# Measuring the masses and spins of astrophysical black holes



Astronomical Observatory of the Jagiellonian University

#### editorial

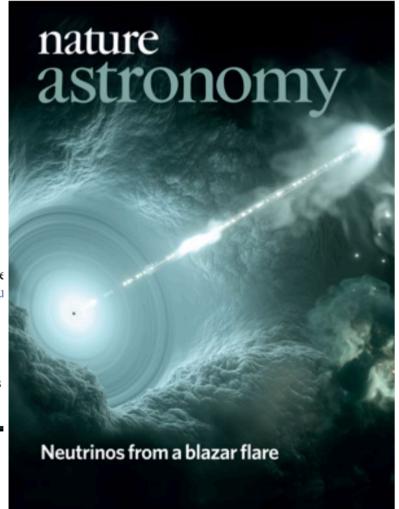
#### Cosmic celebrities with gravitas

Black holes have the distinct honour of being the most popular and potentially the least well-understood objects in the Universe. This issue's Insight explores how far black hole research has come since its inception, though it still

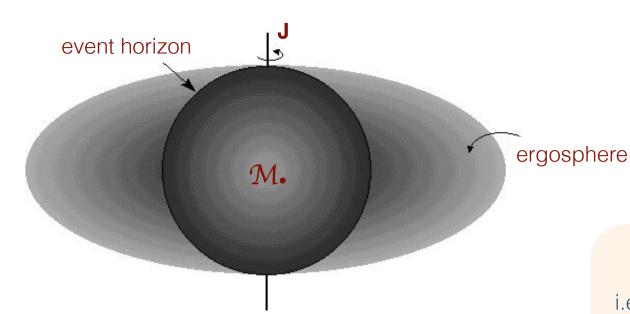
has a long way to go.

ince 1784, when John Michell first thought of the concept of an object so heavy that even light cannot escape its gravitational pull, black holes have fascinated, puzzled and even terrified the public and scientists alike. It was Albert Einstein, with his foundational theory of general relativity (GR), and Karl Schwarzschild, who found the corresponding solution to GR's equations, that placed black holes on a proper physical footing in 1915. In the following century, black holes became a topic of intense scientific research, while at the same time entering the mainstream collective consciousness as one of the staples of the sci-fi genre (with milestones like the dubious 1979 Disney movie The Black Hole and the more acclaimed 2014 Interstellar). Key observational discoveries, like the orbits of stars around Sgr A\*, quasars and the recent

directly probed by classical Newtonian dynamics at scales significantly larger than its Schwarzschild radius. But while such dynamical considerations are only possible for those black holes closest to us (like the one at the centre of M87), we have to rely on much rougher estimates when looking at black holes in most other galaxies. Given their fundamental importance, understanding how black holes grow their mass is key in constraining their formation and evolution and how their growth might impact the growth of their host galaxies (see the News & Views in this issue and our focu issue last March). In the Insight, Thaisa Storchi-Bergmann and Allan Schnorr-Müller review how supermassive black holes are fed, while Marianne Vestergaard discusses current problems and open issues with calculating masses of black holes in other galaxies.



- no-hair theorem/conjecture: black holes can be completely characterized by only three externally observable classical parameters: mass, electric charge, and angular momentum (spin). All other information (for which "hair" is a metaphor) about the matter which formed a black hole or is falling into it, "disappears" behind the black-hole event horizon and is therefore permanently inaccessible to external observers (J.A. Wheeler: "black holes have no hair")
- how exactly can we "observe" black holes in the Universe? how can we measure their masses and spins?
- this talk: black holes as astrophysical objects, interacting with their environment (gravitational potential, but also jets/outflows), and thus observable



angular momentum (spin) expressed in length units

$$a = \frac{J}{c \, \mathcal{M}_{\bullet}}$$

$$J_{\max} = \frac{G\mathcal{M}_{\bullet}^2}{c}$$

$$0 \le a_{\star} \equiv \frac{a}{r_{\rm g}} \le 1$$

gravitational radius, i.e. mass expressed in length units

$$r_{\rm g} = \frac{G\mathcal{M}_{\bullet}}{c^2} \quad \approx \left(\frac{\mathcal{M}_{\bullet}}{M_{\odot}}\right) \, \mathrm{km}$$

$$M_{\odot} = 2 \times 10^{30} \,\mathrm{kg}$$

event horizon (Schwarzchild radius)  $r_{\text{S}}$  and static limit  $r_{\text{C}}$ 

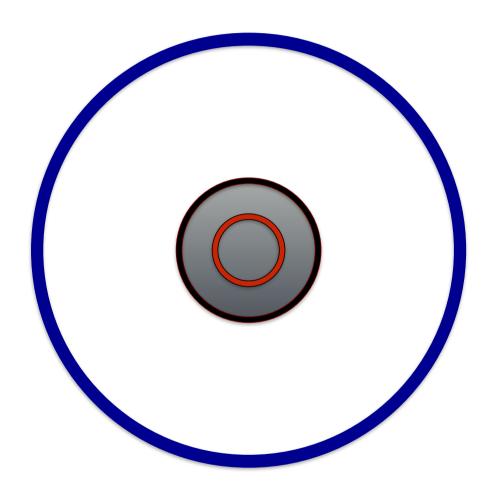
$$r_{\rm S} = r_{\rm g} + \sqrt{r_{\rm g}^2 - a^2}$$

$$r_{\rm C} = r_{\rm g} + \sqrt{r_{\rm g}^2 - a^2 \cos^2 \theta}$$

- charged BH are expected to neutralize quickly by attracting charge of the opposite sign
- $J_{max}$  as you cannot have a "naked singularity", i.e. a singularity exposed to the rest of the Universe
- <u>Lense-Thirring/frame-dragging effect</u>: due to the twisting of a spacetime by rotating BH, an object within the ergosphere cannot appear stationary with respect to an outside observer
- <u>Penrose process</u>: since the ergosphere is outside the event horizon, it is still possible for an object that enters that region to escape from the gravitational pull of the BH, possibly removing the BH rotational energy ("reducible mass", although note that the gravitational mass of the BH has to increase at the same time)
- Blandford–Znajek process: the BH rotational energy and angular momentum transported electromagnetically outwards (jets!) due to the magnetic field supported within the ergosphere by the accreting matter

$$\frac{E_{\text{rot}}}{\mathcal{M}_{\bullet}c^2} = 1 - \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{1 - \left(\frac{a}{r_{\text{g}}}\right)^2}} \leq 0.3$$

#### the innermost stable circular orbit: risco





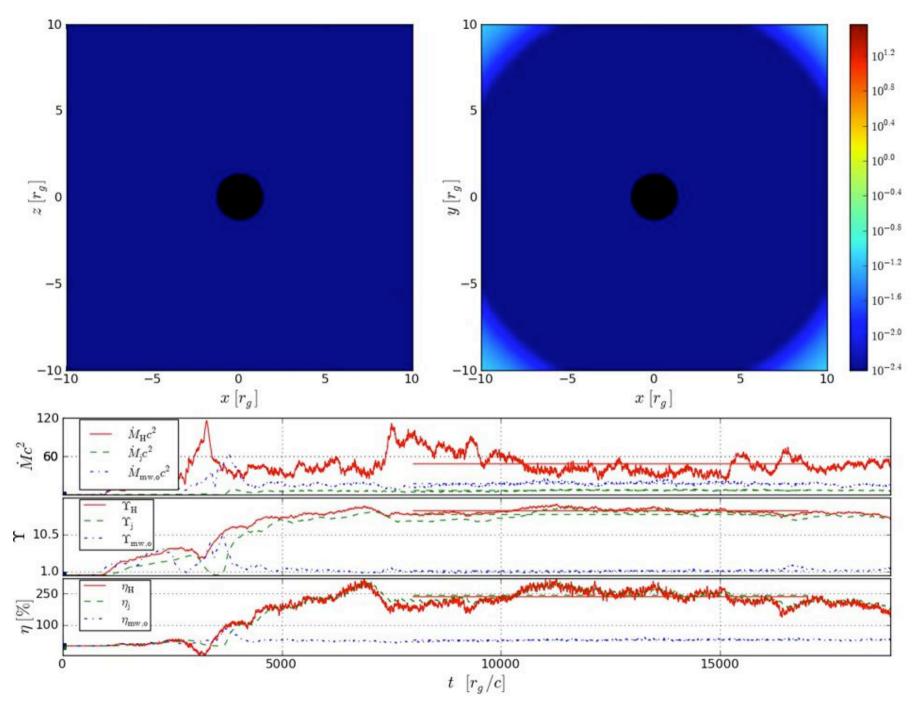
non-spinning BH: a=0

$$r_g = \frac{GM/c^2}{r_S} = 2 r_g$$

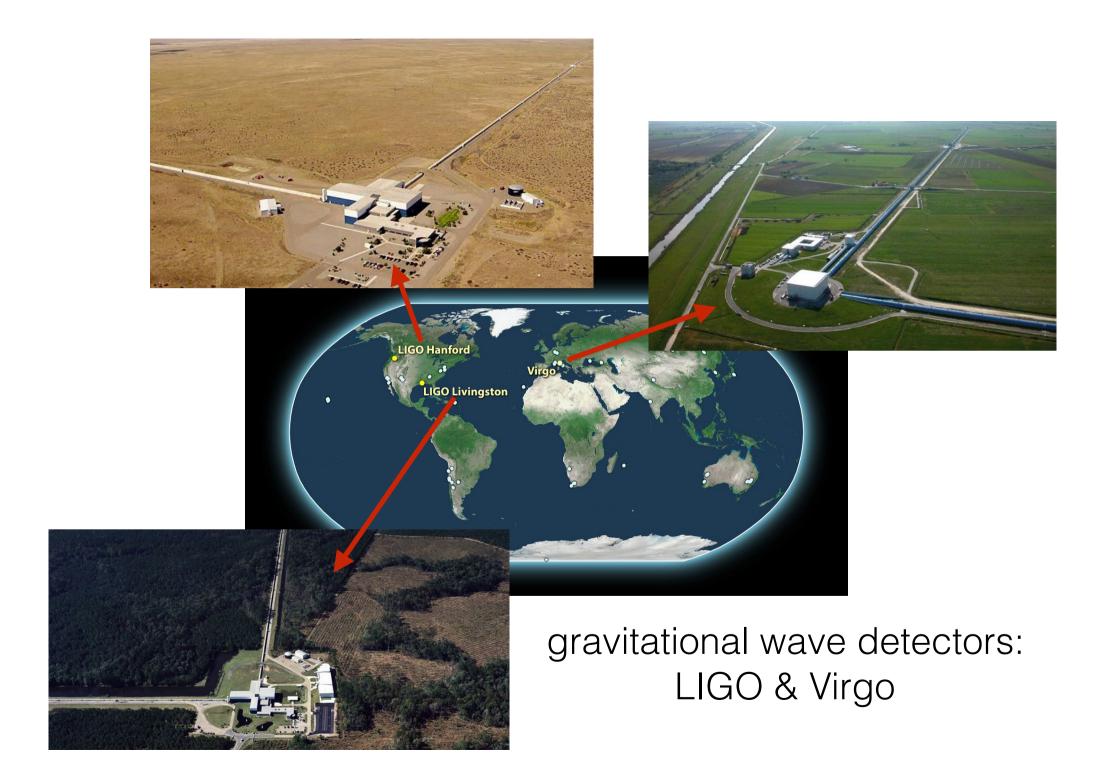
$$r_{ISCO}$$
 = 6  $r_g$ 

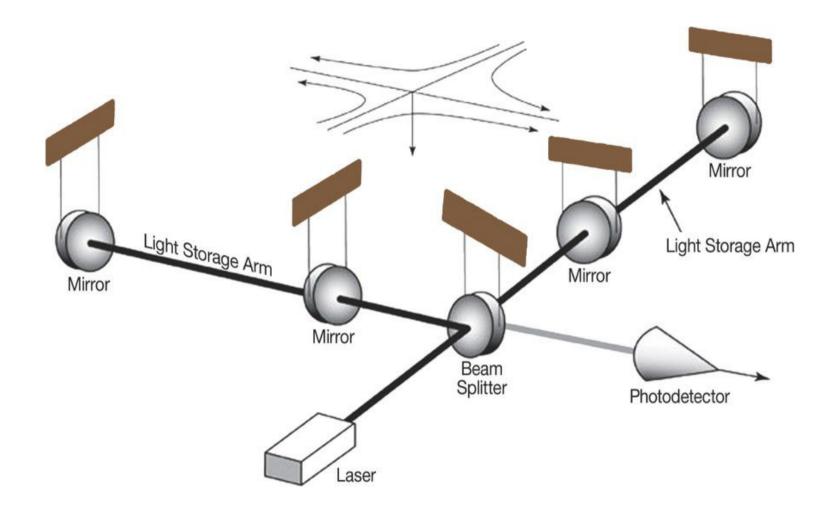
maximally-spinning BH: a~1

$$\begin{aligned} r_{g} &= GM/c^{2} \\ r_{S} &\sim r_{g} \\ r_{C}(\pi/2) &\sim 2 \ r_{g} \\ r_{ISCO} &\sim r_{g} \end{aligned}$$

