Exploring the QCD phase diagram with fluctuations

- Why fluctuations
- Making the connection between experiment and theory (Lattice QCD)

60. Jubilee Cracow School of Theoretical Physics

Collaborators:
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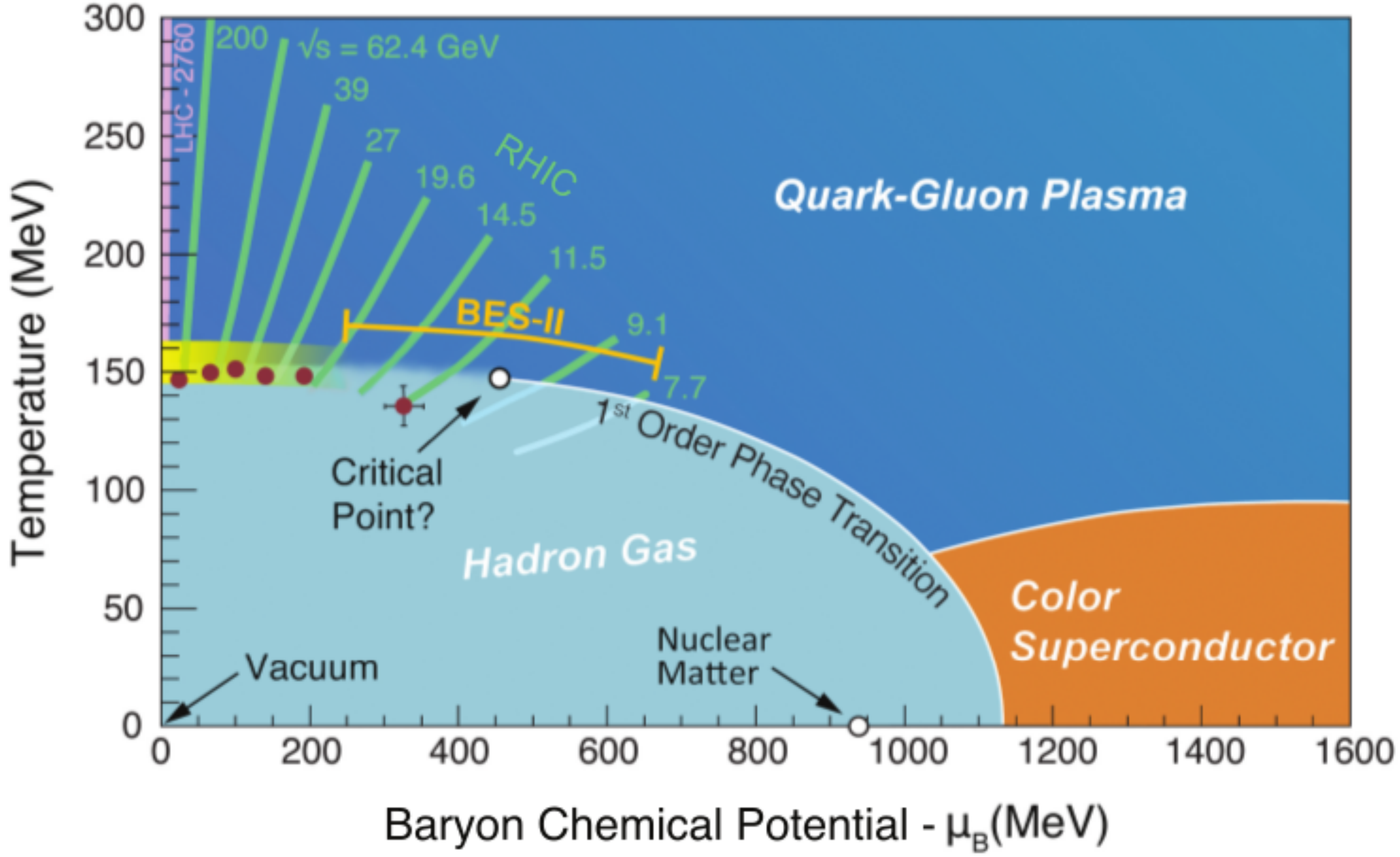
BEST COLLABORATION

The phase diagram



Increase chemical potential by lowering the beam energy

In reality, we add baryons (nucleons) from target and projectile to mid-rapidity

What we know about the Phase Diagram

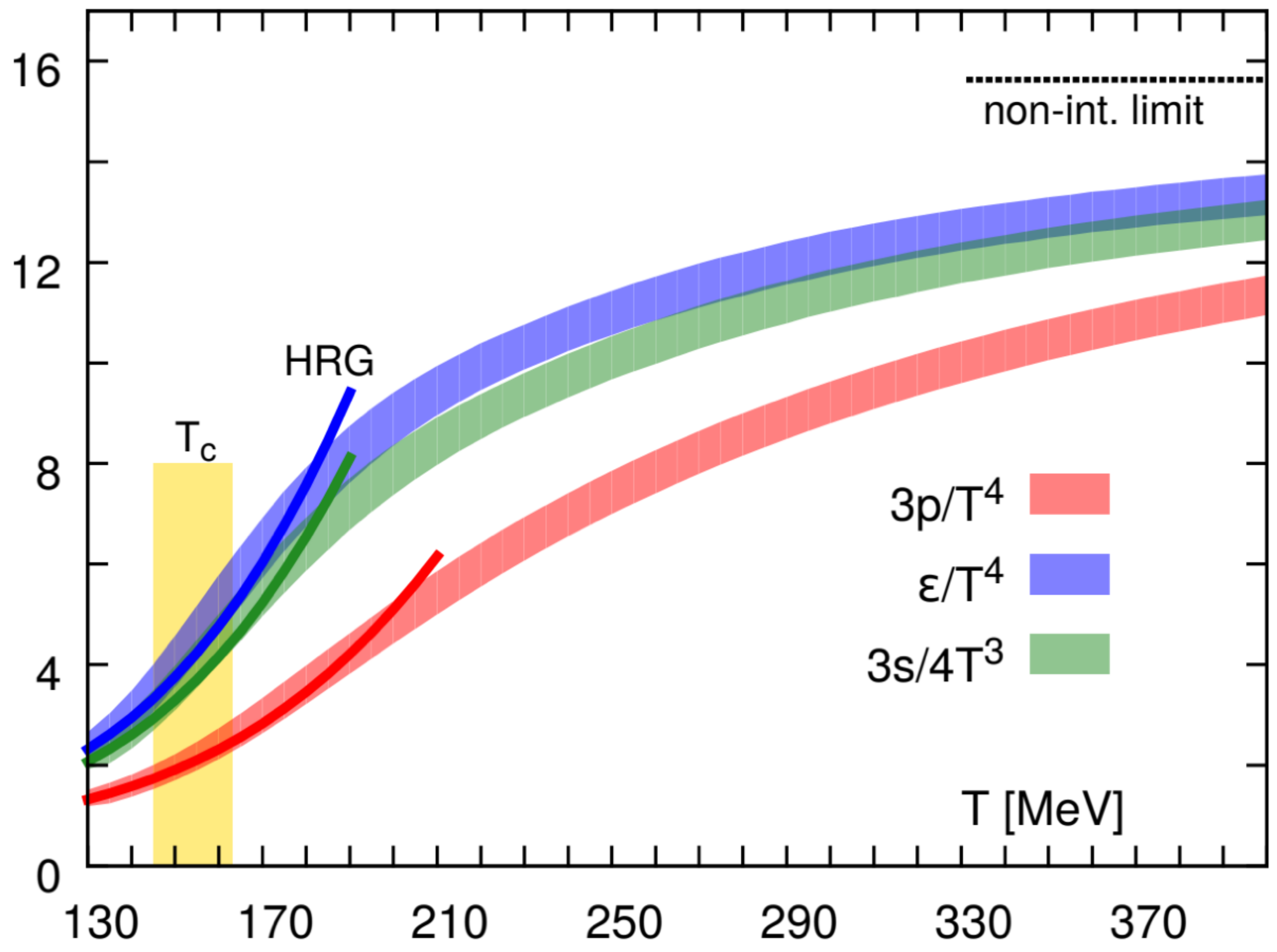
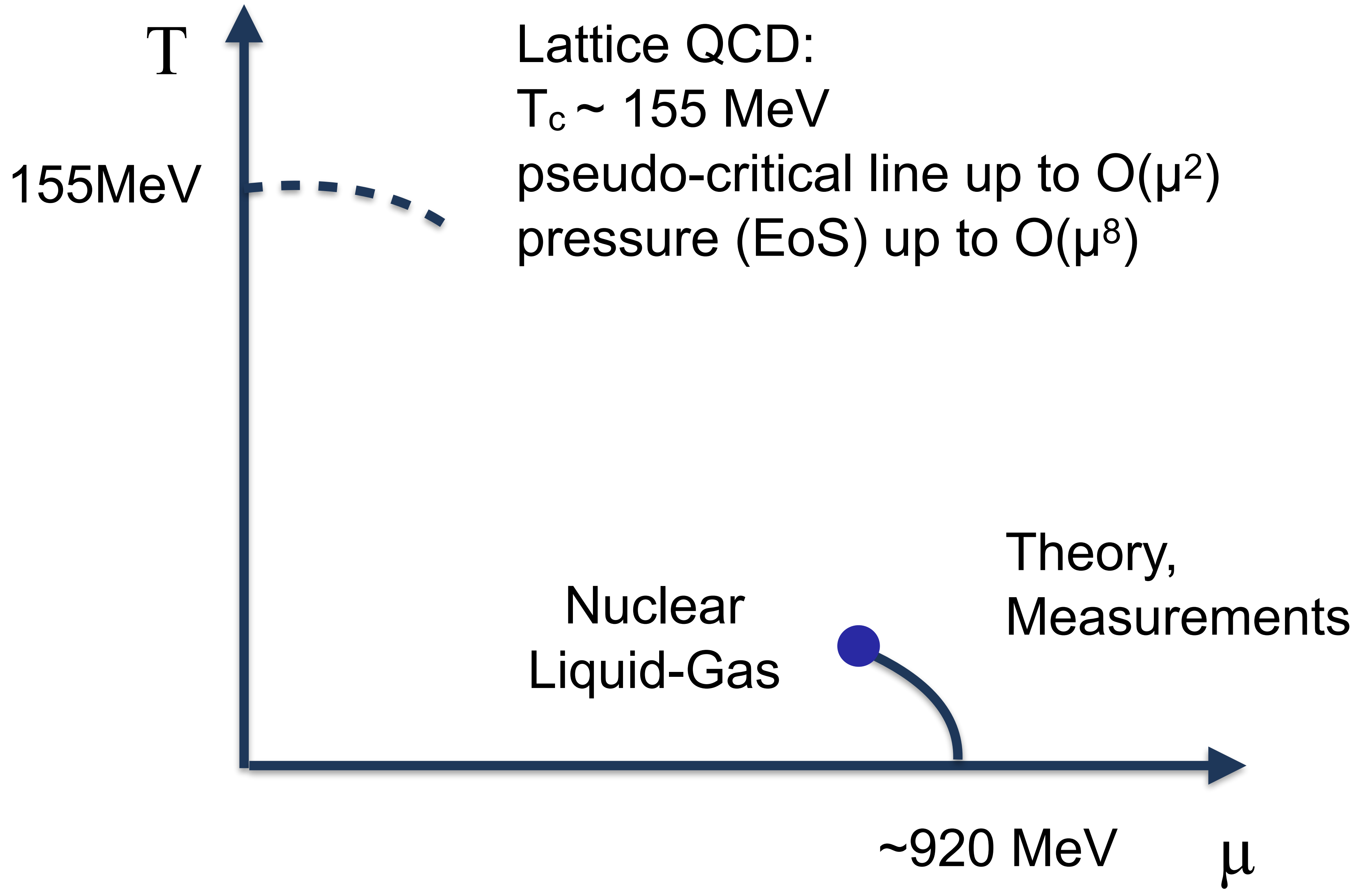
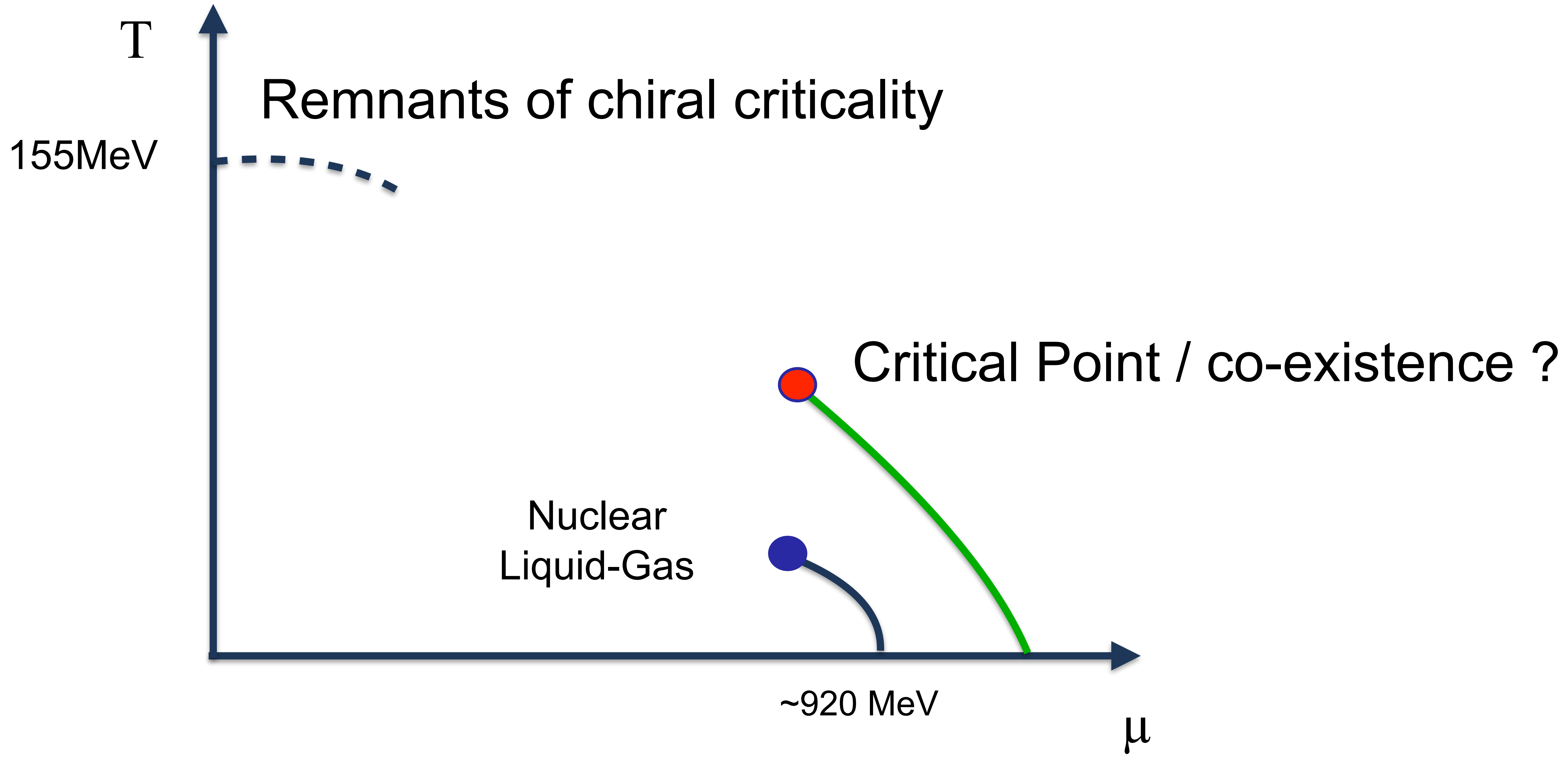


Figure from HotQCD coll., PRD ‘14



What we are looking for

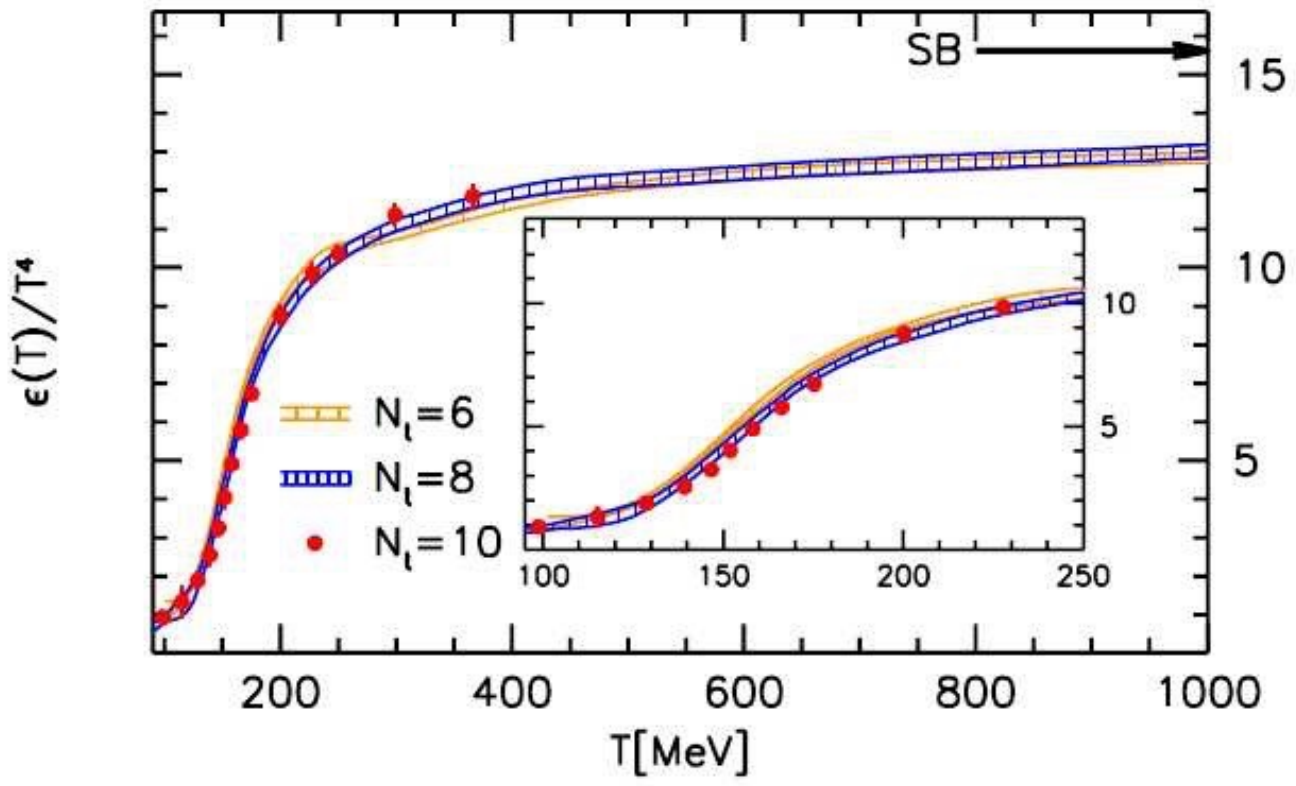


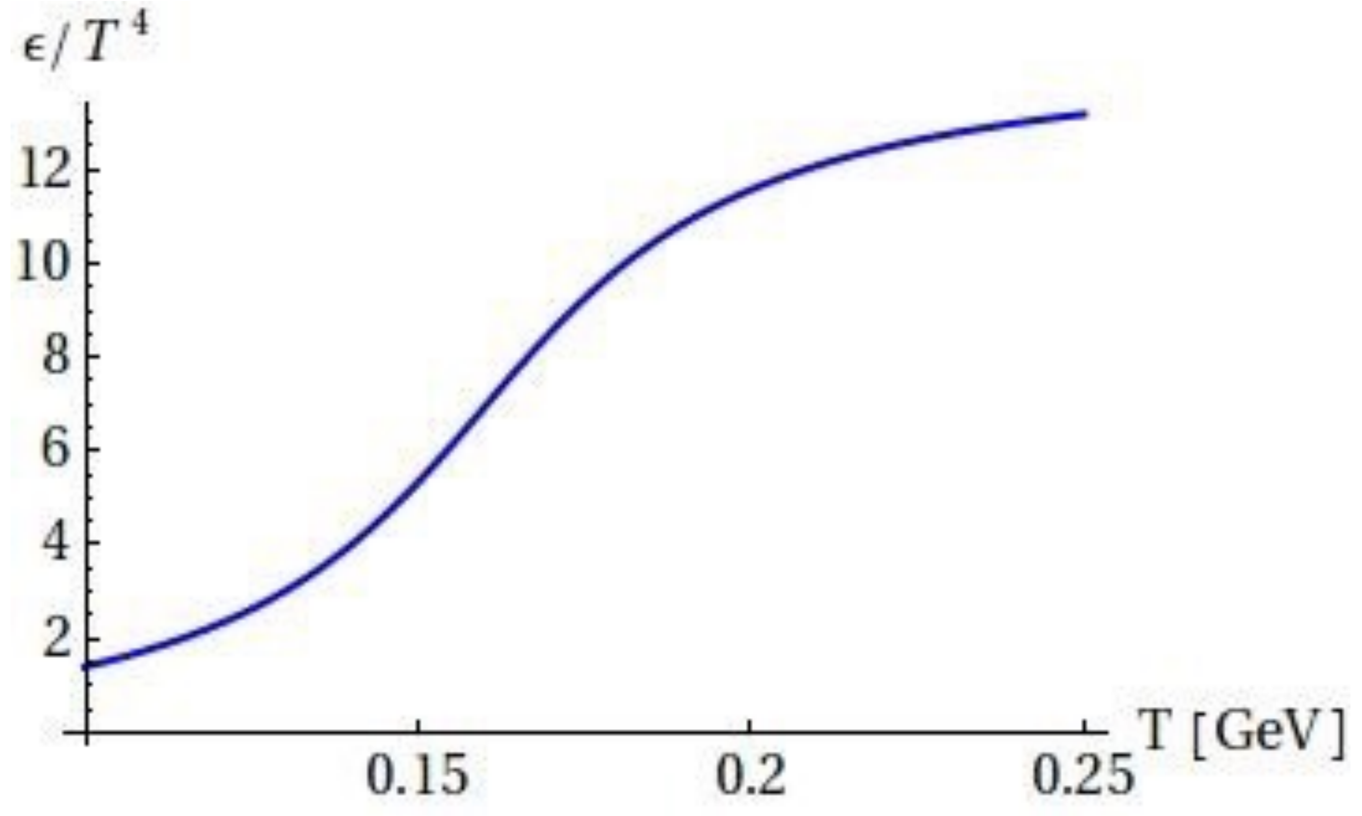
We are dealing with small system of finite lifetime

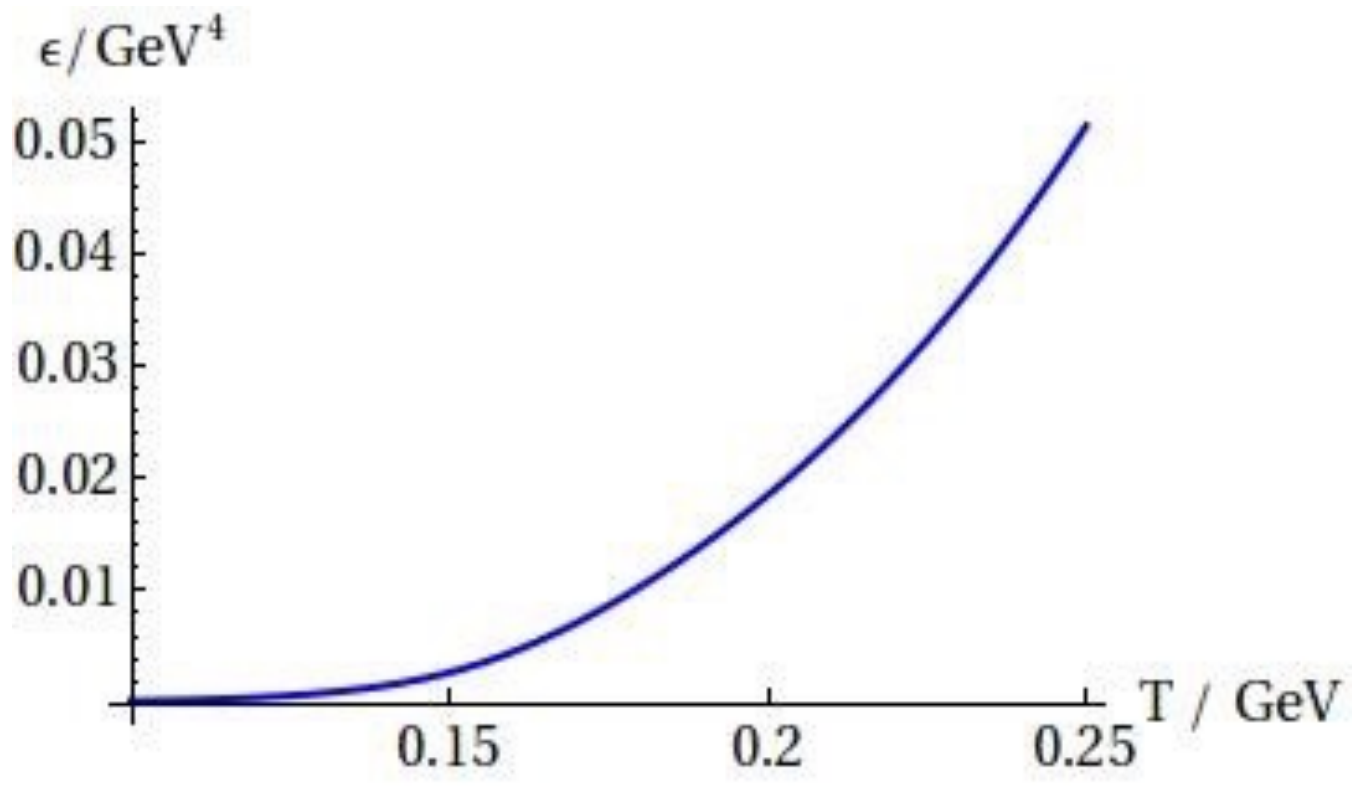
NO real singularities!

