Quarkyonic Matter and HARD Equations of State

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Mass and radii of observed neutron stars and data from neutron star collisions give
an excellent determination of the equation of state of strongly interacting matter

Such equations of state must be hard
The sound velocity squared is greater than or of the order of 1/3 at only a few times nuclear matter density
This is NOT what one expects from a phase transition
Yet relativistic degrees of freedom appear to be important

High Temperature and Low Baryon Density Thermal Matter

Useful to think in the large number of of colors limit

$$N_c \to \infty$$

In this limit, quark loops are suppressed by one over Nc

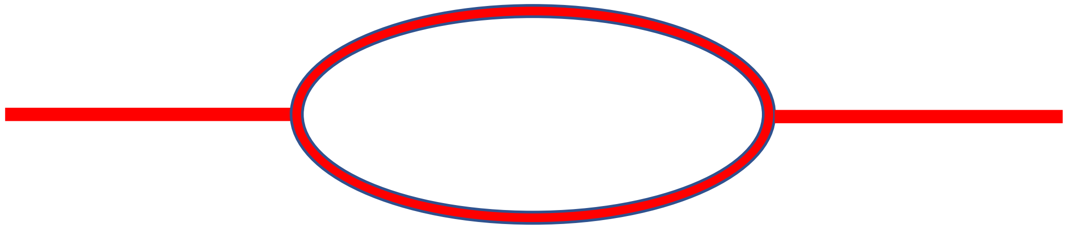
Quark pair production cannot short out the long range linear potential, and so confinement can be expressed as a long range linear potential between quarks

Baryons are heavy and made of N_c quark so their mass is of order N_c

Mesons are weakly interacting, with interaction strength of order 1/N_c

High Temperature World at Zero Baryon Density

The confining force is cutoff at screening length associated with polarization of thermal gluons



$$1/r^2_{Debye} \sim g^2_{'tHooft} T^2$$

At some temperature the Debye length is less than the confinement scale, the potential can no longer become linear

$$T_D \sim \Lambda_{QCD}$$

Can also imagine it from thinking about hadrons

At low temperatures there is a gas of very weakly interacting hadrons, since if

$$T << \Lambda_{QCD}$$

The density of mesons is of order one in powers of N_c because the mesons are color singlet

If there was a gas of gluons, the density would be of order

$$N_c^2$$

This happens because there is an exponentially growing density of mesons that have cross sections of order 1/N_c^2 whose interactions become big when the density is of order N_c^2

What about finite density:

Quarkyonic Matter:

Confinement at finite temperature disappears because the Debye screening length become shorter than the confinement scale. Gluons give

$$1/r^2_{Debye} \sim g^2_{'tHooft} T^2$$

At finite density, there is a typical Fermi momentum or chemical potential associated with quark density

$$\mu_Q$$

The Debye screening length associated with quarks is very large

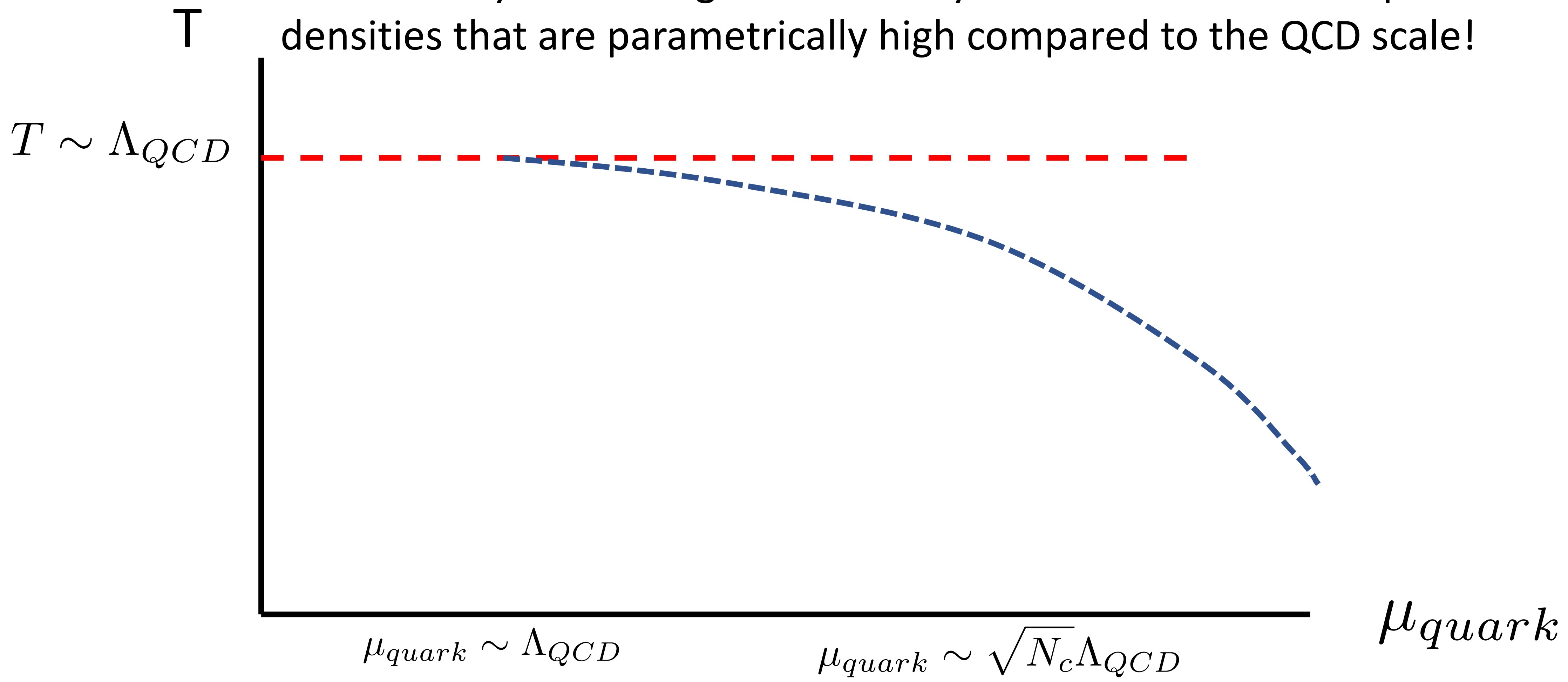
In the large N_c limit, at finite density and zero temperature limit, the deconfinement chemical potential is

