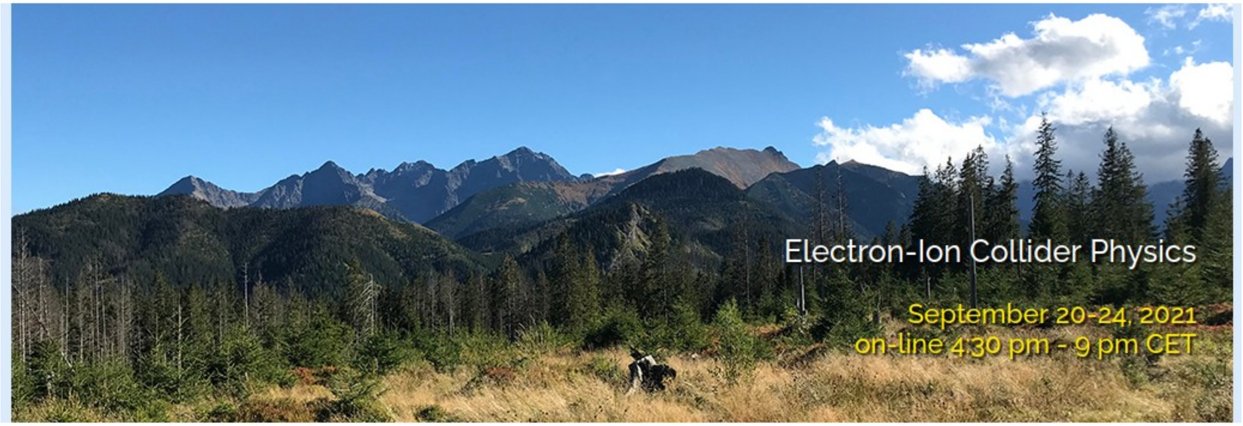
TMD evolution and parton showers

22.09.2021

A. Bermúdez Martínez on behalf of the TMD PB collaboration

61. Cracow School of Theoretical Physics



Why TMDs?

R. A. Martinez et al. [APP B46 (2015) 12, 2501–2534]

- Small transverse momentum phenomena
- Small-x phenomena
- DY, and semi-inclusive DIS
- Transverse momentum effects from intrinsic kt and evolution

Parton Branching (PB) method

FH et al. [PLB 772 (2017) 446–451]
FH et al. [JHEP 2018, 70 (2018)]
ABM et al. [PRD 99, 074008 (2019)]

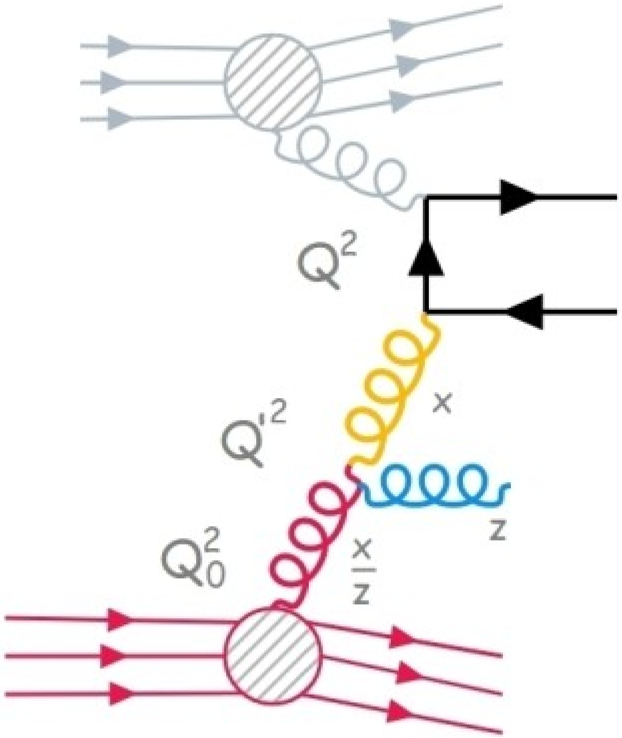
- Evolution of TMDs (and collinear PDFs)
- Resummation of soft gluons at LL and NLL
- Solution valid at LO, NLO and NNLO
- Determination of TMDs from the fully exclusive solution
- **Backward evolution fully determines the TMD shower**

→ consistently treats perturbative and non-perturbative transverse momentum effects

PB method recap

$$A_a^{(1)}(x, \mathbf{k_t}; Q^2) = \Delta_a(Q^2, Q_0^2) A_a(x, \mathbf{k_t}; Q_0^2) +$$

$$+ \sum_b \int_{Q_0^2}^{Q^2} \frac{d^2\mathbf{Q}'}{\pi Q'^2} \frac{\Delta_a(Q^2, Q_0^2)}{\Delta_a(Q'^2, Q_0^2)} \int_x^{z_M} dz P_{ab}^{(R)}\left(z, \alpha_s(Q'^2)\right) \Delta_b(Q'^2, Q_0^2) A_b\left(\frac{x}{z}, \mathbf{k_t} + (1-z)\mathbf{Q}'; Q_0^2\right)$$



- kinematics of the splittings is known
- physics → mapping of evolution variables to splitting kinematics
- TMD from cumulative kt of the branchings in forward PB evolution
- Automatically includes resummation at NLL
- **Initial-state shower fully determined by TMD and its backward PB evolution**
- **Parton shower exactly matches the evolution of the TMD**

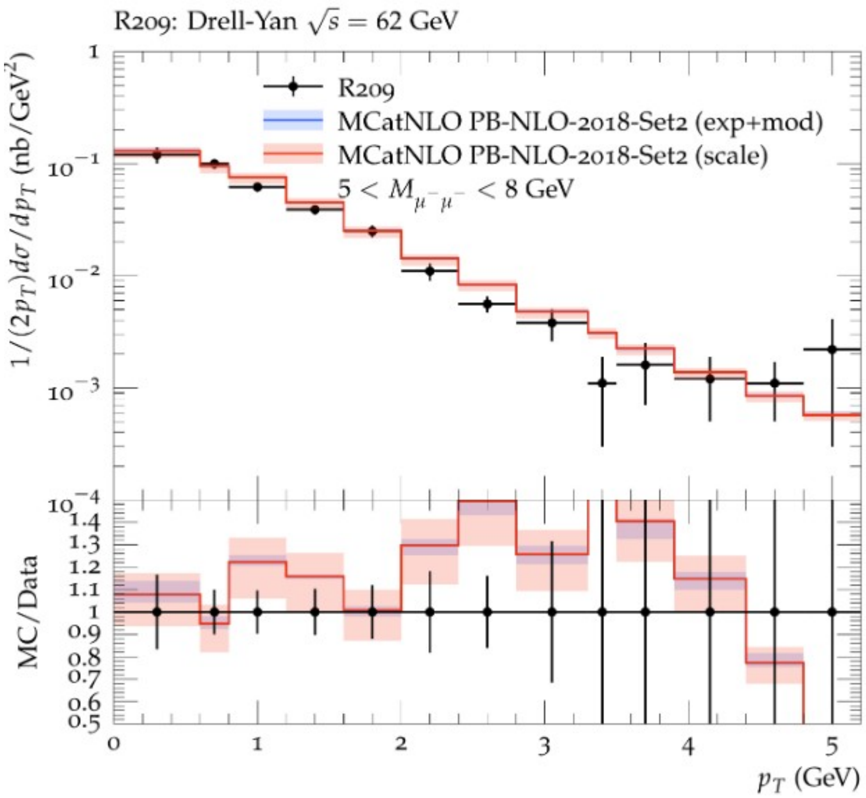
Z pT in a wide range of DY mass

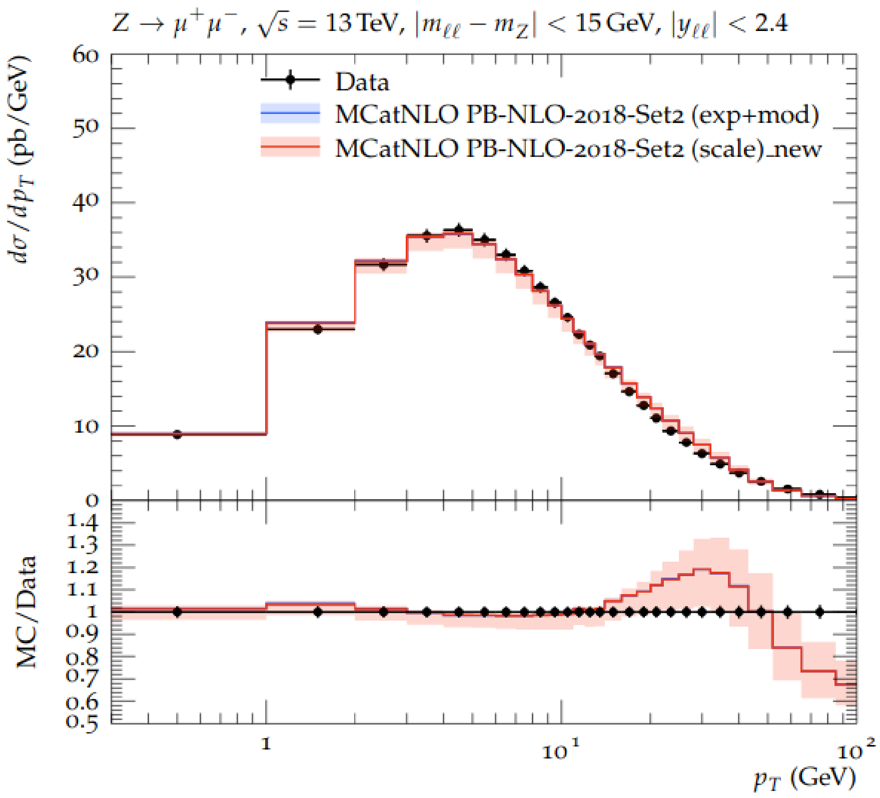
Application to low mass DY production

DY pt spectrum

ABM et al. [PRD 100, 074027 (2019)]
ABM et al. [EPJC 80, 598 (2020)]

- Combined with MC@NLO
- Excellent description of DY pT spectrum
- **First simultaneous description of both low and high-mass DY pT spectrum**
- **No more low pT crisis** Bacchetta et al. [PRD 100 (2019) 014018]; ABM et al. [EPJC 80, 598 (2020)]



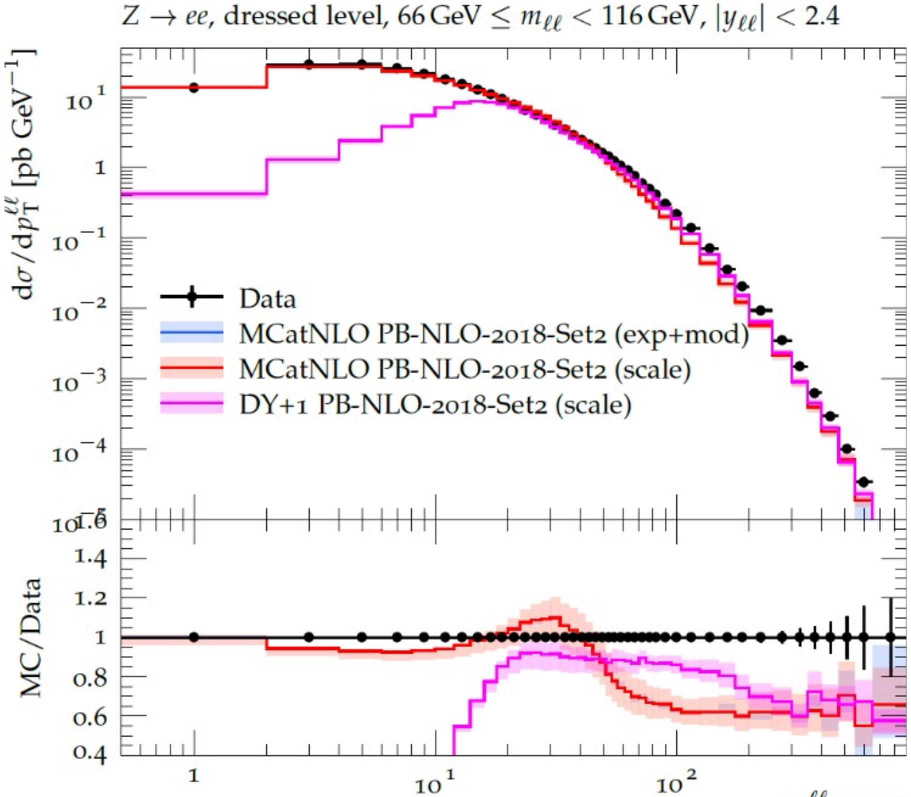


Combining TMD shower with higher orders

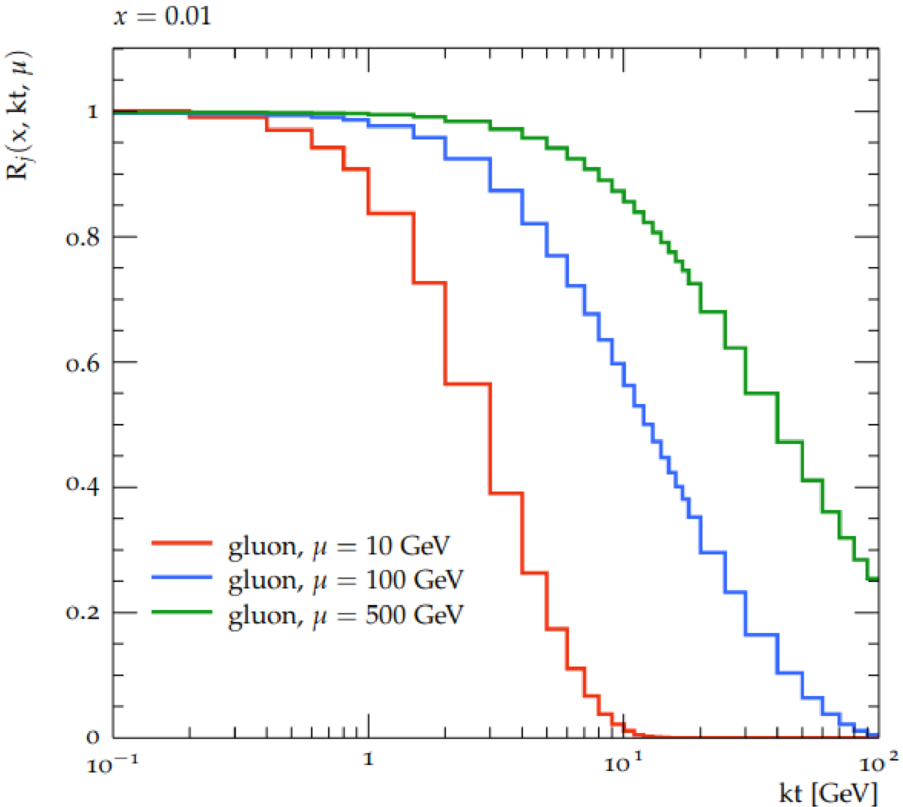
Trying TMD shower with higher orders

DY pt spectrum

- Important deficit at high pT with Z at NLO
- Potentially large corrections by higher orders
- **Try combining high pT TMD effects with multiple higher orders**