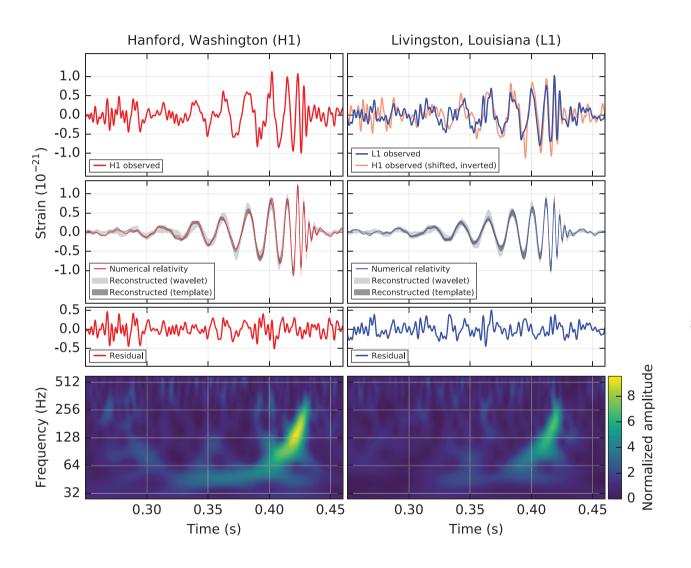


**McKinney et al. 2012:** log of rest-mass density in colour in both the z-x plane at y = 0 (top left-hand panel) and the y-x plane at z = 0 (top right-hand panel). The black lines trace field lines, where the thicker black lines show where field is lightly mass-loaded. Panels below: accretion rate, the horizon's dimensionless magnetic flux, and the jet production efficiency



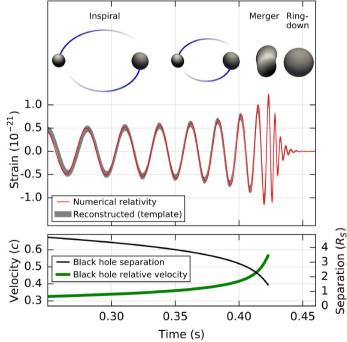


Gravitational waves cause space itself to stretch in one direction and squeeze in a perpendicular direction simultaneously. The effect on LIGO is this: as the wave passes, one arm of an interferometer lengthens while the other shrinks, then vice versa, and so on. The arms will oscillate this way for as long as it takes the wave to pass. As the length of each arm varies, the distance traveled by each laser beam also varies. Consequently, the beam in the shorter arm returns to the beam splitter before the beam in the longer arm. Arriving at different times (one before the other, then after, then before again, etc.) causes the beams to shift in and out of alignment as they merge, experiencing varying levels of destructive and constructive interference as long as the arms change lengths, which is for as long as it takes for the gravitational wave to pass.

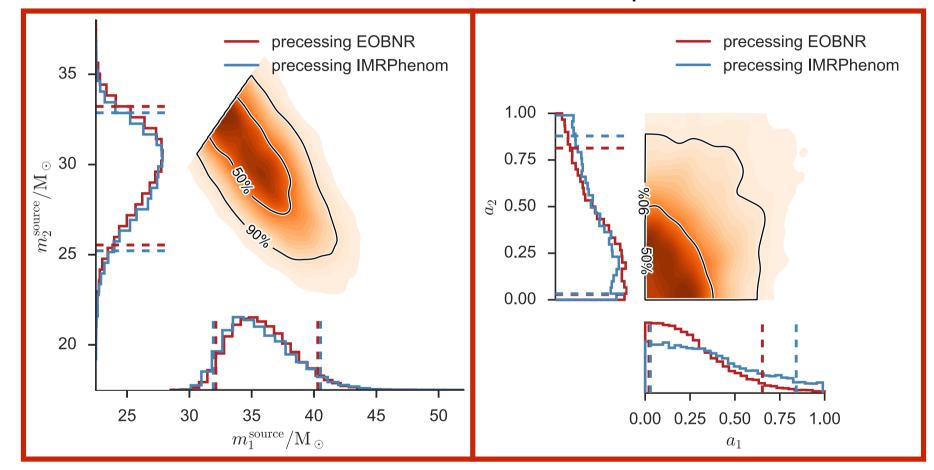


#### First LIGO detection: GW150914

unique characteristics of the signal: direct estimates of the distance, masses, and spins of the merging black holes

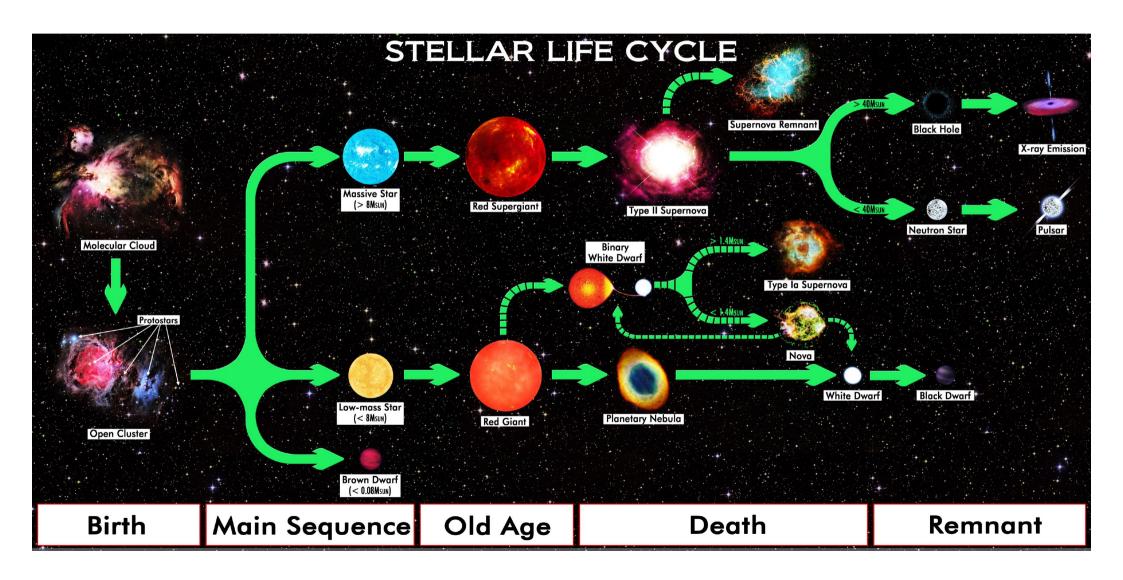


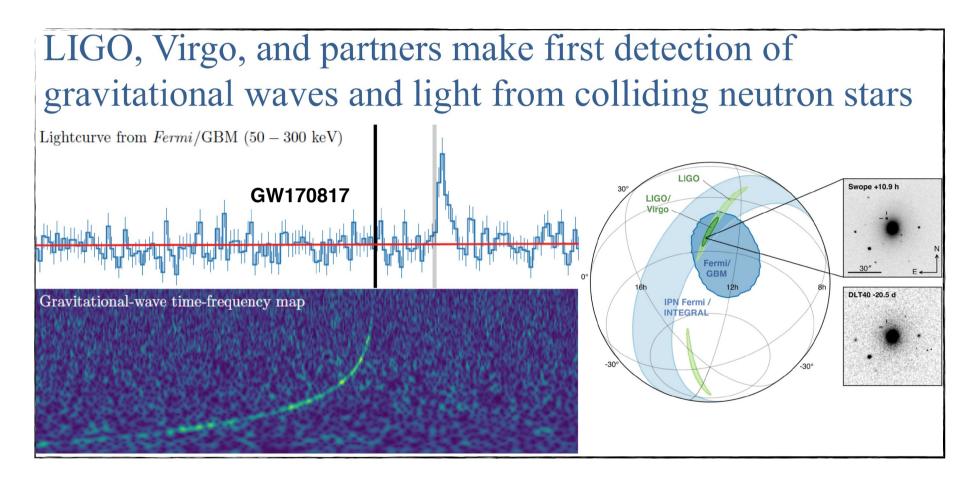
GW150914 — distance 410<sup>+160</sup>-180 Mpc



posterior probability densities

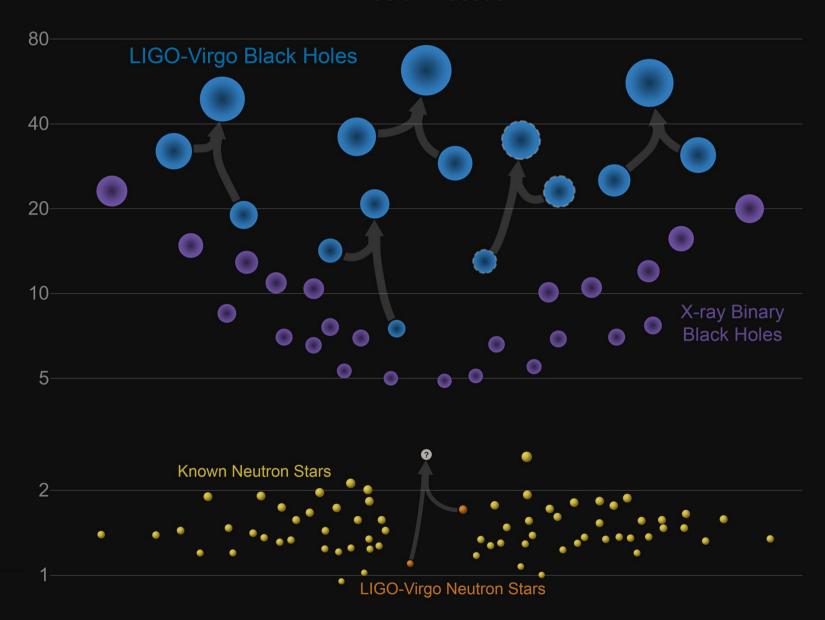
Black holes with masses **3-100 M**⊙ are formed (*exclusively?*) during the **gravitational collapse** of massive stars at the end of their evolution (-> supernovae!)





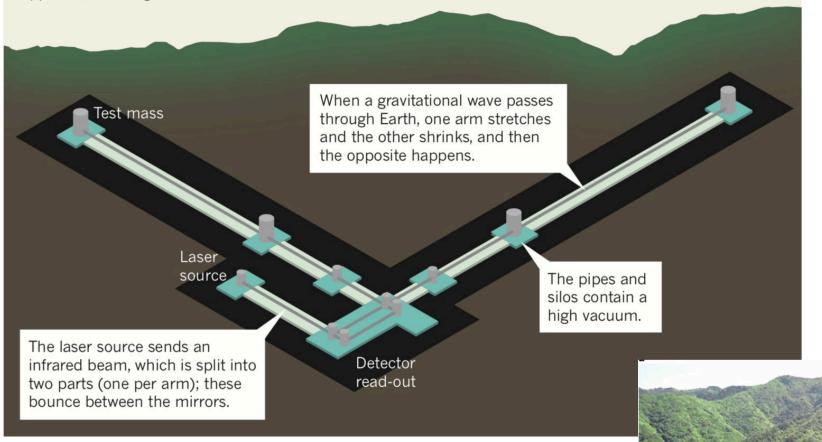


# Masses in the Stellar Graveyard in Solar Masses



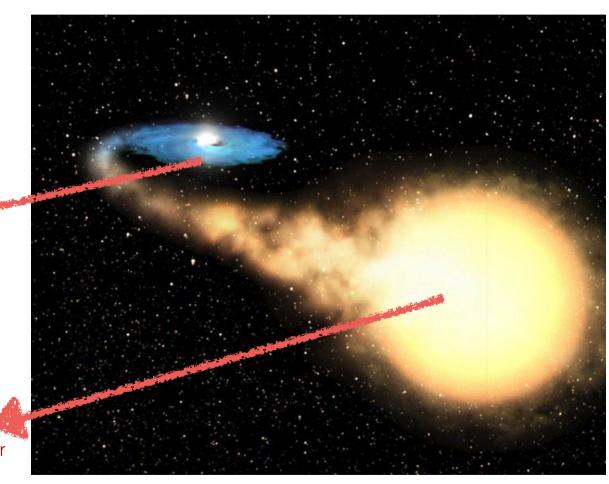
#### **JAPAN'S WAVE HUNTER**

The Kamioka Gravitational Wave Detector (KAGRA) is the world's fourth major gravitational-wave detector — and Asia's first. Due to open in late 2019, it is the first one to be built underground, and to have cryogenically cooled mirrors, operating at around 20 kelvin. Both innovations should help KAGRA to separate the cosmic ripples from background noise.



