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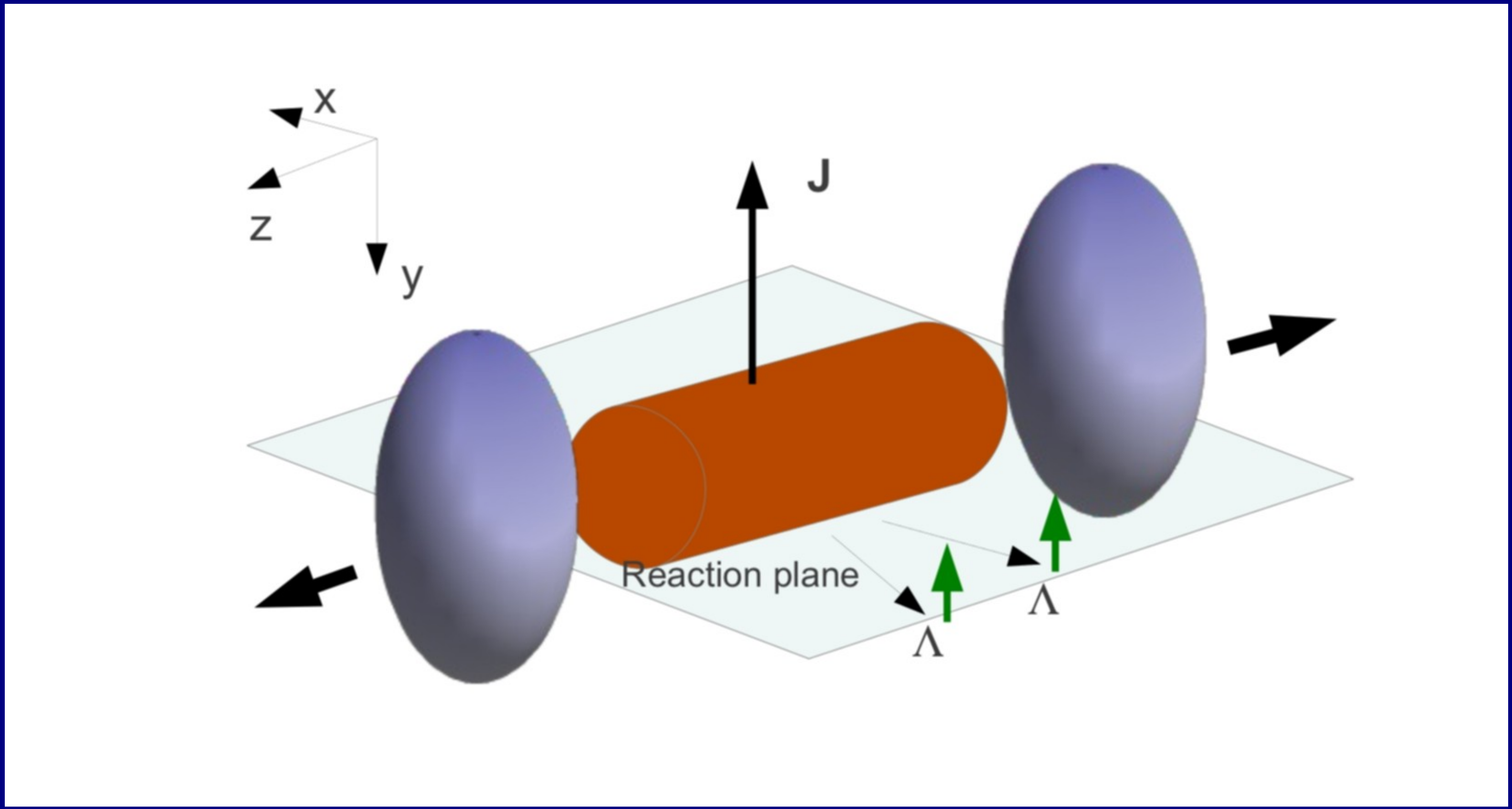
Polarization of the Quark-Gluon-Plasma

OUTLINE

- Introduction
- Polarization by rotation and acceleration: theory
- Λ polarization status
- Polarization and local parity violation
- Conclusions

Peripheral collisions: large angular momentum

Peripheral collisions ➡ Angular momentum ➡ Global polarization w.r.t reaction plane



Theoretical approaches to global polarization

- Polarization estimated at quark level by spin-orbit coupling

Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301

- By local thermodynamic equilibrium of the spin degrees of freedom

F. B., F. Piccinini, Ann. Phys. 323 (2008) 2452; F. B., F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906

Idea: equipartition of angular momentum among momentum
and the spin degrees of freedom

Spin µ (thermal) vorticity

Polarization by rotation

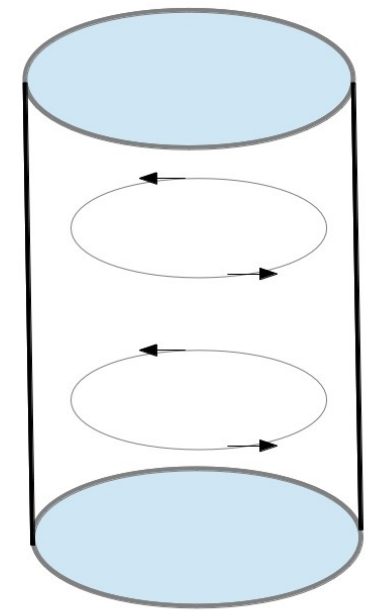
Take an ideal gas in a rigidly rotating vessel. At thermodynamical equilibrium (Landau) the gas will also have a velocity field

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{x}$$

WARNING The potential term has a + sign as it stems from both centrifugal and Coriolis potentials

For the *comoving* observer the equilibrium particle distribution function will be given by:

$$f(\mathbf{x}', \mathbf{p}') \propto \exp[-\mathbf{p}'^2/2mT + m(\boldsymbol{\omega} \times \mathbf{x}')^2/2T]$$



If we calculate the distribution function seen by the external inertial observer

$$f(\mathbf{x}, \mathbf{p}) \propto \exp[-\mathbf{p}^2/2mT + \mathbf{p} \cdot (\boldsymbol{\omega} \times \mathbf{x})/T]$$
$$= \exp[-\mathbf{p}^2/2mT + \boldsymbol{\omega} \cdot \mathbf{L}/T]$$

It seems quite *natural* to extend this to particle with spin

$$f(\mathbf{x}, \mathbf{p}, \mathbf{S}) \propto \exp[-\mathbf{p}^2/2mT + \boldsymbol{\omega} \cdot (\mathbf{L} + \mathbf{S})/T]$$

which implies that particles (and antiparticles) are *POLARIZED*, in a rotating ideal gas, along the direction of the angular velocity vector by an amount

$$P \simeq \frac{S+1}{3} \frac{\hbar\omega}{KT}$$

For a gas at STP with $\omega = 1000$ Hz, P ~ 10^{-11}

For relativistic nuclear collisions:

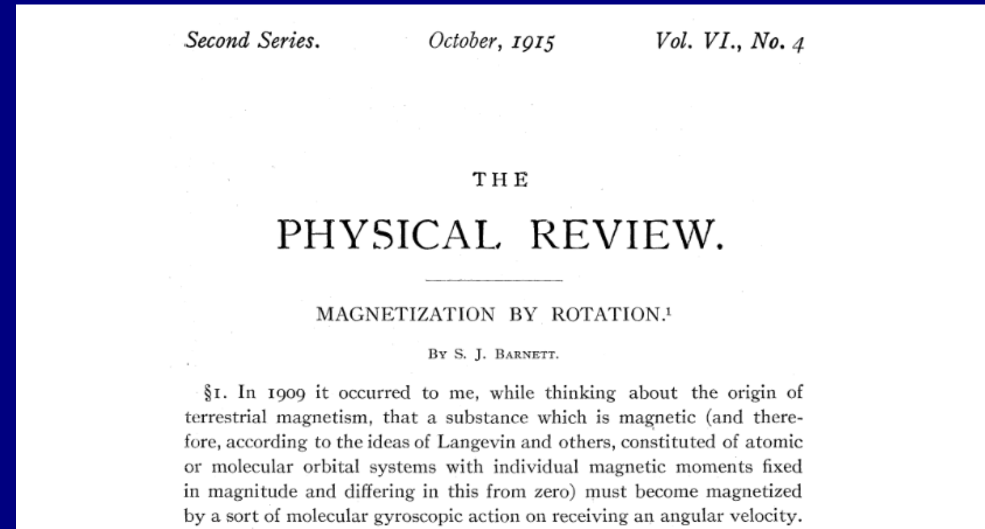
$$\frac{\hbar\omega}{KT} \approx \frac{c}{12\text{fm}200\text{MeV}} \approx 0.08$$

$$a \approx 10^{30} g \implies \frac{\hbar a}{cKT} \approx 0.06$$

Barnett effect

S. J. Barnett, *Magnetization by Rotation,*

Phys. Rev. 6, 239–270 (1915).



Second Series. October, 1915 Vol. VI., No. 4

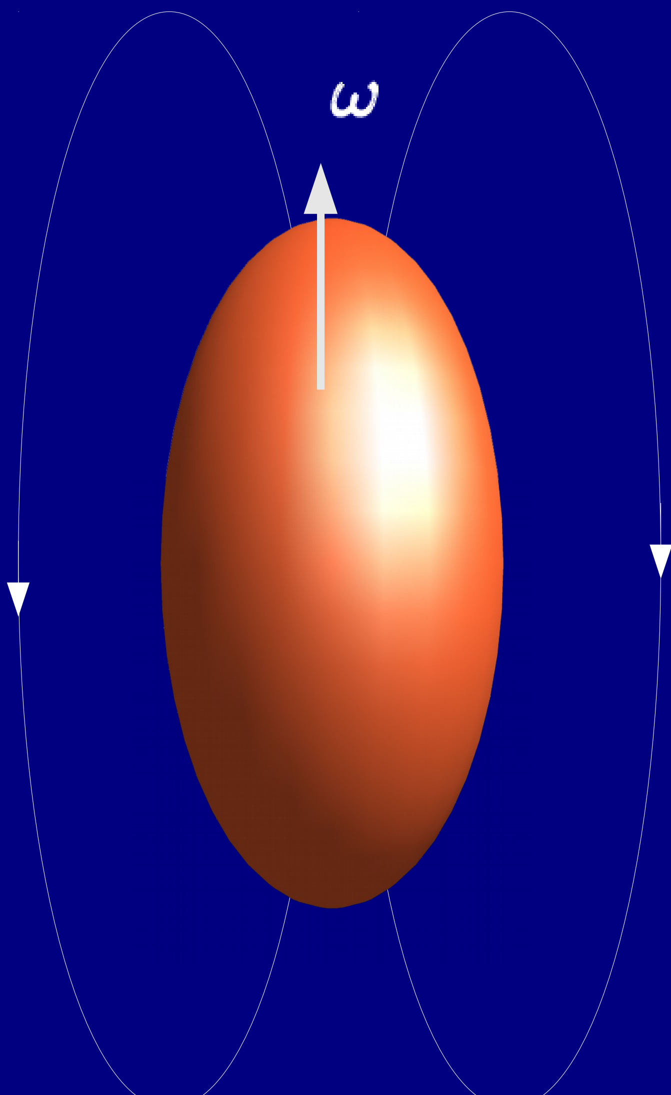
THE

PHYSICAL REVIEW.

MAGNETIZATION BY ROTATION.[1]

By S. J. Barnett.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.



Spontaneous magnetization of an uncharged body when spun around its axis, in quantitative agreement with the previous polarization formula

$$M = \frac{\chi}{g}\omega$$

It can be seen as a dissipative transformation of the orbital angular momentum into spin of the constituents. The angular velocity decreases and a small magnetic field appears; this phenomenon is accompanied by a heating of the sample. Requires a spin-orbit coupling.

Converse: Einstein-De Haas effect

the only Einstein's non-gedanken experiment

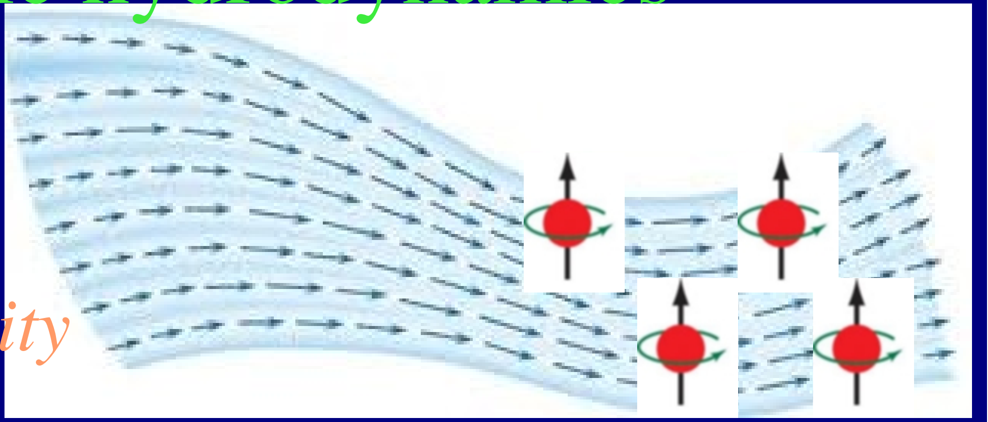
A. Einstein, W. J. de Haas, Koninklijke Akademie van Wetenschappen te Amsterdam, Proceedings, 18 I, 696-711 (1915)

Rotation of a ferromagnet originally at rest when put into an external H field

An effect of angular momentum conservation:
spins get aligned with H (irreversibly) and this must be compensated by a on overall orbital angular momentum

Polarization and relativistic hydrodynamics

F. B., V. Chandra, L. Del Zanna, E. Grossi,
Ann. Phys. 338 (2013) 32
F. B., *Polarization in relativistic fluids: a QFT derivation*
ArXiv:2004:04050 to appear in LNP Springer



Spin, local equilibrium and relativity

It is crucial to use a *quantum-relativistic* formalism from the onset
Definition of a *relativistic spin* four-vector

For a single particle

$$S^\mu = -\frac{1}{2m}\epsilon^{\mu\nu\lambda\rho}\langle \widehat{J}_{\nu\lambda}\widehat{P}_\rho\rangle$$

$$\langle \widehat{X}\rangle = \mathrm{tr}(\widehat{\rho}\widehat{X})$$

Relativistic Spin vs Pauli-Lubanski vs Polarization

$$S^\mu = \frac{1}{m}W^\mu = SP^\mu$$

Local polarization and covariant Wigner function

An useful tool in statistical QFT is the covariant Wigner function

$$W(x,k)_{AB} = -\frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-ik\cdot y}\langle : \Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) :\rangle$$
$$= \frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-ik\cdot y}\langle : \overline{\Psi}_B(x+y/2)\Psi_A(x-y/2) :\rangle$$

It allows to calculate the spin density matrix for spin ½:

$$\langle \widehat{X} \rangle = \mathrm{tr}(\widehat{\rho}\widehat{X})$$

$$\Theta(p)_{rs} = \frac{\int \mathrm{d}\Sigma_\mu p^\mu \bar{u}_r(p) W_+(x,p) u_s(p)}{\sum_t \int \mathrm{d}\Sigma_\mu p^\mu \bar{u}_t(p) W_+(x,p) u_t(p)}$$

And the mean spin vector in these two equivalent forms:

$$S^\mu(p) = -\frac{1}{2m}\epsilon^{\mu\beta\gamma\delta} p_\delta \frac{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4(\Sigma_{\beta\gamma} W_+(x,p))}{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4 W_+(x,p)}$$

$$S^\mu(p) = -\frac{1}{4}\epsilon^{\mu\beta\gamma\delta} p_\delta \frac{\int \mathrm{d}\Sigma_\lambda \mathrm{tr}_4(\{\gamma^\lambda, \Sigma_{\beta\gamma}\} W_+(x,p))}{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4 W_+(x,p)}$$

The density operator

Covariant form of the local thermodynamical equilibrium quantum density operator.

