Calculation of the axion mass based on high-temperature lattice quantum chromodynamics

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Phys. Lett. B752 (2016) 175; quenched results
Nature 539 (2016) 69; dynamical case

Outline

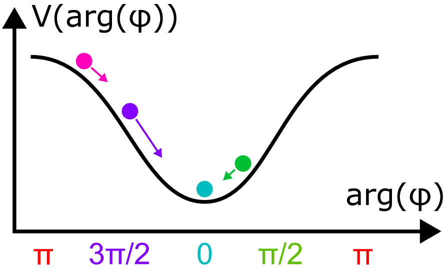
Strong CP problem

- CP is violated in the Standard Model
- QCD may have a CP violating $\frac{\Theta}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$ term
- Experiments constrain $\Theta \lesssim 10^{-10}$
- Possible solutions
  - $m_u = 0$ (ruled out by lattice results)
  - Axion
- Axion: promote Θ to a dynamical field
- Elegant solution: Peccei-Quinn mechanism
  Complex scalar field with a $U(1)$ symmetric “Mexican hat”
- Spontaneous symmetry breaking
  $\rightarrow$ axion is the (pseudo) Goldstone-boson $m_A^2 = \chi_t / f_A^2$

Misalignment

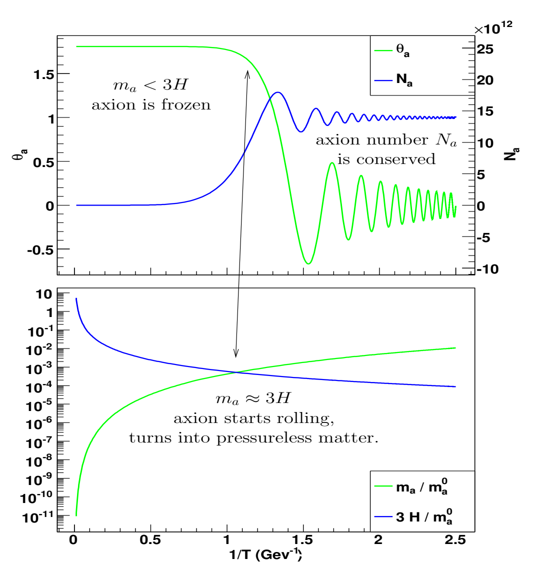
- alignment of misaligned neighbouring patches
  $\rightarrow$ axion radiation
- when χ_t becomes relevant, θ_{eff} "rolls" down to $\theta = 0$
  $\rightarrow$ axion radiation

χ_t influences field dynamics, needed as input for total axion production



Axion: rolldown mechanism

Evolution in the expanding universe



Cosmological Models

Both production mechanisms

- depend on χ_t
- depend on the dynamics over cosmological time scales

$\Rightarrow$ need $\chi_t(t)$ over cosmological time scales

- χ_t is temperature dependent (not explicitly on time)
- the equation of state of QCD gives $T(t)$ for cosmology

$\Rightarrow$ need $\chi_t(T)$, $p(T)$ for cosmologically relevant temperatures
as we will see T up to few GeV is required: lattice QCD

Quenched Study

How far can we go with conventional brute force?

$\rightarrow$ test it in the "cheap" quenched case

- learn how to control all errors and apply it for full QCD
- test bed to improve on the brute force strategy
- roughly the same temperature scaling as for full QCD
- estimate the costs for the full result

Quenched Lattice ↔ DIGA

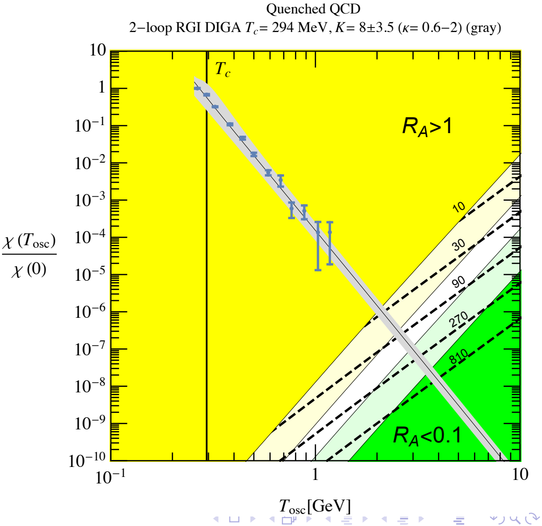
correct T dependence
normalization off by $\mathcal{O}(10)$
fixed by comparison to lattice

