Quantum entanglement within color glass condensate framework

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arXiv.2001.01726, and "Boltzman entropy of Entangeled Color Glass Condensate" in preparation with Alex Kovner and Vladimir V. Skokov

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Main takeaway

- Color Glass Condensate:
  - Hamiltonian is non-perturbative and unknown, so is the wavefunction
  - A model for proton wavefunction

$$|\text{proton}\rangle = \sum_{\rho_a} |v;\rho_a\rangle \otimes |s;\rho_a, A_b\rangle$$

  - Entanglement entropy $S_E = -\,\mathrm{Tr}(\hat{\rho}_r \ln(\hat{\rho}_r))$
- Reduced density matrix diagonalized, and thermal quasi-particle in certain momentum regime.

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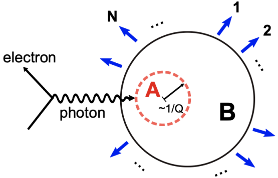
Reduced density matrix

Density matrix $\hat{\rho}(A, B) \rightarrow$ reduced density matrix

$$\hat{\rho}_A = \mathrm{Tr}_B \, \hat{\rho}(A, B)$$

- Here A can be probed in DIS partons of the parton model; B is the unobserved part of the parton wavefunction.
- The property of interest: if $\hat{\rho}(A, B)$ is pure, non-pure reduced density matrix $\hat{\rho}_A \rightarrow$ entanglement



from arXiv.1904.11974

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Quantum entropies

Common entropies in quantum information theory:

- Renyi entropy $S_R^N = \frac{1}{1-N} \ln \mathrm{Tr}\{\hat{\rho}_r^N\}$
- von Neumann entropy $S_V = \lim_{N\to 1} S_R^N = -\mathrm{Tr}\{\hat{\rho}_r \ln \hat{\rho}_r\}$

Entropy of entanglement: entropy of the reduced density matrix

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Proton wave function

CGC model for Proton wavefunction,

$$|\text{proton}\rangle = \sum_{\rho_a} |v; \rho_a\rangle \otimes |s; \rho_a, A_b\rangle$$

where

- $|v\rangle$ describes the valance dof
- $|s\rangle$ stands for soft gluons
- $\rho_a(x)$ is the color charge density of the valance modes
- A_b is the gluon field generated from the source ρ_a

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Reduced density matrix for soft gluons in MV model

Our goal is the reduced density matrix for soft gluons

$$\hat{\rho}_s = \mathrm{Tr}_v(|v\rangle\langle v| \otimes |s\rangle\langle s|)$$

In MV model

$$\langle v|v\rangle = \exp\left\{-\int_k \frac{\rho_a(k)\rho_a^*(k)}{2\mu^2}\right\}$$

$$\mathrm{Tr}_v \Rightarrow \int D[\rho_a]; \quad |s\rangle = \mathcal{C}\,|0\rangle$$

$$\mathcal{C} = \exp\left\{i\int_k b_a^i(k)\phi_a^{*i}\right\}; \quad \phi_a^{*i}(k) = a_a^{i\dagger}(k) + a_a^i(-k)$$

$$b_i^a(x) = \frac{1}{igN_c}\,\mathrm{Tr}\left[T^a U^\dagger(x)\partial_i U(x)\right] = \frac{2}{ig}\,\mathrm{Tr}\left[t^a V^\dagger(x)\partial_i V(x)\right]$$

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Trace out the valance dof

$$\hat{\rho}(\phi_1, \phi_2)$$
$$= \mathcal{N} \int D[\rho_a] e^{-\int_k \frac{\rho_a(k)\rho_a^*(k)}{2\mu^2} + i\int_k b_b^i(k)\phi_{1,b}^{*i}(k) - i\int_k b_c^{*j}(k)\phi_{2,c}^j(k)} \langle \phi_1|0\rangle \langle 0|\phi_2\rangle$$

and in MV model

$$\langle b_i^a(k) b_j^b(-k)\rangle = \frac{(2\pi)^3}{2(N_c^2-1)}\delta^{ab}\, xG_{\mathrm{WW}}^{ij}(x,k) \equiv \frac{S_\perp}{(2\pi)^2}\tilde{M}_{ij}^{ab}(k)$$

where Weizsäcker-Williams gluon distribution in the momentum space has the following form

$$xG_{\mathrm{WW}}^{ij}(x,k) = \frac{1}{2}\delta_{ij} xG^{(1)}(x,k) - \frac{1}{2}\left(\delta_{ij} - 2\frac{k_i k_j}{k^2}\right) xh^{(1)}(x,k)$$