Cumulants and Phase structure

S. Borsanyi et al, JHEP 1011 (2010) 077





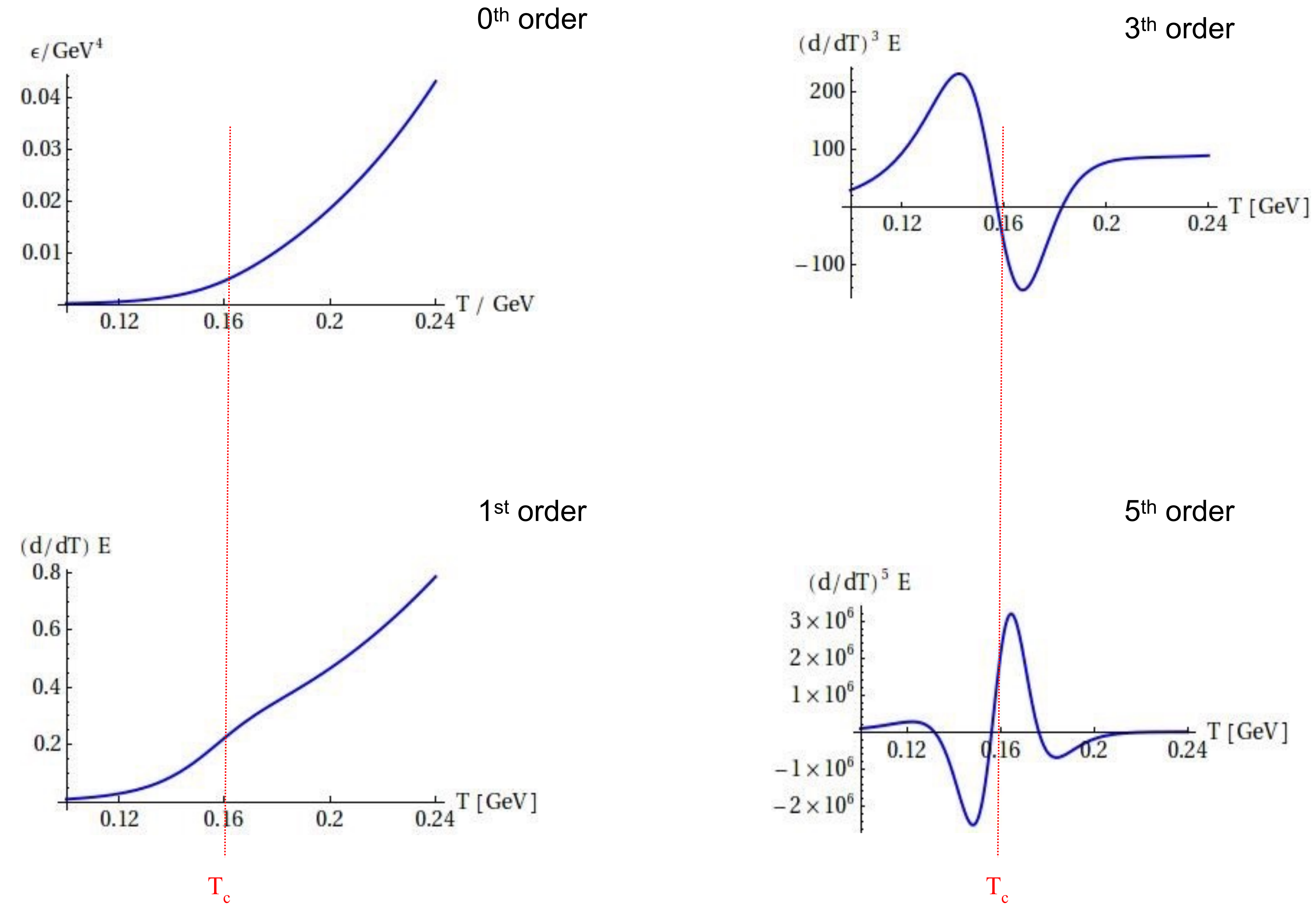


What we always see....

What it really means....

"T_c" ~ 155 MeV

Derivatives



How to measure derivatives

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z} tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

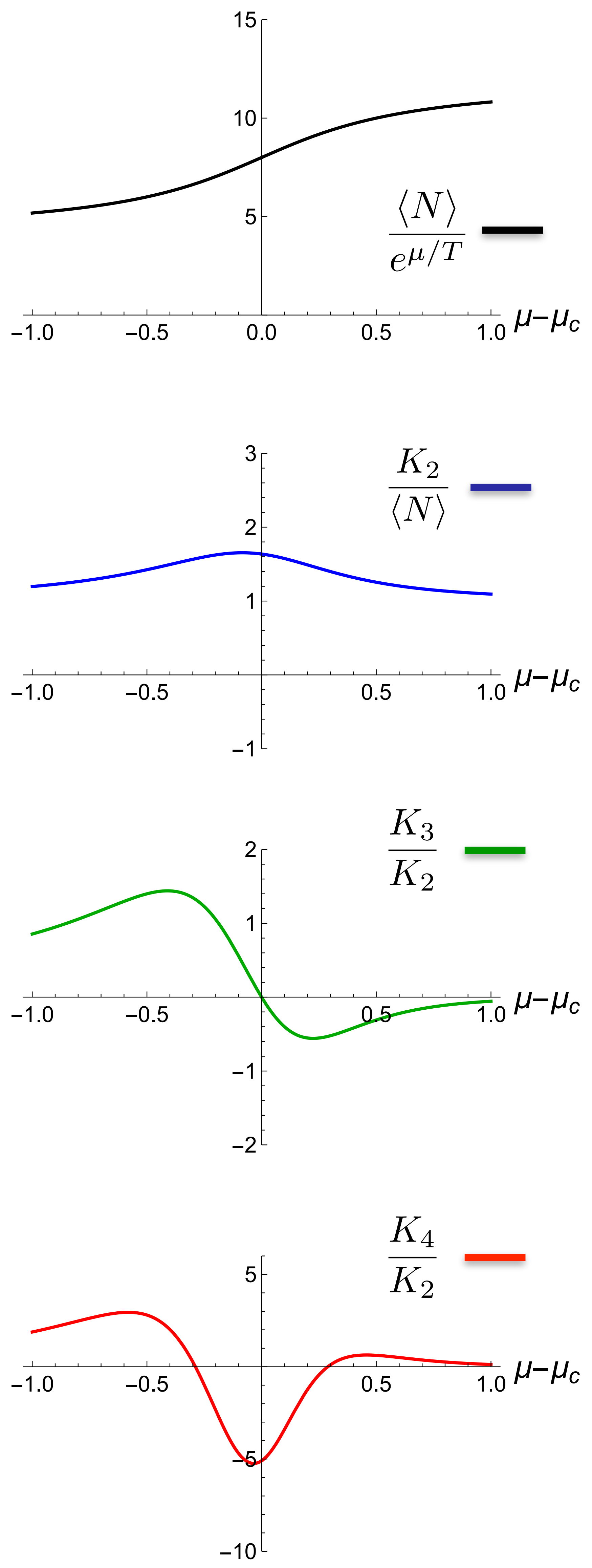
$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

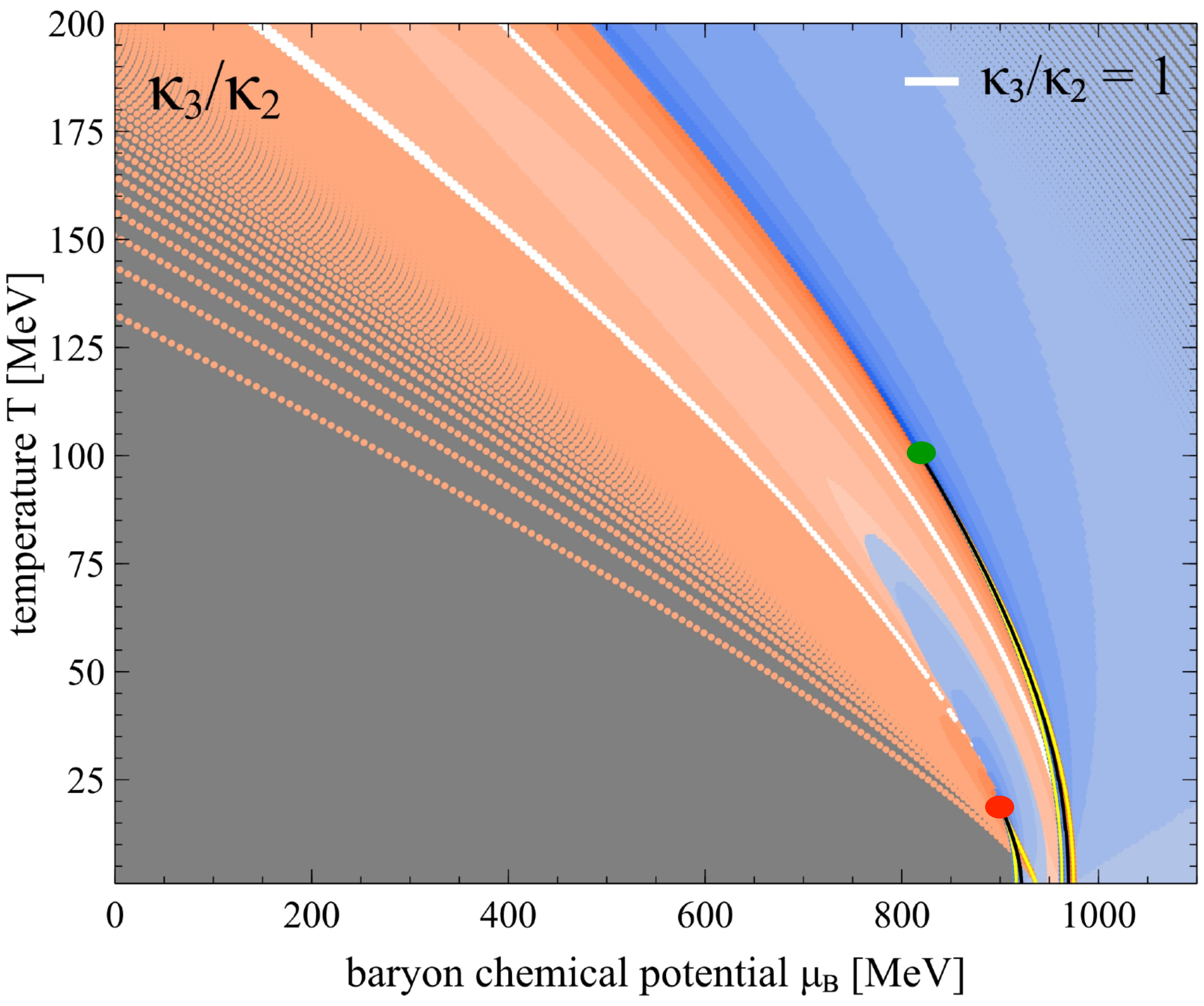
Derivatives 101



$\frac{K_3}{K_2}$ change sign at transition

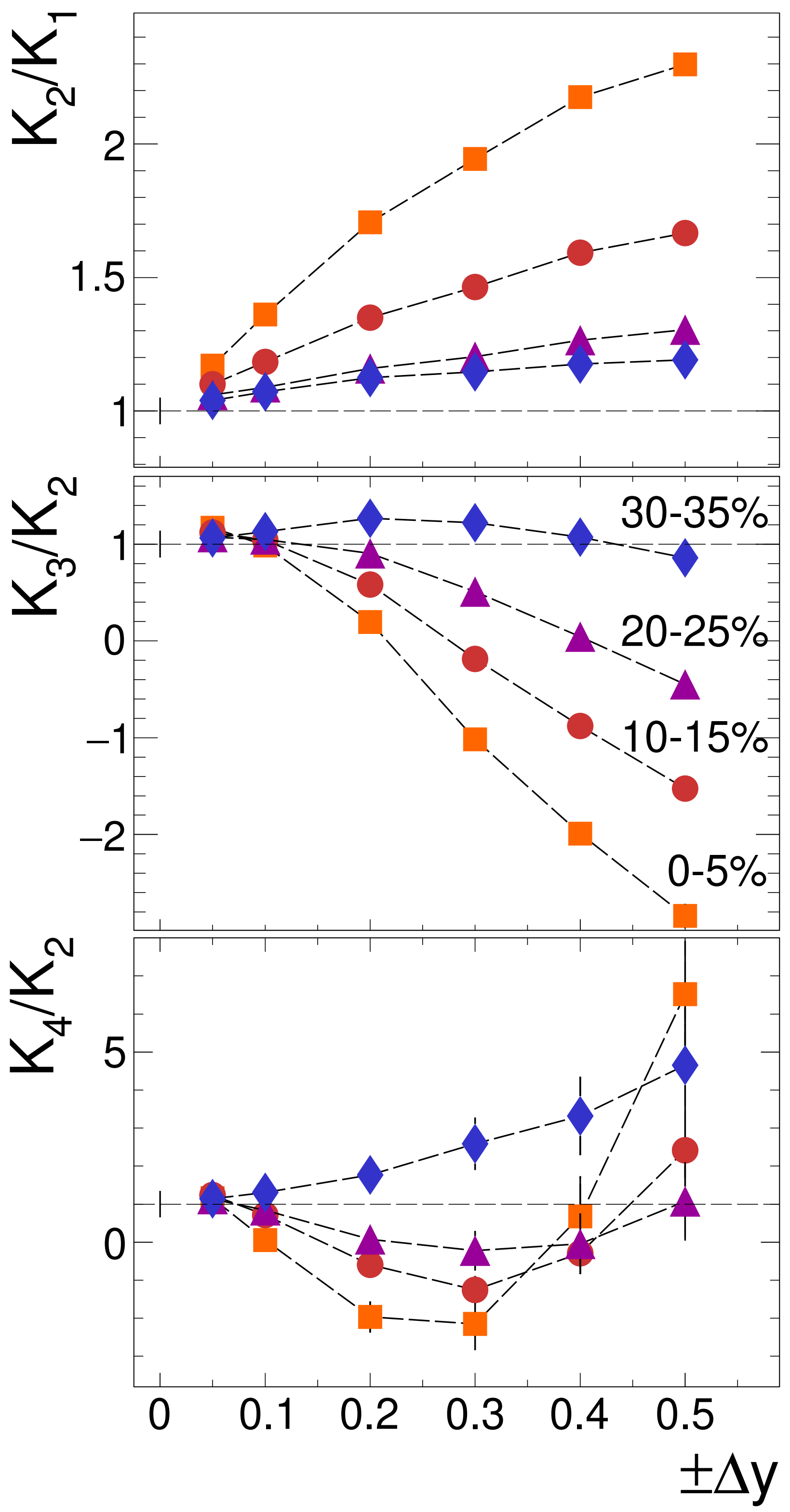
Negative "above" transition

Asakawa et al, arXiv:0904.2089



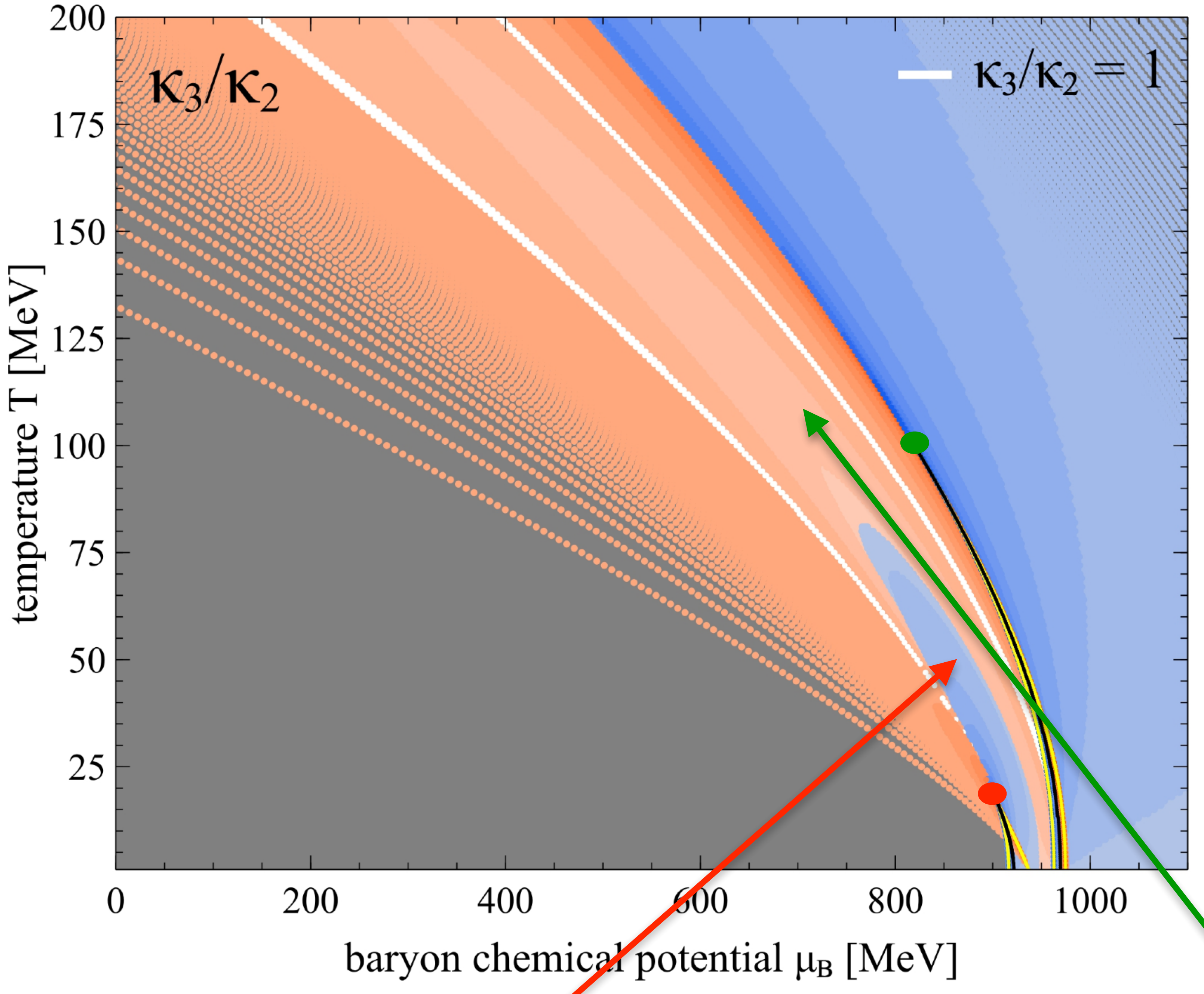
Model calculation by Agnieszka Sorensen (Wergieluk)
arXiv:2011.06635

Cumulants have been measured



HADES arXiv:2002.08701

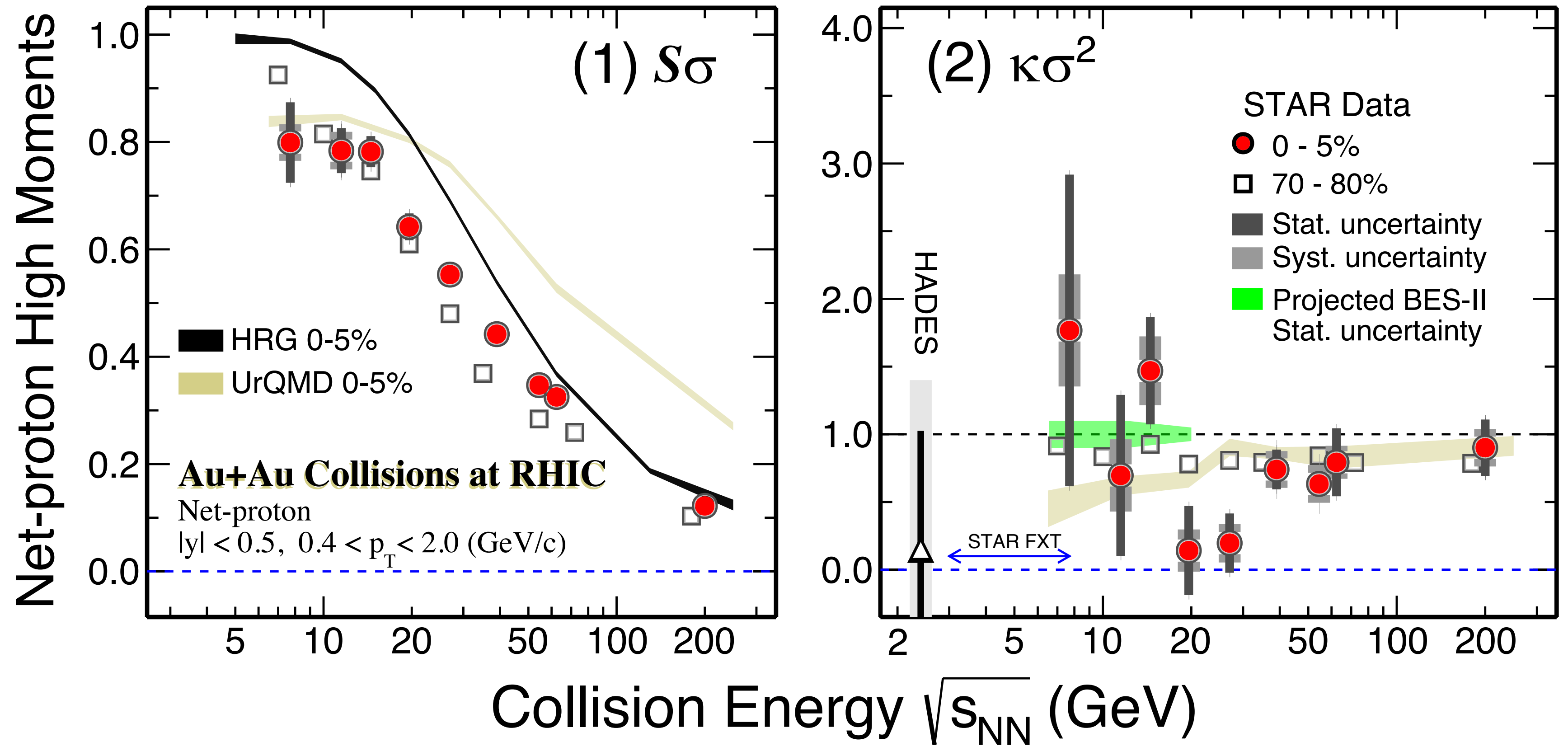
$$\frac{K_3}{K_2} < 0 \quad !!!!!$$



HADES?

