T-Odd GTMDs in quark-diquark model

61^{st} Cracow School of Theoretical Physics (Electron-Ion Collider Physics)

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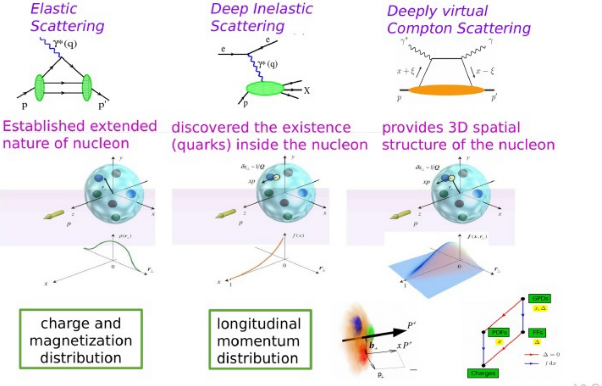
Overview

Probing Nucleon 3D Structure



- Form factors describe the transverse localization of partons in a fast moving nucleon, irrespective of their longitudinal momenta
- Parton densities provides the probability to find partons of a given longitudinal momentum fraction x of the parent

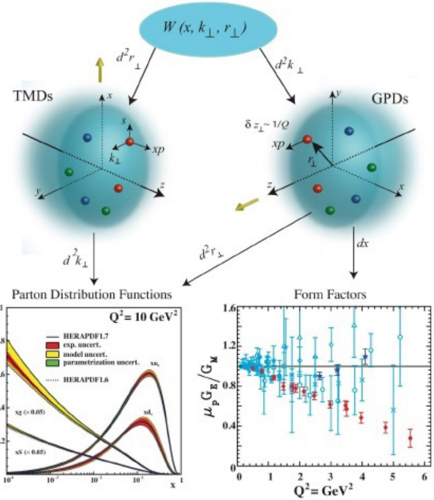


Figure: arXiv:1208.1244

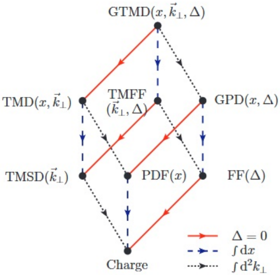


Figure: Lorce, BP, Vanderhaeghen, JHEP05 (2011) 041

- GTMDs and Wigner distributions have gained a lot of attention in the field of hadron physics.
- They are also called as mother distributions.
- One can obtain the GPDs and TMDs by taking the certain limits of the GTMDs.

- GTMDs and Wigner distributions are also shown to be related to elusive quark and gluon orbital angular momentum and spin-orbit correlations.

C. Lorce, *et. al.* PRD 85, 114006

- Although experimental determination of Wigner distribution is challenging but several theoretical studies and model calculations are present.

T. Liu and B. Q. Ma, PRD 91 034019

Y. Hagiwara *et. al.*, PRD 94 094036

N. Kumar and C. Mondal, NPB 931 226

Light-front quark-diquark model (LFQDM) for the nucleon (PRD 94 094020 (2016))

- To investigate the T-odd GTMDs we consider a LFQDM for the proton.
- In this model, we consider proton state as a linear combination of quark-diquark state including both scalar and axial-vector diquarks.
- One can define the proton state as

$$|P;\pm\rangle = C_s|uS^0\rangle^{\pm} + C_v|uA^0\rangle^{\pm} + C_{vv}|dA^1\rangle^{\pm}$$

- The two-particle Fock-state expansion for $J^z = \pm 1/2$ for spin-0 diquark state is given by

$$|u\ S\rangle^{\pm} = \int \frac{dx\ d^2\mathbf{p}_\perp}{16\pi^3\ \sqrt{x(1-x)}}\Big[\psi_+^{\pm}(x,\mathbf{p}_\perp)| + \frac{1}{2}0; xP^+, \mathbf{p}_\perp\rangle + \psi_-^{\pm}(x,\mathbf{p}_\perp)| - \frac{1}{2}0; xP^+, \mathbf{p}_\perp\rangle\Big]$$

- This model has been used to evaluate many interesting properties of the proton.
- Chiral even and odd GPDs, spin densities for the quarks in the proton and how different GPDs cotribute to proton spin densities for different polarizations of the quark and proton.

T. Maji *et. al.*, PRD 96 013006 (2017).

- Axial charge obtained from GPD is in excellent agreement with the observed results.
- This model was shown to predict the single and double spin asymmetries and agree with COMPASS and HERMES data.

T. Maji *et. al.*, PRD 96 114023 (2017).

- Even, after adding FSI in the LFWFs, the model shown to reproduce the Sivers and Boer-Mulders Asymmetries in agreement with data.