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ρ_r in field basis

$$\langle\phi_1|\hat{\rho}_r|\phi_2\rangle = \mathcal{N}\langle\phi_1|0\rangle\langle 0|\phi_2\rangle e^{-\int_k \frac{1}{2}\tilde{M}^{ab}_{ij}(k)(\phi^a_{1j}(k)-\phi^a_{2j}(k))(\phi^b_{1i}(-k)-\phi^b_{2i}(-k))}$$

where the vacuum wave function is

$$\langle 0|\phi\rangle = \exp\left\{-\frac{1}{4}\phi^2\right\}$$

we have two independent modes

$$\tilde{M}_\pm = \frac{(2\pi)^5}{2S_\perp(N_c^2-1)}\frac{xG^{(1)} \pm xh^{(1)}}{2}\,.$$

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We derive the entanglement entropy for both eigenvalues of the matrix $\tilde{M}$, we obtain

$$S^E = \frac{N_c^2 - 1}{2} \sum_{i=\pm} \int_k \left[\ln \tilde{M}_i(k) + \sqrt{1 + 4\tilde{M}_i(k)} \ln\left(1 + \frac{1}{2\tilde{M}_i(k)} + \frac{1}{2\tilde{M}_i(k)} \sqrt{1 + 4\tilde{M}_i(k)} \right) \right]$$

we noticed the result of the previous section can be rewritten in the following form

$$S_E = (N_c^2 - 1) S_\perp \sum_{i=\pm} \int \frac{d^2k}{(2\pi)^2} \Big[(1 + f_i) \ln(1 + f_i) - f_i \ln f_i \Big]$$

with the distribution function $f_\pm = \frac{1}{\exp(\beta\omega_\pm) - 1}$, and

$$\beta\omega_\pm = 2 \ln\left(\frac{1}{2\sqrt{\tilde{M}_\pm}} + \sqrt{1 + \frac{1}{4\tilde{M}_\pm}} \right)$$

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Thermal density matrix

A different perspective, consider the following **reduced** density matrix

$$\hat{\rho}_r = (1 - e^{-\beta\omega_0}) \sum_{n=0} e^{-n\beta\omega_0} |n\rangle\langle n|$$

where n is the energy level, and define $f = \frac{1}{e^{\beta\omega_0} - 1}$. The corresponding von Neumann entropy is

$$S_V = (1 + f)\ln(1 + f) - f\ln(f)$$

This indicates $\hat{\rho}_r$ from CGC can be diagonalized in a new basis.

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After diagonalization

we found the following Bogoliubov transformation

$$c(k) = \frac{1}{2}(\sqrt{\alpha} + \frac{1}{\sqrt{\alpha}})\ a(k) + \frac{1}{2}(\sqrt{\alpha} - \frac{1}{\sqrt{\alpha}})\ a^\dagger(-k)$$
$$c^\dagger(k) = \frac{1}{2}(\sqrt{\alpha} + \frac{1}{\sqrt{\alpha}})\ a^\dagger(k) + \frac{1}{2}(\sqrt{\alpha} - \frac{1}{\sqrt{\alpha}})\ a(-k)$$

we have again two independent modes

$$4\alpha_+ = \sqrt{1 + 4\tilde{M}_+}$$
$$4\alpha_- = \sqrt{1 + 4\tilde{M}_-}$$

and

$$\hat{\rho} = N e^{-\beta\omega_+\hat{n}_+} e^{-\beta\omega_-\hat{n}_-}$$

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Thermalized massless quasi-particle