how $\chi_t(T)$ determines m_A?
start with an m_A e.g. 30μeV
m_A(T=0) gives the value of f_A

known: Hubble constant H(T)
fix T_{osc} by
$3H(T_{\rm osc}) = m_A(T_{\rm osc})$

using T_{osc} calculate
the amount of dark matter

if it is too much/little iterate



About costs: quenched case from T=0 (or T_c) to $4T_c$

Cost of the conventional algorithm at relative error $\delta\chi_t$

$$\text{costs} \propto \frac{1}{(\delta\chi_t)^2 \chi_t(T)}$$

relative cost $(4T_c)/(1T_c)$ (our highest T was $4T_c$: not enough)

from measured $\chi_t(T)$	$4^{7.1} \approx 2 \times 10^4$
from measured $\delta\chi_t$	$10^5 - 10^6$

- quenched $\chi_t(T=0)$ calculated $\sim$ 20 years ago
- Moores law leads to a factor of $\sim 10^5$ in 24 years

$\Rightarrow$ Just possible to do (dynamical case is probably hard)

About costs: dynamical QCD

Dynamic relative cost $\$(7T_c)/\$(1T_c)$ ($7T_c \sim 1200 MeV$)

from estimated $\chi_t(T)$	$7^{7-8} \approx 10^6 - 10^7$
increasing τ_{int} with T	$10^7 - 10^9$

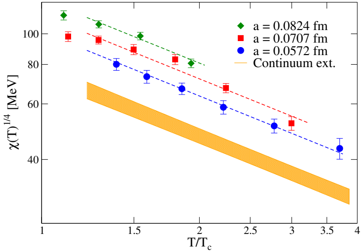
- dynamic $\chi_t(T=0)$ in 2010, Moore factor of ~ 30

$\Rightarrow$ conventional dynamical study not possible (needs 35 years)

Literature: full QCD

C.Bonati, M.d'Elia, G.Martinelli et al. JHEP 229 03, 155 (2016)

- brute force fully dynamic in the continuum up to $\approx 4T_c$



Result: $b \sim 3$ unexpected (DIGA etc. $b \sim 8$)
for T$>$2 GeV is larger than DIGA by 7-8 orders of magnitude
one order of magnitude shift for the axion dark matter window

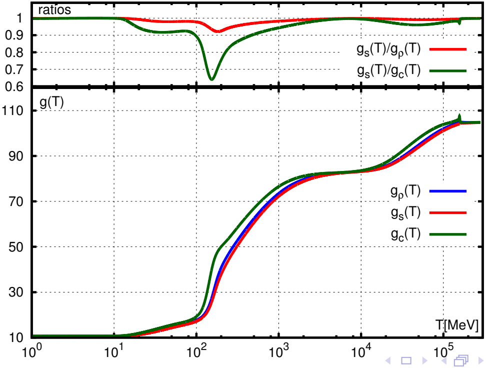
crosses quenched result at $4T_c$ (for quenched $\chi_t^{1/4}(4T_c)$=17 MeV)

$\Rightarrow$ further study is obviously necessary

The equation of state

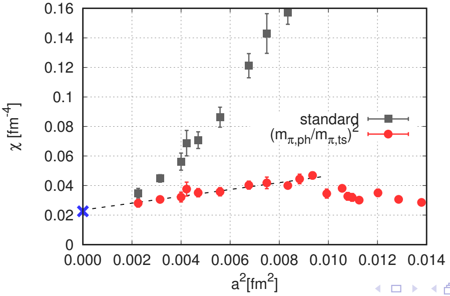
Effective number of degrees of freedom including all SM particles

$$\rho = \frac{\pi^2}{30} g_\rho T^4 \qquad s = \frac{2\pi^2}{45} g_s T^3 \qquad c = \frac{2\pi^2}{15} g_c T^3$$

