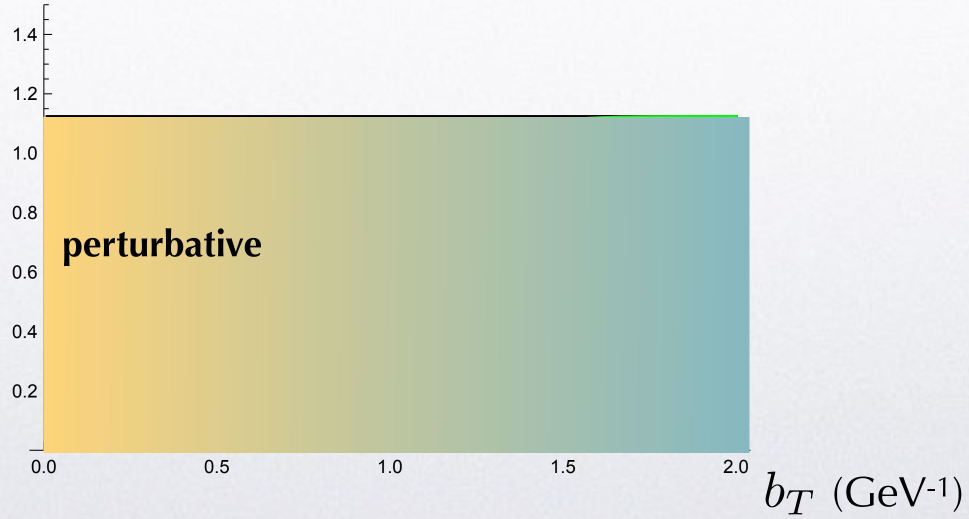
$$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu,\zeta) + K(\mu_0) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\right]$$

↓ OPE on PDFs

$$= \sum_i \left[C_{q\to i}(x, b_T^2; \mu_0, \zeta_0) \otimes f_1^i(x, \mu_0)\right]$$

small b_T (large k_T) from perturbative splitting





More on factorisation → evolution

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DGLAP+CSS eqs. ↓

$$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu,\zeta) + K(\mu_0) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\right]$$

↓

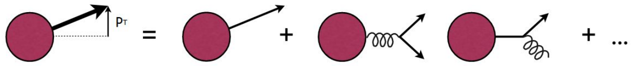
$$K \to K + g_{NP}(b_T)$$

↓ OPE on PDFs

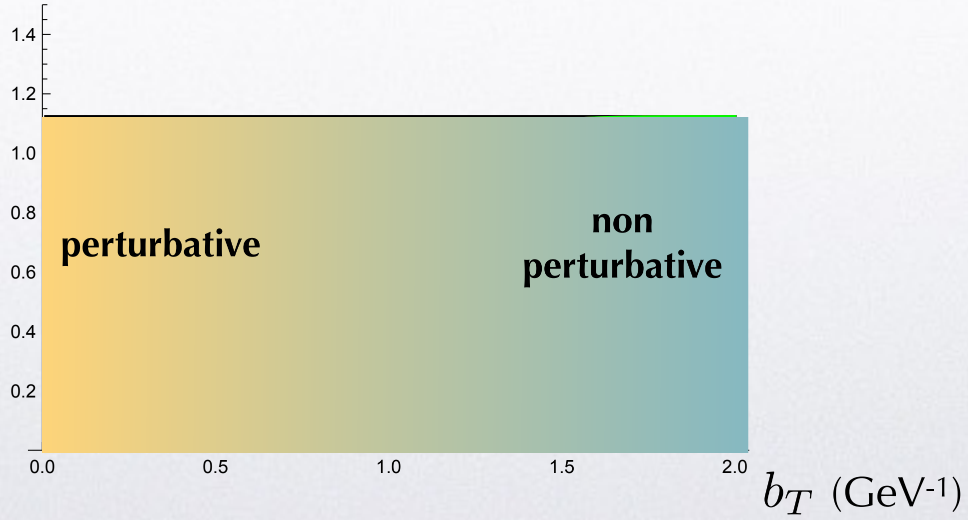
$$= \sum_i \left[C_{q\to i}(x, b_T^2; \mu_0, \zeta_0) \otimes f_1^i(x, \mu_0)\right]$$

$$\times F_{NP}(b_T)$$

small b_T (large k_T) from perturbative splitting



For large b_T perturbation th. breaks down



More on factorisation → evolution

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$$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu, \zeta) + K(\mu_0) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\right]$$

↓

$$K \to K + g_{NP}(b_T)$$

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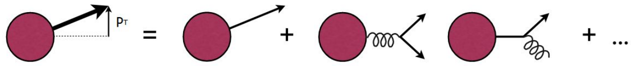


For large b_T perturbation th. breaks down

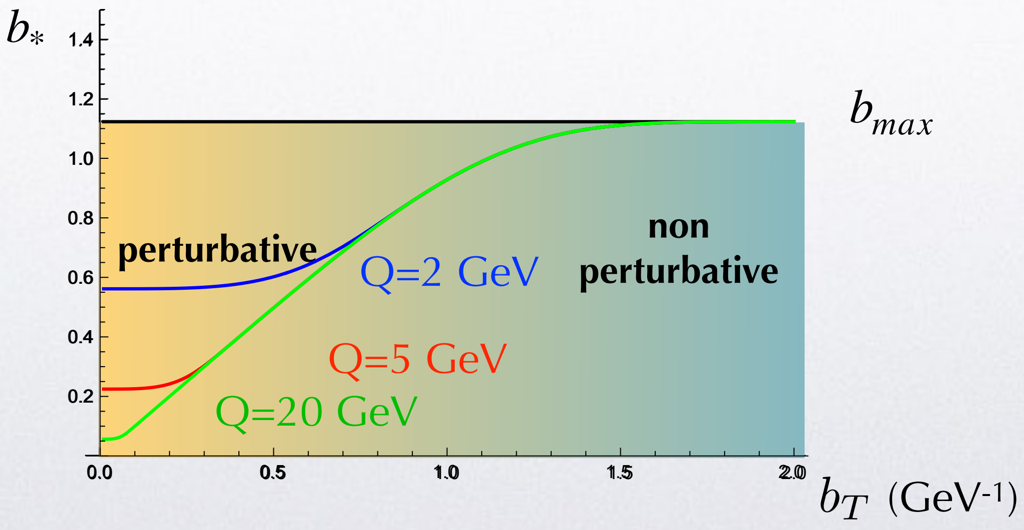
Choice of matching scale

$$\mu = \sqrt{\zeta} = Q$$

$$\mu_0 = \sqrt{\zeta_0} = \mu_b = \frac{2e^{-\gamma_E}}{b_*(b_T)}$$

$$b_{max} = 2e^{-\gamma_E}$$

$$b_{min} = \frac{2e^{-\gamma_E}}{Q}$$



More on factorisation → evolution

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DGLAP+CSS eqs. ↓

$$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu, \zeta) + K(\mu_0) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\right]$$

↓

$$K \to K + g_{NP}(b_T)$$

For large b_T perturbation th. breaks down

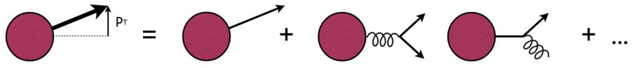
$$f_1^q(x, b_T^2; \mu_0, \zeta_0)$$

↓ OPE on PDFs

$$= \sum_i \left[C_{q \to i}(x, b_T^2; \mu_0, \zeta_0) \otimes f_1^i(x, \mu_0)\right]$$

$$\times F_{NP}(b_T)$$

small b_T (large k_T) from perturbative splitting



Final formula $f_1^q(x, b_T^2; Q^2) = \exp\left[\int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \gamma_D(Q) + K(\mu_b) \log(Q/\mu_b) + g_{NP}(b_T) \log(Q/Q_0)\right] \sum_i \left[C_{q \to i} \otimes f_1^i\right](x, b_T, \mu_b)\, F_{NP}(b_T, Q_0)$

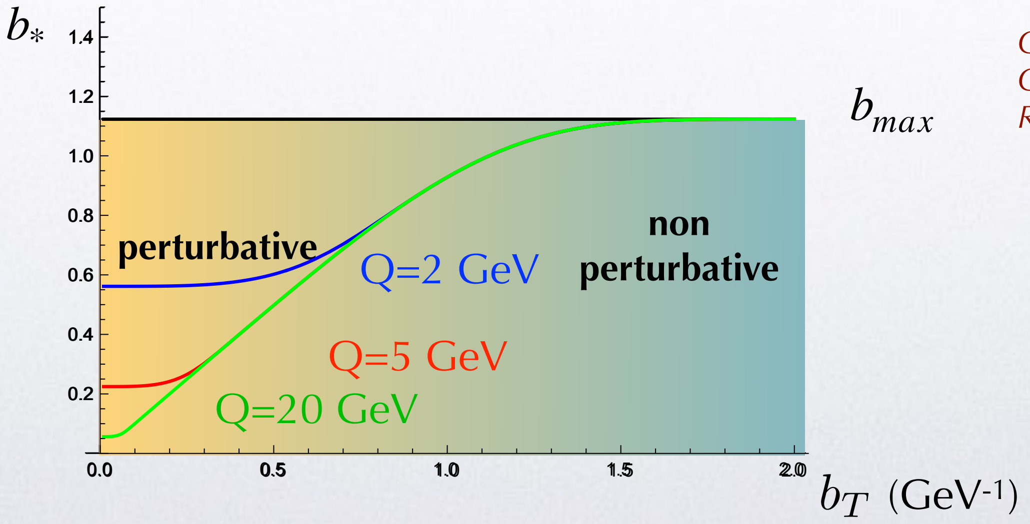
Choice of matching scale

$$\mu = \sqrt{\zeta} = Q$$

$$\mu_0 = \sqrt{\zeta_0} = \mu_b = \frac{2e^{-\gamma_E}}{b_*(b_T)}$$

$$b_{max} = 2e^{-\gamma_E}$$

$$b_{min} = \frac{2e^{-\gamma_E}}{Q}$$



Collins, Soper, Sterman, N.P. ***B250*** *(85)*
Collins, "Foundations of Perturbative QCD" (2011)
Rogers and Aybat, P.R. ***D83*** *(11)*

More on factorisation → evolution

others schemes possible:

Laenen, Sterman Vogelsang, P.R.L. ***84*** *(00)*
Bozzi et al., N.P. ***B737*** *(06)*
Echevarria et al., E.P.J. ***C73*** *(13) …*

CSS evolution formula for TMD

$$f_1^q(x, b_T^2; Q^2) = \exp\left[\int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\,\gamma_D(Q) + K(\mu_b)\log(Q/\mu_b) + g_{NP}(b_T)\log(Q/Q_0)\right] \sum_i \left[C_{q\to i} \otimes f_1^i\right](x, b_T, \mu_b)\, F_{NP}(b_T, Q_0)$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_*(b_T)}$$

More on factorisation → evolution

others schemes possible:

Laenen, Sterman Vogelsang, P.R.L. ***84*** *(00)*
Bozzi et al., N.P. ***B737*** *(06)*
Echevarria et al., E.P.J. ***C73*** *(13) …*

CSS evolution formula for TMD

$$f_1^q(x, b_T^2; Q^2) = \exp\left[\int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\gamma_D(Q) + K(\mu_b)\log(Q/\mu_b) + g_{NP}(b_T)\log(Q/Q_0)\right] \sum_i \left[C_{q\to i} \otimes f_1^i\right](x, b_T, \mu_b) F_{NP}(b_T, Q_0)$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_*(b_T)}$$

arbitrariness of nonperturbative components

- choice of $b_*(b_T)$ functional form
- choice of $g_{NP}(b_T)$ functional form
- choice of $F_{NP}(b_T,Q_0)$ functional form

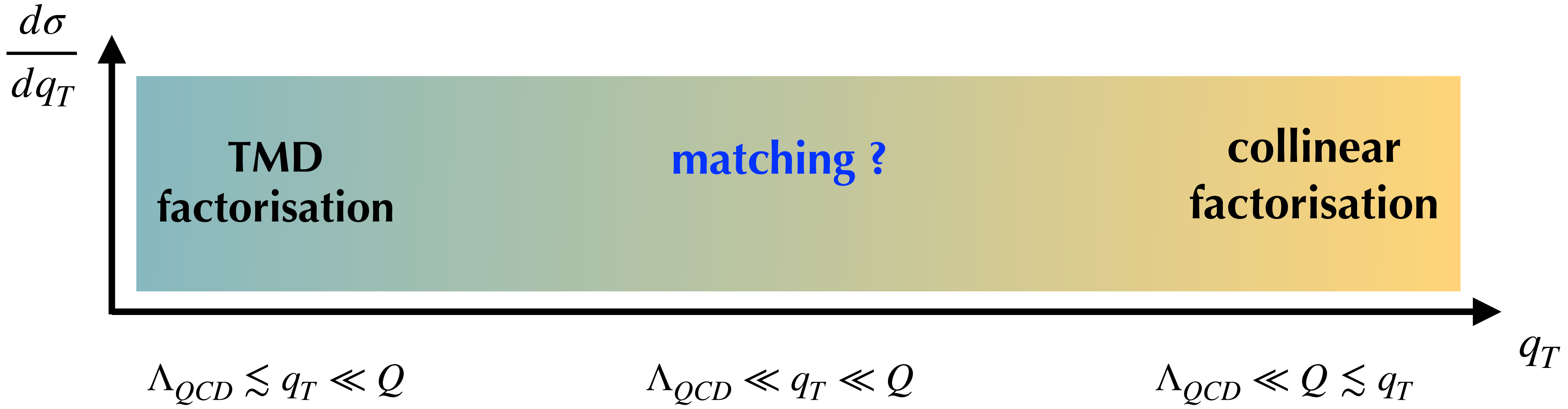
each one affects evolution: how $k_\perp$-distribution changes with scale

→ source of theoretical bias/uncertainty

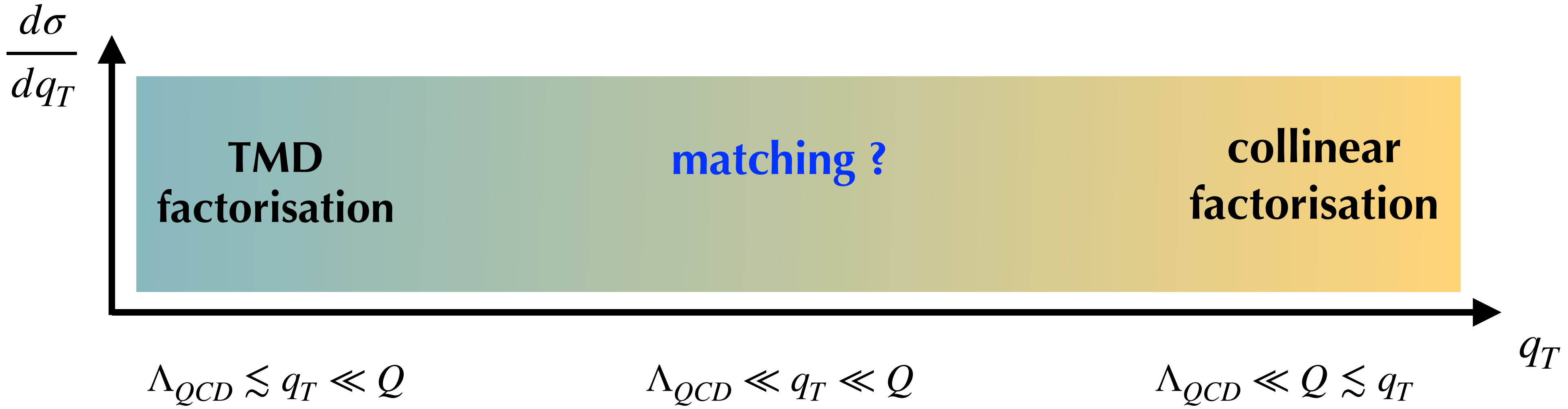
need to be constrained by experimental data with large lever arm in Q^2

EIC is the suitable machine for that

Matching problem



Matching problem



$\frac{d\sigma}{dq_T}$

TMD factorisation

matching ?

collinear factorisation

q_T

$\Lambda_{QCD} \lesssim q_T \ll Q$

$\Lambda_{QCD} \ll q_T \ll Q$

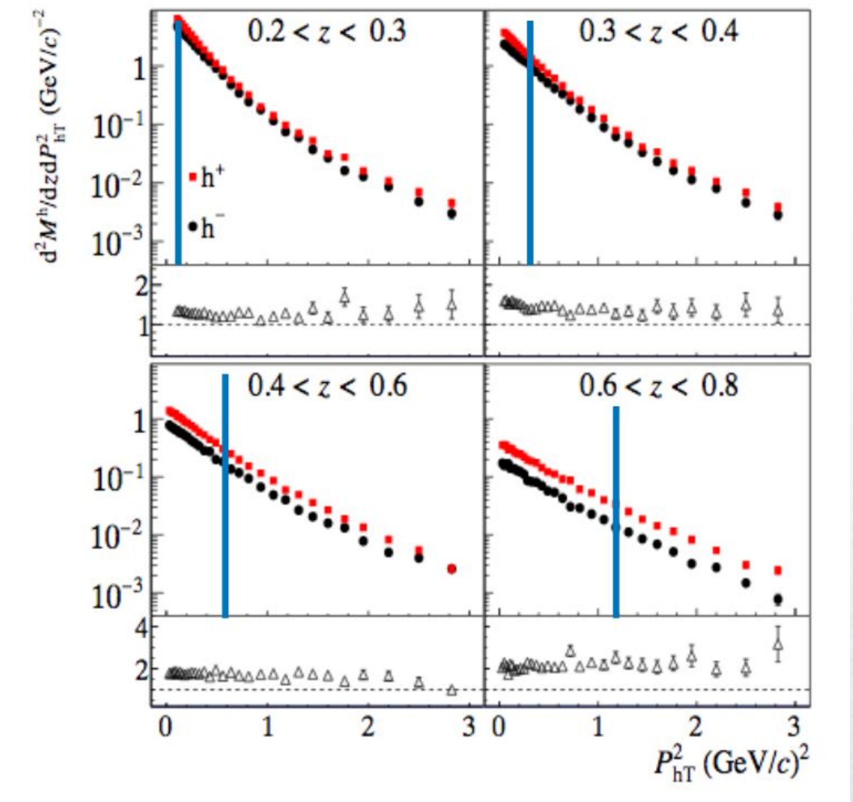
$\Lambda_{QCD} \ll Q \lesssim q_T$

relevant also for phenomenological analysis

Example:
COMPASS unpolarized SIDIS multiplicity
bin $\langle Q^2 \rangle = 9.78$ GeV² , $\langle x \rangle = 0.149$

TMD factorisation valid for $q_T^2 = \frac{P_{hT}^2}{z^2} \ll Q^2$

highlight in picture the $\frac{P_{hT}^2}{z^2} = 0.25\, Q^2$

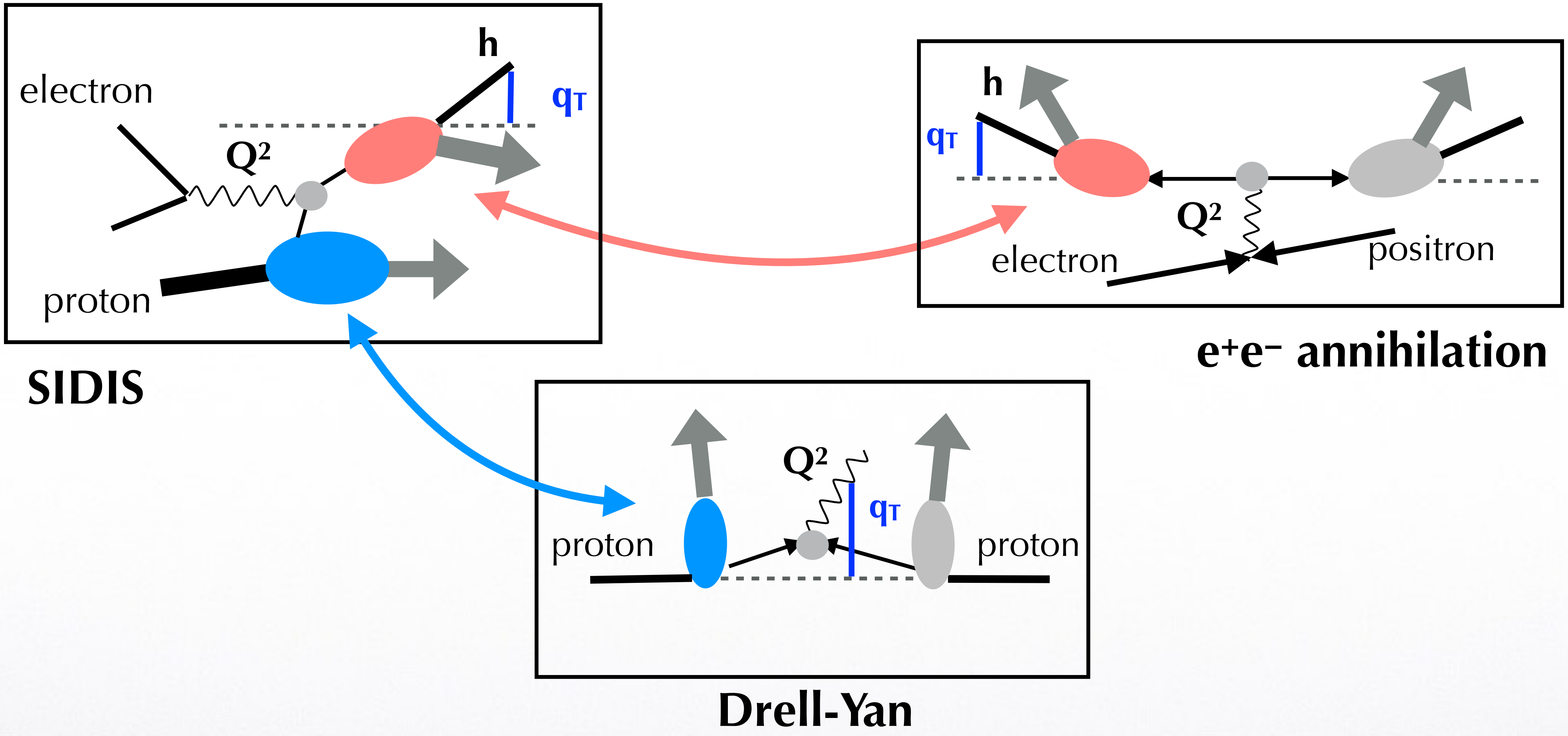


COMPASS, arXiv:1709.07374

Outline

- Where to find TMDs
  - structure functions for various processes
  - prominent examples of phenomenological extractions of TMDs
  - perspectives with the EIC

TMDs : factorisation theorems

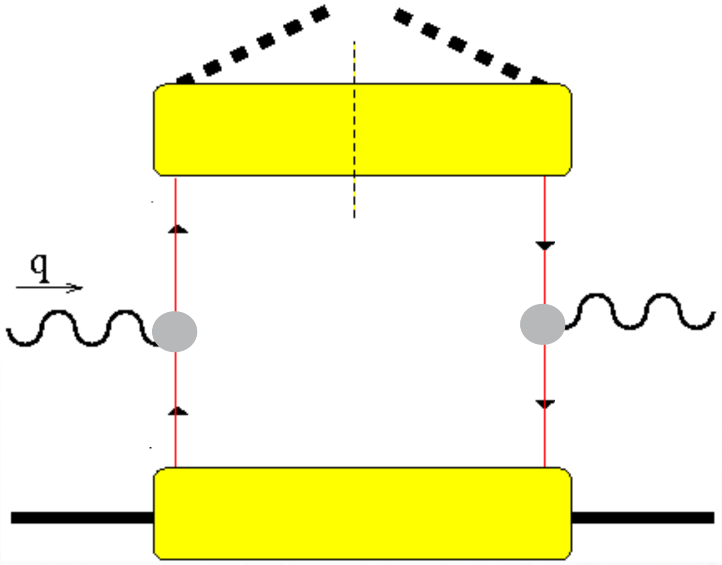


In order to extract information on TMD PDFs and TMD FFs, it is desirable to perform global fits, but this is not yet a standard (also because, for example, very few data on polarized Drell-Yan are currently available)

