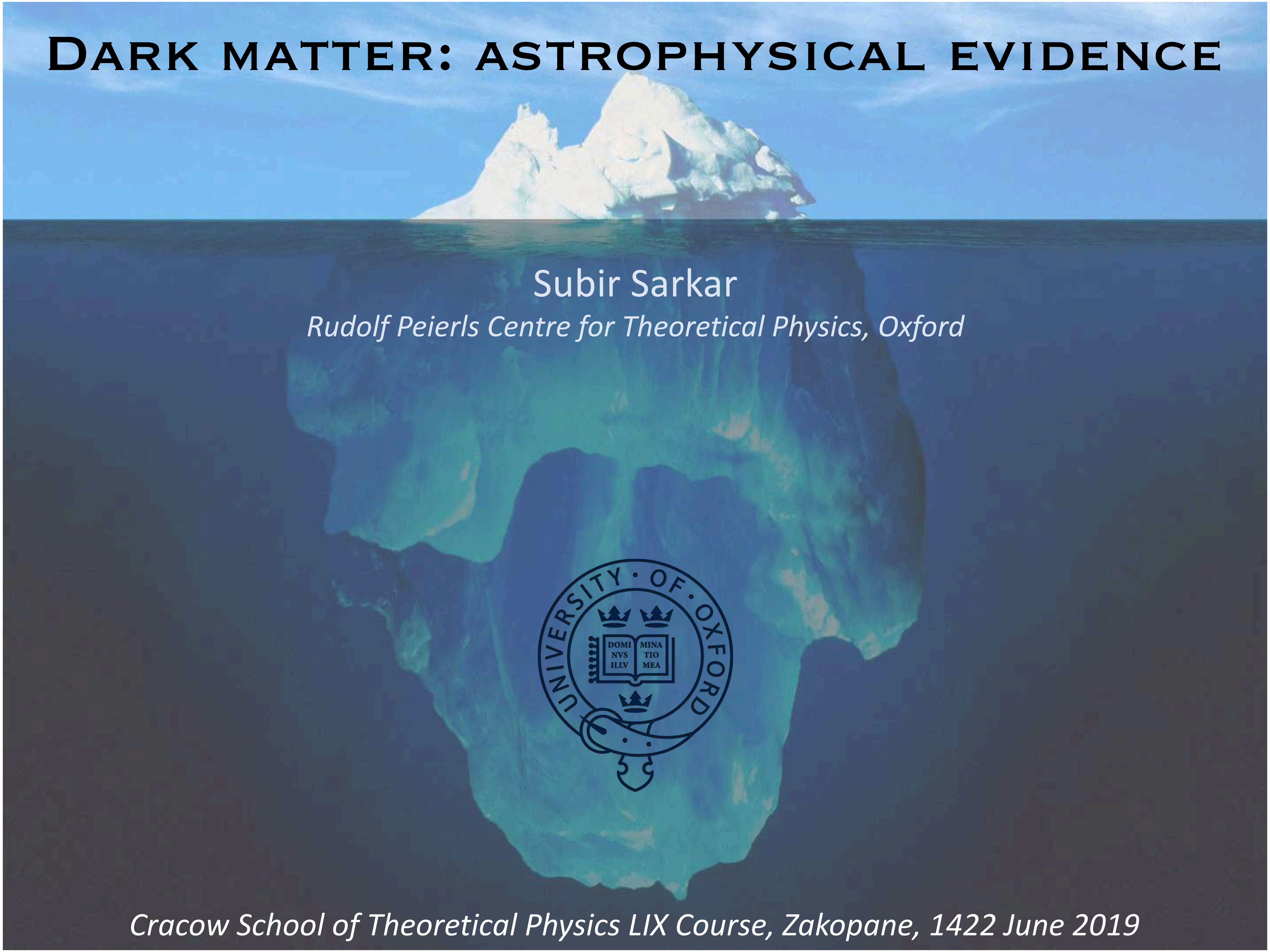


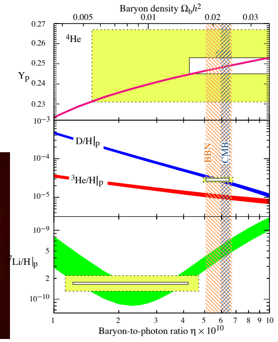
Dark matter: astrophysical evidence

Subir Sarkar

Rudolf Peierls Centre for Theoretical Physics, Oxford

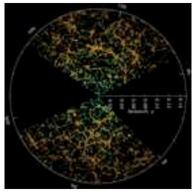
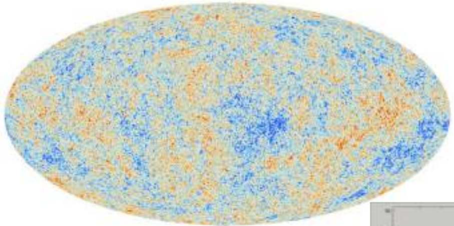
Cracow School of Theoretical Physics LIX Course, Zakopane, 1422 June 2019

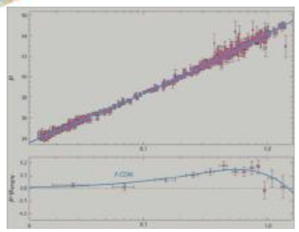
What is the world made of?

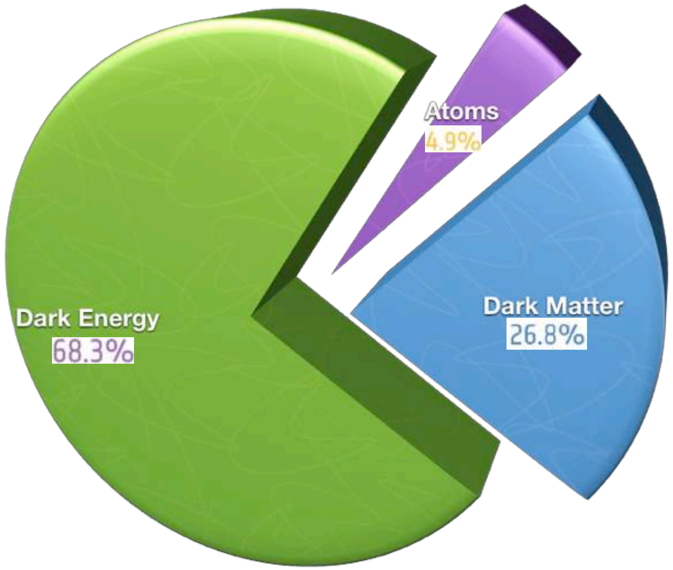



Baryons (*no* anti-baryons)

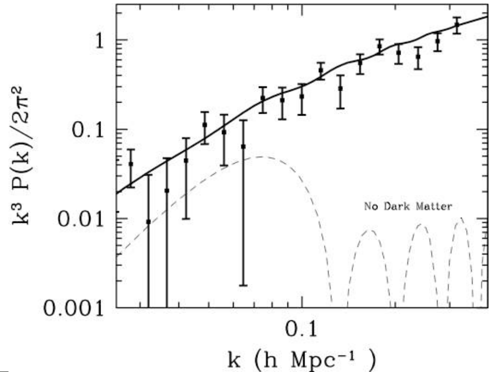
Mainly geometrical evidence:
$\Lambda \sim O(H_0^2)$, $H_0 \sim 10^{-42}$ GeV
… dark energy is *inferred* from the 'cosmic sum rule':
$\Omega_m + \Omega_k + \Omega_\Lambda = 1$

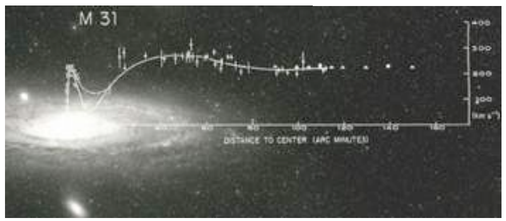


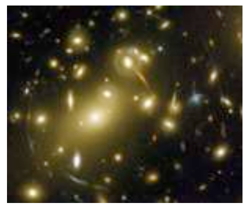


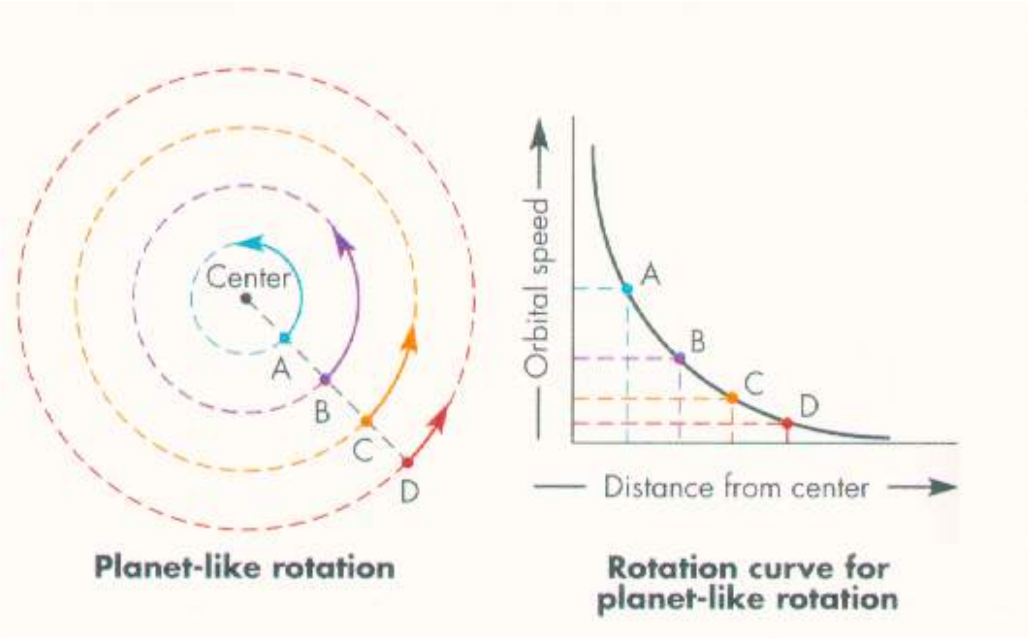
Both geometrical and dynamical evidence (assuming GR to be valid)