STAR?

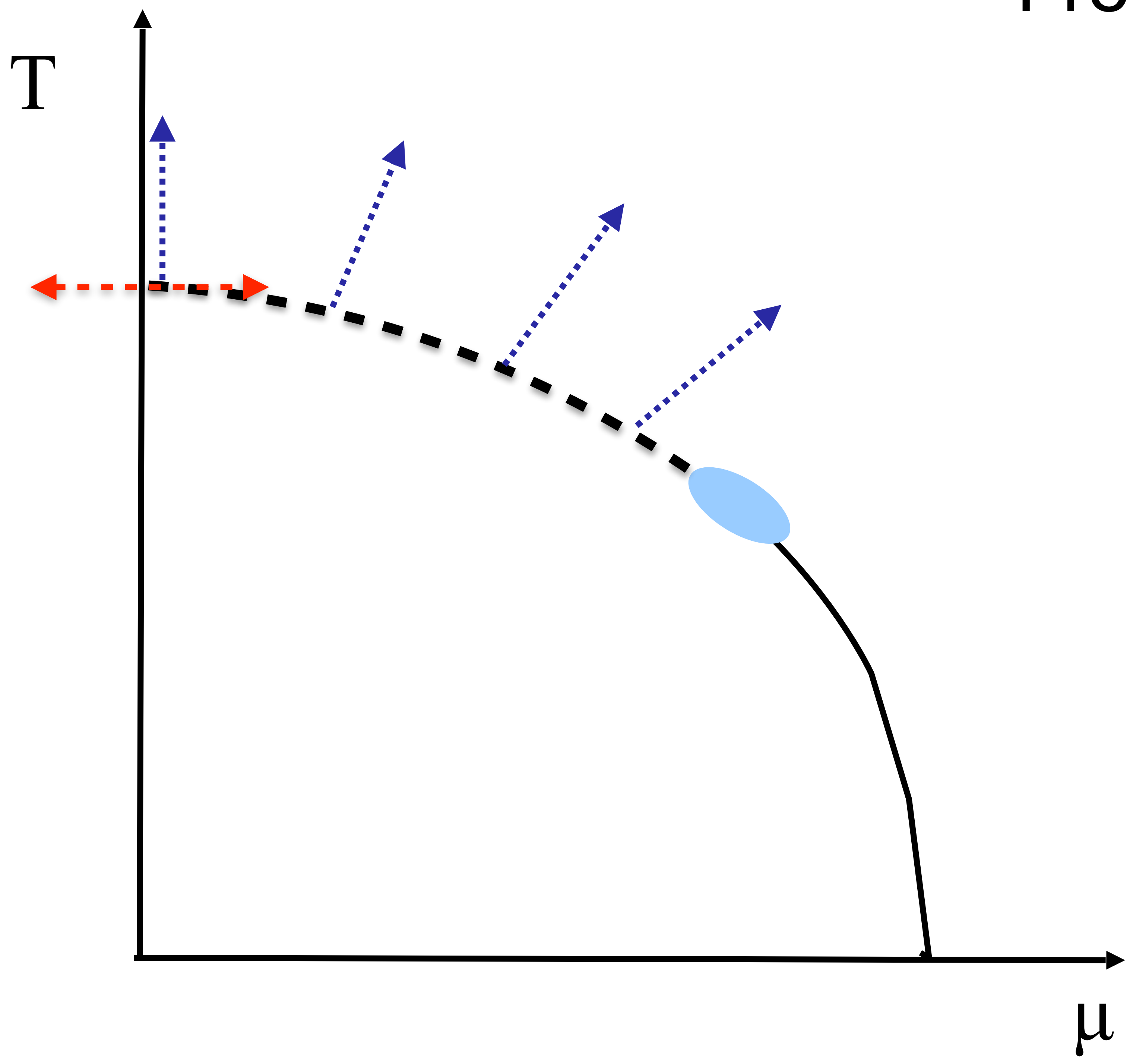
$$\frac{K_3}{K_2} \qquad \frac{K_4}{K_2}$$



STAR arXiv:2001.02852

$$\frac{K_3}{K_2} > 0 \quad !!$$

Close to μ=0



Free energy: $F = F(r), \quad r = \sqrt{T^2 + a\mu^2}$

a ~ curvature of critical line

$$\frac{\partial^2}{\partial \mu^2} F(T,\mu)|_{\mu=0} = \frac{a}{T}\frac{\partial}{\partial T} F(T, \mu=0) \sim \langle E \rangle$$

Needs higher order cumulants (derivatives) at μ ~ 0

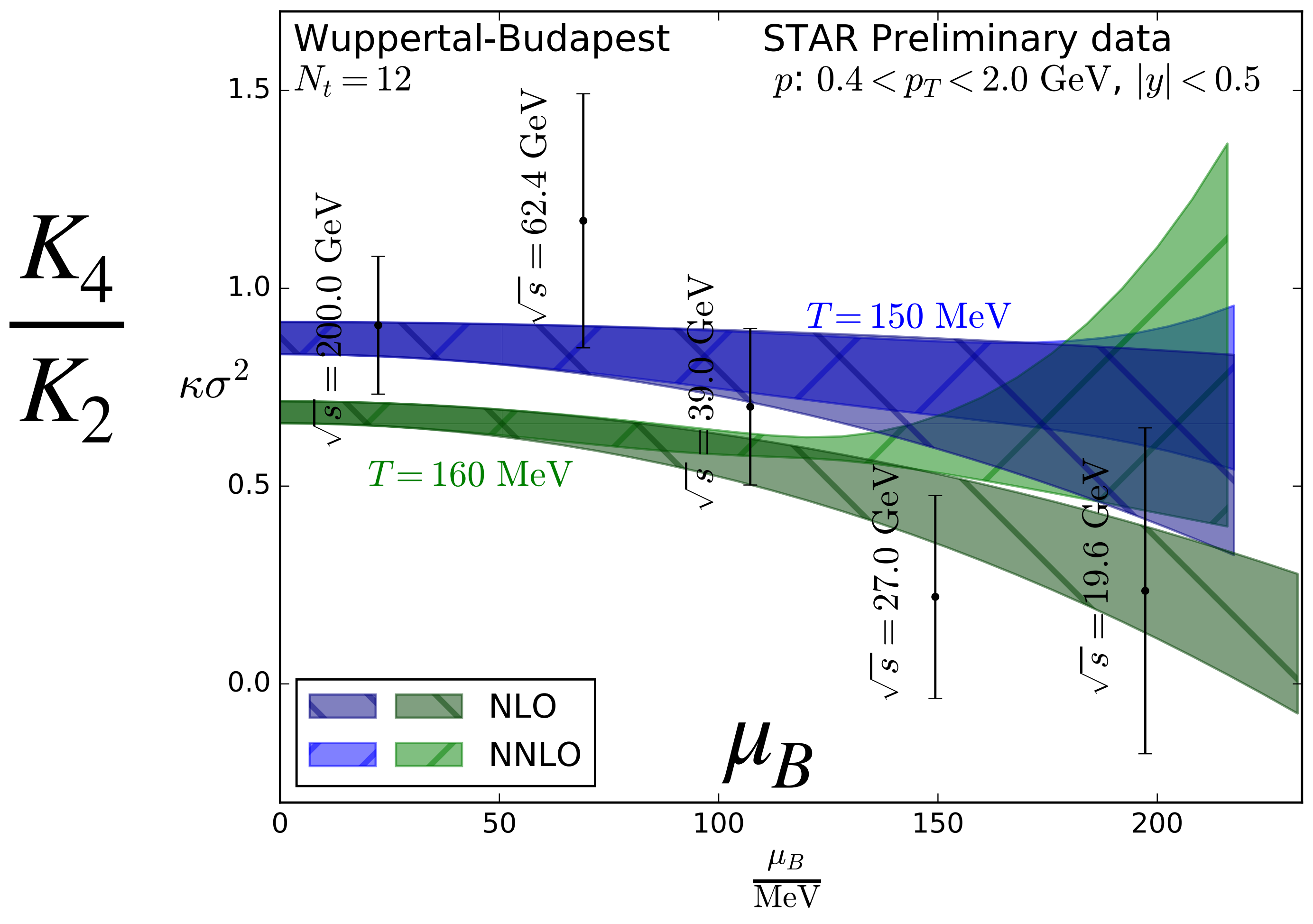
Cumulants at small μ

- Baryon number cumulants can be calculated in Lattice QCD
  - possible test of chiral criticality   Friman et al, ’11

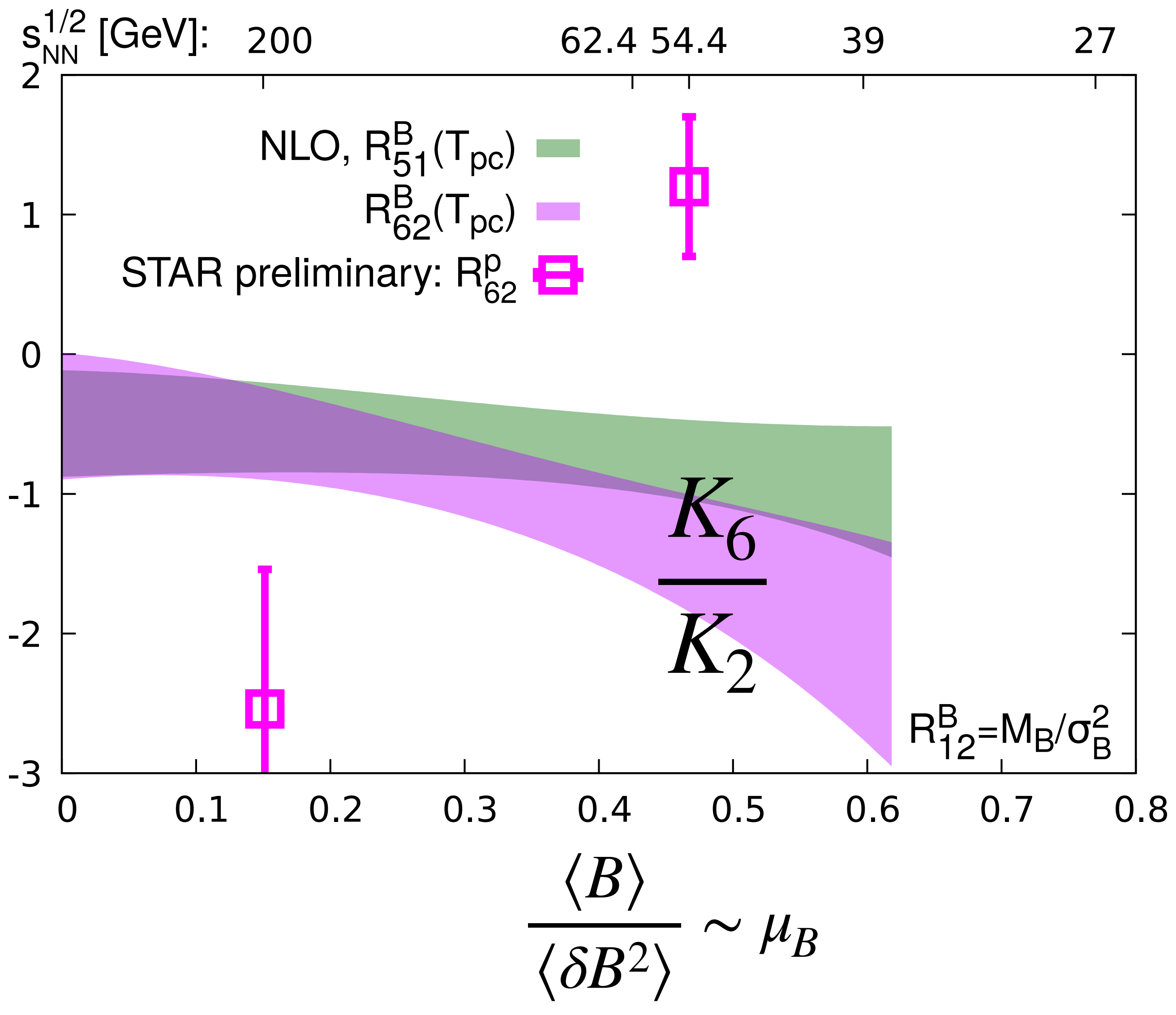
- Lattice:
  - Baryon number cumulants
  - grand canonical ensemble
  - fixed volume

- Experiment
  - Total baryon number is conserved   Bzdak et, ’13, Rustamov et al,’17
  - Proton cumulants   Asakawa, Kitazawa, ’12
  - Volume (N_{part}) fluctuates   Gorenstein et al, 11, Skokov et al, ‘13
  - dynamical: memory effect, hadronic phase   Mukherjee et al, ’15 Steinheimer et al, ‘18

Wuppertal-Budapest, arXiv:1805.04445



HotQCD, arXiv:2001.08530



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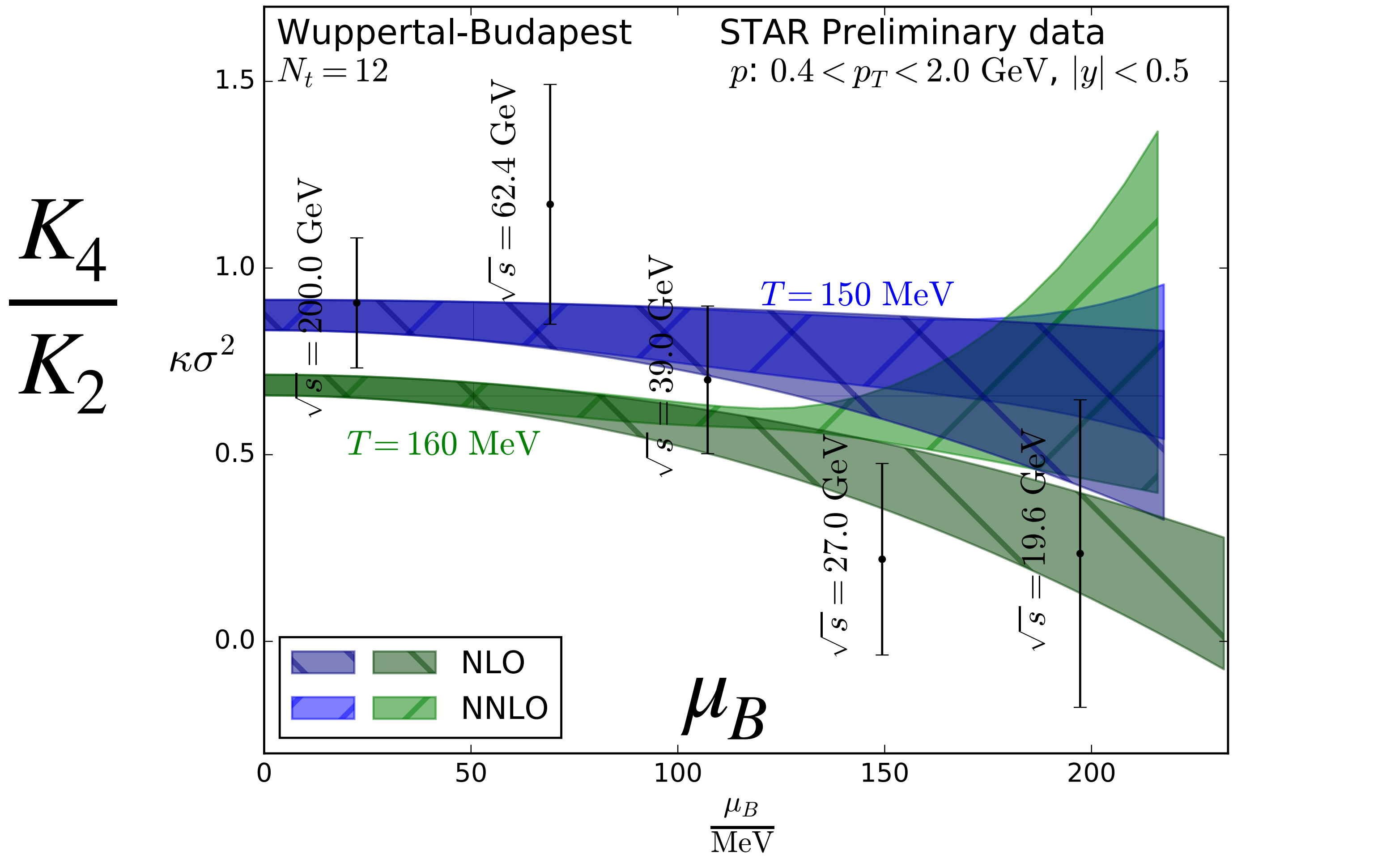

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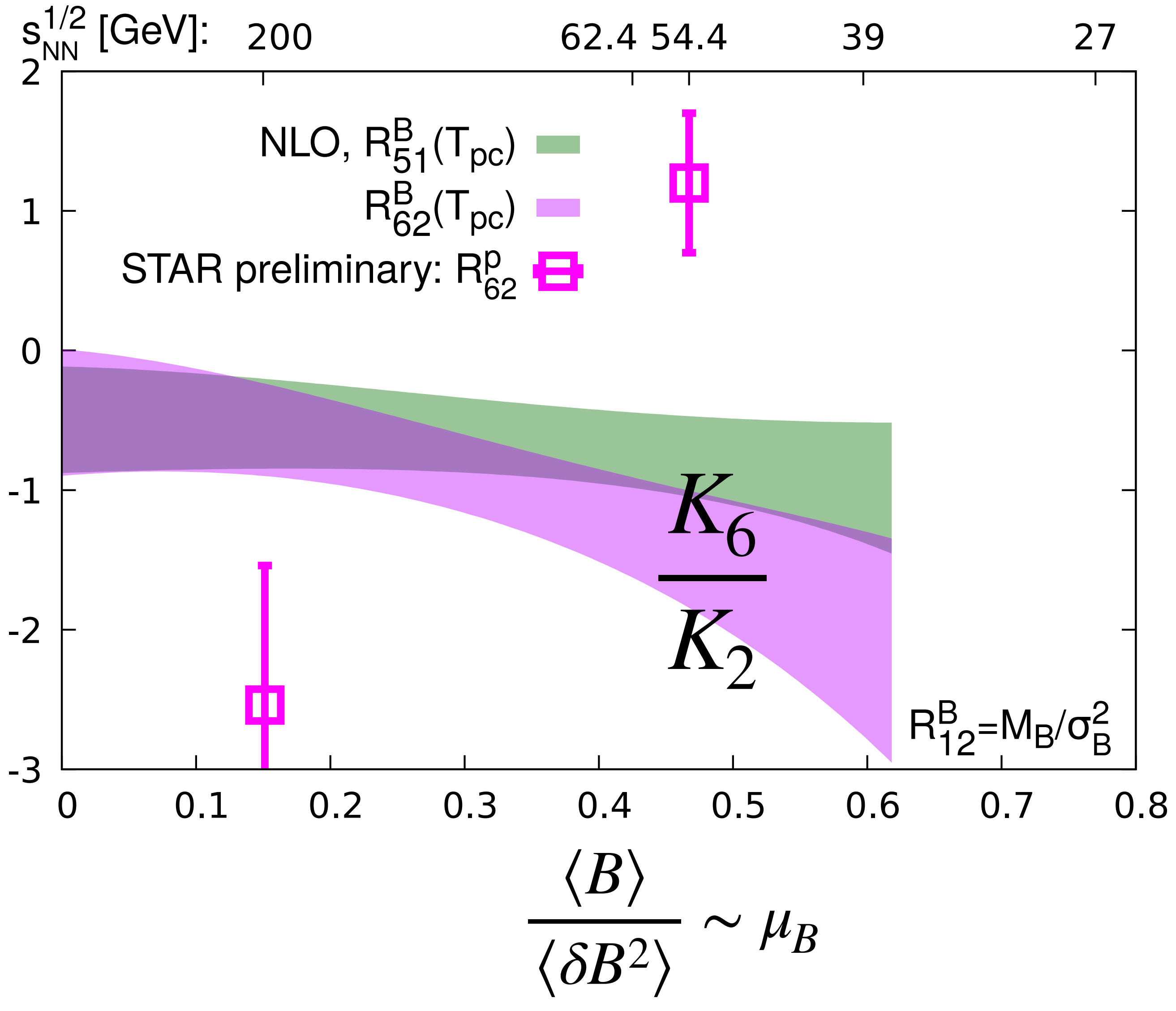
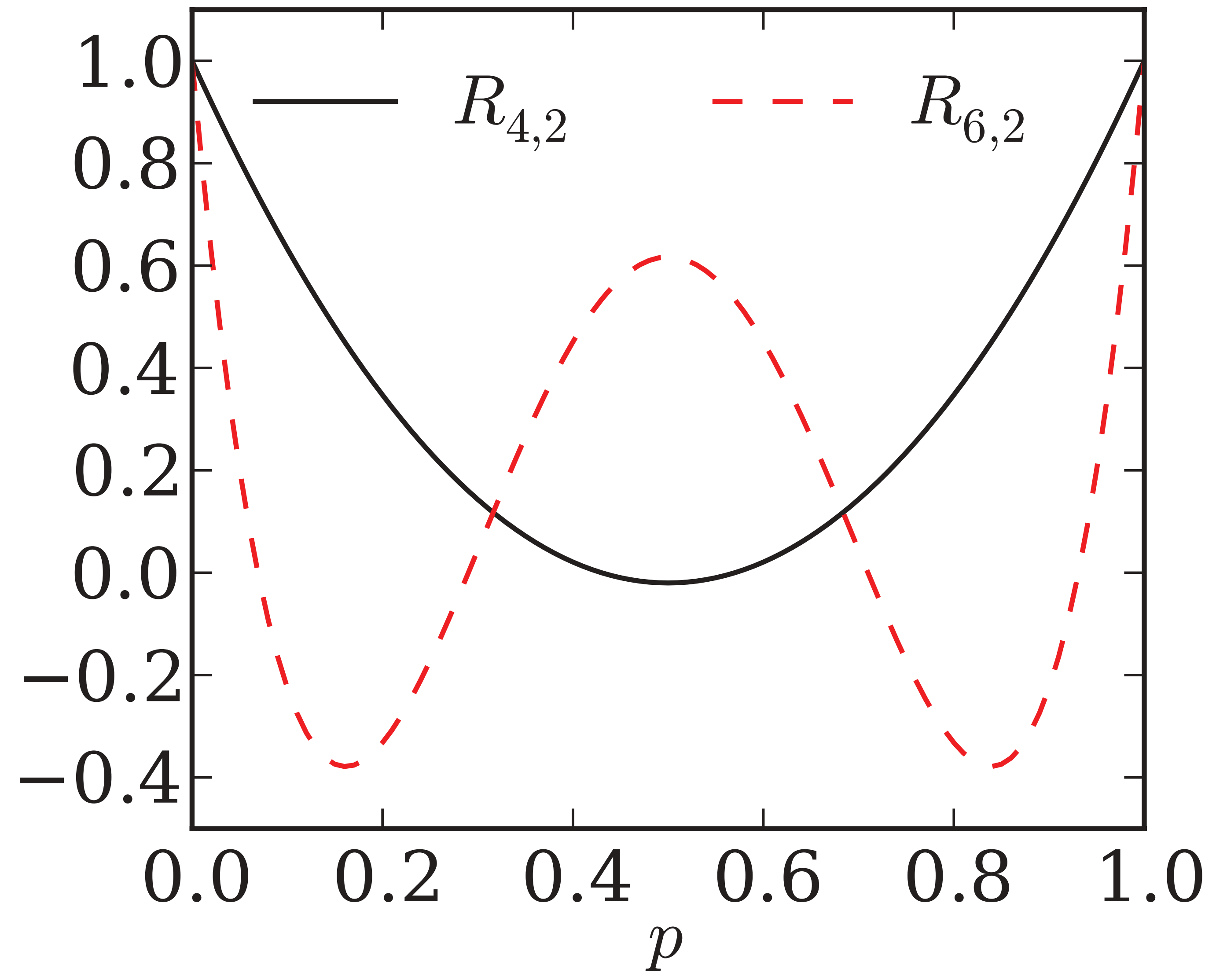


Baryon number conservation and lattice susceptibilities

- Charges (baryon number, strangeness, electric charge) are conserved globally in HI collisions
- Lattice (and most other calculations) work in the grand canonical ensemble: charges may fluctuate
- Effect of charge conservation have been calculated in the **ideal gas/HRG** limit. NON-neglibile corrections especially for higher order cumulants (Bzdak et al 2013, Rustamov et al. 2017,…)
- Wouldn't it be nice to know what the effect of charge conservation on **real QCD** (aka lattice) susceptibilities is?