TMDs : link to structure functions

Example: SIDIS

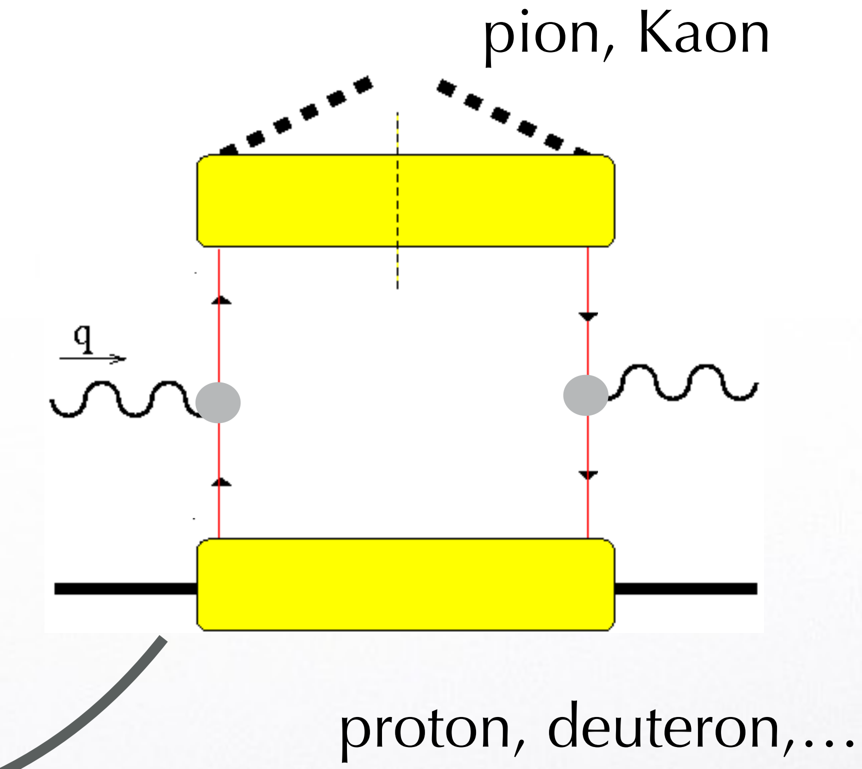
pion, Kaon

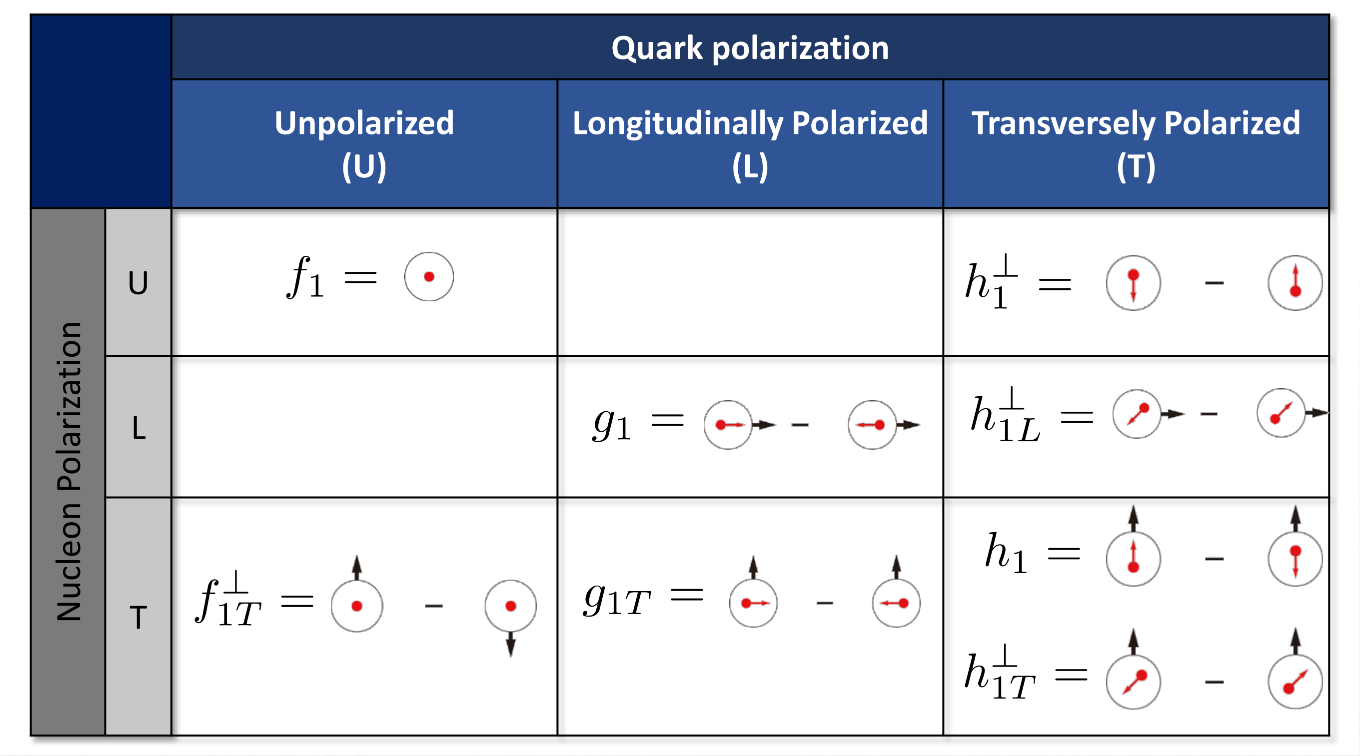


proton, deuteron,…

TMDs : link to structure functions

Example: SIDIS

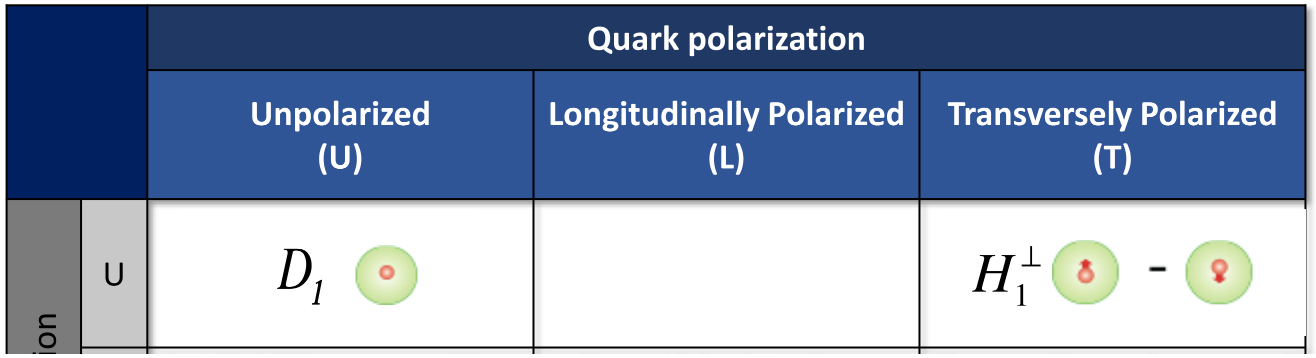




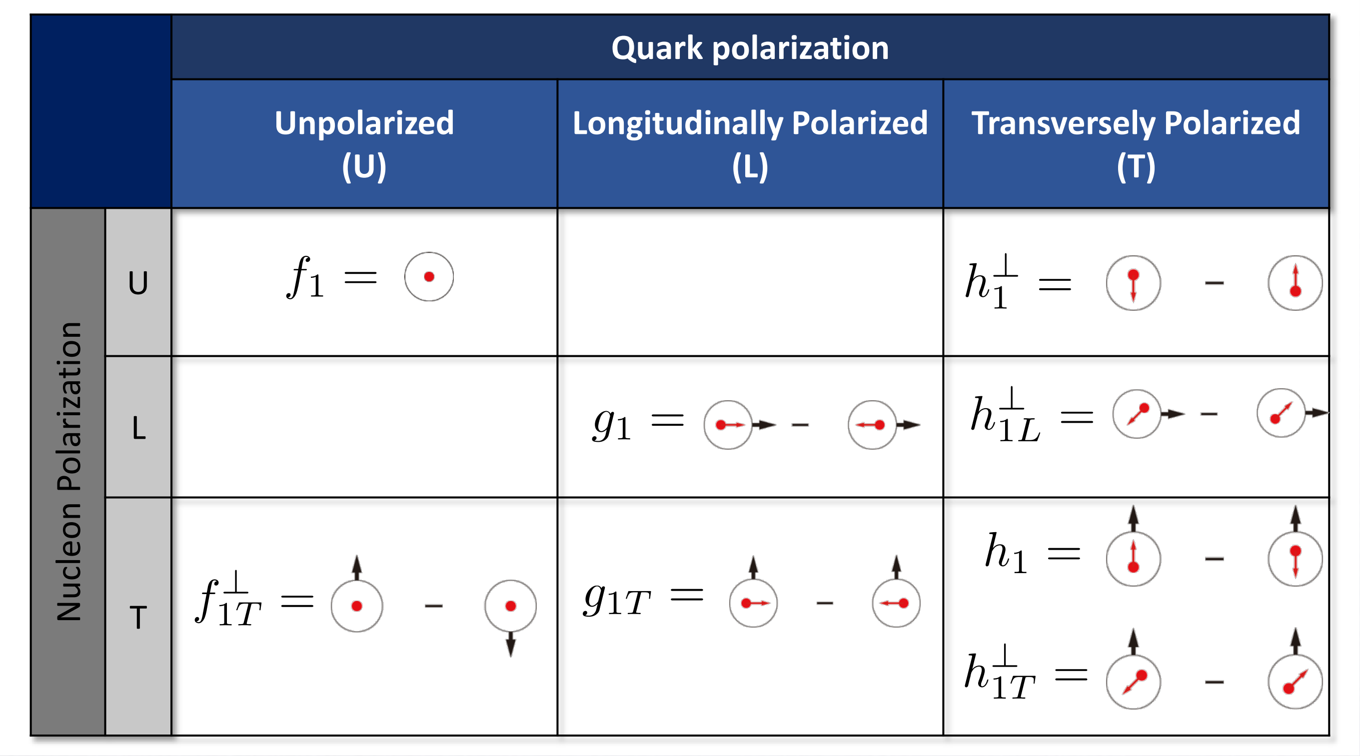
		Quark polarization		
		Unpolarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
Nucleon Polarization	U	$f_1 =$		$h_1^{\perp} =$ -
	L		$g_1 =$ -	$h_{1L}^{\perp} =$ -
	T	$f_{1T}^{\perp} =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^{\perp} =$ -

TMDs : link to structure functions

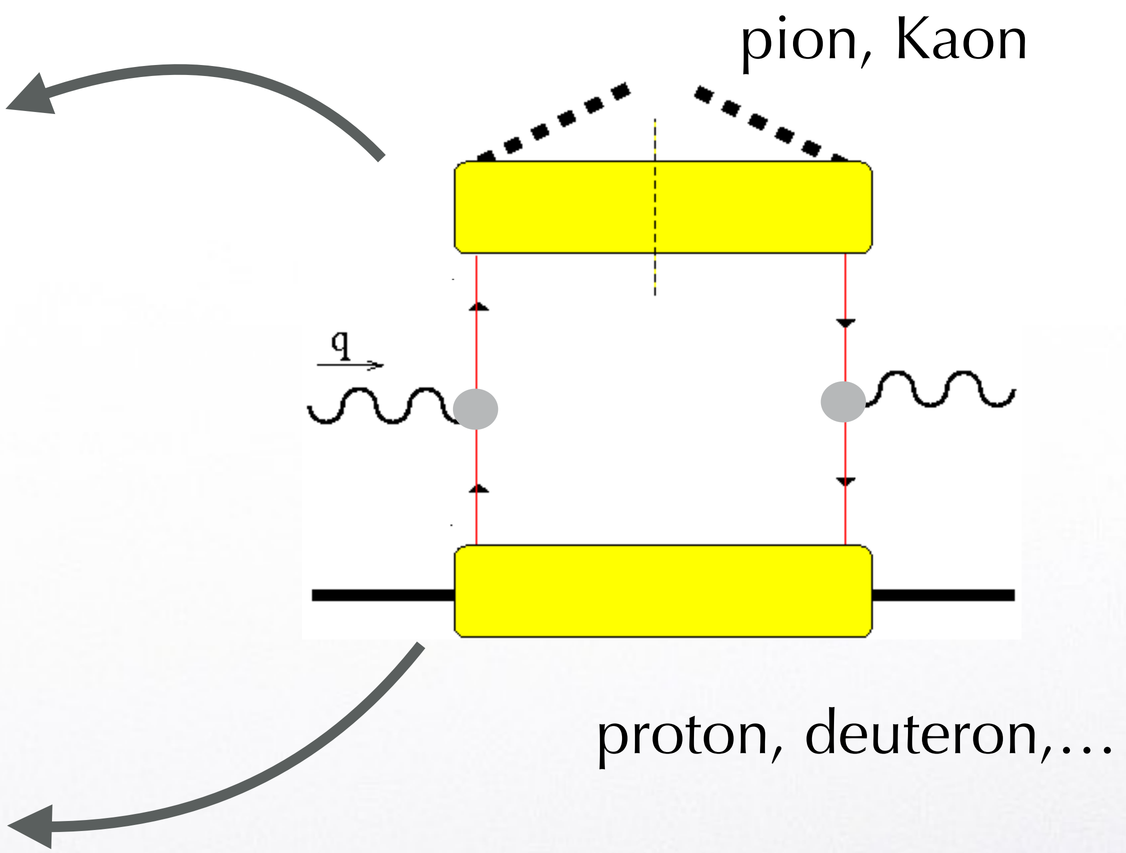
Example: SIDIS



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
ion	U	D_1 ⊙		$H_1^{\perp}$ ⊙ - ⊙



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		$h_1^{\perp} =$ ⊙ - ⊙
	L		$g_1 =$ ⊙ - ⊙	$h_{1L}^{\perp} =$ ⊙ - ⊙
	T	$f_{1T}^{\perp} =$ ⊙ - ⊙	$g_{1T} =$ ⊙ - ⊙	$h_1 =$ ⊙ - ⊙ $h_{1T}^{\perp} =$ ⊙ - ⊙

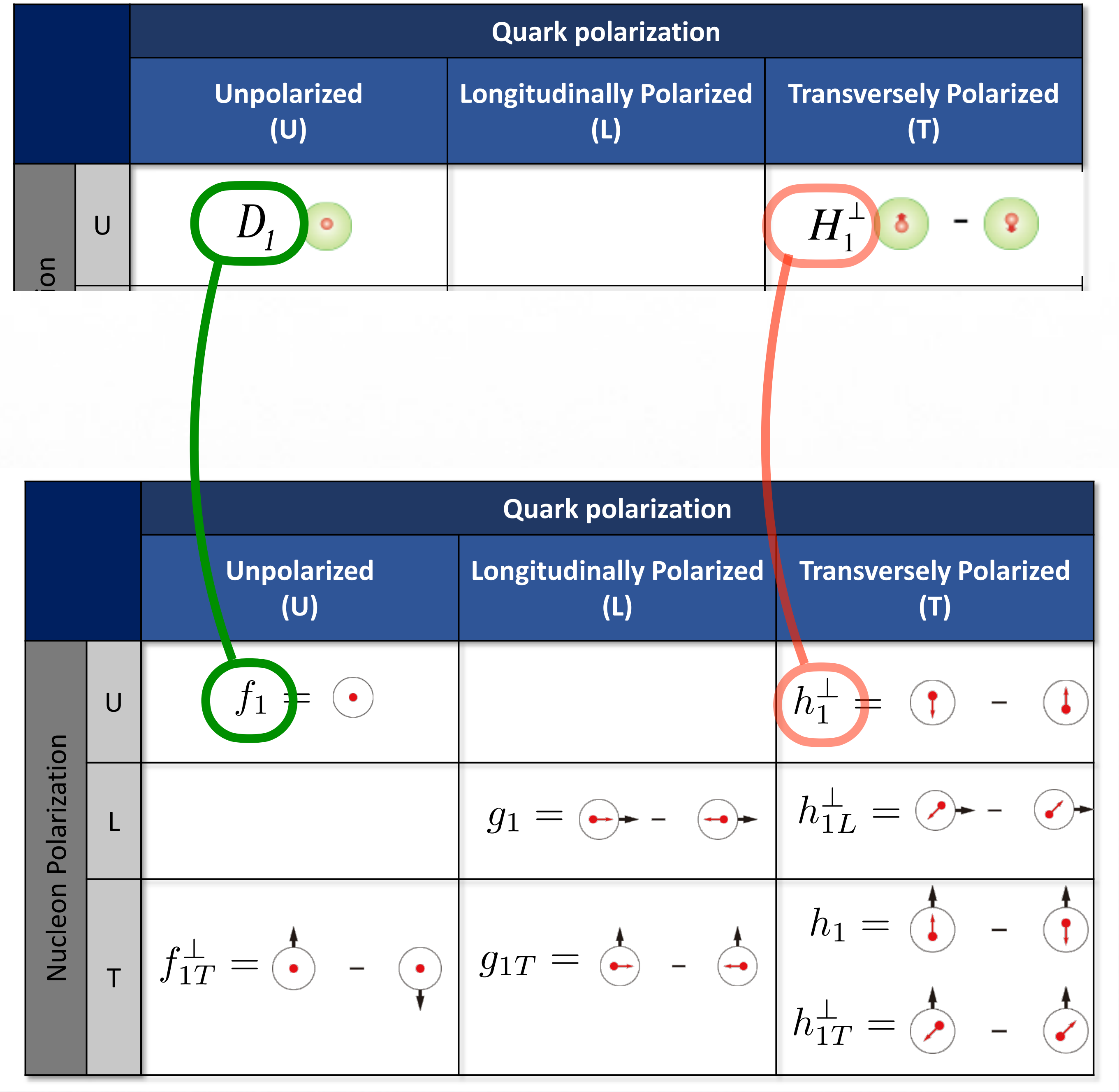


each structure function ~

$$F \sim d\hat{\sigma}(Q^2)\,\mathscr{C}\left[\mathrm{TMDPDF}(x,\mathbf{k}_\perp^2)\,,\mathrm{TMDFF}(z,\mathbf{P}_\perp^2)\right]$$

$$\mathscr{C}[\ldots] = \int d\mathbf{P}_\perp d\mathbf{k}_\perp\, \delta^{(2)}(z\mathbf{k}_\perp + \mathbf{P}_\perp - \mathbf{P}_{hT})\left[\ldots\right]$$

TMDs : link to structure functions



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
ion	U	D_1		$H_1^{\perp}$ -

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Example: SIDIS

target polariz.

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{hT}^2} \sim$$

○

$$A(y)\,F_U \quad + \quad B(y)\,\cos 2\phi_h\,F_U^{\cos 2\phi_h}$$

$$+\,C(y)\,F_{LL} \quad + \quad B(y)\,\sin 2\phi_h\,F_L^{\sin 2\phi_h}$$

$$+\;A(y)\,\sin(\phi_h-\phi_S)\,F_T^{\sin(\phi_h-\phi_S)}$$

$$+\;B(y)\,\sin(\phi_h+\phi_S)\,F_T^{\sin(\phi_h+\phi_S)}$$

$$+\;B(y)\,\sin(3\phi_h-\phi_S)\,F_T^{\sin(3\phi_h-\phi_S)}$$

$$+\;C(y)\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

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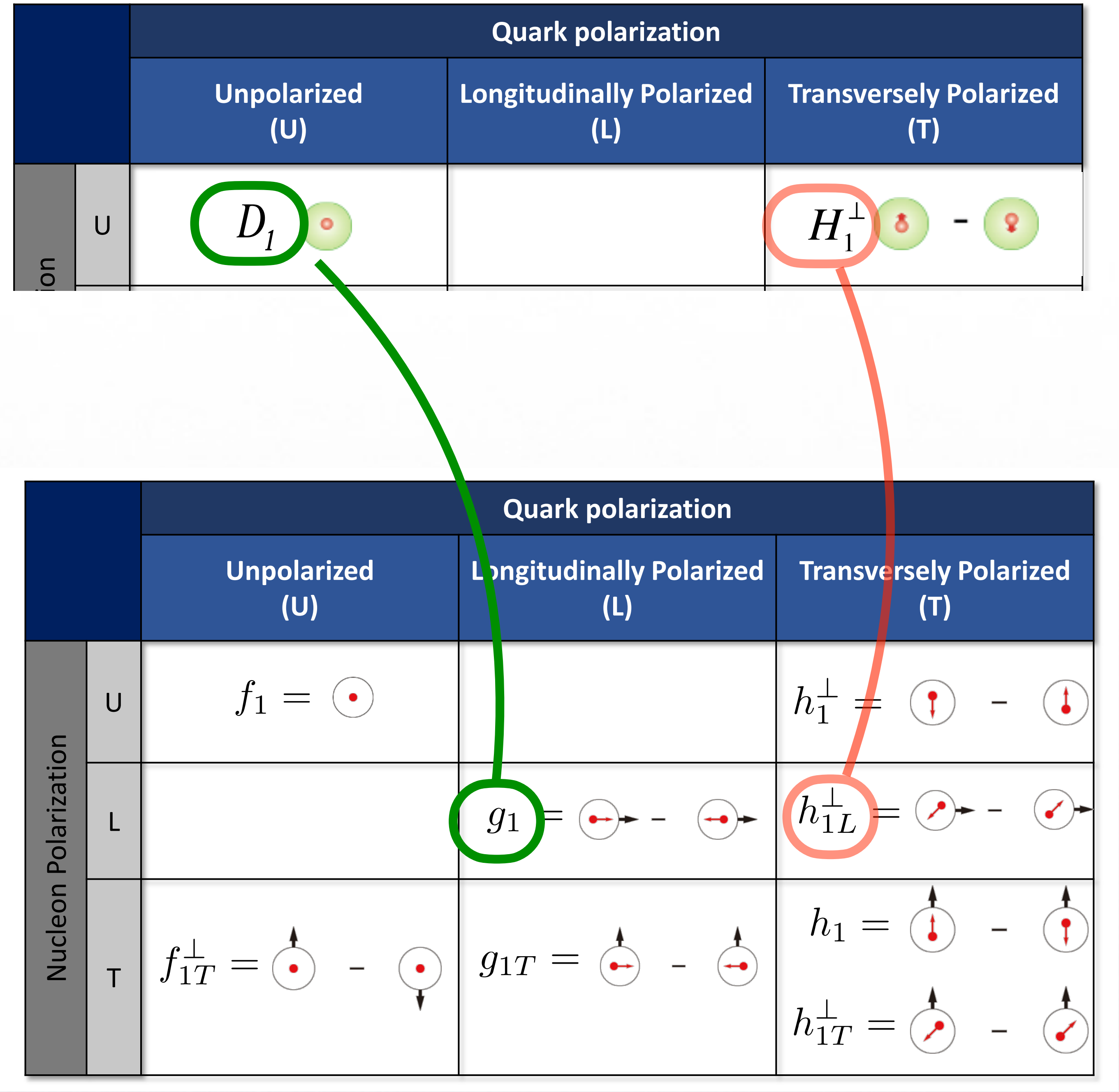
TMDs : link to structure functions



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
ion	U	D_1 ⊙		$H_1^\perp$ ⊙ - ⊙

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		$h_1^\perp =$ ⊙ - ⊙
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	T	$f_{1T}^\perp =$ ⊙ - ⊙	$g_{1T} =$ ⊙ - ⊙	$h_1 =$ ⊙ - ⊙ $h_{1T}^\perp =$ ⊙ - ⊙

Example: SIDIS

target polariz.

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{hT}^2} \sim$$

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○→ $$+\,C(y)\,F_{LL} \quad + \quad B(y)\,\sin 2\phi_h\,F_L^{\sin 2\phi_h}$$

$$+\;A(y)\,\sin(\phi_h-\phi_S)\,F_T^{\sin(\phi_h-\phi_S)}$$

$$+\;B(y)\,\sin(\phi_h+\phi_S)\,F_T^{\sin(\phi_h+\phi_S)}$$

$$+\;B(y)\,\sin(3\phi_h-\phi_S)\,F_T^{\sin(3\phi_h-\phi_S)}$$

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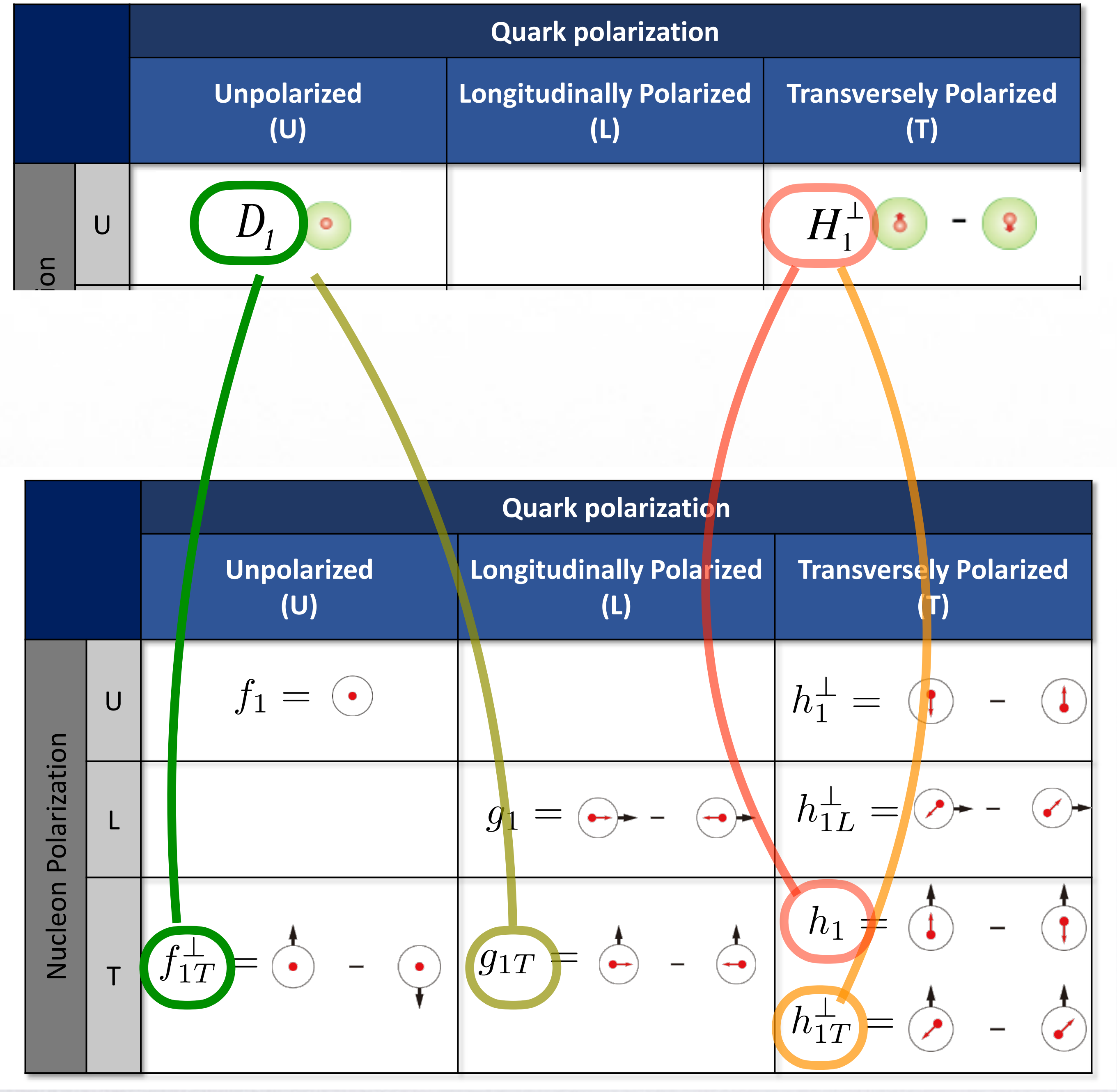
TMDs : link to structure functions



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
ion	U	D_1		$H_1^\perp$ -

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$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{hT}^2} \sim$$