#### **BH X-ray Binaries**:

stellar-mass black holes accreting mater from the companion star



X-ray emission of the accretion disk

optical emission of the companion star

### 0.05 June, 2008 0.04 0.03 0.05 Aug. 02 & 05, 2008 Relative Flux 0.04 0.03 0.05 Aug. 06 & 07, 2008 0.04 0.03 0.5 1.5 Phase

# BH mass estimates in X-ray Binaries

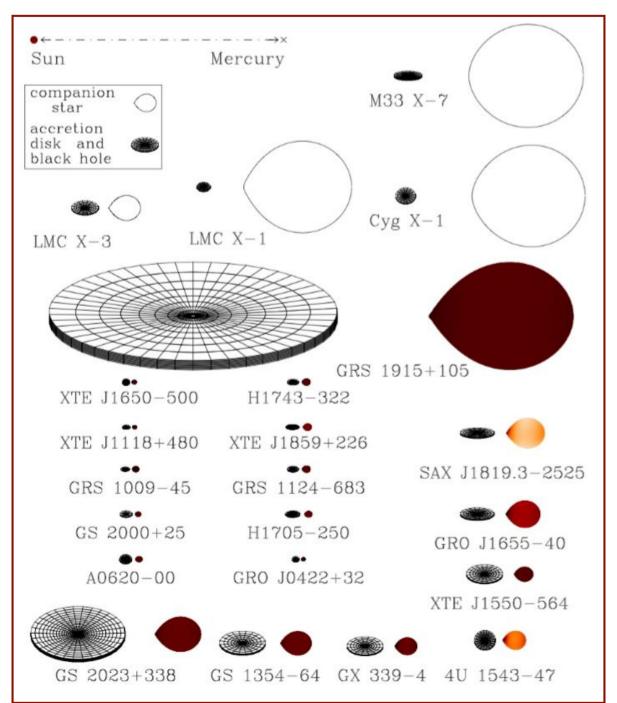
orbital modulation of the optical emission (companion star partly eclipsed by the accretion disk)

given the type of the companion (main sequence) star, one may estimate the other dynamical parameters of the system, including the BH mass

V1408 AQUILAE (Bayless et al. 2011)

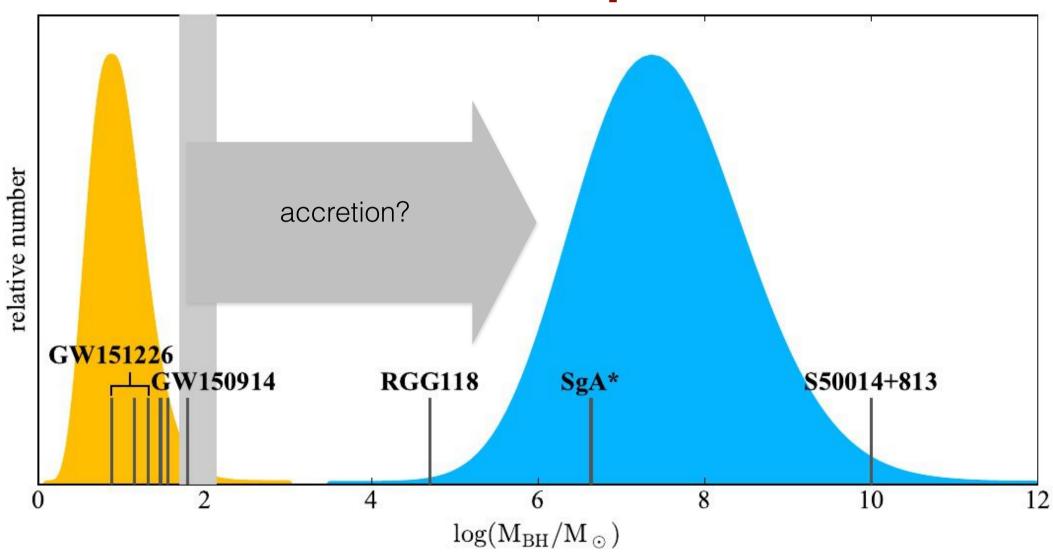
No	Name	$M_{comp}[M_{\odot}]^b$	Spec. type	$ m M_{BH}[M_{\odot}]$
1	XTE J1118+480	$0.22 \pm 0.07$	K7/M1V	$6.9 \div 8.2$
	XTE J1550-564	$0.3 \pm 0.07$	K2/4IV	$10.5 \pm 1.0$
3	GS 2000+25	$0.16 \div 0.47$	K3/6V	$\sim 6.55$
4	GRO J0422+32	$\sim 0.45$	M0/4V	$\sim 10.4$
2 3 4 5 6 7 8 9	GRS 1009-45	$\sim 0.5$	G5/K0V	$\sim 8.5$
6	GRS 1716-249	$\sim 1.6$	K-M	$\gtrsim 4.9$ > 7
7	GX339-4	$0.3 \div 1.1$	KIV	> 7
8	H1705-25	$0.15 \div 1.0$	K3/M0V	$4.9 \div 7.9$
9	A0620-00	$0.68 \pm 0.18$	K2/7V	$6.6 \pm 0.25$
10	XTEJ1650-50(0)	0.7	K4V	$\sim 5.1$
11	XTEJ1859+226	0.7	K5V	$7.7 \pm 1.3$
12	GS2023+338	$0.5 \div 1.0$	K0/3IV	$12\pm2$
13	GRS 1124-68	$0.3 \div 2.5$	K5V	$6.95 \pm 0.6$
14	GRS1915+105	$0.8 \pm 0.5$	K1/5III	$12.9 \pm 2.4$
15	GS 1354-64	1.03	G5IV	$7.6 \pm 0.7$
16	GROJ1655-40	$1.75 \pm 0.25$	F3/G0IV	$5.31 \pm 0.07$
17	4U1543-47	$2.3 \div 2.6$	A2V	$2.7 \div 7.5$
18	XTEJ1819-254	$5.49 \div 8.14$	B9III	$8.73 \div 11.70$
19	CygX-1 b	$19.2\pm1.9$	OI	$14.8\pm0.1$
20	LMC X-1 <sup>c</sup>	$31.79 \pm 3.48$	O7/O8	$10.91 \pm 1.55$
21	LMC X-3 <sup>cf</sup>	$3.72 \pm 0.24$	B5III	$7.00 \pm 0.32$
22	IC 10 X-1 <sup>c</sup>	> 17	WNE	> 23.1
23	NGC 300 X-1°	$26^{+7}_{-5}$	WN5	$20 \pm 4$
24	M33 X-7 <sup>c</sup>	$70.0\pm 6.9$	O7/O8 III	$15.65 \pm 1.45$