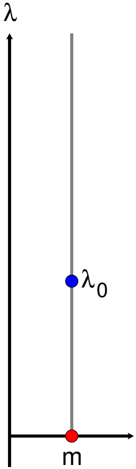


The challenge of computing the susceptibility

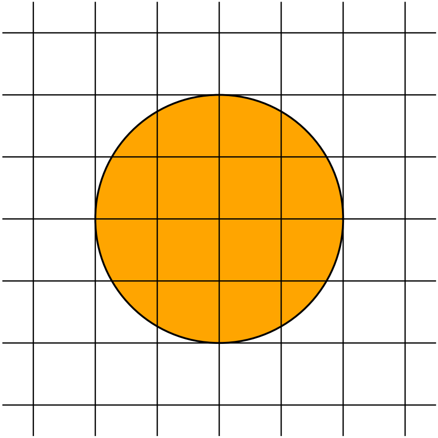
- large autocorrelation of Q on fine lattices (algorithmic problem)
- $\chi(T)$ decreases strongly with temperature
  $\rightarrow$ very few $Q \neq 0$ configurations (physical problem)
  E.g. $\langle Q^2 \rangle = 10^{-6}$ means one $Q = \pm 1$ configuration per million.
  Even $\mathcal{O}(\mathrm{million})$ configurations can lead to large statistical errors
- $\chi(T)$ has large lattice artefacts



T=0 instanton on the lattice: physical units



T=0 towards the continuum limit



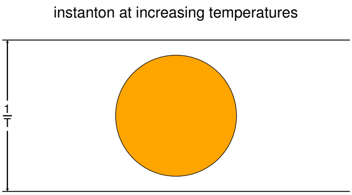
T=0 instanton on the lattice

Goal: compute QCD topological susceptibility $\chi(T)$

- Temperature range: $0 < T < 2\text{GeV}$
- Physical quark masses (m_u, m_d, m_s, m_c)
- Continuum limit
- Using
  - $N_f = 2 + 1 + 1$, with isospin splitting correction
  - staggered and overlap quarks
  - Lattices with $N_t = 8, 10, 12, 16, 20$

Physics to be captured

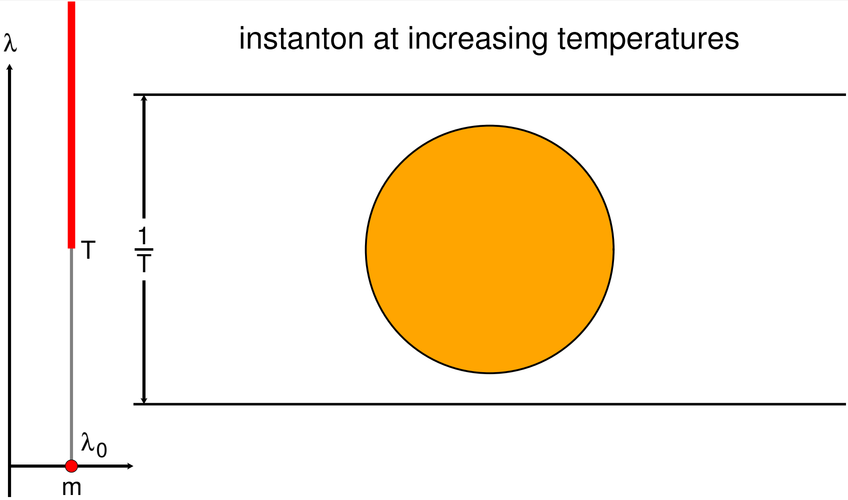
- Typical instanton size $1/T$



- Dilute gas of small ($r \approx 1/T$) instantons remain
- Zero modes in the light quark det suppress topology
- $\Rightarrow \quad \chi(T) \quad$ falls sharply above T_c