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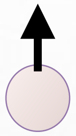
$$+\,C(y)\,F_{LL} \quad + \quad B(y)\,\sin 2\phi_h\,F_L^{\sin 2\phi_h}$$



$$+\;A(y)\,\sin(\phi_h-\phi_S)\,F_T^{\sin(\phi_h-\phi_S)}$$

$$+\;B(y)\,\sin(\phi_h+\phi_S)\,F_T^{\sin(\phi_h+\phi_S)}$$

$$+\;B(y)\,\sin(3\phi_h-\phi_S)\,F_T^{\sin(3\phi_h-\phi_S)}$$

$$+\;C(y)\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

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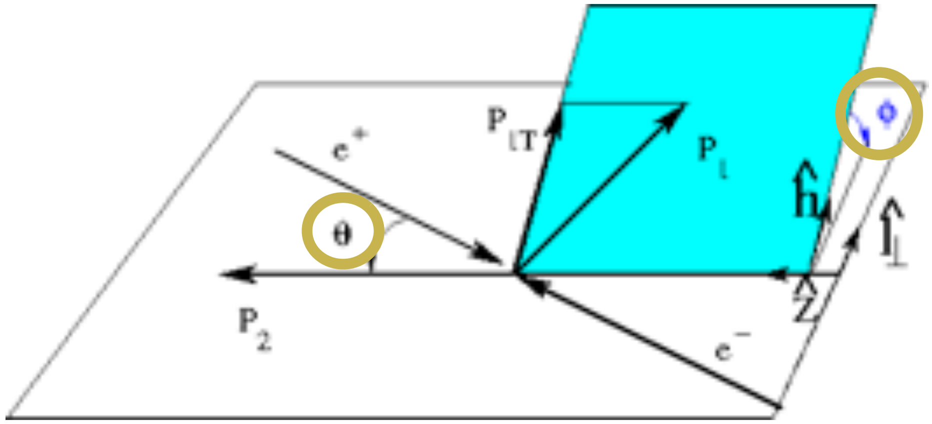
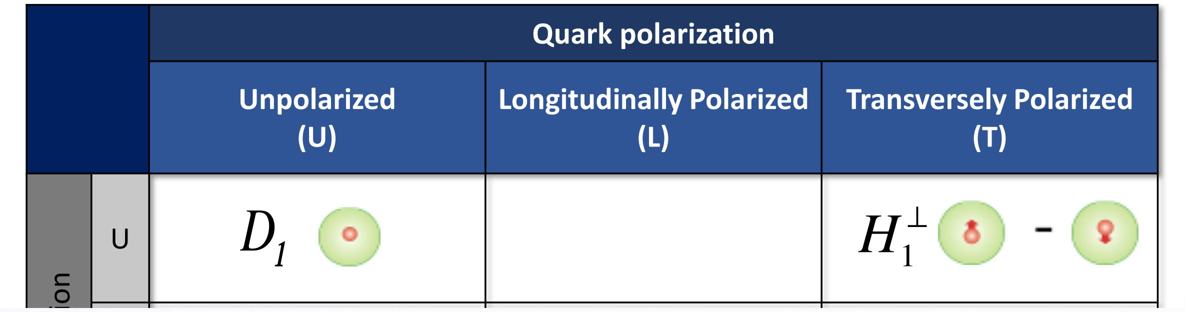
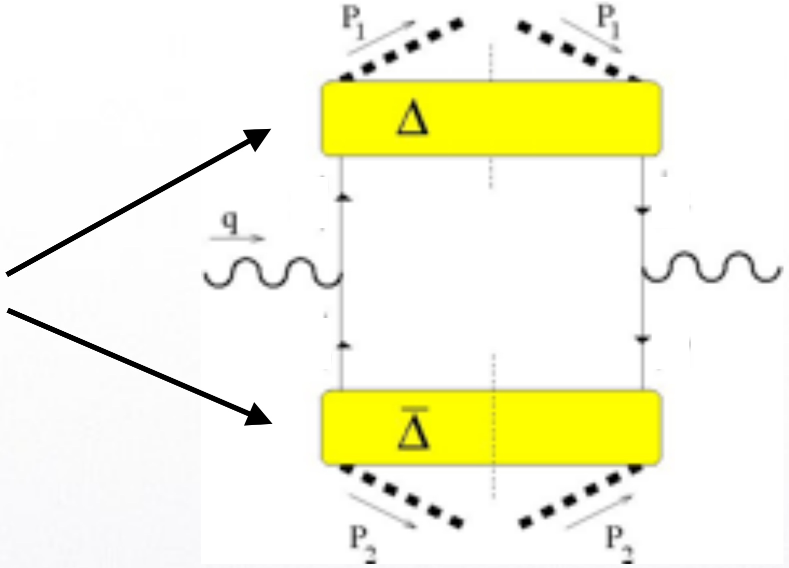
TMDs : link to structure functions

Example: e+ e- to unpolarized hadrons





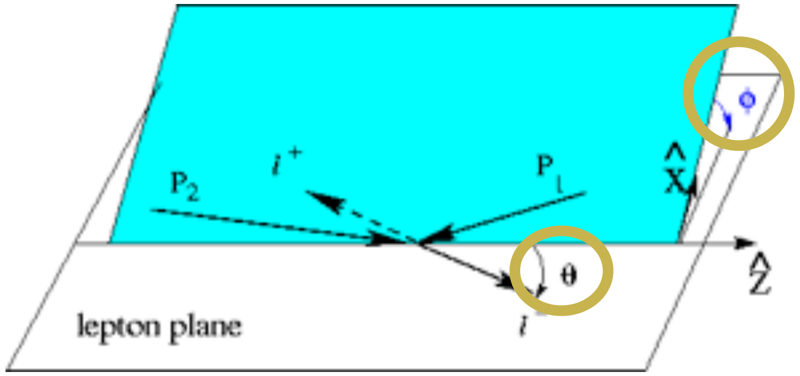
		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
ion	U	D_1 ●		$H_1^\perp$ ● - ●



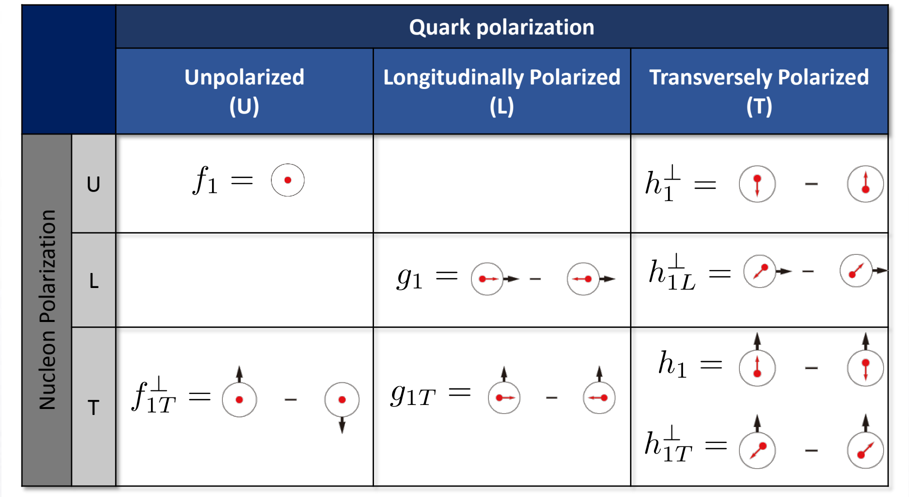
$$\frac{d\sigma}{dz_1 dz_2 d\mathbf{q}_T d\Omega} \sim (1+\cos^2\theta)\,\mathcal{C}\left[D_1(z_1,\mathbf{k}_{1\perp})\,,\bar{D}_1(z_2,\mathbf{k}_{2\perp})\right] + \sin^2\theta\cos^2 2\phi\,\mathcal{C}\left[w_e(\mathbf{k}_{1\perp},\mathbf{k}_{2\perp})\,H_1^\perp(z_1,\mathbf{k}_{1\perp})\,,\bar{H}_1^\perp(z_2,\mathbf{k}_{2\perp})\right]$$

TMDs : link to structure functions

Example: Drell-Yan



Collins-Soper frame
(transv. momenta in xz plane)



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Nucleon Polarization	U	$f_1 =$ ⊙		$h_1^\perp =$ ⊙ - ⊙
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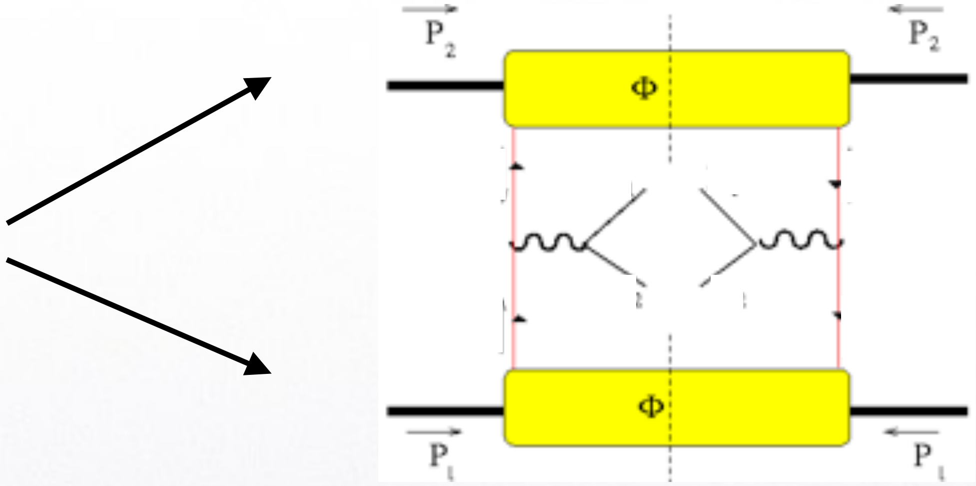
	T	$f_{1T}^\perp =$ ⊙ - ⊙	$g_{1T} =$ ⊙ - ⊙	$h_1 =$ ⊙ - ⊙ $h_{1T}^\perp =$ ⊙ - ⊙

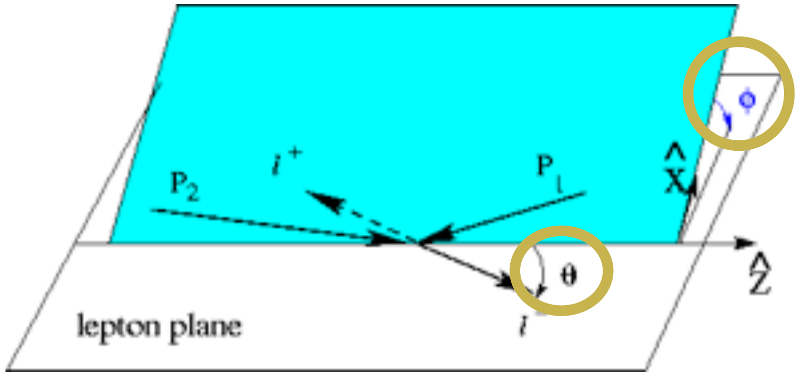


$$\frac{d\sigma}{dx_1 dx_2 d\mathbf{q}_T d\Omega} \sim \underline{(1+\cos^2\theta)}\,\mathscr{C}\left[f_1(x_1,\mathbf{k}_{1\perp})\,,\bar{f}_1(x_2,\mathbf{k}_{2\perp})\right] + \underline{\sin^2\theta\cos^2 2\phi}\,\mathscr{C}\left[w_1(\mathbf{k}_{1\perp},\mathbf{k}_{2\perp})\,h_1^\perp(x_1,\mathbf{k}_{1\perp})\,,\bar{h}_1^\perp(x_2,\mathbf{k}_{2\perp})\right]$$

unpolarized

TMDs : link to structure functions

Example: Drell-Yan



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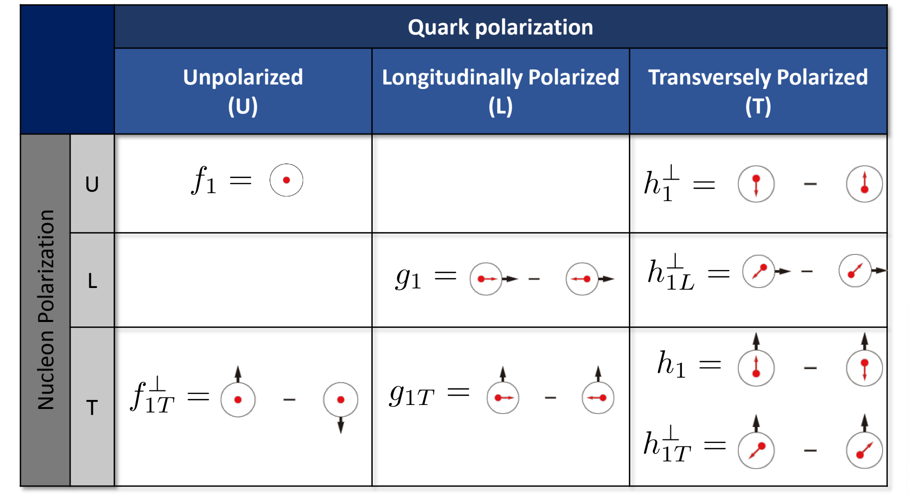

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^{\perp} =$ -
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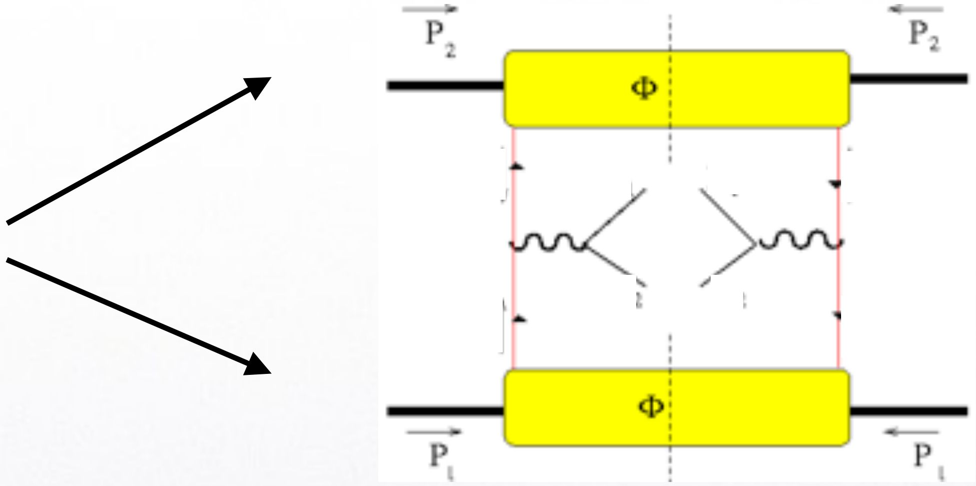
	T	$f_{1T}^{\perp} =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^{\perp} =$ -



$$\frac{d\sigma}{dx_1 dx_2 d\mathbf{q}_T d\Omega} \sim (\underline{1+\cos^2\theta})\, \mathscr{C}\Big[f_1(x_1,\mathbf{k}_{1\perp})\,,\bar{f}_1(x_2,\mathbf{k}_{2\perp})\Big] + \underline{\sin^2\theta\cos^2 2\phi}\, \mathscr{C}\Big[w_1(\mathbf{k}_{1\perp},\mathbf{k}_{2\perp})\, h_1^{\perp}(x_1,\mathbf{k}_{1\perp})\,,\bar{h}_1^{\perp}(x_2,\mathbf{k}_{2\perp})\Big]$$

$$+|\mathbf{S}_{2T}|\,\Big[(\underline{1+\cos^2\theta)\sin(\phi-\phi_{S_2}})\, \mathscr{C}\big[w_1(\mathbf{k}_{1\perp},\mathbf{k}_{2\perp}) f_1(x_1,\mathbf{k}_{1\perp})\,,\bar{f}_{1T}^{\perp}(x_2,\mathbf{k}_{2\perp})\big]$$

$$\underline{-\sin^2\theta\sin(\phi+\phi_{S_2})}\, \mathscr{C}\big[w_2(\mathbf{k}_{1\perp},\mathbf{k}_{2\perp})\, h_1^{\perp}(x_1,\mathbf{k}_{1\perp})\,,\bar{h}_1(x_2,\mathbf{k}_{2\perp})\big]$$

$$-\underline{\sin^2\theta\sin(3\phi-\phi_{S_2})}\, \mathscr{C}\big[w_3(\mathbf{k}_{1\perp},\mathbf{k}_{2\perp})\, h_1^{\perp}(x_1,\mathbf{k}_{1\perp})\,,\bar{h}_{1T}^{\perp}(x_2,\mathbf{k}_{2\perp})\big],\Big] + \ldots$$

hadron "2"
transversely
polarized

Single Spin Asymmetries (SSA)

How to extract a specific structure function ?

Example: SIDIS

$$\frac{d\sigma}{dx_B dy d\phi_S dz_h d\phi_h dP_{hT}^2} = \frac{\alpha^2}{x_B y Q^2} \Big[A(y)\, F_{UU,T} + B(y)\, \cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$

$$+ S_L\, \sin 2\phi_h\, F_{UL}^{\sin 2\phi_h} \quad + \lambda_e\, S_L\, C(y)\, F_{LL}$$

$$+ S_T \big[A(y)\, \sin(\phi_h - \phi_S)\, F_{UT,T}^{\sin(\phi_h - \phi_S)} + B(y)\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h + \phi_S)}$$

$$+ B(y)\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h - \phi_S)}$$

$$+ \lambda_e\, S_T\, C(y)\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h - \phi_S)} \Big] \quad + \mathcal{O}\left(\frac{M}{Q}\right)$$

Single Spin Asymmetries (SSA)

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$$+ S_T \big[A(y)\, \sin(\phi_h - \phi_S)\, F_{UT,T}^{\sin(\phi_h-\phi_S)} + B(y)\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+ B(y)\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+ \lambda_e\, S_T\, C(y)\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} \Big] \quad + \mathcal{O}\left(\frac{M}{Q}\right)$$

if we want to isolate the "Sivers" effect, we build the spin asymmetry

$$A_{UT}^{\sin(\phi_h-\phi_S)} = 2\, \frac{\int d\phi_h d\phi_S\, \sin(\phi_h - \phi_S)\, [d\sigma^\uparrow - d\sigma^\downarrow]}{\int d\phi_h d\phi_S\, [d\sigma^\uparrow + d\sigma^\downarrow]} = \frac{F_{UT,T}^{\sin(\phi_h-\phi_S)}}{F_{UU,T}} = \frac{\mathcal{C}\left[-\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_\perp}{M}\, f_{1T}^\perp\, D_1 \right]}{\mathcal{C}\,[f_1\, D_1]}$$

Single-Spin Asymmetry
(SSA)

Single Spin Asymmetries (SSA)

How to extract a specific structure function ?

Example: SIDIS

$$\frac{d\sigma}{dx_B dy d\phi_S dz_h d\phi_h dP_{hT}^2} = \frac{\alpha^2}{x_B y Q^2} \Big[A(y)\, F_{UU,T} + B(y)\, \cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$

$$+ S_L\, \sin 2\phi_h\, F_{UL}^{\sin 2\phi_h} \quad + \lambda_e\, S_L\, C(y)\, F_{LL}$$

$$+ S_T \big[A(y)\, \sin(\phi_h - \phi_S)\, F_{UT,T}^{\sin(\phi_h - \phi_S)} + B(y)\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h + \phi_S)}$$

$$+ B(y)\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h - \phi_S)}$$

$$+ \lambda_e\, S_T\, C(y)\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h - \phi_S)} \Big] \quad + \mathcal{O}\left(\frac{M}{Q}\right)$$

if we want to isolate the "Sivers" effect, we build the spin asymmetry

$$A_{UT}^{\sin(\phi_h - \phi_S)} = 2\, \frac{\int d\phi_h d\phi_S\, \sin(\phi_h - \phi_S)\, [d\sigma^{\uparrow} - d\sigma^{\downarrow}]}{\int d\phi_h d\phi_S\, [d\sigma^{\uparrow} + d\sigma^{\downarrow}]} = \frac{F_{UT,T}^{\sin(\phi_h - \phi_S)}}{F_{UU,T}} = \frac{\mathcal{C}\left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_\perp}{M}\, f_{1T}^{\perp}\, D_1 \right]}{\mathcal{C}\, [f_1\, D_1]}$$

Single-Spin Asymmetry
(SSA)

any polarized measurement requires
knowledge of unpolarized cross section

Overview of current TMD phenomenology

$$F_{UU} \sim f_1 \otimes D_1$$

Overview of current TMD phenomenology

$$F_{UU} \sim f_1 \otimes D_1$$

N.B. analysis with N^{n}LO and N^{m}LL accuracy means $\mathcal{O}(\alpha_s^n)$ corrections in hard vertex and resummation up to $\alpha_s^n \log^{2n-m}(Q^2/\mu_b^2)$ contributions in the perturbative part of the TMD Evo operator

Overview of current TMD phenomenology

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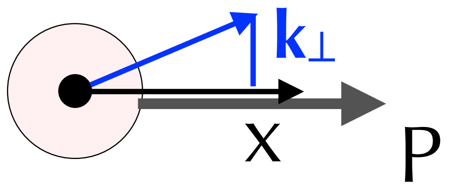
Caveat: unpol. SIDIS data come as multiplicities $\dfrac{\dfrac{d\sigma}{dxdydzd\mathbf{P}_{hT}}}{\dfrac{d\sigma_{DIS}}{dxdy}}$

normalisation problems because due to matching problem $\sum_h \int zdzd\mathbf{P}_{hT} \dfrac{d\sigma}{dxdydzd\mathbf{P}_{hT}} \neq \dfrac{d\sigma_{DIS}}{dxdy}$

Questions



What do we know about $\langle \mathbf{k}_\perp^2 \rangle$?

1. does it depend on x ?
2. does it depend on flavor of quarks ?
3. does it change with Q^2 ?

Recent analyses

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2013 arXiv:1309.3507	extended parton model	✔	✘	✘	✘	1538
Torino 2014 arXiv:1312.6261	extended parton model	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO+N^{2}LL	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO+NLL	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
SIYY 2014 arXiv:1406.3073	NLO+NLL′	✘	✔	✔	✔	200 (?)
Pavia 2017 arXiv:1703.10157	LO+NLL	✔	✔	✔	✔	8059
SV 2017 arXiv:1706.01473	N^{2}LO+N^{2}LL′	✘	✘	✔	✔ (LHC)	309
BSV 2019 arXiv:1902.08474	N^{2}LO+N^{2}LL′	✘	✘	✔	✔ (LHC)	457
Pavia 2019 arXiv:1912.07550	N^{2}LO+N^{3}LL	✘	✘	✔	✔ (LHC)	319
SV 2020 arXiv:1912.06532	N^{2}LO (+N^{3}LO)	✔	✔	✔	✔	1039

Recent analyses

	Framework	HERMES	COMPASS	DY	Z production	N of points
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Torino 201 arXiv:1312.62						576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO+N^{2}LL	✘	✘	✔	✔	223
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First to introduce flavor dependence

Recent analyses

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Pavia 2013 arXiv:1309.3507	extended parton model	✔	✘	✘	✘	1538
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SV 2017 arXiv:1706.01473	N^{2}L					309
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Pavia 2019 arXiv:1912.07550	N^{2}LO+N^{3}LL	✘	✘	✔	✔ (LHC)	319
SV 2020 arXiv:1912.06532	N^{2}LO (+N^{3}LO)	✔	✔	✔	✔	1039

First global fit (>8K data points)

Recent analyses

	Framework	HERMES	COMPASS	DY	Z production	N of points
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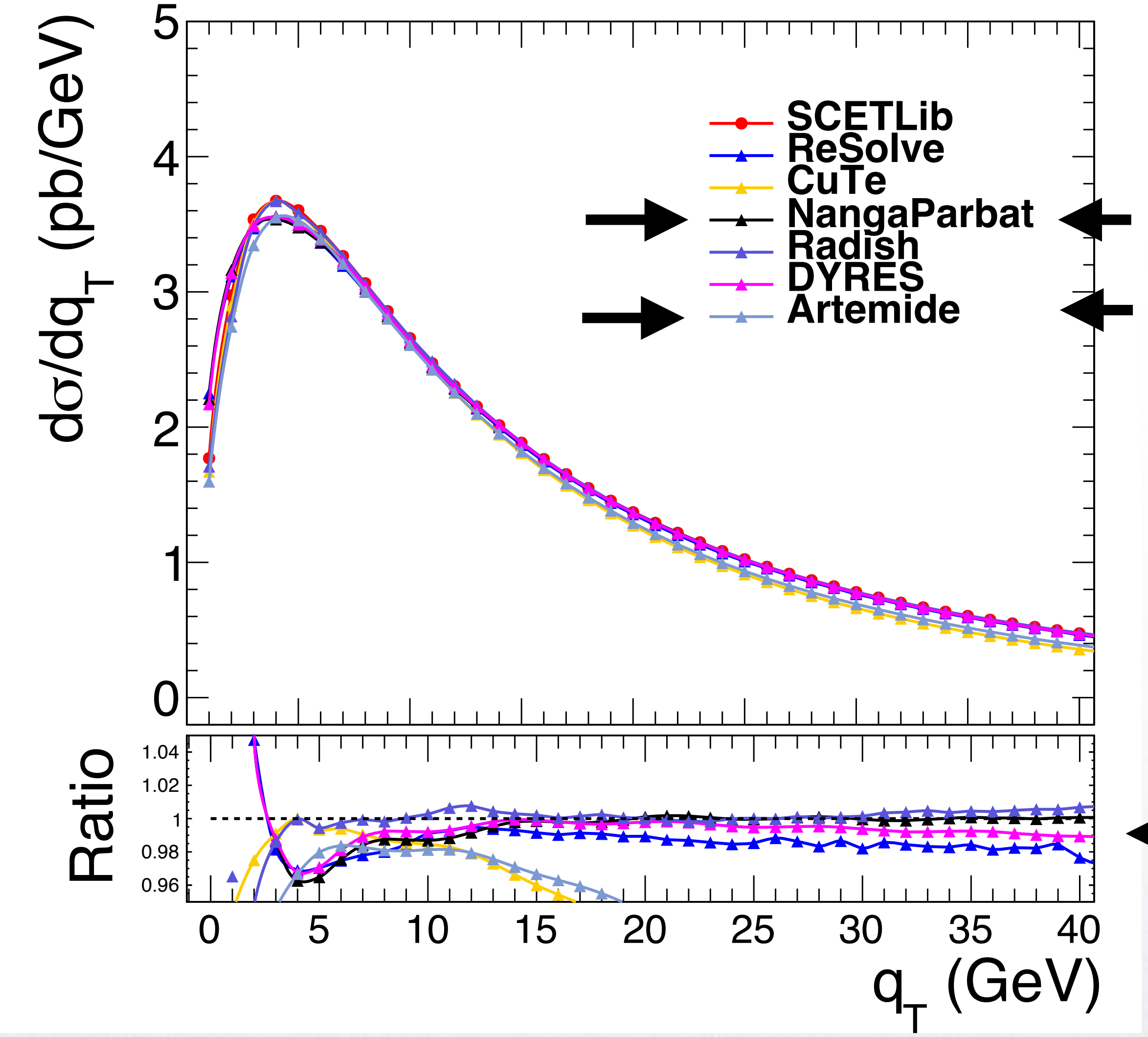
Global fit with current top accuracy

Precision era for TMDs

Z production at rapidity y=0 in ATLAS kin.
benchmarking TMDs (resummed at N^{3}LL) with perturbative calculations