$$1/r^{2\ quarks}_{Debye} \sim \mu_Q^2/N_c$$

In the large N_c limit, at finite density and zero temperature limit, the deconfinement chemical potential is

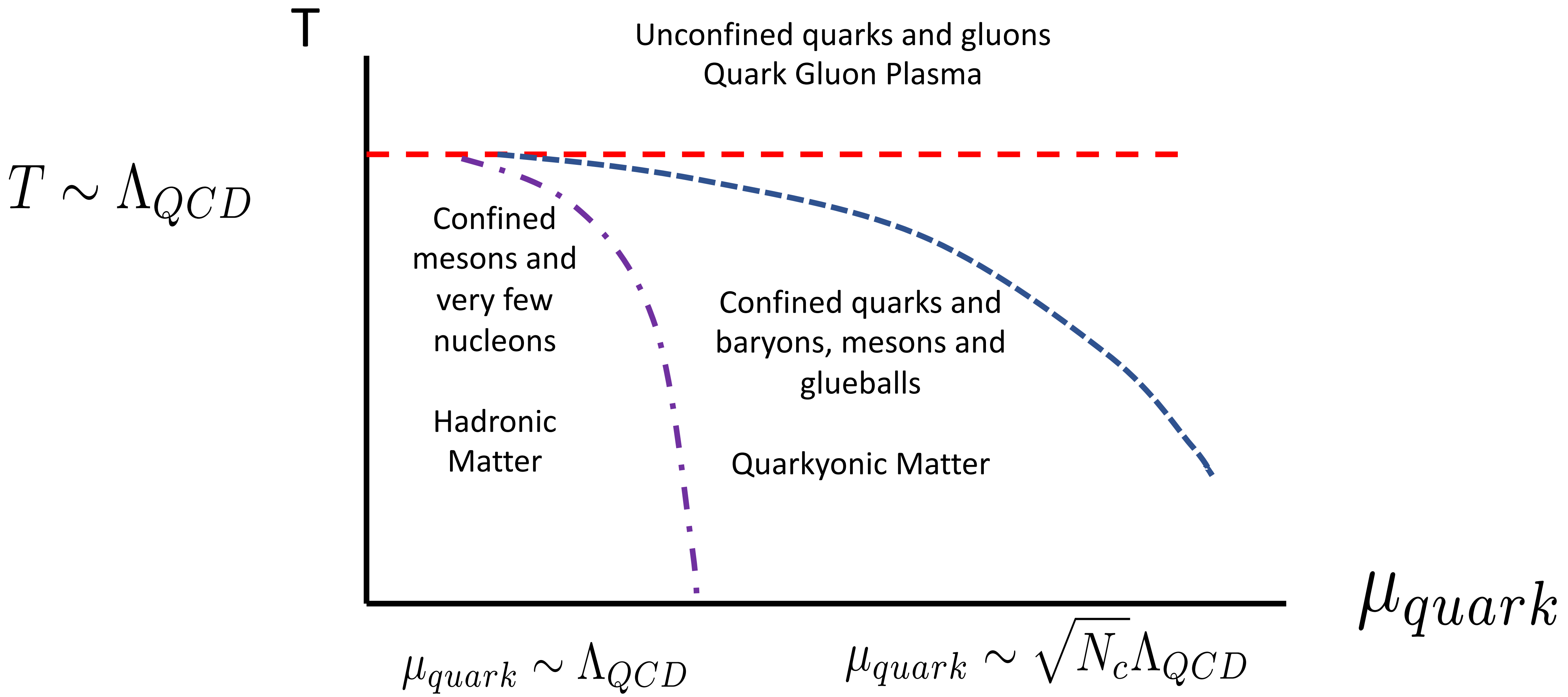
$$\mu_{quark} \sim \sqrt{N_c}\Lambda_{QCD} >> \Lambda_{QCD}$$