Both the baryon asymmetry and dark matter require that there be *new* physics beyond the Standard $SU(3)_c \times SU(2)_L \times U(1)_Y$ Model
… dark energy is even more mysterious (but as yet lacks compelling *dynamical* evidence)



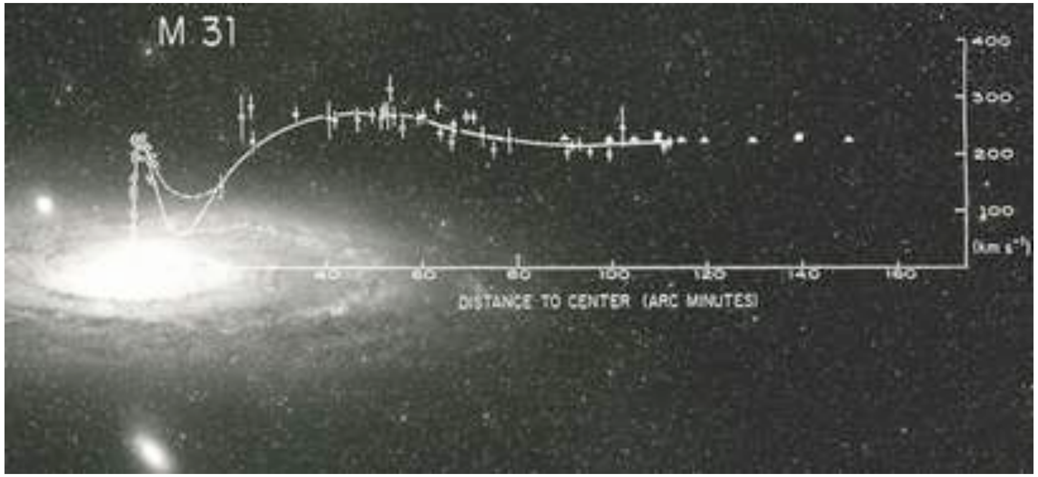




At large distances from the centre, beyond the edge of the visible galaxy, the velocity should fall as $1/\sqrt{r}$ if most of the matter is in the optical disc

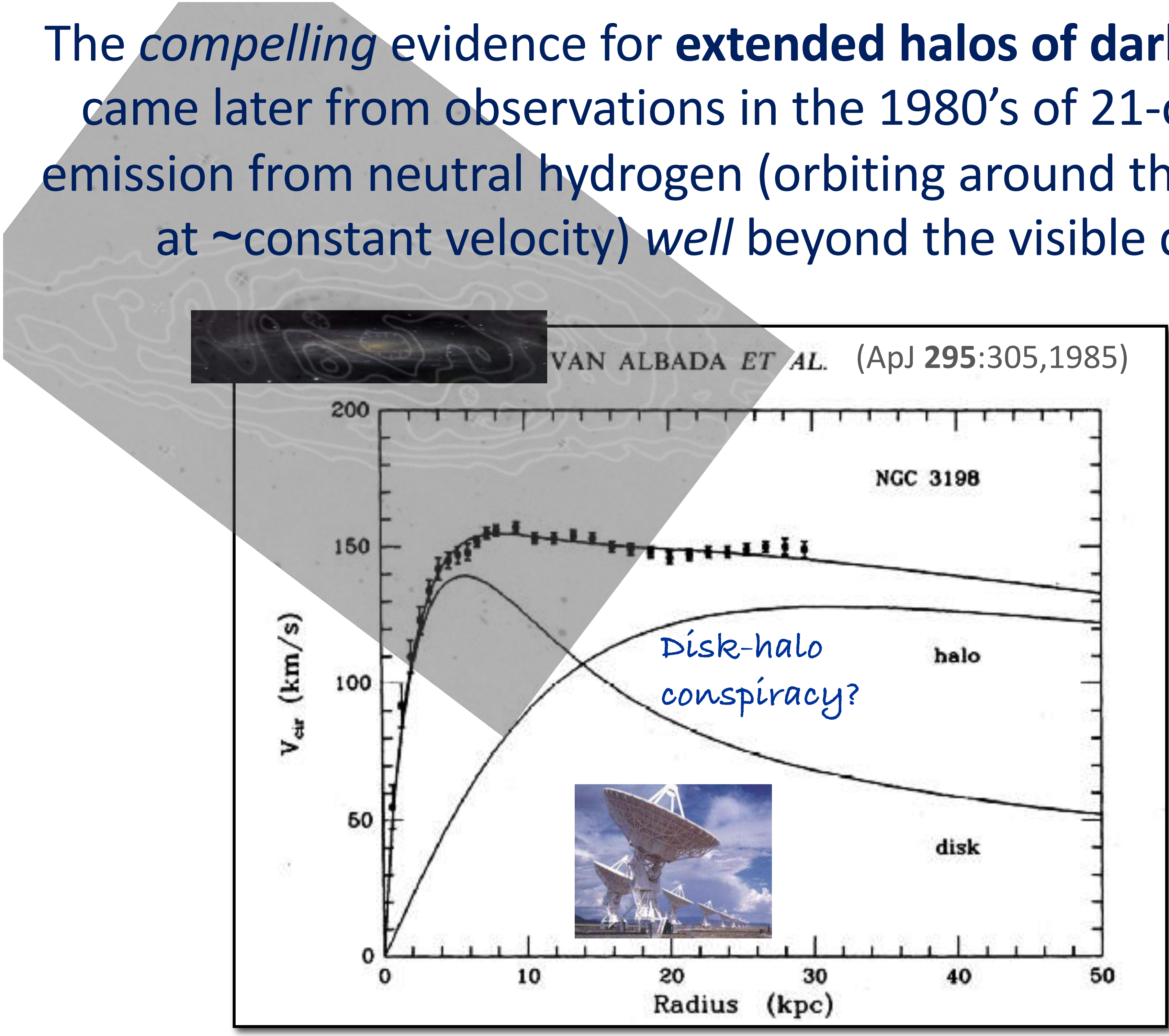
$$v_{\rm circ} = \sqrt{\frac{G_{\rm N} M(<r)}{r}}$$

… but Rubin & Ford (ApJ **159**:379,1970) observed that the rotational velocity remains ~constant in Andromeda – interpreted *later* as implying the existence of an extended (dark) ‘corona’ or halo …



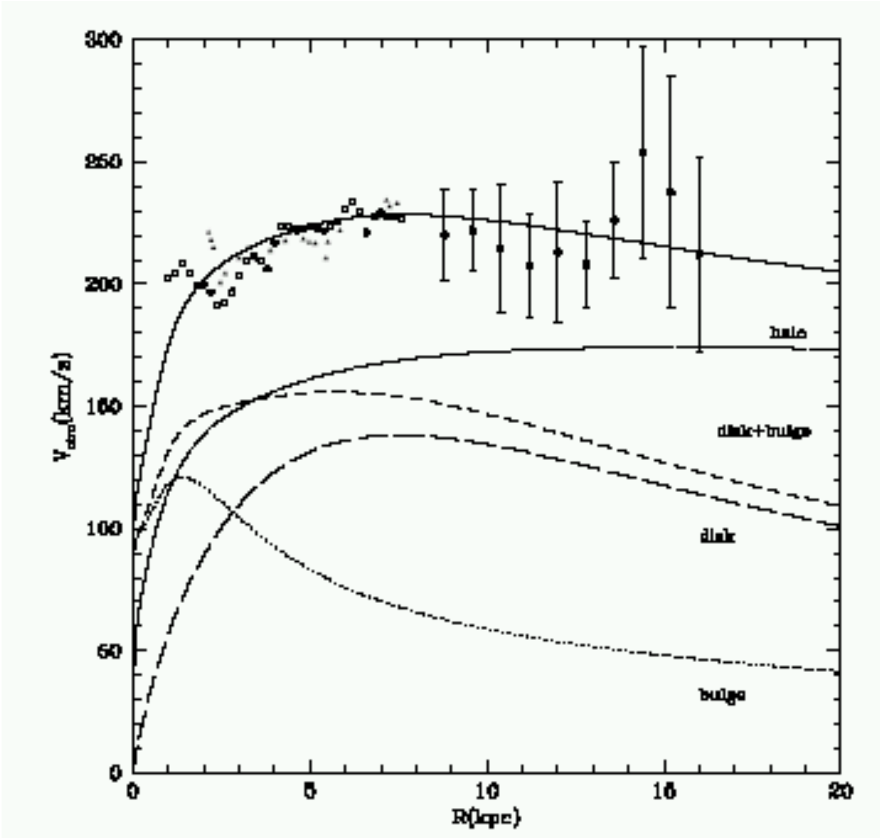
$$v_{\rm circ} \sim {\rm constant} \quad \Rightarrow \quad M(<r) \propto r \quad \Rightarrow \quad \rho \propto 1/r^2$$

The *compelling* evidence for **extended halos of dark matter** came later from observations in the 1980's of 21-cm line emission from neutral hydrogen (orbiting around the Galaxy at ~constant velocity) *well* beyond the visible disk

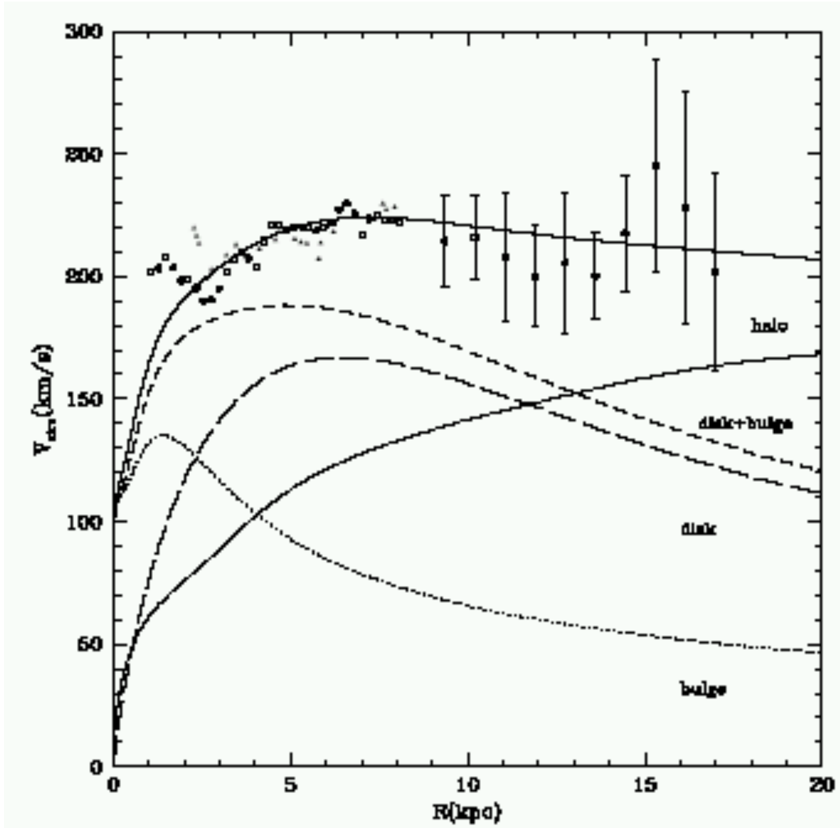


More sophisticated modelling accounts for multiple components and the gravitational coupling between baryonic & dark matter