$\frac{X}{\text{SCETLib}}$

NangaParbat
= Pavia 2019 fit

Artemide
= SV 2020 fit

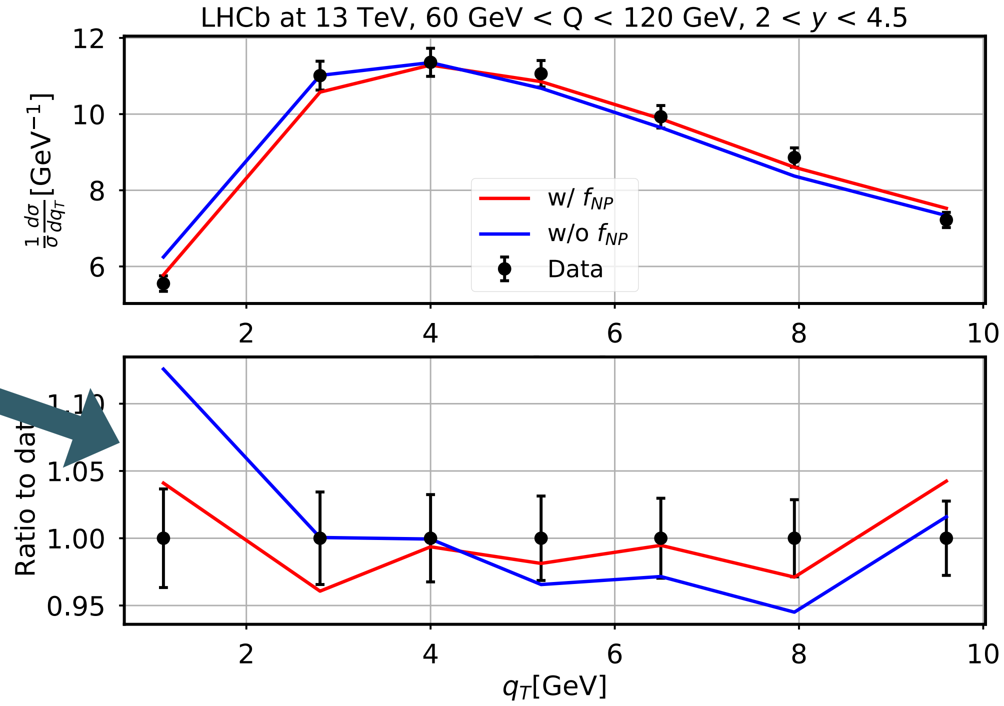
agreement
within ±(1-2)%
for $5 < q_T < 80$ GeV

G. Bozzi, I. Scimemi (eds.) et al.,
Yellow Report of CERN EW Working Group, in preparation

TMD impact at LHC

output of
PV19 fit

Effect of nonperturbative intrinsic k_T
(not included in other benchmark codes)



G. Bozzi, I. Scimemi (eds.) et al.,
Yellow Report of CERN EW Working Group, in preparation

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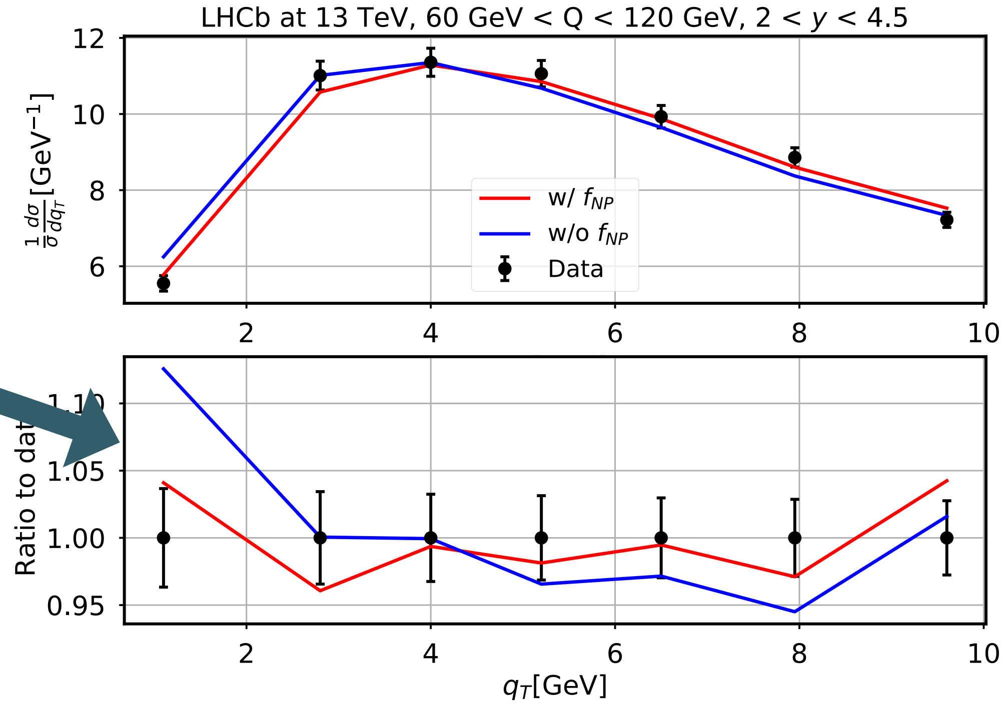


G. Bozzi, I. Scimemi (eds.) et al.,
Yellow Report of CERN EW Working Group, in preparation

Current **extractions of M_W** based on q_T-distribution of decay products
do not include flavor sensitivity

Exercise:

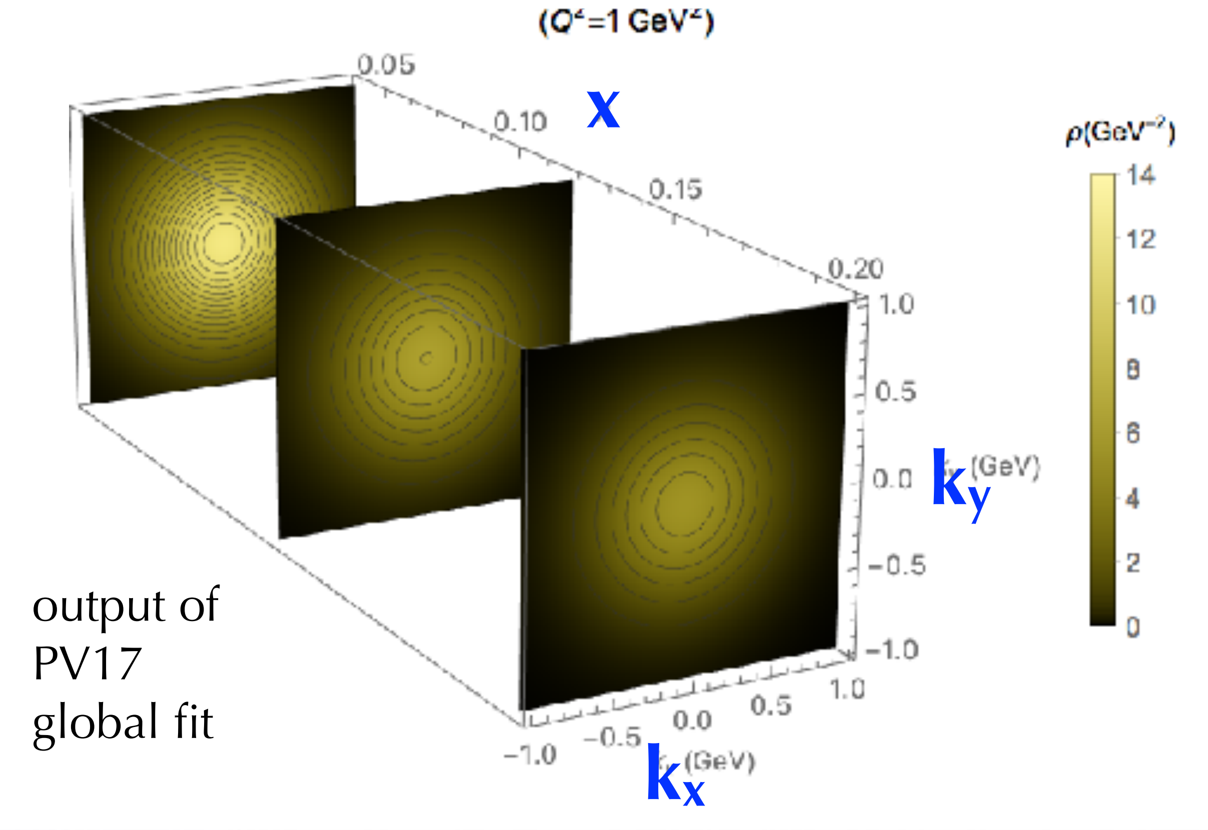
- generate pseudo-data for q_T-spectrum of $W^\pm$ with sets of flavor-dep. parameters that give the same q_T-spectrum of Z^0, from p_T-lepton data and uncertainties of ATLAS and CDF
- make a template fit of these pseudo-data by varying M_W on a set of flavor-independent parameters

→ shifts comparable to world-average uncertainty $-6 \leq \Delta M_{W^\pm} \leq +9$ MeV

$-4 \leq \Delta M_{W^-} \leq +4$ MeV

Bacchetta, Bozzi, Radici, Ritzmann, Signori,
P.L. ***B788*** *(19) 542, arXiv:1807.02101*

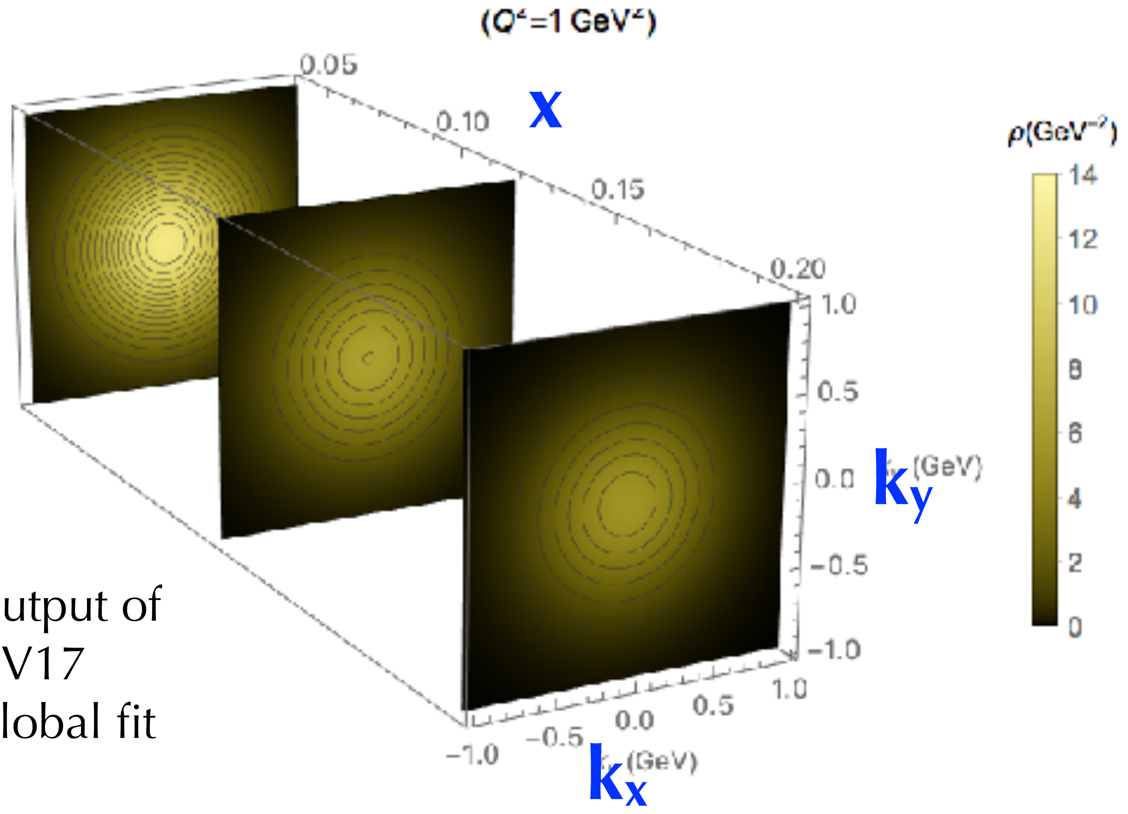
Hadron tomography



output of
PV17
global fit

Answer to question #1:
yes, $\langle \mathbf{k}_\perp^2 \rangle$ does depend on x

Hadron tomography



output of
PV17
global fit

Answer to question #1:
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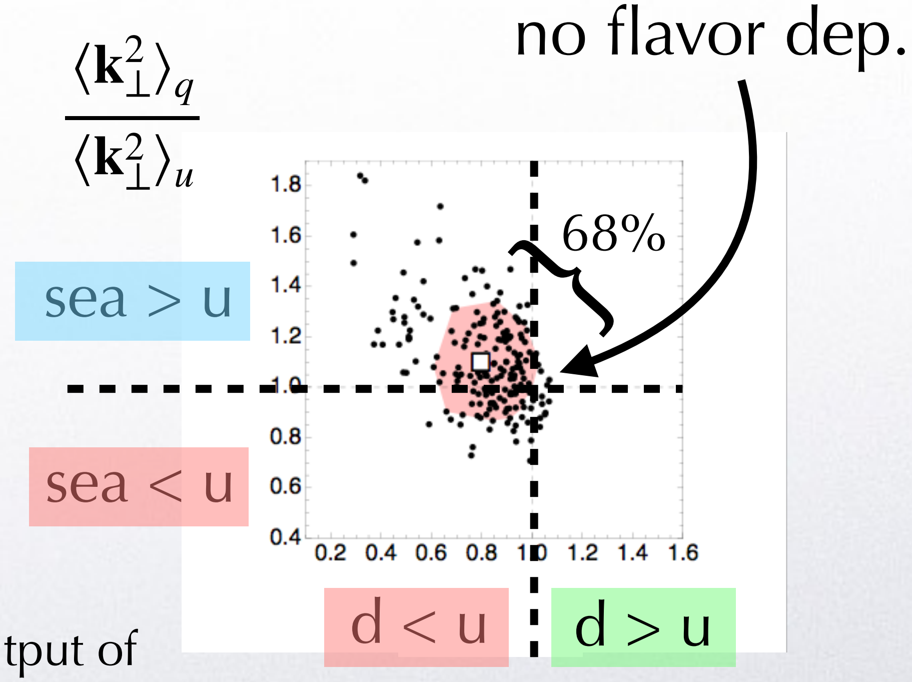
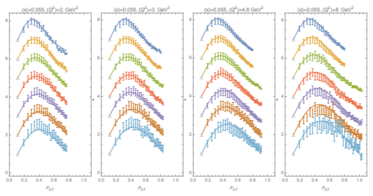


output of
PV13 fit

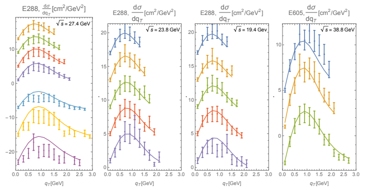
Answer to question #2:
need more and more precise data to assess flavor dependence
(currently, fits w/ and w/o flavor dep. are equivalent)

Hadron tomography

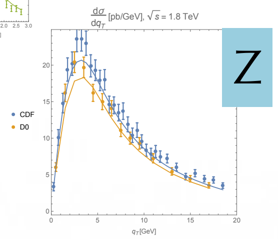
Answer to question #3:
$\langle \mathbf{k}_\perp^2 \rangle$ changes with Q^2 but large uncertainties in TMD evolution


SIDIS

$\langle Q^2 \rangle \sim 2 - 10 \text{ GeV}^2 \rightarrow (P_{hT})_{\text{peak}} \sim$ 0.5 GeV/c


Drell-Yan

$\langle Q^2 \rangle \sim 20 - 150 \text{ GeV}^2 \rightarrow (P_{hT})_{\text{peak}} \sim$ 1 GeV/c



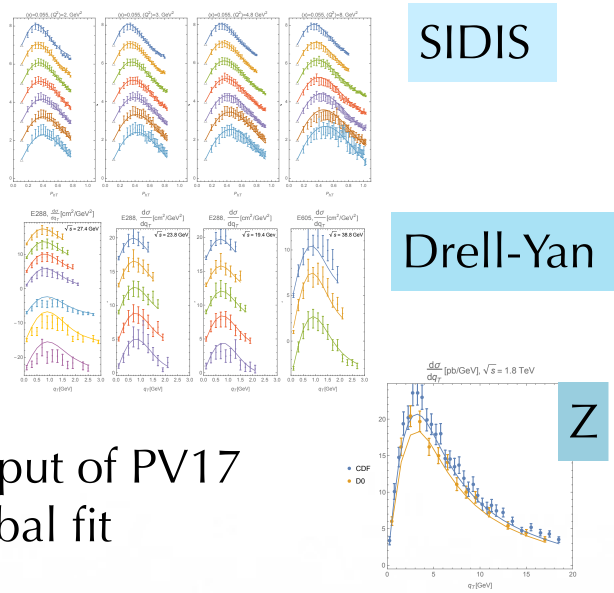
Z

$Q^2 \sim 8200 \text{ GeV}^2 \rightarrow (P_{hT})_{\text{peak}} \sim$ 4 GeV/c

output of PV17
global fit

Hadron tomography

Answer to question #3:
$\langle \mathbf{k}_\perp^2 \rangle$ changes with Q^2 but large uncertainties in TMD evolution

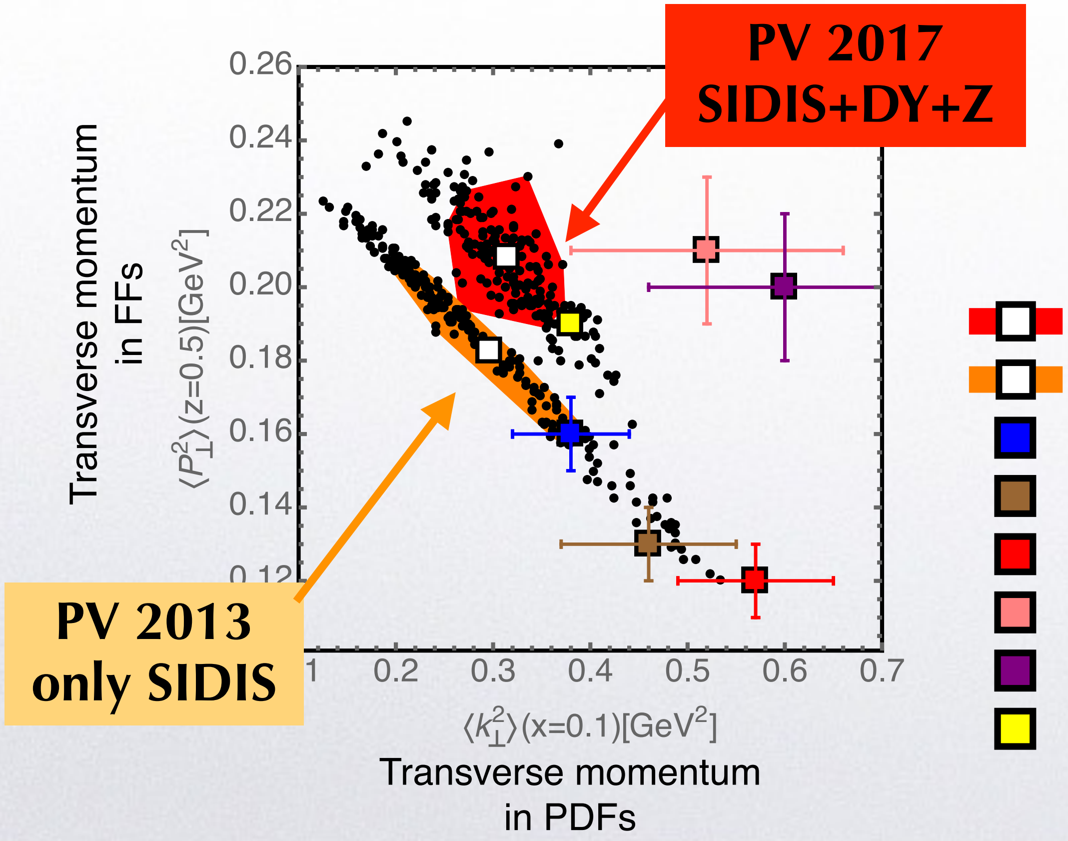


output of PV17 global fit

SIDIS $\langle Q^2\rangle \sim 2-10\ \text{GeV}^2 \to (P_{hT})_{\text{peak}} \sim 0.5\ \text{GeV/c}$

Drell-Yan $\langle Q^2\rangle \sim 20-150\ \text{GeV}^2 \to (P_{hT})_{\text{peak}} \sim 1\ \text{GeV/c}$

Z $Q^2 \sim 8200\ \text{GeV}^2 \to (P_{hT})_{\text{peak}} \sim 4\ \text{GeV/c}$



$\mathbf{P}_{hT} = z\mathbf{k}_\perp + \mathbf{P}_\perp$ problem with **anti-correlation**,
partly mitigated by including DY+Z data
→ include e+e- data in the analysis

- 🟥 Bacchetta, Delcarro, Pisano, Radici, Signori, in preparation (Q = 1 GeV)
- 🟧 Signori, Bacchetta, Radici, Schnell arXiv:1309.3507
- 🟦 Schweitzer, Teckentrup, Metz, arXiv:1003.2190
- 🟫 Anselmino et al. arXiv:1312.6261 [HERMES]
- 🟥 Anselmino et al. arXiv:1312.6261 [HERMES, high z]
- ■ Anselmino et al. arXiv:1312.6261 [COMPASS, norm.]
- 🟪 Anselmino et al. arXiv:1312.6261 [COMPASS, high z, norm.]
- 🟨 Echevarria, Idilbi, Kang, Vitev arXiv:1401.5078 (Q = 1.5 GeV)

Hadron tomography

problem with e+e- data

TMD factorisation theorem exists for production of two back-to-back hadrons

e+e- → h_1+h_2+X

$$d\sigma \sim \mathscr{C}\left[FF(h_1)\,,FF(h_2)\right]$$

Hadron tomography

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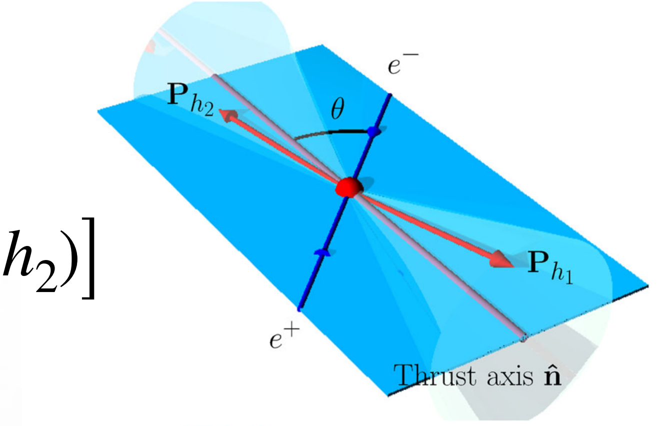
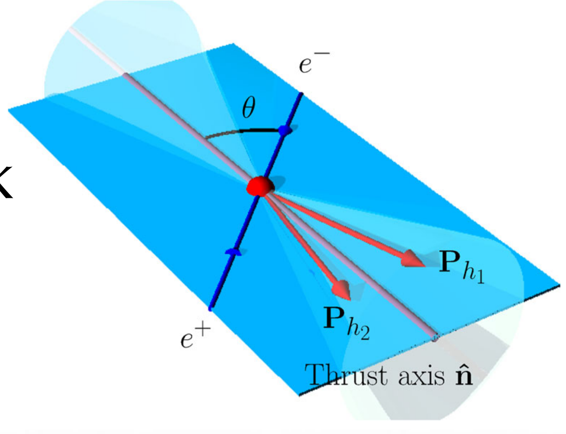


non unique way to distinguish back-to-back from same emisphere (and from Di-hadron FF, DiFF(h_1,h_2))



Belle, PRD **101** (20) 092004

Hadron tomography

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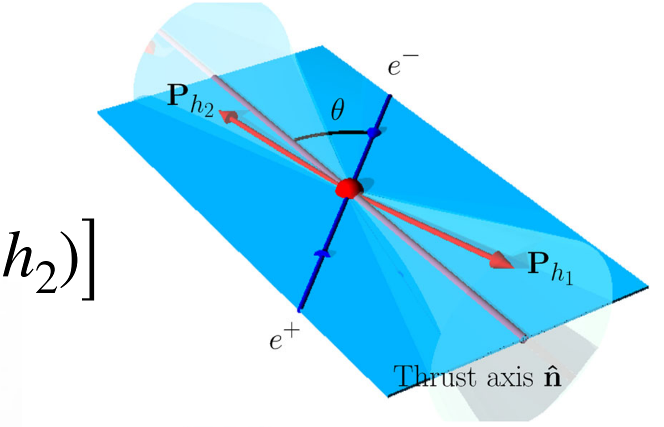

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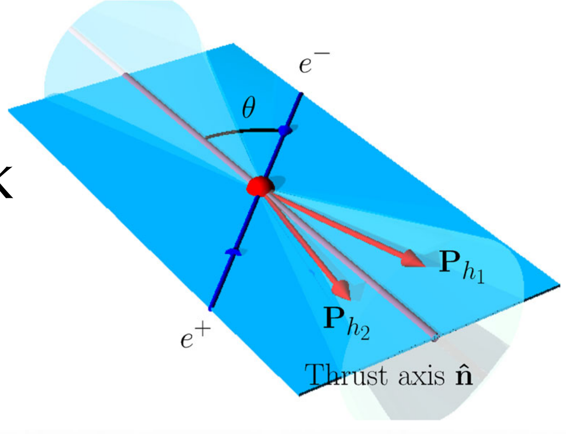
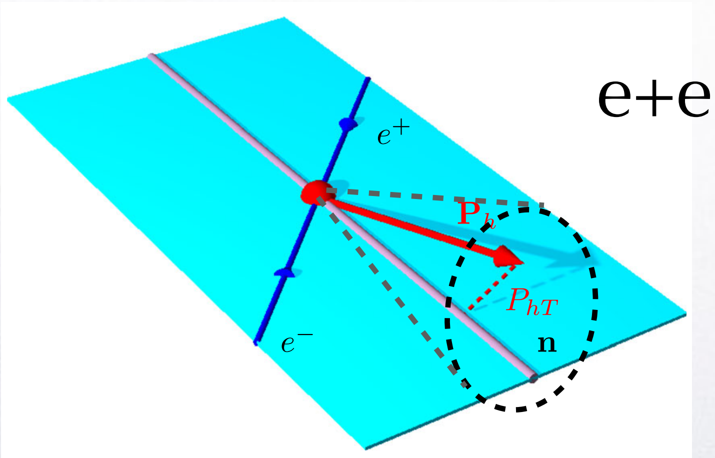