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta \widehat{j}^\mu \right) \right]$$

Extension of the known:

$$\widehat{\rho} = \frac{1}{Z} \exp[-\widehat{H}/T + \mu \widehat{Q}/T]$$

Hydrodynamic limit: Taylor expansion of the β and ζ fields around the point *x* where *Local operators* are to be calculated.

$$\mathrm{tr}(\widehat{O}(x)\widehat{\rho}) = O(x)$$

Local values of *T*,*u*,μ and their local derivatives (antisymmetric part: local thermal vorticity)

In the hydrodynamic limit

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\beta(x)\cdot \widehat{P} + \zeta(x)\widehat{Q} + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x \right.$$
$$\left. + \text{terms vanishing at global equilibrium} \right]$$

$$\beta^\mu = \frac{1}{T} u^\mu \qquad \zeta = \mu/T$$

$$\varpi_{\nu\mu} = -\frac{1}{2}(\partial_\nu \beta_\mu - \partial_\mu \beta_\nu)$$

Thermal vorticity

Adimensional in natural units

Spin mean vector at leading order in thermal vorticity

Approximation at first order in the gradients

$$n_F = (\mathrm{e}^{\beta\cdot p - \xi} + 1)^{-1}$$

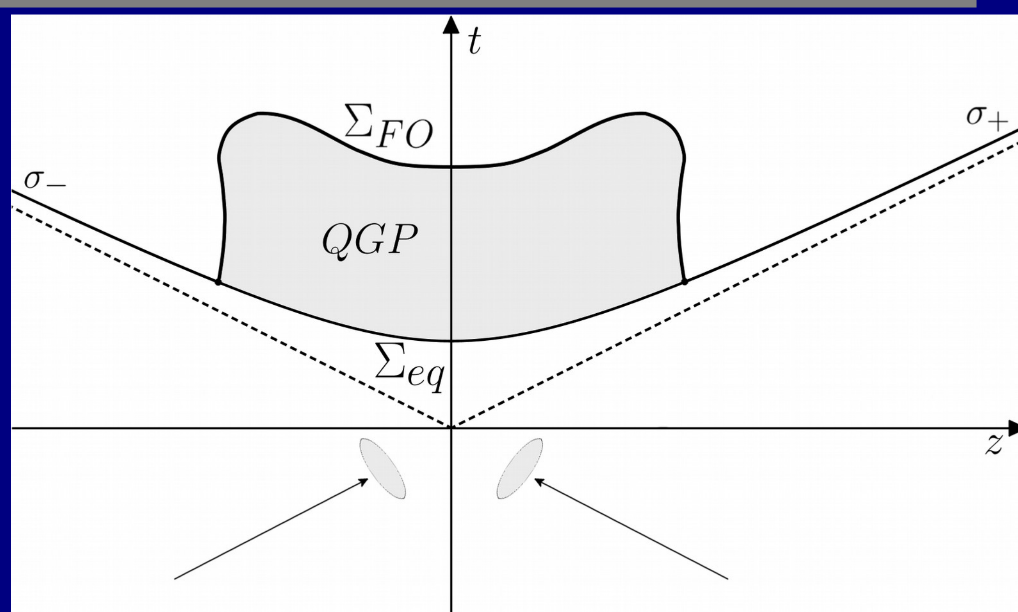
$$S^\mu(x,p) = -\frac{1}{8m}(1-n_F)\epsilon^{\mu\rho\sigma\tau} p_\tau \varpi_{\rho\sigma}$$

$$\varpi_{\nu\mu} = -\frac{1}{2}(\partial_\nu \beta_\mu - \partial_\mu \beta_\nu)$$

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F (1-n_F)\partial_\nu \beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \, n_F}$$

$$\beta^\mu = \frac{1}{T} u^\mu$$

$\zeta = \mu$



Contributions of vorticity, acceleration and Grad T

$$\partial_\mu \beta_\nu = \partial_\mu \left(\frac{1}{T}\right) + \frac{1}{T}\partial_\mu u_\nu$$

$$A^\mu = u \cdot \partial u^\mu$$
$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu u_\rho u_\sigma$$

$$S^\mu(p)\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \, n_F = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\nabla_\nu(1/T)u_\rho$$
Grad T

$$+ \frac{1}{8m}\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F) 2\frac{\omega^\mu u\cdot p - u^\mu \omega \cdot p}{T}$$
Vorticity

$$- \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\frac{1}{T}A_\nu u_\rho$$
Acceleration

In the rest frame of the particle:

$$\mathbf{S}^* \propto \frac{\hbar}{KT^2}\mathbf{u}\times\nabla T + \frac{\hbar}{KT}(\boldsymbol{\omega} - \boldsymbol{\omega}\cdot\mathbf{v}\mathbf{u}/c^2) + \frac{\hbar}{KT}\mathbf{A}\times\mathbf{u}/c^2$$

Thermal term (new)

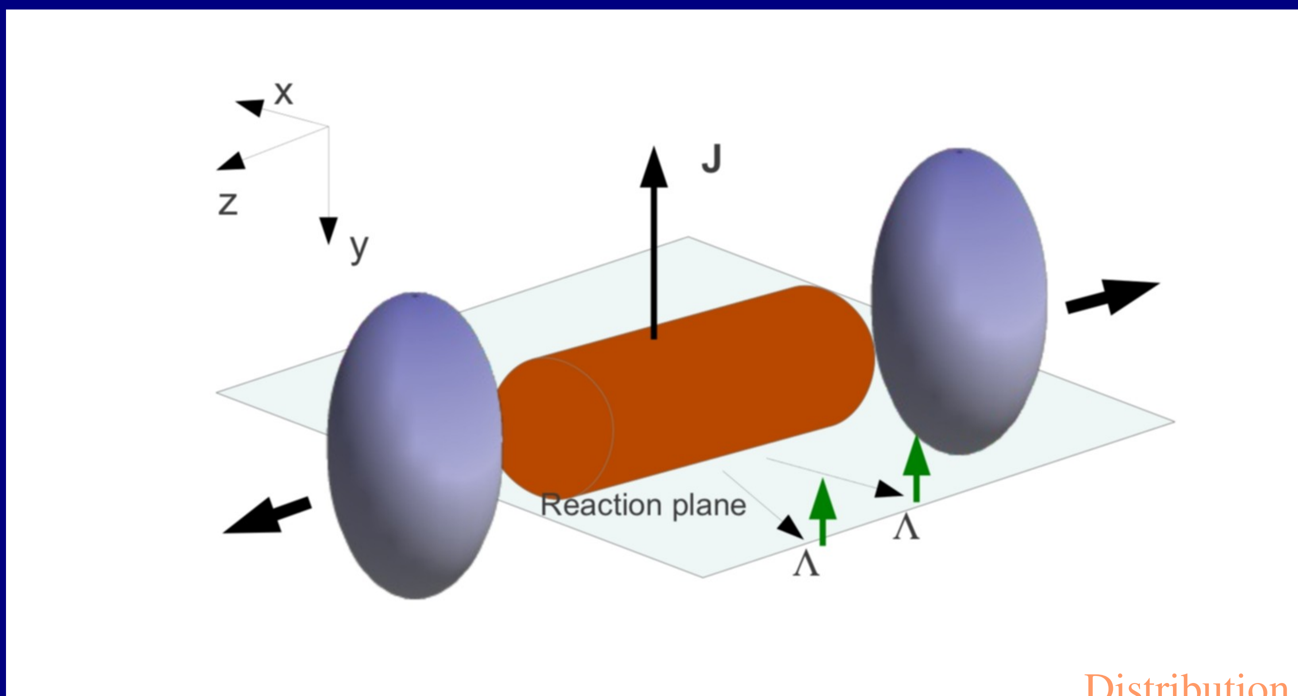
Vorticous term (known)