This can actually be done!



Bzdak et al, 2013

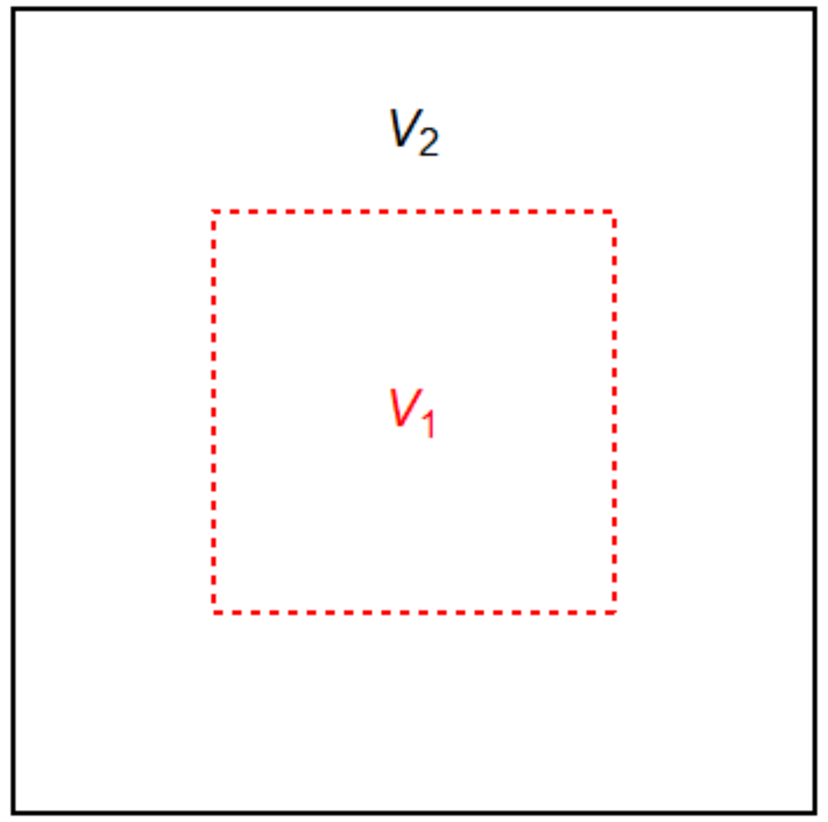
V. Vovchenko, O. Savchuk, R. Poberezhnyuk, M. Gorenstein, V.K., arXiv 2003.13905,

V. Vovchenko, R. Poberezhnyuk, V.K., arXiv:2007.03850

Subensemble acceptance method
(SAM)

Partition a thermal system with a globally conserved charge *B (canonical ensemble)* into two subsystems which can exchange the charge

$$V = V_1 + V_2$$



Assume thermodynamic limit:

$$V, V_1, V_2 \to \infty; \quad \frac{V_1}{V} = \alpha = const; \quad \frac{V_2}{V} = (1-\alpha) = const;$$

$$V_1, V_2 \gg \xi^3 \qquad \xi = correlation\,length$$

The canonical partition function then reads:

$$Z^{ce}(T,V,B) = \sum_{B_1} Z^{ce}(T,V_1,B_1) Z^{ce}(T,V-V_1,B-B_1)$$

The probability to have charge B_1 *in* V_1 is:

$$P(B_1) \sim Z^{ce}(T,\alpha V,B_1) Z^{ce}(T,(1-\alpha)V,B-B_1), \qquad \alpha \equiv V_1/V$$

Subensemble acceptance method
(SAM)

In the thermodynamic limit, $V \to \infty$, Z^{ce} expressed through free energy *density*

$$Z^{\mathrm{ce}}(T,V,B) \overset{V\to\infty}{\simeq} \exp\left[-\frac{V}{T} f(T,\rho_B)\right]$$

Cumulant generating function for B$_1$:

$$G_{B_1}(t) \equiv \ln\langle e^{t\,B_1}\rangle = \ln\left\{\sum_{B_1} e^{t\,B_1} \exp\left[-\frac{\alpha V}{T} f(T,\rho_{B_1})\right] \exp\left[-\frac{\beta V}{T} f(T,\rho_{B_2})\right]\right\} + \tilde{C}$$

Cumulants of B$_1$:

$$\kappa_n[B_1] = \left.\frac{\partial^n G_{B_1}(t)}{\partial t^n}\right|_{t=0} \equiv \tilde{\kappa}_n[B_1(t)]|_{t=0}$$

or

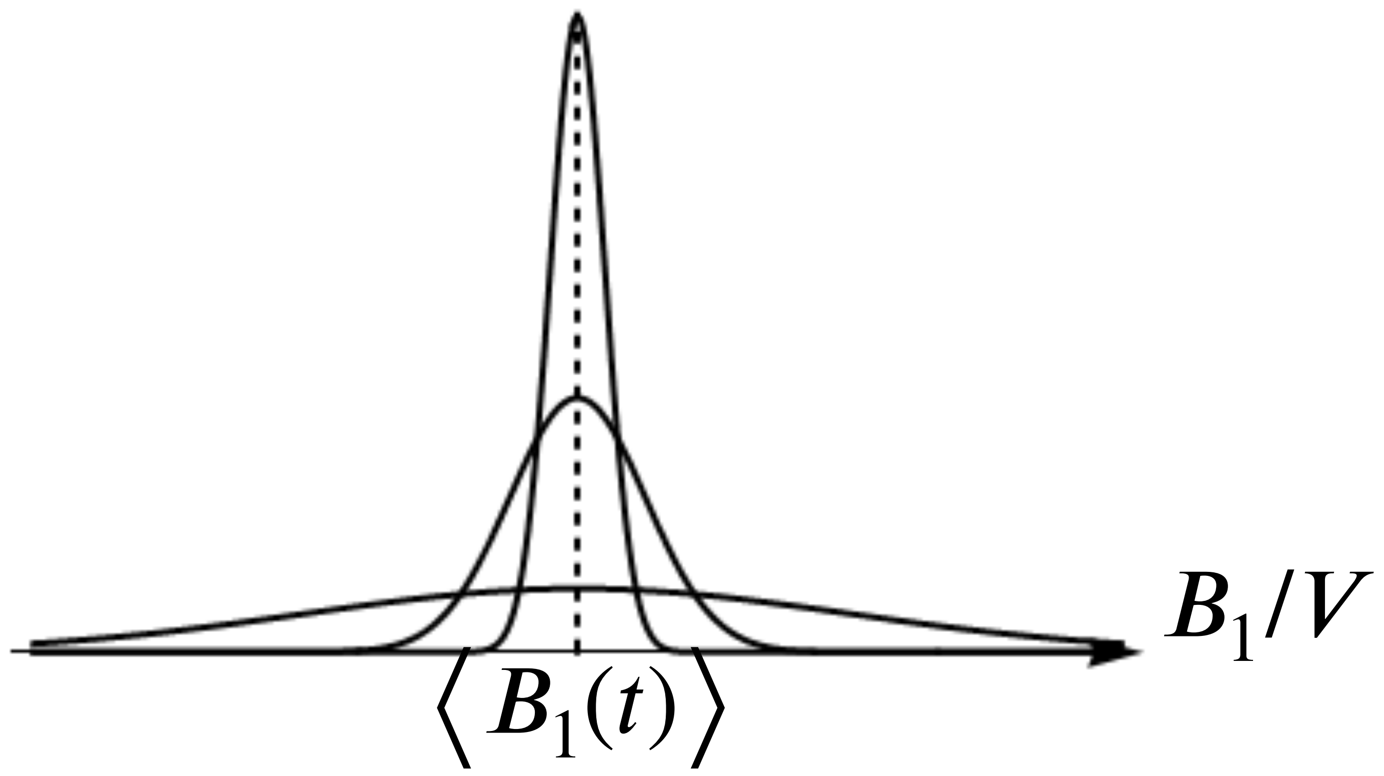
$$\kappa_n[B_1] = \left.\frac{\partial^{n-1}\tilde{\kappa}_1[B_1(t)]}{\partial t^{n-1}}\right|_{t=0}$$

All κ_n can be calculated by determining the *t*-dependent first cumulant $\tilde{\kappa}_1[B_1(t)]$

Making the connection…

$$\tilde{\kappa}_1[B_1(t)] = \frac{\sum_{B_1} B_1 \tilde{P}(B_1;t)}{\sum_{B_1} \tilde{P}(B_1;t)} \equiv \langle B_1(t) \rangle \qquad \text{with} \qquad \tilde{P}(B_1;t) = \exp\left\{ tB_1 - V\frac{\alpha f(T,\rho_{B_1}) + \beta f(T,\rho_{B_2})}{T} \right\}.$$

Thermodynamic limit: $\tilde{P}(B_1;t)$ *highly peaked at* $\langle B_1(t) \rangle$



$\langle B_1(t) \rangle$ is a solution to equation $d\tilde{P}/\mathrm{d}B_1 = 0$:

$$t = \hat{\mu}_B[T,\rho_{B_1}(t)] - \hat{\mu}_B[T,\rho_{B_2}(t)] \qquad \text{with} \qquad \hat{\mu}_B \equiv \mu_B/T, \qquad \mu_B(T,\rho_B) = \partial f(T,\rho_B)/\partial \rho_B$$

t = 0:

$$\rho_{B_1} = \rho_{B_2} = B/V,\ B_1 = \alpha B,$$

i.e. conserved charge uniformly distributed between the two subsystems

Second order cumulant

Differentiate condition for maximum of $\widetilde{P}(B_1;t)$,

$$t = \hat{\mu}_B[T, \rho_{B_1}(t)] - \hat{\mu}_B[T, \rho_{B_2}(t)] \qquad (*)$$

$$\frac{\partial(*)}{\partial t}: \qquad 1 = \left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B1}}\right)_T \left(\frac{\partial \rho_{B1}}{\partial \langle B_1 \rangle}\right)_V \frac{\partial \langle B_1 \rangle}{\partial t} - \left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B2}}\right)_T \left(\frac{\partial \rho_{B2}}{\partial \langle B_2 \rangle}\right)_V \frac{\partial \langle B_2 \rangle}{\partial \langle B_1 \rangle} \frac{\partial \langle B_1 \rangle}{\partial t}$$

$$\left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B1,2}}\right)_T \equiv \left[\chi_2^B\left(T, \rho_{B_{1,2}}\right) T^3\right]^{-1}, \qquad \rho_{B_1} \equiv \frac{\langle B_1 \rangle}{\alpha V}, \qquad \rho_{B_2} \equiv \frac{\langle B_2 \rangle}{(1-\alpha)V}, \qquad \langle B_2 \rangle = B - \langle B_1 \rangle, \qquad \frac{\partial \langle B_1 \rangle}{\partial t} \equiv \tilde{\kappa}_2[B_1(t)]$$

Solve the equation for $\widetilde{\kappa}_2$:

$$\tilde{\kappa}_2[B_1(t)] = \frac{V\,T^3}{[\alpha\,\chi_2^B(T, \rho_{B_1})]^{-1} + [(1-\alpha)\,\chi_2^B(T, \rho_{B_2})]^{-1}}$$

t = 0: $\kappa_2[B_1] = \alpha\,(1-\alpha)\,V\,T^3\,\chi_2^B$

Higher-order cumulants: iteratively differentiate $\widetilde{\kappa}_2$ w.r.t. t

Full result up to sixth order

$$\beta = 1 - \alpha$$

$$\kappa_1[B_1] = \alpha\, VT^3\, \chi_1^B$$

$$\kappa_2[B_1] = \alpha\, VT^3\, \beta\, \chi_2^B$$

$$\kappa_3[B_1] = \alpha\, VT^3\, \beta\,(1-2\alpha)\, \chi_3^B$$

$$\kappa_4[B_1] = \alpha\, VT^3\, \beta\, \left[\chi_4^B \; - 3\alpha\beta \frac{(\chi_3^B)^2 + \chi_2^B\, \chi_4^B}{\chi_2^B}\right]$$

$$\kappa_5[B_1] = \alpha\, VT^3\, \beta\,(1-2\alpha) \left\{[1-2\beta\alpha]\chi_5^B \; - 10\alpha\beta \frac{\chi_3^B \chi_4^B}{\chi_2^B}\right\}$$

$$\kappa_6[B_1] = \alpha\, VT^3\, \beta\,[1-5\alpha\beta(1-\alpha\beta)]\,\chi_6^B + 5\, VT^3\, \alpha^2\, \beta^2 \left\{9\alpha\beta \frac{(\chi_3^B)^2\, \chi_4^B}{(\chi_2^B)^2} - 3\alpha\beta \frac{(\chi_3^B)^4}{(\chi_2^B)^3} - 2(1-2\alpha)^2 \frac{(\chi_4^B)^2}{\chi_2^B} - 3[1-3\beta\alpha] \frac{\chi_3^B\, \chi_5^B}{\chi_2^B}\right\}$$

$$\chi_n^B = \frac{\partial^n (p/T^4)}{\partial(\mu_B/T)^n}$$ – grand-canonical susceptibilities e.g from Lattice QCD!!

Details: Vovchenko, et al. arXiv:2003.13905

Cumulant ratios

Some common cumulant ratios:

scaled variance
$$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$$

skewness
$$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$$

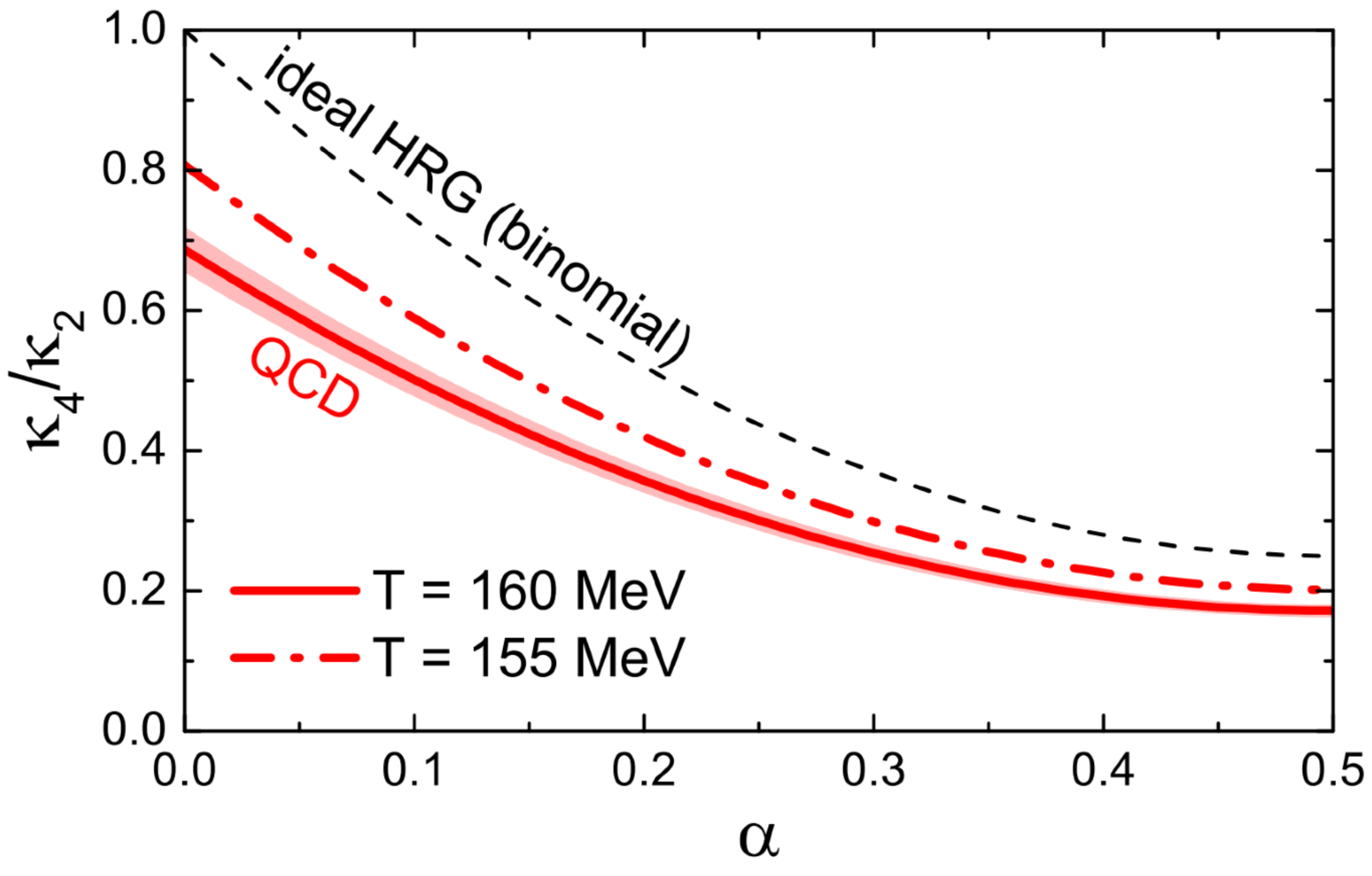
kurtosis
$$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2 .$$

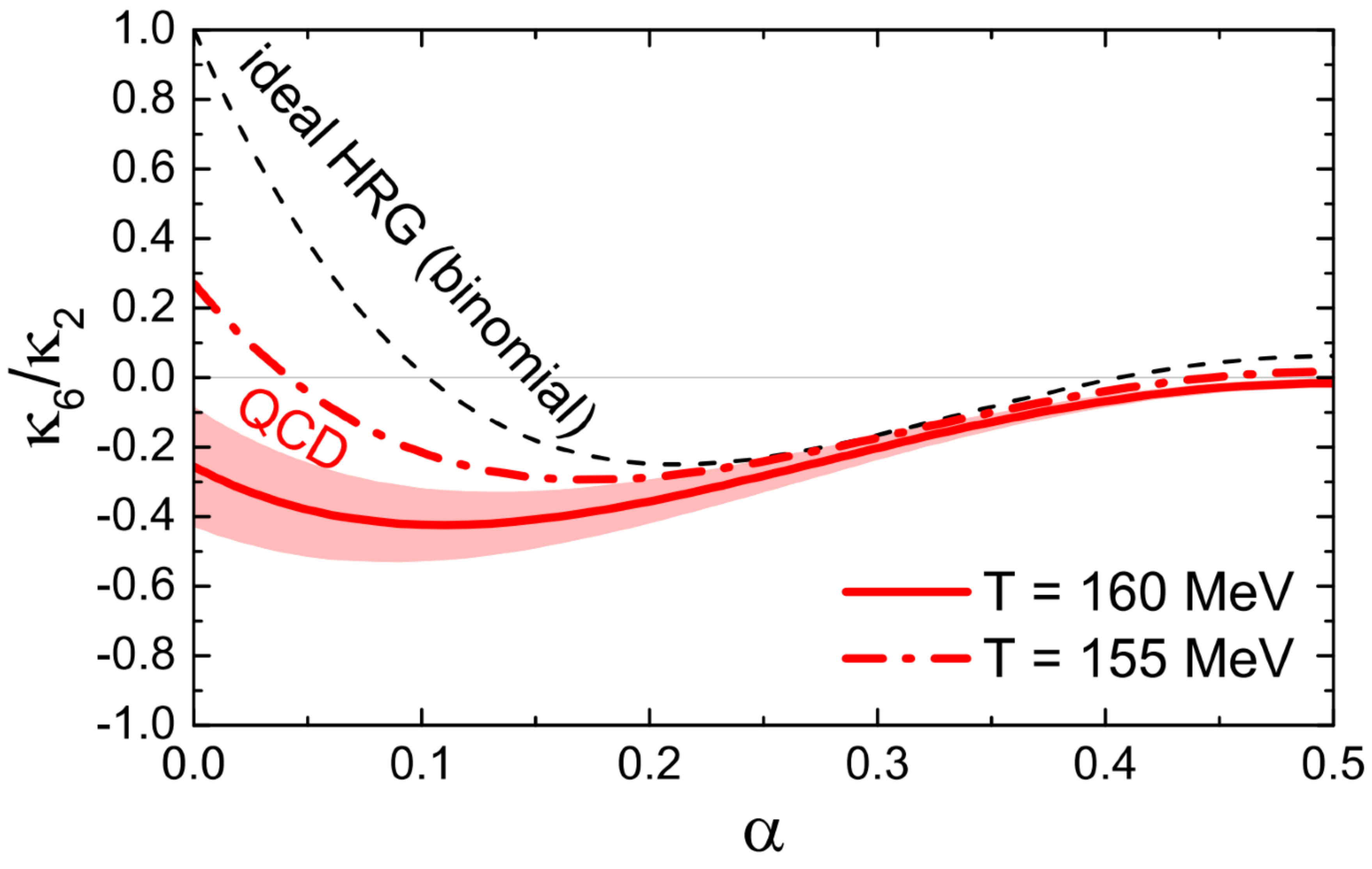
- Global conservation (α) and equation of state (χ_n^B) effects factorize in cumulants up to the 3rd order, starting from κ_4 not anymore
- $\alpha \to 0$: Grand canonical limit
- $\alpha \to 1$: canonical limit
- $\chi_{2n} = \langle N \rangle + \langle \bar{N} \rangle$; $\chi_{2n+1} = \langle N \rangle - \langle \bar{N} \rangle$: recover known results for ideal gas