T. Maji *et. al.*, PRD 97 014016 (2018).

- The state with spin-1 diquark state is given as

$$
\begin{aligned}
|\nu\ A\rangle^{\pm} \;=\; & \int \frac{dx\ d^2\mathbf{p}_\perp}{16\pi^3\ \sqrt{x(1-x)}} \Big[\psi^{\pm}_{++}(x,\mathbf{p}_\perp) | +\frac{1}{2} +1; xP^+, \mathbf{p}_\perp\rangle \\
& + \psi^{\pm}_{-+}(x,\mathbf{p}_\perp) | -\frac{1}{2} +1; xP^+, \mathbf{p}_\perp\rangle + \psi^{\pm}_{+0}(x,\mathbf{p}_\perp) \\
& | +\frac{1}{2}\ 0; xP^+, \mathbf{p}_\perp\rangle + \psi^{\pm}_{-0}(x,\mathbf{p}_\perp) | -\frac{1}{2}\ 0; xP^+, \mathbf{p}_\perp\rangle + \\
& \psi^{\pm}_{+-}(x,\mathbf{p}_\perp) | +\frac{1}{2} -1; xP^+, \mathbf{p}_\perp\rangle + \psi^{\pm}_{--}(x,\mathbf{p}_\perp) \\
& | -\frac{1}{2} -1; xP^+, \mathbf{p}_\perp\rangle \Big]
\end{aligned}
$$

where $|\lambda_q \lambda_A; xP^+, \mathbf{p}_\perp\rangle$ represents a two-particle state with a quark of helicity $\lambda_q = \pm\frac{1}{2}$ and an axial-vector diquark of helicity $\lambda_A = \pm 1, 0$(triplet).

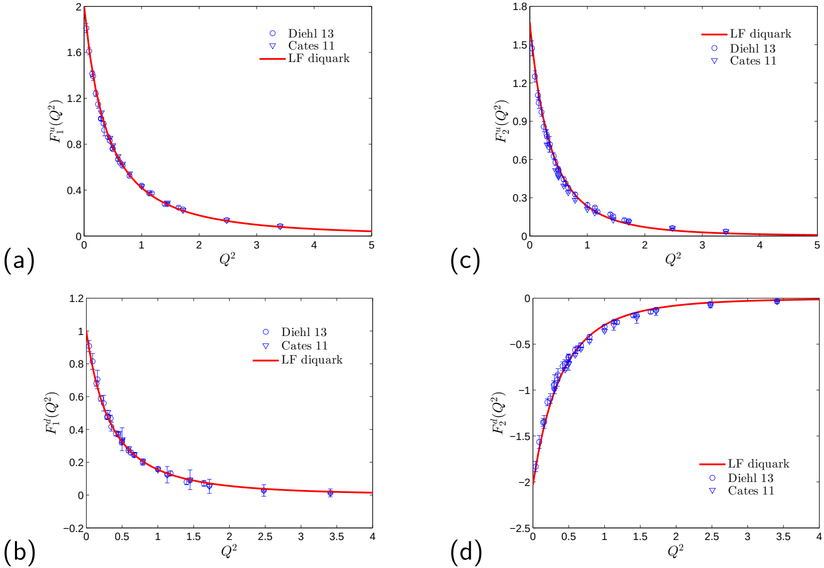
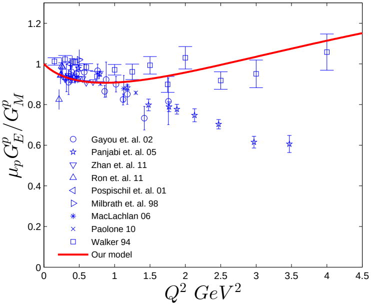
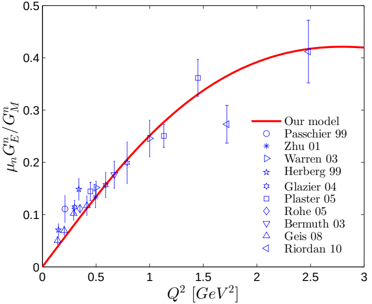


Figure: Flavour form factors fitting in the light-front diquark model.