ABM et al. [PRD 100, 074027 (2019)]



ABM et al. [arXiv:2107.01224]

What we want:

- Treat perturbative and non-perturbative TMD effects
- Include soft gluon resummation
- Include corrections from higher-order fixed-order calculations

→ **Develop a method to combine PB-TMDs with multi-jet calculations**

Multi-jet merging

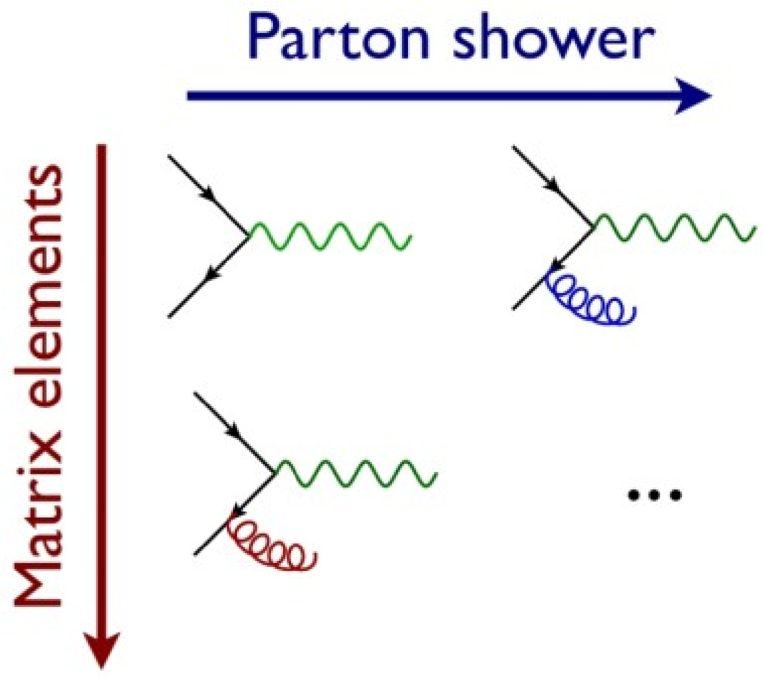
- Make higher-order ME exclusive by Sudakov suppression
- Avoid double counting between PS and ME

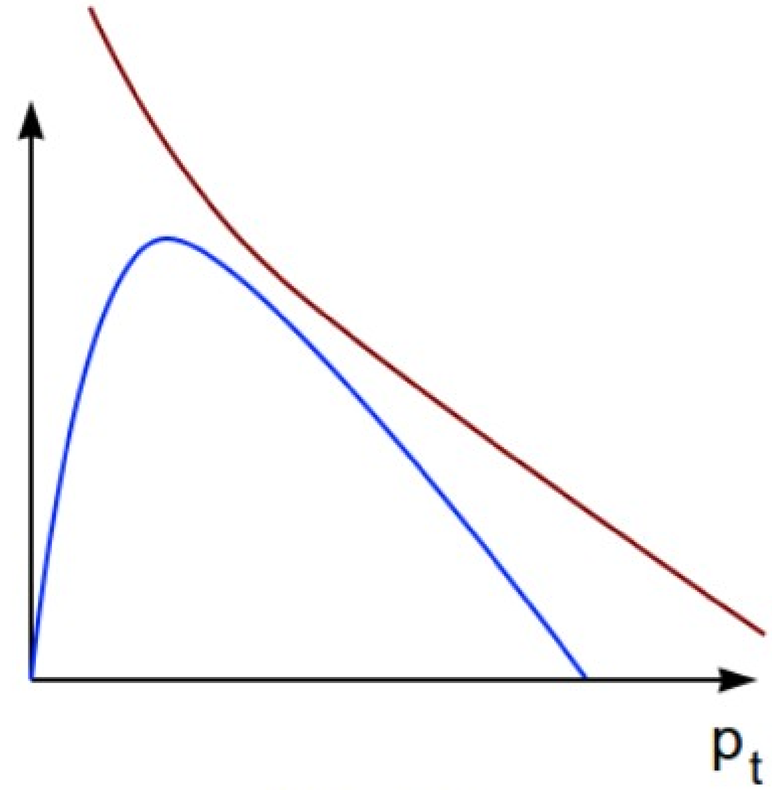
↓

- Improvement of hard, wide-angle emissions
- Description of high-pT phenomena

multi-jet merging

- Z production as an example:





- 1st emission PS: $\mathcal{R}^{PS}(p_t^2) \times \exp\left[-\int_{p_t^2} dp_t'^2 \frac{\mathcal{R}^{PS}(p_t'^2)}{\mathcal{B}}\right]$

- 1st emission ME: $\mathcal{R}(p_t^2)$

multi-jet merging

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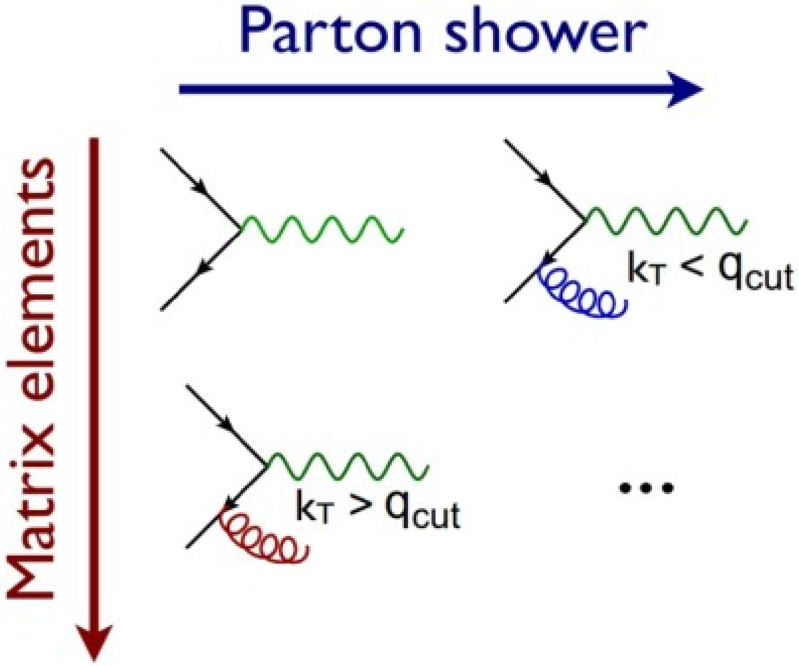

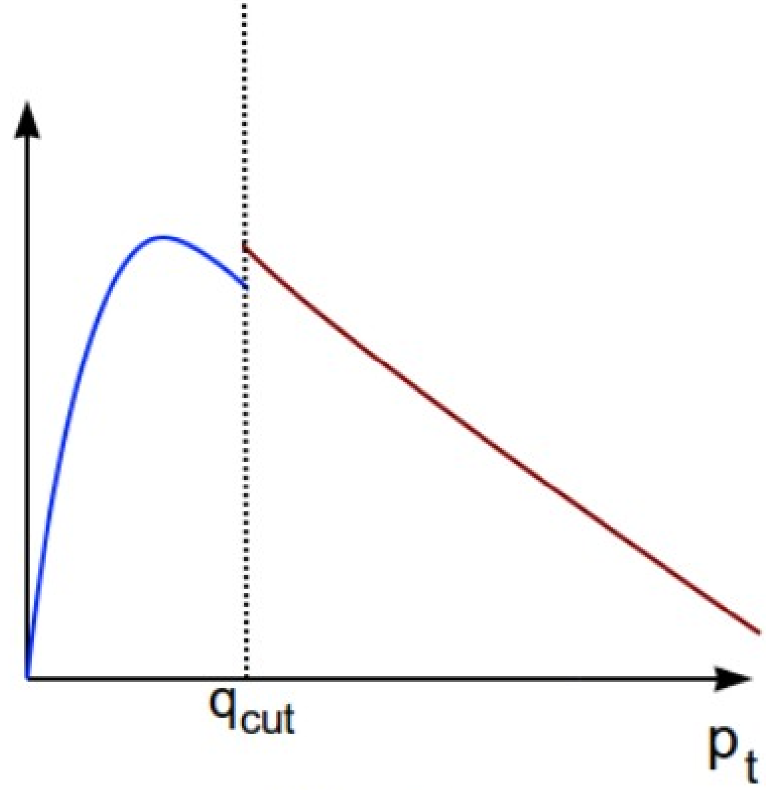

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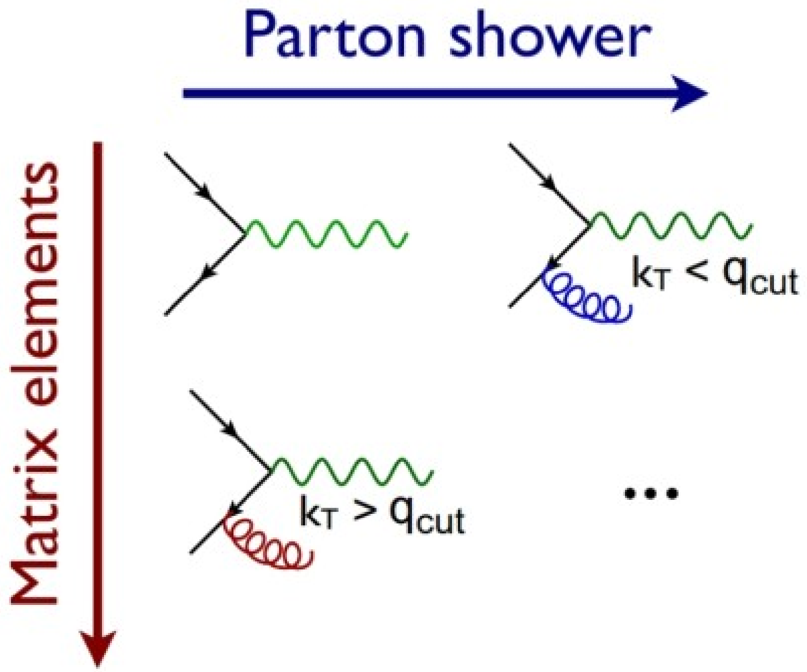

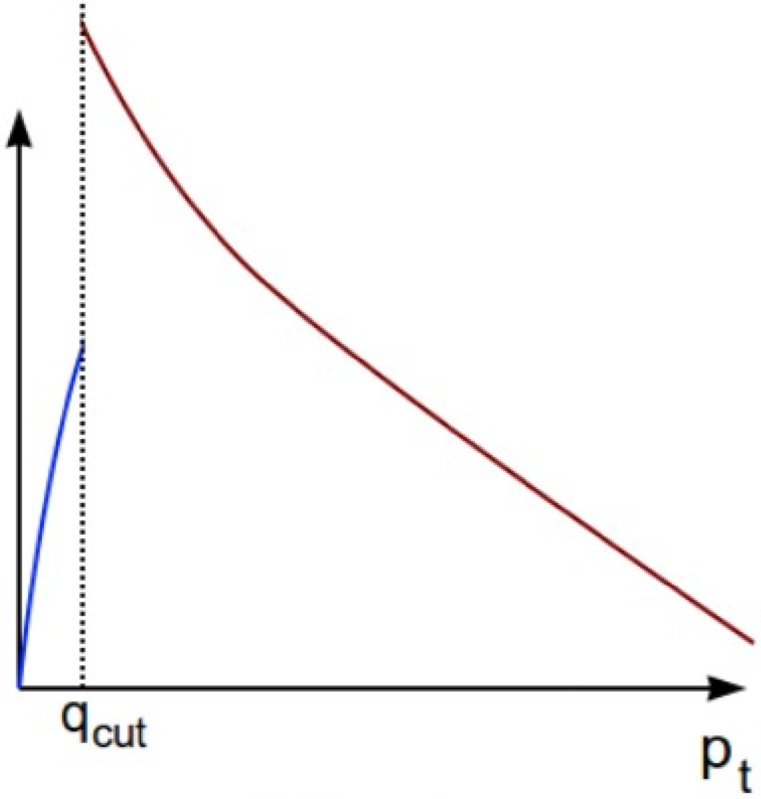

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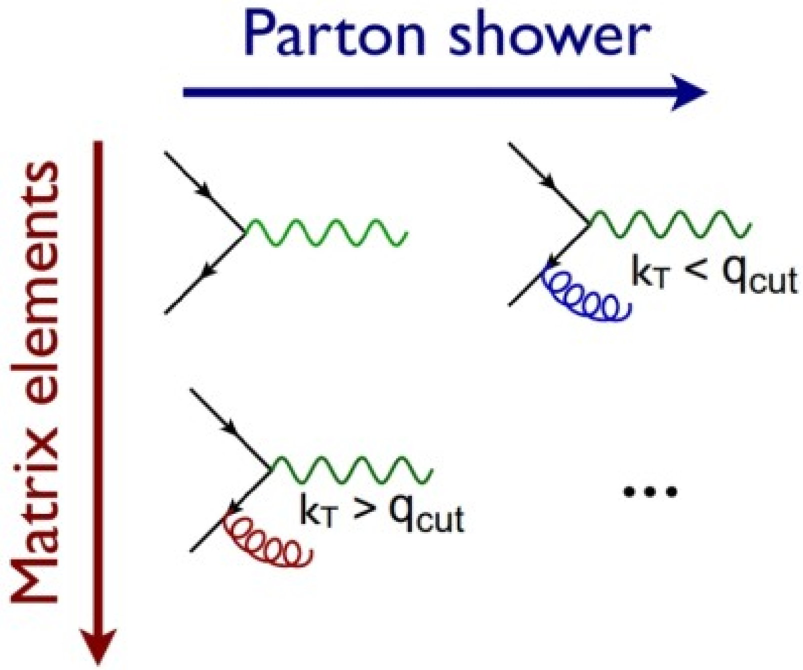