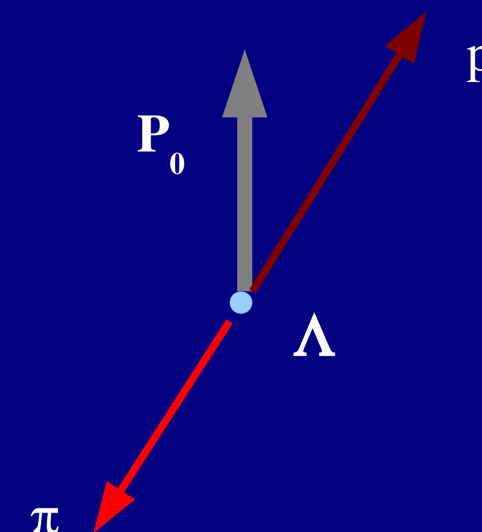
Acceleration term (purely relativistic)

$$\frac{\hbar\omega}{KT} \approx \frac{c}{12\text{fm}200\text{MeV}} \approx 0.08 \qquad a \approx 10^{30} g \implies \frac{\hbar a}{cKT} \approx 0.06$$

How to observe it: global Λ polarization

Because of parity violation, the polarization vector of Λ can be measured in its decay
Into a proton and a pion





Distribution of protons in the Λ rest frame

$$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}\Omega} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}_0 \cdot \hat{\mathbf{p}}^*) \qquad \mathbf{P}_0(p) = \mathbf{P}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon + m)}\mathbf{P}(p) \cdot \mathbf{p}$$

$\alpha = 0.642 \rightarrow 0.75$ (!) PDG 2020

Global Λ polarization prediction at $\sqrt{s_{NN}}$ = 200 GeV

"Minimal" initial
Vorticity scenario

40-80 % centrality

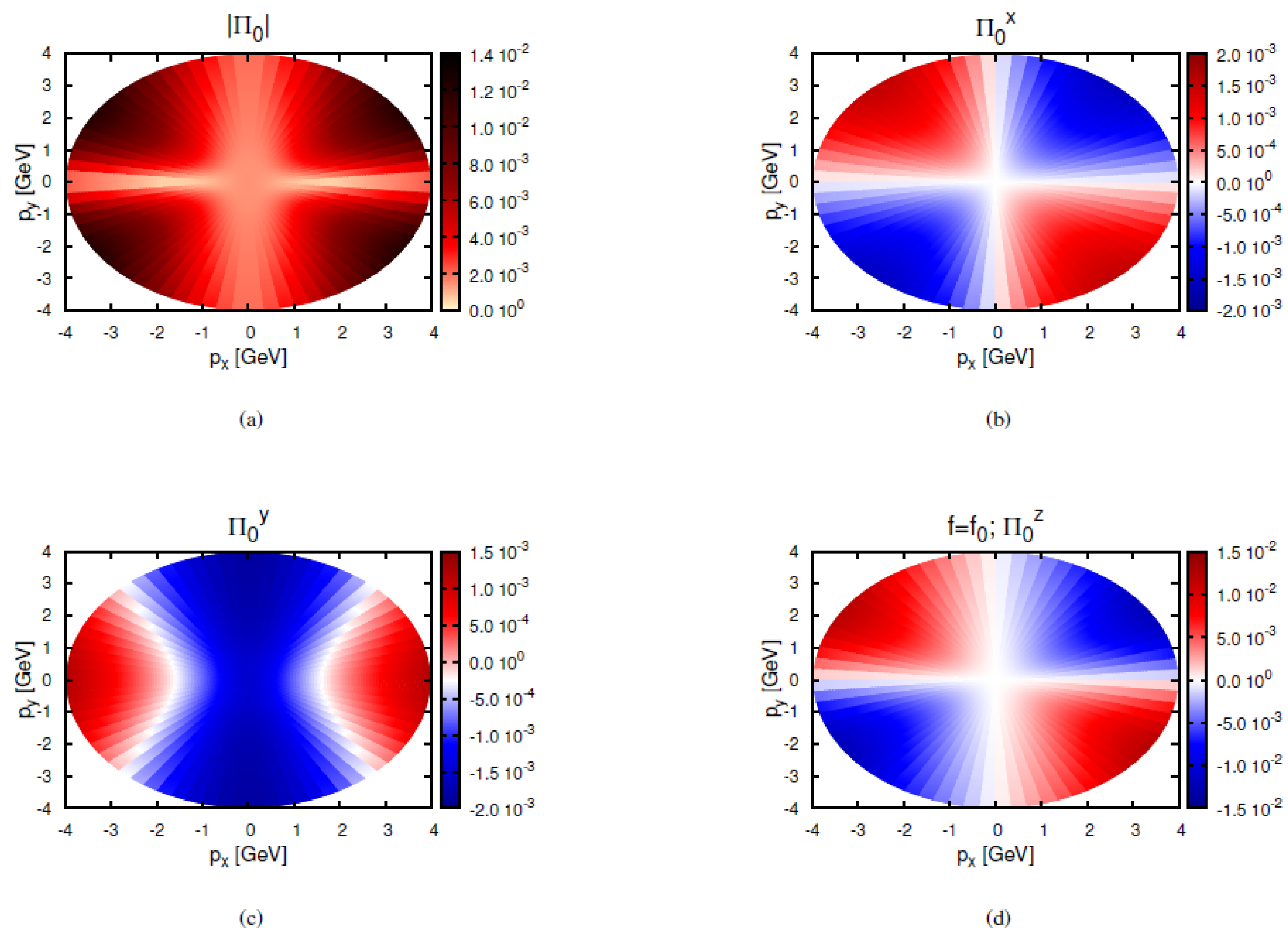
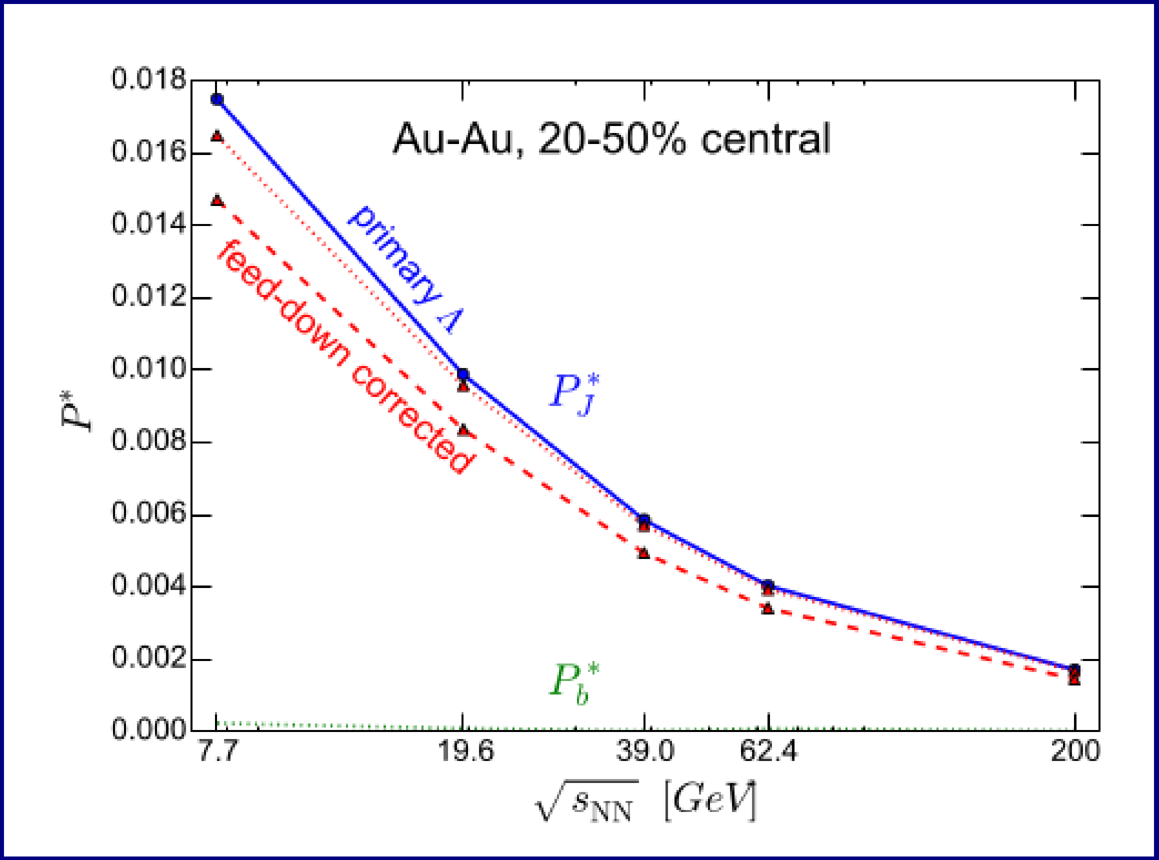


