NLO corrections to DDVCS, DVCS and TCS processes

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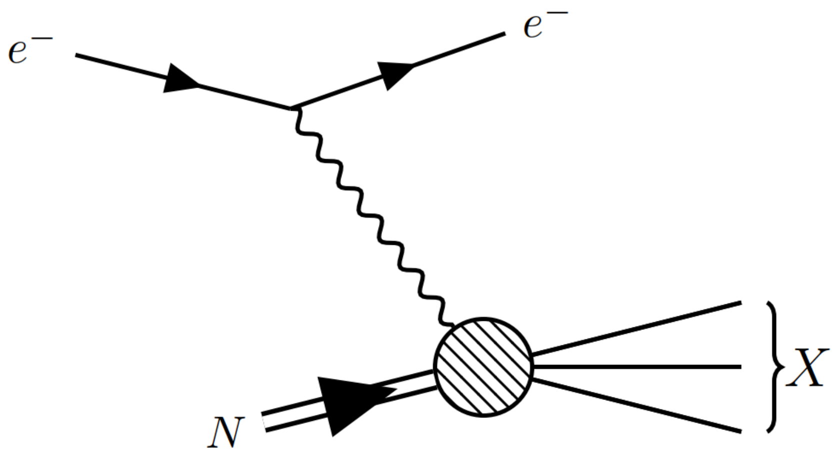
Why NLO corrections?

- Precise determination of GPDs (and so nucleon tomography). Also, gluon GPDs appear at NLO!!

- Universality testing in processes that at Born level look alike: DVCS & TCS
    - **"On timelike and spacelike hard exclusive reactions"**, D. Mueller, B. Pire, L. Szymanowski, J. Wagner - DOI: 10.1103/PhysRevD.86.031502
    - **"Data-driven study of timelike Compton scattering"**, O. Grocholsky, H. Moutarde, B. Pire, P. Sznajder, J. Wagner - DOI: 10.1140/epjc/s10052-020-7700-9

- Apparent divergences of the form $\ln 0$
- Study of feasibility of DDVCS at EIC
- Framework for analysis of JLab 12 GeV data

Refreshing Deep Inelastic Scattering (DIS)



Factorization in cross-section

$$d\sigma_{\mathrm{DIS}} \sim L^{\mu\nu} W_{\mu\nu}$$

pQED

Non-perturbative QCD

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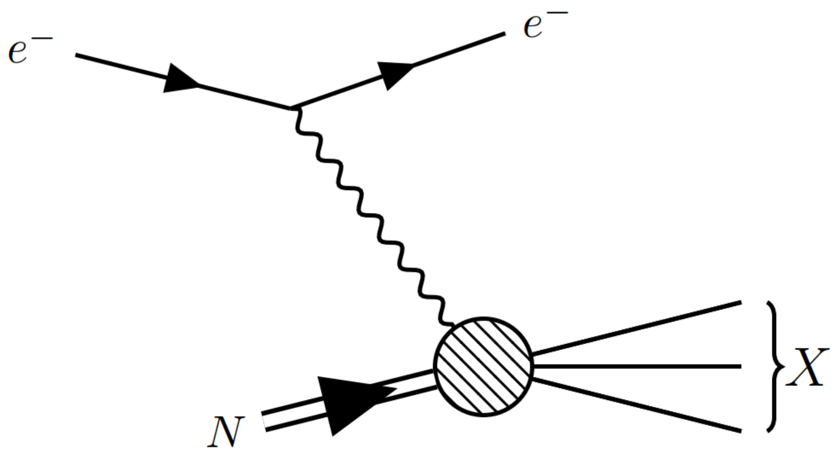

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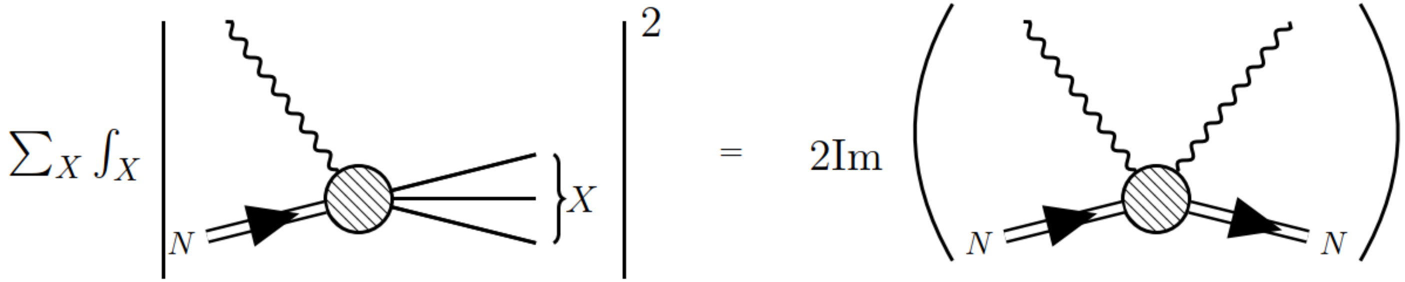
Optical theorem to DIS