This is a very interesting result: the system is confined until quark densities that are parametrically high compared to the QCD scale!

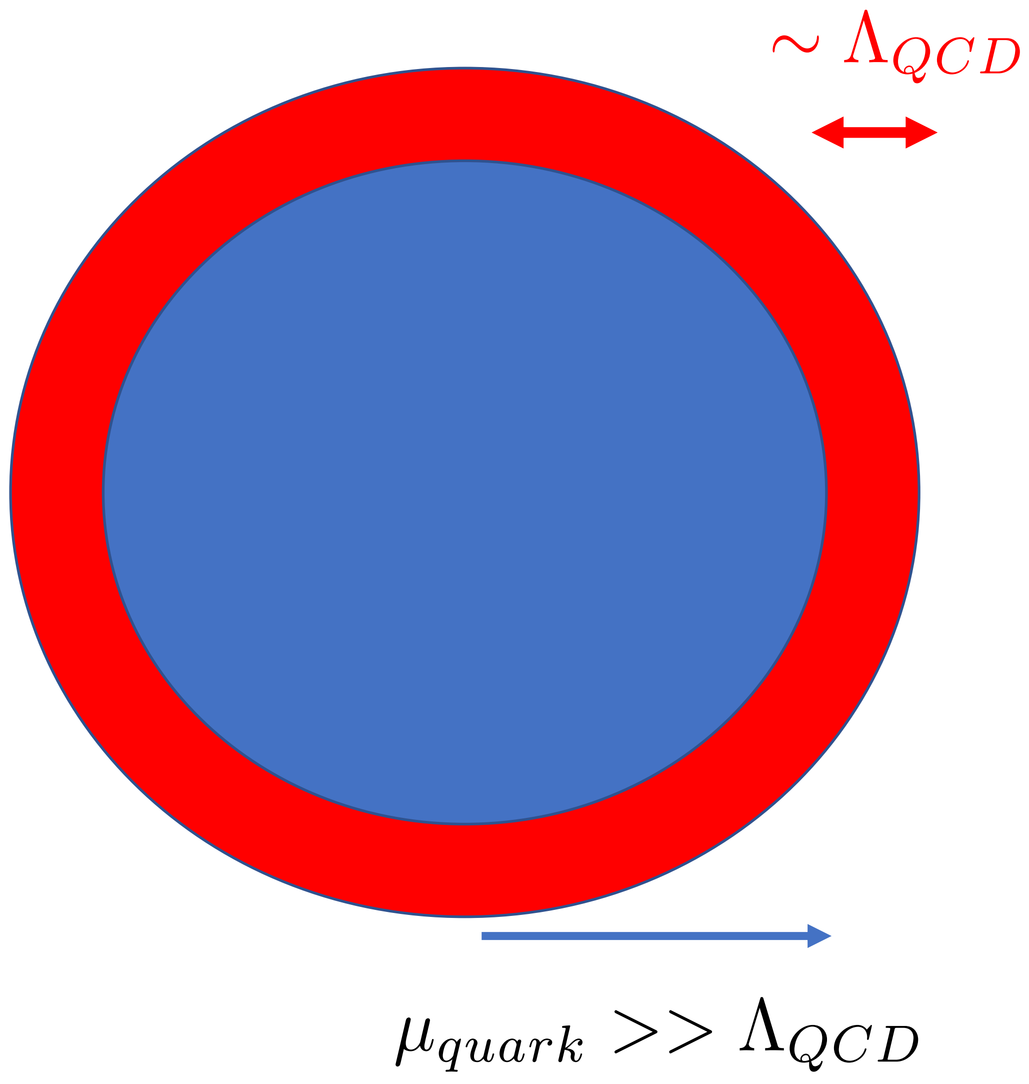


$$n_{baryon} \sim e^{(\mu_B - E)/T} \sim e^{N_c(\mu_q - E_q)/T}$$

No baryons for $\mu_{quark} < M_{nucleon}/N_c \sim \Lambda_{QCD}$



Fermi Surface is Non-perturbative



Fermi Surface
Interactions sensitive to infrared
Degrees of freedom: baryons, mesons and glueballs