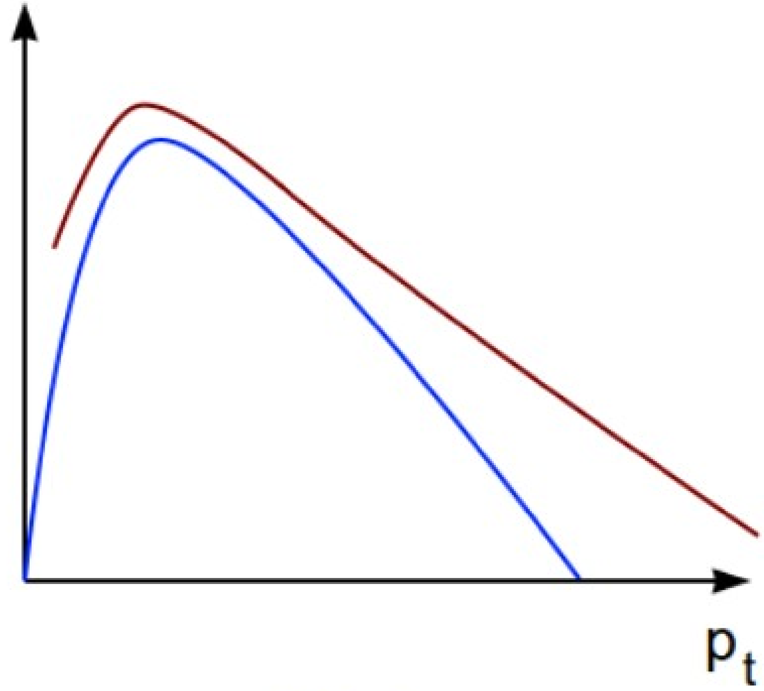

- 1st emission PS: $\mathcal{R}^{PS}(p_t^2) \times \exp\left[-\int_{p_t^2} dp_t'^2 \frac{\mathcal{R}^{PS}(p_t'^2)}{\mathcal{B}}\right]$
- 1st emission ME: $\mathcal{R}(p_t^2) \longrightarrow \mathcal{R}(p_t^2) \times \exp\left[-\int_{p_t^2} dp_t'^2 \frac{\mathcal{R}^{PS}(p_t'^2)}{\mathcal{B}}\right]$

multi-jet merging

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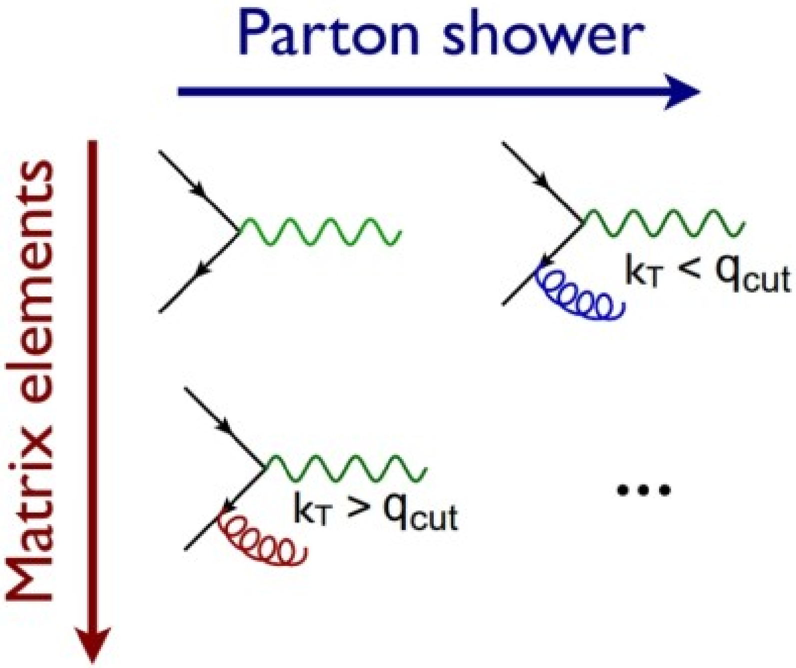

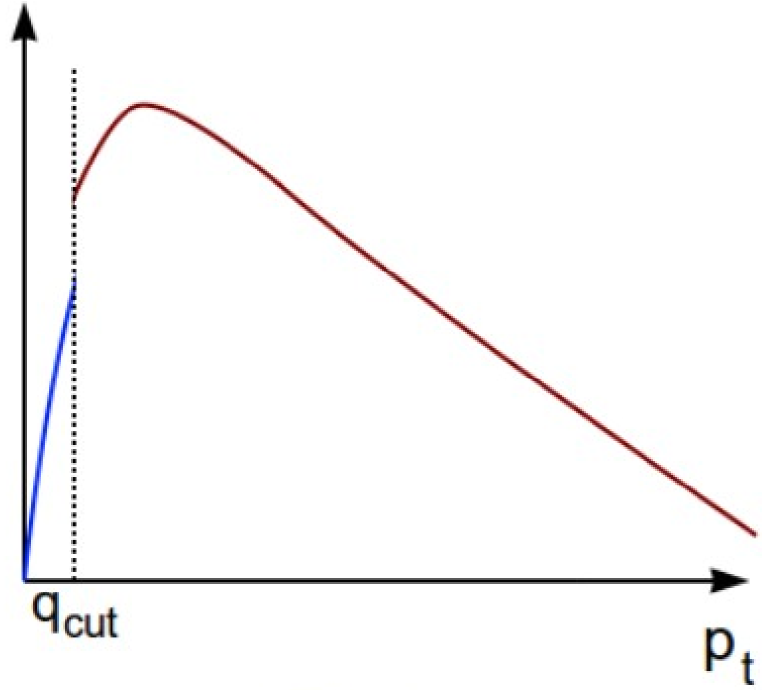

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multi-jet merging

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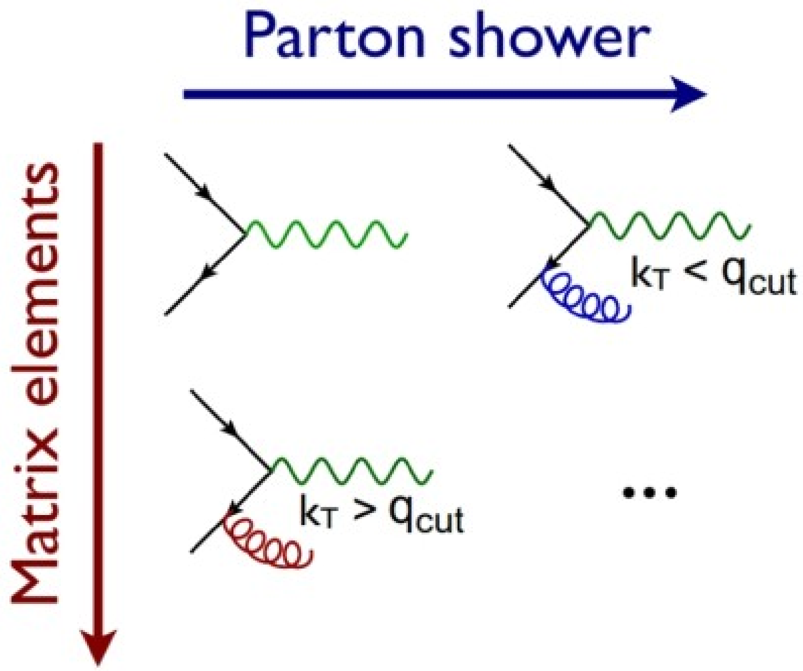

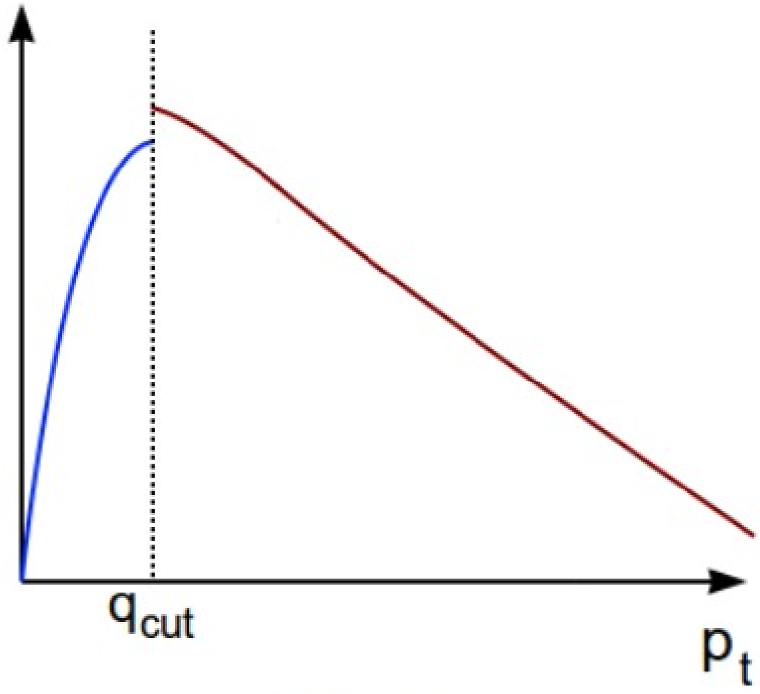

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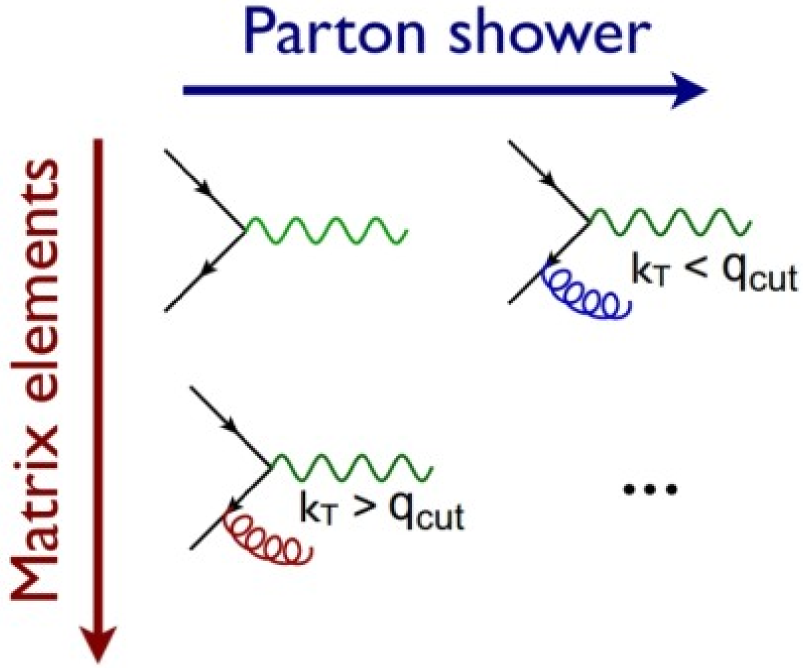
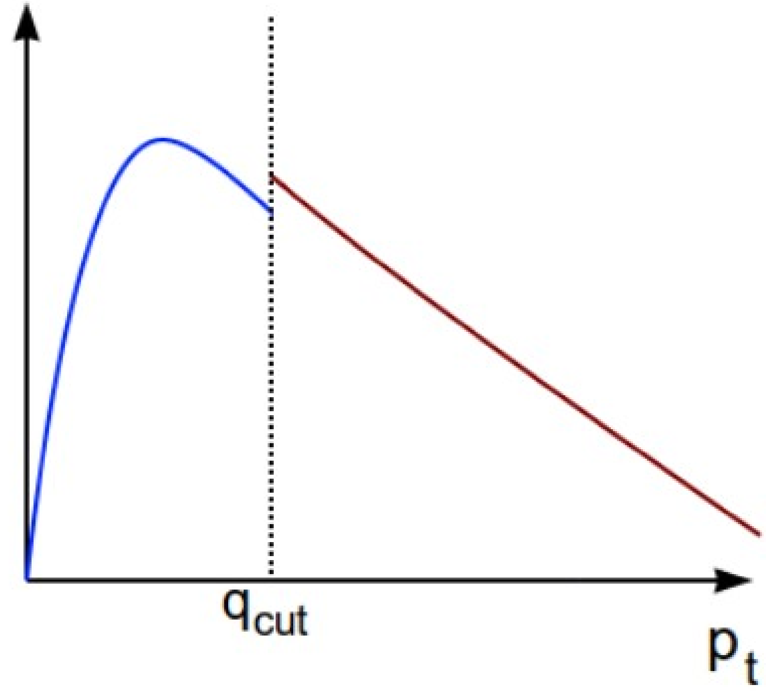
multi-jet merging

○ Z production as an example:



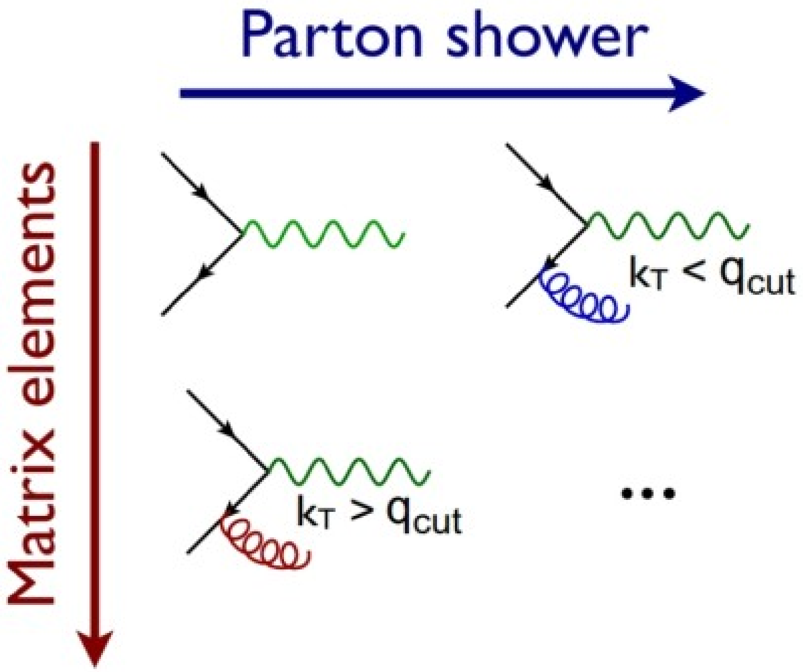


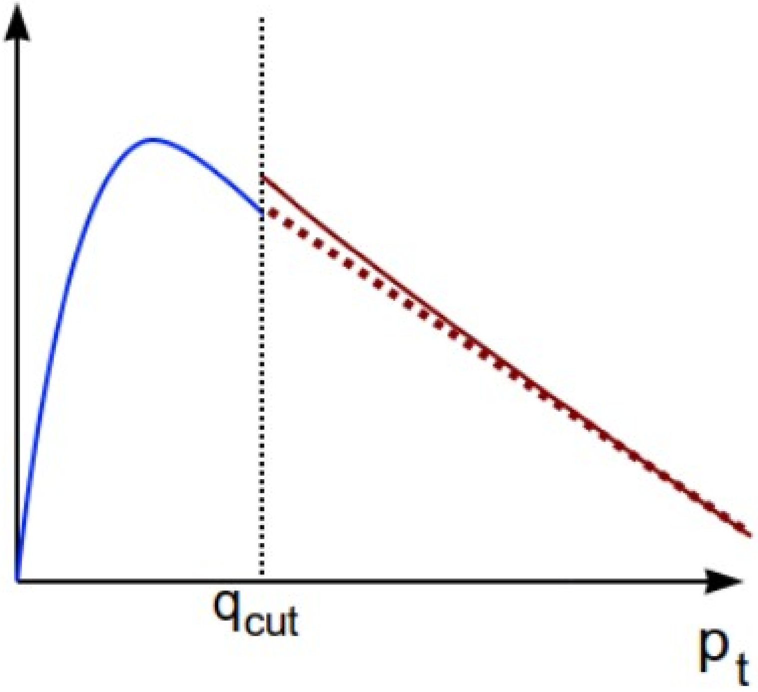
- 1st emission PS: $\mathcal{R}^{PS}(p_t^2) \;\sim \alpha_s(p_t^2)$