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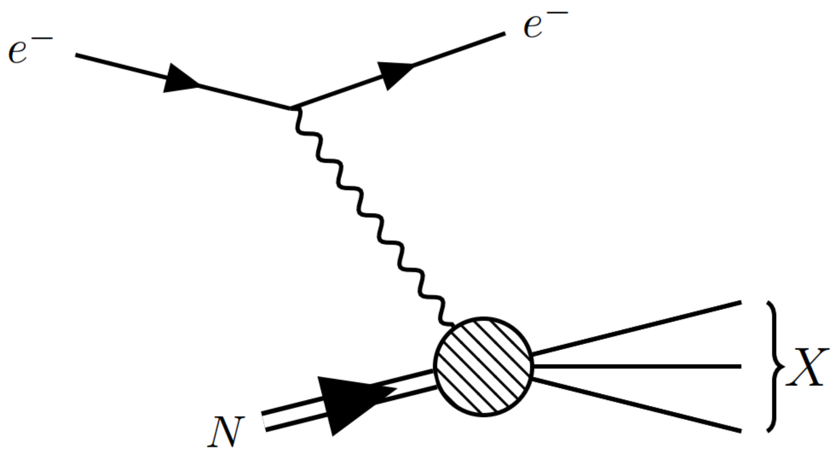

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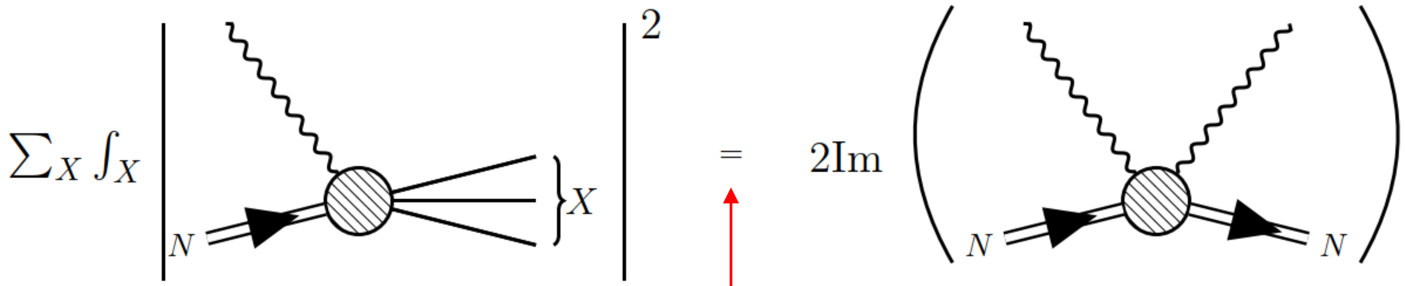

$$\sum_X \int_X |X\rangle\langle X| = 1$$ → DIS is inclusive process

Cf. Riedl's talk for more details in kinematics and experimental results

Parton Distribution Functions (PDFs)

- Extracted from the hadronic tensor $W_{\mu\nu}$, they determine the internal nucleon structure

$$\mathrm{PDF}(x) = \frac{1}{2}\int \frac{dz^+}{2\pi} e^{ixp^- z^+} \langle P|\bar{q}_f(-z/2)\gamma^+ \mathcal{W}[-z/2, z/2] q_f(z/2)|P\rangle \big|_{z_\perp = z^- = 0}$$

Wilson line definition

$$\mathcal{W}[z_1^+, z_2^+] = \mathbb{P}\exp\left[ig\int_{z_2^+}^{z_1^+} d\eta^+ A^-\right]$$

Samuel Wallon. Hard exclusive processes in perturbative QCD: from medium to asymptotical energies. Doctoral Schools ED 107 (Physique de la Région Parisienne) and ED 517 (Particules, Noyaux et Cosmos). 2014.