Figure 14: (color online) Magnitude (panel a) and components (panels b,c,d) of the polarization vector of the Λ hyperon in its rest frame.

F. B., G. Inghirami, V. Rolando, A. Beraudo, L. Del Zanna, A. De Pace, M. Nardi, G. Pagliara, V. Chandra
Eur. Phys. J C 75 (2015) 46

Global Λ polarization: energy dependence prediction

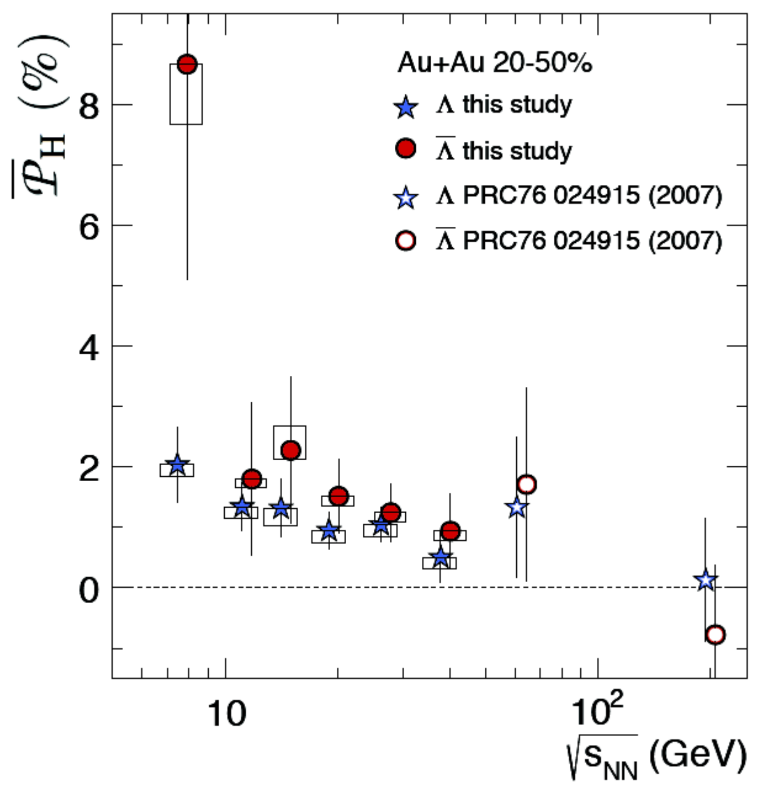
Prediction by 3+1D viscous Hydrodynamics with initial conditions tuned to reproduce spectra,v2 etc.