- 1st emission ME: $\mathcal{R}(p_t^2) \;\; \sim \alpha_s(\mu^2)$

multi-jet merging

- Z production as an example:



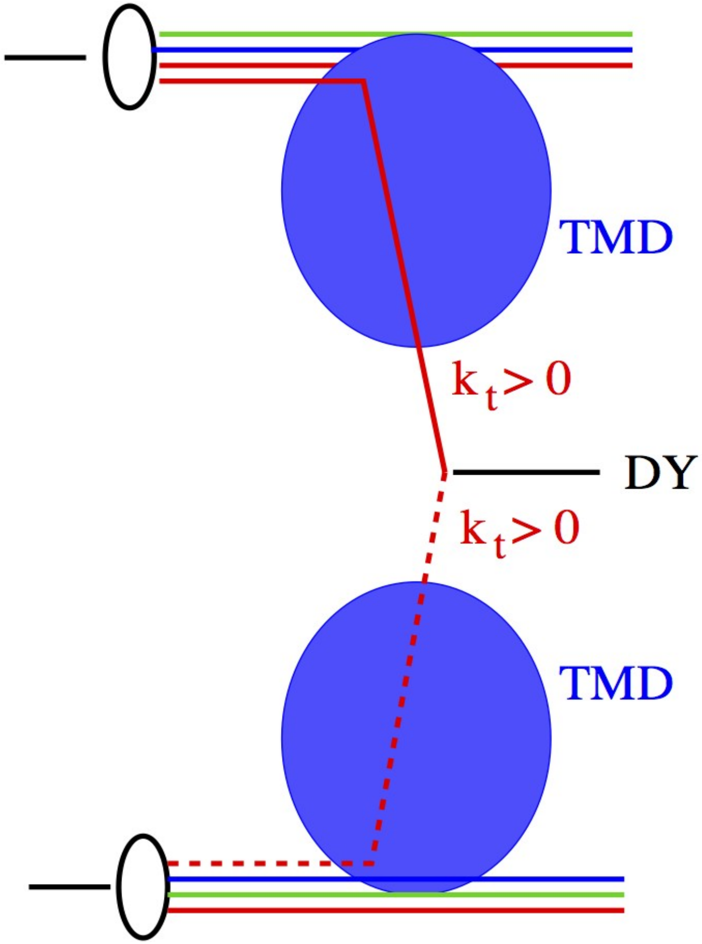


- 1st emission PS: $\mathcal{R}^{PS}(p_t^2) \;\sim \alpha_s(p_t^2)$
- 1st emission ME: $\mathcal{R}(p_t^2) \quad \rightarrow \mathcal{R}(p_t^2)\times\alpha_s(p_t^2)/\alpha_s(\mu^2)$

TMD merging method

ABM et al. [arXiv:2107.01224]

- Evaluate the ME for n-jet cross sections
- Reweight the strong coupling according to shower history
- **Evolve the ME using the TMD PB evolution**
- **Shower the events using the backward PB evolution for ISR**
- **Apply the MLM[1] prescription between the PB-evolved ME and the showered events**



NB: The method could also be applied to merging criteria other than MLM

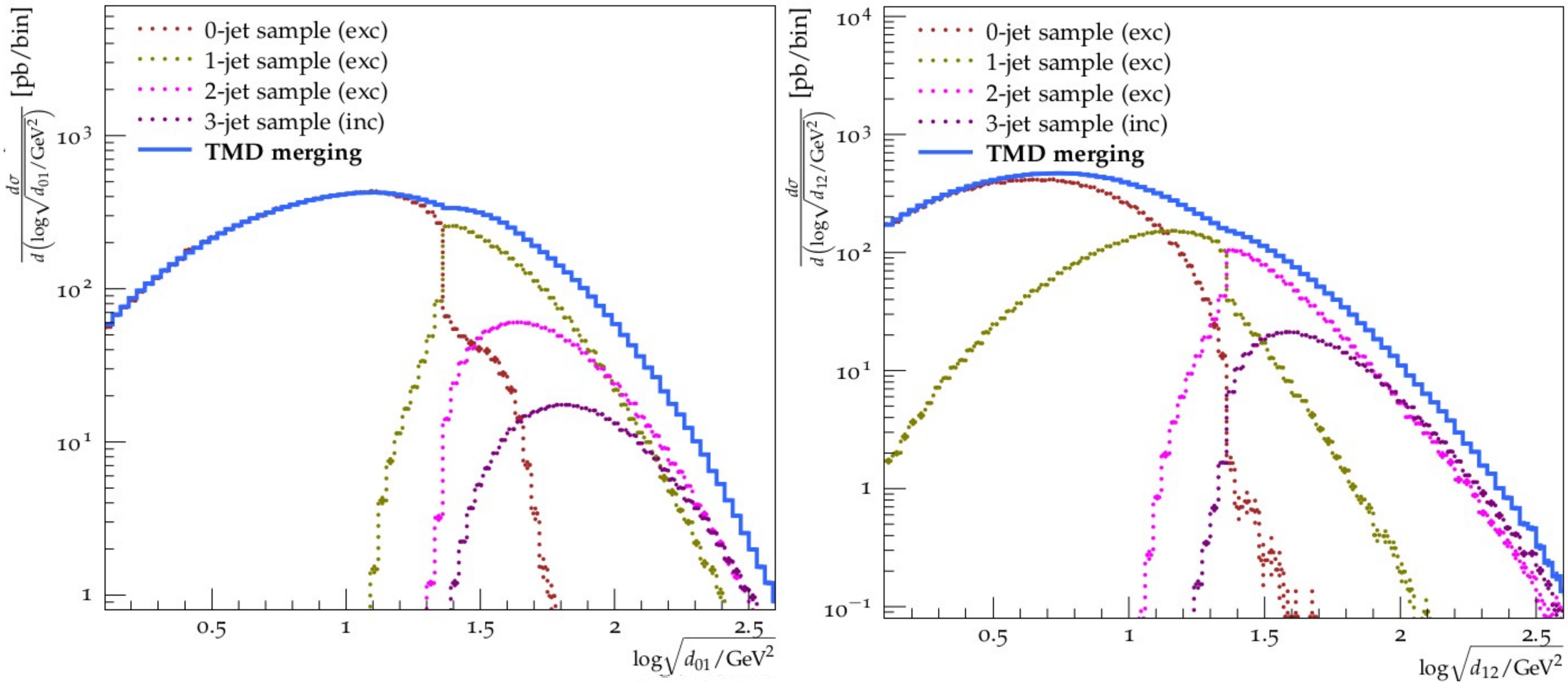
New merging procedure applicable to TMDs!

[1] M. L. Mangano [NPB 632 (2002) 343–362]

Combining TMD shower with higher orders

ABM et al. [paper in preparation]

d(n,n+1): scale at which (n+1)-jet configuration becomes n-jet



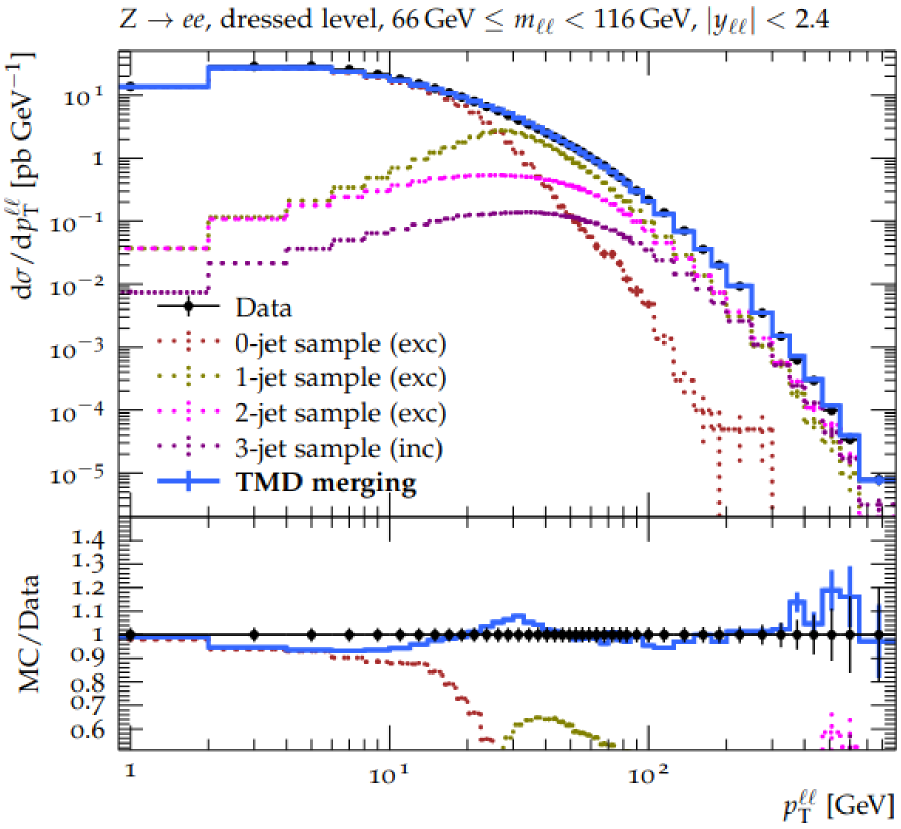
- **Smoothness** ➡ merging follows shower Sudakov suppression
- **Merging scale divides phase space** for different jet multiplicities **avoiding double counting**

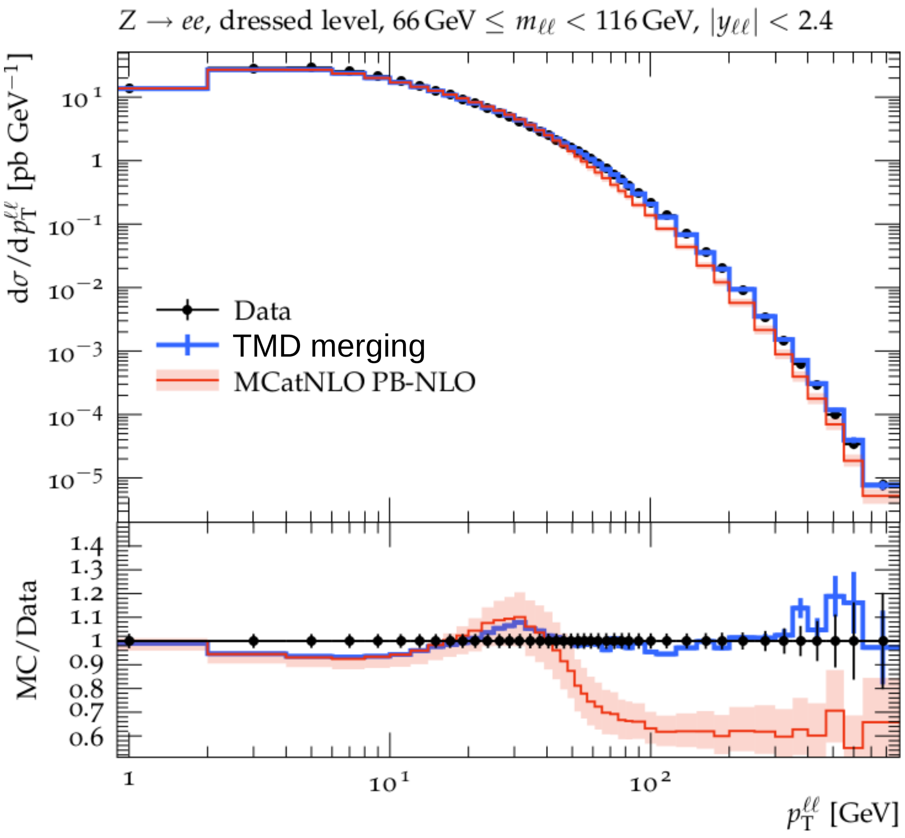
Combining TMD shower with higher orders

DY pt spectrum

- TMD evolution with multi-jet merging achieved at LO
- **Low as well as high-pt now nicely described**
- **Consistent with MCatNLO PB-NLO at low pT**

New! ABM et al. [arXiv:2107.01224]