Net baryon fluctuations at LHC and top RHIC (μ_B=0)

$$\left(\frac{\kappa_4}{\kappa_2}\right)_{LHC} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B}$$

$$\left(\frac{\kappa_6}{\kappa_2}\right)_{LHC} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$





Lattice data for χ_4^B/χ_2^B and χ_6^B/χ_2^B from Borsanyi et al., 1805.04445

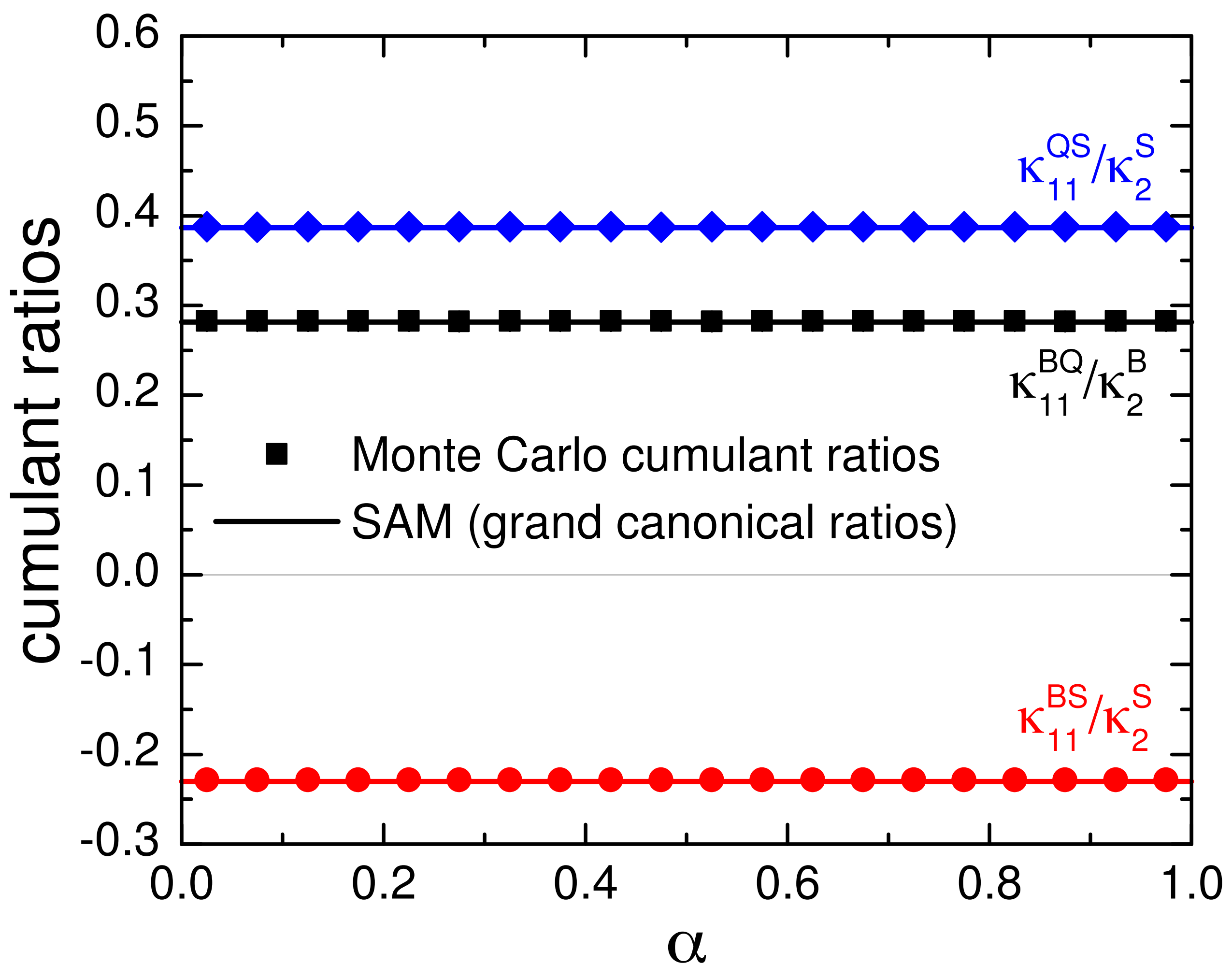
- $\alpha > 0.2$ difficult to distinguish effects of the EoS and baryon conservation in χ_6^B/χ_2^B
- $\alpha \le 0.1$ is a sweet spot where measurements are mainly sensitive to the EoS
- Estimate: $\alpha \approx 0.1$ corresponds to $\Delta Y_{acc} \approx 2(1)$ at LHC (RHIC)

Multiple conserved charges

(Vovchenko, R.Poberezhnyuk, V.K, arXiv:2007.03850)

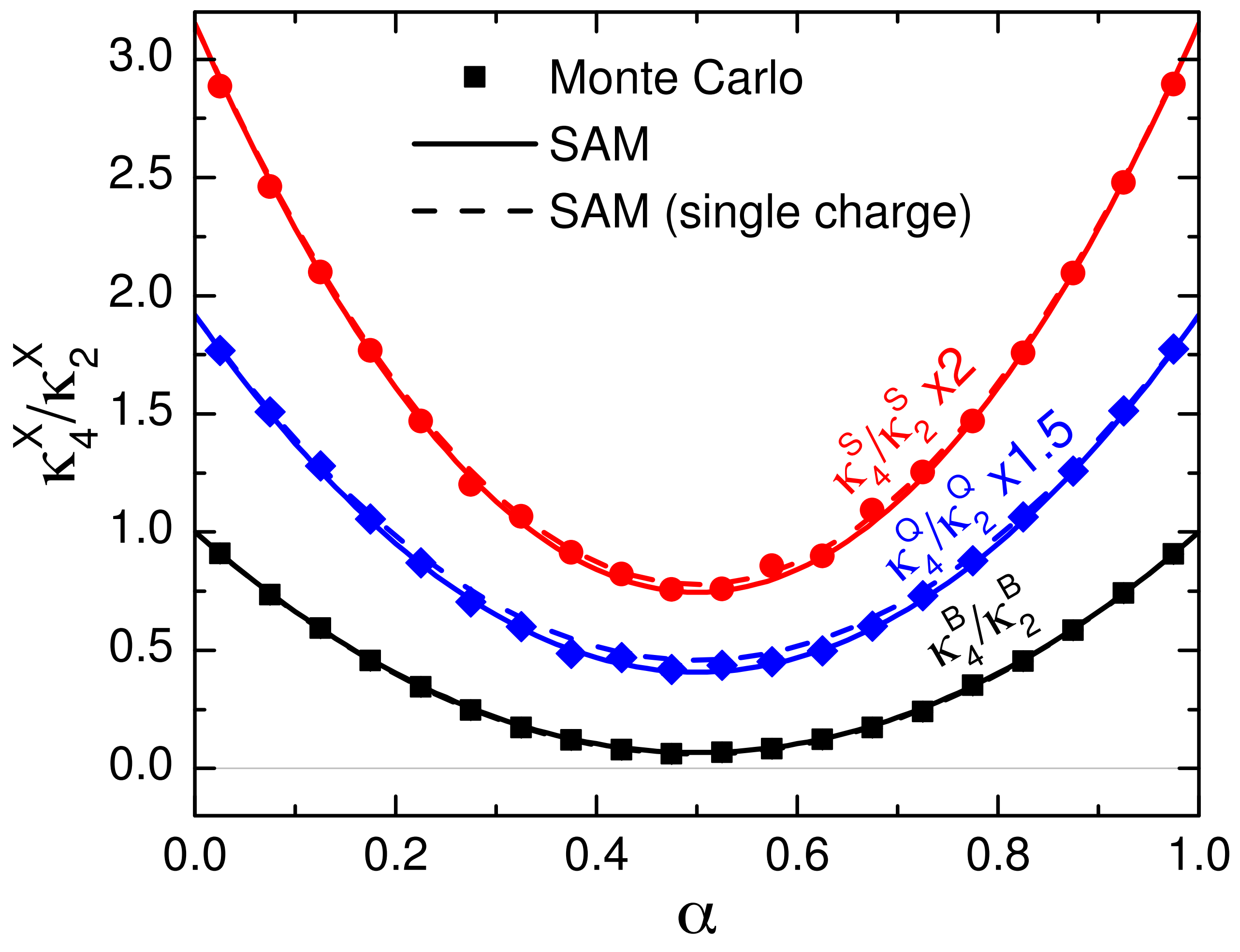
Key findings:

- Ratios of second and third order cumulants are NOT sensitive to charge conservation
  - This is also true for so called "strongly intensive quantities"
  - Requires that acceptance fraction α is the same for both particles (or Q and S)



- For order n>3 charge cumulants "mix". Effect in HRG is tiny

$$\kappa_4[B^1] = \alpha V T^3 \, \beta \left[(1-3\alpha\beta)\, \chi_4^B - 3\,\alpha\,\beta\, \frac{(\chi_3^B)^2 \chi_2^Q - 2\chi_{21}^{BQ}\chi_{11}^{BQ}\chi_3^B + (\chi_{21}^{BQ})^2\chi_2^B}{\chi_2^B\chi_2^Q - (\chi_{11}^{BQ})^2} \right]$$



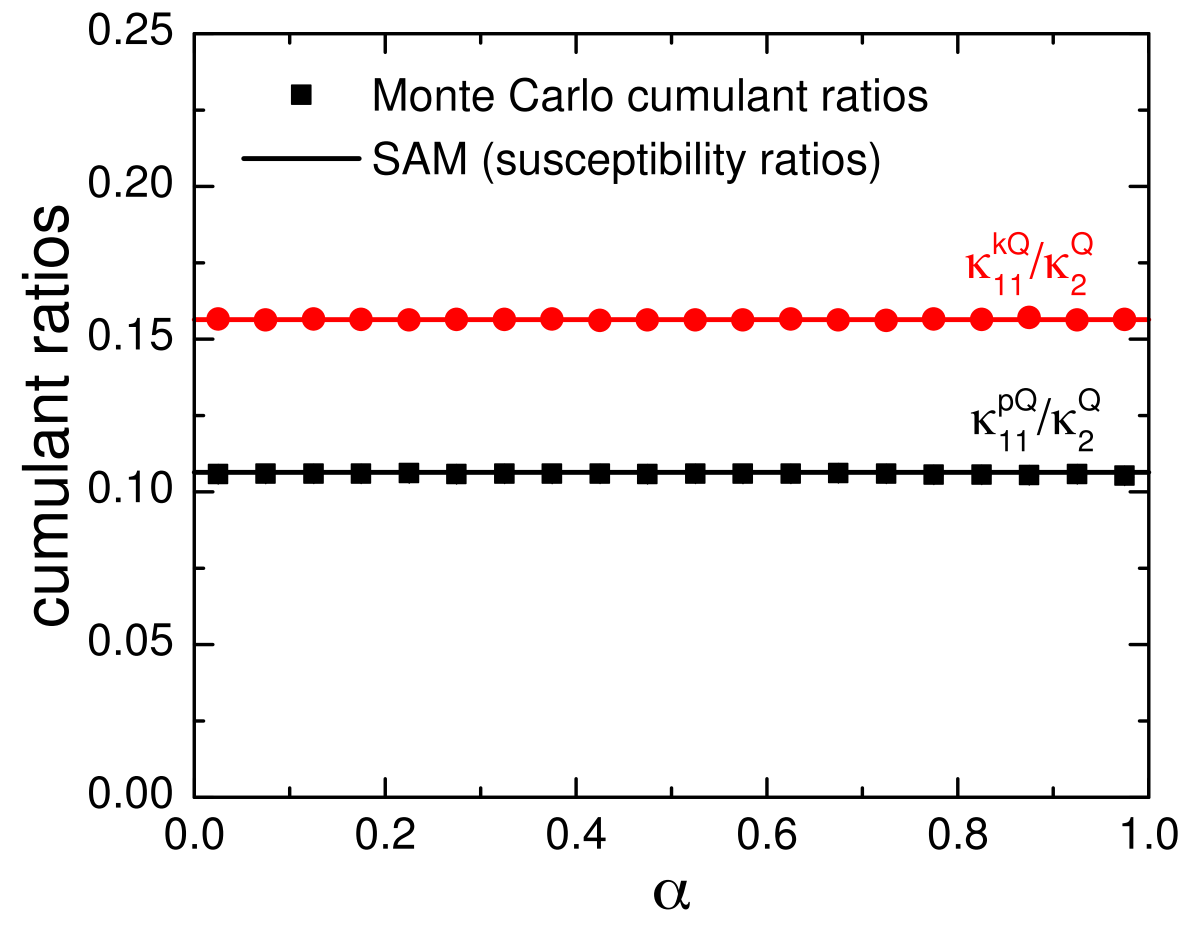
For explicit results up to order n=6, see arXiv:2007.03850

Multiple conserved charges

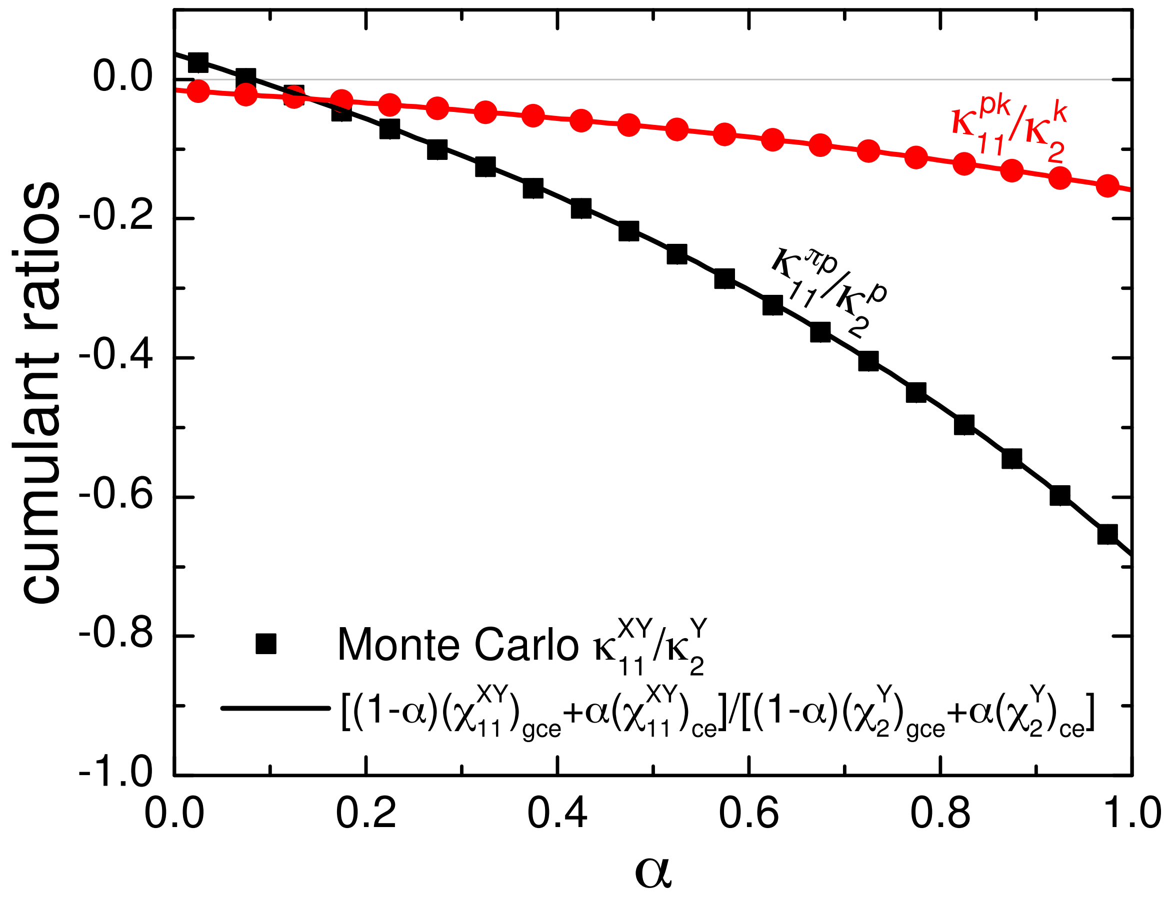
(Vovchenko, R.Poberezhnyuk, V.K, arXiv:2007.03850)

Also works for non-conserved quantities such as protons, K and Λ

- Mixed cumulants involving one conserved charge such as $\sigma_{1,1}^{p,Q}$ scale like second order charge cumulants
  - Again, same acceptance fraction α for both p and Q, or k and Q



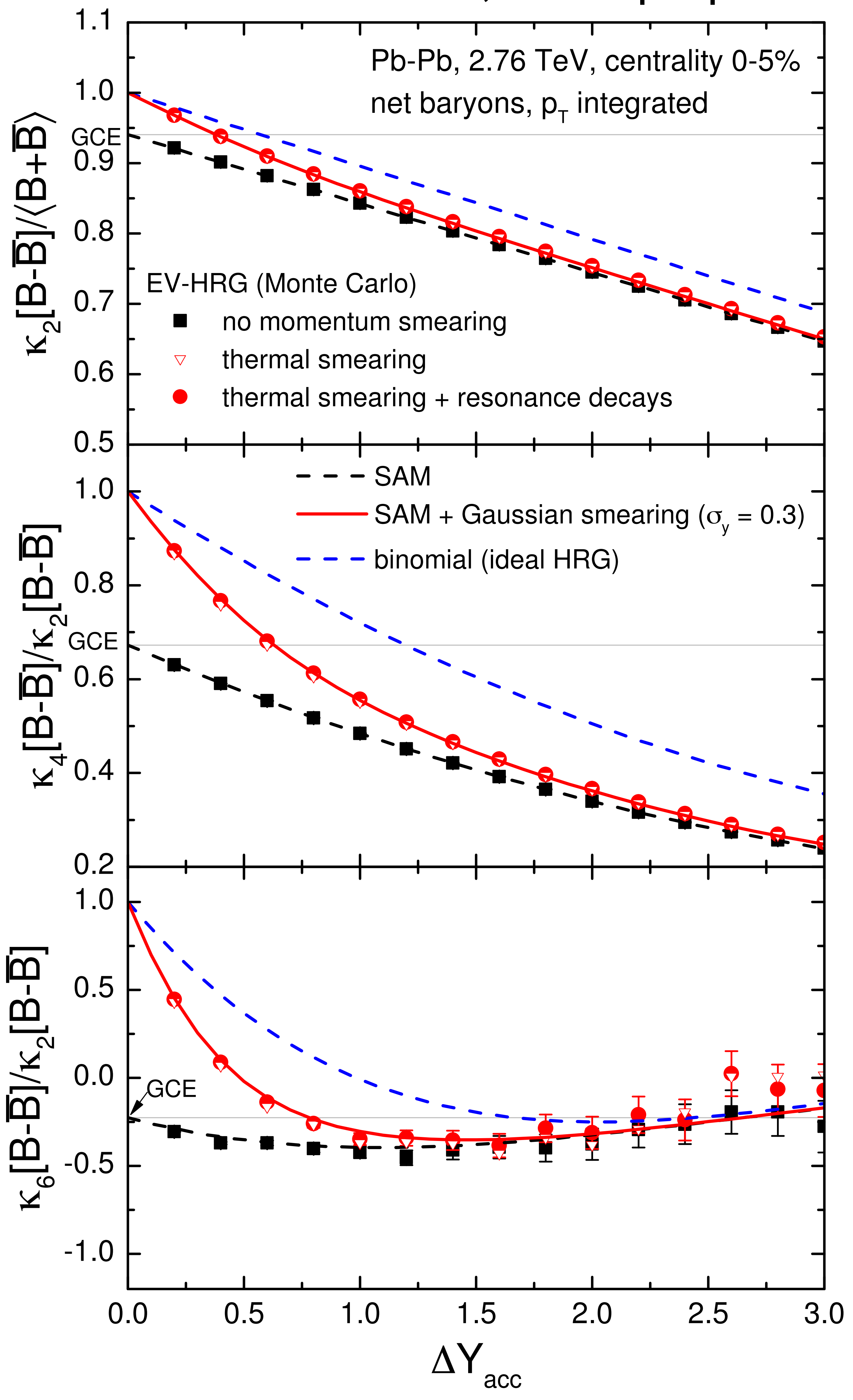
Does NOT work for two non-conserved charges, such as $\sigma_{1,1}^{p,K}$



Thermal smearing

- Subensemble Acceptance Method (SAM) works in configuration space
- Experiment measures momentum space
- OK if perfect space momentum correlations a la Bjorken
- However there is thermal smearing

V. Vovchenko, VK in prep.



Thermal smearing interpolates between ideal gas and true QCD (SAM)

Protons vs Baryons

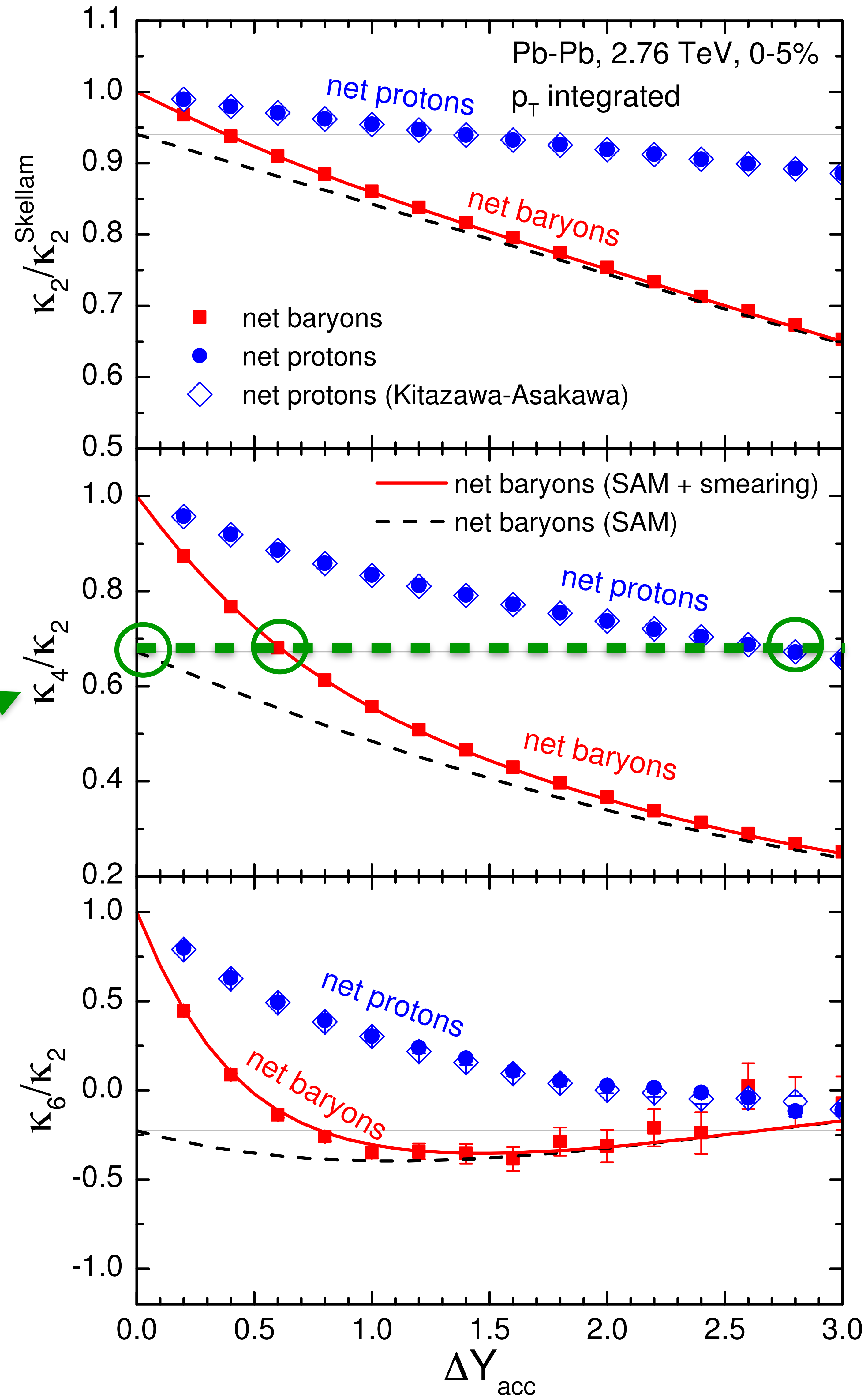
- Proton are subset of all baryons
  - dilutes the signal
  - need to do binomial unfolding
    - Kitazawa, Asakawa PRC '12
  - Otherwise Apples vs. Oranges
  - For example

"lattice QCD" "experiment"

$$\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T=160MeV}^{GCE} \simeq 0.67 \neq \left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{acc}=1}^{HIC} \simeq 0.56 \neq \left.\frac{\chi_4^p}{\chi_2^p}\right|_{\Delta Y_{acc}=1}^{HIC} \simeq 0.83$$

- Unfolding requires factorial moments not directly accessible in Lattice QCD
- Only experiment can ans should do proper corrections

V. Vovchenko, VK in prep.



Applicability and limitations

- Argument is based on partition in ***coordinate*** space; experiments partition in ***momentum*** space
  - Best for high energies where we have Bjorken flow
    - Thermal smearing interpolates between “binomial” and true corrections
  - So far limited applicability for lower energies. Under invenstigation.
- Thermodynamic limit i.e. $V_1, V_2 \gg \xi^3$:
  - Lattice calculations work with $V_{lattice} \simeq (5\,\mathrm{fm})^3 = 125\,\mathrm{fm}^3$.