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$$\gamma^+ = \frac{\gamma^0 + \gamma^3}{\sqrt{2}}$$

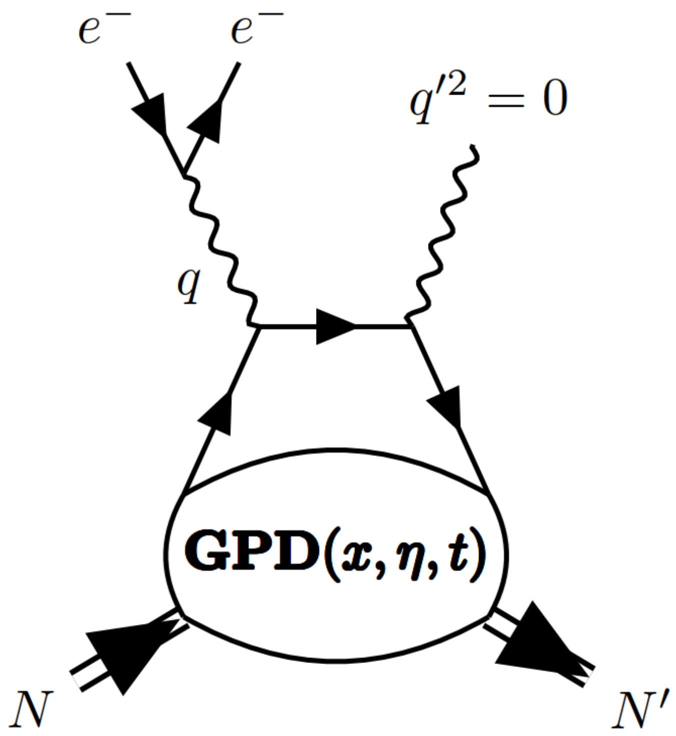
PDF is 1D

x = longitudinal parton momentum fraction

Improving PDF's 1D picture

- In the late '90s, Ji, Müller and Radyushkin introduce Generalized Parton Distributions (GPDs) through Deeply Virtual Compton Scattering (DVCS) process

- The point now is to study the conversion of a virtual photon into a real one



Cf. Ji's talk for theory; and Ent and Newman's talks for phenomenology

DVCS = exclusive process = factorization in amplitude

Sketch of DVCS amplitude

$$\begin{aligned}\mathcal{A}_{\mathrm{DVCS}} &\sim \int_{-1}^{1} dx \, \frac{1}{x-\eta+i0} \mathrm{GPD}(x,\eta,t) + \cdots \\ &= \mathrm{PV}\left(\frac{1}{x-\eta}\right)(\mathrm{GPD}(x,\eta,t)) - \int_{-1}^{1} dx \; i\pi\delta(x-\eta)\mathrm{GPD}(x,\eta,t) + \cdots\end{aligned}$$

So we can measure GPDs at $x=\eta$ only, i.e., we can access $\mathrm{GPD}(\eta,\eta,t)$

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Dispersion relation ⇒ real part can be computed in terms of imaginary:

$$\Re e\mathcal{A} = \mathrm{PV}\int_{-1}^{1} dx \frac{\Im m\mathcal{A}}{x-\eta} + D(t)$$

Cf. Riedl's talk for info. about experimental measurement of Re and Im parts

GPD definition: 3D distribution

$$\mathrm{GPD}(x,\eta,t) = \frac{1}{2}\int \frac{dz^+}{2\pi} e^{ixp^- z^+} \langle P'|\bar{q}_f(-z/2)\gamma^+ \mathcal{W}[-z/2, z/2] q_f(z/2)|P\rangle\Big|_{z_\perp = z^- = 0}$$

Measure the difference between P and P'

$$\eta = -\frac{(q - q')(q + q')}{(P + P')(q + q')}$$

$$t = (P - P')^2$$

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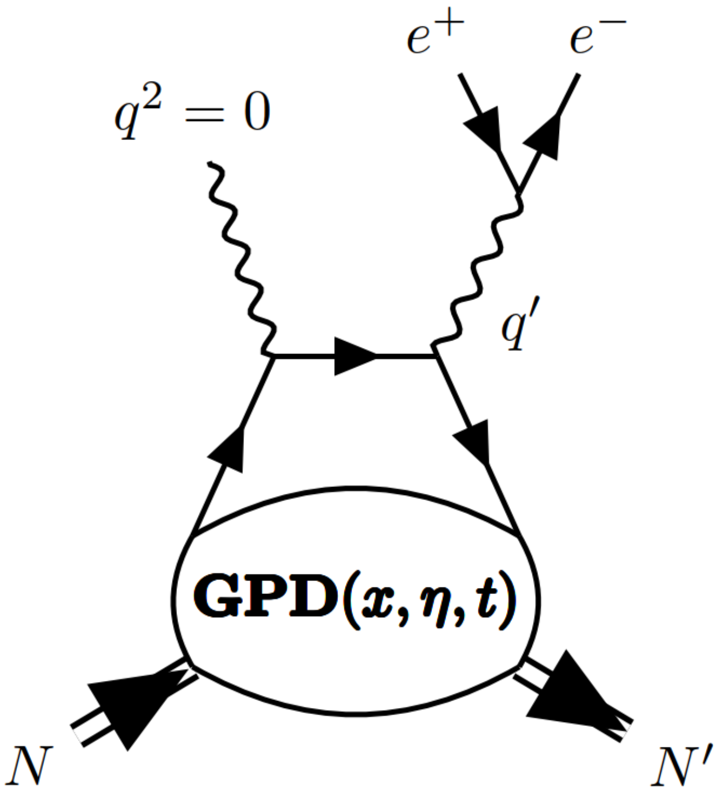
Nucleon tomography via Fourier transform in the plane transverse to proton motion