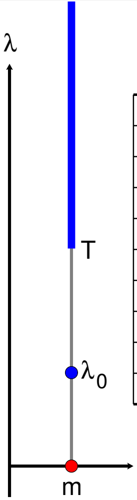
T>0 instanton in the continuum

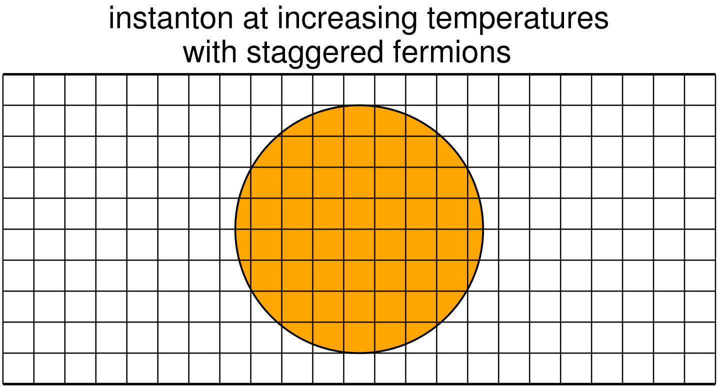
instanton at increasing temperatures



T>0 instanton in the continuum

T>0 instanton on the lattice: physical units





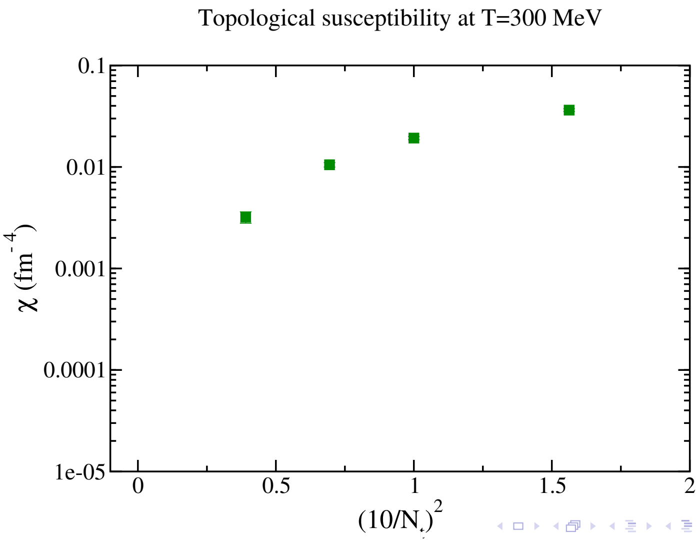
T>0 instanton on the lattice: physical units

Challenge #1: large cut-off effects

- Small-instanton zero modes badly captured by lattice Dirac operator
- Higher Q sectors not properly suppressed
- Cut-off effects much larger at higher T
- Solution: identify would-be zero eigenvalues and shift them to zero $\rightarrow$ reweighting

  $w[U] \sim \frac{m}{m+\lambda_0}$

Unusually large cut-off effects: N_f=2+1+1 with 4-stout



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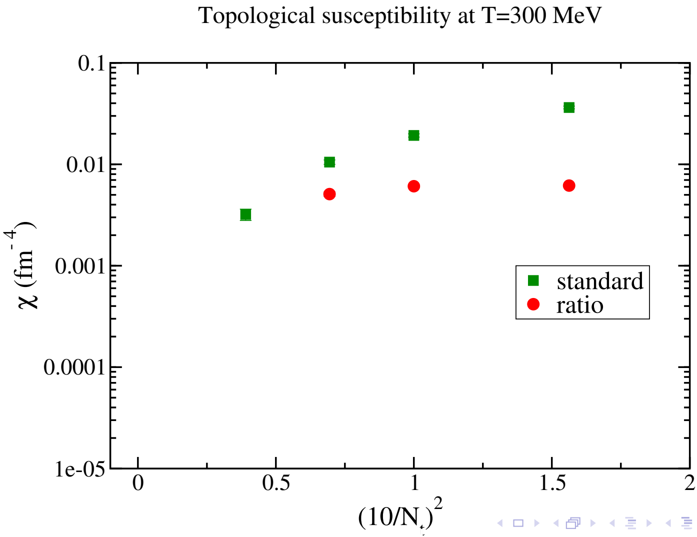

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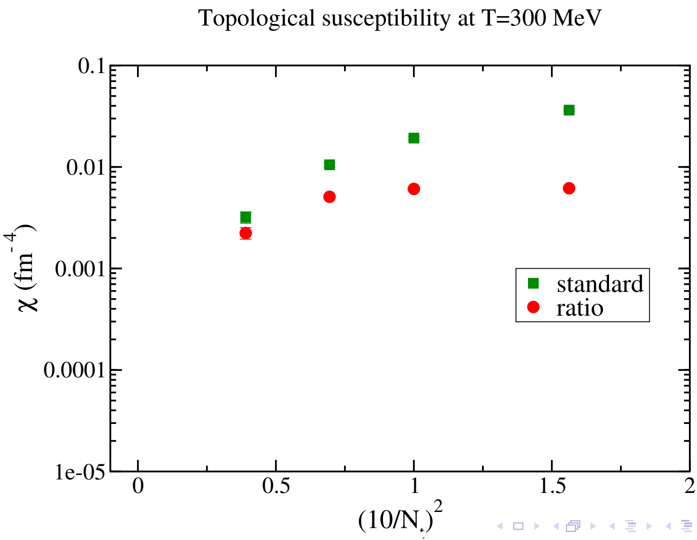