    Chemical freeze out Volume at LHC $\sim 4500\,\mathrm{fm}^3$
- Not addressed: local charge conservation

Summary

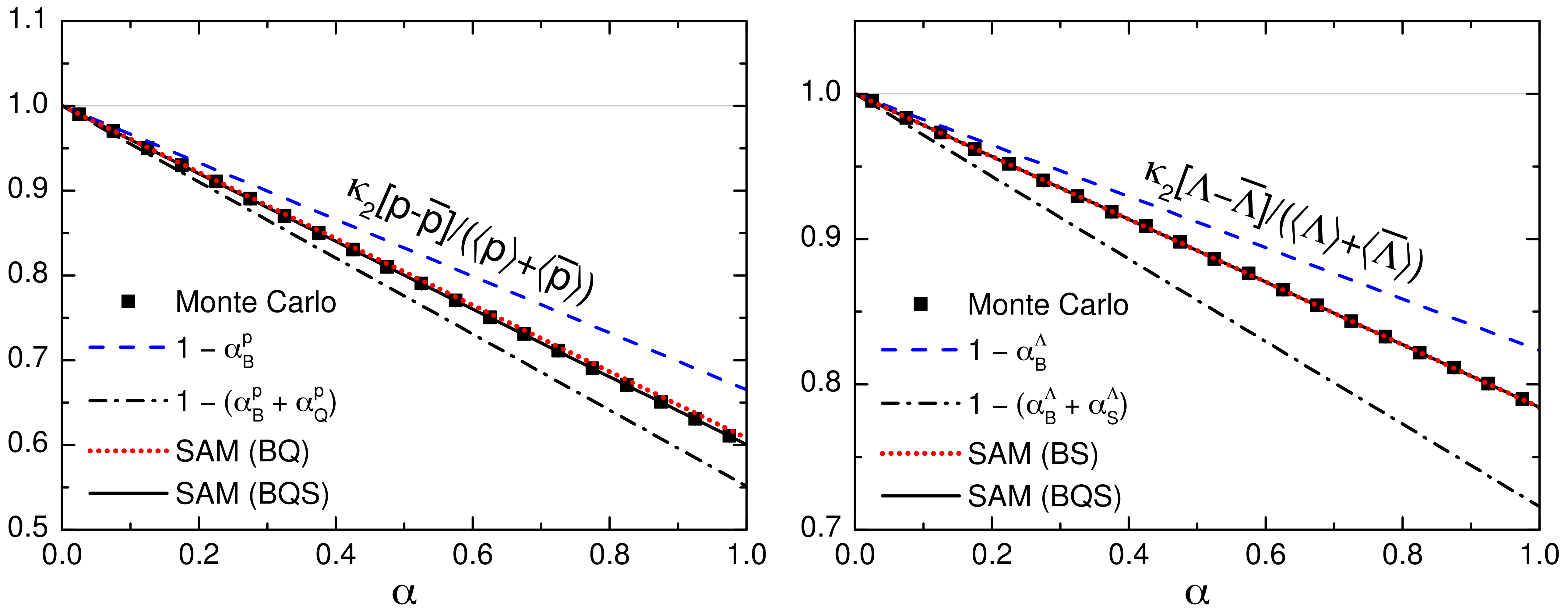
- Fluctuations are a powerful tool to explore QCD phase diagram
  - critical point
  - nuclear liquid gas transition
  - remnants of chiral criticality at $\mu \sim 0$
- HADES reports negative K_3/K_3. Do they see the nuclear liquid gas transition?
- Corrections for global (multiple) charge conservation in terms of grand canonical susceptibilities for ANY equation of state not just ideal gas
  - connection to lattice results
  - Applicable at top RHIC and LHC
  - Ratios of second and third order cumulants insensitive to conservation effects as long as acceptance fraction is the same
- Proton cumulants cannot be directly compared to baryon cumulants
  - unfolding needed which can only done by experiment.

Thank You

Multiple conserved charges

(Vovchenko, R.Poberezhnyuk, V.K, arXiv:2007.03850)

- Allows for corrections due to electric charge (protons) or strangeness (Λ) in addition to baryon number conservation.



Truth lies in between the “naive” corrections. Likely bigger effect for higher orders.

Subensemble acceptance: van der Waals fluid

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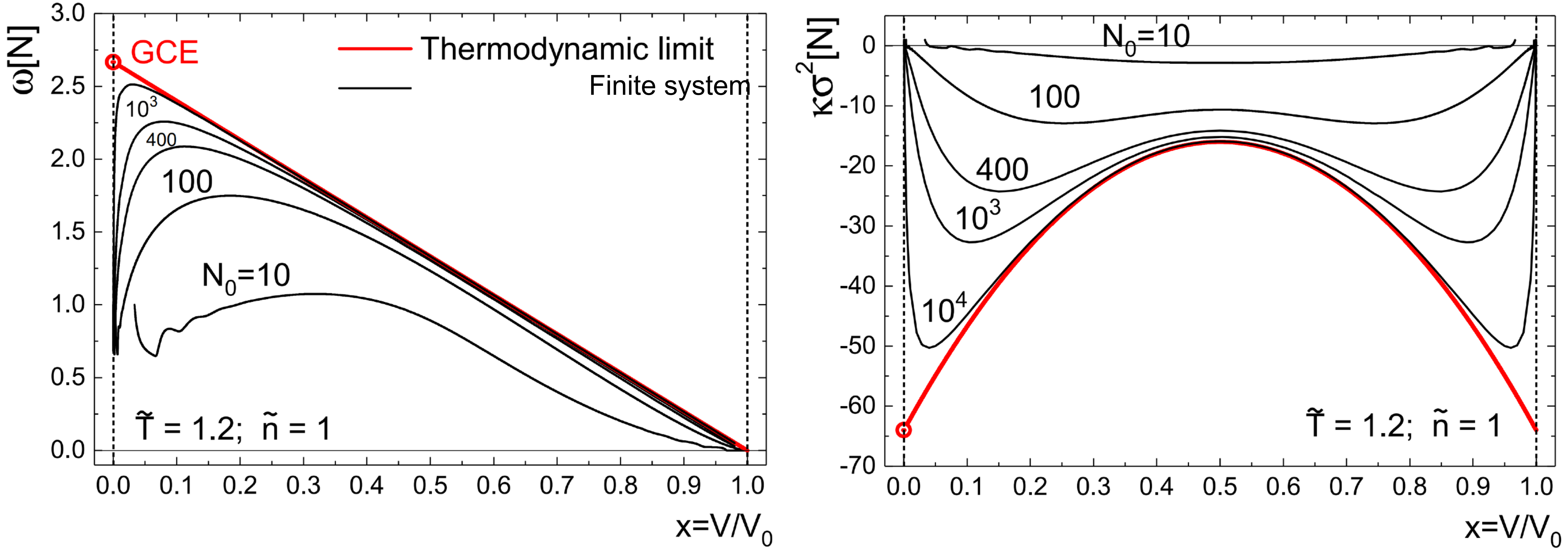
Calculate cumulants $\kappa_n[N]$ in a subvolume directly from the partition function

$$P(N) \propto Z^{\rm ce}_{\rm vdW}(T, xV_0, N)\, Z^{\rm ce}_{\rm vdW}(T, (1-x)V_0, N_0 - N)$$

and compare with the subensemble acceptance results



Results agree with subsensemble acceptance in thermodynamic limit ($N_0 \to \infty$)

Finite size effects are strong near the critical point: a consequence of large correlation length ξ

R. Poberezhnyuk, O. Savchuk, et al., 2004.14358

Binomial acceptance vs actual acceptance

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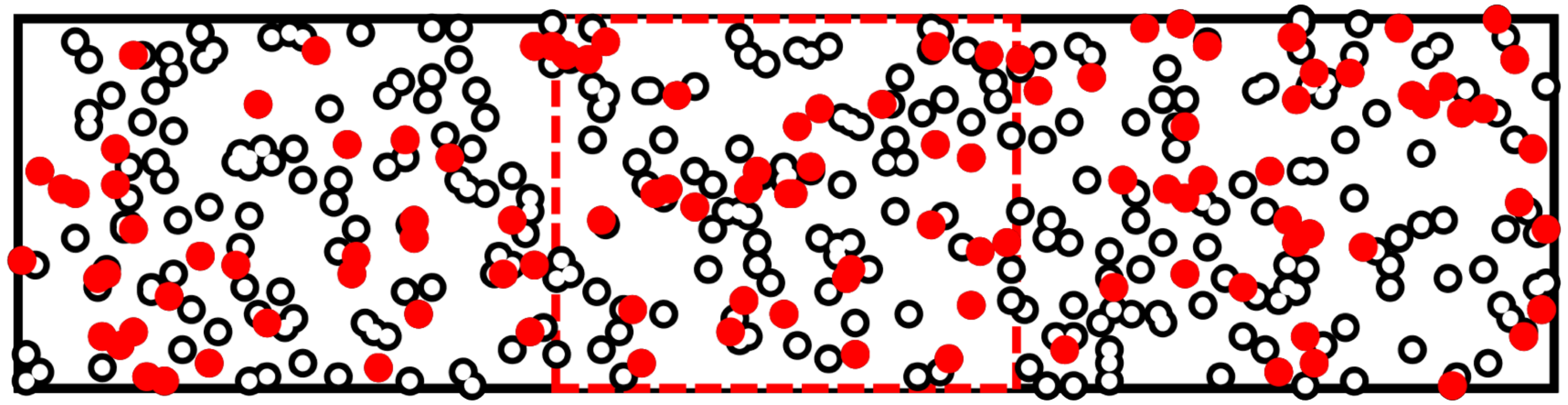
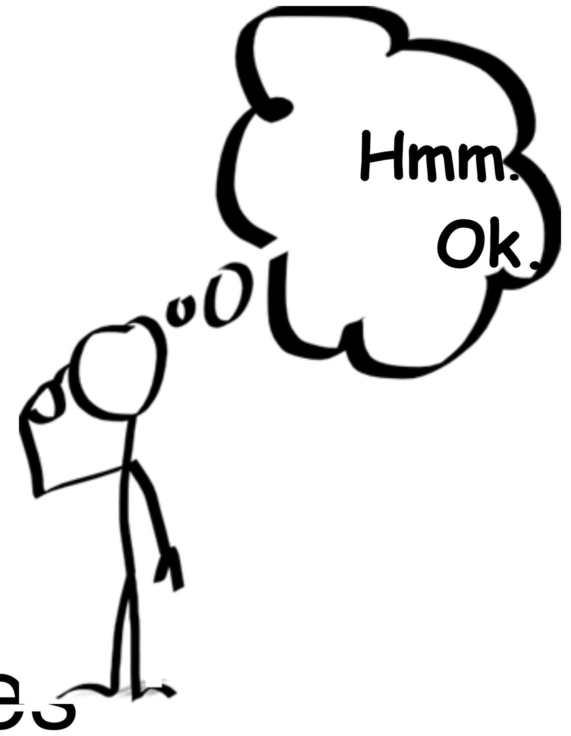
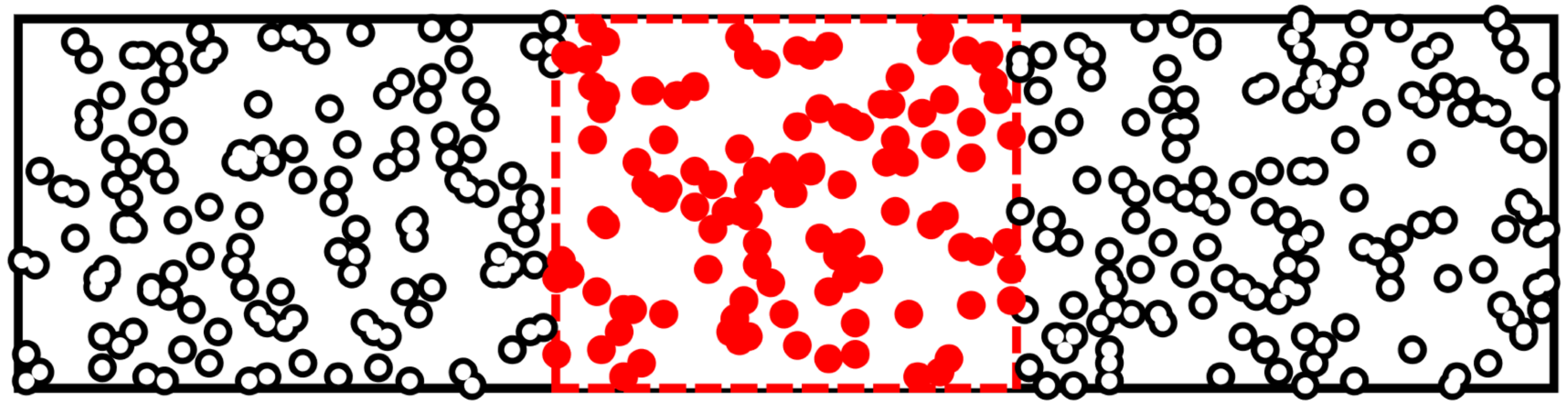




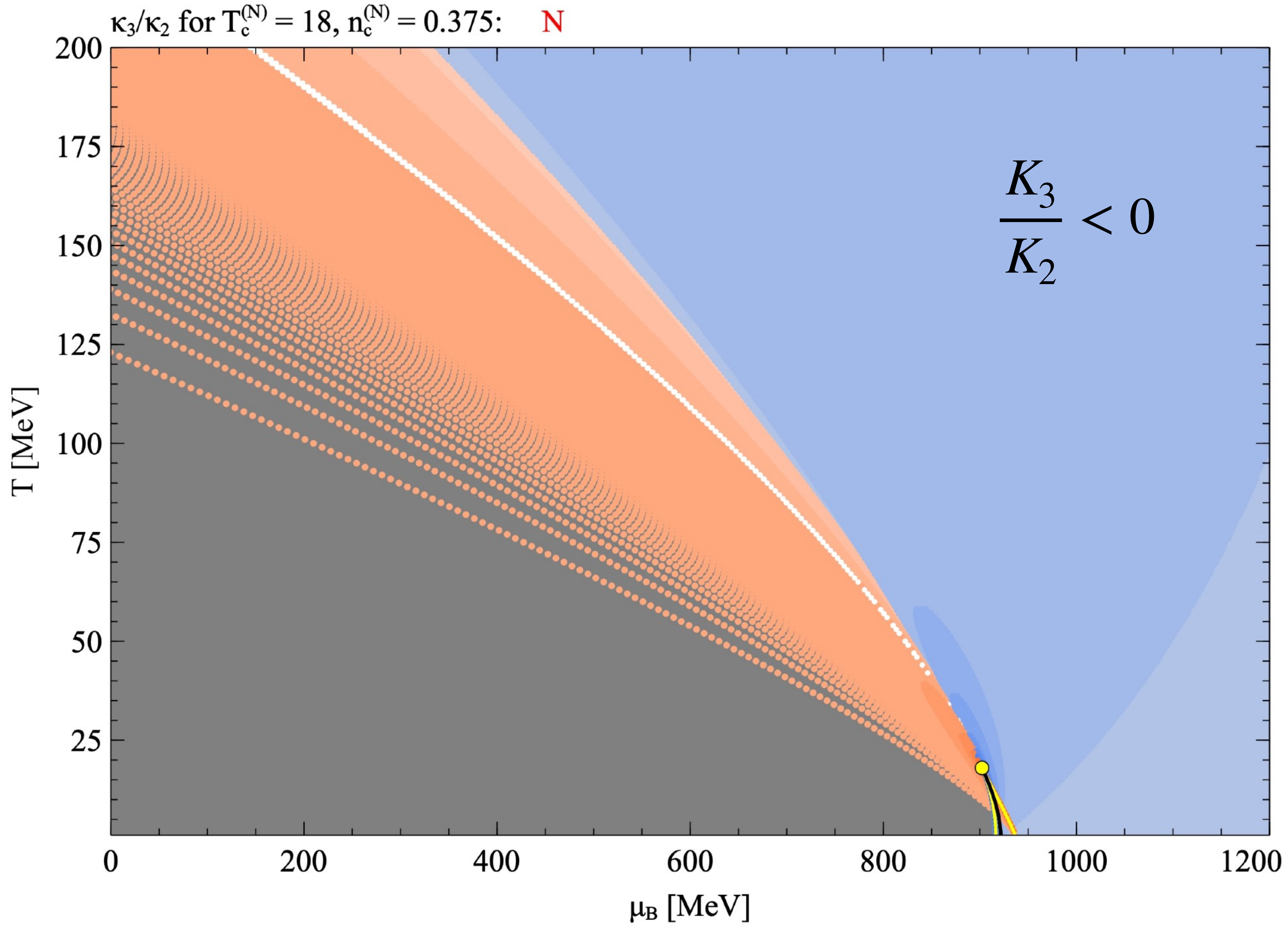
Binomial acceptance: accept each particle (charge) with probability α independently from all other particles

The binomial acceptance will not provide the correct result (except for a gas of uncorrelated particles)