Other 2 “golden channels”: TCS & DDVCS

but experimentally more challenging than DVCS

- TCS or timelike Compton scattering
- Counterpart of DVCS
- A real photon transforms into a virtual one (lepton photo-production)



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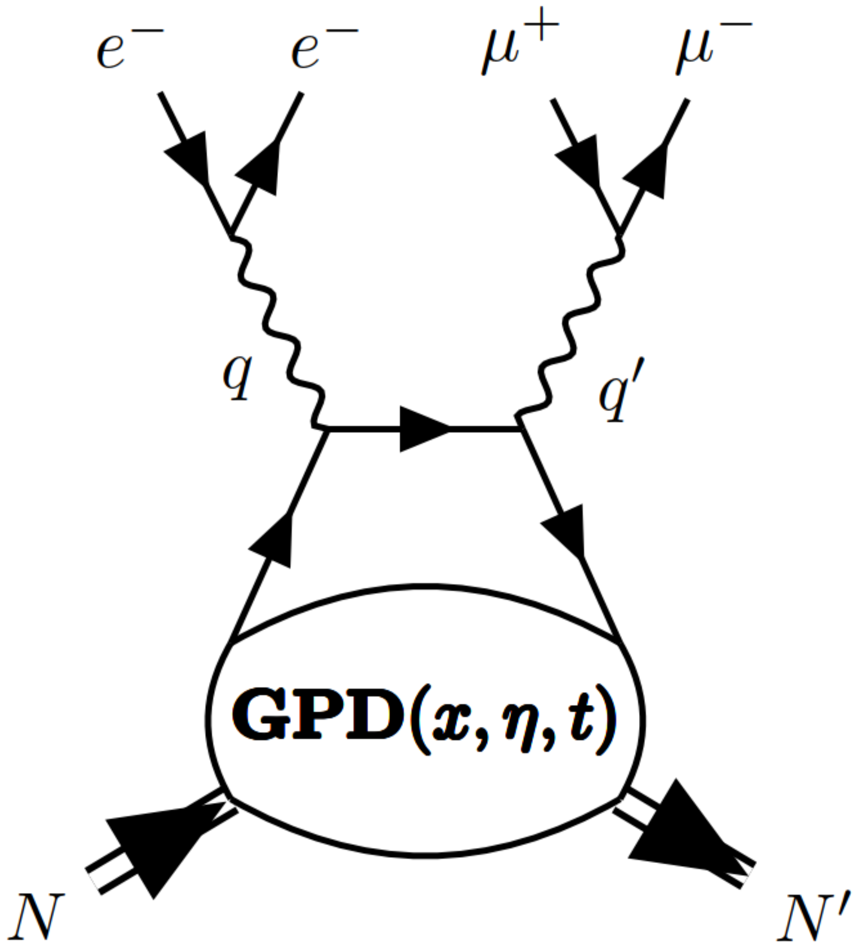
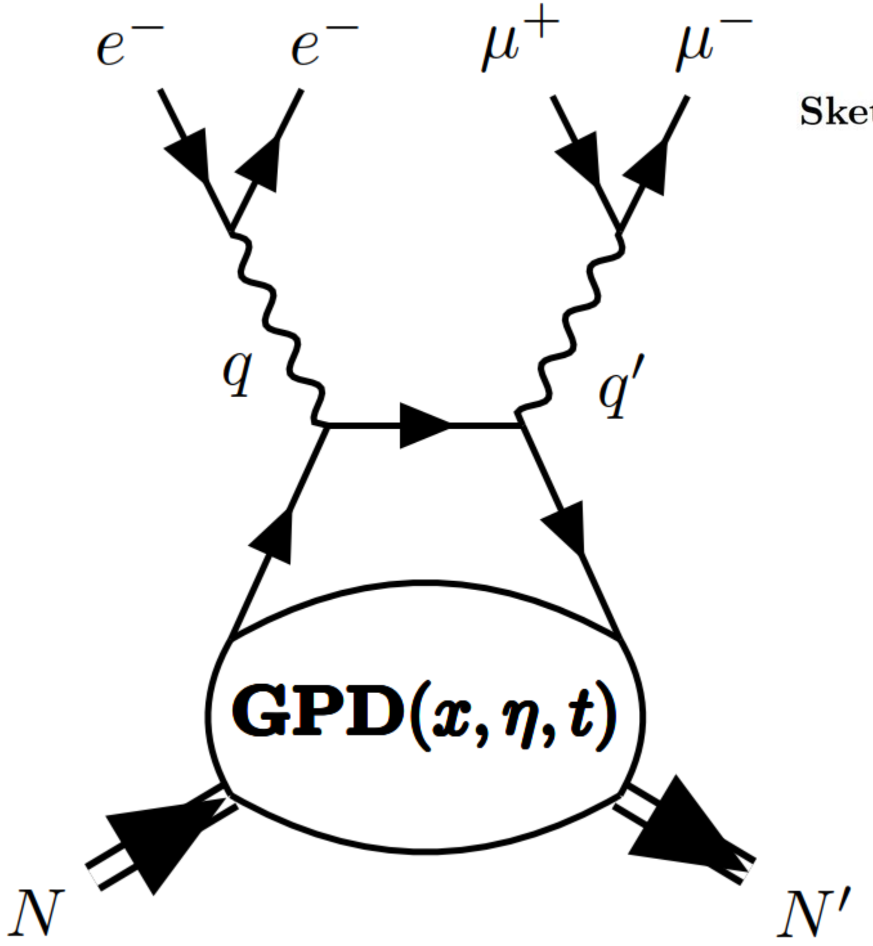
Recent paper (from this August) on the 1st measurement of TCS by CLAS collaboration:

“First-time measurement of Timelike Compton Scattering”, P. Chatagnon et al., arXiv:2108.11746 [hep-ex]

- DDVCS or double DVCS
- 2 virtual photons: spacelike (incoming) and timelike (outgoing)



- DDVCS or double DVCS
- 2 virtual photons: spacelike (incoming) and timelike (outgoing)
- **Allows to measure GPDs outside $x = \eta$**



Sketch of DDVCS amplitude

$$
\begin{aligned}
\mathcal{A}_{\mathrm{DDVCS}} &\sim \int_{-1}^{1} dx\, \frac{1}{x - \xi + i0} \mathrm{GPD}(x, \eta, t) + \cdots \\
&= \mathrm{PV}\left(\frac{1}{x - \xi}\right) (\mathrm{GPD}(x, \eta, t)) - \int_{-1}^{1} dx\ i\pi\delta(x - \xi)\mathrm{GPD}(x, \eta, t) + \cdots
\end{aligned}
$$

So now we can access $\mathrm{GPD}(\xi, \eta, t)$

Related to Orginos' talk: GPDs extraction is not ill-defined problem thanks to DDVCS. LQCD will render GPDs for sure, but DDVCS makes the problem well-defined and experimentally solvable

Details in DDVCS

Here, ξ is the *generalized* Bjorken variable,

$$\xi = \frac{-\bar{q}^2}{2p\bar{q}}, \quad \bar{q} = \frac{q+q'}{2}, \quad p = \frac{P+P'}{2}$$

q, q' are the 4-momenta of incoming & outgoing photon
P, P' are the 4-momentum of incoming & outgoing proton

Experimentally, DDVCS is very demanding: x-sec smaller than DVCS' → EIC will have enough luminosity to accurate measurements

B. Pire, L. Szymanowski, and J. Wagner. NLO corrections to timelike, spacelike and double deeply virtual Compton scattering. *Phys. Rev. D*, 83:034009, 2011.

Renormalization

- Need to renormalize both GPDs and hard part. Amplitude:

$$\mathcal{A}^{\mu\nu} = g_T^{\mu\nu} \int_{-1}^{1} dx \left[\sum_q^{n_F} \tilde{T}^q(x) \tilde{F}^q(x) + \tilde{T}^g(x) \tilde{F}^g(x) \right]$$

Bare GPD

Bare hard-part coefficients

- We can work in DDVCS and use

$$\mathrm{DDVCS}|_{\xi=\eta} \rightarrow \mathrm{DVCS}$$
$$\mathrm{DDVCS}|_{\xi=-\eta} \rightarrow \mathrm{TCS}$$

GPDs at NLO

$$\begin{aligned}\tilde{F}^q(x) &= F^q(x) - \left(\frac{1}{\epsilon} + \frac{1}{2}\ln\frac{e^\gamma \mu_F^2}{4\pi\mu_R^2}\right) K^{qq}(x,x') \otimes F^q(x') \\ &\quad - \left(\frac{1}{\epsilon} + \frac{1}{2}\ln\frac{e^\gamma \mu_F^2}{4\pi\mu_R^2}\right) K^{qg}(x,x') \otimes F^g(x')\end{aligned}$$