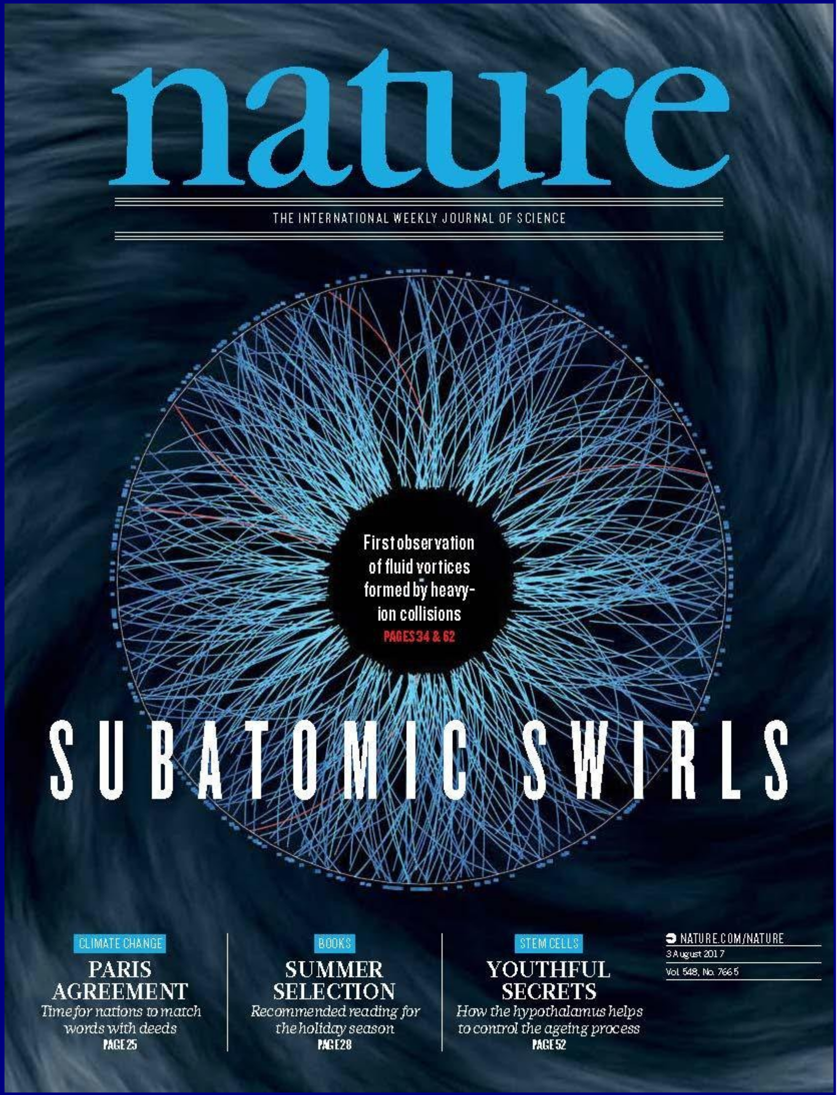


I. Karpenko and F.B., VHLLE code, Eur. Phys. J. C 77 (2017) no.4, 213

First positive signal of this phenomenon found in 2017

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017

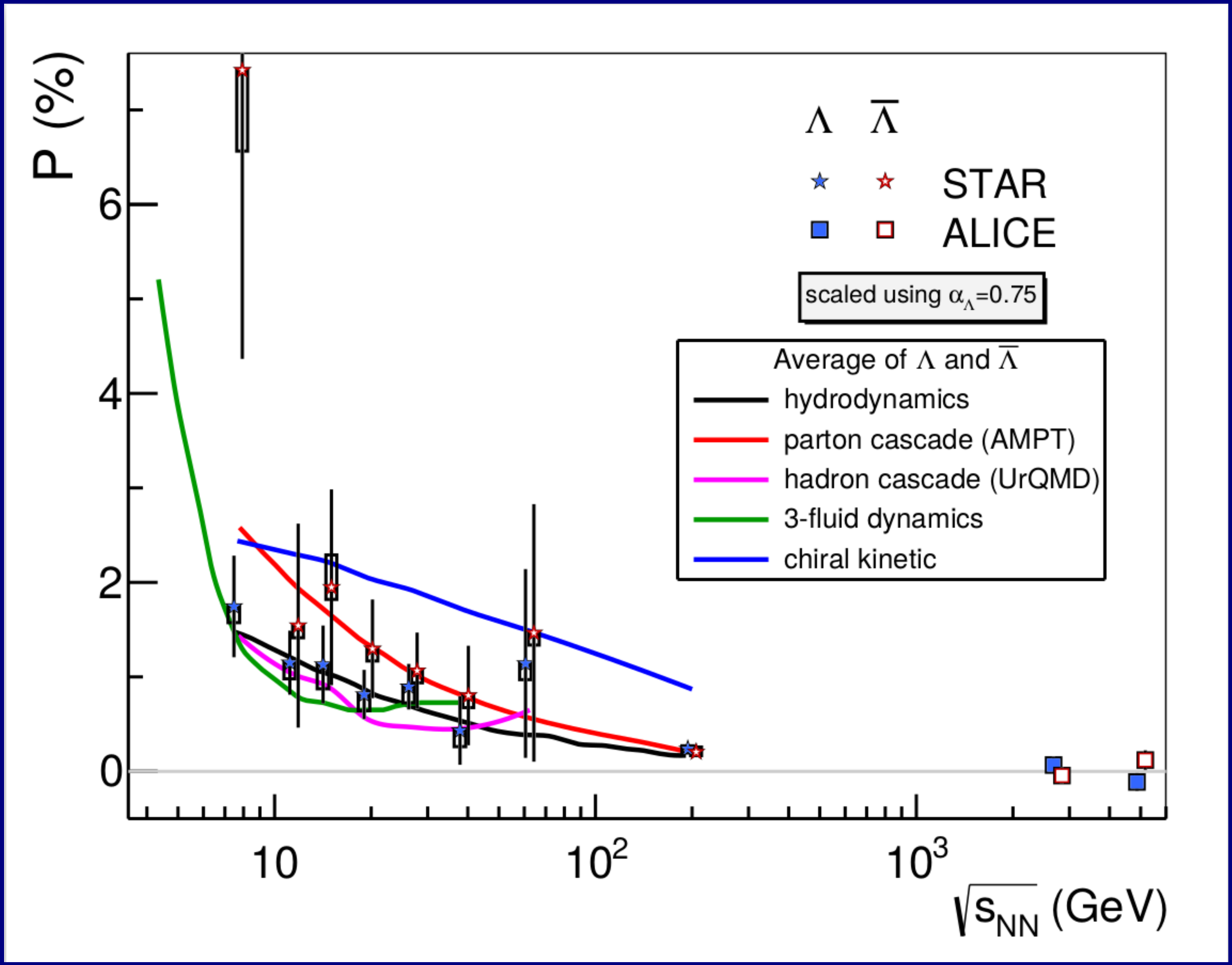




Particle and antiparticle have the same polarization sign.
This shows that the phenomenon cannot be driven
by a mean field (such as EM) whose coupling is *C-odd.*
Definitely favours the thermodynamic (equipartition) interpretation

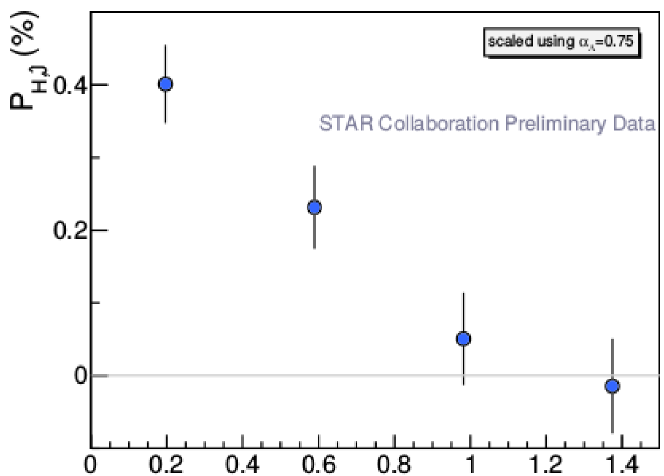
Updated plot

F. B., M. Lisa, Polarization and vorticity in the QGP, arXiv:2003.03640, to appear in Ann. Rev. Part, Nucl.

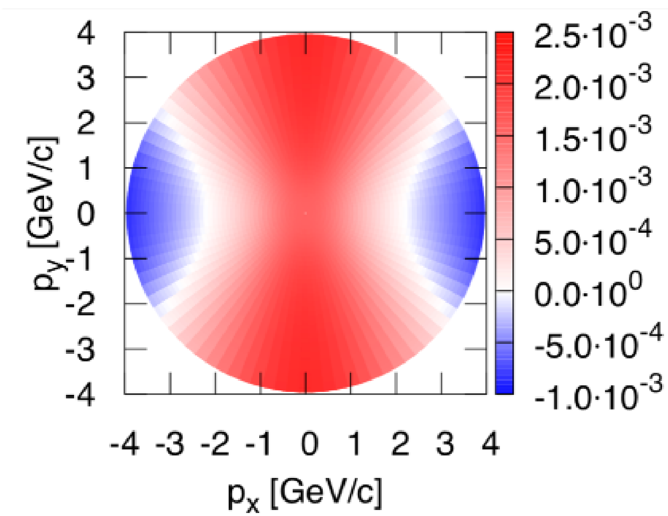


Different models of the collision, same formula for polarization

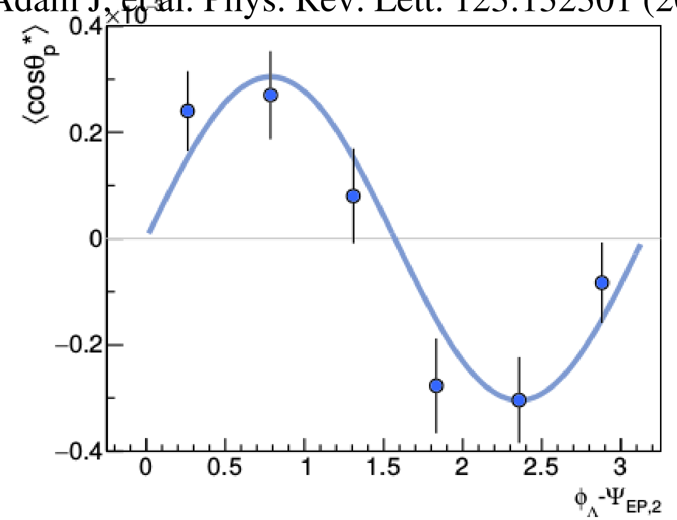
Puzzles: momentum dependence of polarization
(“*local polarization*”)