#### **BH XRBs**



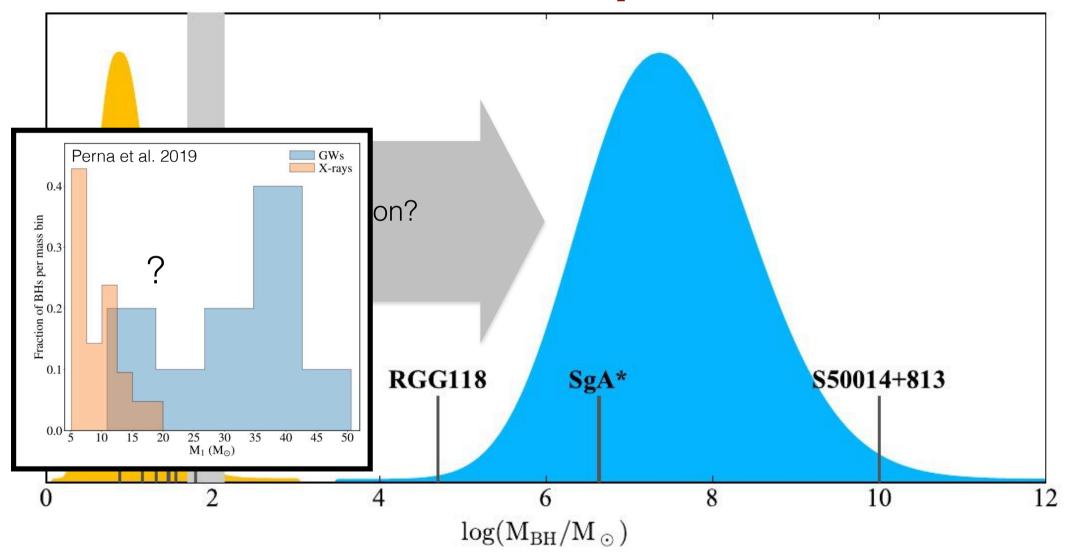


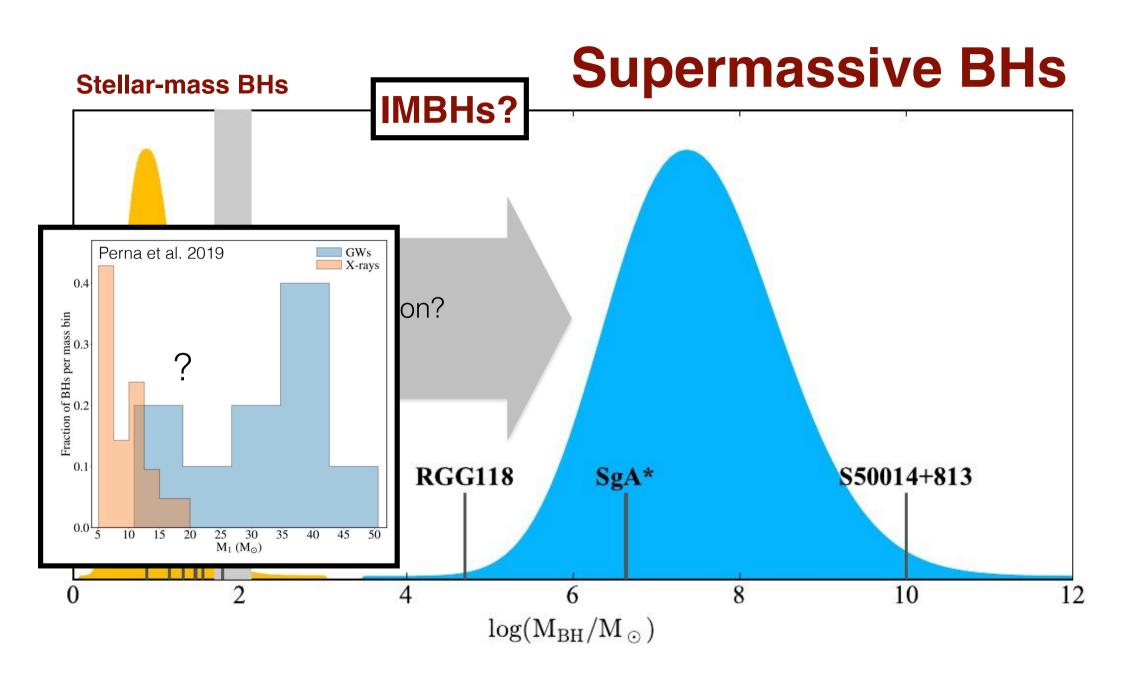
# **Supermassive BHs**

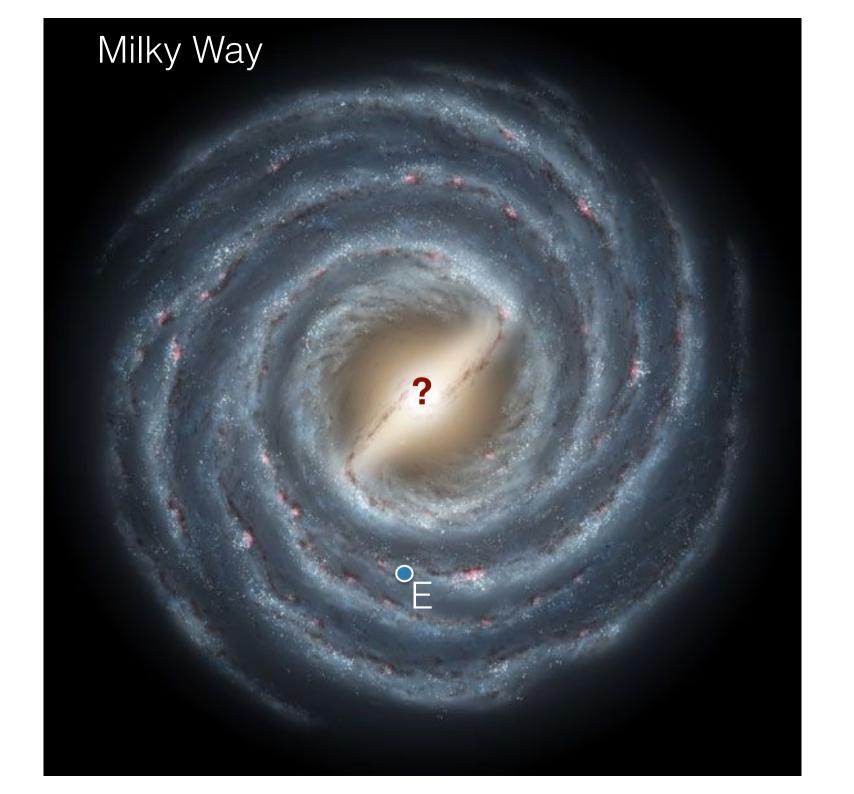


#### **Stellar-mass BHs**

# **Supermassive BHs**









Infrared/optical: lots of stars and dust

#### **Galactic Center**



#### **Galactic Center**

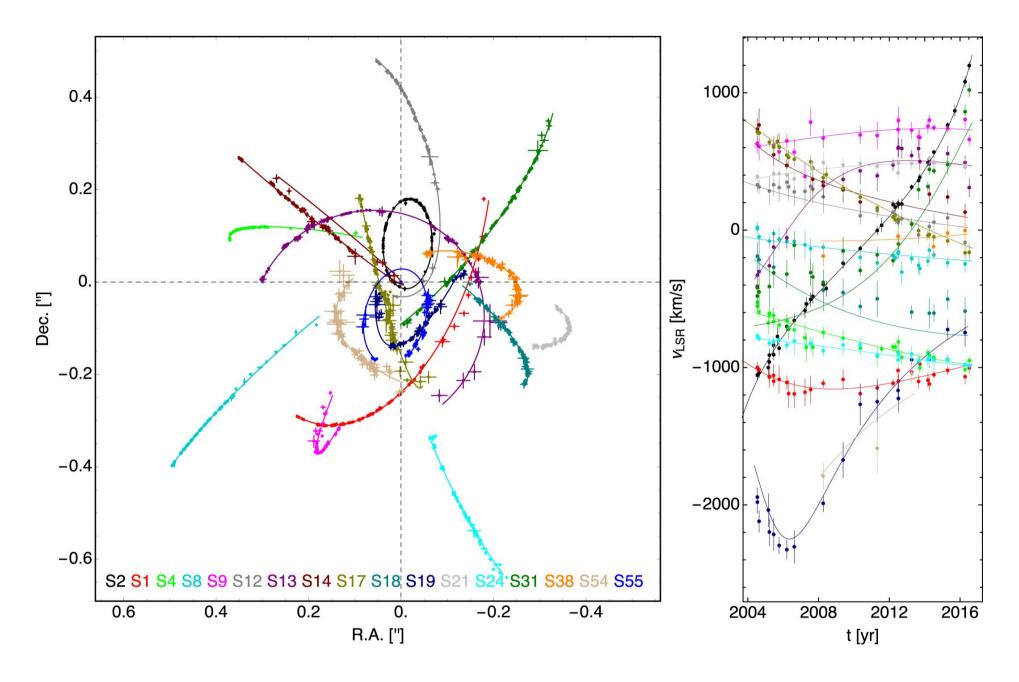
X-rays: hot gas and compact sources (XRBs, SNRs, ...)

# Infrared/optical: lots of stars and dust X-rays:

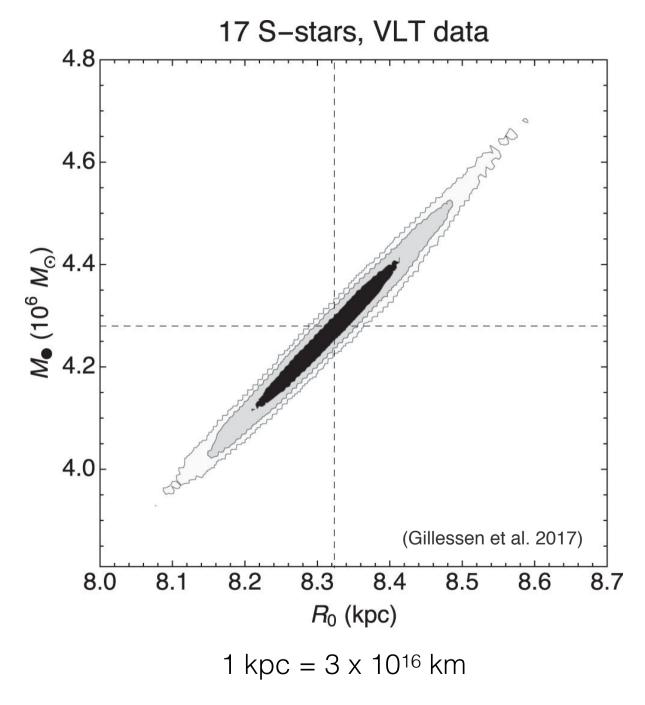
#### **Galactic Center**

X-rays: hot gas and compact sources (XRBs, SNRs, ...)

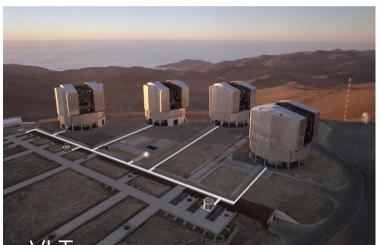
Radio: relativistic magnetised plasma



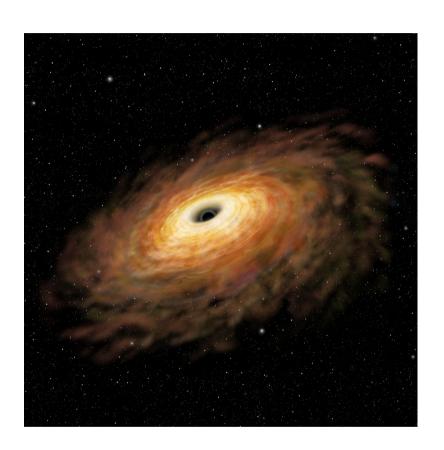
Orbits and radial velocities (along the line of sight) of 17 stars in the closest neighbourhood of Sgr A\* (Gillessen et al. 2017)

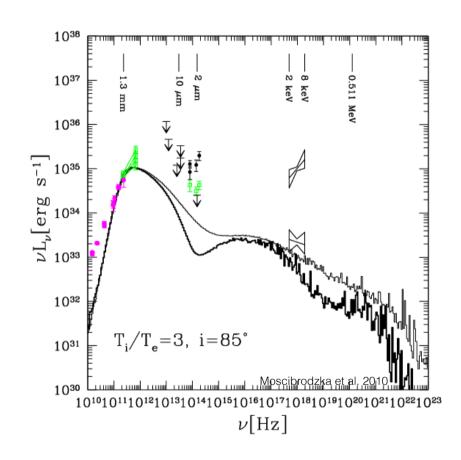


exact measurements of the mass and the distance of the SMBH residing in the center of our Galaxy!



## Can we resolve the Sgr A\* horizon?

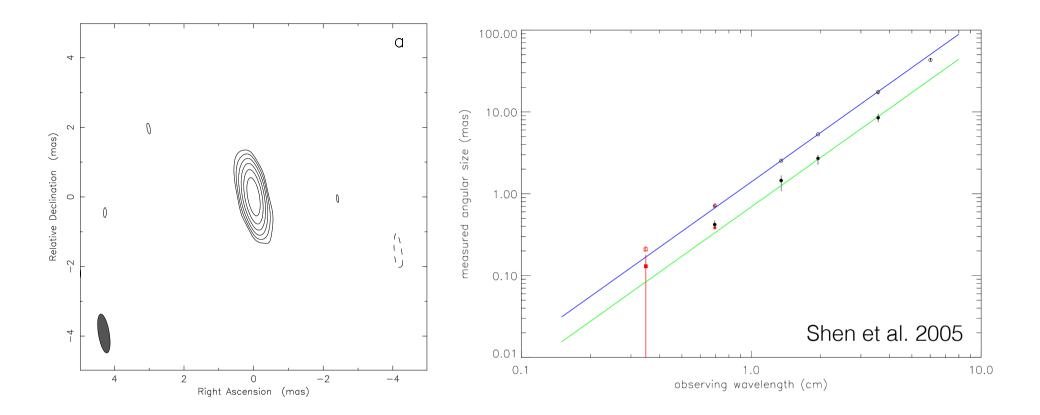




due to very low accretion rate of Sgr A\*, its accretion disk should be pronounced not only in X-rays, but also at radio frequencies ("Radiatively Inefficient Accretion Flow", with decoupled electron and proton temperatures, and hence broad-band emission continuum)

->

good news, as radio interferometers are of a much better angular resolution than the currently operating X-ray telescopes (in particular *Chandra*, with ~arcsec resolution)



VLBA observations with very high angular resolution (~0.2mas) at 3.5mm:

upper limit for the radio size of Sgr A\*

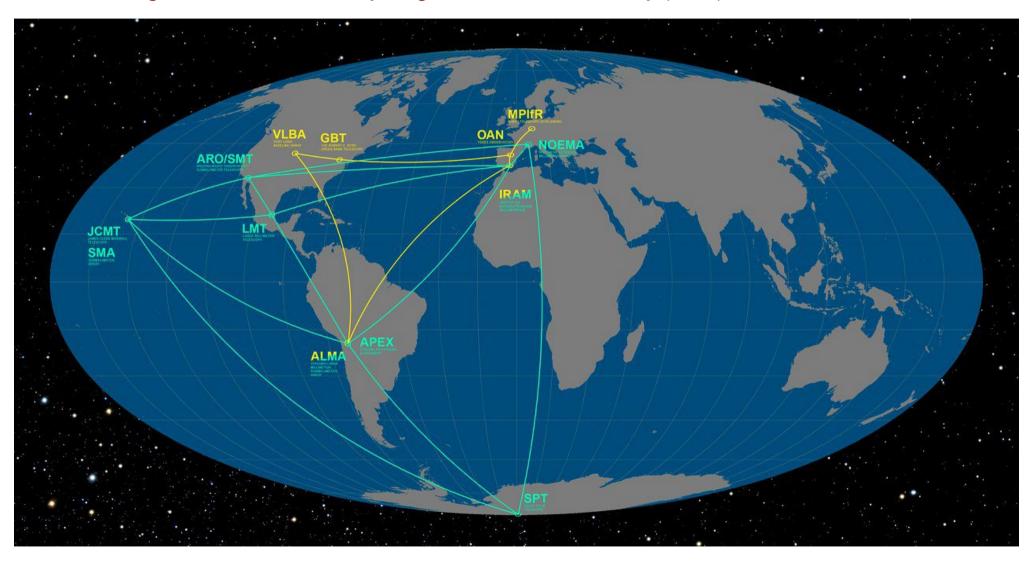
0.1 mas -> ~108 km ~1 AU

event horizon of Sgr A\* (assuming zero angular momentum)  $r_S \sim 10^7 \text{ km} \sim 0.1 \text{ AU} \rightarrow 0.01 \text{ mas}$ 

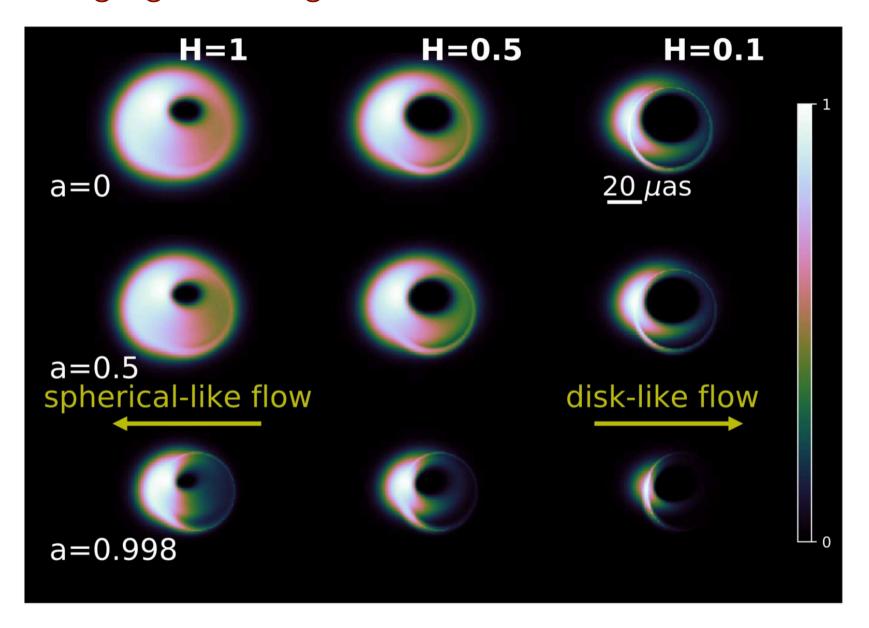


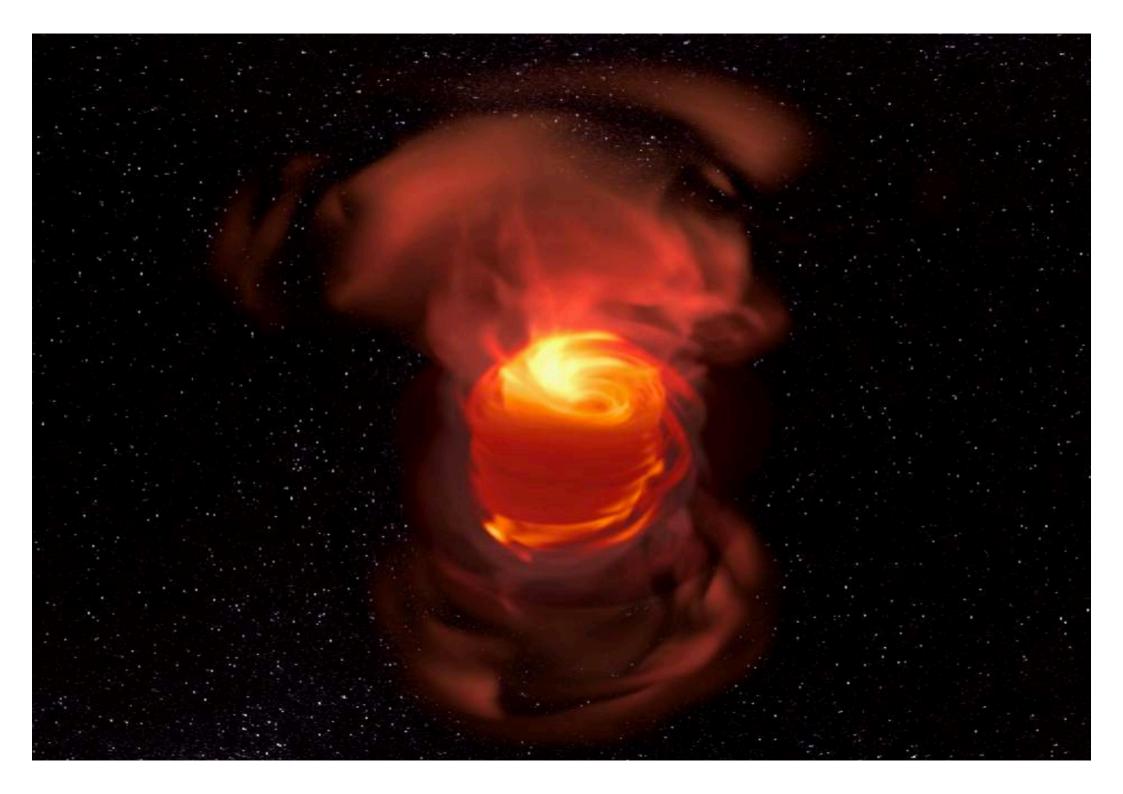
#### The **Event Horizon Telescope** (**EHT**)

is a project to create a large telescope array consisting of a global network of radio telescopes and combining data from several very-long-baseline interferometry (VLBI) stations around the Earth



