Direct CP violation and the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decays from the Standard Model

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Wonderful memories of previous schools

- Turbacz - Site of the first Kraków school, 1961.

1977 “Asymptotic Freedom and Deep Inelastic Electroproduction"

1991 “Heavy Quark Physics from Lattice QCD"

2006 “Lattice Flavour Dynamics"

2014 “Flavour Physics"

“Direct CP violation and the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decays from the Standard Model,”

R.Abbott,T.Blum, P.A.Boyle, M.Bruno, N.H.Christ, D.Hoying, C.Jung, **C.Kelly**, C.Lehner, R.D.Mawhinney, D.J.Murphy, C.T.S, A. Soni, M.Tomii and T.Wang, arXiv:2004.09440 [hep-lat].

The release of this paper allows me to tell a coherent story of RBC-UKQCD's long-standing project on $K \to \pi\pi$ decays.

Outline of talk

1. Directly computing $K \to \pi\pi$ decay amplitudes
2. Evaluation of A_2
3. Evaluation of A_0
4. Conclusions and Outlook

- $K \to \pi\pi$ decays are a very important class of processes for standard model phenomenology with a long and noble history.
  - It is in these decays that both indirect and direct CP-violation was discovered.
- Bose Symmetry $\Rightarrow$ the two-pion state has isospin 0 or 2.

$$ {}_{I=2}\langle \pi\pi | H_W | K^0 \rangle = A_2\, e^{i\delta_2}\,, \qquad {}_{I=0}\langle \pi\pi | H_W | K^0 \rangle = A_0\, e^{i\delta_0}\,. $$

- Among the very interesting issues are the origin of the $\Delta I = 1/2$ rule ($\mathrm{Re}\,A_0/\mathrm{Re}\,A_2 \simeq 22.5$) and an understanding of the experimental value of ε'/ε, the parameter which was the first experimental evidence of direct CP-violation.

- CP-violating experimental amplitudes:

$$\eta_{+-} = \frac{\langle \pi^+\pi^-|H_W|K_L\rangle}{\langle \pi^+\pi^-|H_W|K_S\rangle} = \epsilon + \epsilon'$$

$$\eta_{00} = \frac{\langle \pi^0\pi^0|H_W|K_L\rangle}{\langle \pi^0\pi^0|H_W|K_S\rangle} = \epsilon - 2\epsilon'$$

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \frac{1}{6}\left(1 - \frac{|\eta_{00}|^2}{|\eta_{+-}|^2}\right)$$

- Theoretically (without isospin breaking corrections),

$$\epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right)$$

where $\omega = \mathrm{Re}\,A_2/\mathrm{Re}A_0 \simeq 1/22$.

- Indirect CP-violation: $|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$
- Direct CP-violation: Re $(\epsilon'/\epsilon) = (16.6 \pm 2.3) \times 10^{-4}$

- The effective $\Delta S = 1$ Hamiltonian can be written in the standard form:

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \left\{ z_i(\mu) + \tau\, y_i(\mu) \right\} Q_i(\mu) \,,$$

where

- G_F and V_{ij} are the Fermi Constant and CKM matrix elements respectively;
- τ is the ratio of CKM matrix elements

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \simeq \big(1.558(65) + 0.663(33)i\big) \times 10^{-3};$$

- $Q_i(\mu)$ are four-quark operators defined at the renormalisation scale μ with Wilson Coefficients $z_i(\mu)$ and $y_i(\mu)$.
- Role of lattice computations is to evaluate the hadronic matrix elements $\langle \pi\pi | Q_i(\mu) | K \rangle$ and I briefly mention some of the corresponding theoretical issues.