Fermi Sea: Dominated by exchange interactions which are less sensitive to IR.
Degrees of freedom are quarks

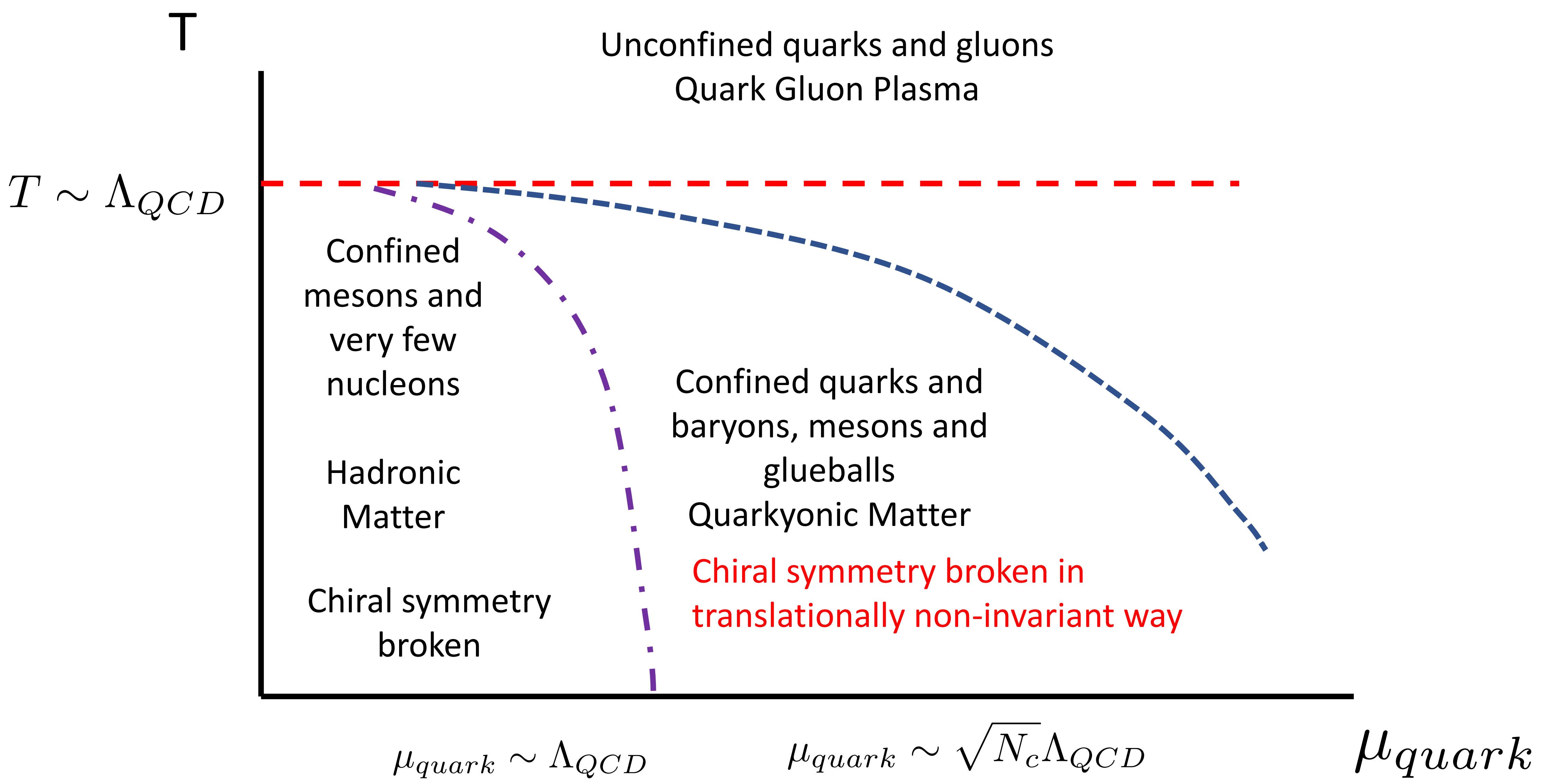
Chiral Symmetry Breaking and the Quarkyonic Phase

Scalar sigma meson condensation near the Fermi surface: particle hole

A small relative momentum of particle antiparticle requires that particle and antiparticle each have momentum of order the fermi momentum

Condensate breaks translational and rotational invariance

In fact there are many possibilities of patching together a discrete number of such condensates, as seen in model computations