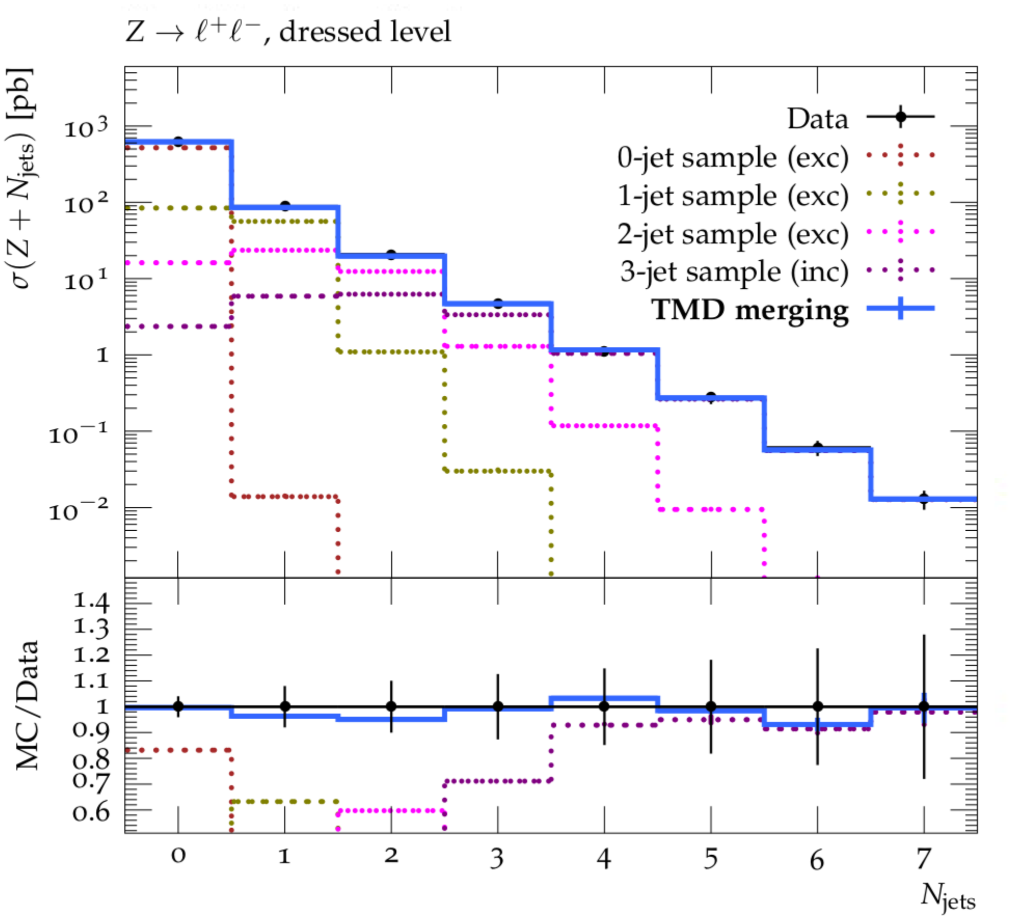




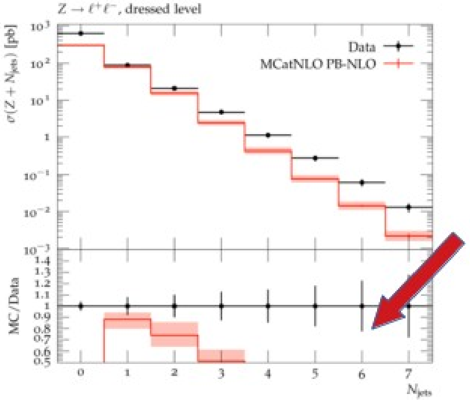
Combining TMD shower with higher orders

Exclusive jet multiplicity in Z events

ATLAS data: [Eur. Phys. J. C77 (2017) 361]



Instead of:



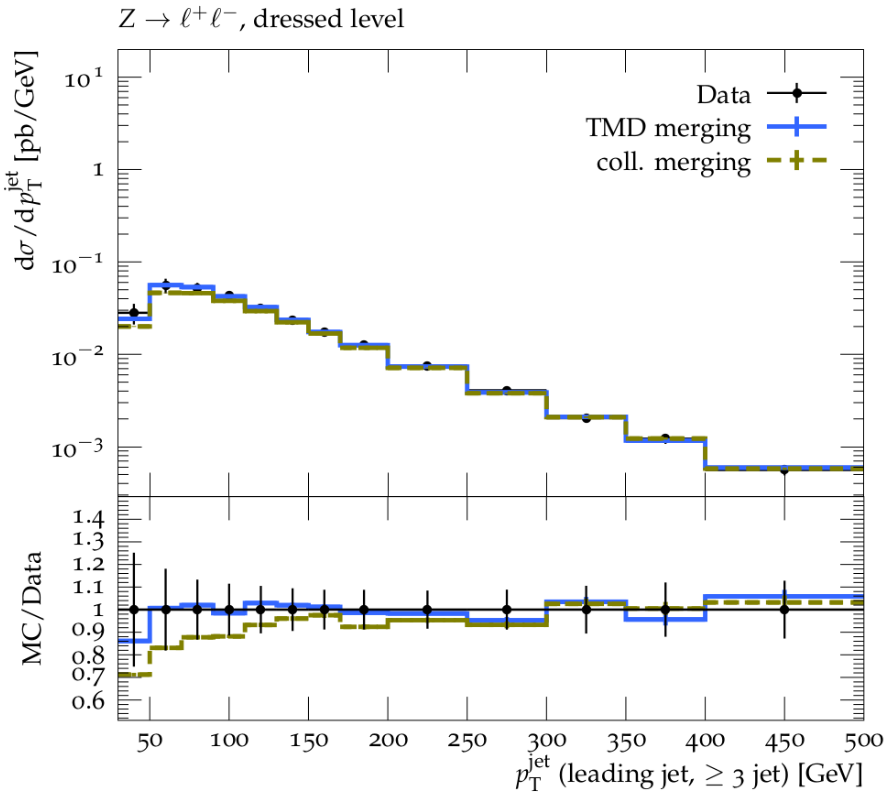
- **Not only the overall recoil but also the number of jets are described**

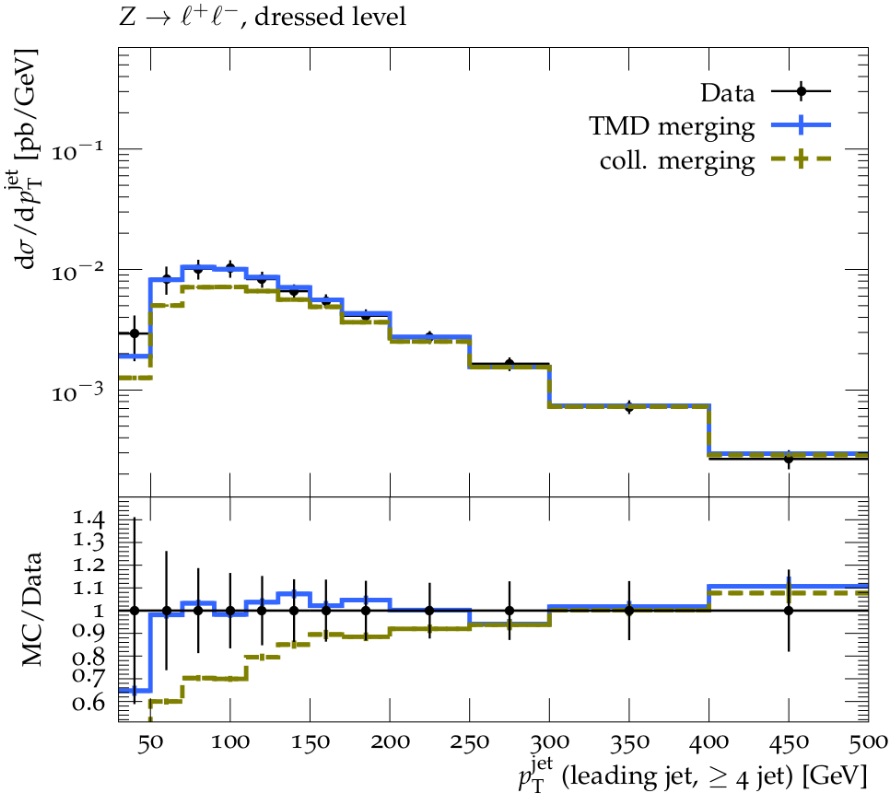
Combining TMD shower with higher orders

Jets pt spectrum

New! ABM et al. [arXiv:2107.01224]

- Not only overall recoil but also jet pT
- **The description of jet pT improves at high multiplicities**





Systematics

Multi-jet cross section in Z production

Merging scale [GeV]	σ[tot] [pb]	σ[≥ 1 jet] [pb]	σ[≥ 2 jet] [pb]	σ[≥ 3 jet] [pb]	σ[≥ 4 jet] [pb]
23	572.98	87.26	20.27	4.84	1.18
33	563.04	86.15	20.48	4.86	1.19

- 10 GeV variation gives **< 2% change** in jets cross sections
- Standard merging algorithms can give over 10 % change for the same variation of the merging scale CF: J. Alwall et al. [EPJC 53, 473–500 (2008)]

→ Dependence on merging scale reduced by treating transverse momentum in the initial state

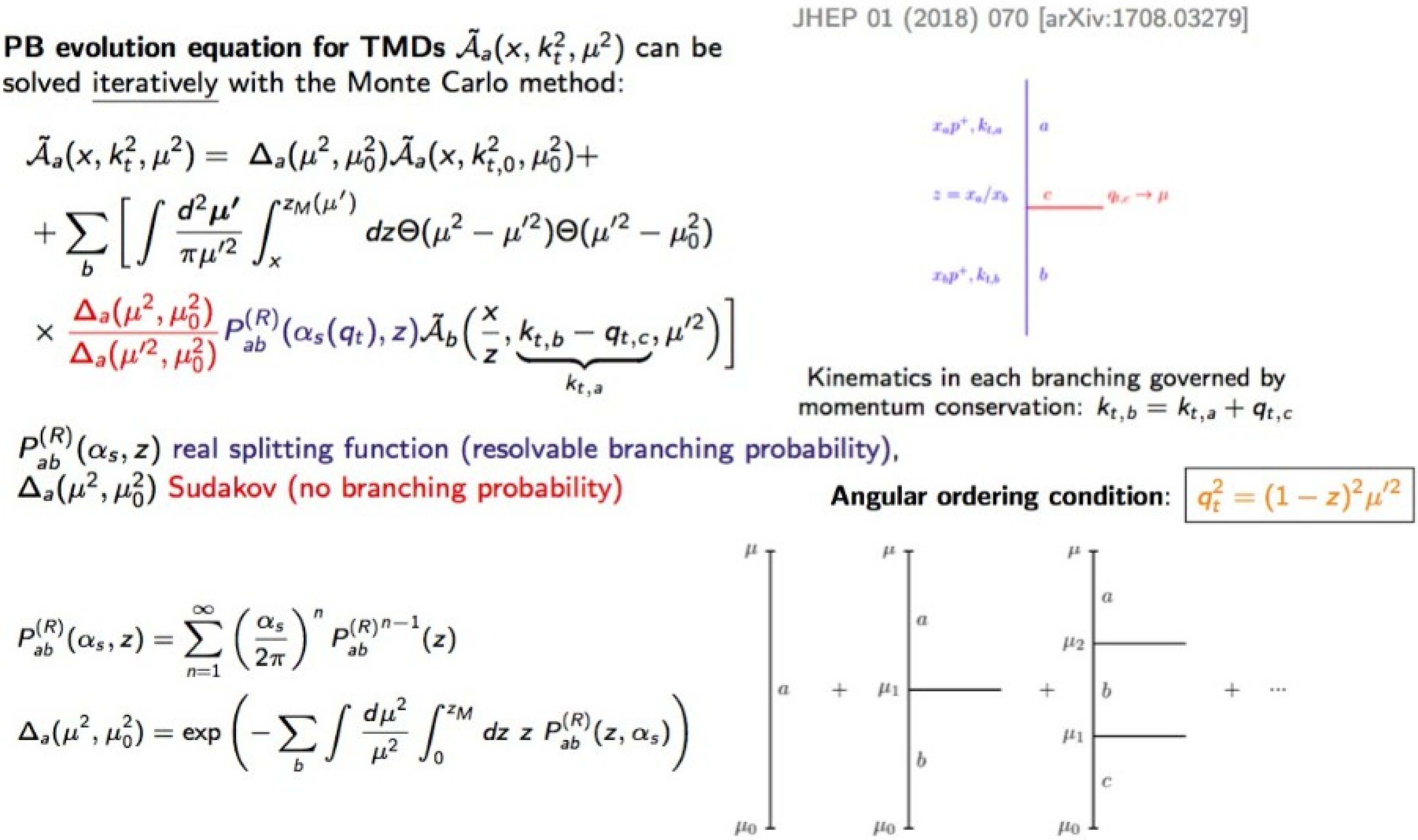
Conclusions

- PB TMD evolution provides excellent description of DY pt spectrum in a wide range of DY mass
- Parton shower from PB TMD evolution have significant contribution to jet multiplicity and jet pt spectra
- Higher fix-order contributions to PB TMD evolution are significant
- First combination of TMD evolution effects with multi-jet merging for Z pt and jet spectra
- Dependence of the results on the merging scale are smaller than that of standard algorithms

Thank you

PB formulation of TMD evolution

[slide by M. van Kampen]



JHEP 01 (2018) 070 [arXiv:1708.03279]

PB evolution equation for TMDs $\tilde{\mathcal{A}}_a(x, k_t^2, \mu^2)$ can be solved iteratively with the Monte Carlo method:

$$\tilde{\mathcal{A}}_a(x, k_t^2, \mu^2) = \Delta_a(\mu^2, \mu_0^2)\tilde{\mathcal{A}}_a(x, k_{t,0}^2, \mu_0^2) +$$
$$+ \sum_b \Bigg[\int \frac{d^2\mu'}{\pi \mu'^2} \int_x^{z_M(\mu')} dz \Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2)$$
$$\times \frac{\Delta_a(\mu^2, \mu_0^2)}{\Delta_a(\mu'^2, \mu_0^2)} P_{ab}^{(R)}(\alpha_s(q_t), z)\tilde{\mathcal{A}}_b\left(\frac{x}{z}, \underbrace{k_{t,b} - q_{t,c}}_{k_{t,a}}, \mu'^2\right)\Bigg]$$

Kinematics in each branching governed by momentum conservation: $k_{t,b} = k_{t,a} + q_{t,c}$

$P_{ab}^{(R)}(\alpha_s, z)$ real splitting function (resolvable branching probability),
$\Delta_a(\mu^2, \mu_0^2)$ Sudakov (no branching probability)