Gayou et. al. 02
Panjabi et. al. 05
Zhan et. al. 11
Ron et. al. 11
Pospischil et. al. 01
Milbrath et. al. 98
MacLachlan 06
Paolone 10
Walker 94
Our model
$\mu_p G_E^p / G_M^p$
$Q^2\ GeV^2$

(a)



Our model
Passchier 99
Zhu 01
Warren 03
Herberg 99
Glazier 04
Plaster 05
Rohe 05
Bermuth 03
Geis 08
Riordan 10
$\mu_n G_E^n / G_M^n$
$Q^2\ [GeV^2]$

(b)

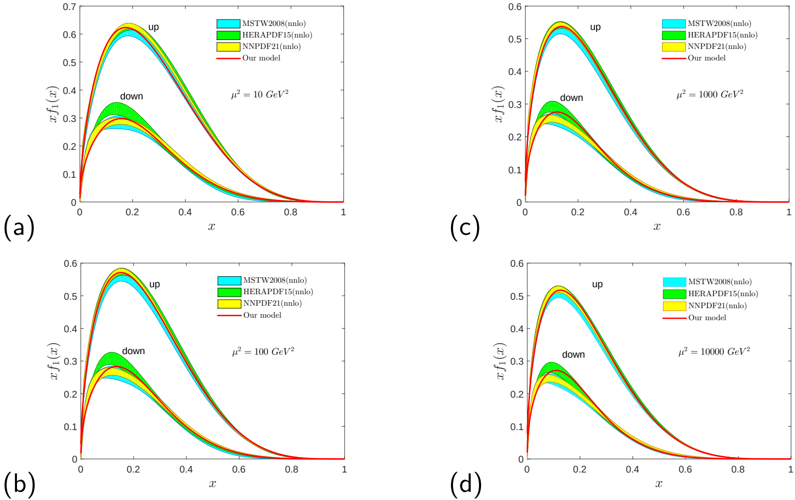


Figure: Evolution of unpolarized PDF in this model at $\mu^2 = 10, 100, 1000$ and 10000 GeV^2 for both u and d quarks.

Final state interactions and T-odd GTMDs

- LFWFs representation of the T-odd TMDs and GTMDs require that the wave functions must have complex phases.
- Brodsky, Hwang and Schmidt showed that the final state interactions (FSI), produces a non-trivial phase in the amplitude which generates the Sivers asymmetry.

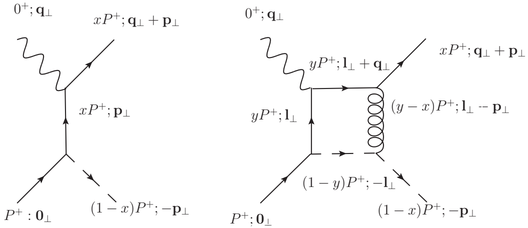


Figure: Left: tree-level diagram. Right: FSI diagram for $\gamma^* P \to q(qq)$.

- Assuming that QCD factorization theorem holds for SIDIS process, Sivers and Boer-Mulders asymmetries can be written as convolutions of T-odd TMDs, a hard part and the fragmentation function.
- To have a wave function representation of T-odd TMDs and GTMDs, we introduce FSI into the light-front wave functions with complex phases.
- The modified wave functions give non-vanishing T-odd GTMDs along with T-even GTMDs.

(For scalar diquark)

$$
\begin{aligned}
\psi_+^{+(u)}(x,\mathbf{p}_\perp) &= N_S\left[1+i\frac{e_1 e_2}{8\pi}(\mathbf{p}_\perp^2+B)g_1\right]\varphi_1^{(u)}(x,\mathbf{p}_\perp),\\
\psi_-^{+(u)}(x,\mathbf{p}_\perp) &= N_S\left(-\frac{p^1+ip^2}{xM}\right)\left[1+i\frac{e_1 e_2}{8\pi}(\mathbf{p}_\perp^2+B)g_2\right]\varphi_2^{(u)}(x,\mathbf{p}_\perp)\\
\psi_+^{-(u)}(x,\mathbf{p}_\perp) &= N_S\left(\frac{p^1-ip^2}{xM}\right)\left[1+i\frac{e_1 e_2}{8\pi}(\mathbf{p}_\perp^2+B)g_2\right]\varphi_2^{(u)}(x,\mathbf{p}_\perp),\\
\psi_-^{-(u)}(x,\mathbf{p}_\perp) &= N_S\left[1+i\frac{e_1 e_2}{8\pi}(\mathbf{p}_\perp^2+B)g_1\right]\varphi_1^{(u)}(x,\mathbf{p}_\perp),
\end{aligned}
$$