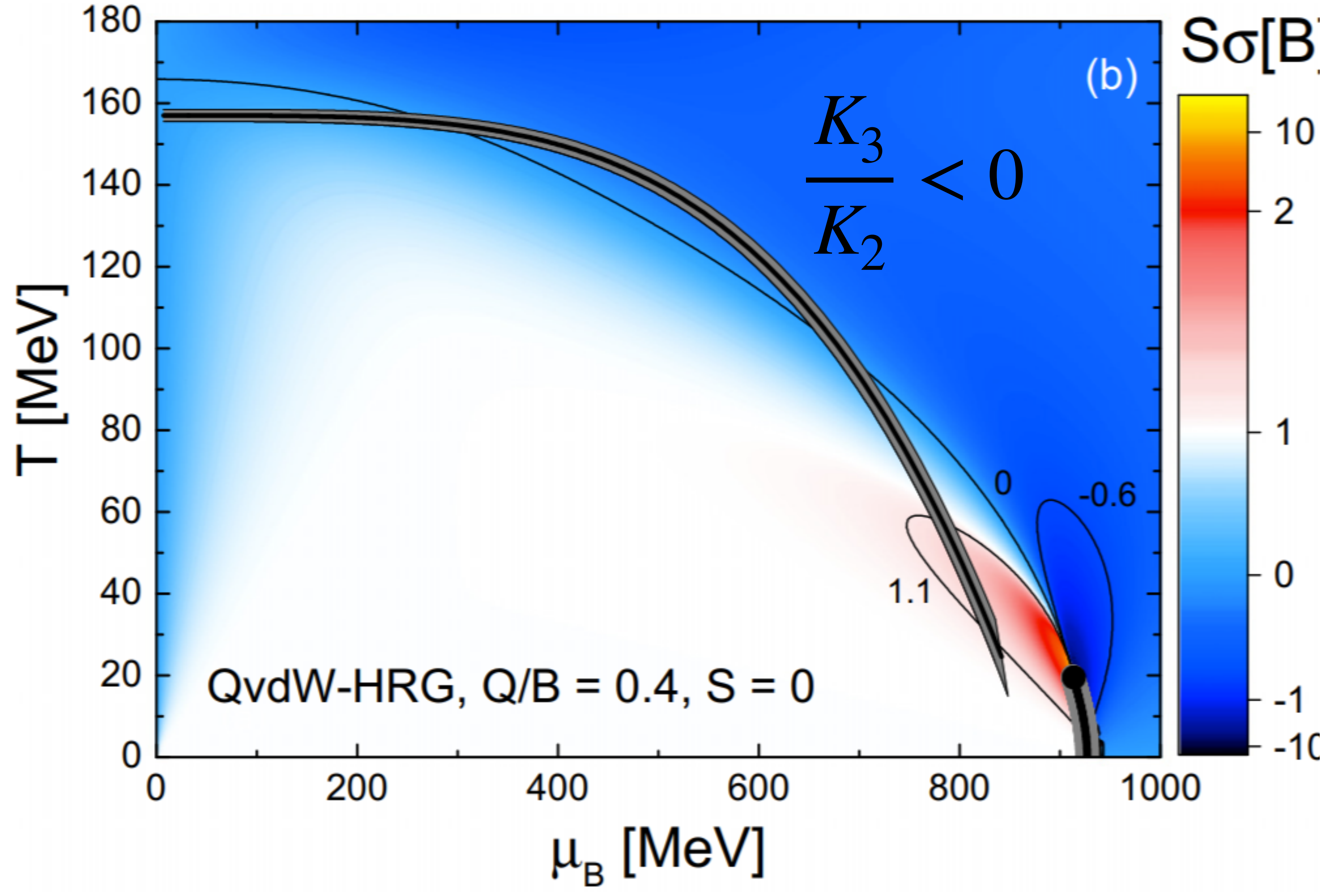
What we really need is



No QCD phase transition



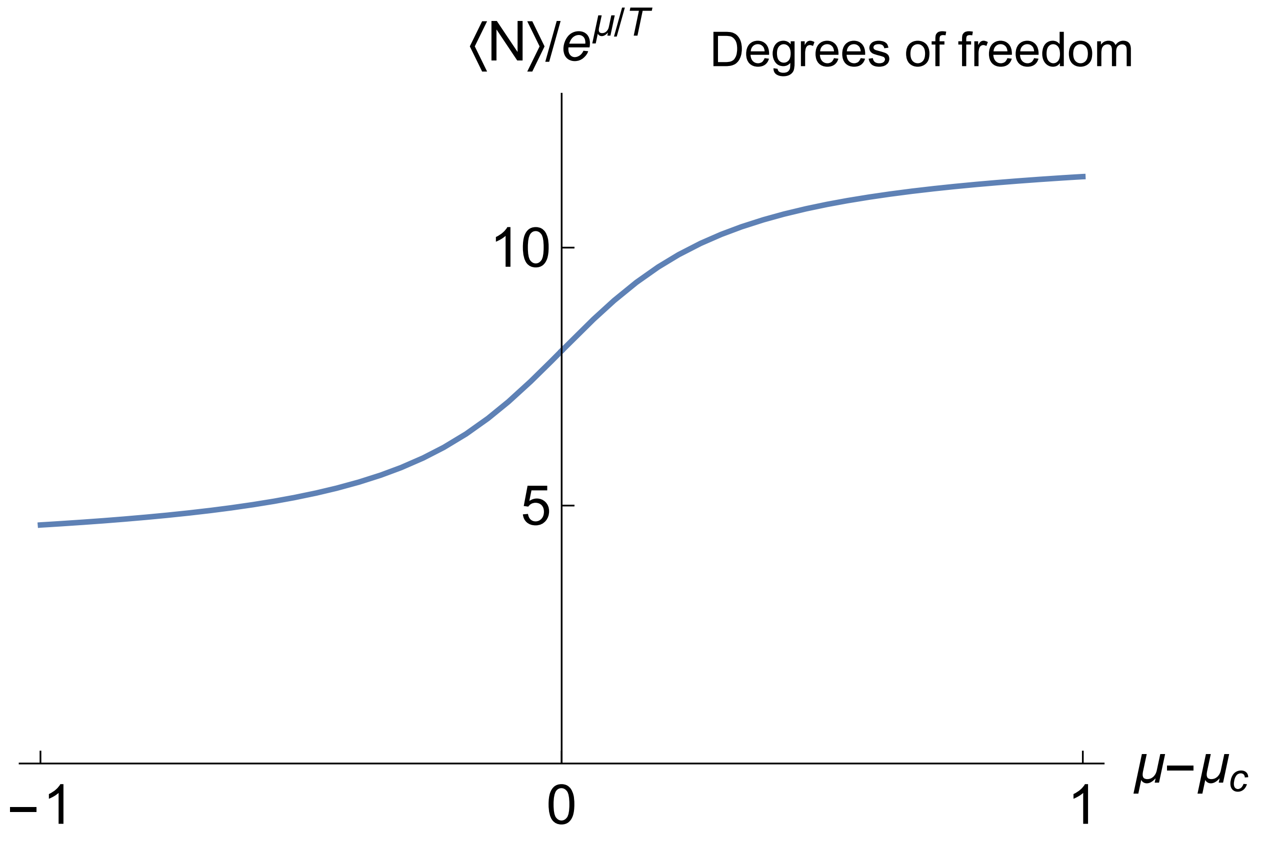
Model by A. Sorensen



V. Vovchenko et al, 1906.01954

Cumulants of (baryon) number distribution

$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

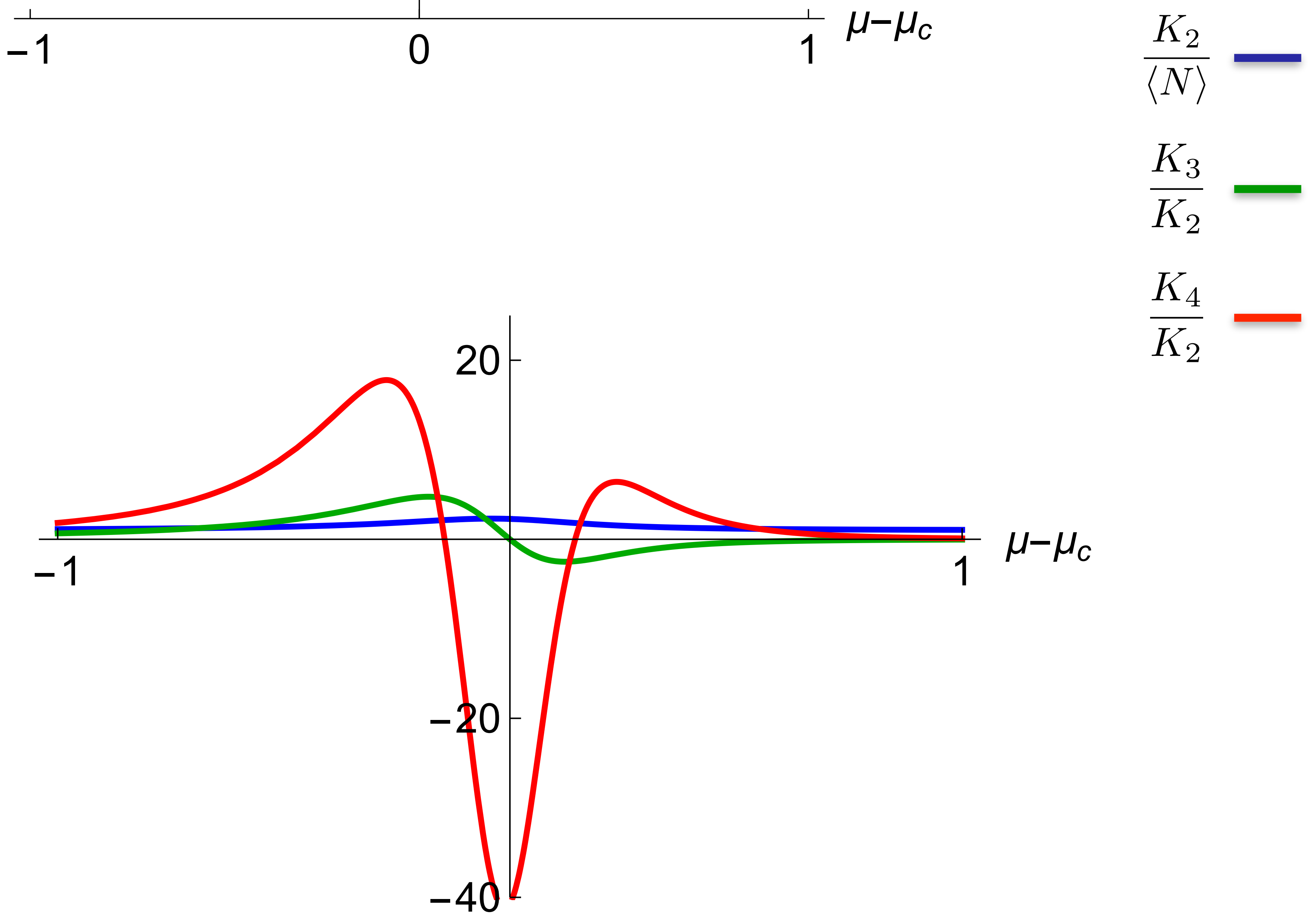


$$K_1 = \langle N \rangle\,, \quad K_2 = \langle N - \langle N \rangle \rangle^2\,, \quad K_3 = \langle N - \langle N \rangle \rangle^3$$

Cumulants scale with volume (extensive): $K_n \sim V$

Volume not well controlled in heavy ion collisions

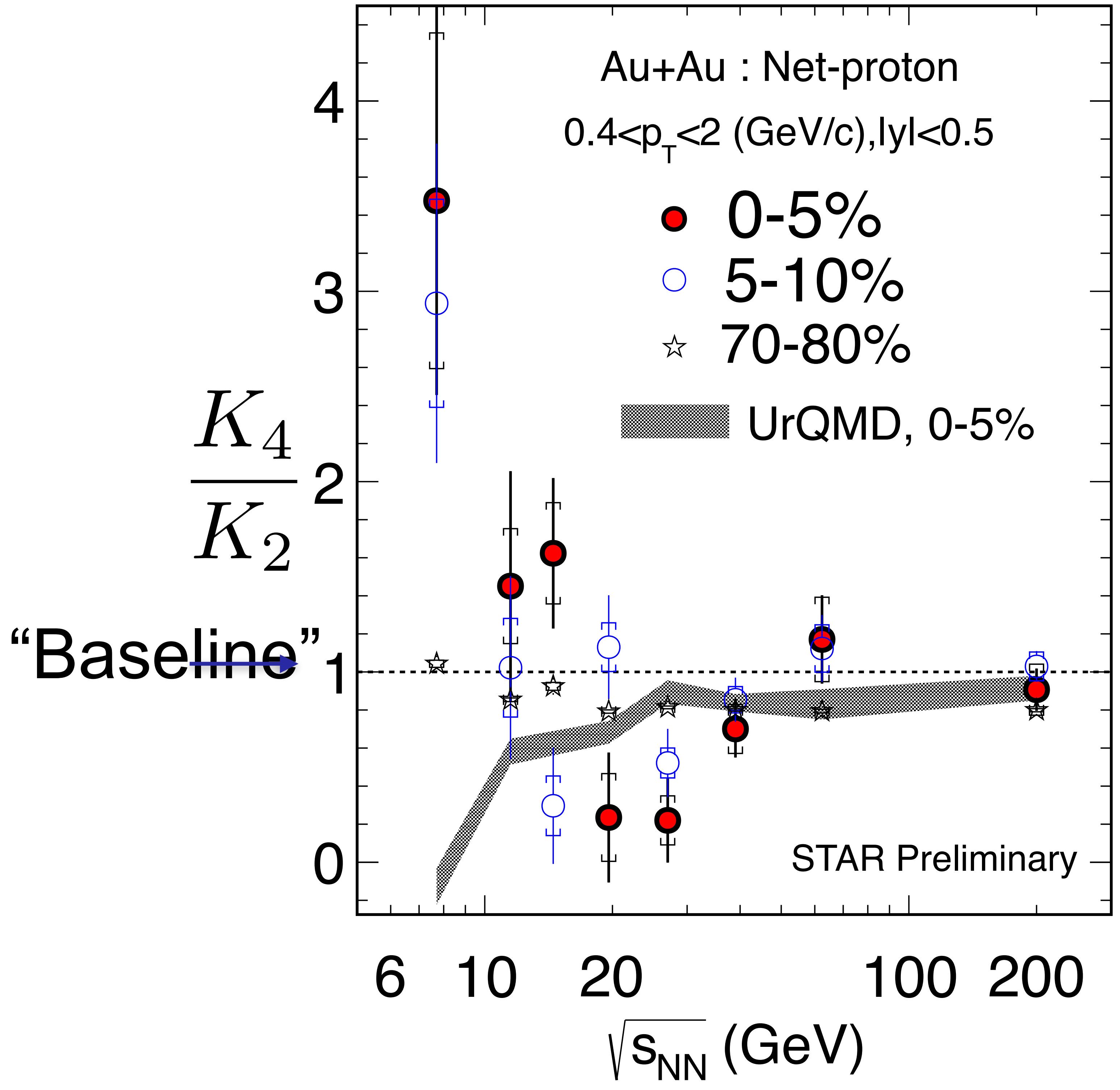
Cumulant Ratios: $\frac{K_2}{\langle N \rangle}, \quad \frac{K_3}{K_2}, \quad \frac{K_4}{K_2}$

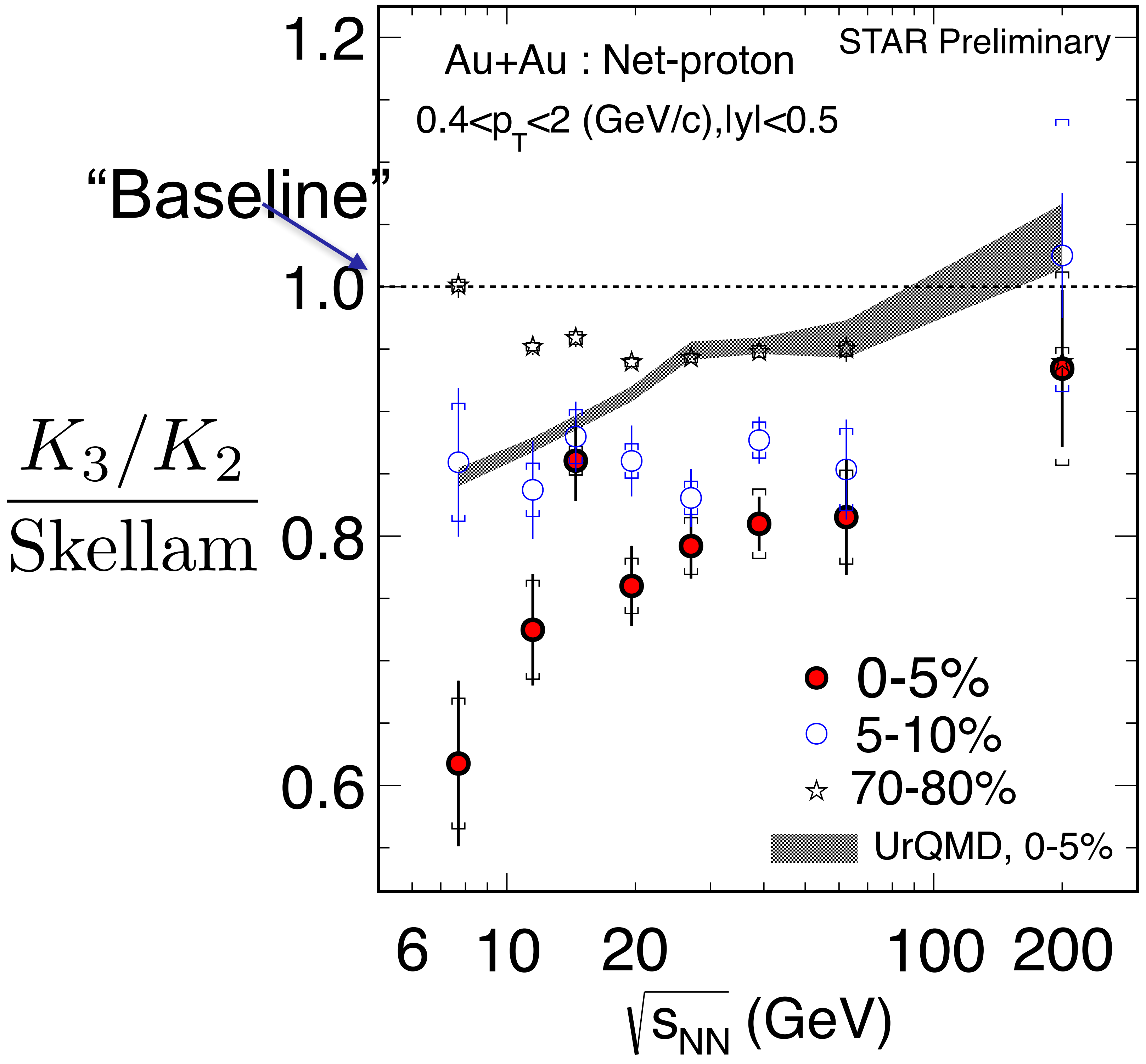


Baryon number cumulants measure derivatives of the EOS w.r.t chemical potential

Latest STAR result on net-proton cumulants

X. Luo, NPA 956 (2016) 75



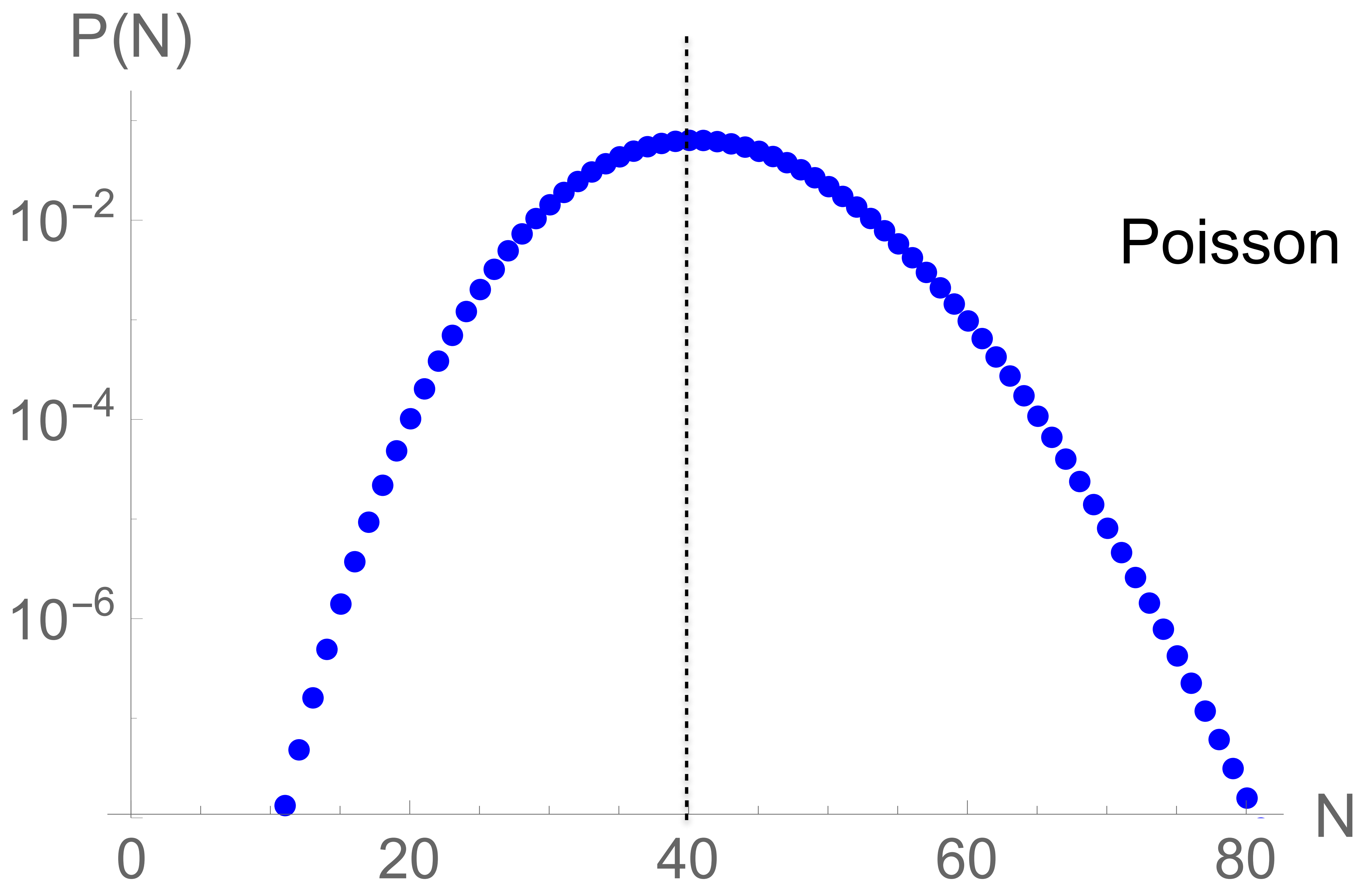


K_4/K_2 above baseline K_3/K_2 below baseline

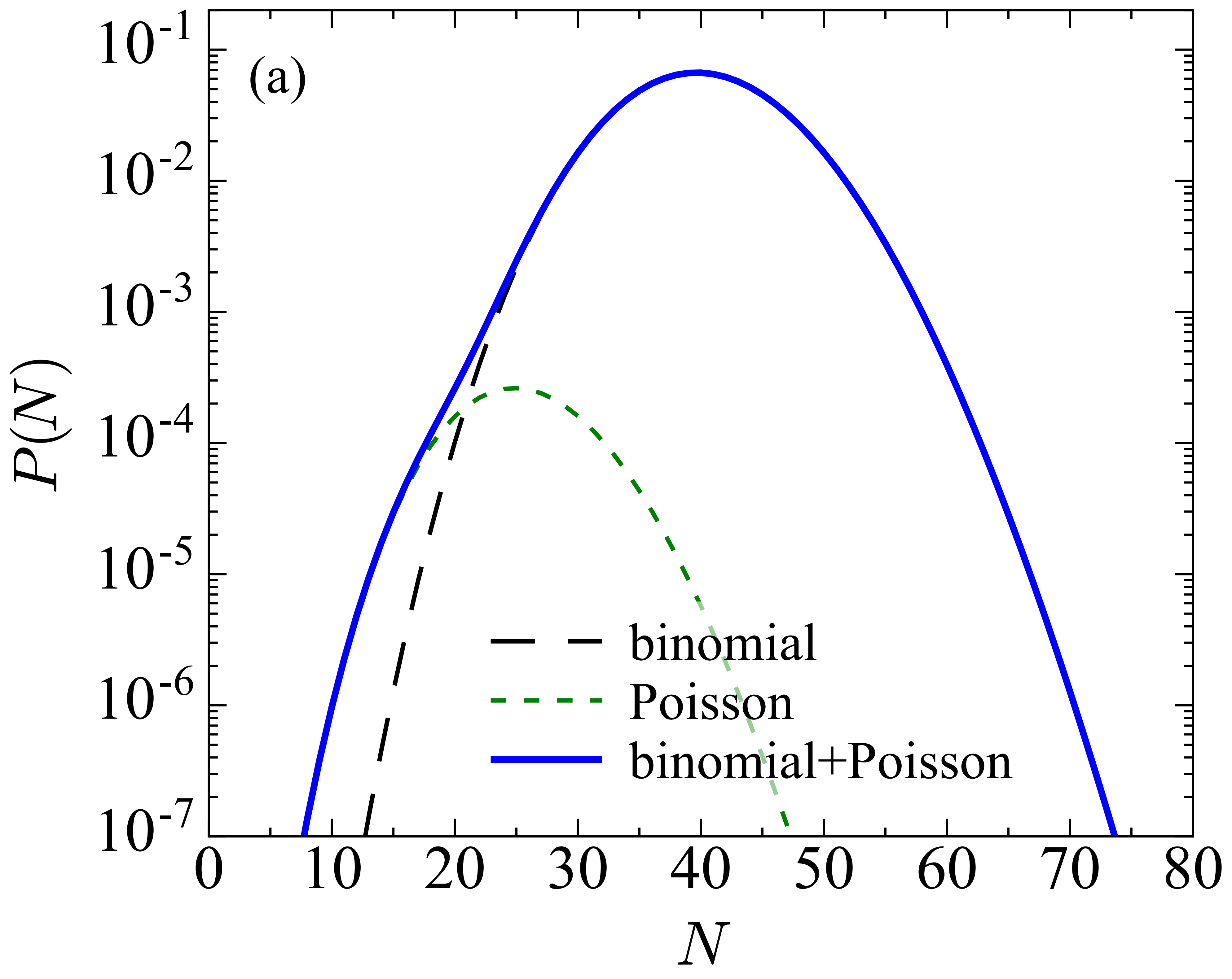
Shape of probability distribution

$$K_3 < \langle N \rangle \qquad K_3 = \langle N - \langle N \rangle \rangle^3$$

$$K_4 > \langle N \rangle \qquad K_4 = \langle N - \langle N \rangle \rangle^4 - 3 \langle N - \langle N \rangle \rangle^2$$