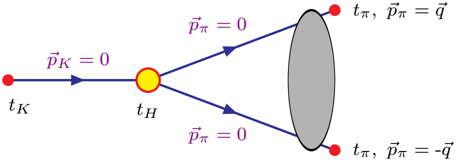
$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \{z_i(\mu) + \tau\, y_i(\mu)\}\, Q_i(\mu)$$

Schematic structure of the calculation:

$$A_I = F\,\frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \sum_{j,k=1}^{7} \{z_i(\mu) + \tau\, y_i(\mu)\}\, Z_{ij}^{\mathrm{RI}\to\overline{\mathrm{MS}}}\, Z_{jk}^{\mathrm{Latt}\to\mathrm{RI}}\, \langle(\pi\pi)_I|Q_k^{\mathrm{Latt}}|K\rangle$$

$$= F\,\frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \sum_{j=1}^{7} \{z_i(\mu) + \tau\, y_i(\mu)\}\, Z_{ij}^{\mathrm{RI}\to\overline{\mathrm{MS}}}\, \langle(\pi\pi)_I|Q_j^{\mathrm{RI}}|K\rangle$$

- F is the Lellouch-Lüscher factor, necessary because the computations are performed in a finite-volume.
- RI is a “Regularisation Independent" renormalisation scheme which can be defined non-perturbatively (not $\overline{\mathrm{MS}}$).
- Lattice computations provide $\langle(\pi\pi)_I|Q_j^{\mathrm{RI}}|K\rangle$ and F.
- The Wilson coefficients z_i, y_i and the matching matrix $Z_{ij}^{\mathrm{RI}\to\overline{\mathrm{MS}}}$, necessarily calculated in perturbation theory.

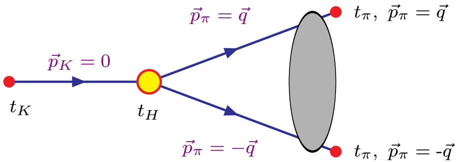


- $K \to \pi\pi$ correlation function is dominated by the lightest state, i.e. the state with two-pions at rest (or the vacuum for $I = 0$). Maiani and Testa, PL B245 (1990) 585

$$C(t_\pi) = A + B_1 e^{-2m_\pi t_\pi} + B_2 e^{-2E_\pi t_\pi} + \cdots$$

  - Solution 1: Study an excited state. Lellouch and Lüscher, hep-lat/0003023
  - Solution 2: Introduce suitable boundary conditions such that the $\pi\pi$ ground state is $|\pi(\vec{q})\pi(-\vec{q})\rangle$. RBC-UKQCD, C.h.Kim hep-lat/0311003
    N.Christ, C.Kelly, D.Zhang, arXiv:1908.08640

For B-decays, with so many intermediate states below threshold, this is the main obstacle to producing reliable calculations.



- Imagine now that we chosen the boundary conditions so that the ground state is $|\pi(\vec{q})\pi(-\vec{q})\rangle$.
  - In a finite volume each component of $\vec{q}$ is quantised, with allowed values separated by $2\pi/L$.
  - Thus in order to obtain the physical value of $|\vec{q}|$ the volume must be chosen appropriately.
  - Moreover, the s-wave, $I = 0$ and $I = 2$ channels are attractive and repulsive respectively and so the two cases must be studied on lattices of different volumes.

2. Evaluation of A_2

- The amplitude A_2 is considerably simpler to evaluate that A_0.
- Our first results for A_2 at physical kinematics were obtained at a single, rather coarse, value of the lattice spacing ($a \simeq 0.14$ fm). Estimated discretization errors at 15%. arXiv:1111.1699, arXiv:1206.5142
- Our latest results were obtained on two new ensembles, 48^3 with $a \simeq 0.11$ fm and 64^3 with $a \simeq 0.084$ fm so that we can make a continuum extrapolation:

$$\begin{aligned} \mathrm{Re}(A_2) &= 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{syst}} \times 10^{-8}\ \mathrm{GeV}. \\ \mathrm{Im}(A_2) &= -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{syst}} \times 10^{-13}\ \mathrm{GeV}\,. \end{aligned}$$

arXiv:1502.00263

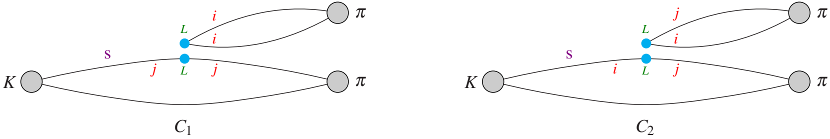
- The experimentally measured value is $\mathrm{Re}(A_2) = 1.479(4) \times 10^{-8}$ GeV.
- Although the precision can still be significantly improved (partly by perturbative calculations), the calculation of A_2 at physical kinematics can now be considered as standard.
- We are not currently working towards improving this result.