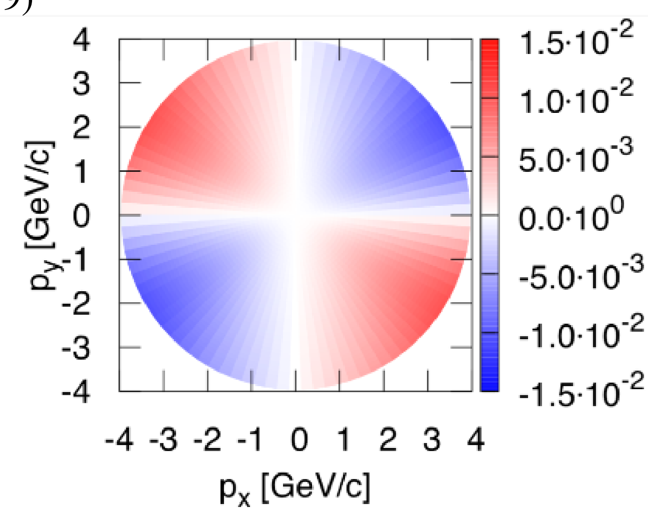


Niida T. Nucl. Phys. A982:511 (2019)



Adam J, et al. Phys. Rev. Lett. 123:132301 (2019)





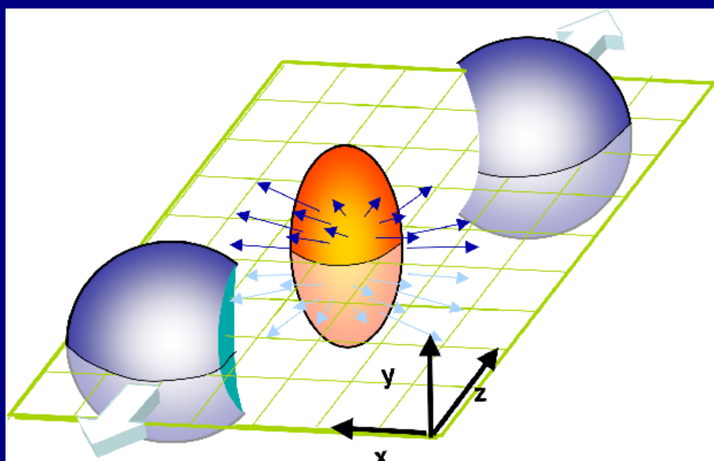
Similar results obtained with AMPT:
H. Z. Wu, L. G. Pang,
X. G. Huang and Q. Wang,
Phys. Rev. Research. \textbf{

Not the effect of decays:
F.B,, G. Cao and E. Speranza,
Eur. Phys. J. C 79 (2019) 741
X.L. Xia et al., Phys. Rev. C 100 (2019) 014913

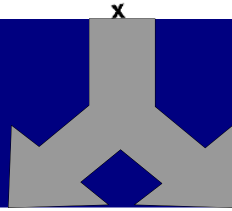
UNDER INTENSE INVESTIGATIONS BY SEVERAL GROUPS

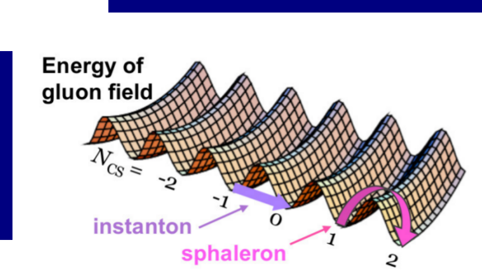
Polarization and local parity violation

F.B., M. Buzzegoli, A. Palermo, G. Prokhorov 2009.13449

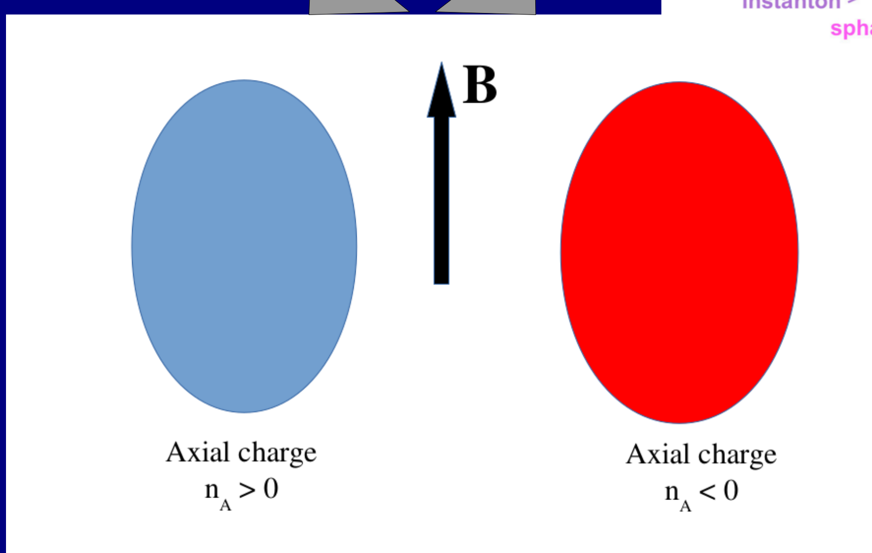


A peripheral collision is a system invariant by reflection. However, there might be event-by-event ("local") parity breaking due to non-perturbative QCD topological transition induced by high T





Axial imbalance = parity violation long sought in relativistic heavy ion collisions through the Chiral Magnetic Effect



$$\mathbf{j} = \frac{\mu_A}{2\pi}\mathbf{B}$$

Local parity violation has become a synonimous of CME

Investigating local parity violation with spin without the mediation of the EM field

$$S^\mu(p) = S^\mu_\chi(p) + S^\mu_\varpi(p)$$

$$S^\mu_\chi(p) \simeq \frac{g_h}{2} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, \zeta_\mathrm{A} n_\mathrm{F}\,(1-n_\mathrm{F})}{\int_\Sigma \mathrm{d}\Sigma \cdot p\; n_\mathrm{F}} \frac{\varepsilon p^\mu - m^2 \hat{t}^\mu}{m\varepsilon} \quad \leftarrow \quad \text{Axial imbalance}$$

$$S^\mu_\varpi(p) = \frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma_\lambda p^\lambda n_F (1-n_F)\partial_\rho \beta_\sigma}{\int_\Sigma \mathrm{d}\Sigma_\lambda p^\lambda n_F} \quad \leftarrow \quad \text{Thermal vorticity}$$

$$\zeta_A = \frac{\mu_A}{T}$$

g_h

axial charge of hadron h

ζ_A changes sign event by event

Average over multiple events

$$\langle\langle S^\mu(p)\rangle\rangle = \cancel{\langle\langle S^\mu_\chi(p)\rangle\rangle} + \langle\langle S^\mu_\varpi(p)\rangle\rangle$$

$$\langle\langle \zeta_\mathrm{A}\rangle\rangle = 0 \qquad \langle\langle \zeta_\mathrm{A}^2\rangle\rangle \neq 0$$