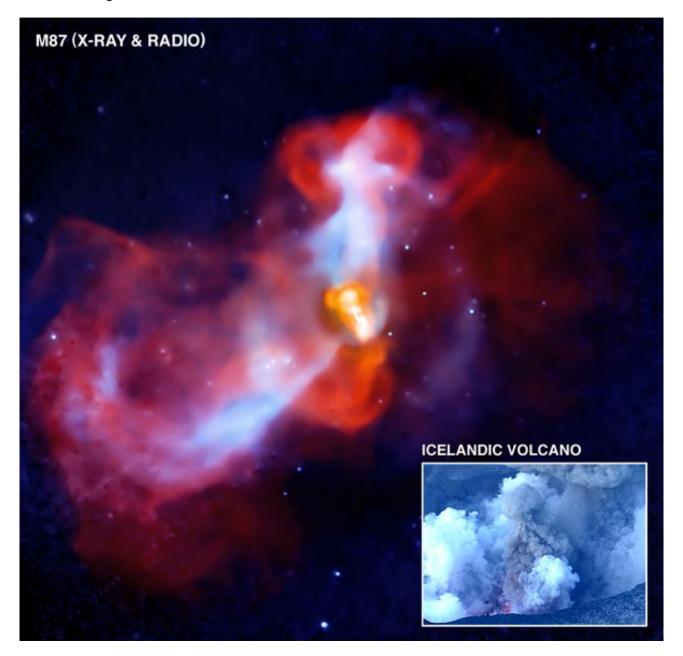
#### Imaging of the Sgr A\* event horizon with the EHT





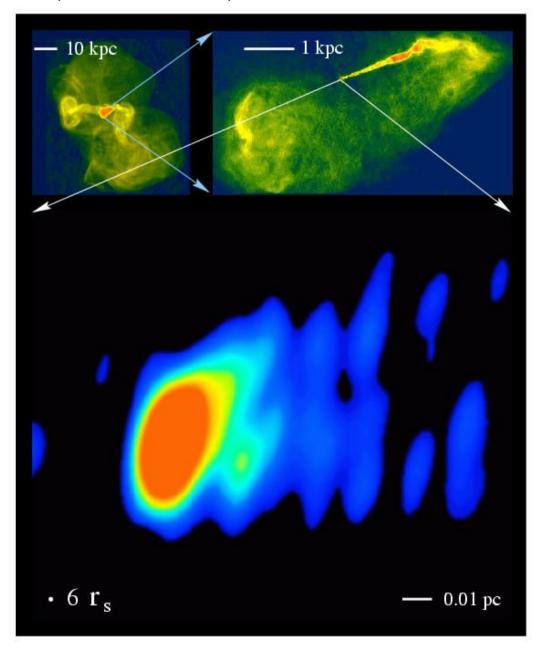
#### Virgo A (M87)

- · low-power but nearby radio galaxy, hosted by the dominant galaxy in the Virgo cluster
- · complex radio outflow interacting with the intracluster medium

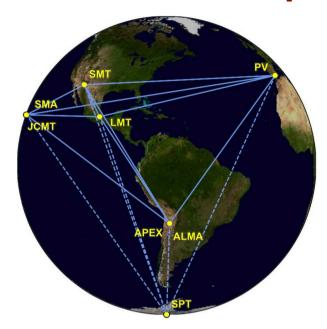


#### Virgo A (M87), distance 16 Mpc

· radio outflow can be traced from kpc scale down to sub-pc scale



#### The EHT 2017 campaign

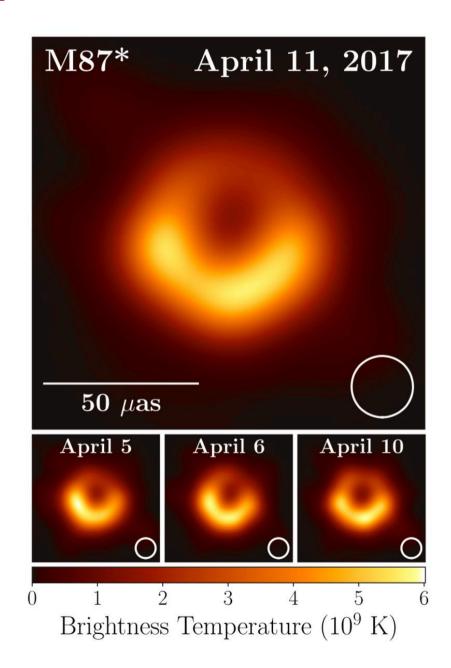


#### The EHT Collab. 2019:

"Comparing the data with an extensive library of synthetic images obtained from GRMHD simulations covering different physical scenarios and plasma conditions (...) allows us to derive an estimate for the black hole mass of

#### $M_{\bullet}=(6.5\pm0.7)\times10^9\ M_{\odot}$

Based on our modeling and information on the inclination angle, we derive the sense of rotation of the black hole to be in the clockwise direction, i.e., the spin of the black hole points away from us. The brightness excess in the south part of the emission ring is explained as relativistic beaming of material rotating in the clockwise direction as seen by the observer, i.e., the bottom part of the emission region is moving toward the observer."



## SMBH sphere of influence

event horizon (for a=0)

$$r_{\rm S} = \frac{2G\mathcal{M}_{\bullet}}{c^2} \approx 10^{-8} \left(\frac{\mathcal{M}_{\bullet}}{10^8 M_{\odot}}\right) \,\mathrm{kpc}$$

radius of the "sphere of influence" of the SMBH, i.e. the distance at which its gravitational potential significantly affects the motion of the stars or of the interstellar medium (for the velocity dispersion  $\sigma$  of the stars within the inner parts of the galaxy, i.e. spheroidal component whose stellar dynamics is dominated by random motions, not by rotation)

$$\frac{1}{2}\sigma^2 = \frac{G\mathcal{M}_{\bullet}}{r_{\rm i}} \longrightarrow r_{\rm i} = \frac{2G\mathcal{M}_{\bullet}}{\sigma^2} \approx 10^{-2} \left(\frac{\mathcal{M}_{\bullet}}{10^8 M_{\odot}}\right) \left(\frac{\sigma}{200 \, \rm km \, s^{-1}}\right)^{-2} \, \rm kpc$$

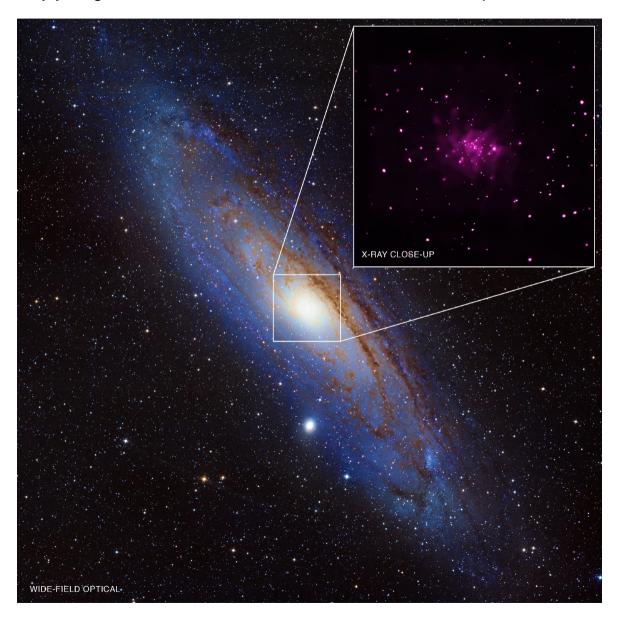
linear scale of a typical galaxy

$$r_{\rm G} \sim 10 - 100 \, {\rm kpc}$$

the direct dynamical influence of central SMBHs is relevant only in the innermost regions of the galaxies, and— due to the limited angular resolution of the optical telescopes— could be observable only in most massive galaxies located at distances <100 Mpc (effectively ~100 systems)

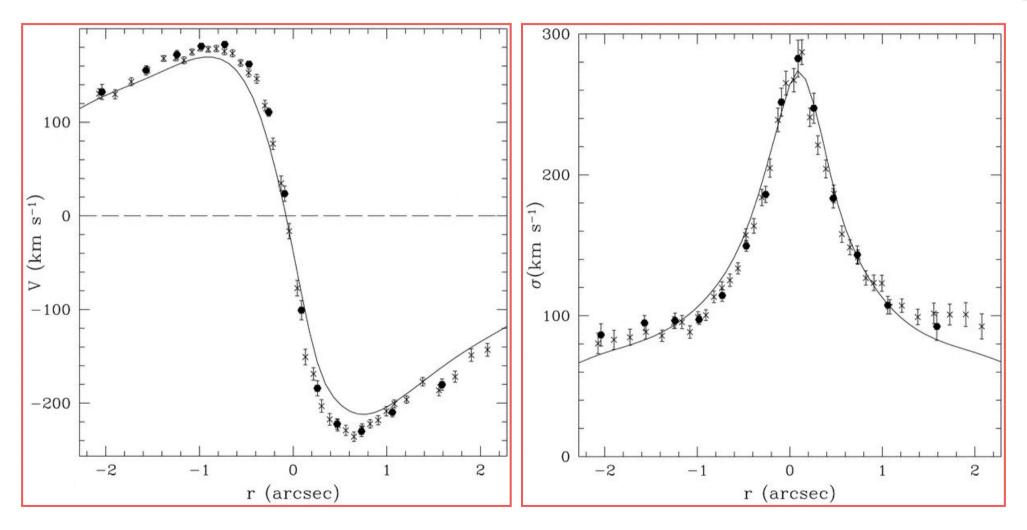
#### Andromeda (M31)

- relatively large angular size of the sphere of influence of the central SMBH
- small amount of gas, relatively young stars in the center, with older stars located at Keplerian orbits within the outer disk



#### Andromeda (M31), distance 0.7 Mpc

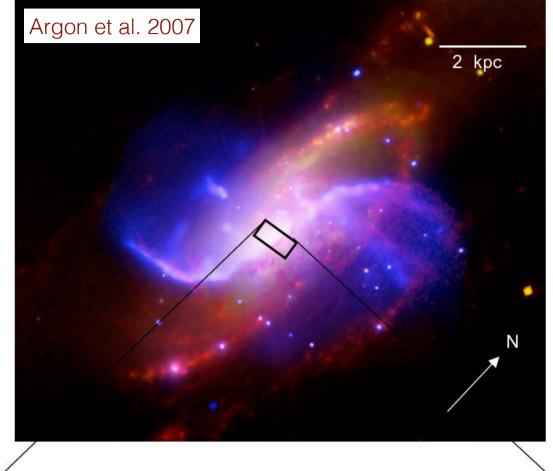
radial velocities and velocity dispersion of the stars in the central parts of the galaxy gives  $M_{\bullet}$  = (1.1-2.3) x 10<sup>8</sup>  $M_{\odot}$ 



Peiris & Tremaine 2003 Bender et al. 2005

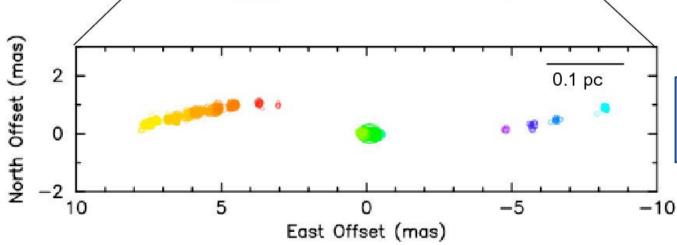
**NGC 4258** (M106)

(X-ray – blue, Optical – gold, IR – red, Radio – purple)



An atom or molecule may absorb a photon and move to a higher energy level, or the photon may stimulate emission of another photon of the same energy causing a transition to a lower energy level.