RBC-UKQCD Collaboration, arXiv:1212.1474

- Re A_2 is dominated by a simple operator:

$$O^{3/2}_{(27,1)} = (\bar{s}^i d^i)_L \left\{ (\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L \right\} + (\bar{s}^i u^i)_L \, (\bar{u}^j d^j)_L$$

and two diagrams:



- Re A_2 is proportional to $C_1 + C_2$.
- Colour counting might suggest that $C_2 \simeq \frac{1}{3} C_1$.
- We find instead that $C_2 \approx -0.7\, C_1$ so that A_2 is significantly suppressed!
- The strong suppression of Re A_2 is a major factor in the $\Delta I = 1/2$ rule.
- The contribution to Re A_0 from Q_2 is proportional to $2C_1 - C_2$ and that from Q_1 is proportional to $C_1 - 2C_2$ with the same overall sign.

- In 2015 RBC-UKQCD published our first result for ϵ'/ϵ computed at physical quark masses and kinematics, albeit still with large relative errors:

  Z.Bai et al. (RBC-UKQCD), arXiv:1505.07863

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{RBC-UKQCD}} = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$$

  to be compared with

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{Exp}} = (16.6 \pm 2.3) \times 10^{-4}\,.$$

  - Is this 2.1σ deviation real? $\Rightarrow$ must reduce the uncertainties.
  - The matrix elements themselves are calculated with a smaller relative error.
- This is by far the most complicated project that I have ever been involved with.
- Puzzle: For the $I=0$ s-wave $\pi\pi$ phase shift we obtained $\delta_0 = (23.8 \pm 4.9 \pm 2.2)^\circ$, to be compared with the dispersive results of about 34°. G.Colangelo et al.

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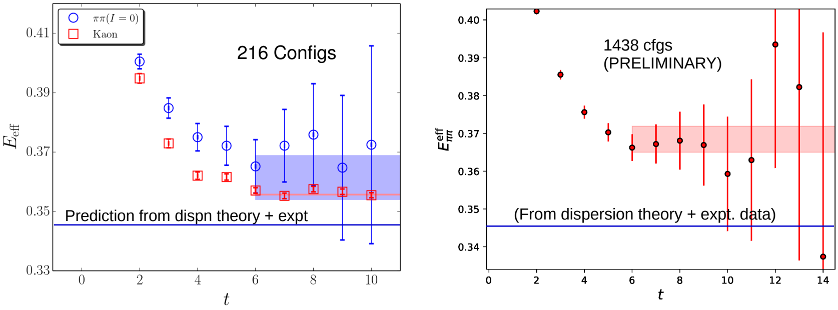
2015

- $32^3 \times 64$ ensemble (Möbius DWF and Iwasaki + DSDR gauge action)
- $a^{-1} = 1.3784(68)$GeV, $L = 4.53$ fm.
- G-parity boundary conditions in 3-directions
- 216 configurations
- Almost physical kinematics: ($m_\pi = 143.1(2.0)$ MeV, $m_K = 490.6(2.2)$ MeV, $E_{\pi\pi} = 498(11)$ MeV).

Extension and Improvement in 2020

- Increase the statistics: 216 $\rightarrow$ 1438 configurations.
  - Reduce the statistical error;
  - Improved statistics allows for an in-depth study of the systematics.
- Use an expanded set of operators to create the $\pi\pi$ state. (741 configurations)
- Improve the non-perturbative renormalisation, including step-scaling to match at a higher energy.
- Significantly improve the analysis techniques.