No angular momentum exchange



With angular momentum exchange



Klypin, Zhao & Somerville, ApJ **573**:597,2002

The *local* halo dark matter density is inferred from such models to be ~0.3 GeV cm^{-3}
(uncertainty at least a factor of ~2)

With the $1/r^2$ density profile, the solution of the *collisionless* Boltzmann equation is the 'Maxwellian distribution':

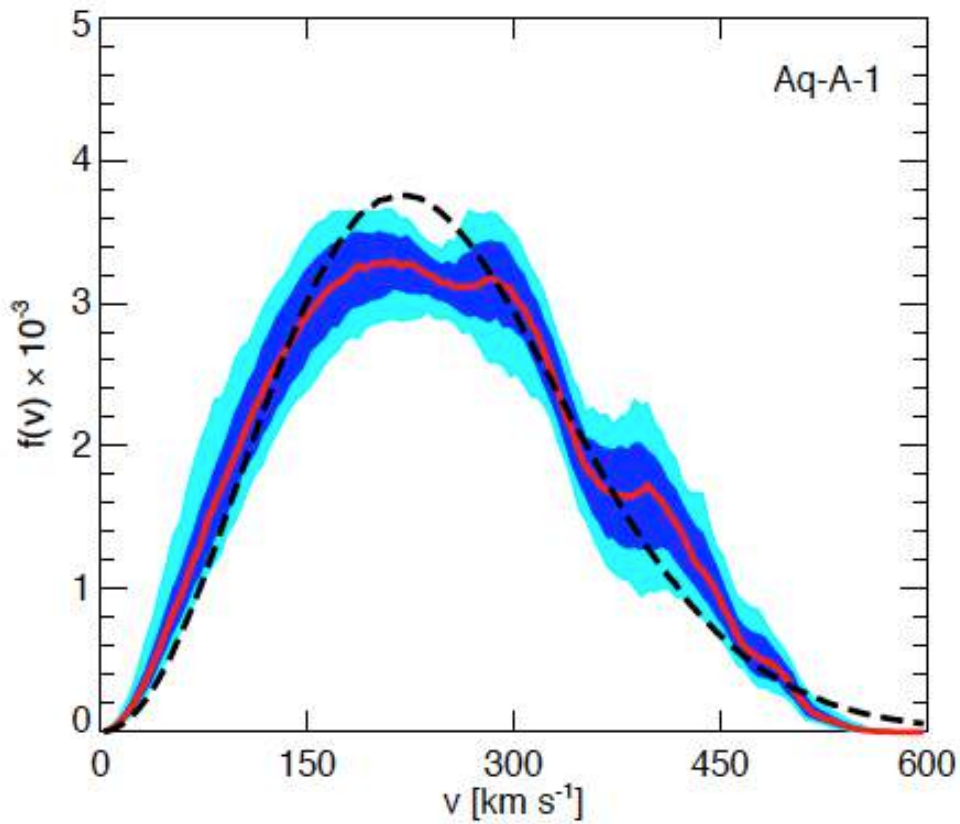
$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}}\frac{\partial f}{\partial \mathbf{v}} = 0$$

$$f(\mathbf{v}) = N \exp\left(-\frac{3|\mathbf{v}|^2}{2\sigma^2}\right)$$

$$\sigma = \sqrt{3/2}\, v_c$$

The 'standard halo model' has v_c = 220 km/s and is truncated at v_{esc} = 544 km/s (both numbers have large observational uncertainties)

High resolution numerical simulations however suggest significant deviations from the Maxwellian distribution, particularly at high velocities (this has important implications for direct detection experiments)

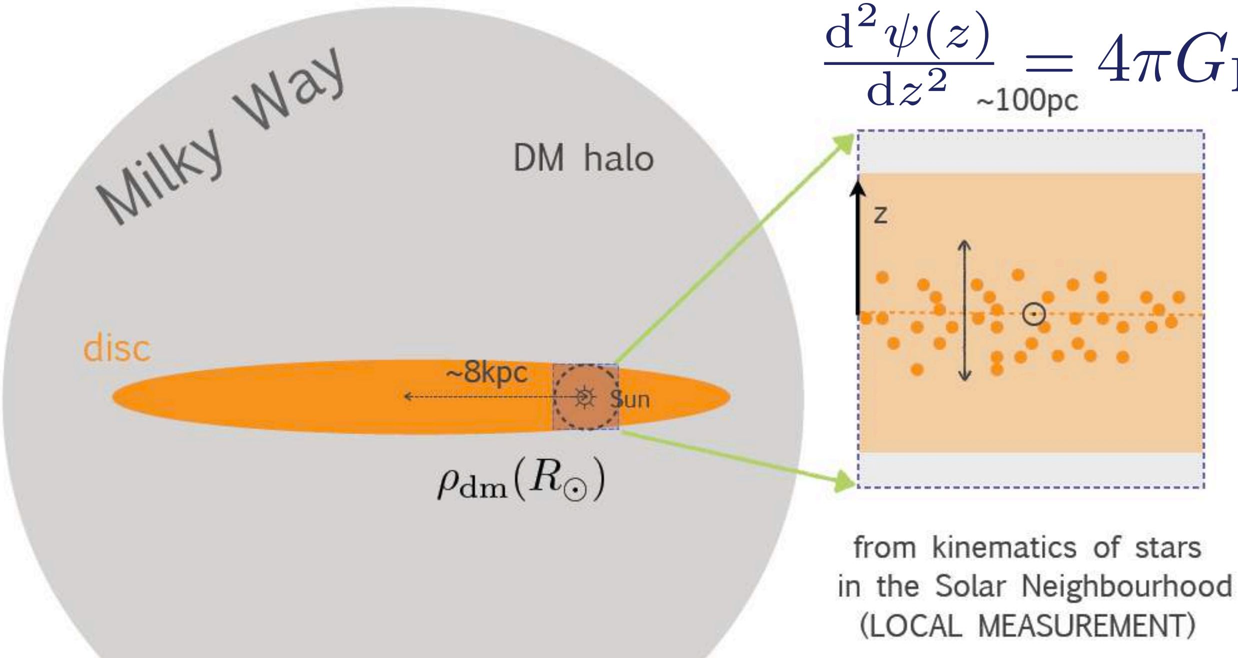


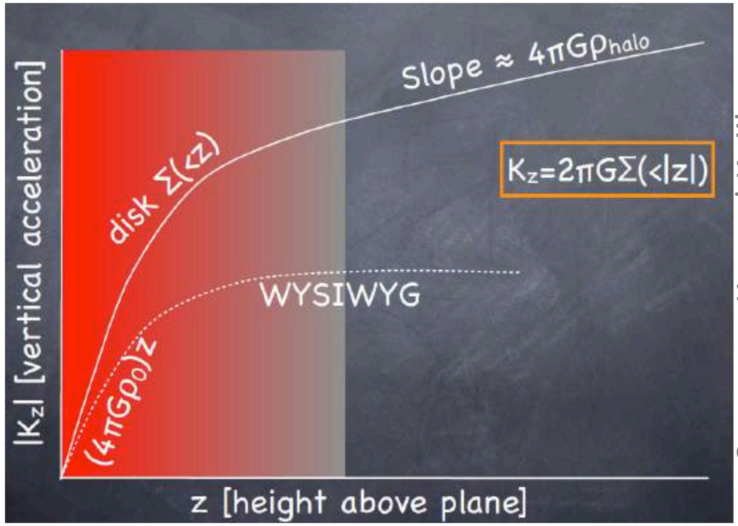
Vogelsberger *et al*, MNRAS **395**:797,2009

We can determine the *local* dark matter density by measuring vertical distribution of stars … pioneered by Kapetyn (1922) and Oort (1932)

If galaxy is approximated as thin disk, then orthogonal to the Galactic plane:

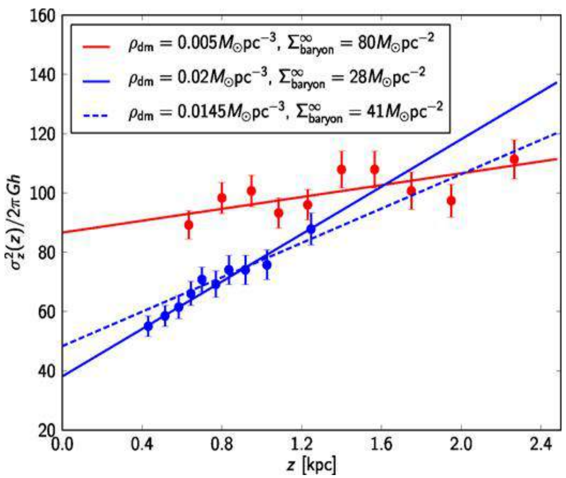
$$\frac{\mathrm{d}^2\psi(z)}{\mathrm{d}z^2} = 4\pi G_\mathrm{N}\rho_\mathrm{m} \rightarrow \frac{\mathrm{d}\psi(z)}{\mathrm{d}z} = 2\pi G_\mathrm{N}\Sigma_\mathrm{m}$$