Belle, PRD **101** (20) 092004



e+e- → h+X data available Belle, PRD **99** (19) 112006

dσ depends on z, $\mathbf{P}_T$, and thrust

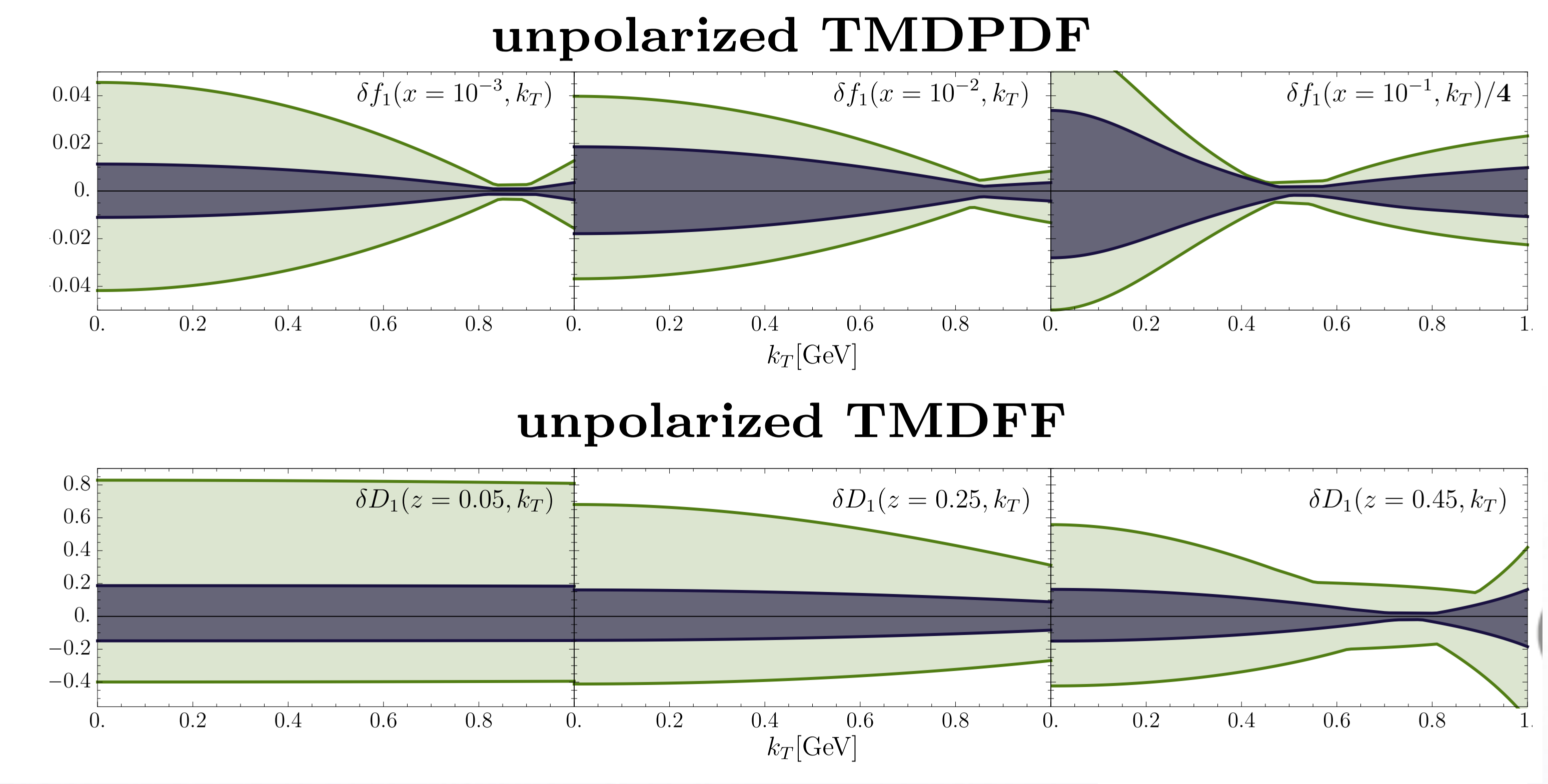
$$T = \frac{\sum_i \mathbf{P}_i \cdot \hat{\mathbf{n}}}{\sum_i |\mathbf{P}_i|}$$

depending on where h is inside the jet → different factorisation th.'s
some not well established

work in progress, see

Boglione, Simonelli, JHEP 02 (21) 076
Makris, Ringer, Waalewijn, JHEP 02 (21) 070
Kang, Shao, Zhao, JHEP 12 (20) 127
Boglione, Simonelli, in preparation

EIC impact on unpolarised TMD



using SV19 parametrisation

Vladimirov, talk at Snowmass 2021 EF06-EF07 meeting, 28 Oct. 2020

see also EIC Yellow Report, arXiv:2103.05419

Overview of current TMD phenomenology

$$A_{UT}^{\sin(\phi_h-\phi_S)} \propto \frac{F_{UT}^{\sin(\phi_h-\phi_S)}}{F_{UU}} \sim \frac{f_{1T}^{\perp} \otimes D_1}{f_1 \otimes D_1}$$

Sivers effect

Overview of current TMD phenomenology

$$A_{UT}^{\sin(\phi_h-\phi_S)} \propto \frac{F_{UT}^{\sin(\phi_h-\phi_S)}}{F_{UU}} \sim \frac{f_{1T}^{\perp}\otimes D_1}{f_1\otimes D_1}$$

Sivers effect

Long record of extractions of k_T-moment of Sivers

$$f_{1T}^{\perp(1)}(x) = \int d\mathbf{k}_\perp \, \frac{\mathbf{k}_\perp^2}{2M^2} \, f_{1T}^{\perp}(x,\mathbf{k}_\perp^2)$$

from k_T-weighted SSA A_{UT}

*Vogelsang & Yuan, P.R. D**72** (05) 054028*
*Collins et al., P.R. D**73** (06) 014021*
*Bacchetta & Radici, P.R.L. **107** (11) 212001*
*Anselmino, Boglione, Melis, P.R. D**86** (12) 014028*
*Aybat, Prokudin, Rogers, P.R.L. **108** (12) 242003*
*Sun & Yuan, P.R. D**88** (13) 034016*
*Boer, N.P. **B874** (13) 217*
*Echevarria et al., P.R. D**89** (14)*
*Boglione et al., JHEP **07** (18) …*

Recent analyses

	Framework	SIDIS	DY	W/Z production	e+e-	N of points
JAM 20 arXiv:2002.08384	extended parton model	✔	✔	✔	✔	517
Pavia 2020 arXiv:2004.14278	LO+NLL	✔	in progress	in progress	✘	118 (+32)
EKT 2020 arXiv:2009.10710	NLO+N^{2}LL	✔	✔	✔	✘	243
BPV 2020 arXiv:2012.05135	?	✔	✔	✔	✘	76

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First global fit (but simplified analysis)

Recent analyses

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[illegible]	[illegible]					[illegible]
BPV 2020 arXiv:2012.05135	?	✔	✔	✔	✘	76

First consistent extraction of $f_{1T}^{\perp}$ and f_1 in TMD framework (use PV17)

Recent analyses

	Framework	SIDIS	DY	W/Z production	e+e-	N of points
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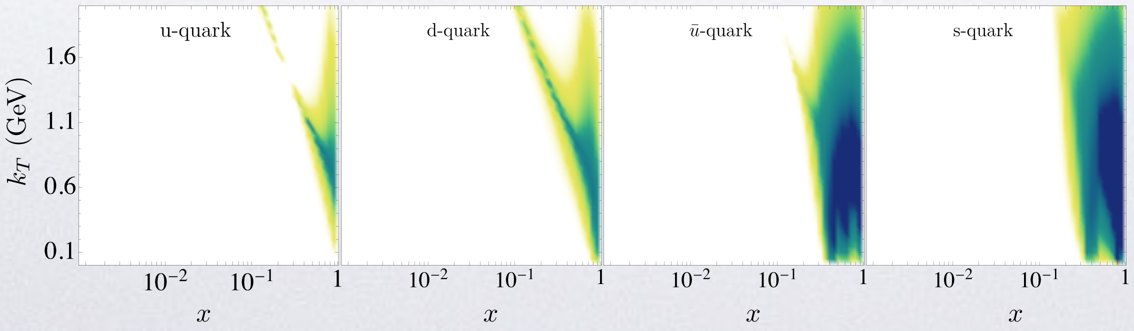
Perturbative matching coeffs. of $f_{1T}^{\perp}$ onto collinear function at small b$_T$ are known only at NLO.
Authors replace matching formula with fitting parametrisation, and use SV19 for f_1 and D_1
The claim is that in their (ζ-prescription) scheme they can mix different descriptions of numerator and denominator of the asymmetry, hence overall perturbative accuracy is same as SV19, namely N^{2}LO (+N^{3}LO).
Also, resulting Sivers function violates positivity bounds at medium-large x

$$\frac{f_{1T}^{\perp}\otimes D_1}{f_1\otimes D_1}$$

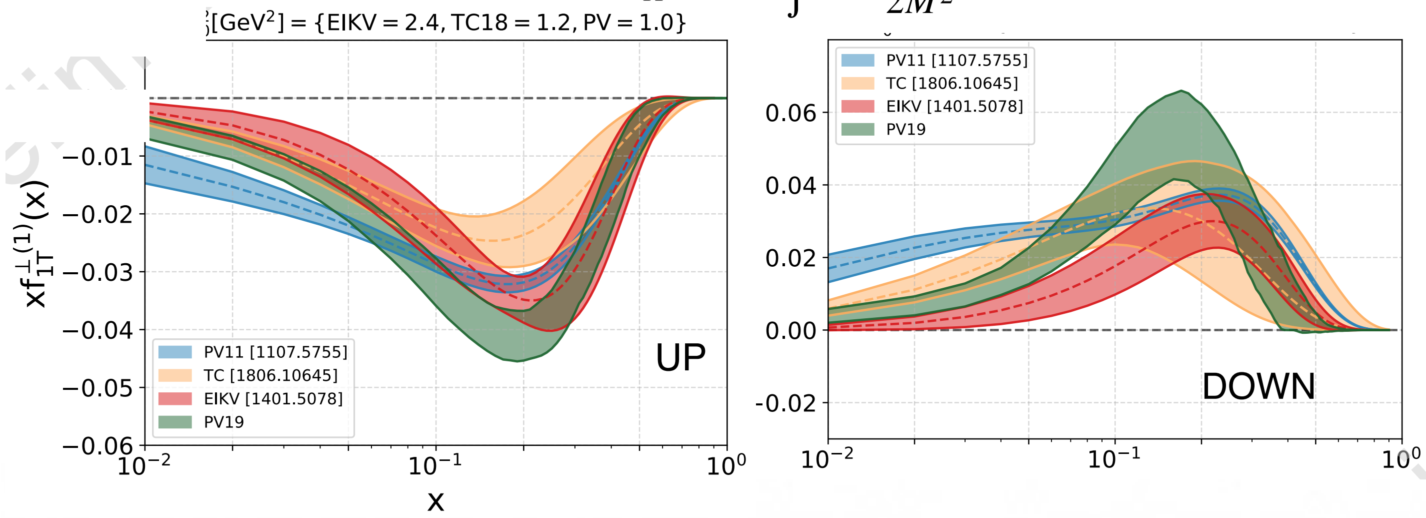


$$\frac{\mathbf{k}_\perp^2}{M^2}\left[f_{1T}^{\perp}(x,\mathbf{k}_\perp^2)\right]^2 \cancel{\leq} \left[f_1(x,\mathbf{k}_\perp^2)\right]^2$$

in coloured areas

Sivers extractions

$$f_{1T}^{\perp(1)}(x) = \int d\mathbf{k}_\perp \frac{\mathbf{k}_\perp^2}{2M^2} f_{1T}^\perp(x,\mathbf{k}_\perp^2)$$



PV11

*Bacchetta & Radici, P.R.L. **107** (11)*

EIKV14

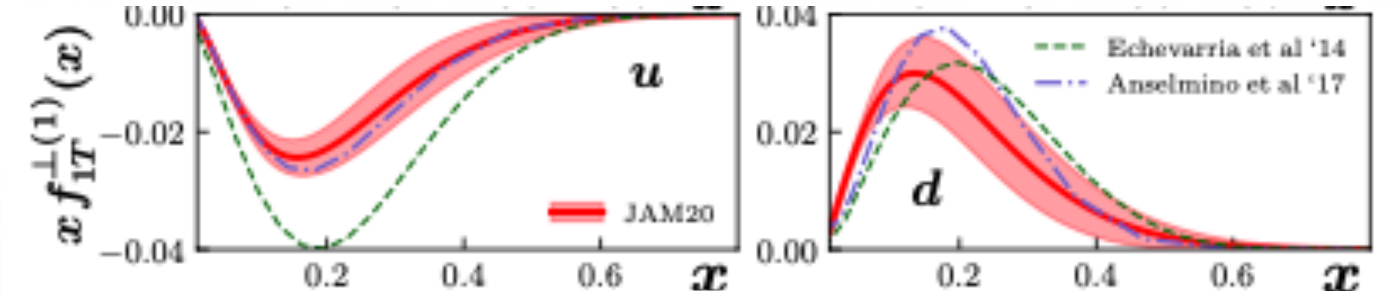
*Echevarria et al., P.R. D**89** (14)*

TC18

*Boglione et al., JHEP **1807** (18)*

PV20

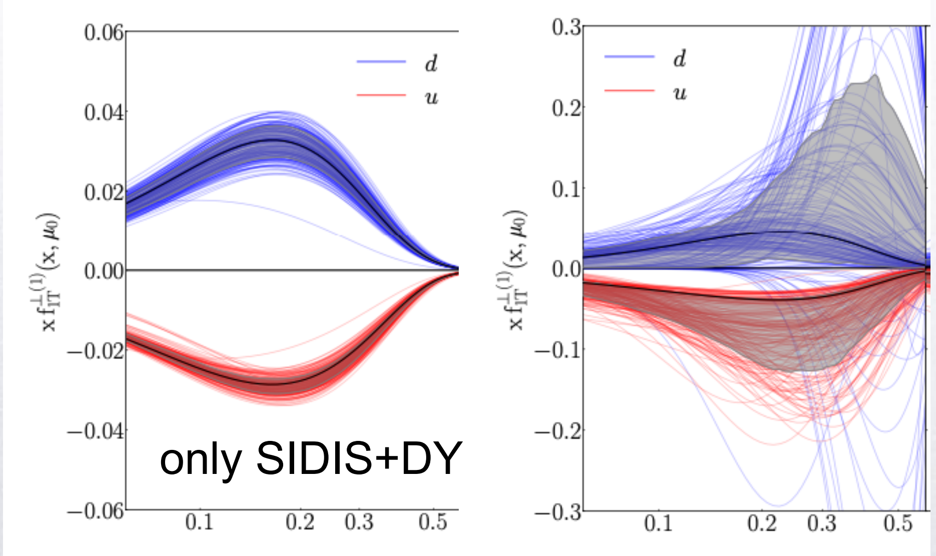
Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278



JAM 20

Cammarota et al. (JAM20), arXiv:2002.08384

sea quark
~ 10^{-1} × smaller



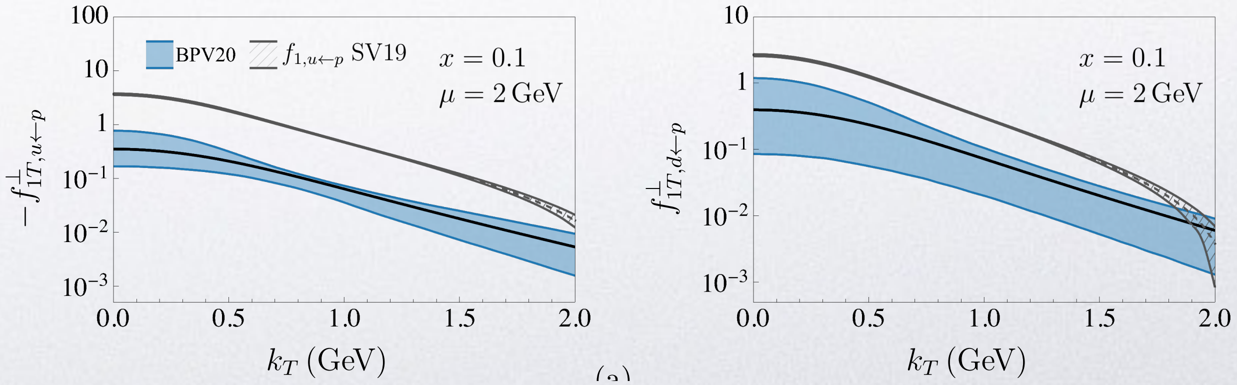
EKT 20

Echevarria, Kang, Terry, arXiv:2009.10710

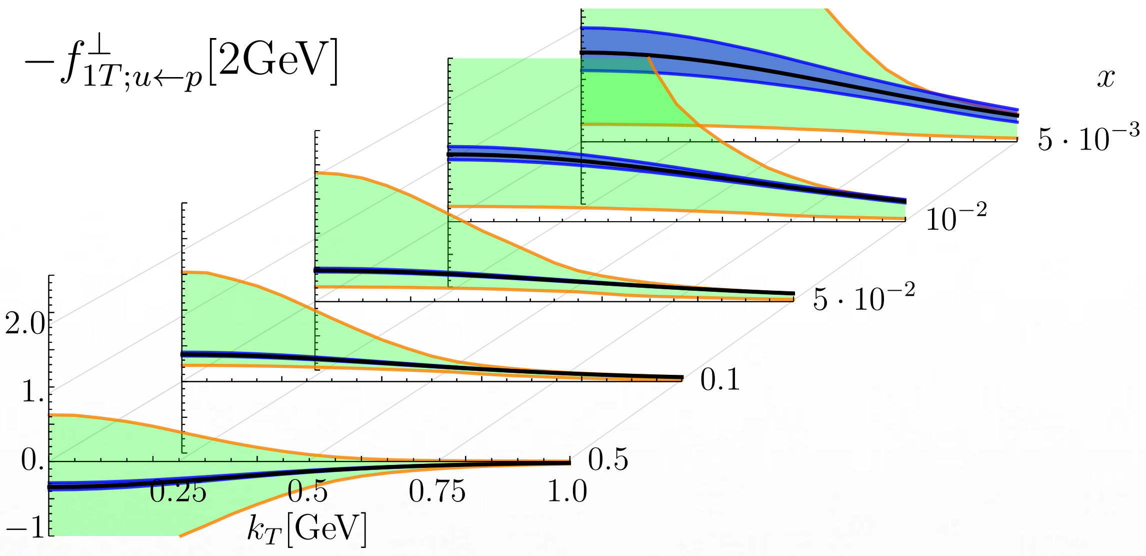
tensions with W/Z data

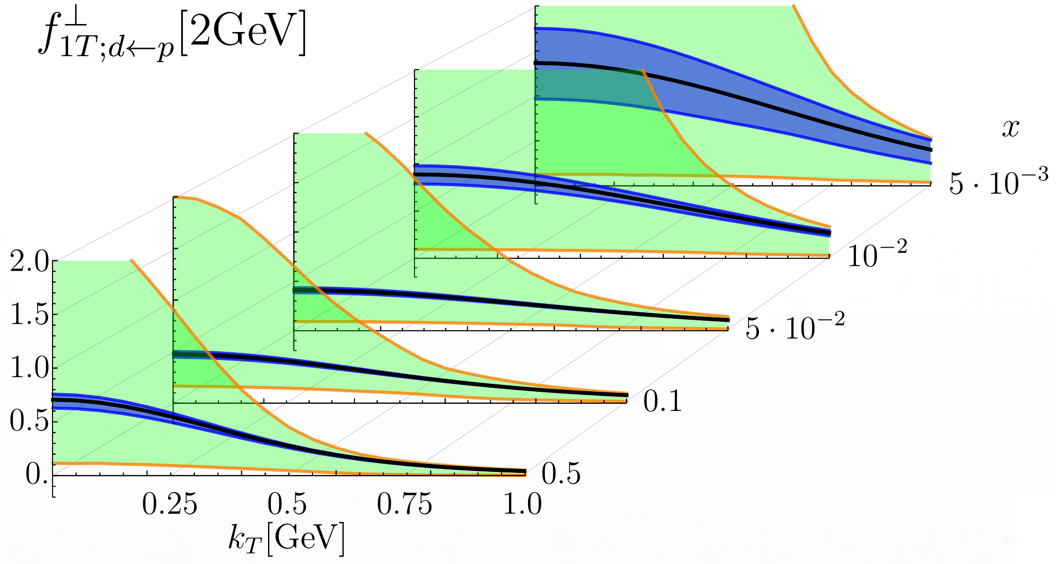
BPV 20

Bury, Prokudin, Vladimirov, arXiv:2012.05135 and 2103.03270



EIC impact on Sivers





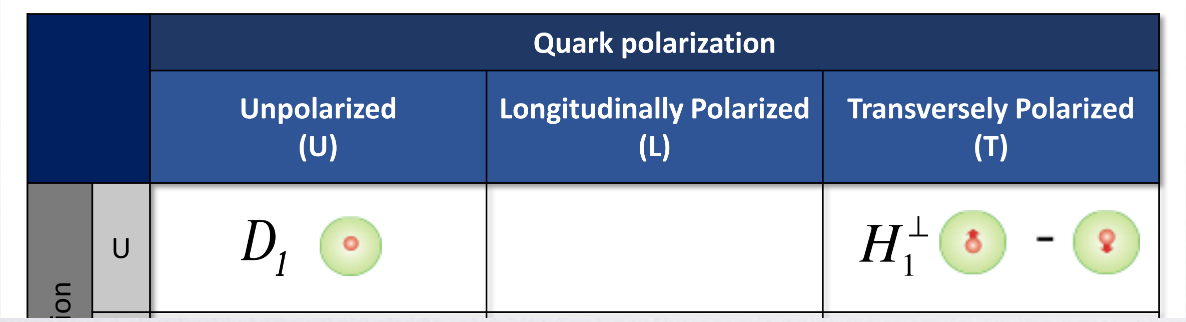
using BPV20 parametrisation

EIC Yellow Report, arXiv:2103.05419

Overview of current TMD phenomenology

$$A_{UT}^{\sin(\phi_h+\phi_S)} \propto \frac{F_{UT}^{\sin(\phi_h+\phi_S)}}{F_{UU}} \sim \frac{h_1 \otimes H_1^\perp}{f_1 \otimes D_1}$$

transversity

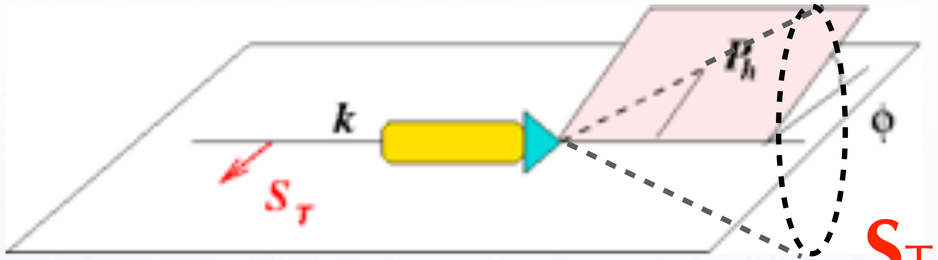


		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
ion	U	D_1 ●		$H_1^\perp$ ● - ●

Overview of current TMD phenomenology

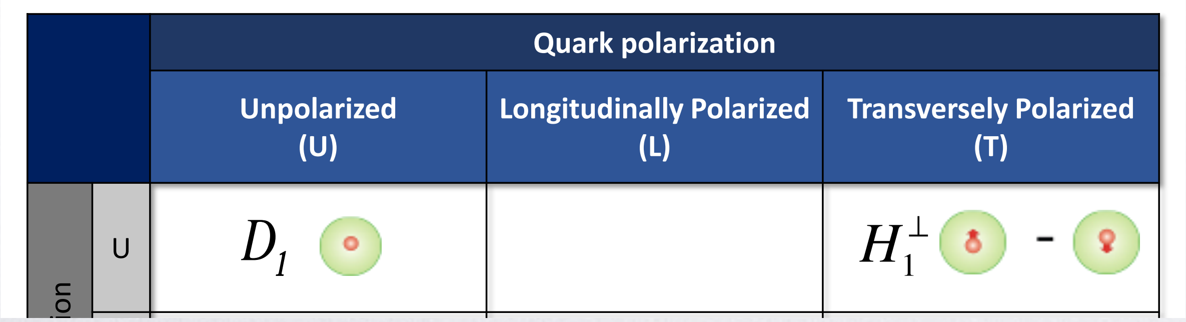
$$A_{UT}^{\sin(\phi_h+\phi_S)} \propto \frac{F_{UT}^{\sin(\phi_h+\phi_S)}}{F_{UU}} \sim \frac{h_1 \otimes H_1^{\perp}}{f_1 \otimes D_1}$$

requires knowledge of $H_1^{\perp}$ from e+e- → h_1+h_2+X data



$S_T \cdot \mathbf{k}\times\mathbf{P}_{hT}$

transversity
based on Collins effect



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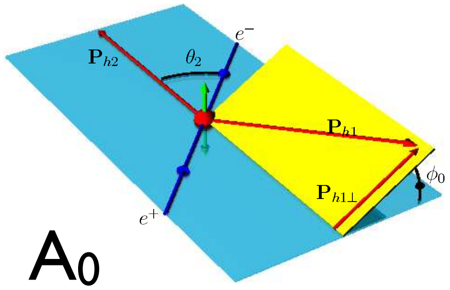
ion	U	D_1 ⊙		$H_1^{\perp}$ ⊙ - ⊙

non recent extractions

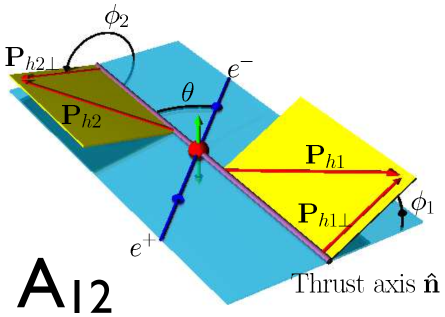
*Anselmino et al., P.R. D**87** (13) 094019*
Anselmino et al., P.R. D92 (15) 114023
*Martin,Bradamante,Barone, P.R. D**91** (15) 014034*
*Kang et al., P.R. D**93** (16) 014009*
*Lin et al., P.R.L. **120** (18) 152502 …*

Collins extractions

2 exp. frames:



A_0



A_{12}

problems with
TMD factoriz. th.
because of thrust

2 kinds of data:

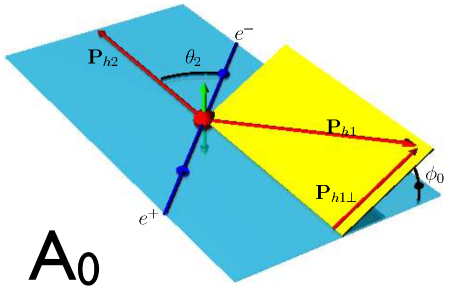
$$A_0^{UL} \equiv \frac{A_0(\pi^\pm \pi^\mp)}{A_0(\pi^\pm \pi^\pm)}$$

$$A_0^{UC} \equiv \frac{A_0(\pi^\pm \pi^\mp)}{A_0(\text{all } \pi)}$$

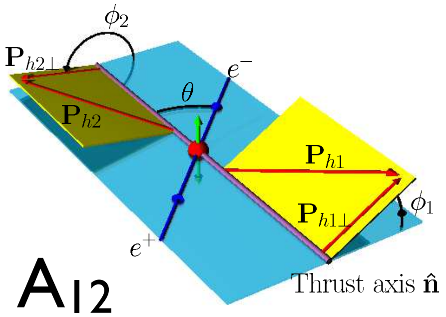
and similarly for A_{12}

Collins extractions

2 exp. frames:



A_0



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Belle data: $A_{0,12}^{UL,UC}(z_1, z_2)$ *Abe et al., P.R.L. **96** (06) 232002; S* *Seidl et al., P.R. D**78** (08) 032011, D**86** (12) 039905(E)*

$A_{12}^{UL,UC}(z_1, z_2, \mathbf{P}_{1T}, \mathbf{P}_{2T})$ *P.R. D**100** (19) 092008*