Similar formula for gluons

Kernels $K^{qq,qg}$ can be read from

M. Diehl, Phys. Rept. **388** (2003) 41;
A. V. Belitsky and A. V. Radyushkin, Phys. Rept. **418**, 1 (2005)

Corrections and $\ln 0$

- When calculating hard part you find expressions that can be written down as

$$\widetilde{T}^q \sim \text{prefactors} \times \ln\left(\frac{\bar{Q}^2}{\xi\mu_R^2}(\xi - x) - i0\right) + \cdots \qquad \bar{Q}^2 = \frac{Q^2 - Q'^2}{2}$$

If $Q^2 \sim Q'^2$, then $\bar{Q}^2 \to 0$ and ln is divergent

Solution: focus on $\frac{\bar{Q}^2}{\xi}$

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Complete expressions in

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$$\frac{\bar{Q}^2}{\xi} = \frac{Q^2}{x_B} - \hat{Q}^2 \xrightarrow[Q^2 \to Q'^2]{} 2Pq - Q^2 = s - P^2 = s - M^2 > 0$$

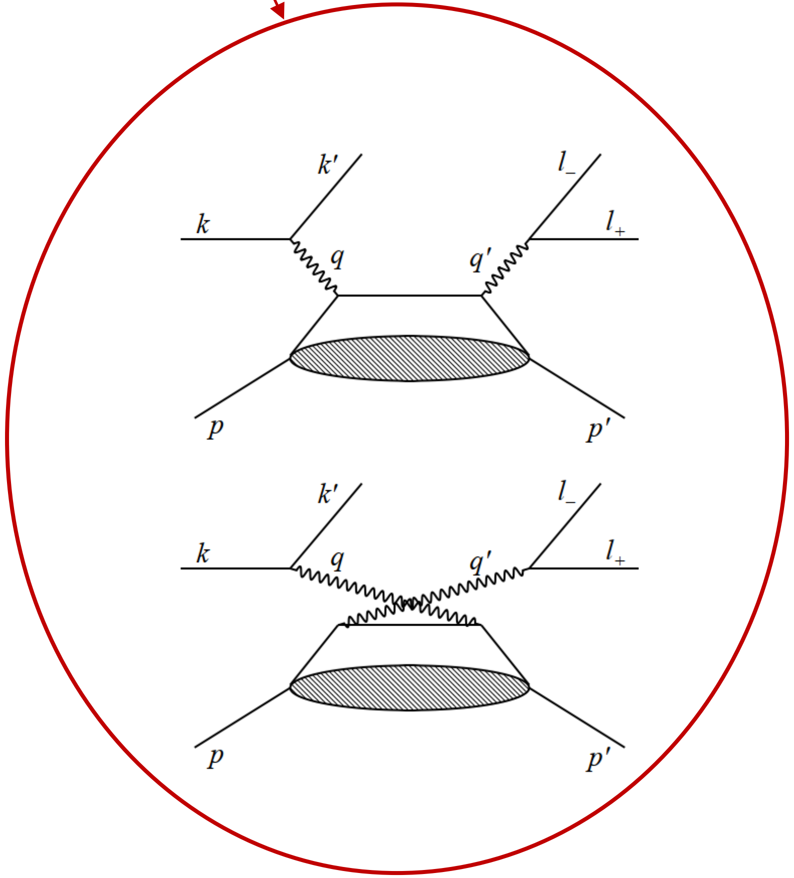
$$\Rightarrow \frac{\bar{Q}^2}{\xi} \not\to 0$$