Courtesy: Konrad Kuijken

K-dwarfs: ρ_{DM} = 0.85 ± 0.6 GeV/cm³ (Kuijken & Gilmore MNRAS **239**:605,1989)

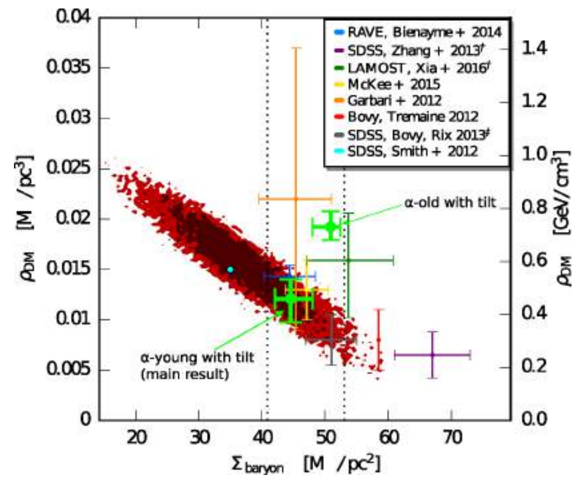


Further observations/analyses indicate multiple stellar populations (with different scale heights):

ρ_{DM} = 0.46 ± 0.08 GeV/cm³

… but with systematic uncertainties

Sivertsson *et al*, MNRAS **478**:1677,2018



Modelling dark mater halos

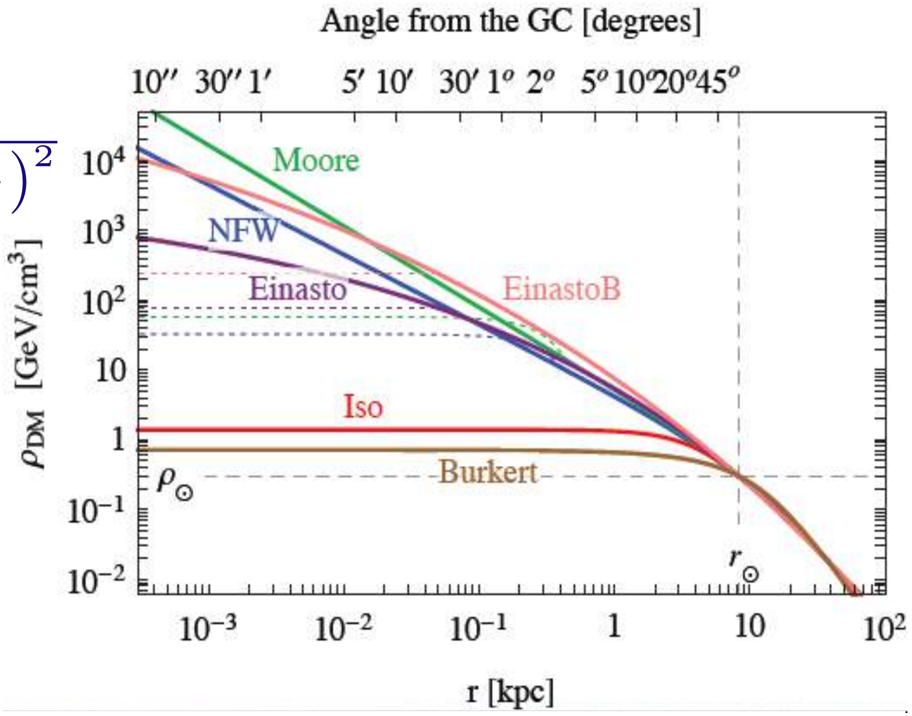
Cored isothermal sphere: $\rho_{\text{isothermal}} = \frac{\rho_s}{\left(1+\frac{r}{r_s}\right)^2}$

Navarro-Frenk-White profile: $\rho_{\text{NFW}} = \frac{\rho_s}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$
(indicated by CDM simulations)

Burkert profile: $\rho_{\text{Burkert}} = \frac{\rho_s}{\left(1+\frac{r}{r_s}\right)\left[1+\left(\frac{r}{r_s}\right)^2\right]}$
(fits observations better)

Einasto profile:

$$\rho_{\text{Einasto}} = \rho_s \exp\left\{-d_n\left[\left(\frac{r}{r_s}\right)^{1/n} - 1\right]\right\}$$



where d_n is defined such that ρ_s is the density at the radius r_s enclosing half the total mass

… more generally define the **Hernquist profile**: $\rho_{\text{Hernquist}} = \rho_s \left(\frac{r}{r_s}\right)^{-\gamma}\left[1+\left(\frac{r}{r_s}\right)^{\alpha}\right]^{\frac{\gamma-\beta}{\alpha}}$

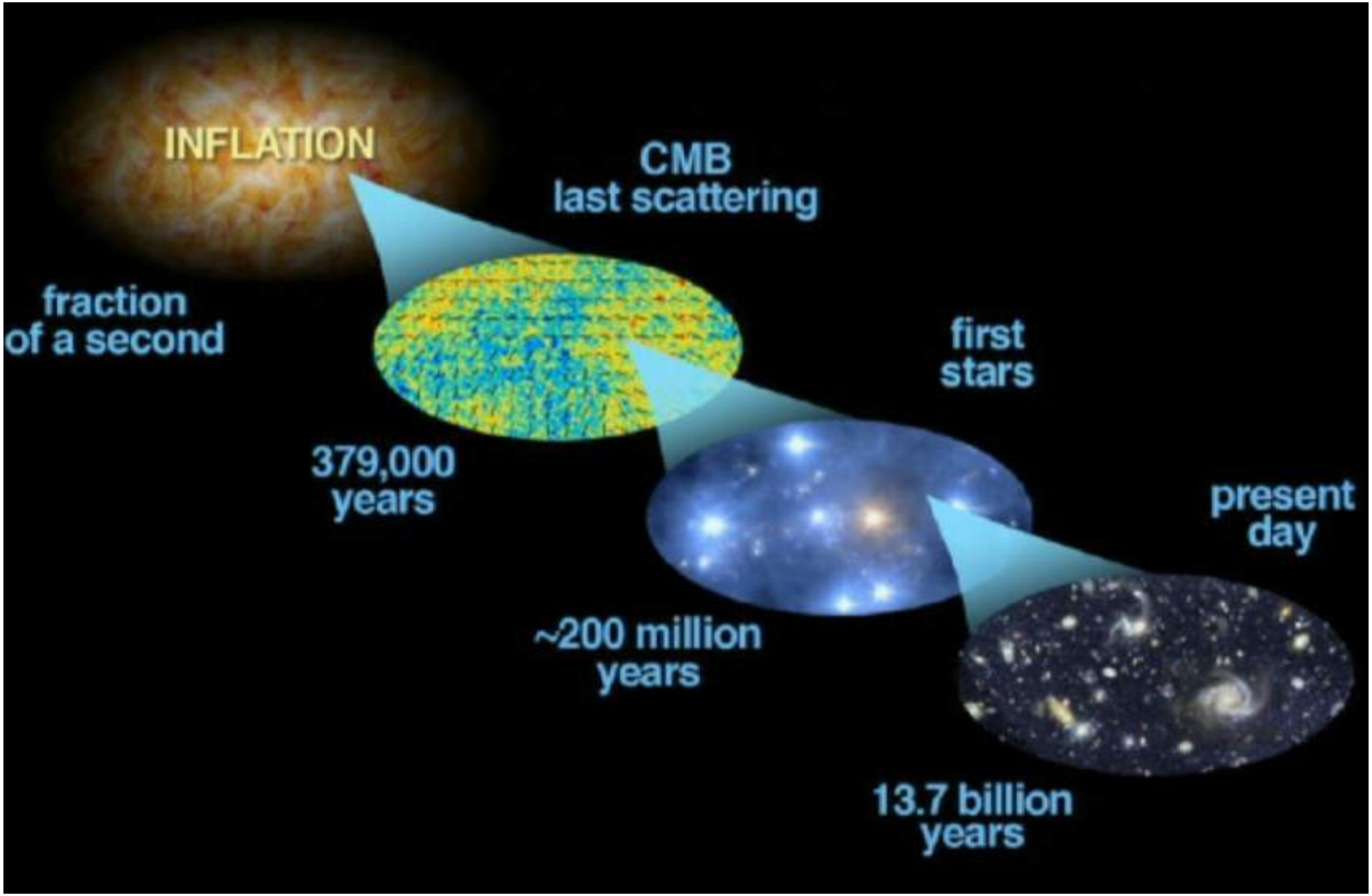
Here r_s is a characteristic scale and α controls the sharpness of the transition from the inner slope $\lim_{r\to 0} \mathrm{d}\ln(\rho)/\mathrm{d}\ln(r) = -\gamma$ to the outer slope $\lim_{r\to\infty} \mathrm{d}\ln(\rho)/\mathrm{d}\ln(r) = -\beta$

… e.g. the NFW profile corresponds to choosing $\alpha = 1, \beta = 3, \gamma = 1$, whereas a cored isothermal profile corresponds to choosing $\alpha = 1, \beta = 2, \gamma = 0$, and a Moore profile corresponds to $\alpha = 1.5, \beta = 2, \gamma = 1.5$ *etc*

For the Milky Way, the fit parameters are:

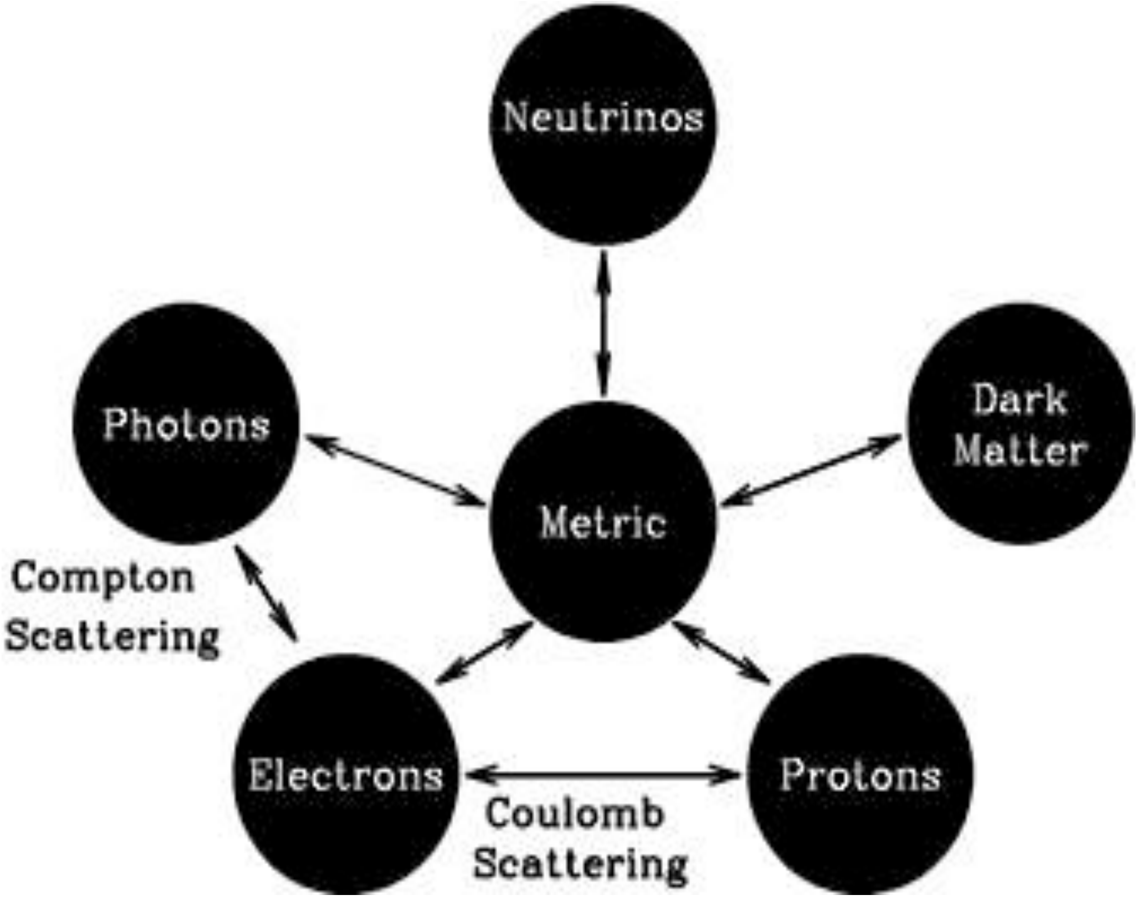
DM halo	α	r_s [kpc]	ρ_s [GeV/cm³]
NFW	–	24.42	0.184
Einasto	0.17	28.44	0.033
EinastoB	0.11	35.24	0.021
Isothermal	–	4.38	1.387
Burkert	–	12.67	0.712
Moore	–	30.28	0.105

A KEY argument for DM comes from considerations of structure formation



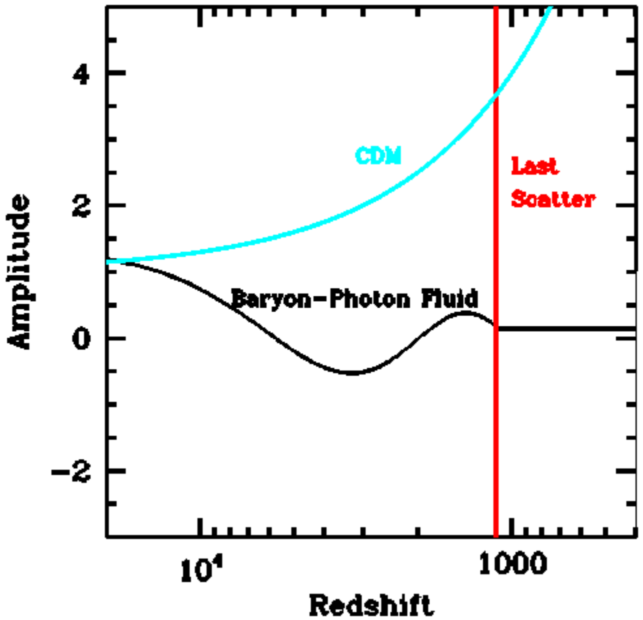
These temperature fluctuations are understood as due to **scalar density perturbations** with an ~scale-invariant spectrum which were generated during an early phase of inflationary expansion … these perturbations have subsequently grown into the **large-scale structure** of galaxies observed today through **gravitational instability** in a sea of **dark matter**

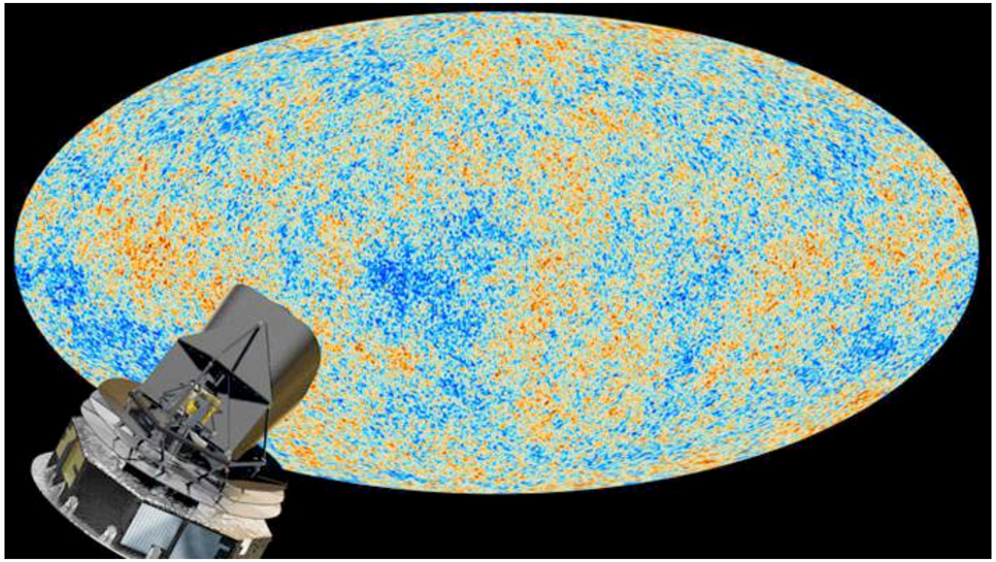
Perturbations in metric (generated during inflation) induce perturbations in photons and (dark) matter



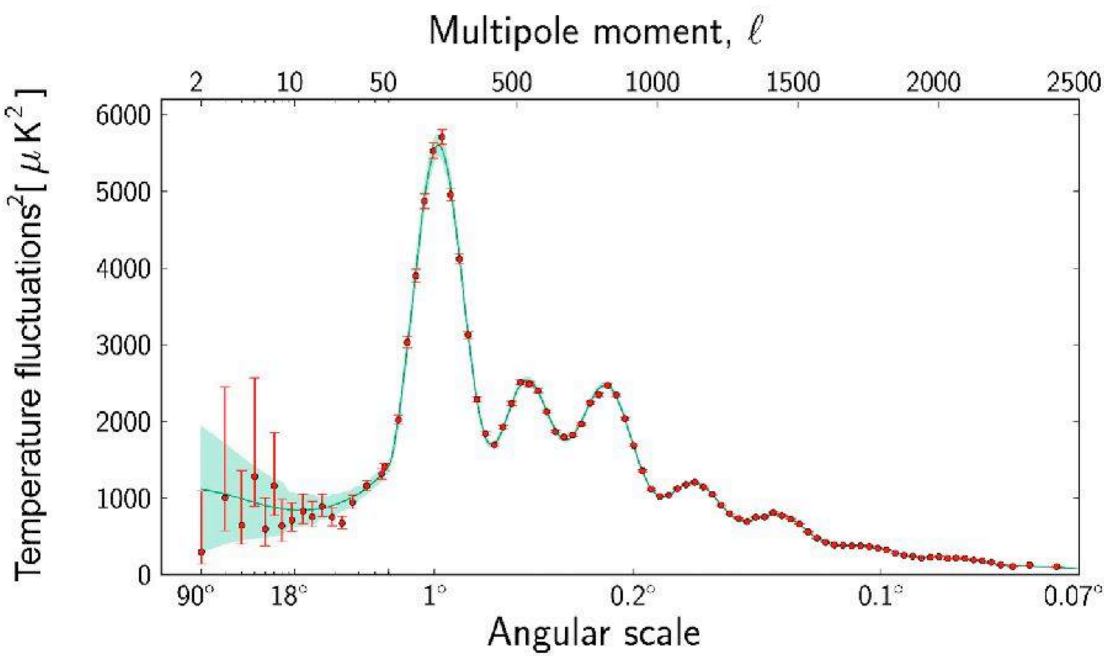
These perturbations begin to grow through gravitational instability after matter domination

Before recombination, the primordial fluctuations just excite sound waves in the plasma, but can start growing already in the sea of collisionless dark matter …