- For axial-vector diquark

$$
\begin{aligned}
\psi^{+(\nu)}_{+\,+}(x,\mathbf{p}_\perp) &= N_1^{(\nu)}\sqrt{\frac{2}{3}}\left(\frac{p^1-ip^2}{xM}\right)\left[1+i\frac{e_1e_2}{8\pi}(\mathbf{p}_\perp^2+B)g_2\right]\varphi_2^{(\nu)}(x,\mathbf{p}_\perp),\\
\psi^{+(\nu)}_{-\,+}(x,\mathbf{p}_\perp) &= N_1^{(\nu)}\sqrt{\frac{2}{3}}\left[1+i\frac{e_1e_2}{8\pi}(\mathbf{p}_\perp^2+B)g_1\right]\varphi_1^{(\nu)}(x,\mathbf{p}_\perp),\\
\psi^{+(\nu)}_{+\,0}(x,\mathbf{p}_\perp) &= -N_0^{(\nu)}\sqrt{\frac{1}{3}}\left[1+i\frac{e_1e_2}{8\pi}(\mathbf{p}_\perp^2+B)g_1\right]\varphi_1^{(\nu)}(x,\mathbf{p}_\perp),\\
\psi^{+(\nu)}_{-\,0}(x,\mathbf{p}_\perp) &= N_0^{(\nu)}\sqrt{\frac{1}{3}}\left(\frac{p^1+ip^2}{xM}\right)\left[1+i\frac{e_1e_2}{8\pi}(\mathbf{p}_\perp^2+B)g_2\right]\varphi_2^{(\nu)}(x,\mathbf{p}_\perp),\\
\psi^{+(\nu)}_{+\,-}(x,\mathbf{p}_\perp) &= 0,\\
\psi^{+(\nu)}_{-\,-}(x,\mathbf{p}_\perp) &= 0,
\end{aligned}
$$

T-odd GTMDs

- The GTMD correlator is defined as

$$W^{\nu[\Gamma]}_{\lambda''\lambda'}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x) = \frac{1}{2}\int \frac{dz^-}{(2\pi)}\frac{d^2 z_T}{(2\pi)^2} e^{ip.z}\langle P''; \lambda''| \times \bar{\psi}^\nu(-z/2)\Gamma \mathcal{W}_{[-z/2,z/2]}\psi^\nu(z/2)|P'; \lambda'\rangle\Big|_{z^+=0}$$

- There are altogethre 16 GTMDs at the leading twist and each GTMDs has an even and odd part under the time reversal invariance.
- In this work, we will concentrate on two leading twist T-odd GTMDs which reduce to Sivers and Boer-Mulder TMD in the limit $\mathbf{\Delta}_\perp = 0$.

$$W^{\nu[\gamma^+]}_{\lambda''\lambda'}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x) = ... + \frac{1}{2M}\bar{u}(P'', \lambda'')\left(\frac{i\sigma^{i+}p^i_\perp}{P^+}\right) \times \left[F^{e\nu}_{1,2} + iF^{o\nu}_{1,2}\right] u(P', \lambda') + ...,$$

$$W^{\nu[i\sigma^{j+}\gamma^5]}_{\lambda''\lambda'}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x) = \frac{1}{2M}\bar{u}(P'', \lambda'')(-\frac{i\epsilon_T^{ij} p_\perp^i}{M})\Big[H^{e\nu}_{1,1} + iH^{o\nu}_{1,1}\Big]u(P', \lambda') +$$

where the superscripts e, o stand for T-even and T-odd part respectively.

- In this model using the wave functions as described previously, the results for GTMDs are

$$\begin{aligned}
F^{e\nu}_{1,2}, H^{e\nu}_{1,1}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x) &= 0 \\
F^{o\nu}_{1,2}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x) &= \Big(C_S^2 N_S^{\nu 2} - C_A^2 \frac{1}{3} N_0^{\nu 2}\Big), \\
H^{o\nu}_{1,1}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x) &= \Big(C_S^2 N_S^{\nu 2} + C_A^2 (\frac{1}{3} N_0^{\nu 2} + \frac{2}{3} N_1^{\nu 2})\Big) \\
&\times \frac{1}{x}\Big\{(D_1' - D_2') + (D_1'' - D_2'')\Big\} \\
&\times \frac{1}{16\pi^3} A_1^\nu(x) A_2^\nu(x) \exp[-a(x)\tilde{\mathbf{p}}_\perp^2]
\end{aligned}$$

- Note that T-even part vanishes due to vanishing of some components of LFWFs.

D. Chakrabarti et. al., PRD 95 074028 (2017)

- T-odd parts became non-zero due to the incorporation of final state interaction.
- We neglect the higher order terms and present the result up to $\mathcal{O}(\alpha_s)$.
- The explicit results for the T-odd GTMDs are

$$
\begin{aligned}
F^{o\nu}_{1,2}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x) \quad = \quad & \left(C_S^2 N_S^{\nu 2} - C_A^2 \frac{1}{3} N_0^{\nu 2} \right) (-C_F \alpha_s) \\
& \times \frac{1}{2x} \Bigg\{ (\mathbf{p}'^2_\perp + B) \frac{1}{\mathbf{p}'^2_\perp} \ln\Big(\frac{\mathbf{p}'^2_\perp + B}{B}\Big) \\
& + (\mathbf{p}''^2_\perp + B) \frac{1}{\mathbf{p}''^2_\perp} \ln\Big(\frac{\mathbf{p}''^2_\perp + B}{B}\Big) \Bigg\} \\
& \times \frac{1}{16\pi^3} A_1^\nu(x) A_2^\nu(x) \exp[-a(x) \tilde{\mathbf{p}}^2_\perp],
\end{aligned}
$$