Axial contribution modifies helicity pattern

In the rest frame of the hadron

$$\mathbf{S}_{0,\chi} = \frac{g_h}{2} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, \zeta_A n_\mathrm{F} (1 - n_\mathrm{F})}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_\mathrm{F}} \hat{\mathbf{p}} \equiv F_\chi(\mathbf{p}) \hat{\mathbf{p}}$$

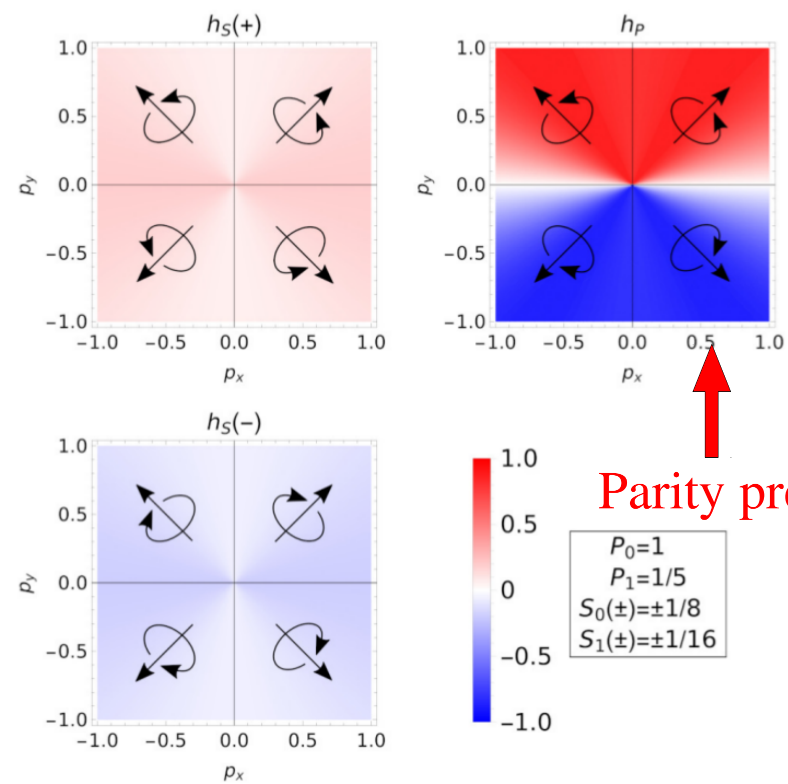
MODEL-INDEPENDENT ANALYSIS

$h = h_\mathrm{P} + h_\mathrm{S}$

From rotational symmetry $\phi \to \pi - \phi$ and reflection properties $\phi \to \pi + \phi$:

$$h_P(p_T, \phi) = \sum_k P_k(p_T) \sin[(2k+1)\phi]$$

$$h_S(p_T, \phi) = \sum_k S_k(p_T) \cos[2k\phi]$$



Parity preserving

Parity breaking

Local parity violation $S_k(p_T) \neq 0$

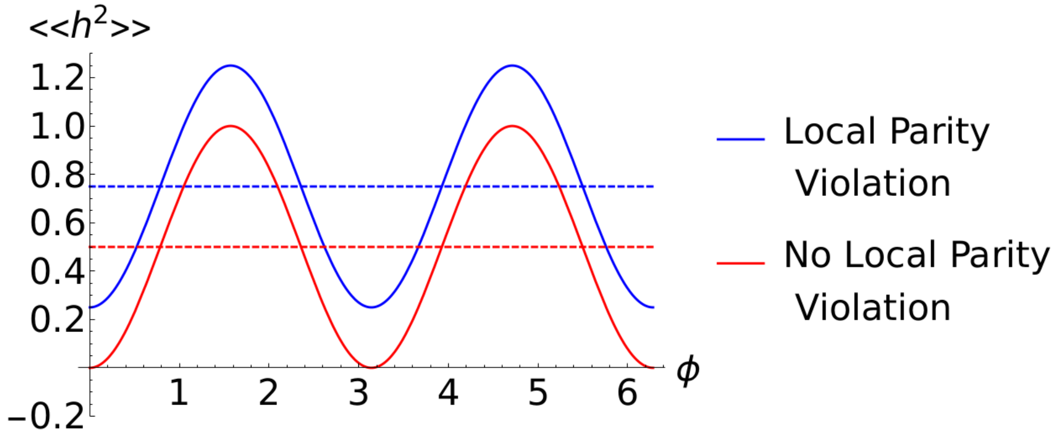
Global parity conservation $\langle\langle S_k(p_T) \rangle\rangle = 0$

Azimuthal analysis of helicity and helicity correlations

Helicity can be measured by projecting the proton momentum in the Λ rest frame onto the momentum of the Λ in the QGP frame

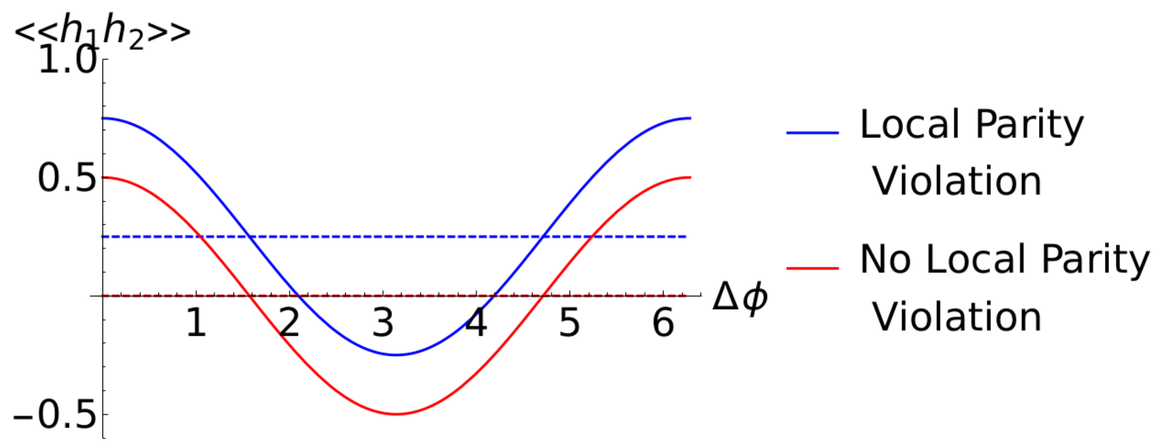
$$h^2(\mathbf{p}_T) = (S_0 + P_0 \sin\phi + \cdots)^2 = S_0^2 + P_0^2 \sin^2\phi + 2S_0 P_0 \sin\phi + \cdots$$

$$\langle\langle h^2 \rangle\rangle = \langle\langle S_0^2 \rangle\rangle + \langle\langle P_0^2 \rangle\rangle \sin^2\phi + \cdots$$



Helicity squared

$$\langle h_1 h_2(\Delta\phi)\rangle \simeq \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi \left(\bar{S}_0^2 + \bar{P}_0^2 \sin^2\phi \cos\Delta\phi\right) = \bar{S}_0^2 + \frac{1}{2}\bar{P}_0^2 \cos\Delta\phi$$



Helicity-helicity azimuthal 2Pcorrelation

Summary and outlook

- Polarization driven by acceleration, vorticity and temperature gradients: 1st order quantum effect in (relativistic) hydrodynamics
- Evidence for global particle-antiparticle polarization in relativistic nuclear collisions in agreement with the predictions of relativistic hydrodynamics and local thermodynamic equilibrium/equipartition of angular momentum.
- Puzzles in the local polarization pattern, which may lead to a deeper understanding of the physics of the QGP
- New degree of freedom to study the QGP dynamics and finite temperature QCD
- Helicity as a probe of local parity violation in QCD other than CME