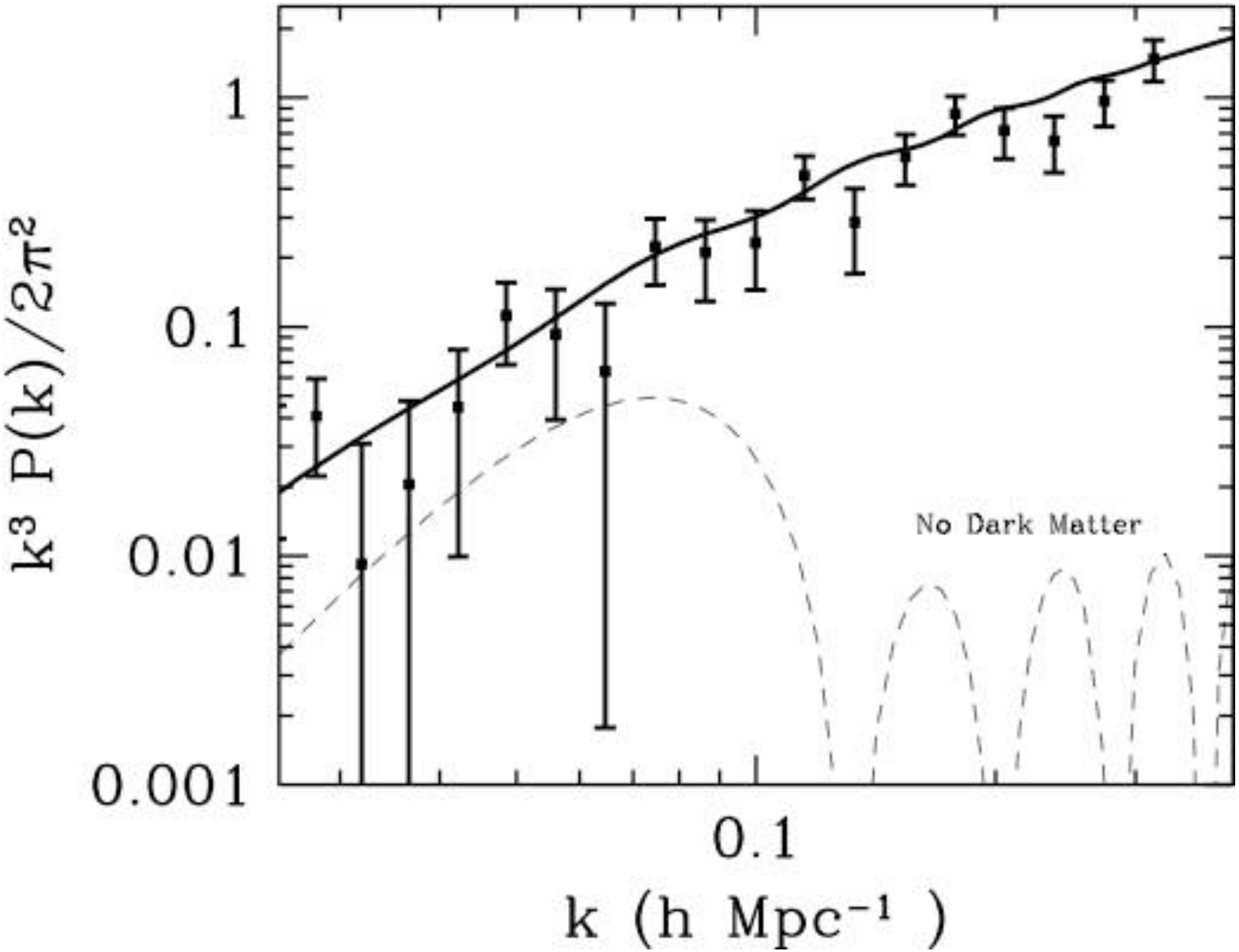


These sound waves leave an imprint on the last scattering surface as the universe turns neutral and transparent … sensitive to the baryon/CDM densities



The angular power spectrum of the fluctuations can be well described only if dark matter *dominates* over baryonic matter ('Silk damping')

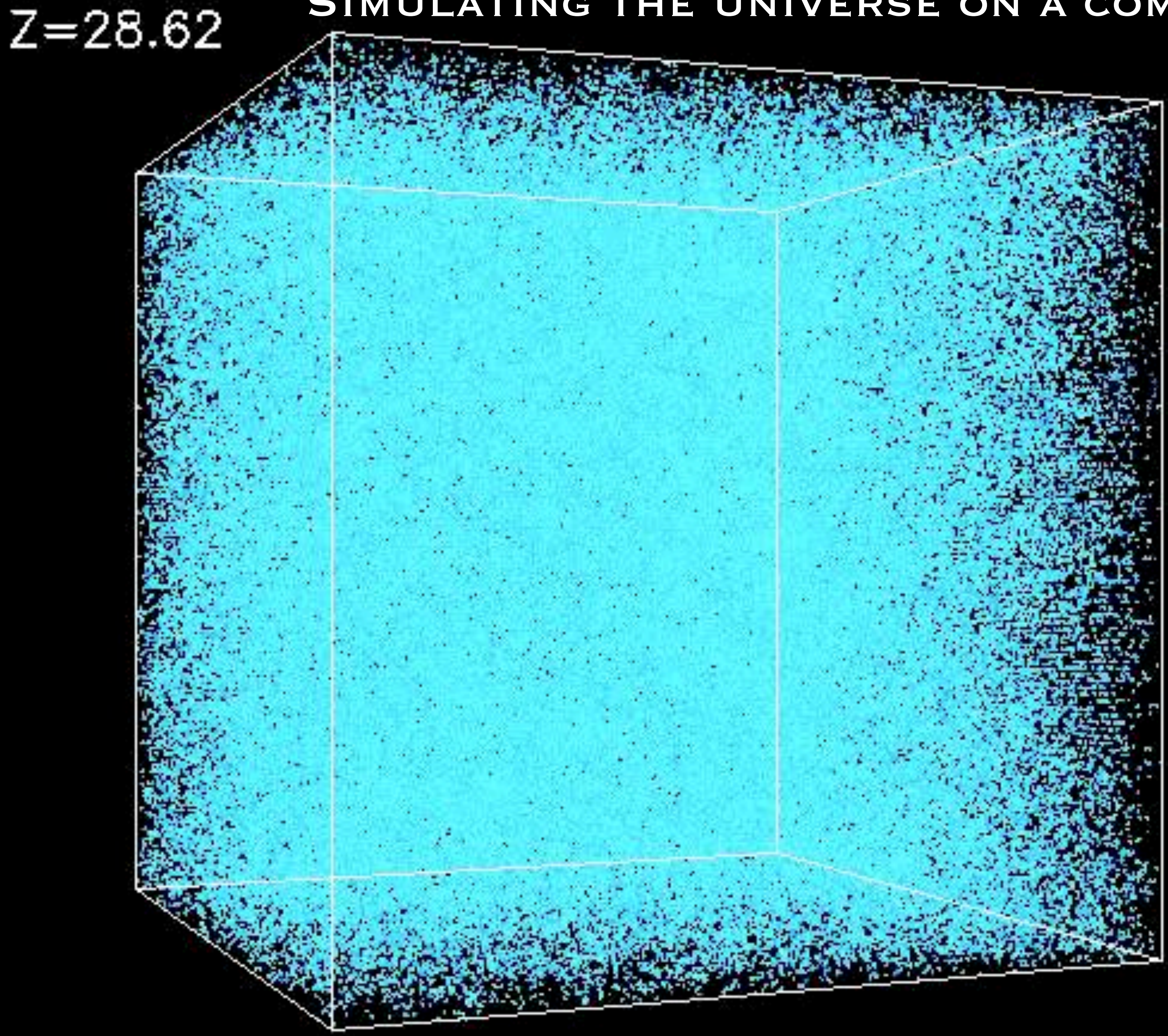
The observed large-scale structure *requires* $\Omega_M >> \Omega_B$ if it has resulted from growth under gravity of initial density fluctuations ... which left their imprint on the CMB at last scattering



Dodelson & Liguori, PRL 97:231301,2006

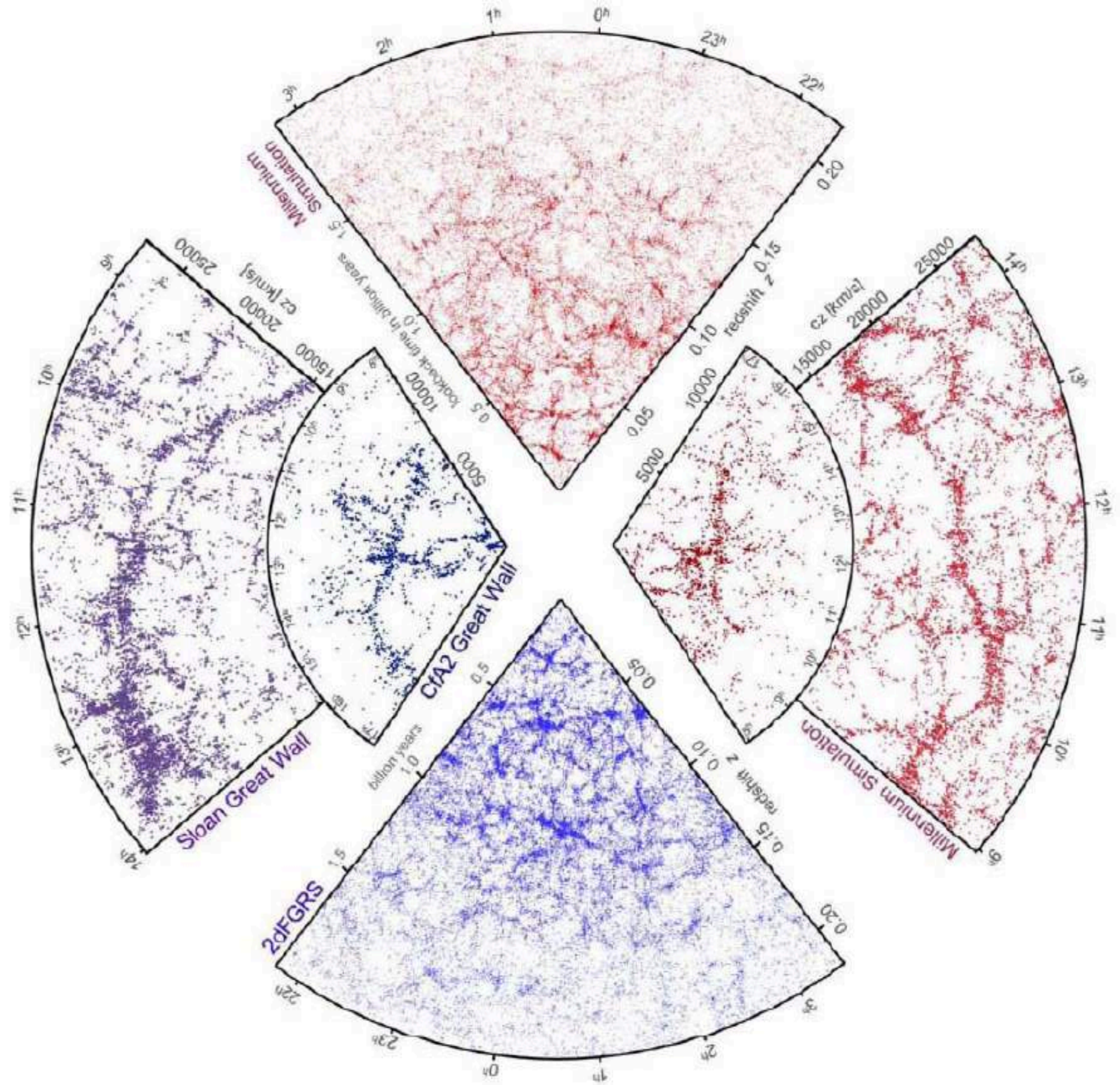
Detailed modelling of WMAP/Planck and 2dF/SDSS ⇒ $\Omega_m \sim 0.3$, $\Omega_B \sim 0.05$