$$
\begin{aligned}
H_{1,1}^{o\nu}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x) &= \Big(C_S^2 N_S^{\nu 2} + C_A^2\big(\frac{1}{3}N_0^{\nu 2} + \frac{2}{3}N_1^{\nu 2}\big)\Big)(-C_F \alpha_s) \\
&\times \frac{1}{2x}\Big\{(\mathbf{p}_\perp'^2 + B)\frac{1}{\mathbf{p}_\perp'^2}\ln(\frac{\mathbf{p}_\perp'^2 + B}{B}) \\
&+(\mathbf{p}_\perp''^2 + B)\frac{1}{\mathbf{p}_\perp''^2}\ln(\frac{\mathbf{p}_\perp''^2 + B}{B})\Big\} \\
&\times \frac{1}{16\pi^3} A_1^\nu(x) A_2^\nu(x) \exp[-a(x)\tilde{\mathbf{p}}_\perp^2]
\end{aligned}
$$

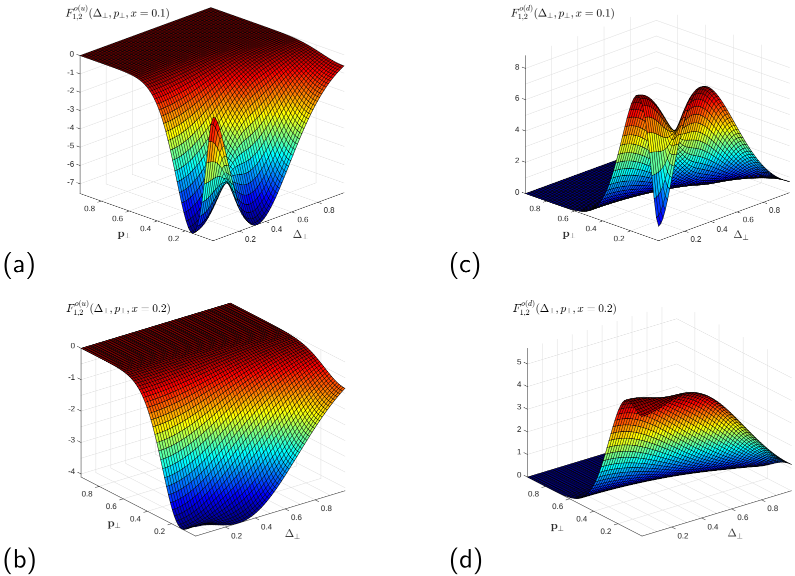


Figure: $F^{o\nu}_{1,2}(\Delta_\perp, \mathbf{p}_\perp)$ for three different $x = 0.1$ and 0.2.

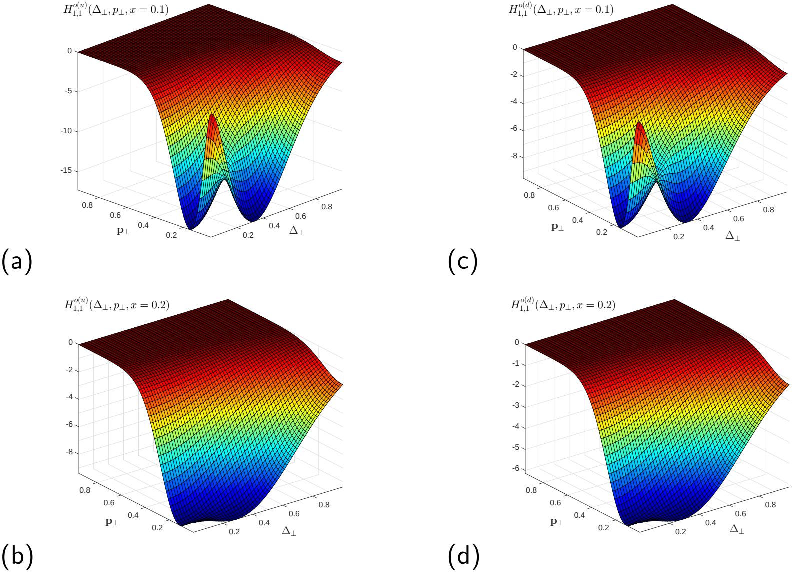


Figure: $H^{o\nu}_{1,1}(\Delta_\perp, \mathbf{p}_\perp)$ for three different $x = 0.1$ and 0.2.