T
Unconfined quarks and gluons
Quark Gluon Plasma
$T \sim \Lambda_{QCD}$
Confined mesons and very few nucleons
Hadronic Matter
Chiral symmetry broken
Confined quarks and baryons, mesons and glueballs
Quarkyonic Matter
Chiral symmetry broken in translationally non-invariant way
$\mu_{quark} \sim \Lambda_{QCD}$
$\mu_{quark} \sim \sqrt{N_c}\Lambda_{QCD}$
μ_{quark}

Simple large N_c considerations

Near nuclear matter density

$$k_F \sim \Lambda_{QCD}$$

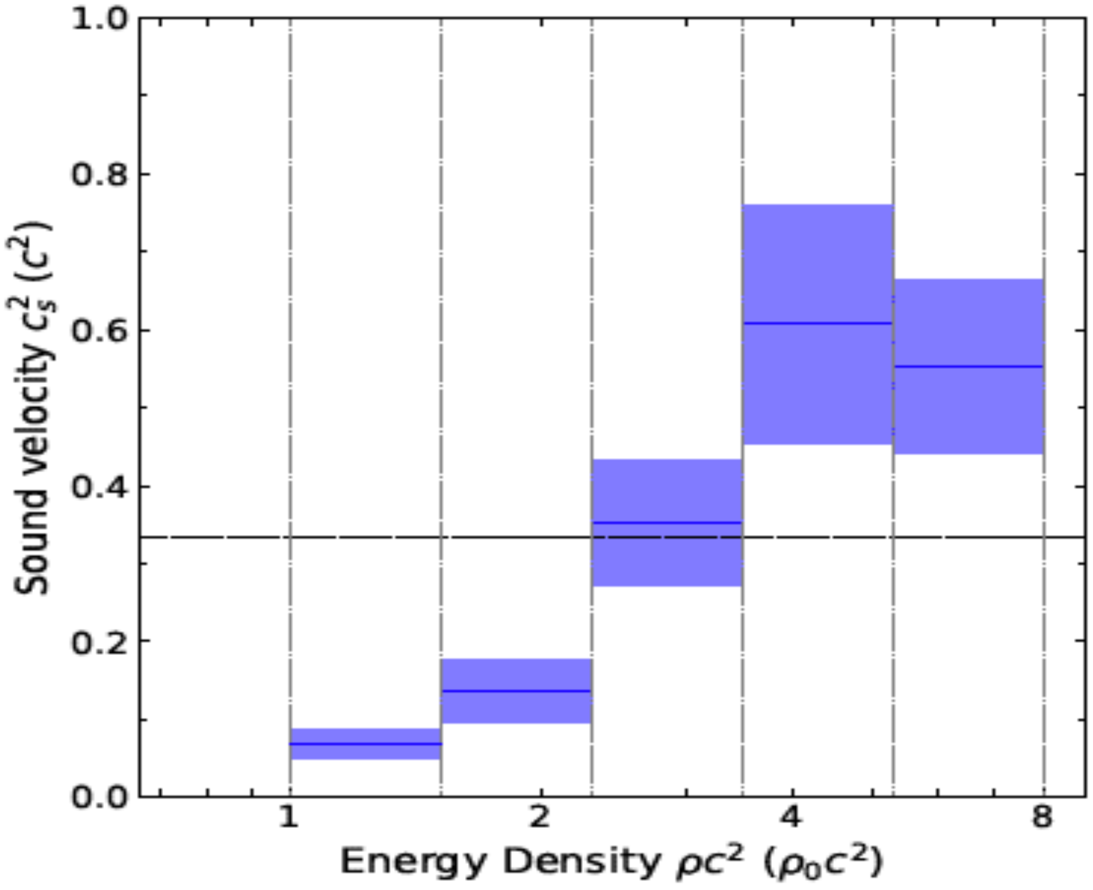
$$\epsilon/n - M_N \sim \Lambda^2_{QCD}/2M_N \sim \Lambda_{QCD}/N_c$$

On the other hand, short distance QCD interactions are of order N_c

$$\epsilon/n - M_N \sim N_c\Lambda_{QCD}$$

But the density of hard cores is also parametrically of order

$$\Lambda^3_{QCD}$$



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As a result of LIGO experiments, and more precise measurement of neutron star masses, the equation of state of nuclear matter at a few time nuclear matter density is tightly constrained

Typically sound velocity approaches and perhaps exceeds

$$v_s^2 = 1/3$$

at a few times nuclear matter density

Sound velocity of order one has important consequences

For zero temperature Fermi gas:

$$\frac{n_B}{\mu_B dn_B/d\mu_B} = v_s^2$$

where the baryon chemical potential includes the effects of nucleon mass

$$\frac{\delta\mu_B}{\mu_B} \sim v_s^2 \; \frac{\delta n_B}{n_B}$$

So if the sound velocity is of order one, an order one change in the baryon density generates a change in the baryon number chemical potential of order the nucleon mass