Simulating the universe on a computer



ΛCDM

Such numerical simulations provide a good match to the observed large-scale structure of galaxies in the universe

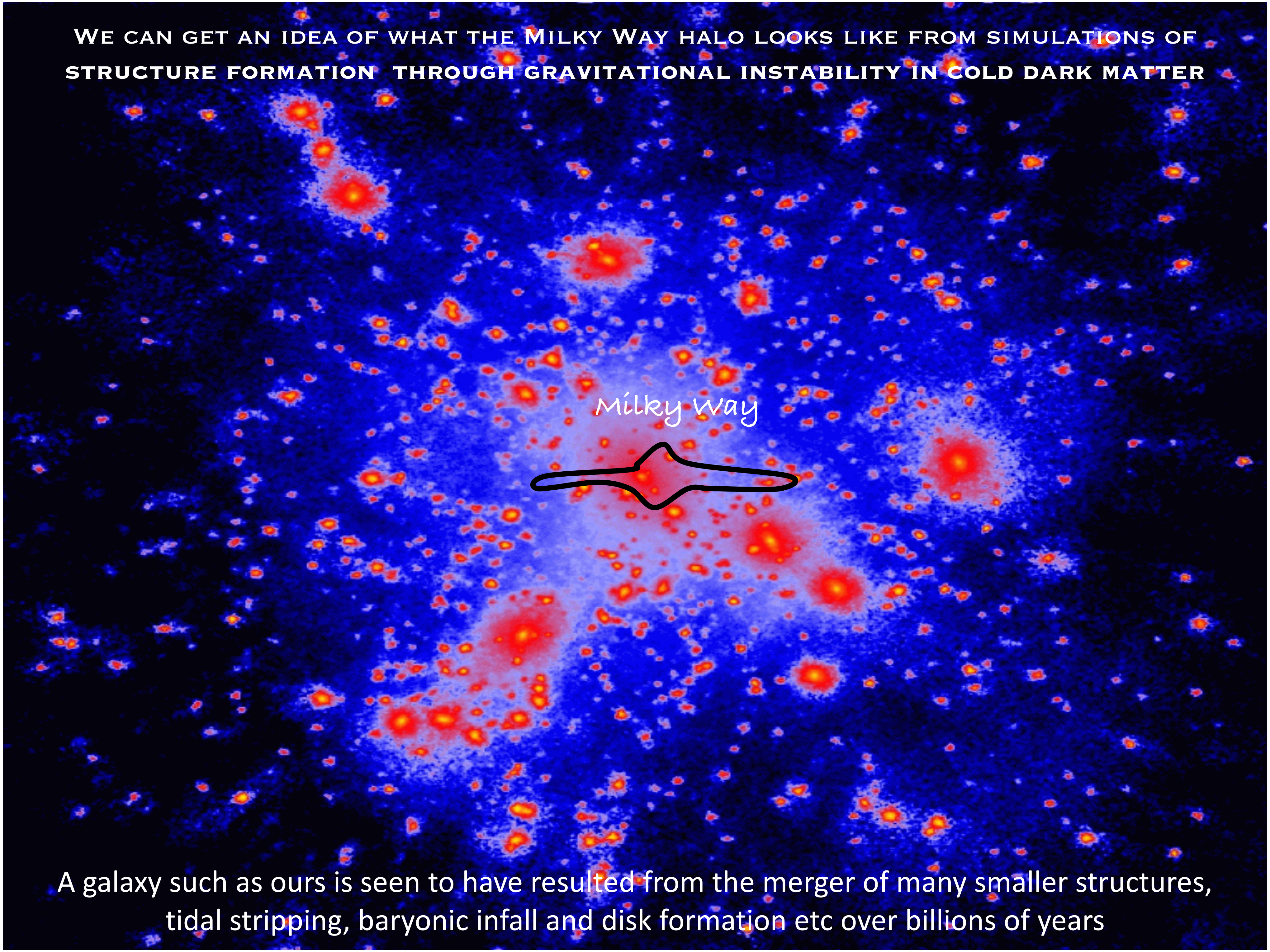


Springel, Frenk & White, Nature **440**:1137,2006

The dark matter particles are assumed to be *cold* (non-relativistic at decoupling) & *collisionless*

… this is true of both weakly interacting massive particles (WIMPs) and ultralight axions!

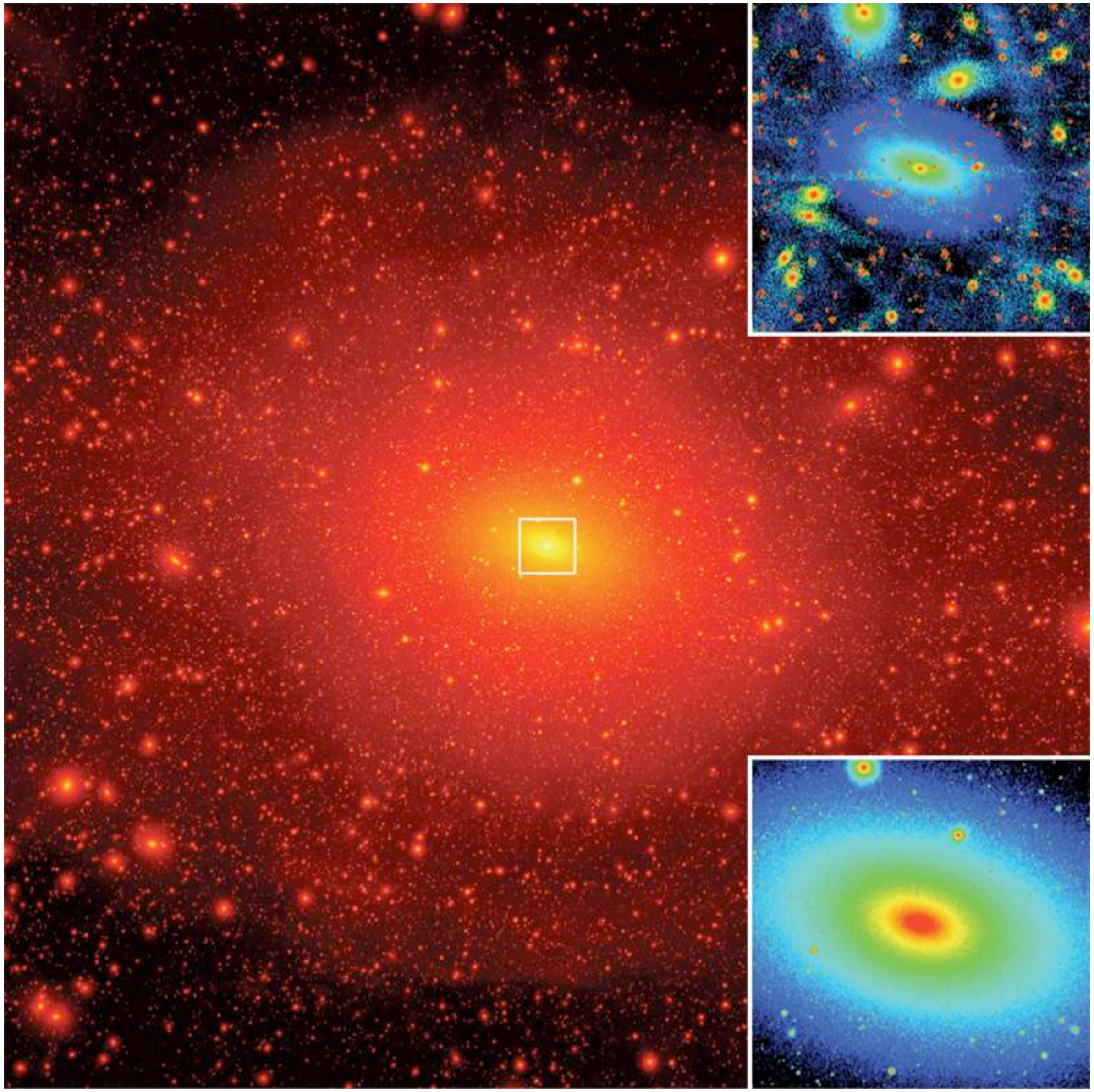
We can get an idea of what the Milky Way halo looks like from simulations of structure formation through gravitational instability in cold dark matter



A galaxy such as ours is seen to have resulted from the merger of many smaller structures, tidal stripping, baryonic infall and disk formation etc over billions of years

The phase space structure of the dark CDM halo is pretty complicated

Via Lactea II projected dark matter (squared-) density map



phase space

real space

Diemand, Kuhlen, Madau, Zemp, Moore, Potter & Stadel, Nature **454**:735,2008

But real galaxies appear to be simpler than expected!