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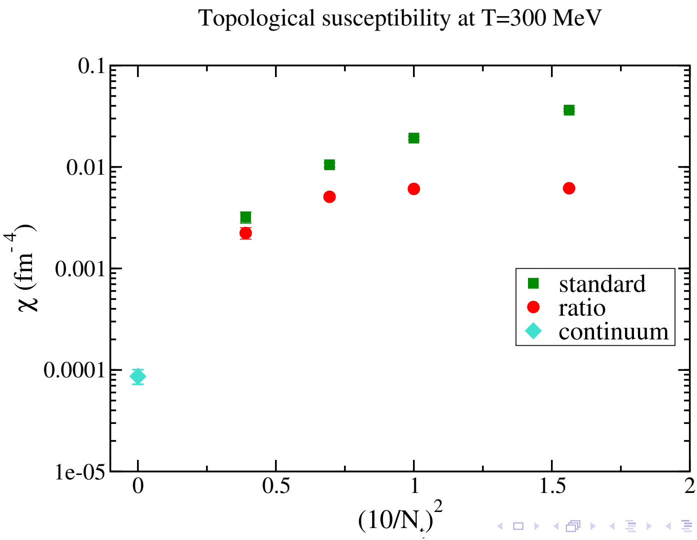


Reason for bad scaling

- Would-be zero eigenvalues too big
- Weight in det is

  Lattice: $\lambda_0 + m_f$

  Instead of continuum: m_f

- Even if $a \propto 1/T$ (fix N_t, increase β)

  λ_0/m increases with T

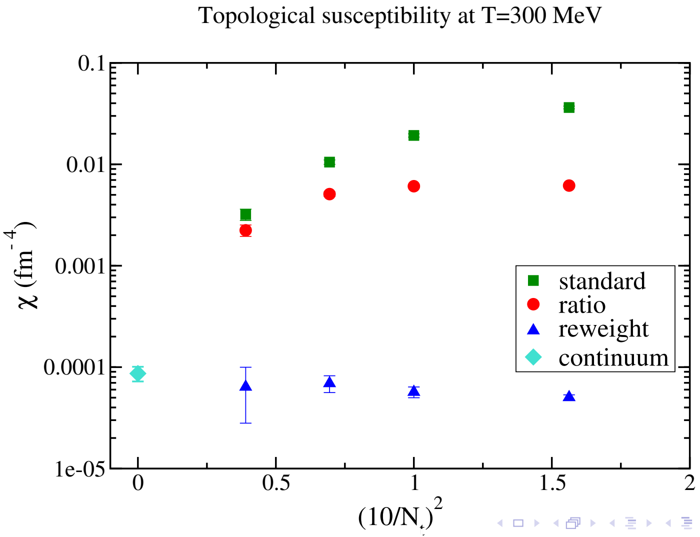
Reweighting

Strong cut-off effects are related to the lack of exact zero-modes.

- In the continuum non-trivial sectors are suppressed by the contribution of zero-modes to the fermion determinant, ie. by the quark mass.
- On the lattice the suppression is altered: $m \to m + \lambda_0$, where λ_0 is a would be zero-mode. Weaker suppression $\to \chi(T)$ overestimated.
- To improve 1. identify would be zero-modes
  2. restore the continuum weight $\to$ reweight

$$w[U] \sim \frac{m}{m + \lambda_0}$$

T=300 MeV: susceptibility after reweighting



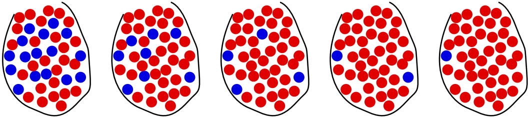
Challenge #2: tiny χ hard to measure

- No statistics for $Q \neq 0$ sectors (dictated by physics)
- Topology change slow on fine lattices (algorithmic)
- Solution:
  Derivative of $\chi(T)$ much easier to measure than χ
- Measure $\chi(T_0)$ at low enough T_0
- Using $d\chi/dT$ integrate up to T $\rightarrow$ integral method
  Also suggested for the quenched case by [Frison et al '16]

Determine topological susceptibility/axion potential

Challenge

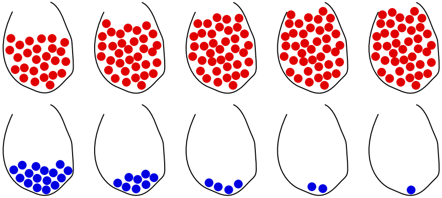
Determine the blue/red ratio by random pick!



$\longrightarrow$ getting very difficult with T $\longrightarrow$

Solution

Separate colors and determine the rate of change with T!