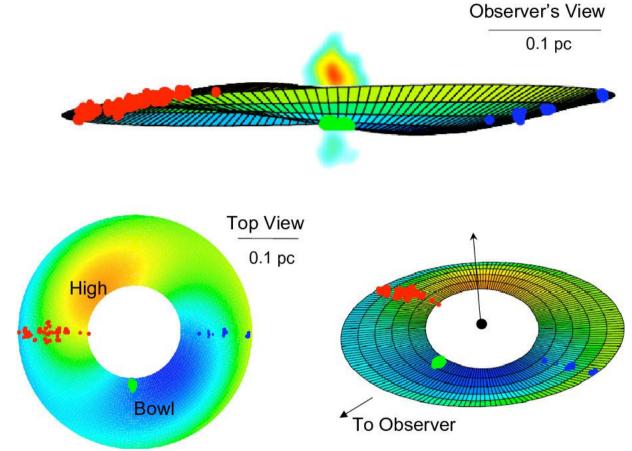
Producing a maser requires population inversion, i.e., a system with more members in a higher energy level relative to a lower energy level. Water maser emission is observed primarily at 22 GHz, due to a transition between rotational energy levels in the water molecule.



MASER = Microwave Amplification by Stimulated Emission of Radiation

**NGC 4258**, distance 7.6 Mpc

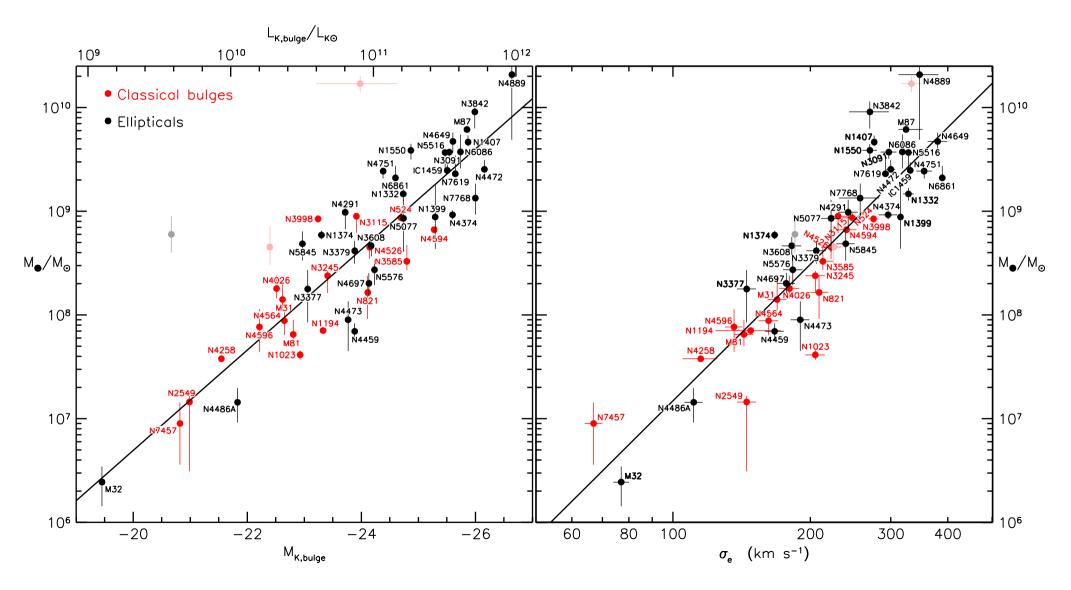
$$M_{\bullet}=(3.9\pm0.1)\times10^{7}\,M_{\odot}$$



The phenomenon is observed in edge disks hosting molecular clouds with water molecules. The rest frame frequency of the maser line is 22 GHz and by measuring the Doppler shift due to the Keplerian motion of the clouds, the rotation curve of the disk can be measured, allowing the estimate of the BH mass.

Maser emission occurs only along the major axes (at 90deg with respect to the observer line of sight) of the edge on disk, and the nearest semi minor axis (if a photon at 22 GHz is emitted at an angle with respect to the observer, it encounters molecules with different radial velocities along the line of sight to the observer and Doppler shifts prevent maser amplification to occur).

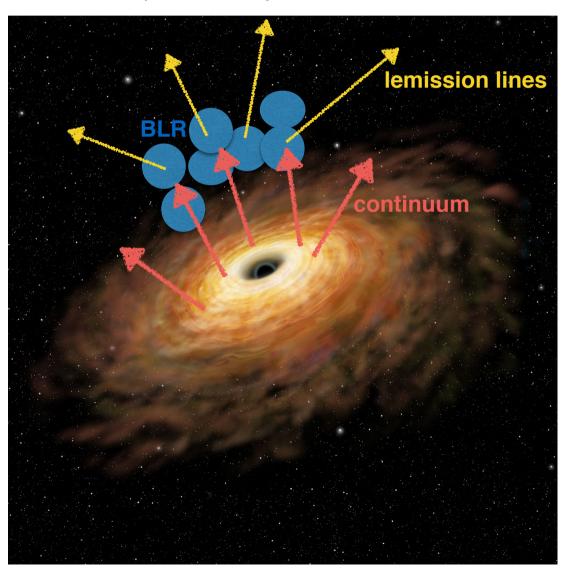
# Empirical <u>correlations</u> between the stellar velocity dispersion or the luminosity of a galaxy bulge and the mass of the central SMBH



#### Continuum and line emission in type I AGN

<u>continuum:</u> dominated by the innermost parts of the accretion disk, around the innermost stable circular orbit, i.e. a few/several r<sub>S</sub>

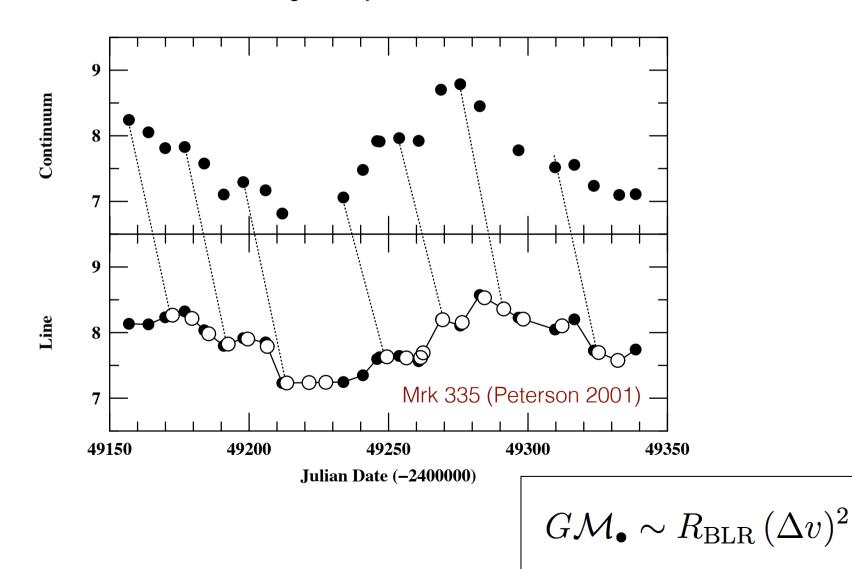
broad emission lines: emitted from atomic gas in the "broad line region" (BLR), i.e. from dense clouds located at some distance from the central engine (but within the SMBH sphere of influence), and photoionized by the disk continuum

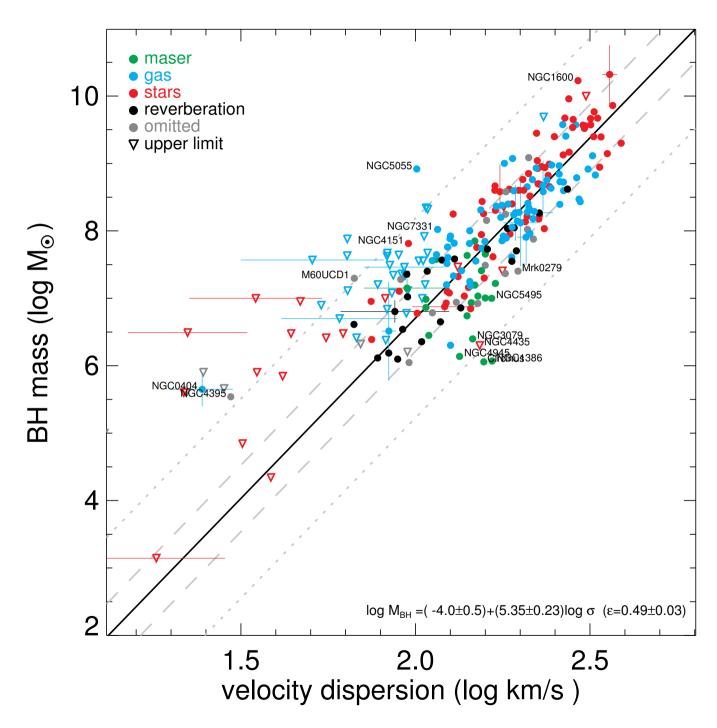


**Reverberation mapping**: the emission line variability shows a time lag with respect to the continuum — line intensity varies in response to the continuum variability, with a time lag  $\Delta t$  that can be associated to the distance of the BLR from the central engine via RBLR = c $\Delta t$ .

From the full width half maximum (FWHM) of the broad lines the velocity  $\Delta v$  of the gas can be inferred.

Main issues here: geometry and structure of the BLR!

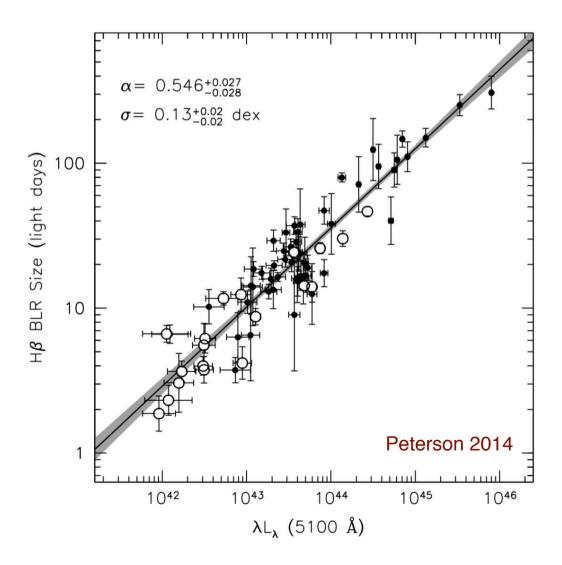




#### What if $\Delta t$ cannot really be measure?

Empirical correlation between the characteristic broad-line region size R<sub>BLR</sub> and the optical (continuum) AGN luminosity!

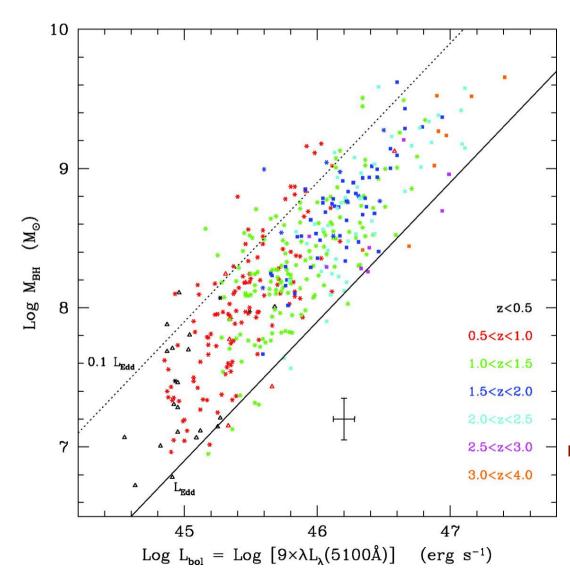
This relationship allows us to estimate R<sub>BLR</sub> from the luminosity in a single AGN spectrum, thus bypassing the resource-intensive RM technique



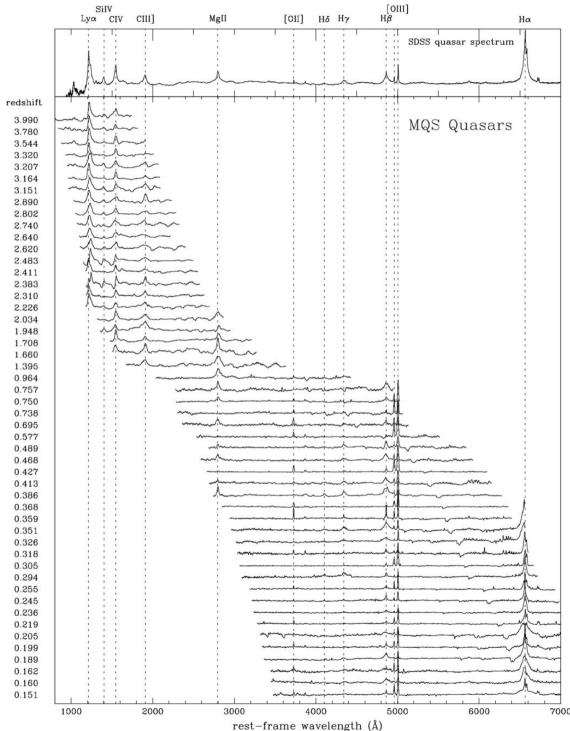
This single-epoch method is used for large samples of distant AGN using any of the prominent emission lines. The absolute mass uncertainty is evaluated statistically to be a factor ~4 to 5 on an absolute scale and a factor ~3 relative to the reverberation mapping-based mass. Individual mass estimates can be uncertain by a factor of ~10, however.