Wigner Distributions

- In light-front framework, the 5-dimensional quark Wigner distribution is

$$\rho^{\nu[\Gamma]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; S) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp . \mathbf{b}_\perp} W^{\nu[\Gamma]}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x; S).$$

- The correlator $W^{[\Gamma]}$ relates the GTMDs and in the Drell-Yan-West frame ($\Delta^+ = 0$) and fixed light-cone time $z^+ = 0$ is given by

$$W^{\nu[\Gamma]}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x; S) = \frac{1}{2}\int \frac{dz^-}{(2\pi)} \frac{d^2 z_T}{(2\pi)^2} e^{ip.z} \langle P''; S|\bar{\psi}^\nu(z/2) \times \Gamma \mathcal{W}_{[-z/2, z/2]} \psi^\nu(z/2)|P'; S\rangle \Big|_{z^+=0}.$$

- Depending on the various polarization configurations of the proton(X) and quark (Y), there are 16 independent twist-2 quark Wigner distributions ρ^{ν}_{XY}.
- We discuss $\rho^{\iota\nu}_{TU}$ and $\rho^{j\nu}_{UT}$ which are

$$\begin{aligned}
\rho^{i\nu}_{TU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) &= \frac{1}{2}[\rho^{\nu[\gamma^+]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; +\hat{S}_i) \\
&- \rho^{\nu[\gamma^+]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; -\hat{S}_i)], \\
\rho^{j\nu}_{UT}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) &= \frac{1}{2}[\rho^{\nu[i\sigma^{j+}\gamma^5]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; +\hat{S}_z) \\
&+ \rho^{\nu[i\sigma^{j+}\gamma^5]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; -\hat{S}_z)].
\end{aligned}$$

- These two Wigner distributions can be parametrized in terms of the $F_{1,2}$ and $H_{1,1}$ GTMDs as

$$\begin{aligned}
\rho^{i\nu}_{TU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) &= \frac{1}{2M}\epsilon^{ij}_\perp \frac{\partial}{\partial b^j_\perp}\Big[\mathcal{F}^\nu_{1,1}(x, 0, \mathbf{p}^2_\perp, \mathbf{p}_\perp.\mathbf{b}_\perp, \mathbf{b}^2_\perp) \\
&- 2\mathcal{F}^\nu_{1,3}(x, 0, \mathbf{p}^2_\perp, \mathbf{p}_\perp.\mathbf{b}_\perp, \mathbf{b}^2_\perp)\Big] \\
&+ i\frac{1}{M}\epsilon^{ij}_\perp p^j_\perp \mathcal{F}^\nu_{1,2}(x, 0, \mathbf{p}^2_\perp, \mathbf{p}_\perp.\mathbf{b}_\perp, \mathbf{b}^2_\perp), \\
\rho^{j\nu}_{UT}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) &= -i\frac{1}{M}\epsilon^{ij}_\perp p^i_\perp \mathcal{H}^\nu_{1,1}(x, 0, \mathbf{p}^2_\perp, \mathbf{p}_\perp.\mathbf{b}_\perp, \mathbf{b}^2_\perp) \\
&+ \frac{1}{M}\epsilon^{ij}_\perp \frac{\partial}{\partial b^i_\perp}\mathcal{H}^\nu_{1,2}(x, 0, \mathbf{p}^2_\perp, \mathbf{p}_\perp.\mathbf{b}_\perp, \mathbf{b}^2_\perp),
\end{aligned}$$

- where the $\chi^\nu = \mathcal{F}^\nu_{1,1}, \mathcal{F}^\nu_{1,2}, \mathcal{F}^\nu_{1,3}$ and $\mathcal{H}^\nu_{1,1}, \mathcal{H}^\nu_{1,2}$ can be expressed as Fourier transform of GTMDs $X^\nu = F^\nu_{1,1}, F^\nu_{1,2}, F^\nu_{1,3}$ and $H_{1,1}, H_{1,2}$ respectively.