BaBar data: $A_{0,12}^{UL,UC}(z_1, z_2)\, , A_0^{UL,UC}(z_1, z_2, \mathbf{P}_{1T})\, , A_{12}^{UL,UC}(z_1, z_2, \mathbf{P}_{1T}, \mathbf{P}_{2T})$ *Lees et al., P.R. D**90** (14) 052003;* *Lees et al., P.R. D**92** (15) 111101*

BES data: $A_0^{UL,UC}(z_1, z_2, \mathbf{P}_{1T})$ *Ablikim et al., P.R.L. **116** (16) 042001*

Collins extractions

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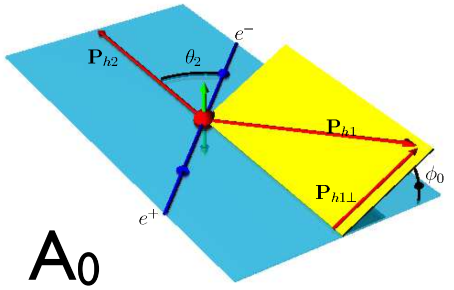
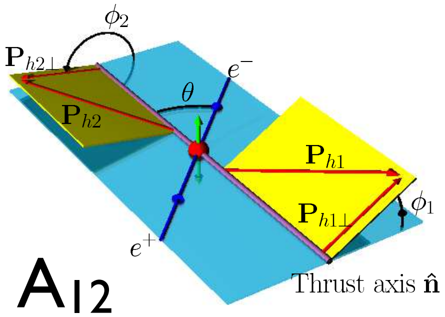


A0



A12

problems with
TMD factoriz. th.
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$$A_0^{UL} \equiv \frac{A_0(\pi^\pm \pi^\mp)}{A_0(\pi^\pm \pi^\pm)} \qquad A_0^{UC} \equiv \frac{A_0(\pi^\pm \pi^\mp)}{A_0(\text{all } \pi)}$$

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*Seidl et al., P.R. D****78*** *(08) 032011, D****86*** *(12) 039905(E)*
$A_{12}^{UL,UC}(z_1, z_2, \mathbf{P}_{1T}, \mathbf{P}_{2T})$ *P.R. D****100*** *(19) 092008*

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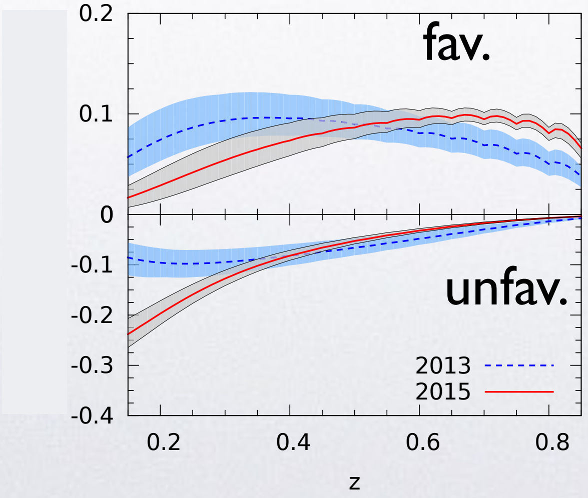
*Lees et al., P.R. D****92*** *(15) 111101*

BES data: $A_0^{UL,UC}(z_1, z_2, \mathbf{P}_{1T})$ *Ablikim et al., P.R.L.* ***116*** *(16) 042001*

2 main fits:

Torino 2013 *Anselmino et al.,*
*P.R. D****87*** *(13) 094019*

Torino 2015 *Anselmino et al.,*
*P.R. D****92*** *(15) 114023*



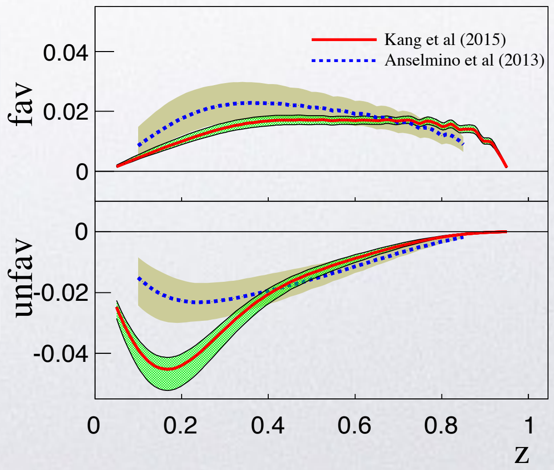
extended parton model

Torino 2013

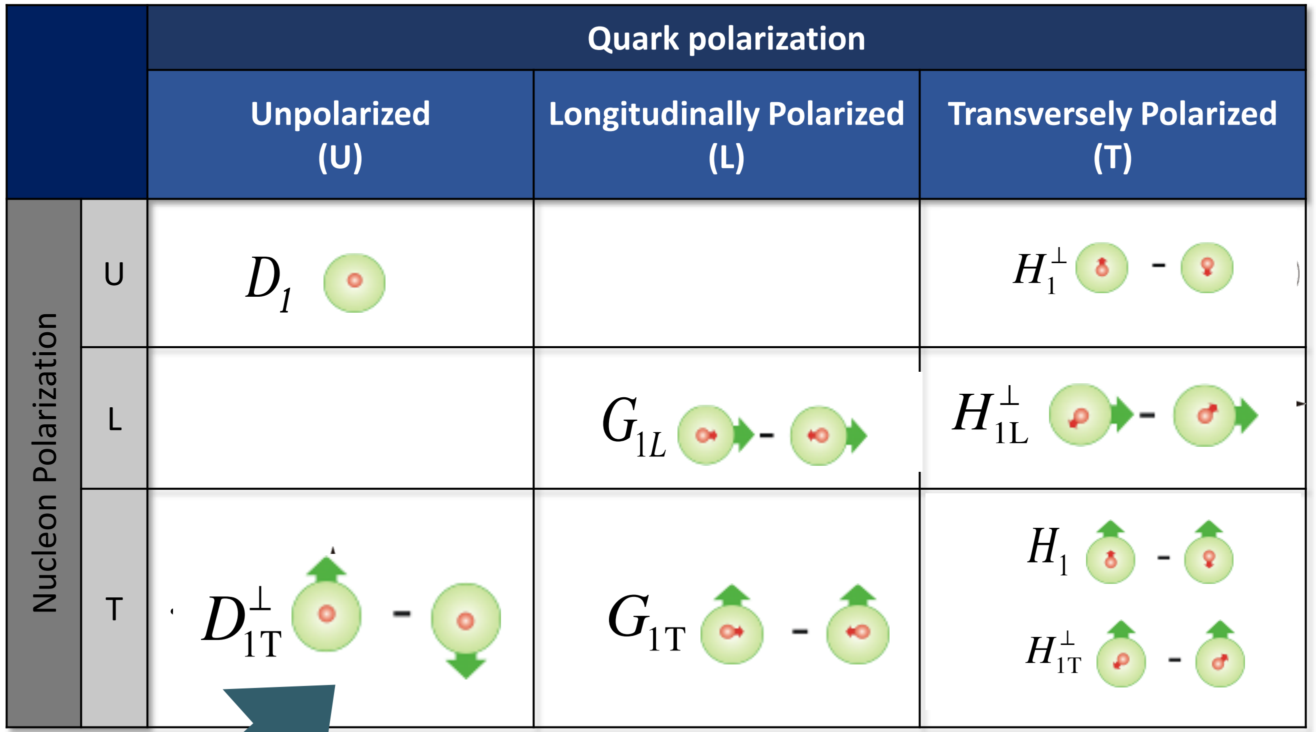
KPSY 2015

Kang et al.,
*P.R. D****93*** *(16) 014009*

TMD framework



Polarizing Fragmentation Function



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1		$H_1^\perp$ -
	L		G_{1L} -	$H_{1L}^\perp$ -
	T	$D_{1T}^\perp$ -	G_{1T} -	H_1 - $H_{1T}^\perp$ -

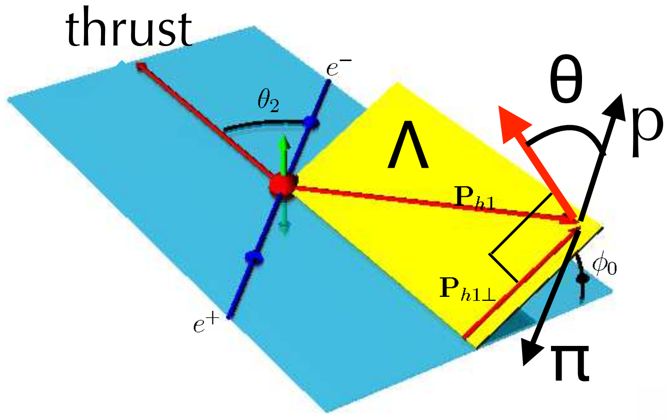
Belle data:

Guan et al. (BELLE), arXiv:1808.05000

$e^+ e^- \rightarrow \Lambda^\uparrow + X$

and also

$e^+ e^- \rightarrow \Lambda^\uparrow + \pi + X$



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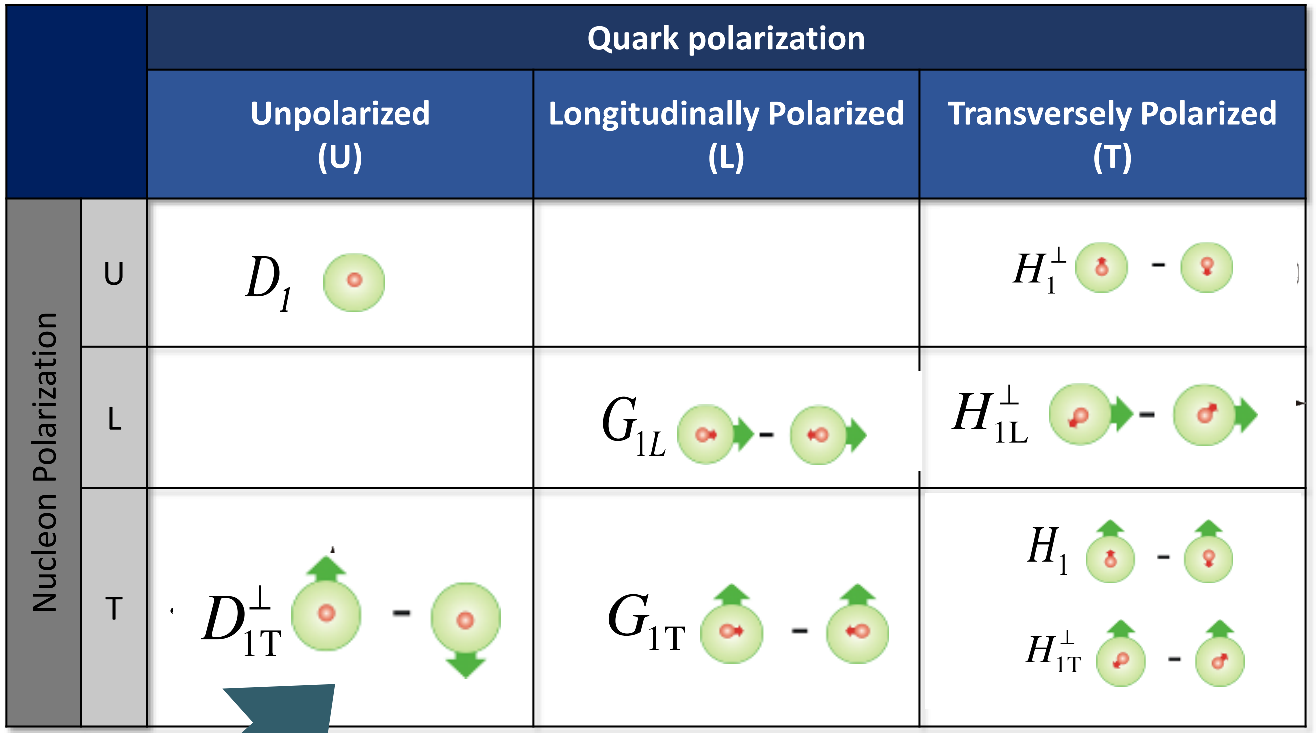

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Nucleon Polarization	U	D_1		$H_1^\perp$
	L		G_{1L}	$H_{1L}^\perp$
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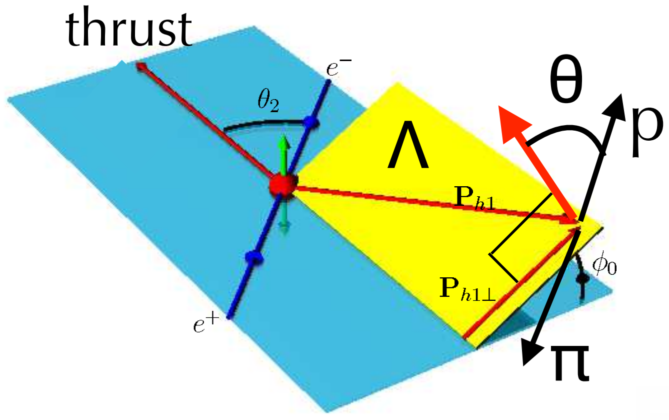
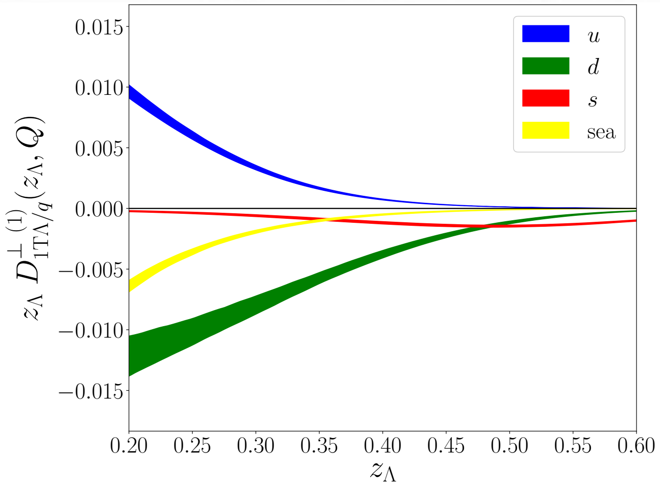


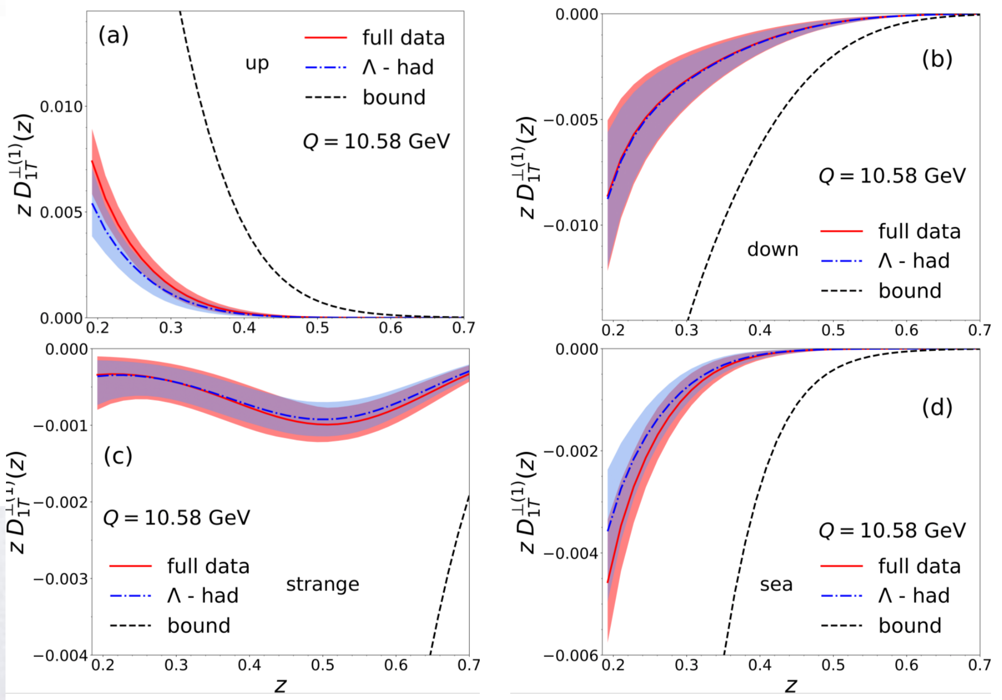
2 fits: both based on extended parton model

Callos, Kang, Terry, arXiv:2003.04828



$$D_{1T}^{\perp(1)}(z) = \int d^2\mathbf{p}_\perp \frac{p_\perp}{2zm_h} \Delta D_{h^\uparrow/q}(z, p_\perp)$$

D'Alesio, Murgia, Zaccheddu, arXiv:2003.01128



TMD factorisation th. for single Λ + thrust

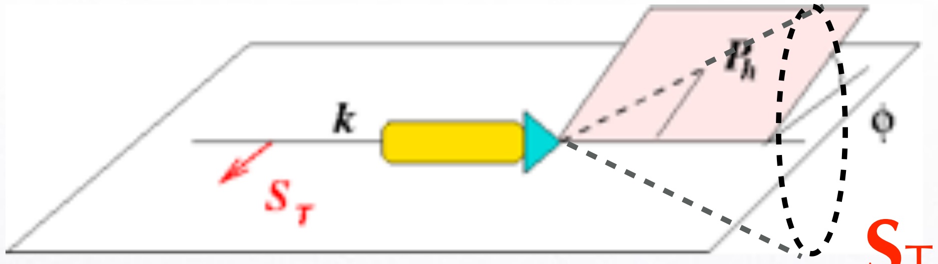
Gamberg et al., arXiv:2102.05553

work in progress for a fit within TMD factorization

Overview of current TMD phenomenology

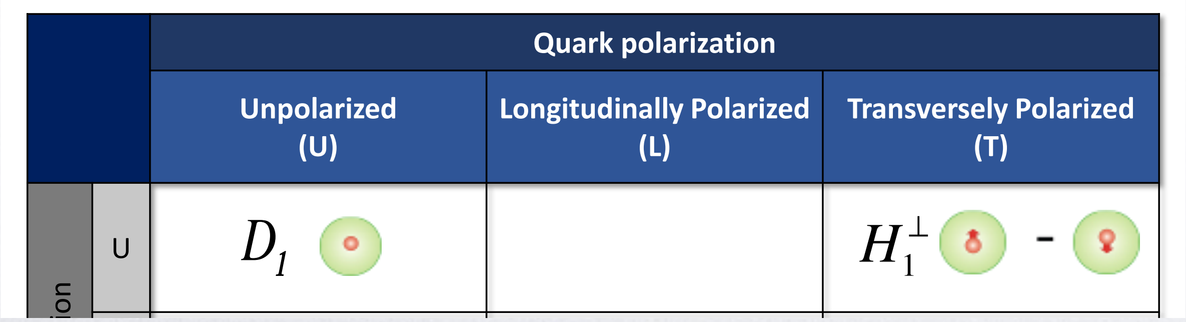
$$A_{UT}^{\sin(\phi_h+\phi_S)} \propto \frac{F_{UT}^{\sin(\phi_h+\phi_S)}}{F_{UU}} \sim \frac{h_1 \otimes H_1^\perp}{f_1 \otimes D_1}$$

complicated convolution
upon transverse momenta



based on Collins effect

$\mathbf{S}_T \cdot \mathbf{k}\times\mathbf{P}_{hT}$



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non recent extractions

*Anselmino et al., P.R. D**87** (13) 094019*
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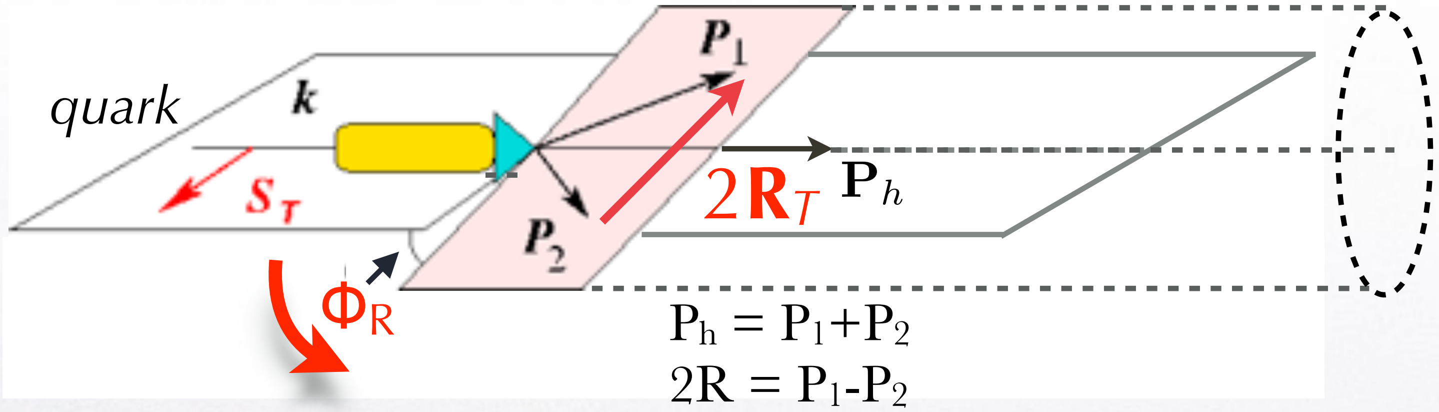
*Kang et al., P.R. D**93** (16) 014009*
*Lin et al., P.R.L. **120** (18) 152502 …*

But transversity is also a collinear PDF

$$A_{UT}^{\sin(\phi_R+\phi_S)} \propto \frac{h_1\, H_1^{\sphericalangle}}{f_1\, D_1}$$



di-hadron mechanism

$$\mathbf{S}_T \cdot \mathbf{P}_2\times\mathbf{P}_1 = \mathbf{S}_T \cdot \mathbf{P}_h\times\mathbf{R}_T$$

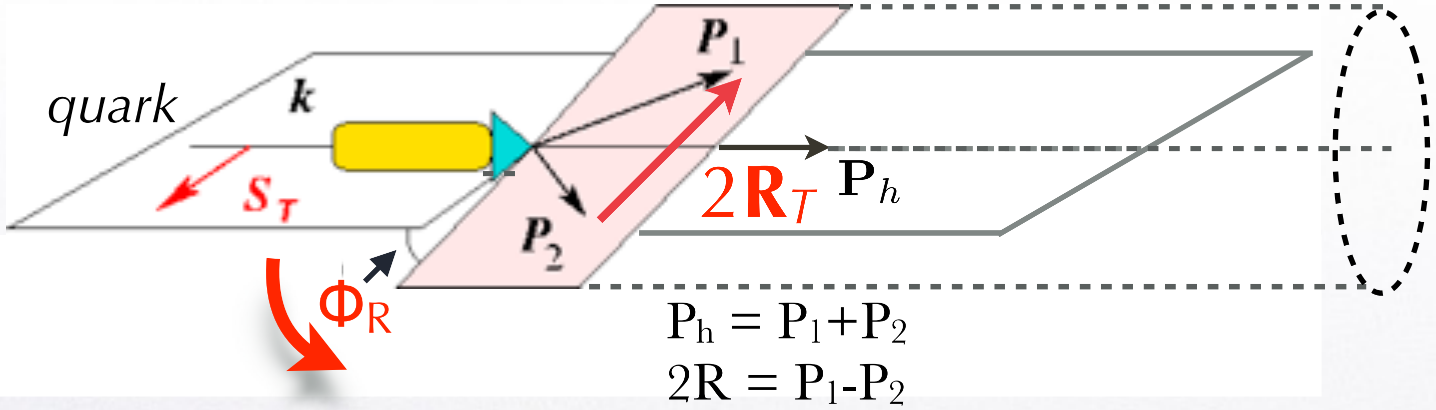
Collins, Heppelman, Ladinsky, N.P. ***B420*** *(94)*

collinear: $\mathbf{P}_h \,||\, \mathbf{k}$

But transversity is also a collinear PDF

$$A_{UT}^{\sin(\phi_R+\phi_S)} \propto \frac{h_1\, H_1^{\sphericalangle}}{f_1\, D_1}$$

requires knowledge of $H_1^{\sphericalangle}$
from e+e- → $(h_1 h_2)$+X data



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*Collins, Heppelman, Ladinsky, N.P. **B420** (94)*

collinear: $\mathbf{P}_h \,||\, \mathbf{k}$

only for $R_T^2 \propto M_{h_1h_2}^2 \ll Q^2$

define Di-hadron Fragmentation Functions (DiFF)

$\mathrm{DiFF}(z = z_1 + z_2, M_{h_1h_2}^2; Q^2)$

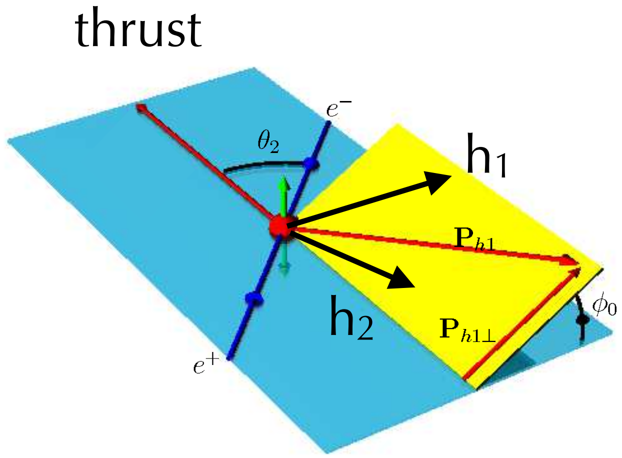
non recent extractions

*Jaffe, Jin, Tang, P.R.L. **80** (98) 1166*
*Radici, Jakob, Bianconi, P.R. D**65** (02) 074031*
*Bacchetta, Courtoy, Radici, P.R.L. **107** (11) 012001*
*Bacchetta, Courtoy, Radici, JHEP **03** (13) 119*
*Radici et al., JHEP **05** (15) 123*