$$\log M_{\rm BH}({\rm H}\beta) = \log \left[ \left( \frac{\rm FWHM(H\beta)}{1000 \text{ km s}^{-1}} \right)^2 \left( \frac{\lambda L_{\lambda}(5100 \text{ Å})}{10^{44} {\rm erg s}^{-1}} \right)^{0.50} \right] + (6.91 \pm 0.02)$$

e.g., Vestergaard & Peterson 2006



e.g., Kollmeier et al. 2006 (point types denote the emission line used for the mass measurement, with open triangles, asterisks, and filled squares corresponding to H-beta, Mg II, and C IV, respectively)

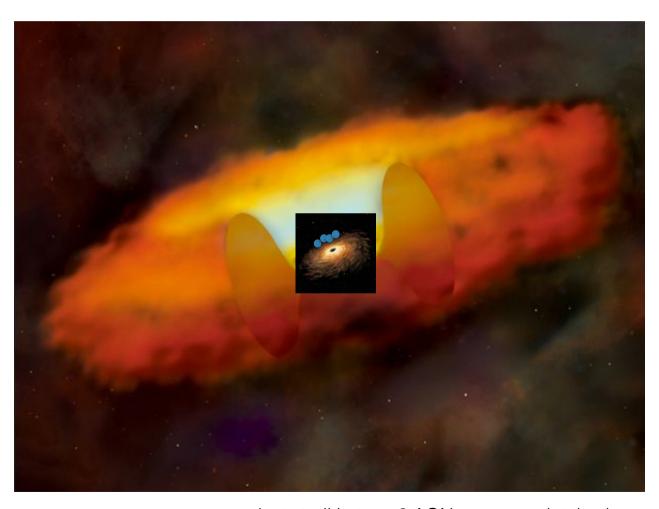


Kozlowski et al. 2013 spectroscopy of OGLE quasars

# Active galaxies: accretion disks



# Active galaxies: accretion disks and dusty torii



dusty torii in type 2 AGN may completely obscure central engines, i.e. accretion disks and BLRs...

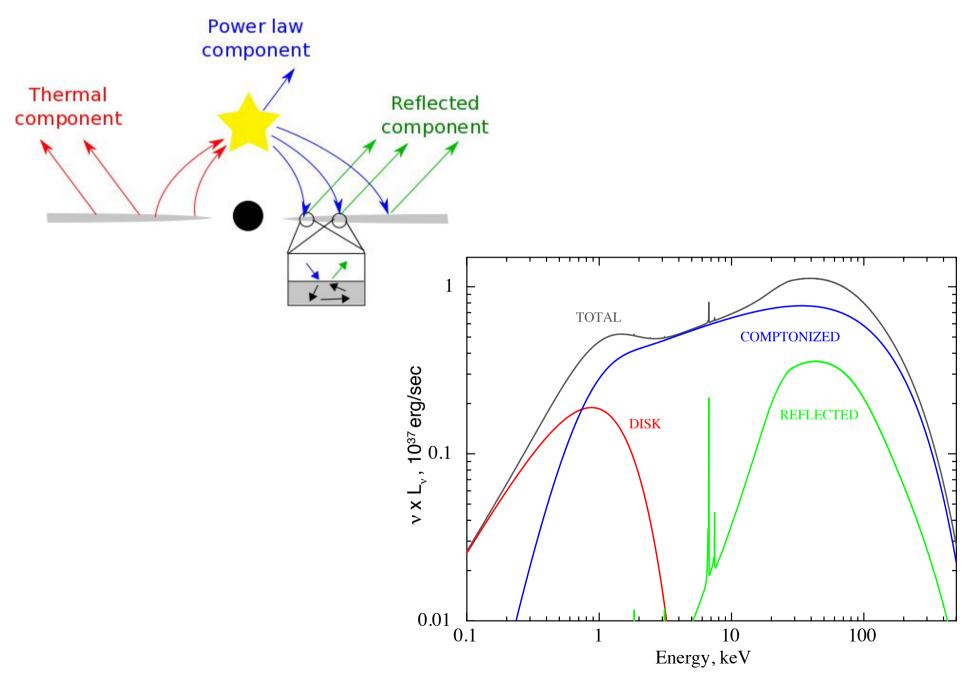
# "Hidden" active galactic nuclei



# X-ray spectroscopy



#### X-ray emission of accretion disk coronae

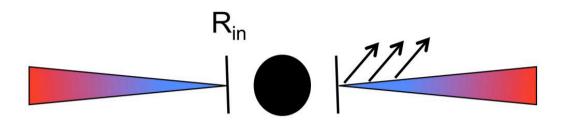


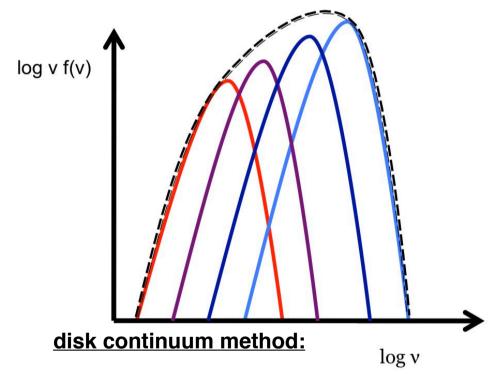
## Spin measurements in BH XRBs

To date, there are three methods that have been widely applied in estimating the spins of stellar-mass BHs (Remillard & McClintock 2006), namely,

- · fitting the thermal continuum spectrum of the accretion disk,
- · modeling the disk reflection spectrum with a focus on the Fe K line, and
- modeling high-frequency (~100–450 Hz) quasi-periodic oscillations (HFQPOs).

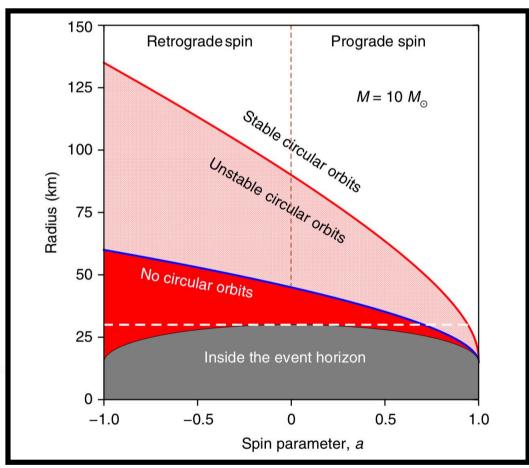
While there are well-established models underpinning the first two methods, there is no agreed upon, or even leading, model of HFQPOs. Many classes of models have been proposed including several types of resonance models; global oscillation ("disko- seismic") modes of the accretion disk; orbiting hot spots; tidal disruption of large inhomogeneities in the accretion flow; and the "relativistic precession" model.

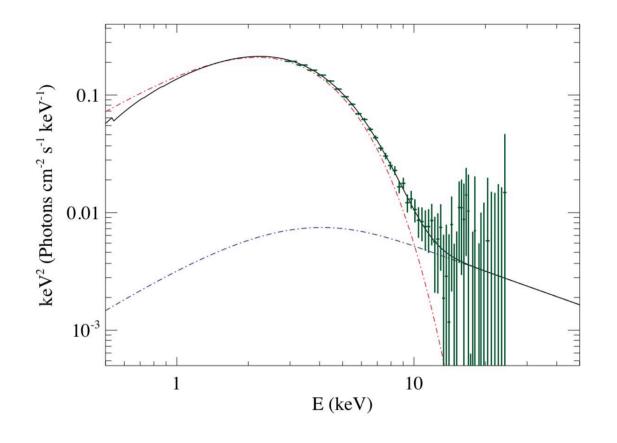




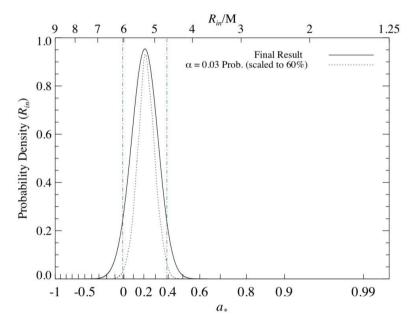
disk temperature depends on the radius, while at the same time the innermost stable circular orbit is a function of the BH spin:

risco ~6rg for a=0, and ~rg for a=1

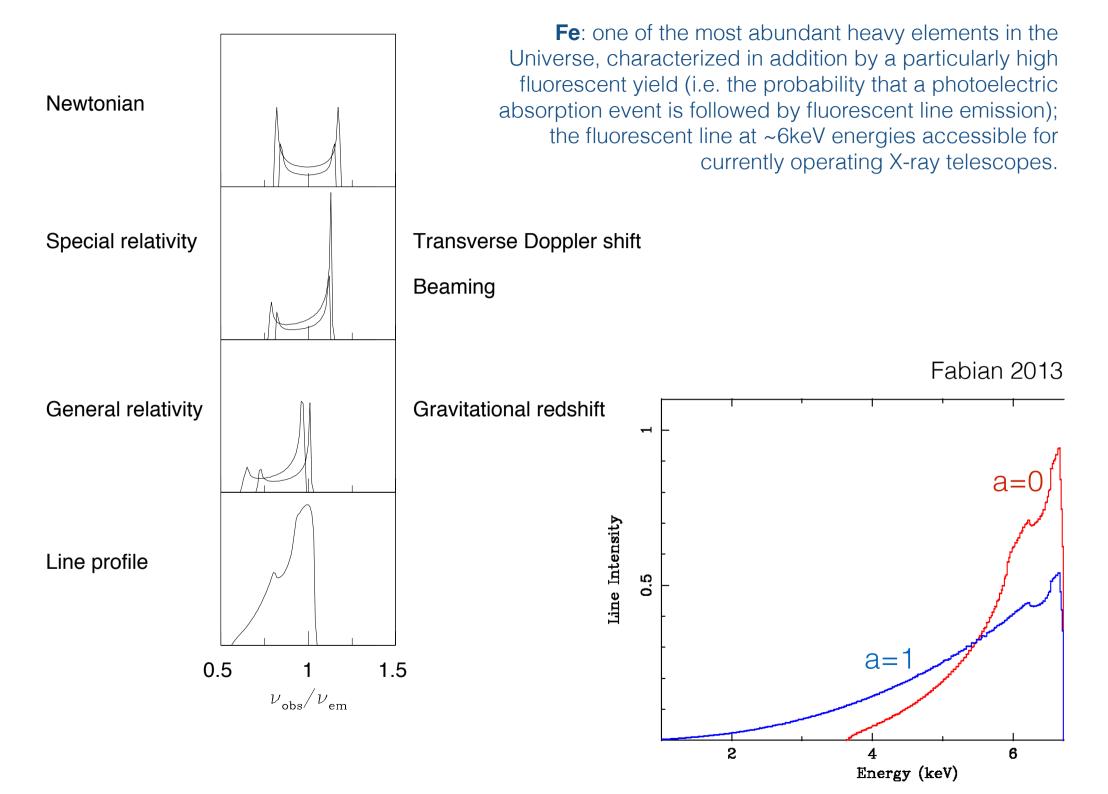


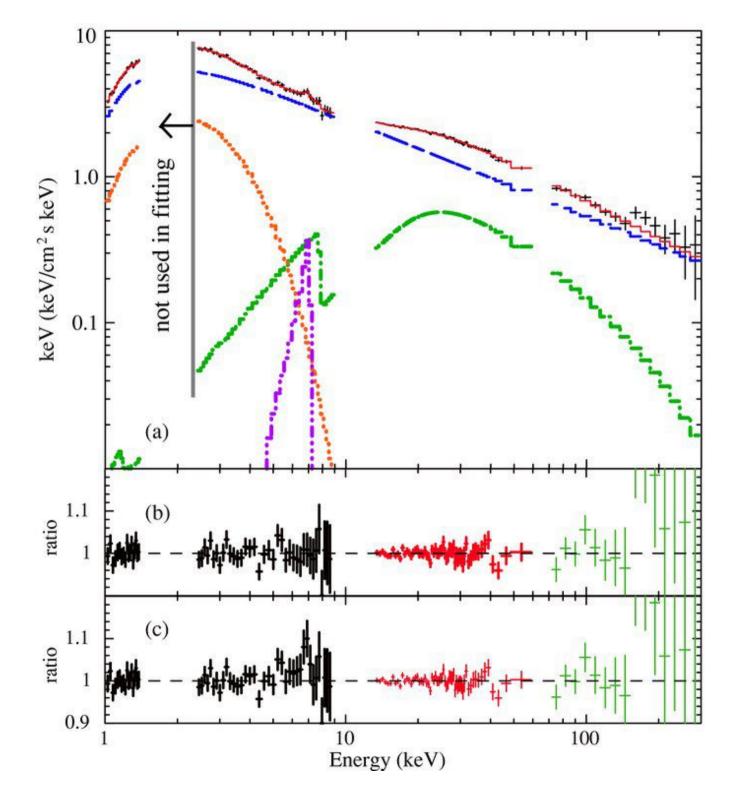


LMC X-3
Steiner et al. 2014



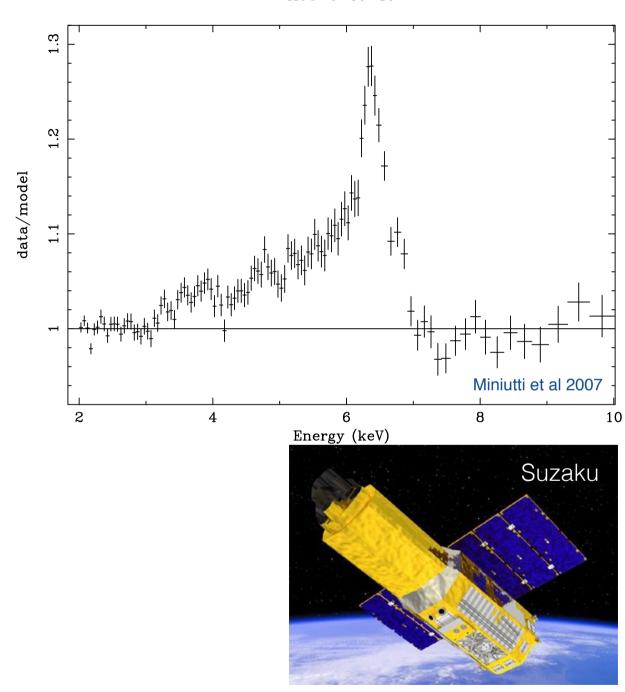
Method not applicable to AGN, as the BH mass estimates for AGN are much less accurate, and also the AGN disk continuum peaks at UV frequencies which are hardly accessible observationally





GX 339-4 Yamada et al. 2009

$a_*$ (Continuum)	$a_*$ (Iron)
> 0.98	$0.98\pm0.01$
> 0.98	> 0.95
_	> 0.98
_	> 0.98
_	> 0.96
$0.92 \pm 0.06$	$0.97^{+0.02}_{-0.25}$
< 0.9	$0.95 \pm 0.03$
_	$0.92 \pm 0.06$
_	$0.88 \pm 0.03$
$0.84 \pm 0.05$	×
$0.80 \pm 0.10^{\star}$	_
$\gtrsim 0.7$	_
_	$0.76^{+0.11}_{-0.15}$
_	$0.84 \sim 0.98$
$0.70\pm0.10^{\star}$	> 0.9
$0.63^{+0.16}_{-0.19}$	_
_	< 0.5
$0.34 \pm 0.28$	$0.55^{+0.15}_{-0.22}$
$0.25 \pm 0.15$	
$0.2 \pm 0.3$	_
$0.12 \pm 0.19$	×
< -0.2	
	$> 0.98$ $  0.92 \pm 0.06$ $< 0.9$ $ 0.84 \pm 0.05$ $0.80 \pm 0.10^*$ $\gtrsim 0.7$ $ 0.70 \pm 0.10^*$ $0.63^{+0.16}_{-0.19}$ $ 0.34 \pm 0.28$ $0.25 \pm 0.15$ $0.2 \pm 0.3$ $0.12 \pm 0.19$



## **AGN**

Object	$a_*$ (Iron)
IRAS 13224-3809	> 0.99
Mrk 110	> 0.99
NGC 4051	> 0.99
1H0707-495	> 0.98
RBS 1124	> 0.98
NGC 3783	> 0.98
NGC 1365	$0.97^{+0.01}_{-0.04}$
Swift J0501-3239	> 0.96
PDS 456	> 0.96
Ark 564	$0.96^{+0.01}_{-0.06}$
3C120	> 0.95
Mrk 79	> 0.95
NGC 5506	$0.93^{+0.04}_{-0.04}$
MCG-6-30-15	$0.91^{+0.06}_{-0.07}$
Ton S180	$0.91^{+0.02}_{-0.09}$
1H0419-577	> 0.88
IRAS 00521-7054	> 0.84
Mrk 335	$0.83^{+0.10}_{-0.13}$
Ark 120	$0.81^{+0.10}_{-0.18}$
Swift J2127+5654	$0.6^{+0.2}_{-0.2}$
Mrk 841	> 0.56
Fairall 9	$0.52^{+0.19}_{-0.15}$
	0.15

## Accretion history of SMBHs (Soltan 1982)

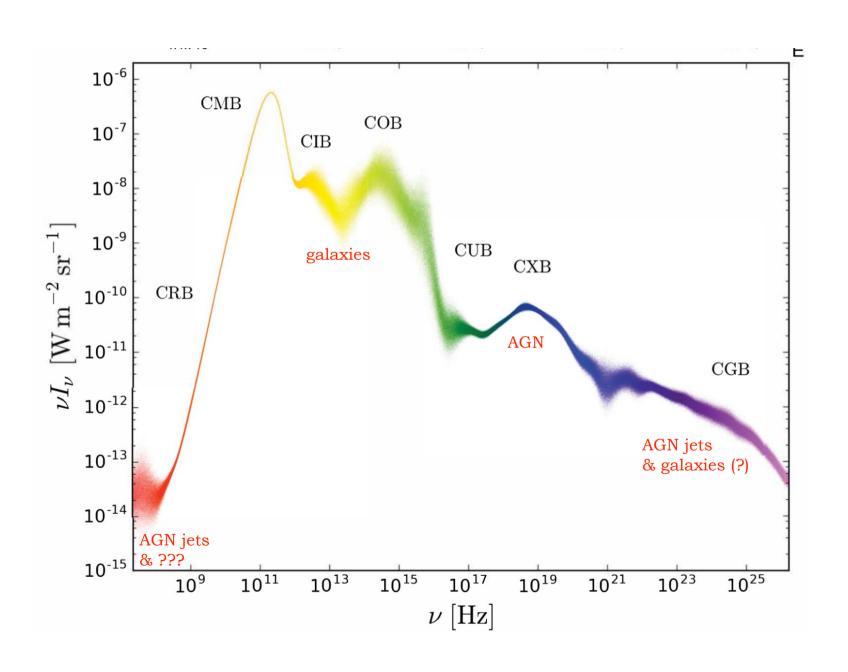
Since the energy radiated away by a quasar after accreting the mass  $M_{acc}$  during the quasar lifetime is  $\eta_d$   $M_{acc}c^2 \sim \int\!\!dt\;L$ , while at the same time the BH growth is  $M_{BH}$  =(1- $\eta_d$ )  $M_{acc}$ , knowing the total electromagnetic energy density produced by quasars in the entire Universe (the X-ray background!), U =  $\int\!\!dt\int\!dL\;L\;\Phi(L,z)$ , and the mass density of SMBHs in the local Universe,  $\rho_{BH}$ , one may estimate the average radiative efficiency of the accreting matter,  $\eta_d$ , and hence the average BH spin for the quasar population:  $\rho_{BH}\,c^2$  = (1- $\eta_d$ ) U /  $\eta_d$ 

```
\eta_d = 0.07 non-rotating BH accreting at the Eddington rate (a = 0) \eta_d = 0.42 maximally-rotating BH accreting at the Eddington rate (a = 1)
```

 $(\eta_d \ll 0.1 \text{ for BH accreting at the sub-Eddington rate})$ 

The luminosity function of quasars as a function of redshift, or equivalently the extragalactic background light, reflects the accretion history of local remnant SMBHs (Small & Blandford 1992).

## Extragalactic Background Light



## SMBH demographics

#### Graham & Driver 07

				Granam a Birror or
Study	Method	$\rho_{\mathrm{bh,0}} \; (\mathrm{E/S0})$	$\rho_{\mathrm{bh,0}}~(\mathrm{Sp})$	$\rho_{ m bh,0} \; ({ m total})$
		$10^5 M_{\odot}~{ m Mpc^{-3}}$	$10^5 M_{\odot}~{ m Mpc^{-3}}$	$10^5 M_{\odot}~{ m Mpc^{-3}}$
Graham et al. (2007)	$M_{ m bh}\!\!-\!\!n$	$(3.46 \pm 1.16)h_{70}^3$	$(0.95 \pm 0.49)h_{70}^3$	$(4.41 \pm 1.67)h_{70}^3$
Wyithe (2006)	$M_{ m bh}\!\!-\!\!\sigma$			$(1.98 \pm 0.38)h_{70}^{3}$
Fukugita & Peebles (2004)	$ ho_{ m spheroid}$	$(3.4^{+3.4}_{-1.7})h_{70}$	$(1.7^{+1.7}_{-0.8})h_{70}$	$(5.1^{+3.8}_{-1.9})h_{70}$
Marconi et al. (2004)	$M_{ m bh}$ – $(L,\sigma)$	$3.3h_{70}^{0.74}f(h)$	$1.3h_{70}^{0.74}f(h)$	$(4.6^{+1.9}_{-1.4})h_{70}^{0.74}f(h)$
Shankar et al. (2004)	$M_{ m bh}\!\!-\!\!L$	$(4.3^{+1.3}_{-1.1})h^{0.5}_{70}f(h)$	$(1.5^{+0.7}_{-0.7})h^{0.5}_{70}f(h)$	$(5.9^{+1.5}_{-1.5})h_{70}^{0.5}f(h)$
Shankar et al. (2004)	$M_{ m bh}$ – $\sigma$	$(3.4^{+1.1}_{-0.7})h^3_{70}$	$(1.4^{+0.5}_{-0.3})h^3_{70}$	$(4.8^{+1.2}_{-0.8})h^3_{70}$
McLure & Dunlop (2004)	$M_{ m bh}\!\!-\!\!L$	$(4.8 \pm 0.7) h_{70}^{0.5} f(h)$		
Wyithe & Loeb (2003)	$M_{ m bh}$ – $\sigma$	•••		$(2.1^{+3.4}_{-1.3})h_{70}^3$
Aller & Richstone (2002)	$M_{ m bh}$ – $\sigma$	$(4.5 \pm 1.5)h_{70}^{0.39}f(h)$	$(1.4 \pm 1.3)h_{70}^{0.39}f(h)$	$(5.9 \pm 2.0) h_{70}^{0.39} f(h)$
Yu & Tremaine (2002)	$M_{ m bh}$ – $\sigma$	$(2.0 \pm 0.2) h_{70}^3$	$(0.9 \pm 0.2) h_{70}^3$	$(2.9 \pm 0.4) h_{70}^3$
Merritt & Ferrarese (2001)	$ ho_{ m spheroid}$			$4.6h_{70}$
Salucci et al. (1999)	$ ho_{ m spheroid}$	$6.2h_{70}^2$	$2.0h_{70}^2$	$8.2h_{70}^2$

Soltan (1982), Fabian and Iwasawa (1999), Elvis et al. (2002), Yu and Tremaine (2002), Marconi et al. (2004):

## **Conclusions**

#### **Stellar-mass BHs:**

BH masses in XRBs —
orbital modulation (companion star)
BH spins in XRBs —
accretion disk continua (accretion disks)
Fe fluorescence lines (disk coronae; X-ray reflection)
QPOs in accretion disk lightcurves ?
BH masses & spins in binary BHs —
GW detections with LIGO/Virgo/KAGRA

#### **SMBHs:**

BH masses — individual stars (Sgr A\*) stellar/gas velocity dispersion, maser emission (nearby non-active galaxies) reverberation mapping (nearby type 1 AGN; geometry and structure of the BLR) broad emission lines (type 1 AGN; geometry and structure of the BLR)

BH spin:

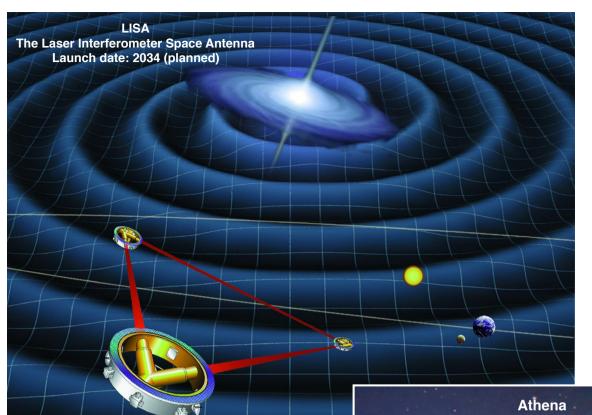
Fe fluorescence line (broad-band X-ray coverage with fine energy resolution) CXB (population; quasar luminosity function & cosmic background radiation)

**Stellar-mass BHs:** mass distribution and binary merger rates?

**IMBHs:** where are they? how many?

**SMBHs:** change their masses and spins during cosmological co-evolution with host galaxies!

**EHT:** radio imaging of the event horizon in Sgr A\* (and M87?) **ATHENA:** broad-band X-ray spectroscopy with <10eV energy resolution (earlier: XRISM) **LISA:** mergers of IMBHs and SMBH?



## **FUTURE**