- As discussed before, each GTMD can be written by separating the even and odd part under time reversal

JHEP 0908:056,2009

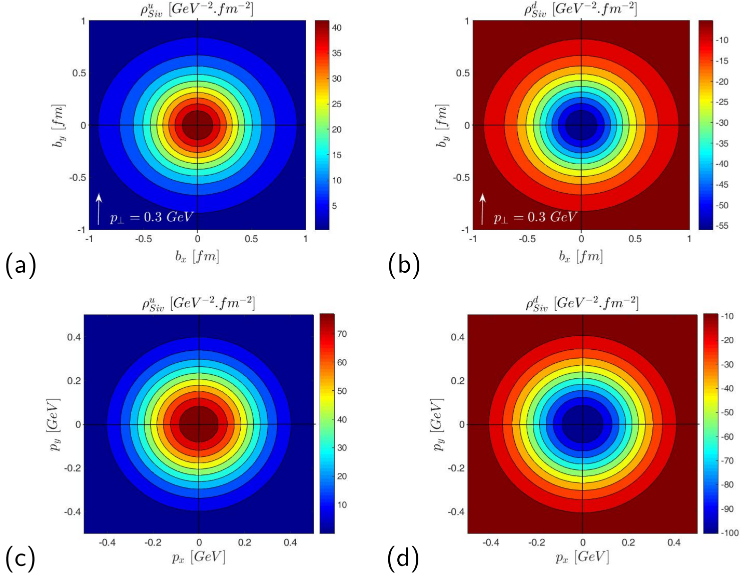
$$X^{\nu}(x,0,\mathbf{p}_{\perp}^2,\mathbf{p}_{\perp}.\mathbf{\Delta}_{\perp},\mathbf{\Delta}_{\perp}^2) = X^{e\nu}(x,0,\mathbf{p}_{\perp}^2,\mathbf{p}_{\perp}.\mathbf{\Delta}_{\perp},\mathbf{\Delta}_{\perp}^2) + iX^{o\nu}(x,0,\mathbf{p}_{\perp}^2,\mathbf{p}_{\perp}.\mathbf{\Delta}_{\perp},\mathbf{\Delta}_{\perp}^2)$$

- GTMDs have non -vanishing odd part which contribute to the Wigner distributions.
- Hermiticity property of the GTMDs ensures that the Wigner distributions are real-valued functions.
- Hermiticity of the GTMDs is satisfied in our model and as a result, the phase space distributions are real.

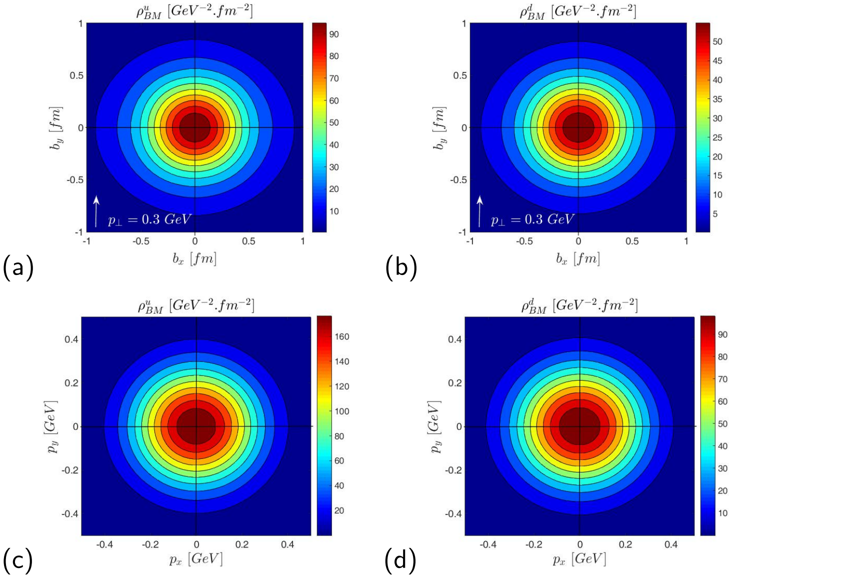
- One can define the Fourier transform of these leading twist T-odd GTMDs as

$$\rho^{\nu}_{Siv}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = -\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp . \mathbf{b}_\perp} F^{o\nu}_{1,2}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x),$$

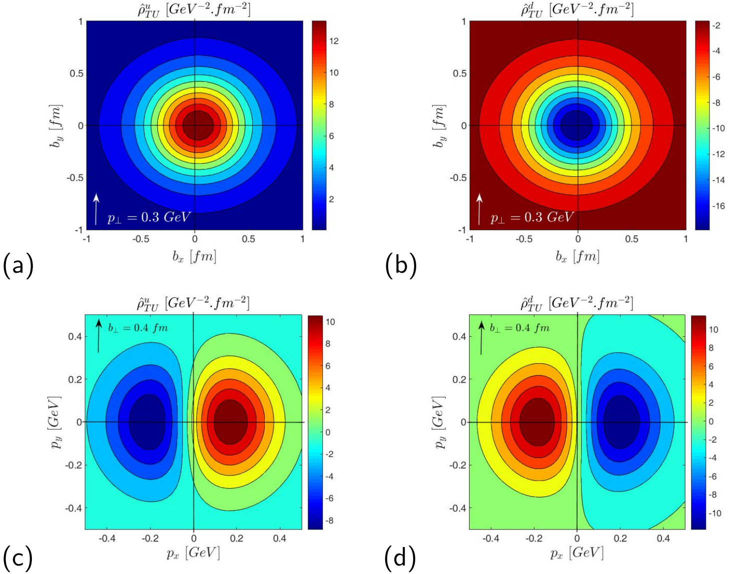
$$\rho^{\nu}_{BM}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = -\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp . \mathbf{b}_\perp} H^{o\nu}_{1,1}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x),$$