DiFF extraction

Belle data for $A^{\cos\phi}$

*Vossen et al., P.R.L. **107** (11) 072004*



“thrust-axis” frame

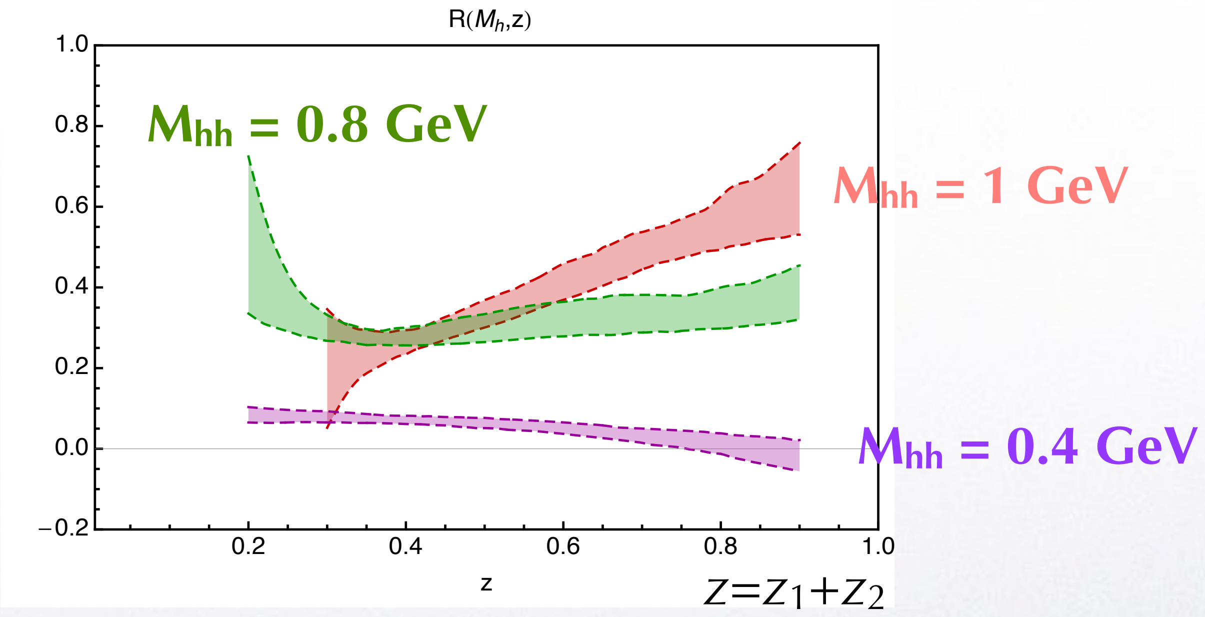
1 fit :

Pavia 2012

*Courtoy et al., P.R. D**85** (12) 114023*

refined in

*Radici et al., JHEP **05** (15) 123*



based on predictions

*Boer, Jakob, Radici, P.R.D**67** (03) 094003*

see also

Matevosyan, Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas, arXiv:1802.01578

Recent analyses

	Mechanism	Framework	SIDIS	e+e-	p-p collisions	N of points
Pavia 2018 arXiv:1802.05212	collinear DiFF	LO	✔	✔	✔	78
JAM 2020 arXiv:2002.08384	TMD Collins effect	extended parton model	✔	✔	✔	517
MEX 2019 arXiv:1912.03289	collinear DiFF	LO	✔	✔	✘	68
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First global fit

Recent analyses

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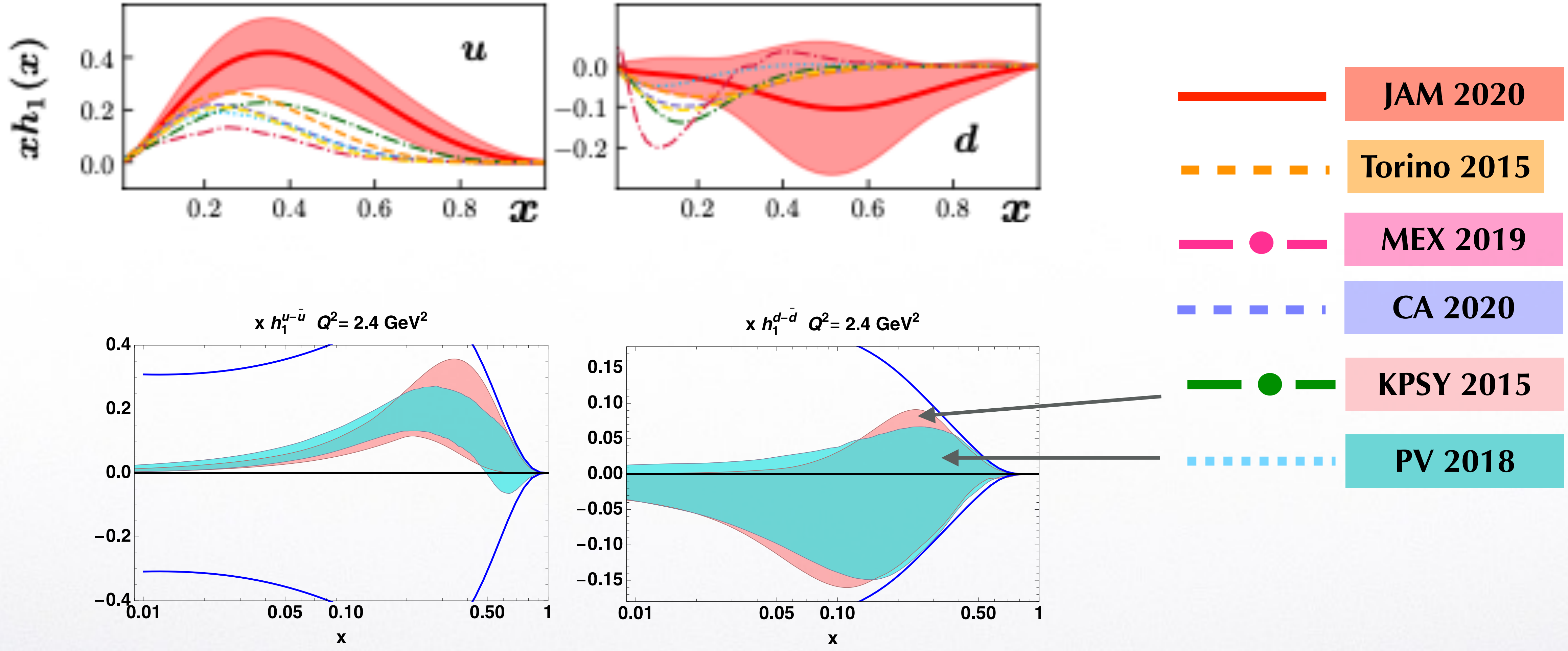
MEX 2019 arXiv:1912.03289	collinear DiFF				✘	68
CA 2020 arXiv:2001.01573	TMD Collins effect	extended parton model	✔	✔	✘	76

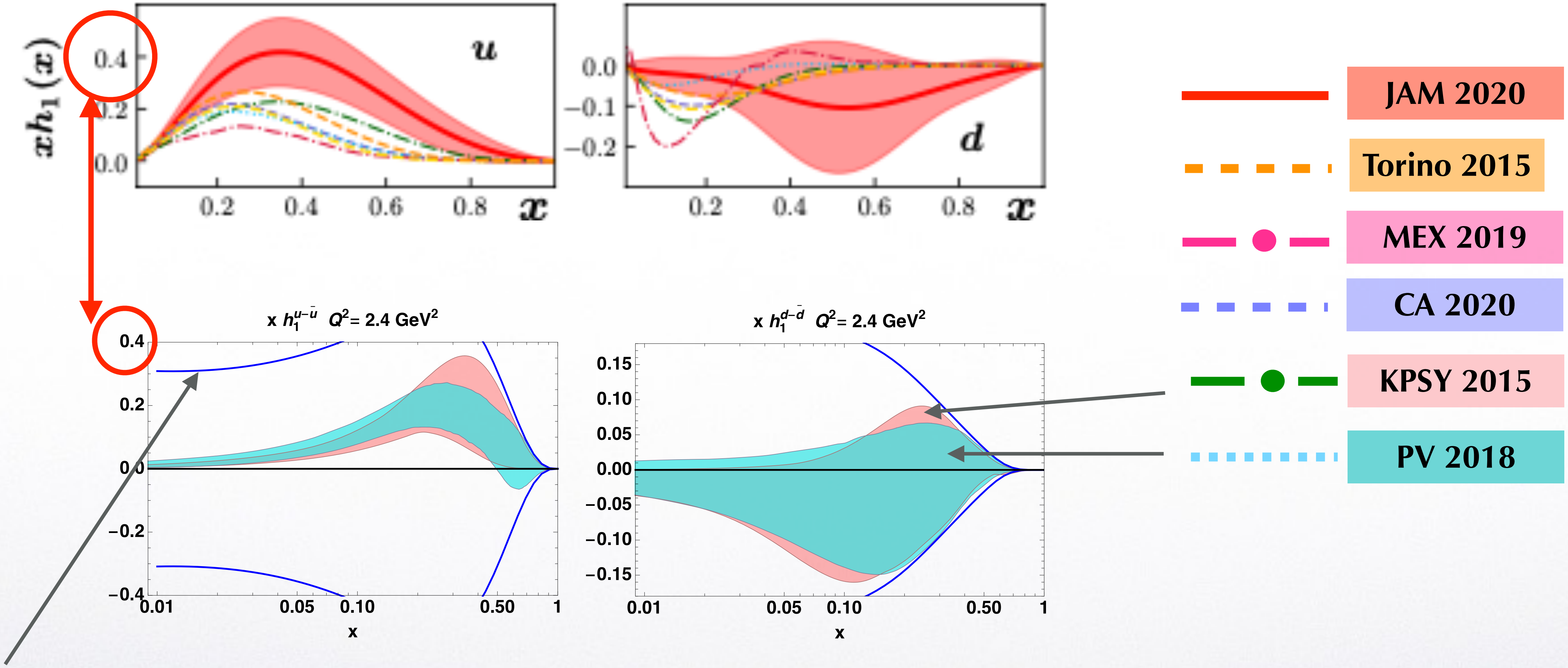
global fit (but simplified analysis)

- very few data
- need to improve accuracy of formalism

Transversity extractions



Transversity extractions



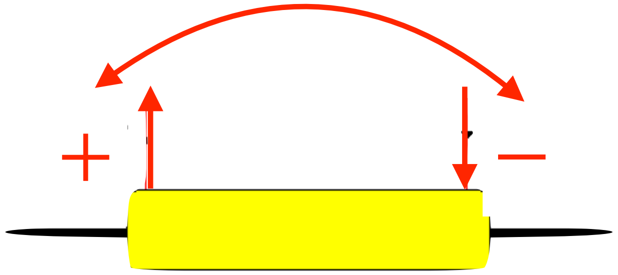
Soffer bound violated $|h_1(x,Q^2)| \leq \frac{1}{2}|f_1(x,Q^2)+g_1(x,Q^2)|$ (Soffer bound ↔ positivity)

PV 2018 fullfils Soffer bound by construction

Why is transversity important?

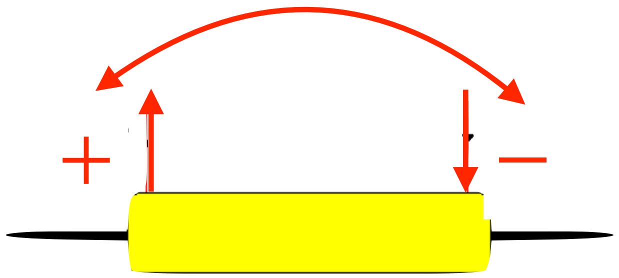
- chiral-odd structure in spin-1/2 hadron
  no gluon transversity → h_1 is a non-singlet object

$$\max \Delta s_L = |S'_L\text{-}S_L|$$



Why is transversity important?

max $\Delta s_L = |S'_L - S_L|$



- chiral-odd structure in spin-1/2 hadron
  no gluon transversity → h_1 is a non-singlet object

- doorway to BSM physics:
  - SM EFT of CP violation from neutron EDM d_n

$$d_n = \boxed{\delta u}\, d_u + \boxed{\delta d}\, d_d + \boxed{\delta s}\, d_s$$

bounds from exp.

tensor charge

= 1st Mellin moment of h_1

$$\delta^q(Q^2) = \int_0^1 dx\, h_1^{q-\bar{q}}(x, Q^2)$$

  - SM EFT with tensor operators → tensor coupling in nucleon β-decay

<u>hadron level</u> : $n \to p\, e^-\, \nu_e$

<u>quark level</u> : $d \to u\, e^-\, \nu_e$

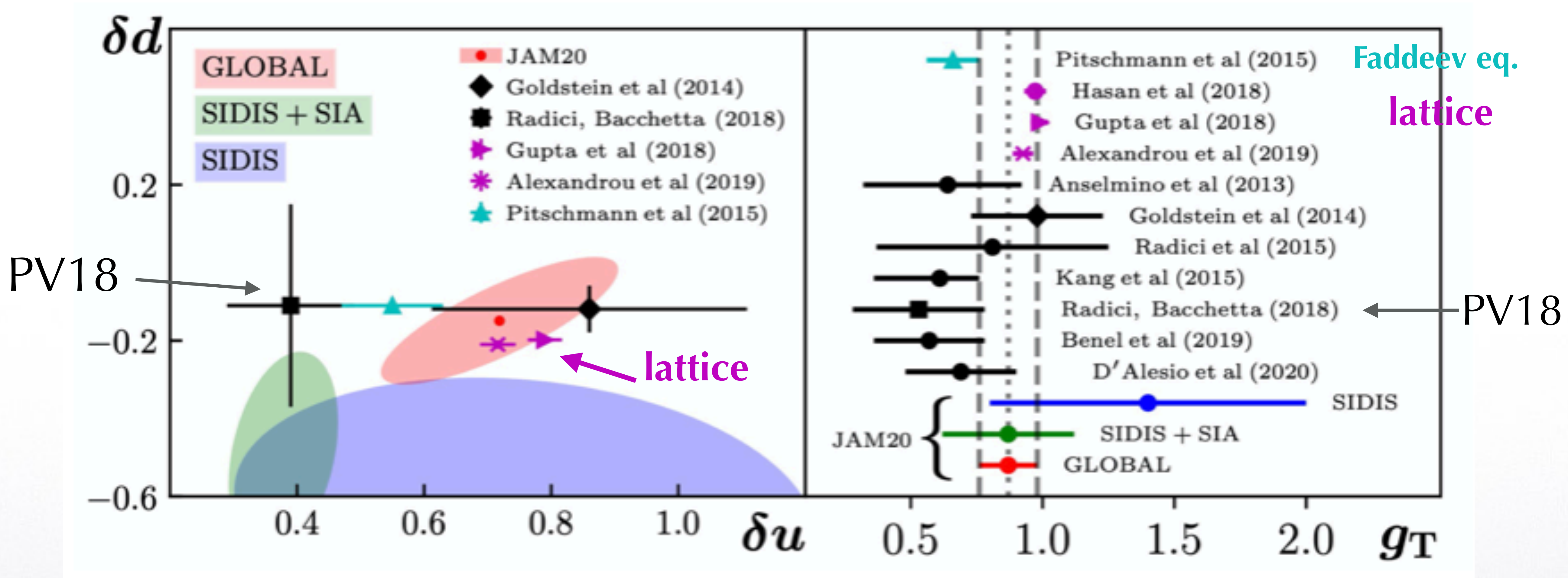
$$\langle p|\, \bar{u}\, \sigma^{\mu\nu}\, d\, |n\rangle \quad \epsilon_T\ \bar{e}\sigma_{\mu\nu}(1-\gamma_5)\nu_e$$

$$\text{bounds from exp.} \longrightarrow C_T \leftrightarrow \boxed{g_T}\,\epsilon_T \longleftarrow \text{unknown}$$

$$g_T = \delta u - \delta d$$

isovector tensor charge

Tensions on tensor charge



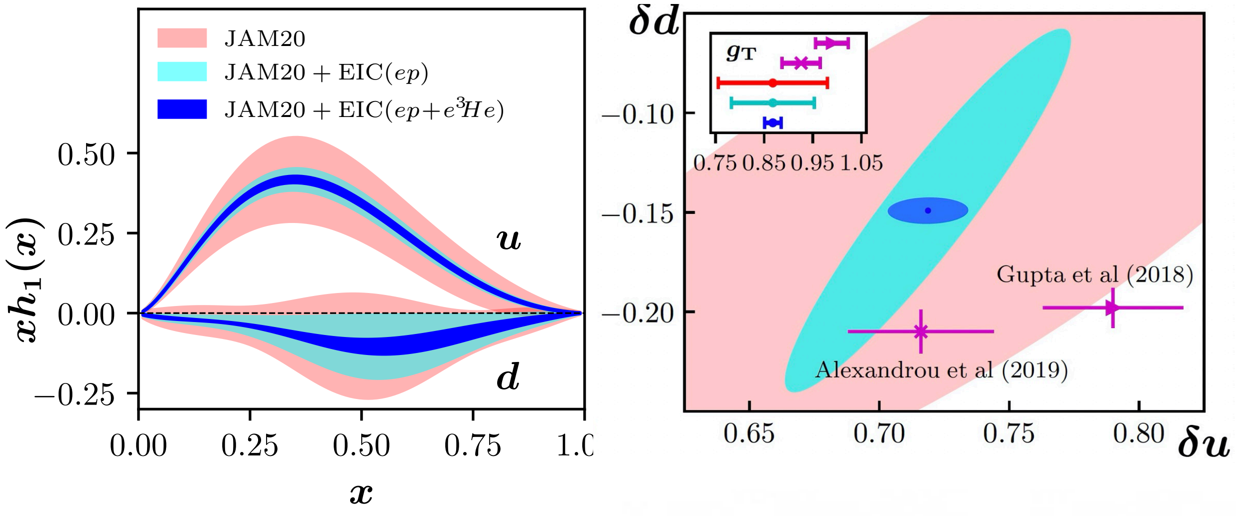
*Cammarota et al. (JAM20), PR D**102** (20) 054002, arXiv:2002.08384*

sometimes discrepancy with lattice

role of Soffer bound? *D'Alesio et al., P.L. **B802** (20) 135347, arXiv:2001.01573*

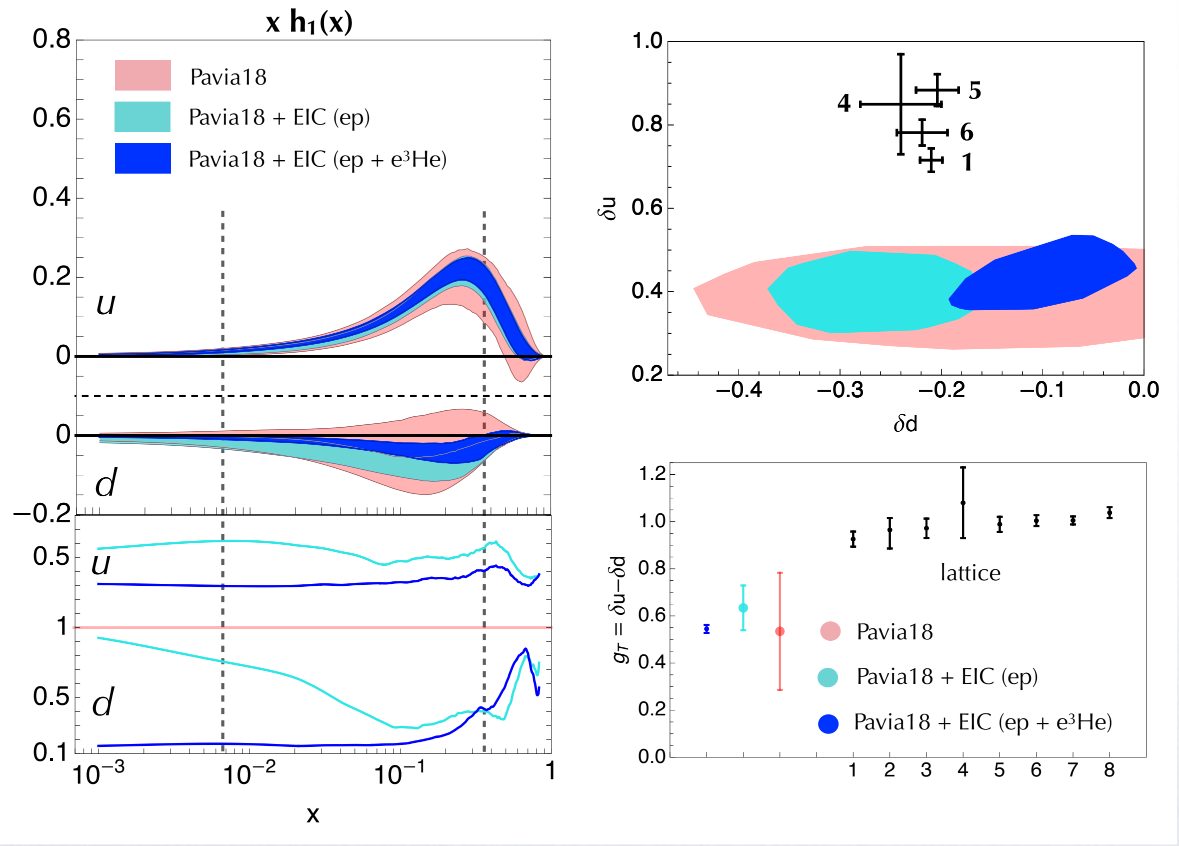
data in the $0.006 \lesssim x \lesssim 0.3$ range → need to constrain extrapolation $\delta q = \int_0^1 dx \ldots$

EIC impact on tensor charge



using JAM20 parametrisation (Collins effect)

EIC Yellow Report, arXiv:2103.05419



using PV18 parametrisation (DiFF mechanism)

1) **ETMC '19** *Alexandrou et al., arXiv:1909.00485*

2) **Mainz '19** *Harris et al., P.R. D**100** (19) 034513*

3) **LHPC '19** *Hasan et al., P.R. D**99** (19) 114505*

4) **JLQCD '18** *Yamanaka et al., P.R. D**98** (18) 054516*

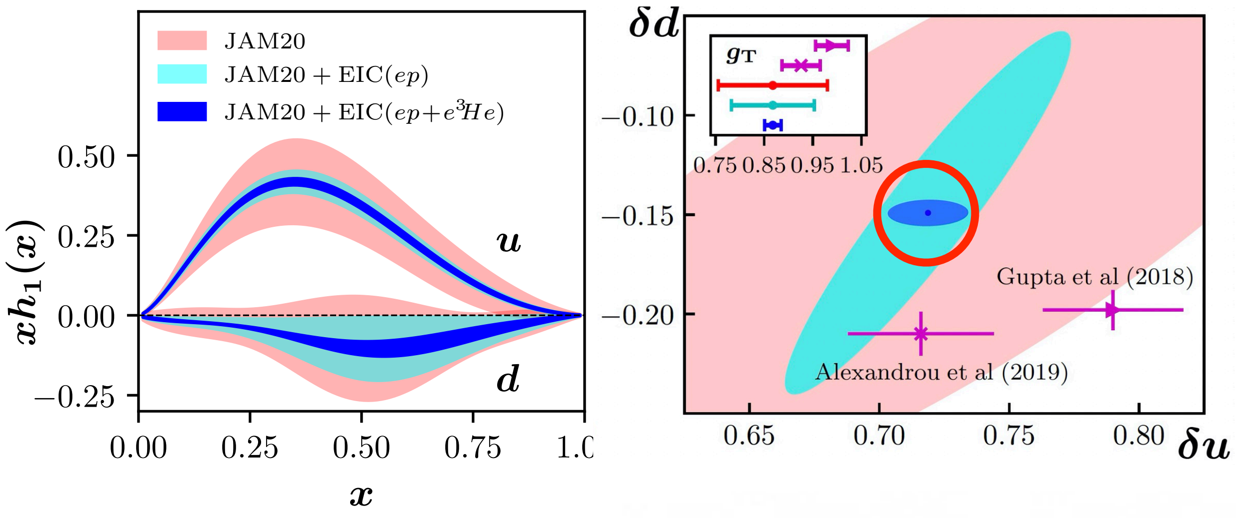
5) **PNDME '18** *Gupta et al., P.R. D**98** (18) 034503*

6) **ETMC '17** *Alexandrou et al., P.R. D**95** (17) 114514; (E) P.R. D**96** (17) 099906*

7) **RQCD '14** *Bali et al., P.R. D**91** (15) 054501*

8) **LHPC '12** *Green et al., P.R. D**86** (12) 114509*

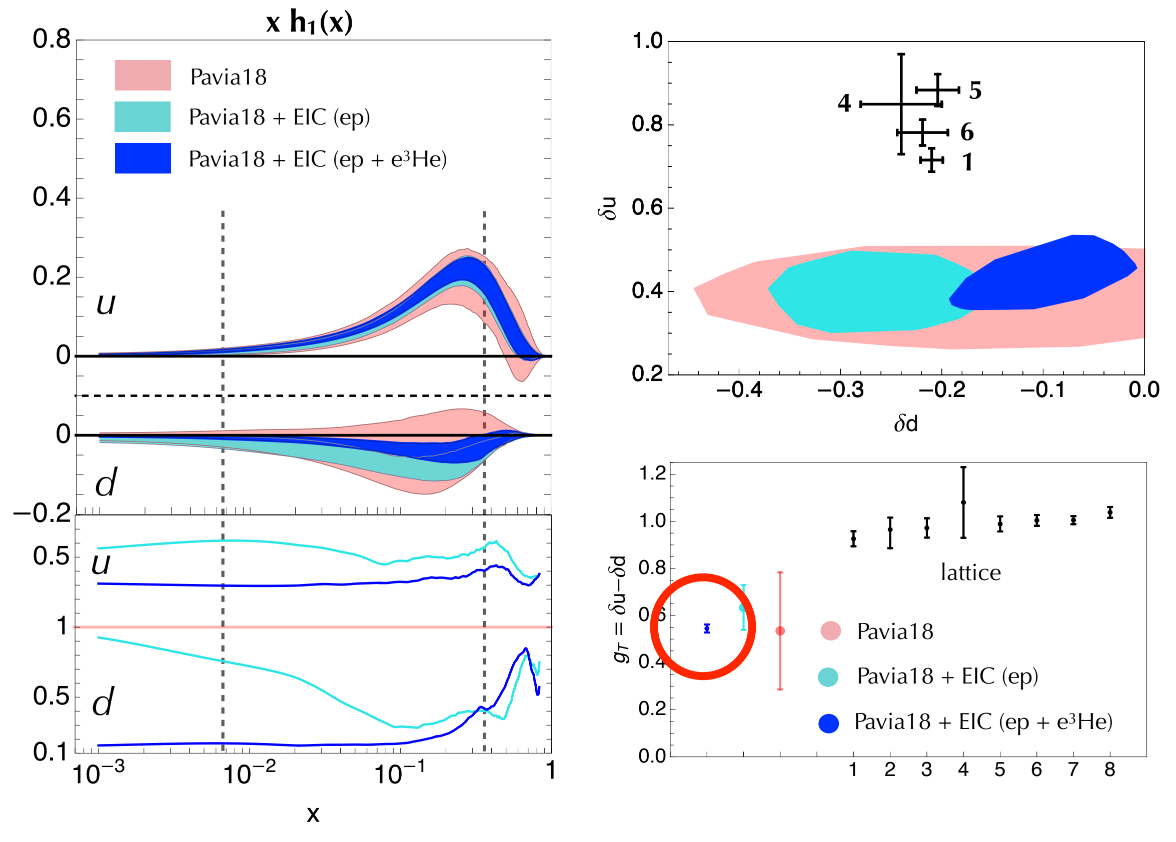
EIC impact on tensor charge



using JAM20 parametrisation (Collins effect)

expected precision close to (or higher than) lattice

EIC Yellow Report, arXiv:2103.05419



using PV18 parametrisation (DiFF mechanism)

1) **ETMC ’19** *Alexandrou et al., arXiv:1909.00485*

2) **Mainz ’19** *Harris et al., P.R. D**100** (19) 034513*

3) **LHPC ’19** *Hasan et al., P.R. D**99** (19) 114505*

4) **JLQCD ’18** *Yamanaka et al., P.R. D**98** (18) 054516*

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7) **RQCD ’14** *Bali et al., P.R. D**91** (15) 054501*

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gluon TMDs

Gluon TMDs are phenomenologically unknown.
Why ?