C.Kelly and T.Wang, arXiv:1911.04582



- Increasing the statistics from 216 to 1438 configurations, the $\pi\pi$ correlation function is still well described by a single $\pi\pi$ state.
  - It does not solve the δ_0 puzzle however:

$$\delta_0 = (23.8 \pm 4.9 \pm 2.2)^\circ \quad \to \quad \delta_0 = (19.1 \pm 2.5 \pm 1.2)^\circ \qquad (\chi^2/\text{dof} = 1.6)$$

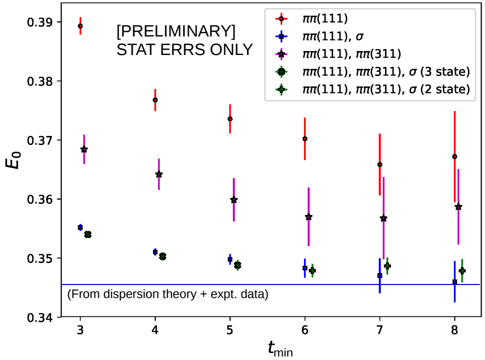
Adding more $\pi\pi$ interpolating operators

- The δ_0-puzzle has been resolved by adding more interpolating operators for the $\pi\pi$ states.
  Originally we only had a single $\pi\pi$ operator with each pion being given a momentum $\pm(1,1,1)\pi/L$ (with total momentum $\vec{0}$).
- In particular the inclusion of a σ-like two-quark operator $(\bar{u}u+\bar{d}d)$ has exposed a second state, e.g. for $t_f - t_i = 5$

$$\det\begin{pmatrix}\langle\pi\pi(t_f)\pi\pi(t_i)\rangle & \langle\pi\pi(t_f)\sigma(t_i)\rangle\\ \langle\sigma(t_f)\pi\pi(t_i)\rangle & \langle\sigma(t_f)\sigma(t_i)\rangle\end{pmatrix} = 0.439(50)\neq 0$$

- We have also included a third operator giving each pion a larger momentum $\pm(3,1,1)\pi/L$.
- At present we have only analysed 741 configurations with the additional operators. Remainder will be done in the future.

Adding more $\pi\pi$ interpolating operators (cont.)



- $\delta_0 = (32.3 \pm 1.0 \pm 1.8)^\circ$ from a fit in the range $t = 5$ - 15 (statistical error only).
- We have now analysed the $K \to \pi\pi$ matrix elements with multiple operators.

arXiv:1505.07863

Description	2015 Error	2020 Error
Operator normalisation	15%	5%[1]
Wilson coefficients	12%	unchanged
Finite lattice spacing	12%	unchanged
Lellouch - Lüscher factor	11%	1.5%[2]
Residual FV corrections	7%	unchanged
Parametric errors	5%	6%[3]
Excited state contamination	5%	negligible[4]
Unphysical kinematics	3%	5%
Total	**27%**	**21%**

- [1] As a result of step scaling from $\mu = 1.53$ GeV $\to$ 4.00 GeV.
- [2] Better control of $\pi\pi$ system due to additional operators.
- [3] Largest uncertainty is due to $\tau \sim 5\%$.
- [4] Significantly underestimated in 2015.

- For leptonic decays, the precision of lattice calculations is such that $O(1\%)$ isospin breaking corrections (including electromagnetism) are becoming important.
- Extension to $K \to \pi\pi$ decays is much more complicated.
- At present we are not concerned with including $O(1\%)$ corrections.
  - However, because of the $\Delta I = \frac{1}{2}$ rule, the corrections are expected to be amplified.
- Recently a detailed updated study of IB corrections was presented in the framework of ChPT and the large N_C approximation. V.Cirigliano et al., arXiv:1911.01359

$$\epsilon' = \frac{i\omega_+ e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left[\frac{\mathrm{Im}\,A_2^{\mathrm{ewp}}}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\left(1-\hat{\Omega}_{\mathrm{eff}}\right)\right], \qquad \hat{\Omega}_{\mathrm{eff}} = \left(17.0^{+9.1}_{-9.0}\right)\times 10^{-2},$$

where $\omega_+ = \mathrm{Re}\,A_2^+/\mathrm{Re}\,A_0$ and A_2^+ is A_2 obtained from $K^+ \to \pi^+\pi^0$ at NLO.