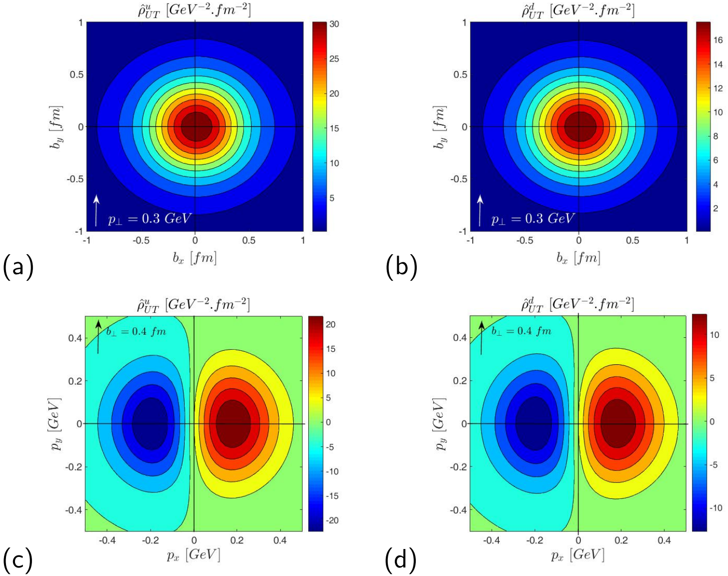
x integrated Sivers Wigner distribution $\rho^{\nu}_{Siv}((\mathbf{b}_\perp, \mathbf{p}_\perp))$: in the impact-parameter plane(a,b) for $\mathbf{p}_\perp = 0.3$ GeV along $\hat{y}$ and in the transverse momentum plane (c,d) with $\mathbf{b}_\perp = 0.4$ fm along $\hat{y}$.



x integrated Boer-Mulders Wigner distribution $\rho_{BM}(\mathbf{b}_\perp, \mathbf{p}_\perp)$: in the impact-parameter plane (a,b) for $\mathbf{p}_\perp = 0.3\ GeV$ along $\hat{y}$ and in the transverse momentum plane (c,d) with $\mathbf{b}_\perp = 0.4\ fm$ along $\hat{y}$.



The modified WD $\hat{\rho}^{\nu}_{TU}(\mathbf{b}_\perp, \mathbf{p}_\perp)$ in the impact-parameter plane (a,b) for $\mathbf{p}_\perp = 0.3\ GeV$ along $\hat{y}$ and in the transverse momentum plane (c,d)for $\mathbf{b}_\perp = 0.4\ fm$ along $\hat{y}$.



The modified WD $\hat{\rho}^{\nu}_{UT}(\mathbf{b}_\perp, \mathbf{p}_\perp)$ in the impact-parameter plane (a,b) for $\mathbf{p}_\perp = 0.3\ GeV$ along $\hat{y}$ and in the transverse momentum plane (c,d) for $\mathbf{b}_\perp = 0.4\ fm$ along $\hat{y}$.

- Wigner distributions and the GTMDs have drawn a lot of attention in recent years as various experiments are probing the internal structure of proton
- T-even GTMDs and TMDs are studied in several models but T-odd GTMDs are not investigated much
- We present first model calculations of T-odd Wigner functions and the GTMDs, by incorporating the effect of final state interaction at the level of one gluon exchange
- The T-odd GTMDs $F^{o}_{1,2}$ and $H^{o}_{1,1}$ reduce to Sivers and Boer-Mulders functions in the TMD limit ($\mathbf{\Delta}_{\perp} = 0$)
- It is interesting to note that both these T-odd GTMDs have exactly same functional dependence on $\mathbf{\Delta}_{\perp}, \mathbf{p}_{\perp}$ and x, they only differ in the overall normalization factors which also depend on the quark flavor in our model.
- We have also shown that the FSI contribution can be factored out for the GTMDs in our model, as observed in other spectator type models.

- It would be interesting to compare these results with other model calculations, once they become available.
- So far GTMDs and Wigner functions have not been accessed experimentally, however such model calculations are of importance as they can help to visualize the behaviour of these functions in $b_\perp$ and $k_\perp$ spaces.

Thank You