For nuclear matter densities

$$\mu_B - M \sim \frac{\Lambda^2_{QCD}}{2M} \sim 100 \;\; MeV$$

Large sound velocities will require very large intrinsic energy scales, and a partial occupation of available nucleon phase space because density is not changing much while Fermi energy changes a lot

How can the density stay fixed if the Fermi momentum goes up.? The nucleon must sit on a shell of varying thickness as the density increases. The added baryon number has to come in the form of new degrees of freedom: quarks

This can be understood in large N_c arguments:

$k_f \sim \Lambda_{QCD}$ requires $\mu_B - M_N \sim \Lambda_{QCD}/N_c$

Quarks should become important when

$$\mu_Q = \mu_B/N_c \sim \Lambda_{QCD}$$

The hypothesis of quarkyonic matter implies there need be no first order phase transition, The quarkyonic hypothesis requires a transition when the baryon Fermi energy is very close to the nucleon mass, so the transition may in principle occur quite close to nuclear matter density.

$$n_B^n = \frac{2}{3\pi^2} k_n^{F\ 3}$$

Density is of order one in power of N_c for baryon density computed both by both quark and nucleon degree of freedom

$$n_B^q = \frac{2}{3\pi^2} k_f^{q\ 3}$$

The problem is that if we take a constituent quark model

$$k_n^F = \sqrt{\mu_B^2 - M_N^2} = \sqrt{N_c^2 \mu_q^2 - N_c^2 M_q^2} = N_c k_q^F$$

Baryon number chemical potential must jump, while density
In nucleons stays constant and density in quarks turns on

If there is a continuous transition then the baryon density will have to remain fixed, so the chemical potential will change by of order N_c. The sound velocity is changing for a very non-relativistic system to a very relativistic one.