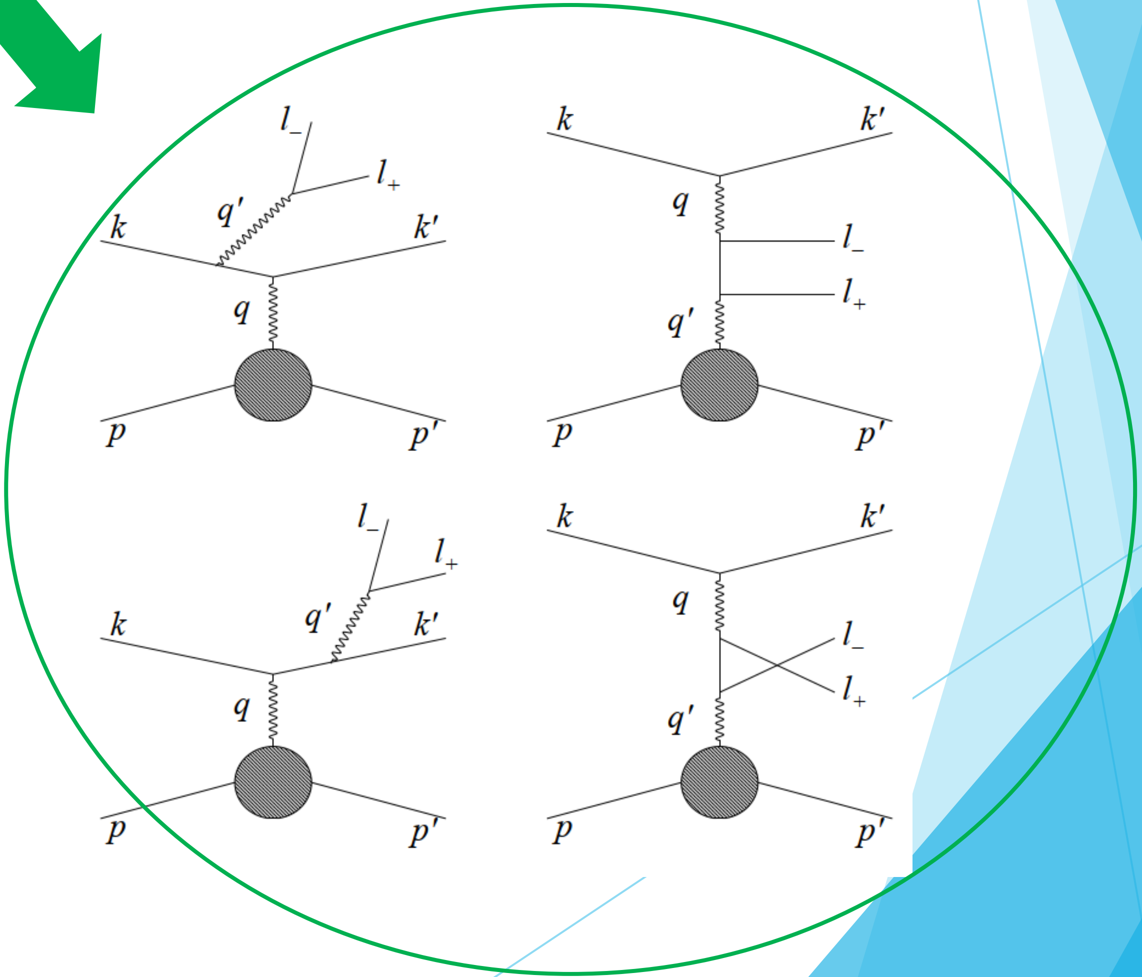
Actually,

$$\frac{\bar{Q}^2}{\xi} = \frac{\hat{Q}^2}{\eta} > 0 \Rightarrow \textbf{no} \text{ divergent } \ln$$

Not only Compton scattering counts…

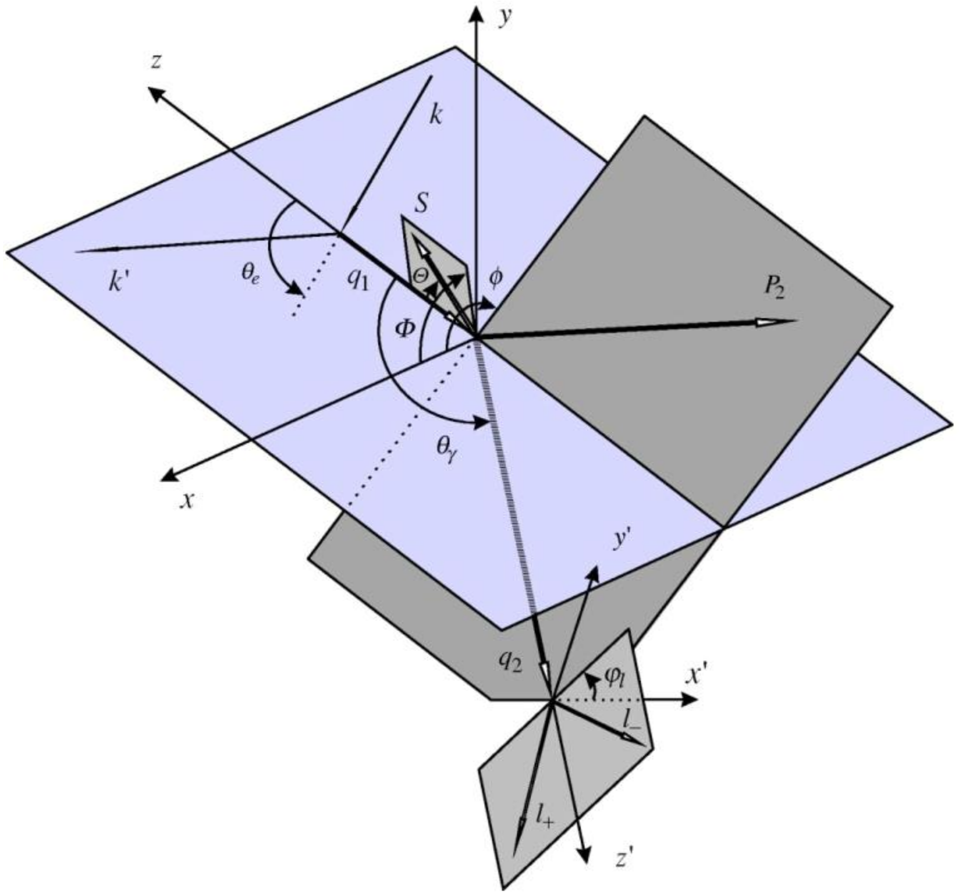
- Compton scattering + BH (Bethe-Heitler diagrams) is a precise representation of DDVCS