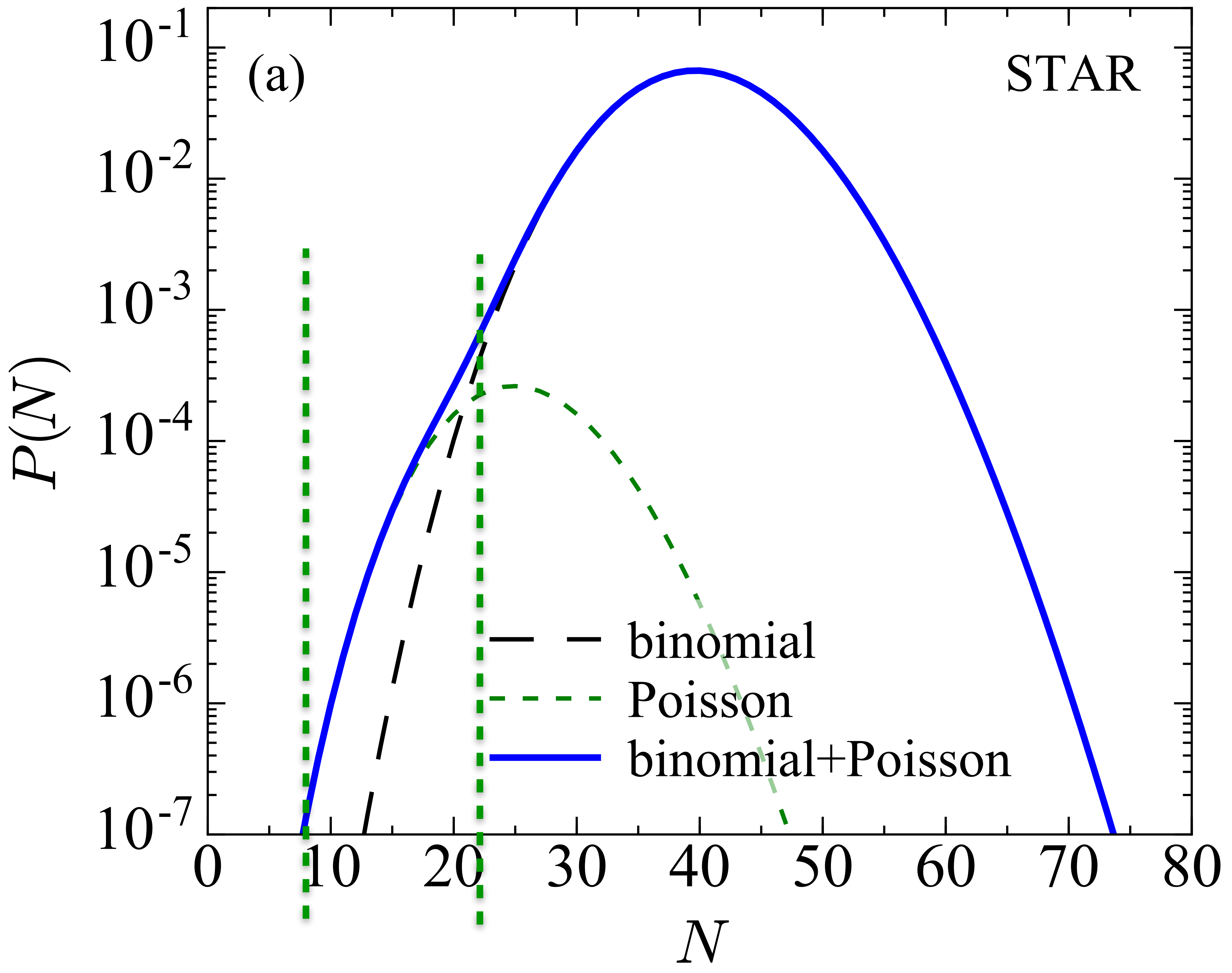


Simple two component model



Weight of small component: ~0.3%

Simple two component model



Analyse data for N_p <20

- Is flow etc different?
- “Inspect by eye (<1% of all events)

Two component model

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

$$\bar{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle > 0$$

For $P_{(a)}$, $P_{(b)}$ Poisson, or (to good approximation) Binomial

$$C_n = (-1)^n K_n^B \bar{N}^n \quad n \geq 2$$

C_n : Factoral cumulant

K_n^B : Cumulant of Bernoulli distribution

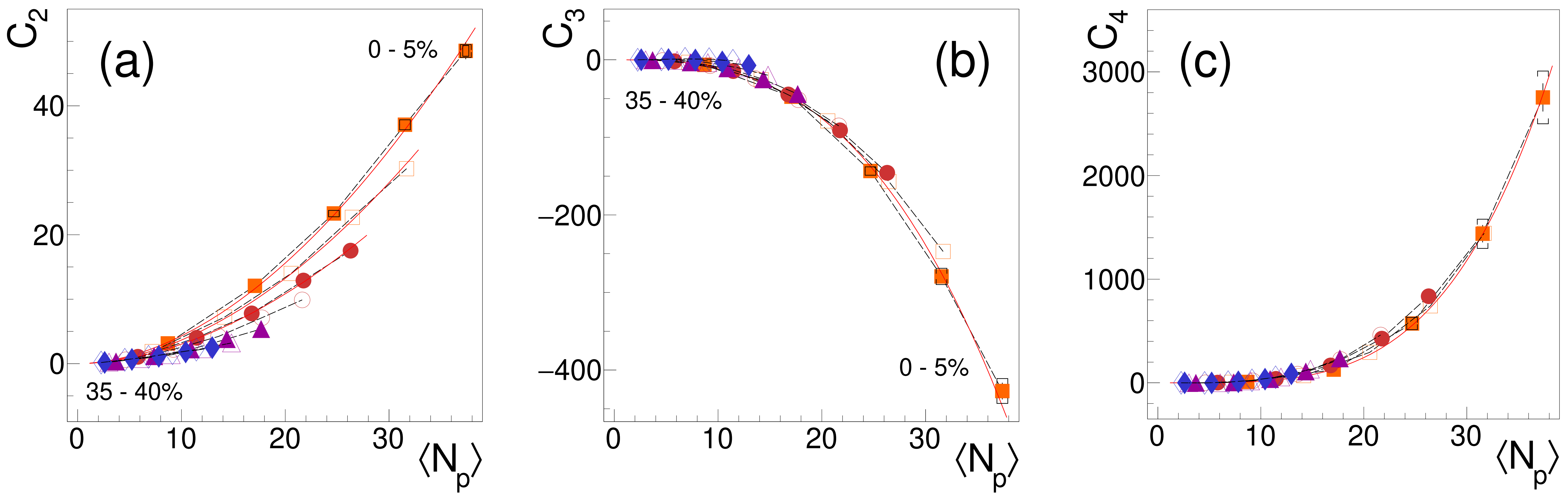
$$\alpha \ll 1, K_n^B = \alpha \;\Rightarrow\; C_n \simeq \alpha(-1)^n \bar{N}^n$$

$\Rightarrow |C_n| \sim \langle N \rangle^n$ as seen by STAR (i.e. "infinite" correlation length)

predict: $$\frac{C_4}{C_3} = \frac{C_5}{C_4} = \frac{C_{n+1}}{C_n} = -\bar{N}$$ $\bar{N} \simeq 15$

Clear and falsifiable prediction: $C_5 \approx -2650 \quad C_6 \approx 41000$

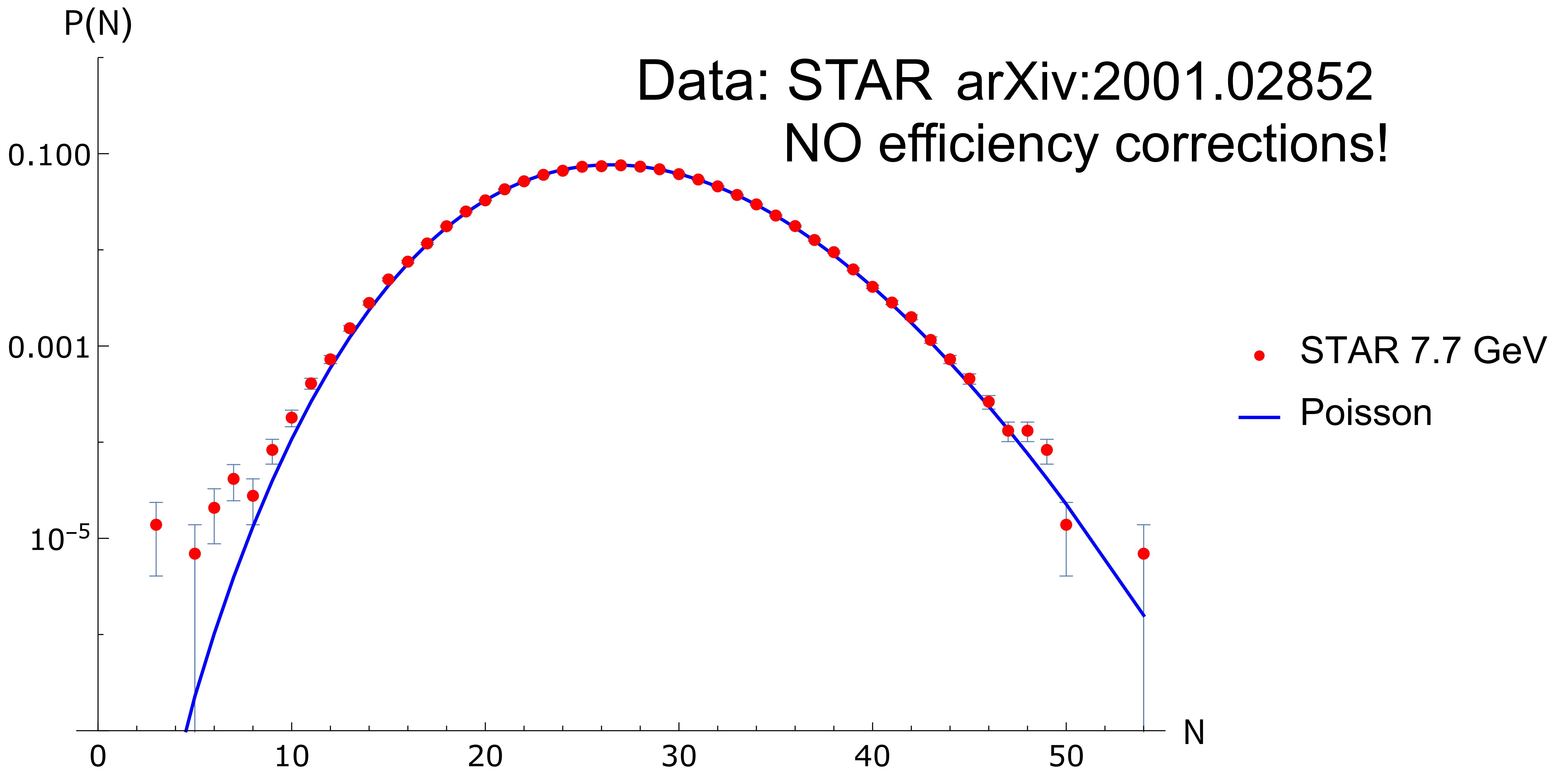
Hades see similar trend
(arXiv:2002.08701)



$$\frac{C_{n+1}}{C_n} \simeq -10$$

Caveat: rather significant N_{part} fluctuations to be corrected for

Multiplicity distribution @ 7.7 GeV



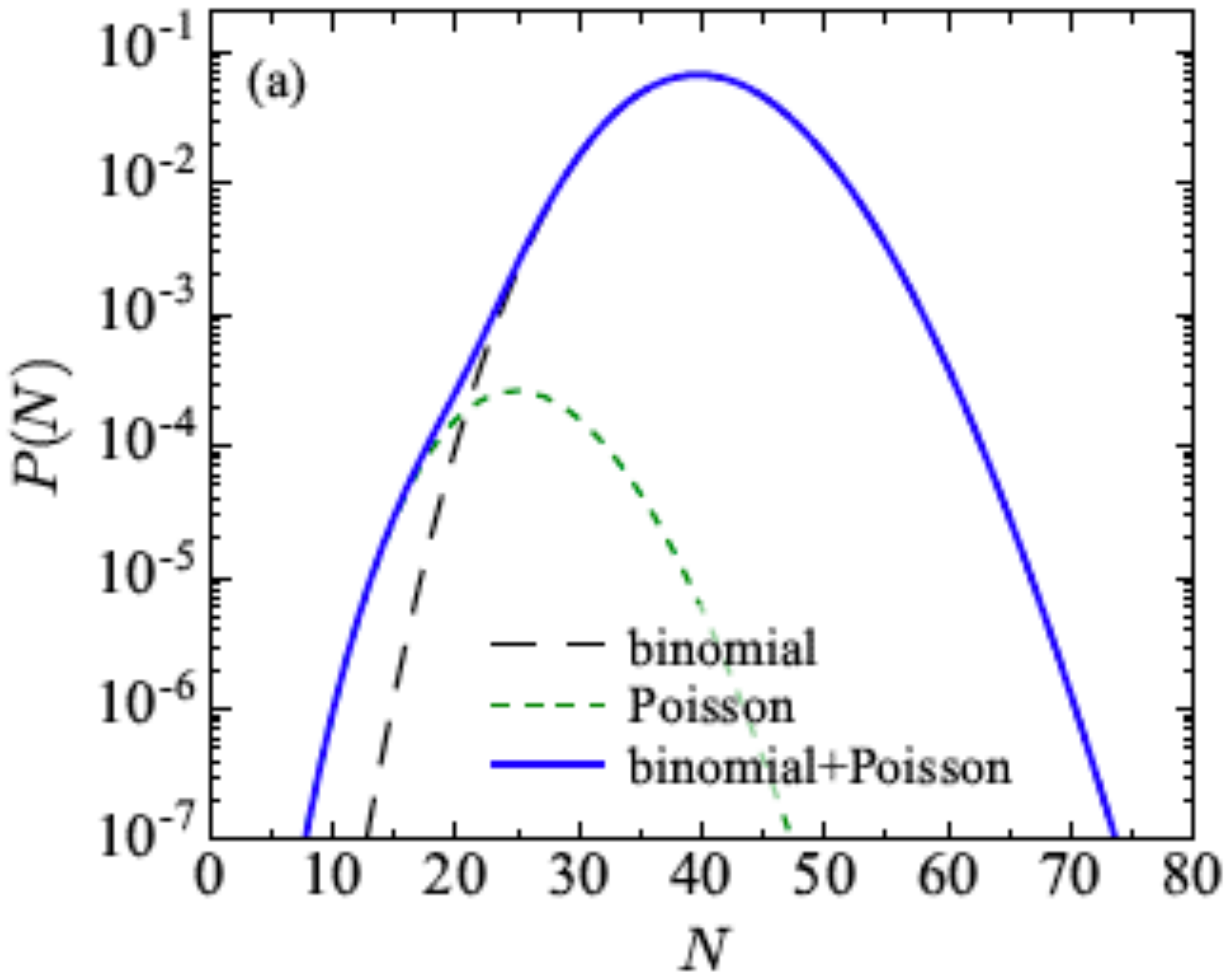
Data: STAR arXiv:2001.02852
NO efficiency corrections!

Now we need to figure out what this means….

First question: How does it look in the revised data?

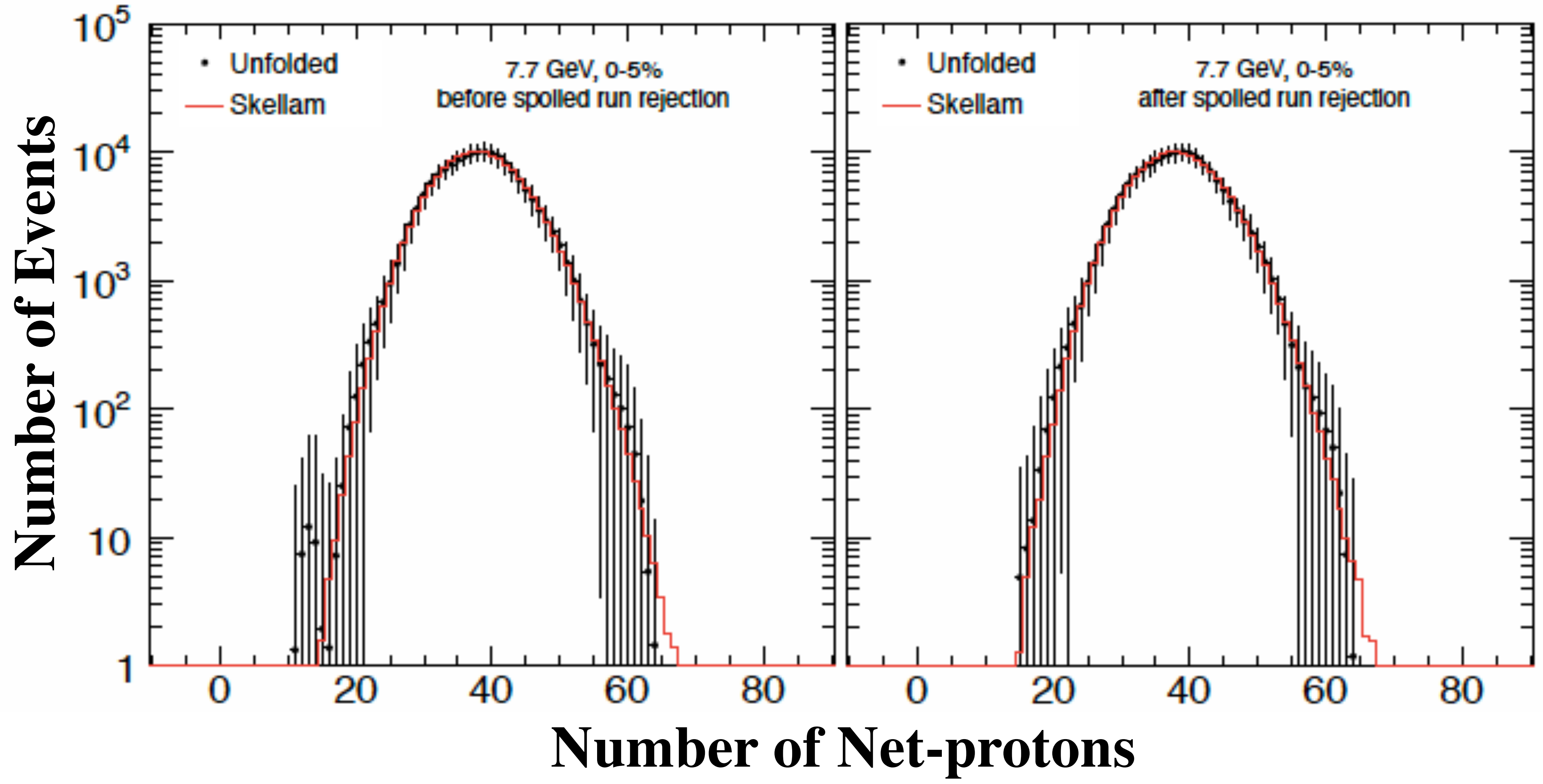
- STAR: arXiv: 2001.02852
- A. Bzdak, V. Koch, D. Oliinychenkov, and J. Steinheimer, Phys. Rev. **C98**, 054901(2018).

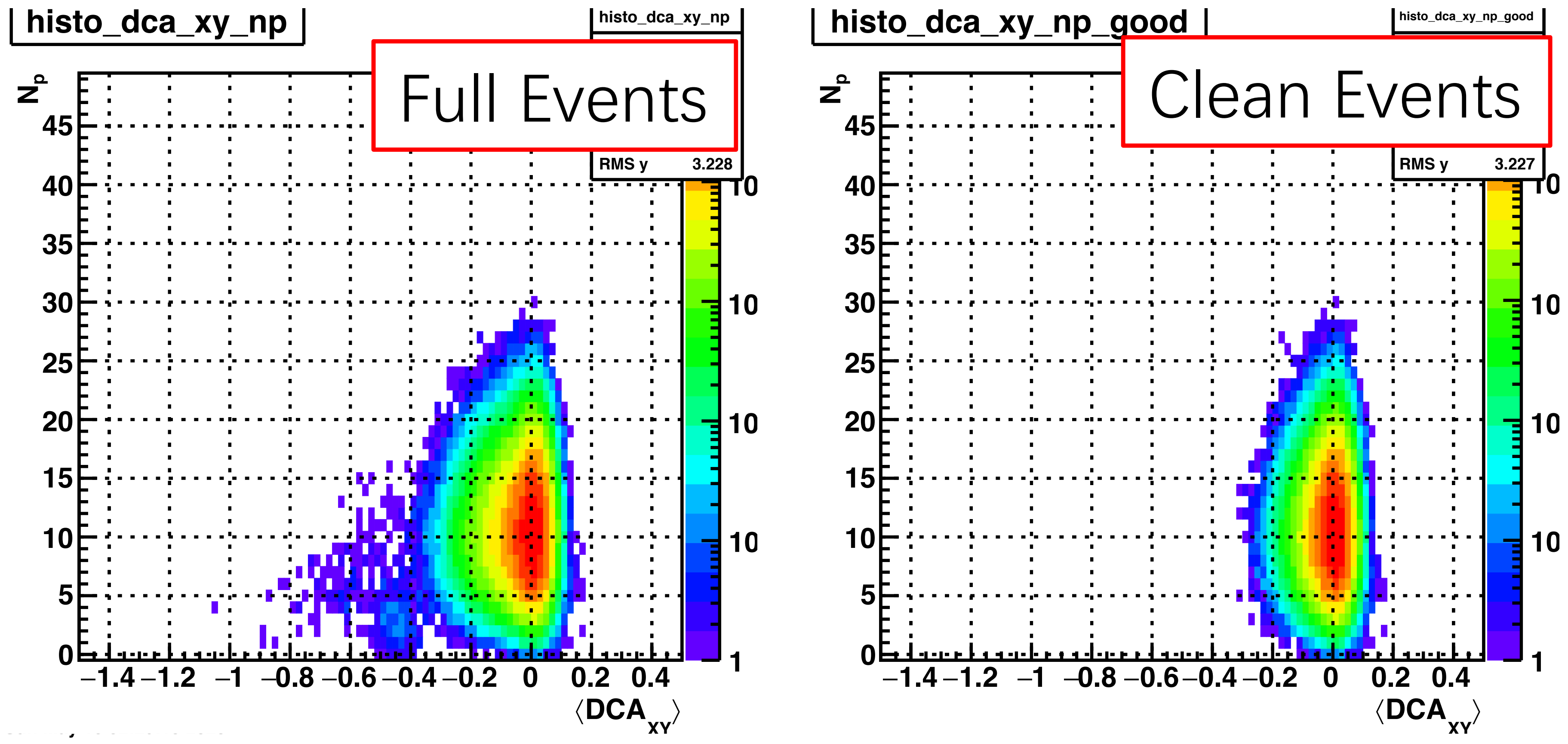
"Phase Boundary" vs. Spoiled Events



Given the fit, we can also predict the factorial cumulants, C_2, C_5, C_6 and we obtain:

$$C_2 \approx -3.85,$$
$$C_5 \approx -2645,$$
$$C_6 \approx 40900,$$





- For the 7.7 GeV collisions, after cleaning up the spoiled events, the 2nd bump is gone, C5 becomes close to zero;
- We made scan of the DCA_{XY} vs. run number for all collisions. All systematic uncertainties are also re-evaluated