$$\epsilon_B = M_N \Lambda_{QCD}^3 \sim N_c \Lambda_{QCD}^4$$

$$\epsilon_Q \sim \Lambda_{QCD} n_q \sim N_c \Lambda_{QCD}^4$$

$$P \sim \frac{k_F}{M_B}\epsilon_N$$

The pressure on the other hand must jump by order N_c squared

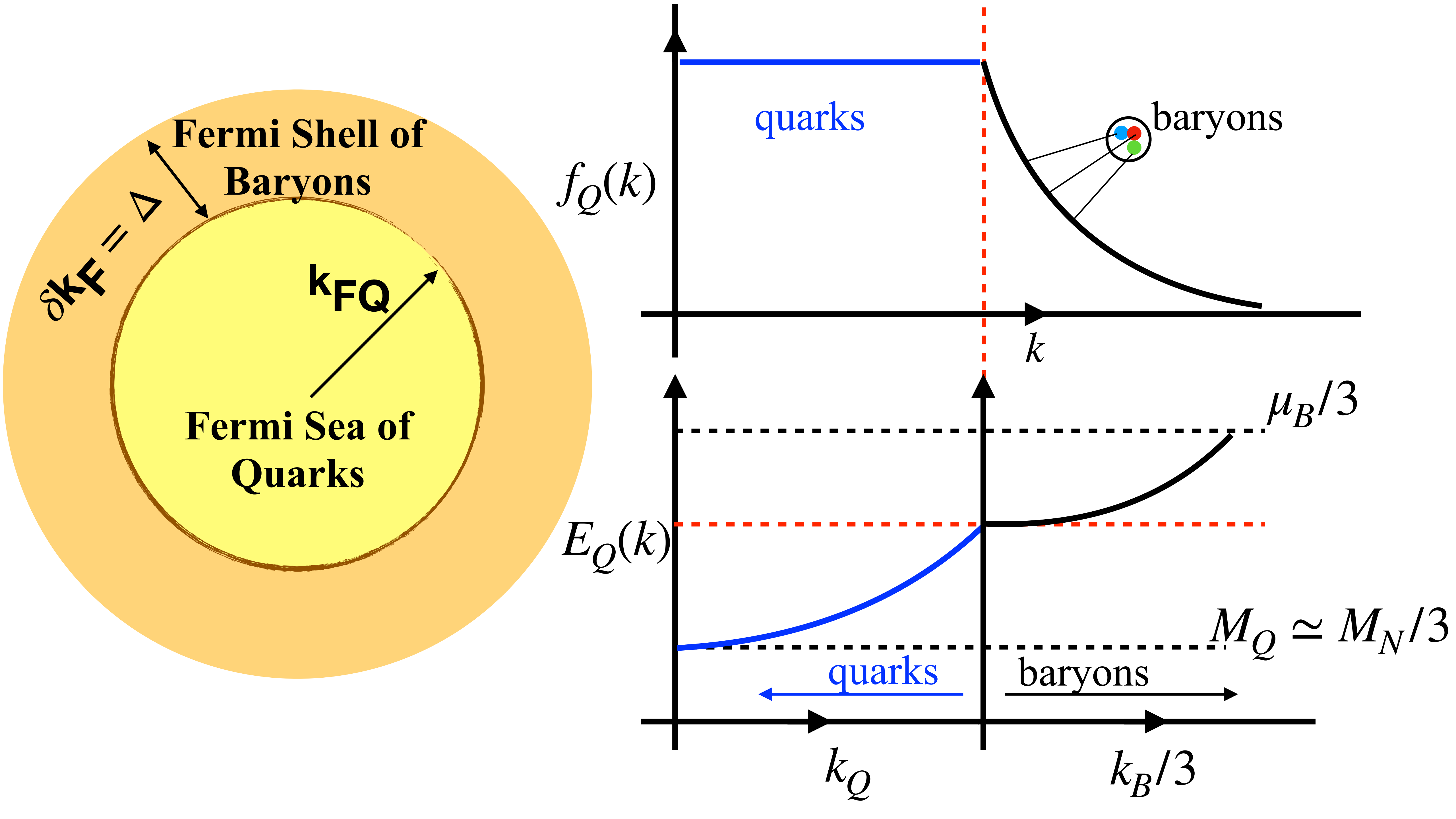
$$P \sim \epsilon_q$$

In ordinary first order phase transitions, the energy density and density jump but the pressure and chemical potential remain fixed

Here the energy density and density do not jump but the pressure and chemical potential jump.

The transition from nuclear matter to quarkyonic matter therefore involves a

First Order Un-Phase Transition


Fermi Shell of Baryons
δkF = Δ
kFQ
Fermi Sea of Quarks
quarks
baryons
fQ(k)
k
μB/3
EQ(k)
MQ ≃ MN/3
quarks
baryons
kQ
kB/3

Simplest model for a First Order Un-Phase Transition with Quarkyonic Matter

Above some Fermi momentum, assume a Fermi surface shell develops. For momenta

$$k_F^B > k > k_F^B - \Delta$$

the degrees of freedom are nucleons. Near the Fermi surface there can be non-perturbative low momentum interactions.

For momentum

$$k < k_F^B - \Delta$$

The degrees of freedom are quarks.

When the Fermi shell first appear, Delta is of the order of the QCD scale. At high densities, it must narrow to be of order 1/N_C^2 because we require that we smoothly match to a QCD limit for a a finite shell thickness

$$k_F^{B\ 3} - (k_F^B - \Delta)^3 \sim k_F^{B\ 2}\Delta \sim N_C^2 \Lambda^2 \Delta \sim \Lambda^3$$