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Why ?

- gluons carry no electric charge → in SIDIS they appear only at higher orders
- gluons carries “two color charges” → in general, difficult to neutralise them all
- in hadronic collisions, gluons appear at tree level, but :
  - factorisation theorem available only for Drell-Yan processes
  - for $H_1+H_2 \rightarrow h+X$ no factor. th. but also no counterexample disproving it

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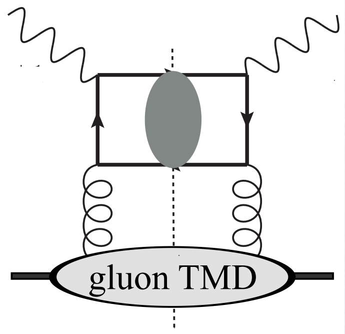
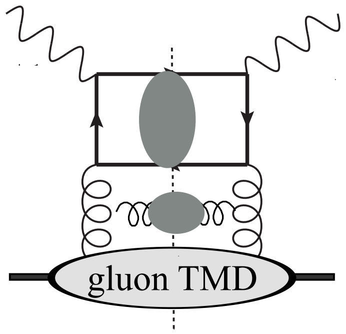
- useful processes under study:

e p↑→ e+J/ψ+X

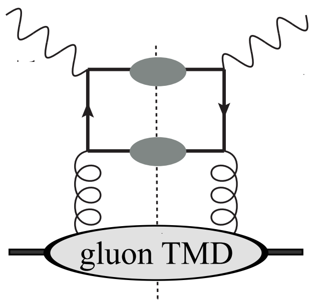


e p↑→ e+J/ψ+jet+X

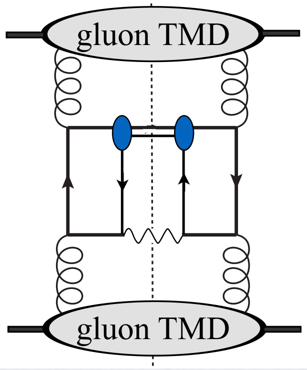


$e\, p \rightarrow e$ jet jet X

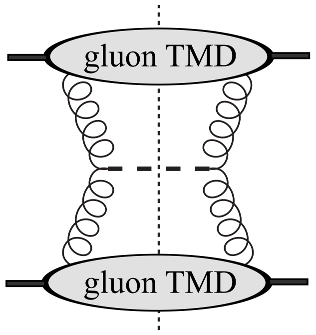
e p↑→ $e+h_1+h_2+X$



$p\, p \rightarrow J/\psi\, \gamma\, X$

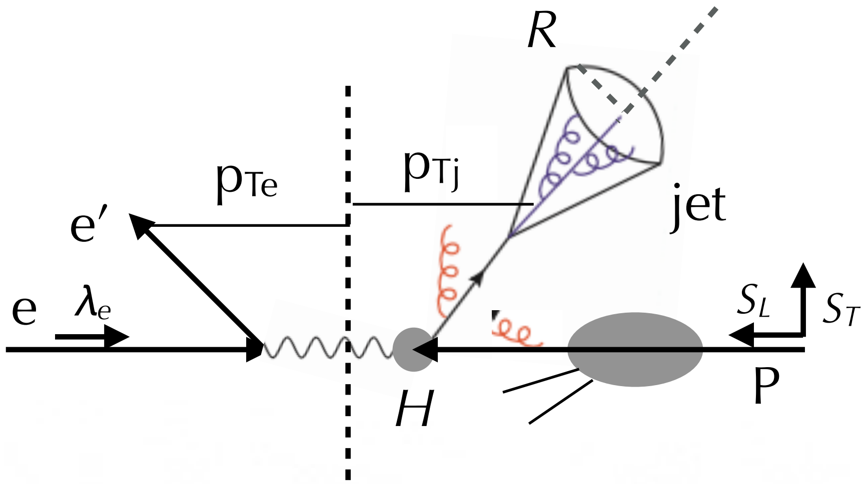


$p\, p \rightarrow \eta_c\, X$



Boer et al., P.R.L. ***108*** *(12) 032002*
den Dunnen et al., P.R.L. ***112*** *(14) 212001*
Mukherjee & Rajesh, arXiv:1609.05596
Boer et al., arXiv:1605.07934
Godbole et al., arXiv:1703.01991
D'Alesio et al., arXiv:1705.04169
Rajesh et al., arXiv:1802.10359
Zheng et al., arXiv:1805.05290
Bacchetta et al., arXiv:1809.02056
D'Alesio et al., arXiv:1908.00446
D'Alesio et al., arXiv:1910.09640
....

TMDs with jets: SIDIS



$$\mathbf{q}_T = \mathbf{p}_{Te} + \mathbf{p}_{Tj} \ll \mathbf{p}_T = \frac{\mathbf{p}_{Te} - \mathbf{p}_{Tj}}{2}$$

$$F_{UU} \sim \underset{\text{hard}}{H(Q)}\, \underset{\text{jet}}{J(p_T R, Q)}\, \underset{\text{“dressed” TMD}}{\{f_1(x, q_T, Q)\}}$$

similarly for other *F*..

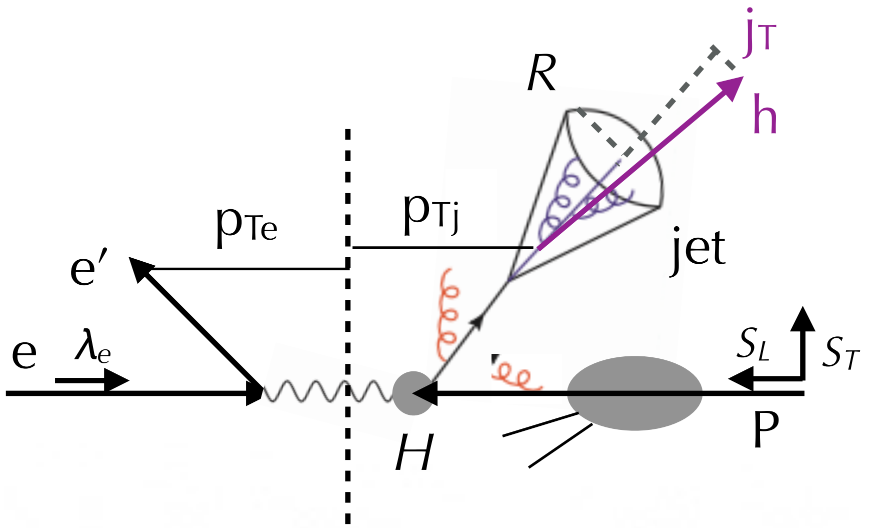
“familiar” expression

$$\frac{d\sigma}{dy_j d\mathbf{p}_T d\mathbf{q}_T} = \overset{f_1}{F_{UU}} + \lambda_e S_L \overset{g_{1L}}{F_{LL}}$$

$$+ S_T \sin(\phi_j - \phi_S)\, \overset{f_{1T}^{\perp}}{F_{UT}^{\sin(\phi_j - \phi_S)}} + \lambda_e S_T \cos(\phi_j - \phi_S)\, \overset{g_{1T}}{F_{LT}^{\cos(\phi_j - \phi_S)}}$$

Kang et al., arXiv:2106.15624

TMDs with jets: SIDIS



$$\mathbf{q}_T = \mathbf{p}_{Te} + \mathbf{p}_{Tj} \ll \mathbf{p}_T = \frac{\mathbf{p}_{Te} - \mathbf{p}_{Tj}}{2}$$

$$\underset{\text{hard}}{F_{UU} \sim H(Q)}\ \underset{\text{jet}}{\mathrm{TMDJFF}(z_h, p_T R, Q)}\ \underset{\text{“dressed” TMD}}{\{f_1(x, q_T, Q)\}}$$

similarly for other *F*..

“familiar” expression

$$\frac{d\sigma}{dy_j d\mathbf{p}_T d\mathbf{q}_T} = \overset{f_1\,\mathscr{D}_1}{F_{UU}} + \cos(\phi_j - \phi_h)\,\overset{h_1^\perp\,\mathscr{H}_1^\perp}{F_{UU}^{\cos(\phi_j-\phi_h)}} + \lambda_e S_L\,\overset{g_{1L}\,\mathscr{D}_1}{F_{LL}}$$

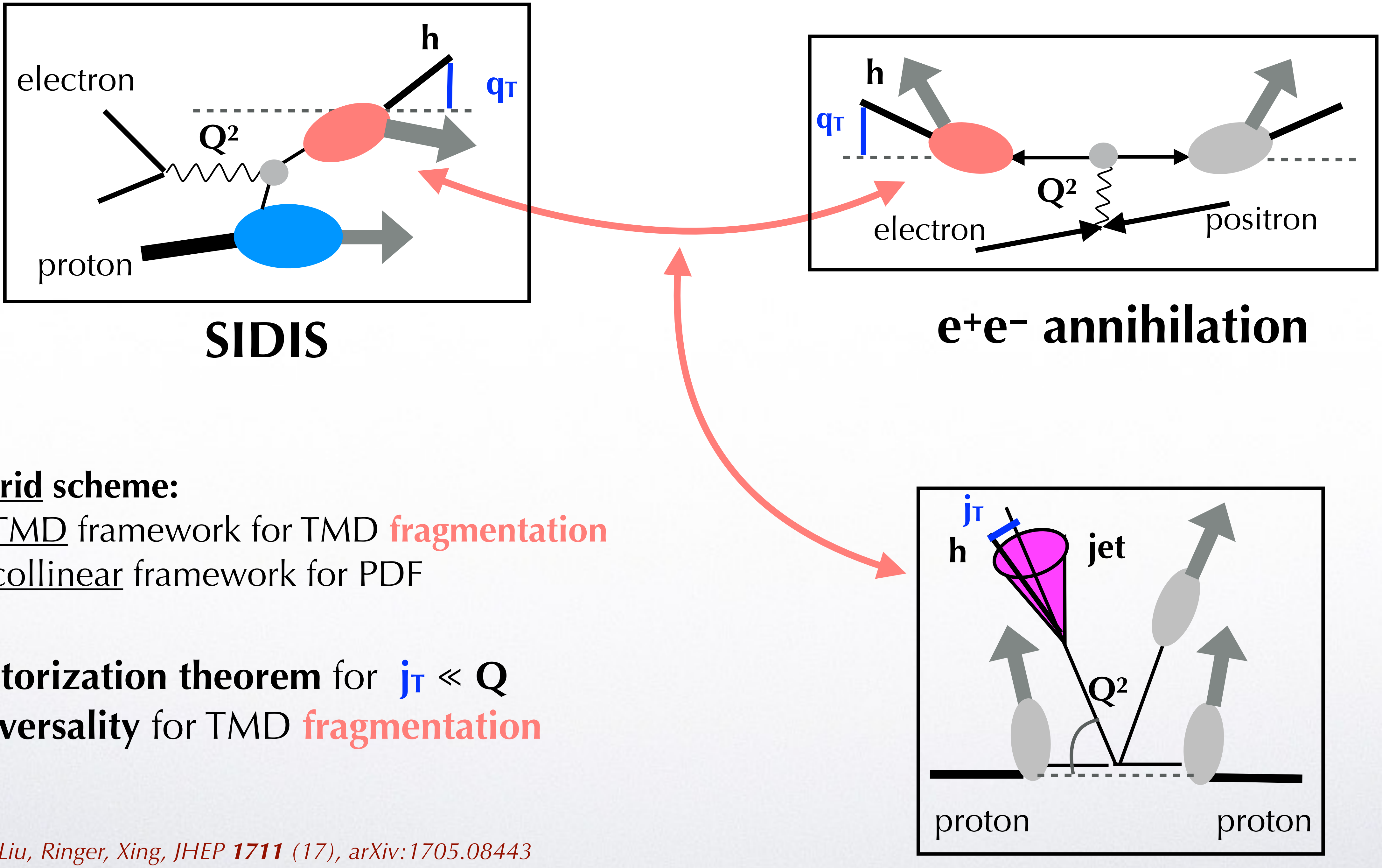
$$+ S_L \sin(\phi_j - \phi_h)\,\overset{h_{1L}^\perp\,\mathscr{H}_1^\perp}{F_{UL}^{\sin(\phi_j-\phi_h)}}$$

$$+ S_T \sin(\phi_j - \phi_S)\,\overset{f_{1T}^\perp\,\mathscr{D}_1}{F_{UT}^{\sin(\phi_j-\phi_S)}} + \lambda_e S_T \cos(\phi_j - \phi_S)\,\overset{g_{1T}\,\mathscr{D}_1}{F_{LT}^{\cos(\phi_j-\phi_S)}}$$

$$+ S_T \sin(\phi_h - \phi_S)\,\overset{h_1\,\mathscr{H}_1^\perp}{F_{UT}^{\sin(\phi_h-\phi_S)}} + S_T \sin(2\phi_j - \phi_h - \phi_S)\,\overset{h_{1T}^\perp\,\mathscr{H}_1^\perp}{F_{UT}^{\sin(2\phi_j-\phi_h-\phi_S)}}$$

Kang et al., arXiv:2106.15624

TMDs with jets: hybrid factorisation



SIDIS

e^+e^- annihilation

hybrid scheme:

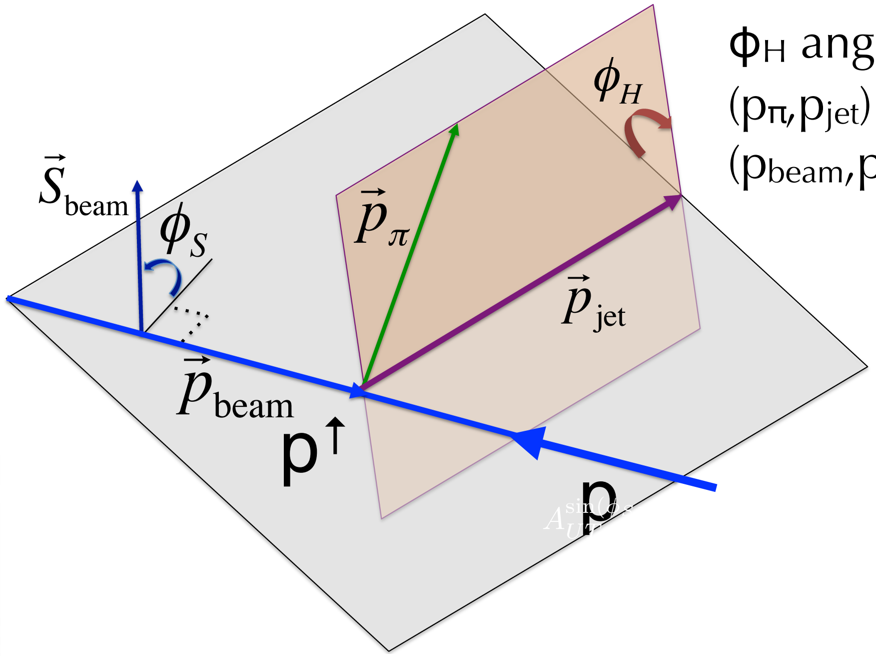
- TMD framework for TMD **fragmentation**
- collinear framework for PDF

Factorization theorem for $j_T \ll Q$

universality for TMD **fragmentation**

*Kang, Liu, Ringer, Xing, JHEP **1711** (17), arXiv:1705.08443*
*Kang, Prokudin, Ringer, Yuan, P.L. **B774** (17), arXiv:1707.00913*

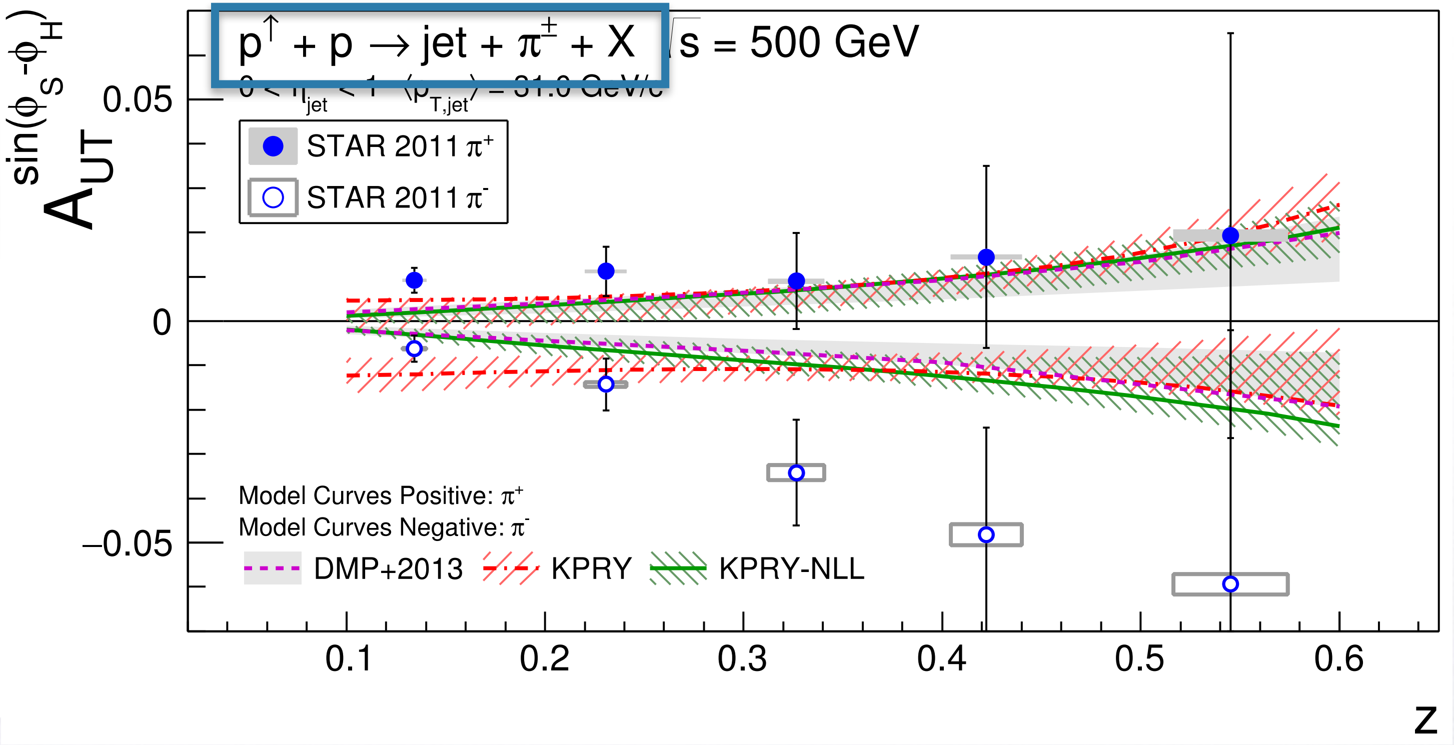
hadron-in-jet Collins effect



ϕ$_H$ angle of
(p$_\pi$,p$_{jet}$) plane and
(p$_{beam}$,p$_{jet}$) plane

$$A_T^{\sin(\phi_S-\phi_H)} \propto \frac{h_1^q \otimes f_1^{\bar{q}} \otimes H_1^{\perp q}}{f_1^q \otimes f_1^{\bar{q}} \otimes D_1^q}$$

*Adamczyk et al. (STAR), P.R. D**97** (18) 032004*



PDF & TMDFF from
SIDIS + e$^+$e$^-$ analysis

(no evolution)

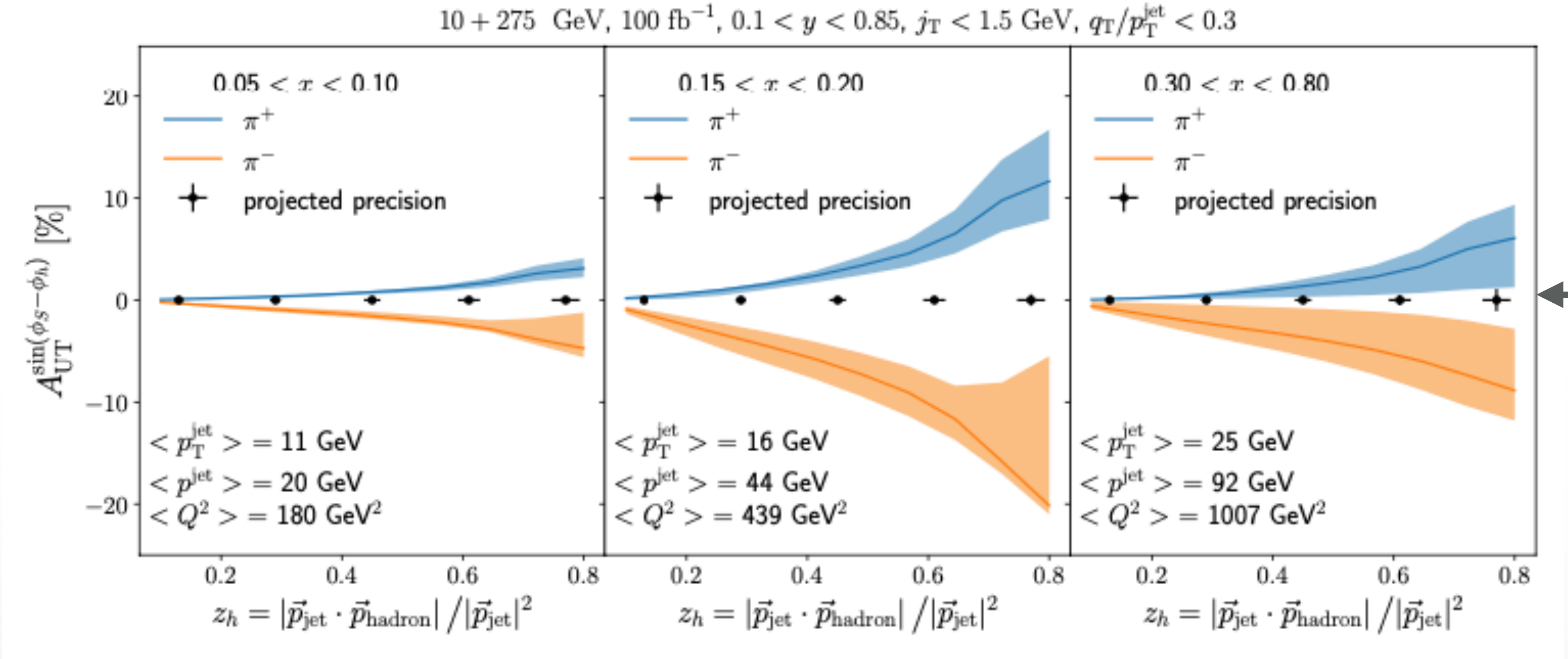
*D'Alesio et al., P.L. **B773** (17) 300*

no evolution

TMD evolution

*Kang et al., P.L. **B774** (17) 635*

EIC impact on hadron-in-jet Collins effect

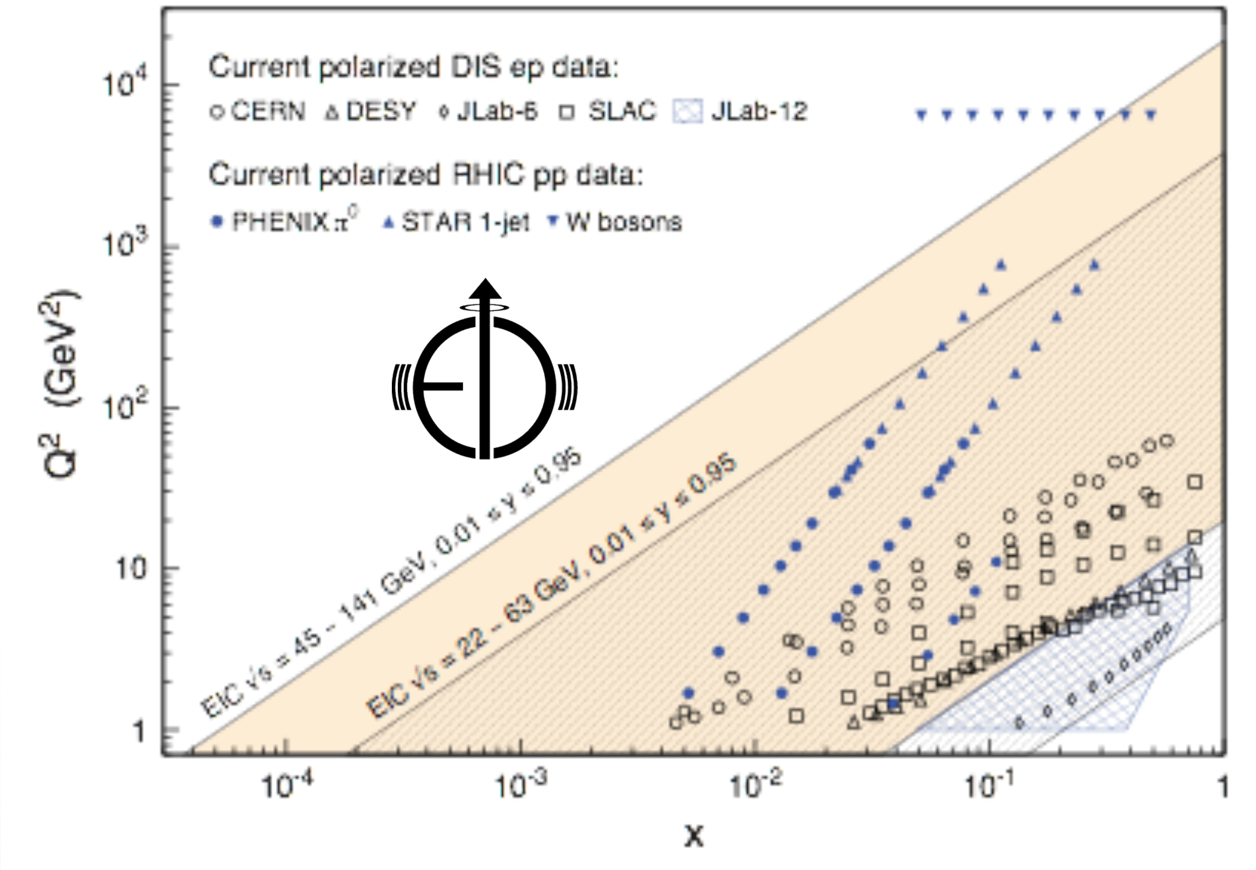


projected uncertainties

*Arratia et al., P.R. D**102** (20) 074015, arXiv:2007.07281*

based on *Kang et al., P.R. D**93** (16) 014009*

The EIC project and TMDs



The EIC from the TMD point of view:

- enlarging phase-space coverage
- high polarization
- high statistics

Benefits:

- deepening knowledge of all (un)polarized TMDs
- explore new channels involving jets in final state
- explore unknown territory of gluon TMDs