Amplitude structure

- DDVCS is a 2 → 4 problem, i.e., complex amplitude dependent on several angles
- Set-up:



Amplitude can be written as a Fourier expansion in the angles.

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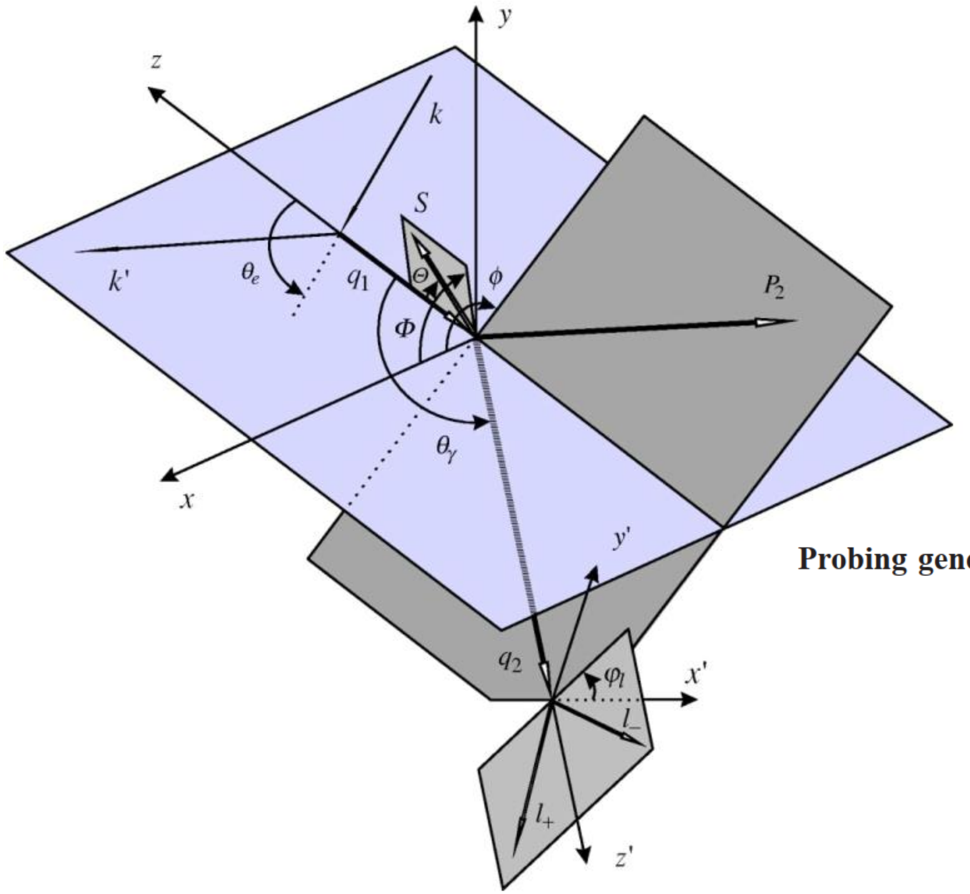

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LO expressions can be found in

PHYSICAL REVIEW D **68**, 116005 (2003)

Probing generalized parton distributions with electroproduction of lepton pairs off the nucleon

A. V. Belitsky D. Müller

Specialized software for this study

- PARTONS platform: open-source C++ program
  - Contains several GPD models
  - Leading twist… but higher twists will be included in near future
  - Can be used by theorists and experimentalists
  - Provides x-secs, Compton form factors, etc

PARtonic Tomography Of Nucleon Software

To download and for tutorials
http://partons.cea.fr

For detail description of architecture see:
Eur. Phys. J. C78 (2018), 478

Summary

- NLO is needed to described not only quark GPD but also gluon GPD, that appear at $\mathcal{O}(\alpha_s)$ in hard part
- DDVCS is a remarkable process since it allows for GPD measurement outside $x = \eta$, at can be reduced to DVCS and TCS (measurements are reported for both)
- I have presented a framework for the study of DDVCS feasibility at EIC, and extraction of GPDs through JLab 12GeV data. All these, at NLO accuracy $\rightarrow$ precise determination of GPDs and observables (asymmetries, xsecs, etc)