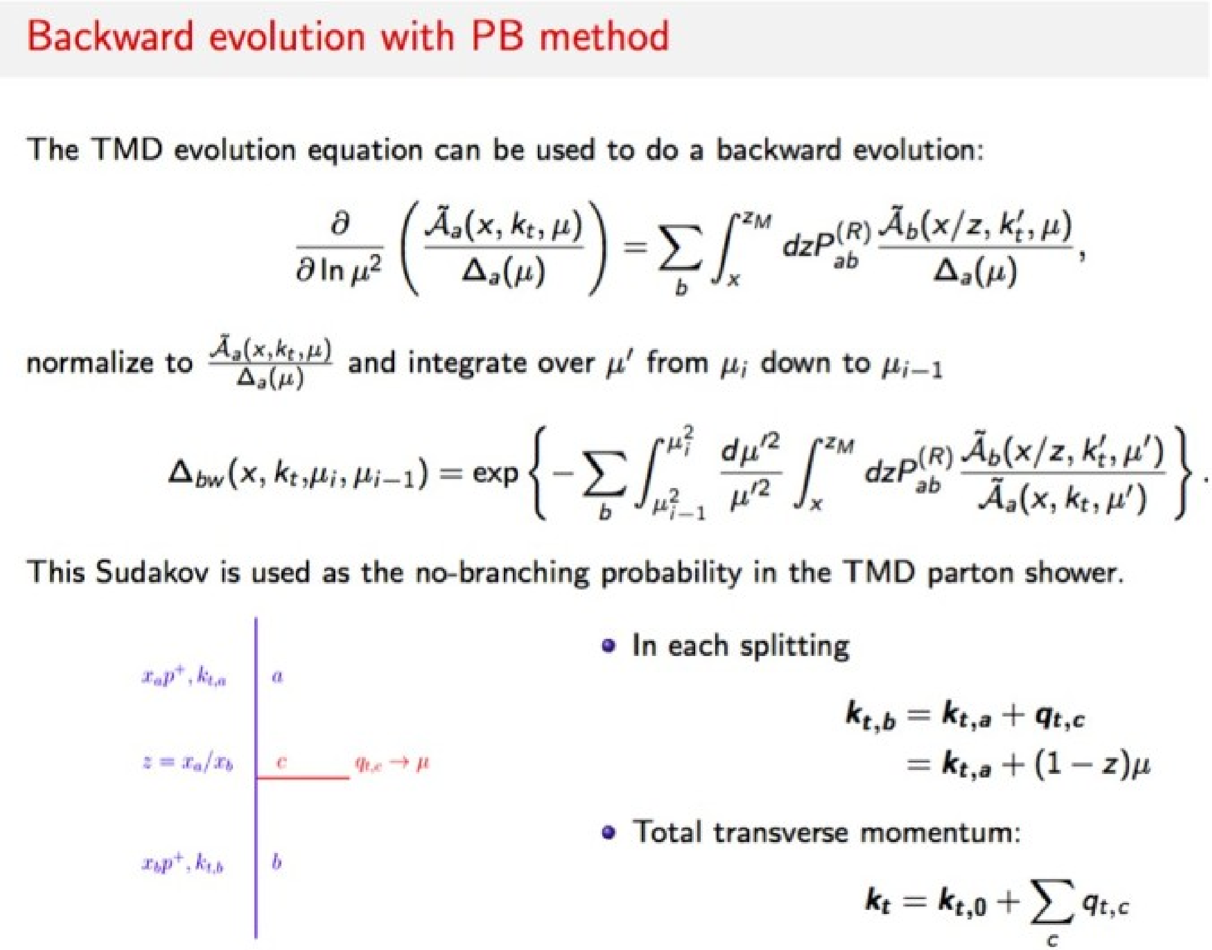
Angular ordering condition: $\boxed{q_t^2 = (1-z)^2\mu'^2}$

$$P_{ab}^{(R)}(\alpha_s, z) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n P_{ab}^{(R)\,n-1}(z)$$

$$\Delta_a(\mu^2, \mu_0^2) = \exp\left(- \sum_b \int \frac{d\mu^2}{\mu^2} \int_0^{z_M} dz \; z \; P_{ab}^{(R)}(z, \alpha_s) \right)$$

PB formulation of TMD evolution

[slide by M. van Kampen]



Backward evolution with PB method

The TMD evolution equation can be used to do a backward evolution:

$$\frac{\partial}{\partial \ln \mu^2}\left(\frac{\tilde{A}_a(x,k_t,\mu)}{\Delta_a(\mu)}\right) = \sum_b \int_x^{z_M} dz P_{ab}^{(R)} \frac{\tilde{A}_b(x/z,k_t',\mu)}{\Delta_a(\mu)},$$

normalize to $\frac{\tilde{A}_a(x,k_t,\mu)}{\Delta_a(\mu)}$ and integrate over μ' from μ_i down to μ_{i-1}

$$\Delta_{bw}(x,k_t,\mu_i,\mu_{i-1}) = \exp\left\{-\sum_b \int_{\mu_{i-1}^2}^{\mu_i^2} \frac{d\mu'^2}{\mu'^2} \int_x^{z_M} dz P_{ab}^{(R)} \frac{\tilde{A}_b(x/z,k_t',\mu')}{\tilde{A}_a(x,k_t,\mu')}\right\}.$$

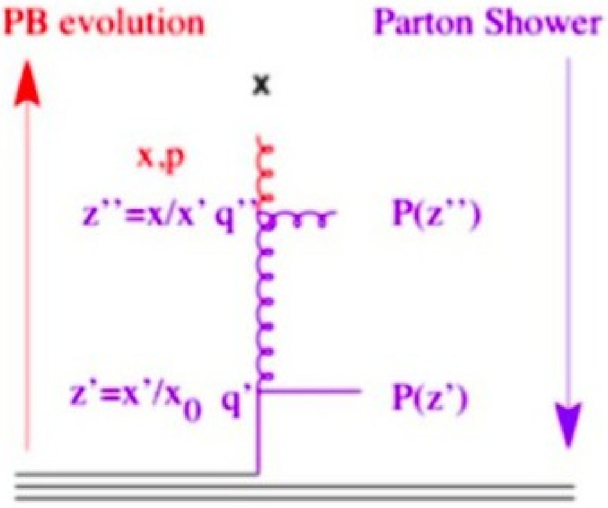
This Sudakov is used as the no-branching probability in the TMD parton shower.

- In each splitting

$$\begin{aligned} \mathbf{k}_{t,b} &= \mathbf{k}_{t,a} + \mathbf{q}_{t,c} \\ &= \mathbf{k}_{t,a} + (1-z)\mu \end{aligned}$$

- Total transverse momentum:

$$\mathbf{k}_t = \mathbf{k}_{t,0} + \sum_c q_{t,c}$$



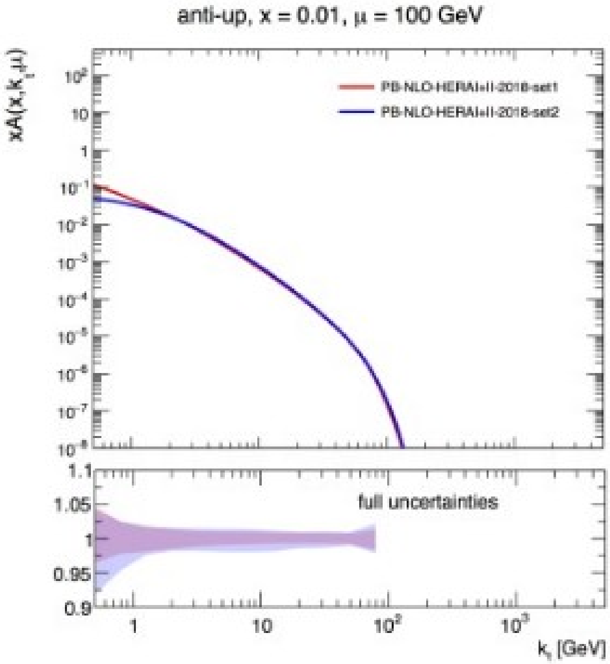
Implemented in the CASCADE event generator

S. Baranov et al. [Eur. Phys. J. C 81 (2021) 425]

PB framework

Phys. Lett. B 772:446451 (2017)
JHEP 01:070 (2018)
Phys. Rev. D 99, 074008 (2019)

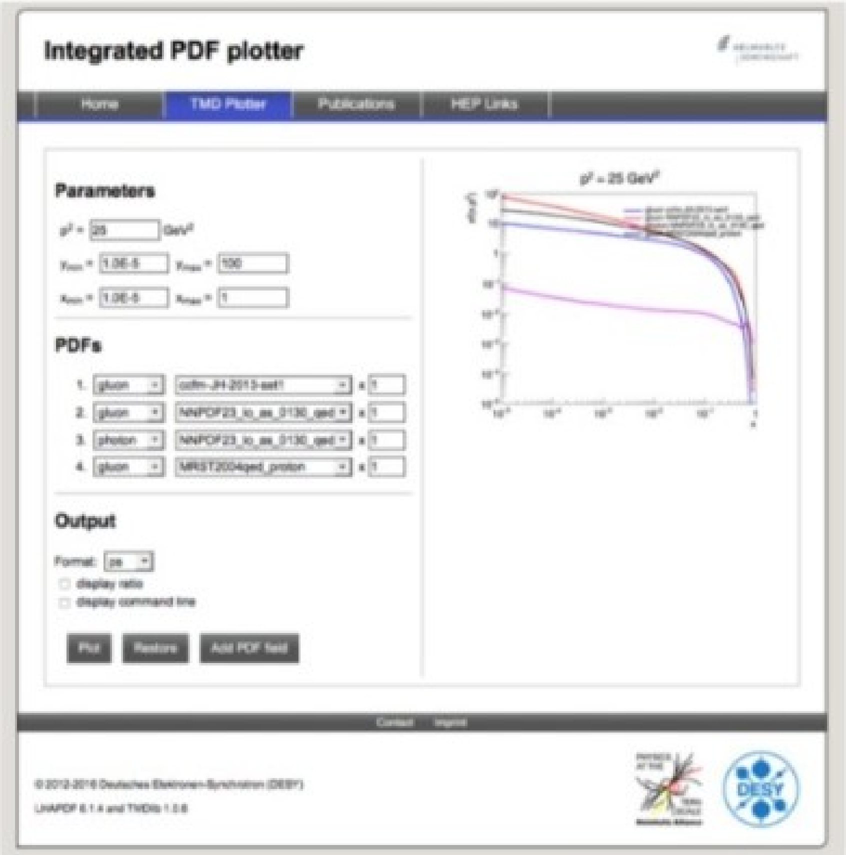
- TMD determined, no extra parameters
- Full access to splitting kinematics
- TMD evolution implemented in xFitter



Where to find them:

arXiv:2103.09741 (accepted for publication in EPJC)

- TMDlib: library of parametrization of TMDs and uPDFs
- TMDplotter: TMD plotting tool



Integrated PDF plotter

Parameters

μ^2 = 25 GeV²

y_{min} = 1.0E-5 y_{max} = 100

x_{min} = 1.0E-5 x_{max} = 1

PDFs

1. gluon ccfm-JH-2013-set1 x 1
2. gluon NNPDF23_lo_as_0130_qed x 1
3. photon NNPDF23_lo_as_0130_qed x 1
4. gluon MRST2004qed_proton x 1

Output

Format: ps

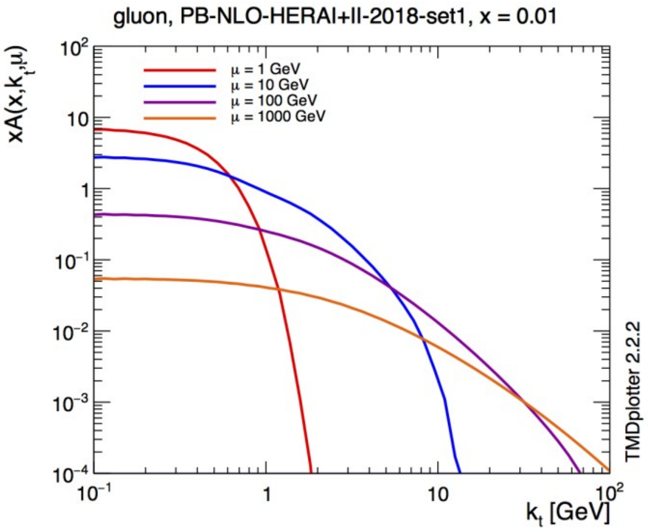
☐ display ratio
☐ display command line

μ^2 = 25 GeV²

© 2012-2016 Deutsches Elektronen-Synchrotron (DESY)

LHAPDF 6.1.4 and TMDlib 1.0.8

Pert. and non-pert. PB TMD contributions



- **ISR broadens initial distribution**

ABM et al. [PRD 99, 074008 (2019)]

- DIS measurements from HERA I+II
- fitting procedure in a nutshell:
  - parametrize the integrated distribution at Q_0
  - with the PB method evolve the TMD to $Q > Q_0$ (implemented in xFitter)
  - fit the measurements and extract the initial parametrization
  - store the TMD in a grid for later use (TMDlib, complementary slides)

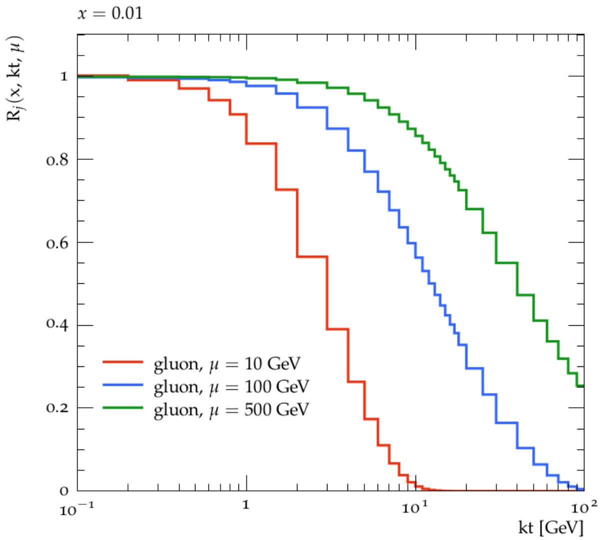
N. A. Abdulov et al. [Eur. Phys. J. C 81 (2021) 752]

Consider the integrated distribution above the jet pT scale:

$$a_j(x, \boldsymbol{k}, \mu^2) = \int \frac{d^2\boldsymbol{k}'}{\pi} \mathcal{A}_j(x, \boldsymbol{k}', \mu^2)\, \Theta(\boldsymbol{k}'^2 - \boldsymbol{k}^2)$$

- **e.g. probability of 0.3 that the gluon develops a kt larger than 20 GeV, for μ = 100 GeV**

- **TMD evolution effects crucial at describing jet production**



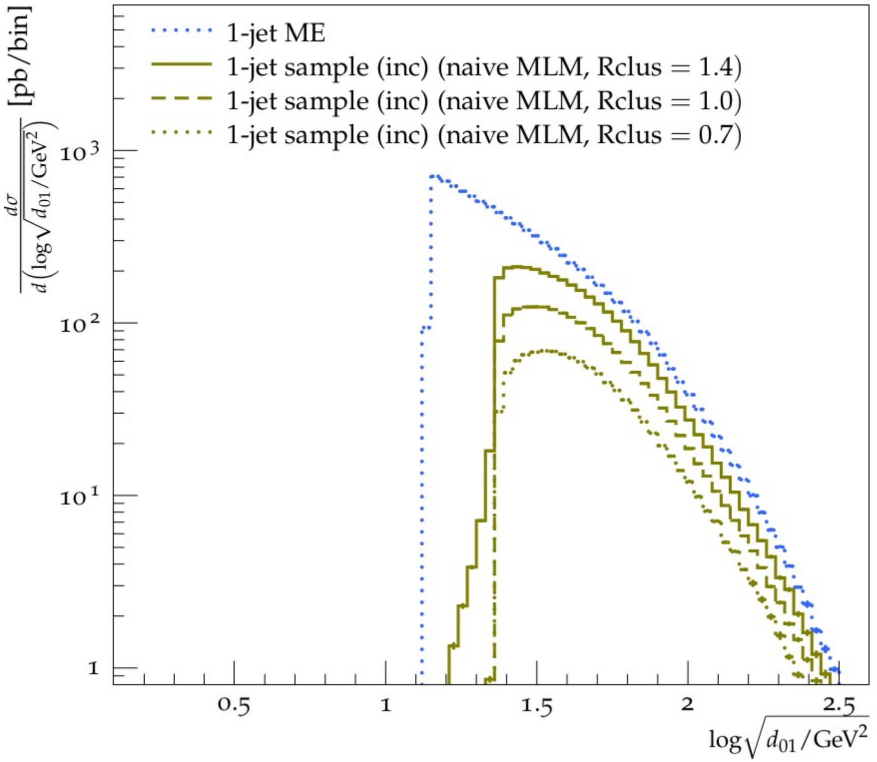
ABM et al. [arXiv:2107.01224]

From MLM to TMD merg.