- A careful discussion of arXiv:1911.01359, and the determination of the LECs at NLO in particular, is beyond the scope of our work and we include the central value as a further 23% systematic error on our result.

- Re $A_0 = 2.99(0.32)(0.59) \times 10^{-7}$ GeV (Experiment: $3.3201(18) \times 10^{-7}$ GeV)

  Im $A_0 = -6.98(0.62)(1.44) \times 10^{-11}$ GeV .

- Combining our new result for Re A_0 with our 2015 one for ReA_2 we find:

$$\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2} = 19.9 \pm 2.3 \pm 4.4\,,$$

  in good agreement with the experimental result of 22.45(6).

- Combining our new result for Im A_0 with our 2015 result for Im A_2 and using the experimental results for the real parts, we find

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = 0.00217(26)(62)(50),$$

  where the third uncertainty is due to isospin breaking effects. This result is consistent with the experimental value of $0.00166(23)$.

  - Note that if, instead of treating the isospin correction from arXiv:1911.01359 as a component of the systematic uncertainty, we were to implement on our result, we would obtain a central value Re$(\epsilon'/\epsilon) = 0.00167$, coincidentally identical to the experimental result.

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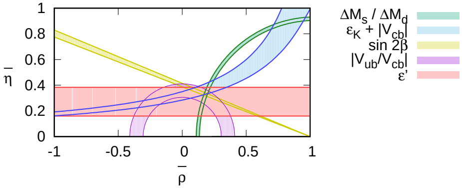

- We have completed the update on our 2015 lattice determination of A_0 and ϵ'/ϵ with:
  - a 3.2 times increase in statistics;
  - the use of multi-operator techniques in order to essentially remove the systematic error due to excited state contamination;
  - the use of step-scaling to reduce significantly the systematic error in the renormalisation.
- We reproduce the experimental value of ReA_0/ReA_2 demonstrating that, within our uncertainties, QCD is sufficient to solve this decades-old puzzle.
- Our result for Re ϵ'/ϵ is consistent with the experimental value, with an error which is about 3.5 times larger. This quantity remains a promising avenue in which to search for new physics but more precision is required.

- The collaboration intends to perform measurements on two larger lattices with different lattice spacings to the perform continuum limit. This will require the next generation of supercomputers.
- A project is currently underway to perform the $4 \to 3$ flavour matching in the Wilson coefficients non-perturbatively. M.Tomii, arXiv:1901.04107
- We are also working on laying the groundwork for the lattice calculation of isospin breaking and electromagnetic effects. N.Christ and Xu Feng, arXiv:1711.09339
- The collaboration is actively investigating the potential for multi-operator fits to circumvent need for G-parity BCs, allowing for more reuse of ensembles and eigenvectors from other RBC&UKQCD projects. D.Hoying, PoS LATTICE2018 (2019) 064

Hala Gąsienicowa, 2006

The RBC & UKQCD collaborations

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Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

1. A_0 and A_2 amplitudes with unphysical quark masses and with the pions at rest.

"K to $\pi\pi$ decay amplitudes from lattice QCD,"
T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Lehner, Q.Liu, R.D. Mawhinney, C.T.S, A.Soni, C.Sturm, H.Yin and R. Zhou, Phys. Rev. D **84** (2011) 114503 [arXiv:1106.2714 [hep-lat]].