A reasonable parameterization is

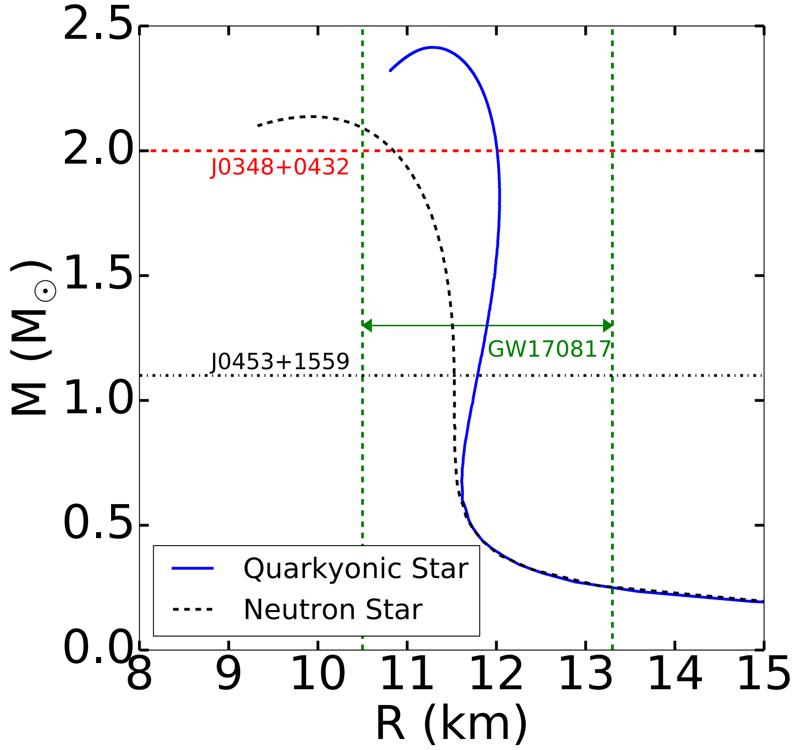
$$\Delta = \frac{\Lambda^3}{k_B^2}$$

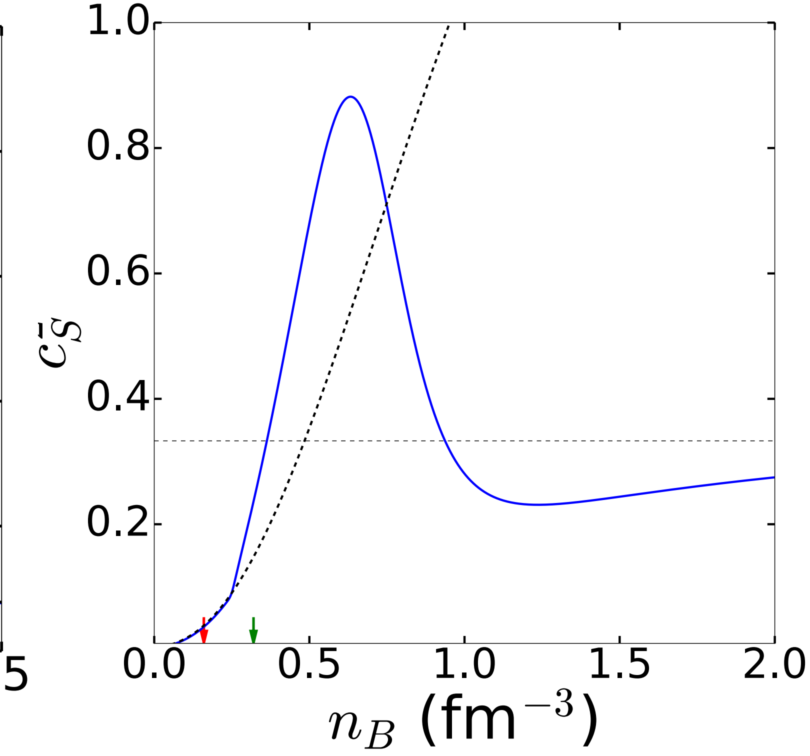
But if the density really becomes a constant then the sound velocity diverges. to have a finite limit with v^2 < 1 need to include a correction

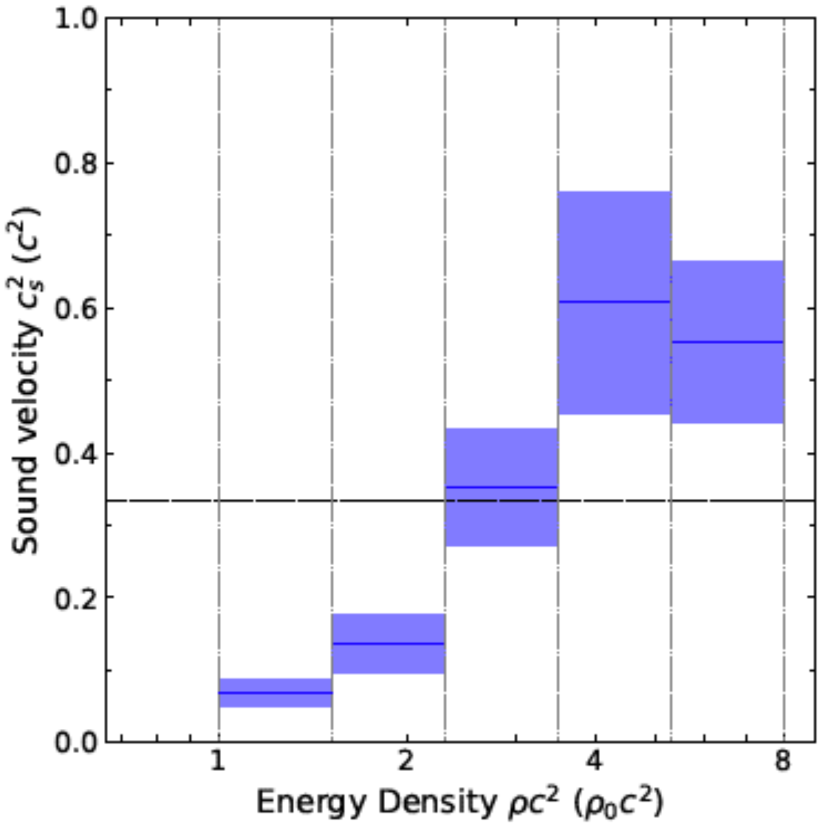
$$\Delta = \frac{\Lambda^3}{k_B^2} + \kappa \frac{\Lambda}{N_c^2}$$

This correction is important only at very high density where the quark contribution is of the order of the baryons

In explicit computation, we used a phenomenological equation of state for nuclear matter that is







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