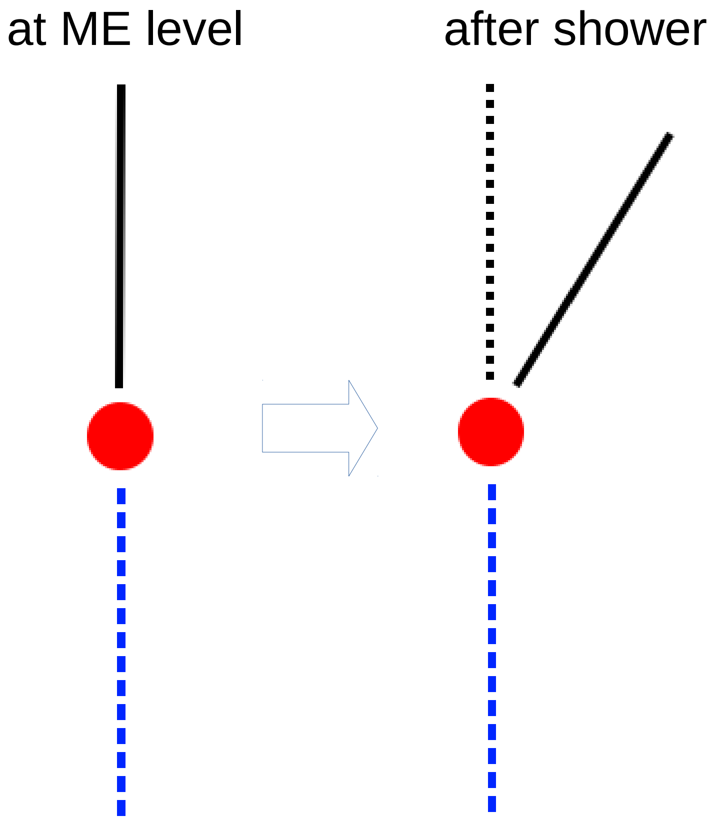
ABM et al. [paper in preparation]

What about the original MLM applied to TMD events?

- very strong dependence on Rclus
- at large scales ME accuracy lost!



d(n,n+1): square of scale at which (n+1)-jet configuration becomes n-jet



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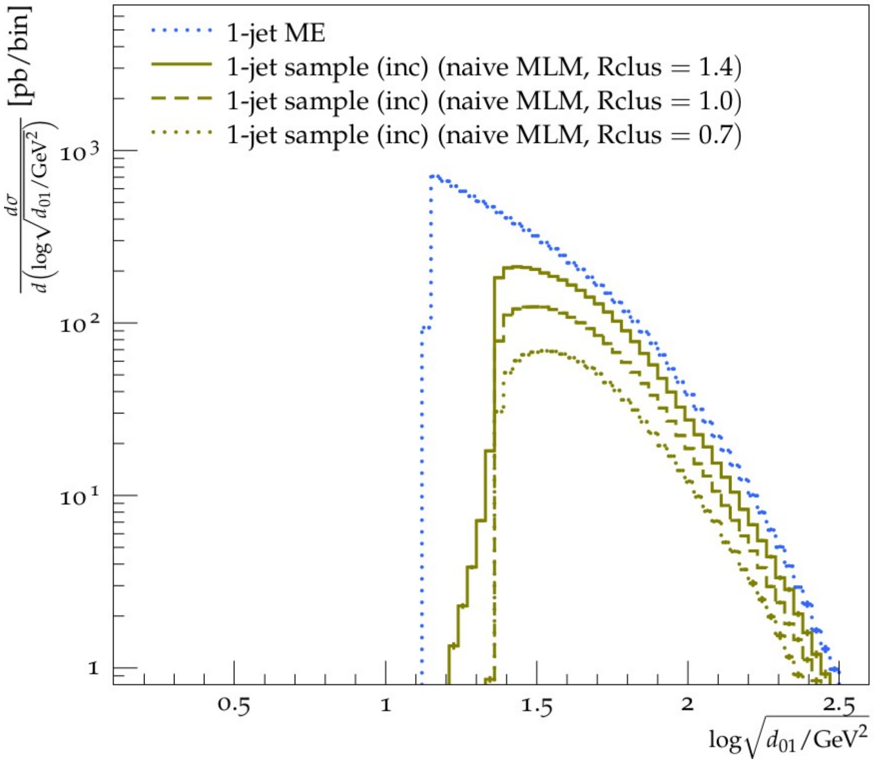

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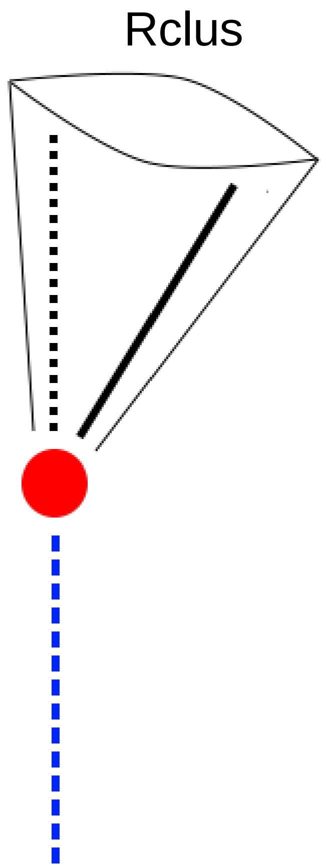

reject non-matched events

From MLM to TMD merg.

ABM et al. [paper in preparation]

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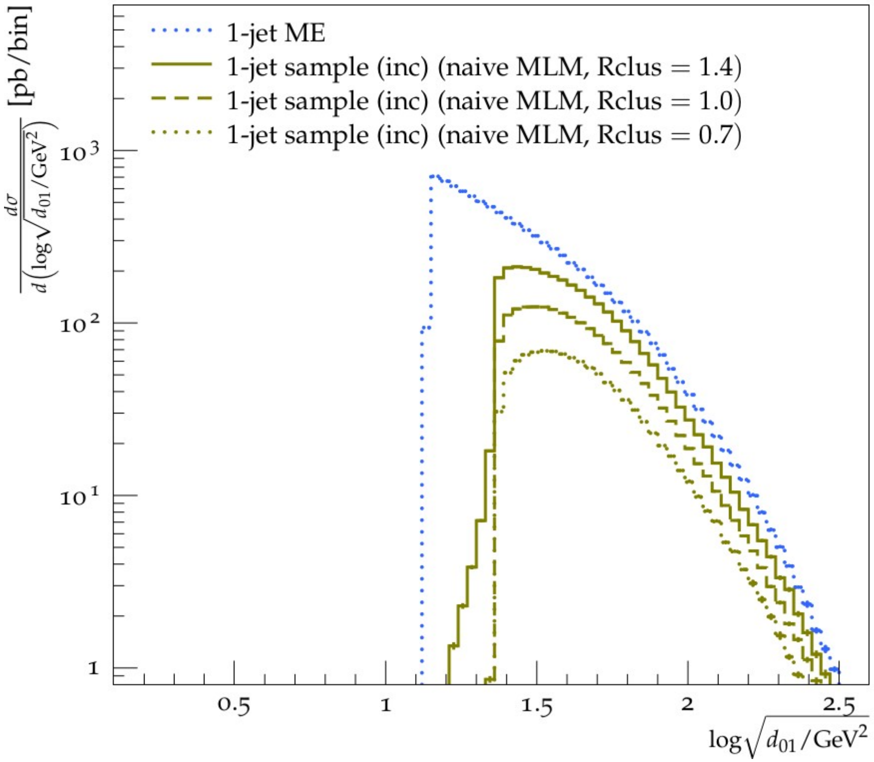

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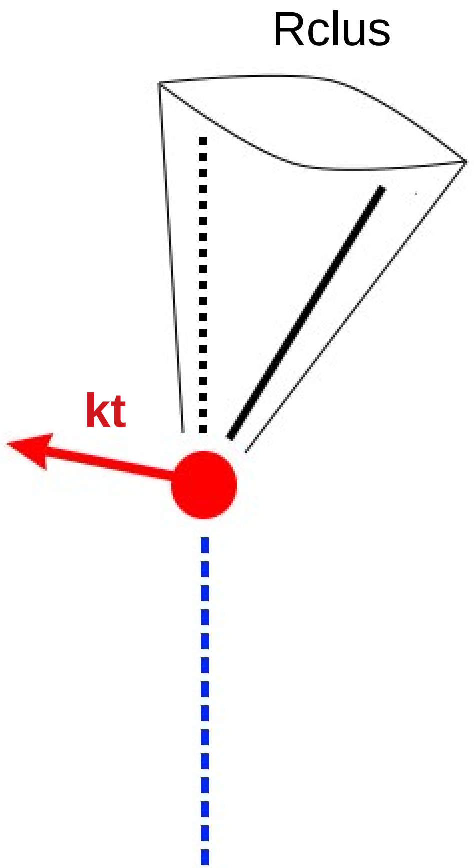


ktmax = **kt**max(Rclus) !

Rclus translates into a maximum TMD evolution scale

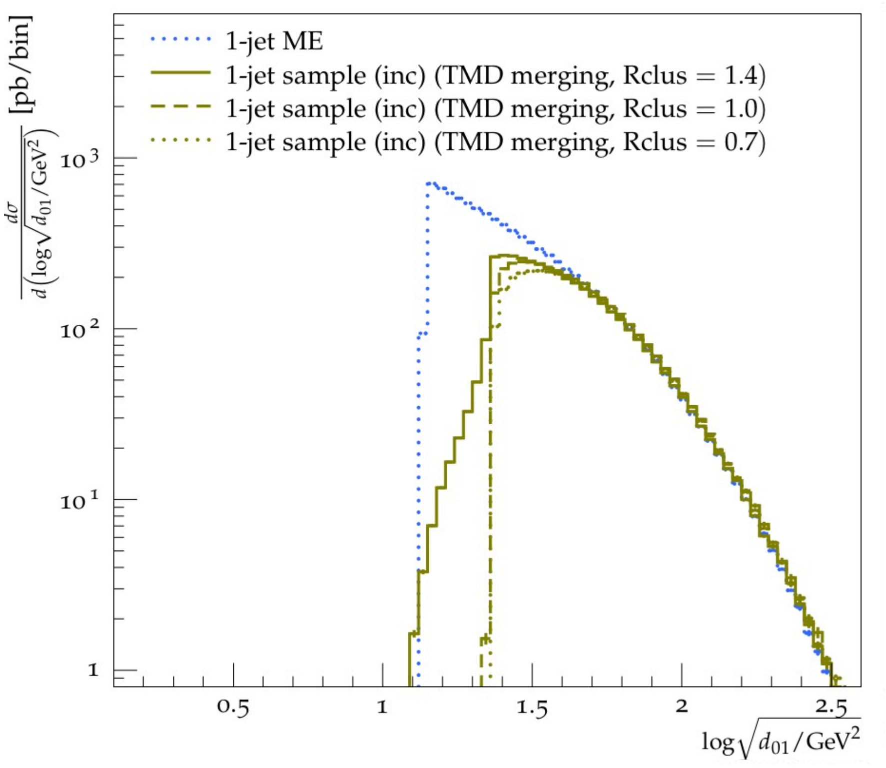
⇨ naive MLM incompatible with PB TMD evolution

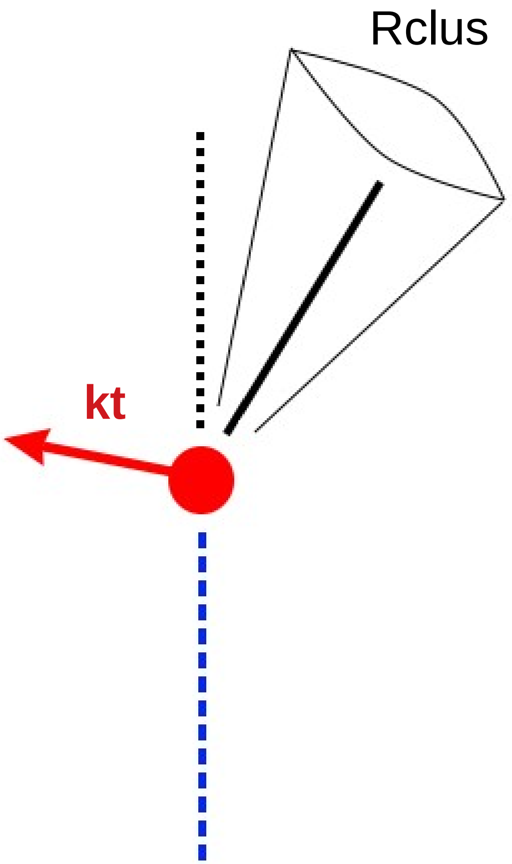
From MLM to TMD merg.

ABM et al. [paper in preparation]

TMD merging

- little dependence on Rclus
- at large scales ME accuracy recovered!