Fixed sector integral

Instead of waiting for tunneling events,
we make simulations in fixed Q sectors. How to get

$$Z_1/Z_0 = ?$$

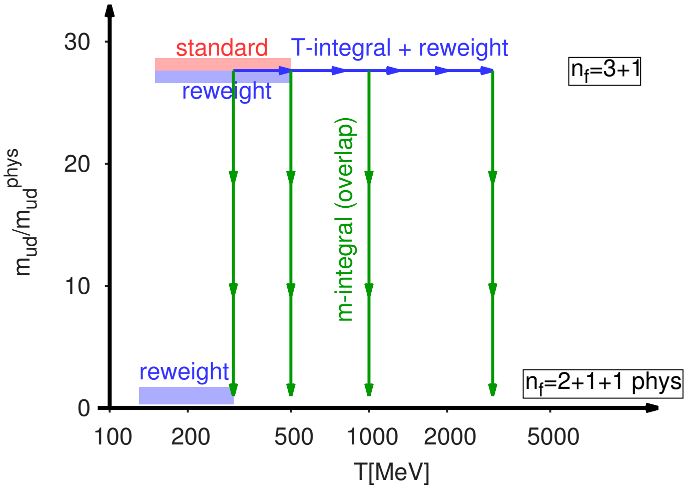
First calculate derivative of $\log Z_1/Z_0$:

$$b_1(T) \equiv \frac{d \log Z_1/Z_0}{d \log T}$$

Use fixed N_t-approach, ie. $T = (aN_t)^{-1}$ is changed by β:

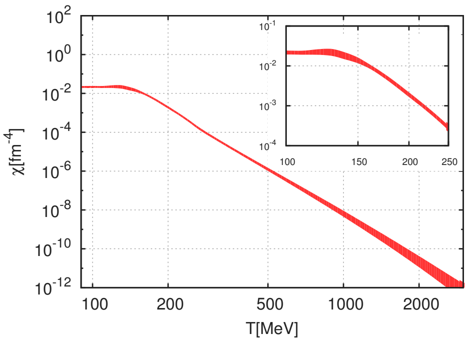
$$b_1(T) = \frac{d\beta}{d \log a} \left(\langle S_g \rangle_1 - \langle S_g \rangle_0 \right)$$

Map of simulations



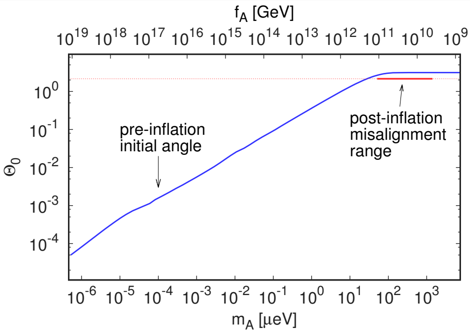
Topological susceptibility at the physical point

very few topology changes (hard): S. Borsanyi et al. Nature 539 (2016) 69



absolute lower limit (all DM from misalignment): $m_A \gtrsim 28(2)\ \mu\text{eV}$
assuming 50-99% other (e.g. strings): $m_A = 50 - 1500\ \mu\text{eV}$

Constraints on the axion mass



- Pre-inflation scenario: m_A unambiguously determines the Θ_0 initial condition of our Universe
- Post-inflation: Θ_0 average equivalent to $\Theta \approx 2.15$
  absolute lower limit (all DM from misalignment): $m_A \gtrsim 28(2)\ \mu\mathrm{eV}$
  assuming 50-99% other (e.g. strings): $m_A = 50 - 1500\ \mu\mathrm{eV}$

Summary: results

- Axion: a solution to a) strong CP b) dark matter problems
- Calculating axion production in the early universe requires the EoS and $\chi(T)$
- Brute force approach expensive: estimates using pure SU(3)
- Both were determined using lattice calculations up to high T
- Axion mass in the post-inflation scenario:
  lower bound: 28(2) μeV
  estimated mass range: $m_A = 50 - 1500\ \mu\text{eV}$

Summary: methods

Calculated T-dependence of the QCD topological susceptibility

- Temperature range: $0 \leq T \leq 2$ GeV
  (follow change of χ over 10 orders of magnitude)
- Physical quark masses
- Continuum limit

Main lesson: keep in mind the physics of the problem

- Large cut-off effects due to instanton zero-modes
- At high T : tiny χ $\rightarrow$ ideal instanton gas