"Kaon to two pions decay from lattice QCD, $\Delta I = 1/2$ rule and CP violation"
Q.Liu, Ph.D. thesis, Columbia University (2010)

2. A_2 at physical kinematics and a single coarse lattice spacing.

"The $K\to(\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD,"
T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, R.D.Mawhinney, C.T.S., A.Soni, and C.Sturm
Phys. Rev. Lett. **108** (2012) 141601 [arXiv:1111.1699 [hep-lat]],

"Lattice determination of the $K\to(\pi\pi)_{I=2}$ Decay Amplitude A_2"
Phys. Rev. D **86** (2012) 074513 [arXiv:1206.5142 [hep-lat]]

"Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD,"
P.A. Boyle, N.H. Christ, N. Garron, E.J. Goode, T. Janowski, C. Lehner, Q. Liu, A.T. Lytle, C.T. Sachrajda, A. Soni, and D.Zhang, Phys. Rev. Lett. **110** (2013) 15, 152001 [arXiv:1212.1474 [hep-lat]].

3. A_2 at physical kinematics on two finer lattices $\Rightarrow$ continuum limit taken.
   "$K \to \pi\pi$ $\Delta I = 3/2$ decay amplitude in the continuum limit,"
   T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Janowski, C.Jung, C.Kelly, C.Lehner, A.Lytle, R.D.Mawhinney, C.T.S., A.Soni, H.Yin, and D.Zhang
   Phys. Rev. D **91** (2015) 7, 074502 [arXiv:1502.00263 [hep-lat]].

4. A_0 at physical kinematics and a single coarse lattice spacing.
   "Standard-model prediction for direct CP violation in $K \to \pi\pi$ decay,"
   Z.Bai, T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, R.D.Mawhinney, C.T.S, A. Soni, and D. Zhang,
   Phys. Rev. Lett. **115** (2015) 21, 212001 [arXiv:1505.07863 [hep-lat]].

5. Improved and Updated version of Item 4.
   "Direct CP violation and the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decay from the Standard Model,"
   R.Abbott,T.Blum, P.A.Boyle, M.Bruno, N.H.Christ, D.Hoying, C.Jung, C.Kelly, C.Lehner, R.D.Mawhinney, D.J.Murphy, C.T.S, A. Soni, M.Tomii and T.Wang, arXiv:2004.09440 [hep-lat].

Backup Slides

- For A_2, there is no vacuum subtraction and we can use the Wigner-Eckart theorem to write

$$\underbrace{\langle(\pi\pi)^{I=2}_{I_3=1}\,|}_{\frac{1}{\sqrt{2}}(\langle\pi^+\pi^0|+\langle\pi^0\pi^+|)}\quad Q^{\Delta I=3/2}_{\Delta I_3=1/2,i}\,|\,K^+\rangle=\frac{3}{2}\,\underbrace{\langle(\pi\pi)^{I=2}_{I_3=2}\,|}_{\langle\pi^+\pi^+|}\,Q^{\Delta I=3/2}_{\Delta I_3=3/2,i}\,|\,K^+\rangle\,,$$

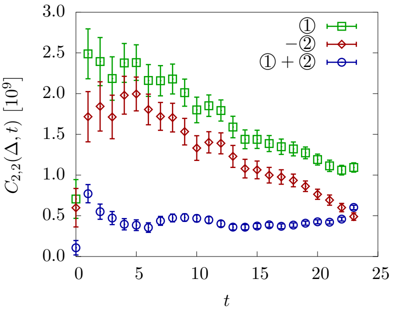
  and impose anti-periodic conditions on the d-quark in one or more directions.

- If we impose the anti-periodic boundary conditions in all 3 directions then the ground state is

$$\Big|\pi\left(\frac{\pi}{L},\frac{\pi}{L},\frac{\pi}{L}\right)\,\pi\left(\frac{-\pi}{L},\frac{-\pi}{L},\frac{-\pi}{L}\right)\Big\rangle\,.$$

- With an appropriate choice of L and the number of directions, we can arrange that $E_{\pi\pi}=m_K$.
- Isospin breaking by the boundary conditions is harmless here.

CTS & G.Villadoro, hep-lat/0411033



Physical Kinematics

- Notation ⓘ $\equiv C_i,\ i = 1, 2.$