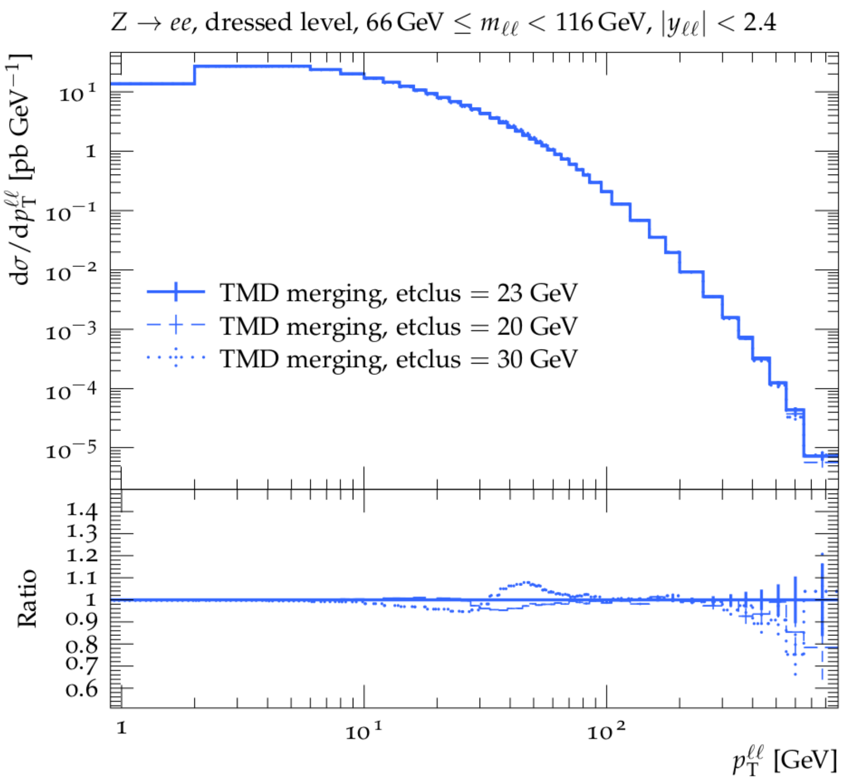


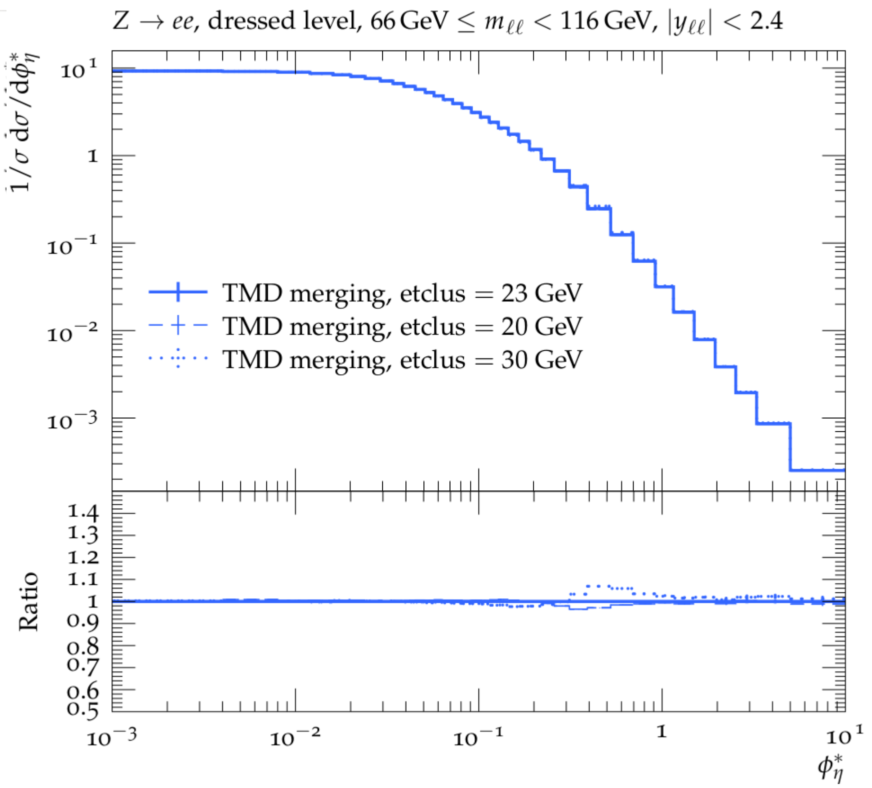


TMD PB evolution decouples from MLM matching

Systematics

Z pt and phi*

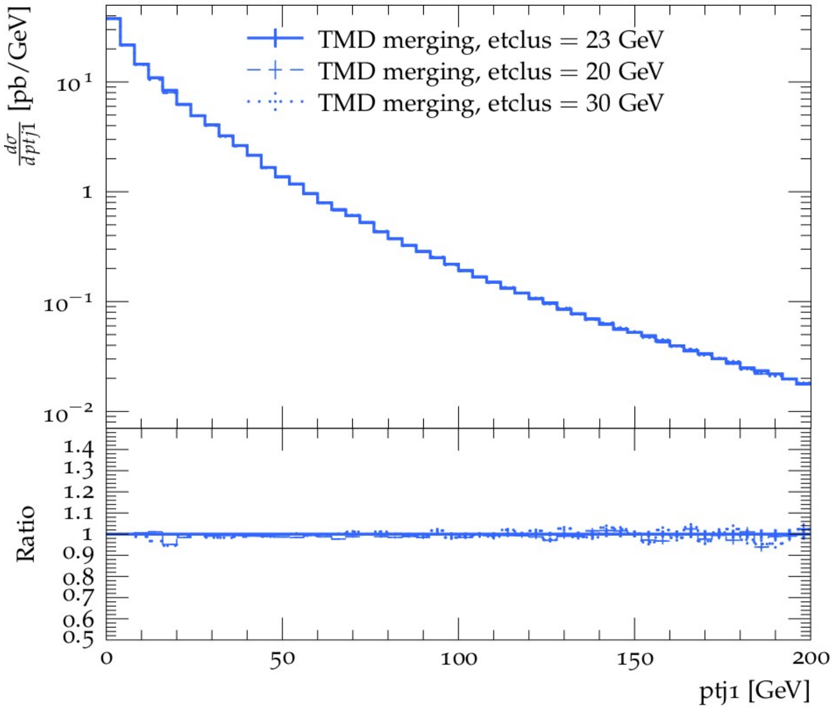


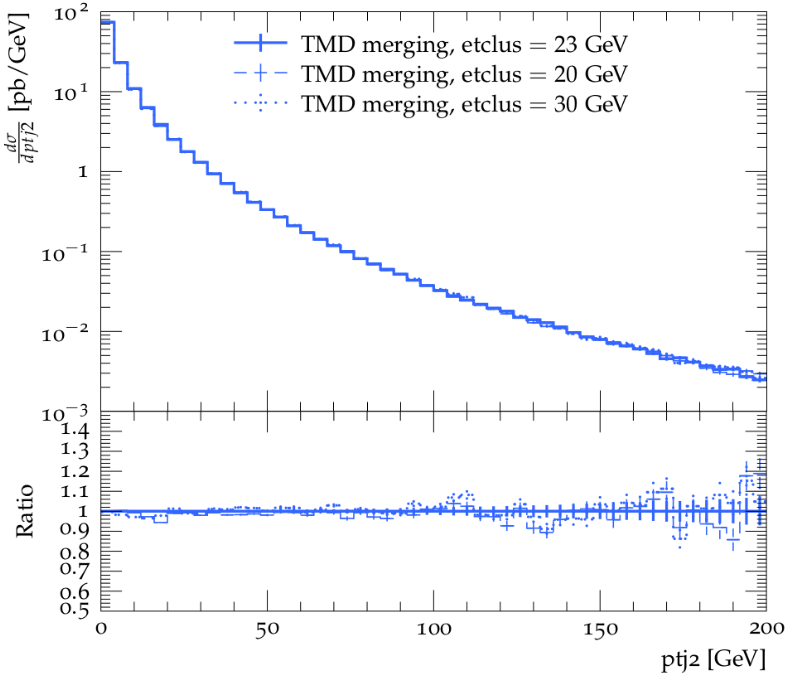


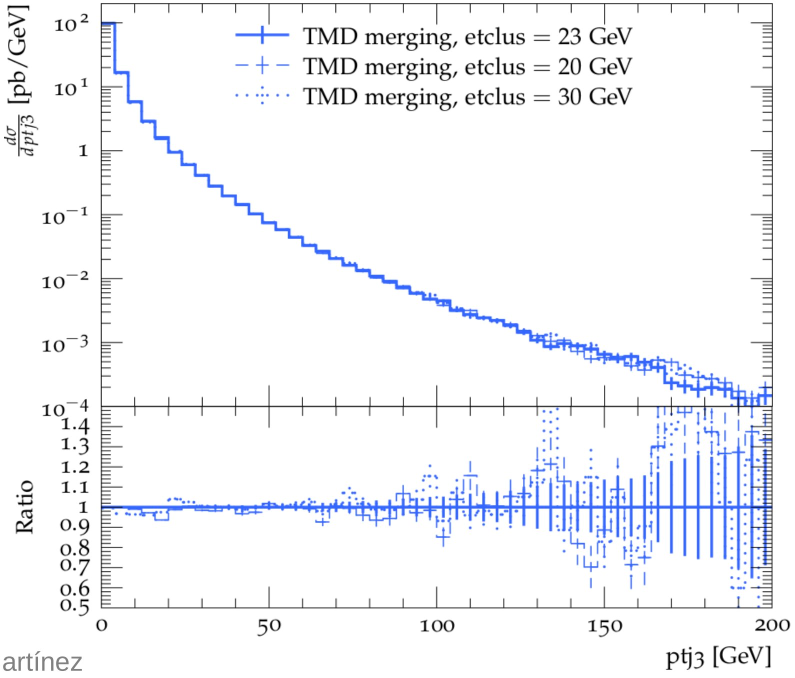
Less than 10% effect localized around the merging scale

Systematics

Pt of the jets



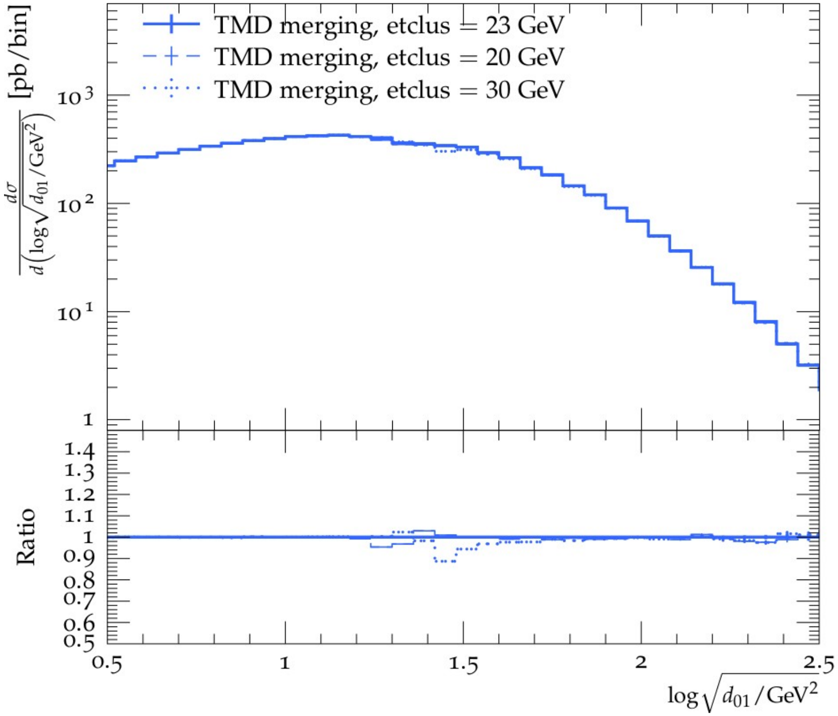


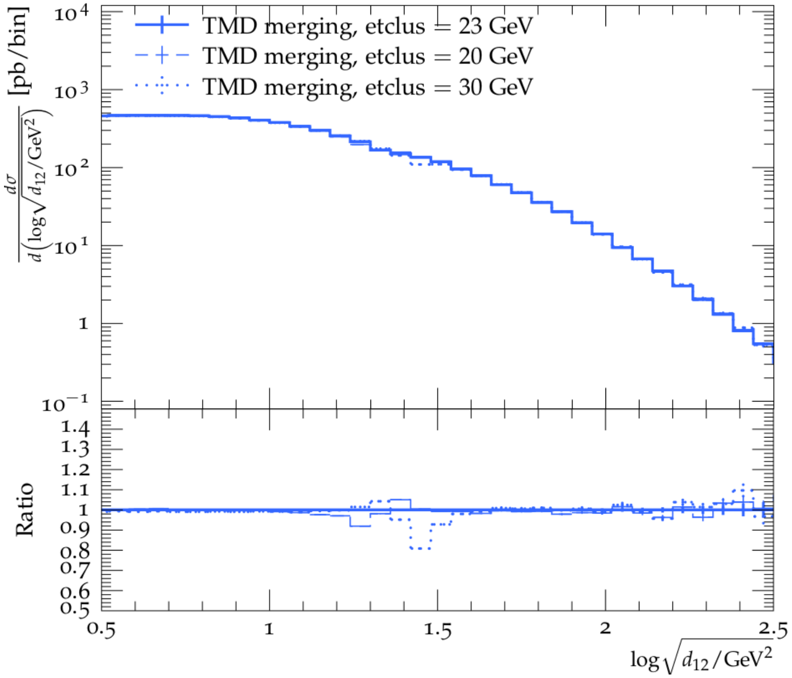


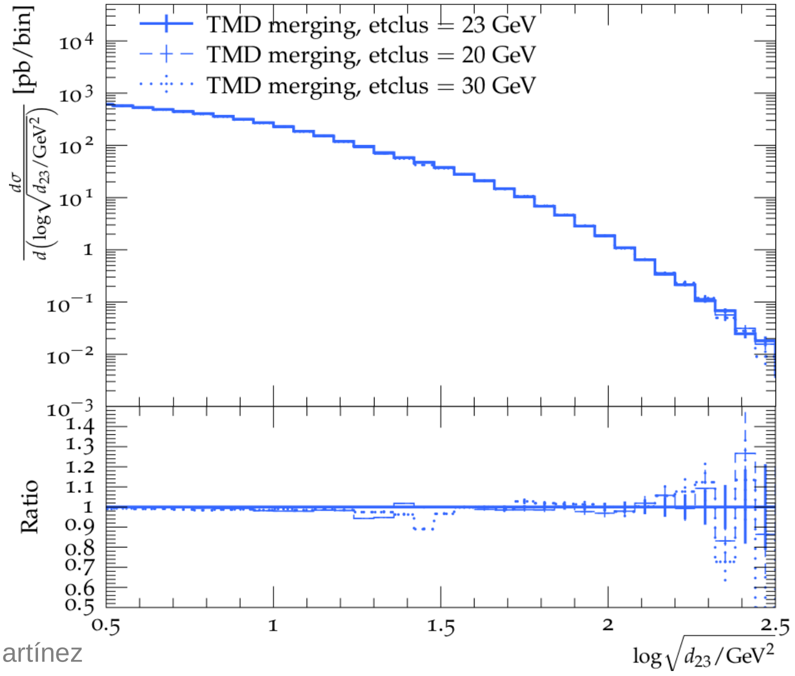
Less than 10% effect localized around the merging scale

Systematics

Differential jet rates







Up to 20% effect localized around the merging scale