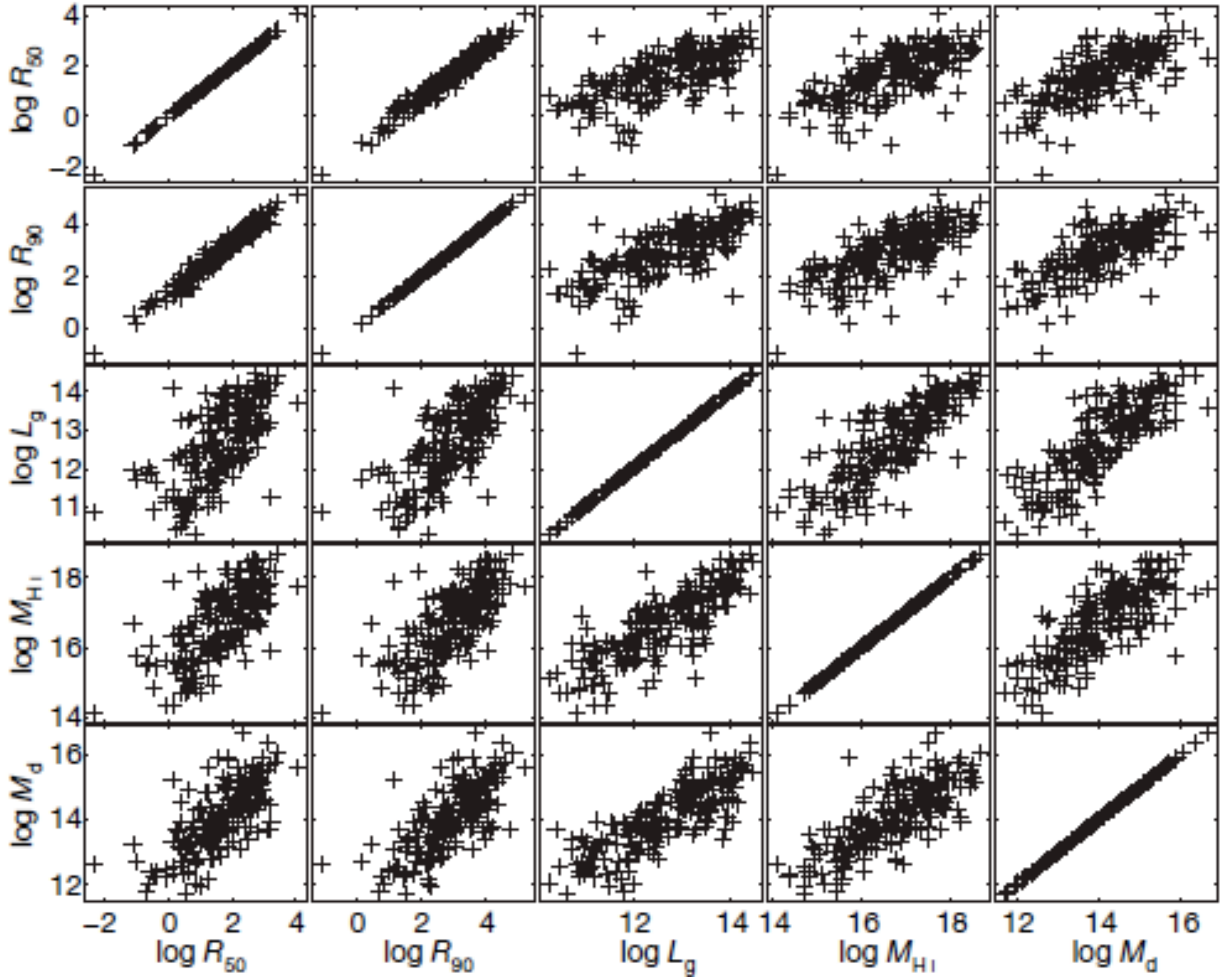
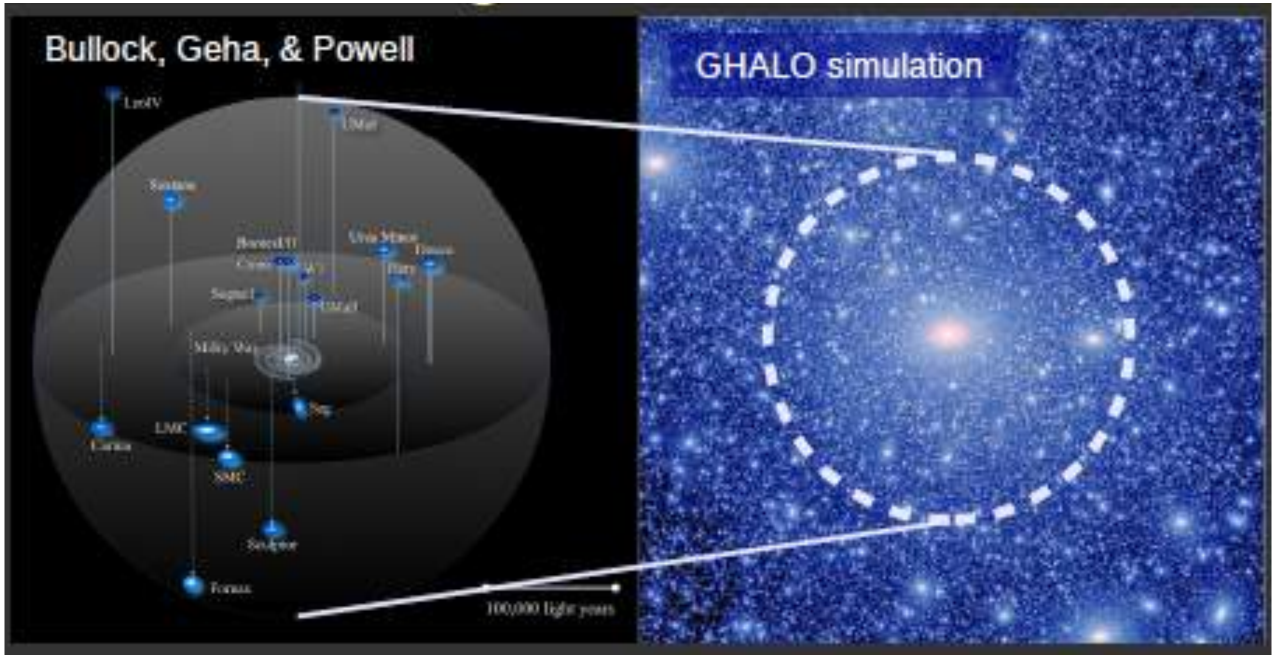
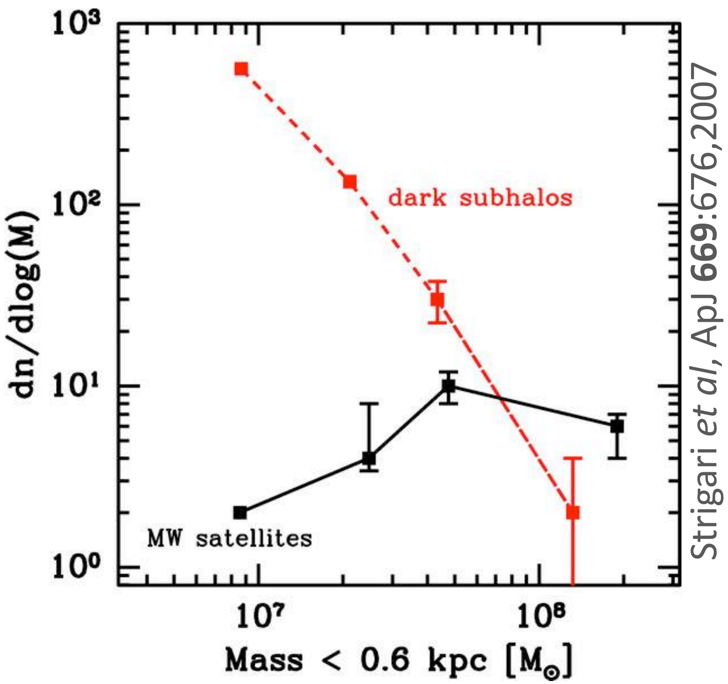


Figure 1 | Scatter plots showing correlations between five measured variables, not including colour. The variables are two optical radii, R_{50} and R_{90} (in parsecs), respectively containing 50 and 90% of the emitted light; and luminosity, L_g; neutral hydrogen mass, M_{HI}; and dynamical mass, M_d (inferred from the 21-cm linewidth, the radius and the inclination in the

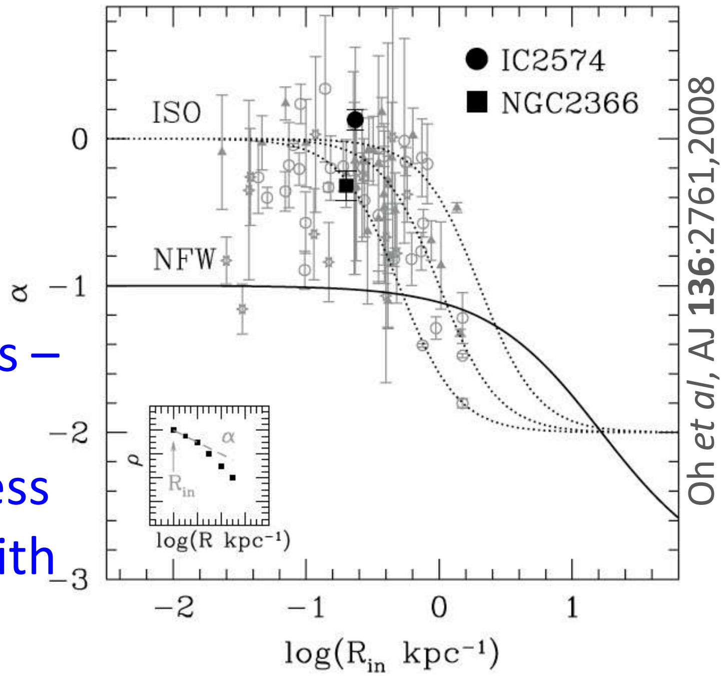
Disney, Romano, Garcia–Appadoo, West, Dalcanton & Cortese, Nature **455**:1082,2008

whereas the Milky Way does have satellites and substructure, There is a lot *less* than is expected from CDM simulations





Also, the halo density profile for collisionless dark matter is predicted to be 'cuspy', whereas observations suggest 'cored' isothermal profiles

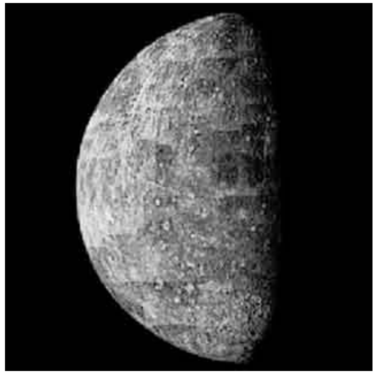


This could be because of the 'feedback effect' of baryons – computer simulations are just beginning to test this – or it could even be because dark matter is *not* collisionless but *self-interacting* (e.g. `dark baryons') or `warm' i.e. with non-zero velocity dispersion (e.g. keV-mass neutrinos)

Inferences of dark matter are not *always* right
... it may instead be a change in the dynamics

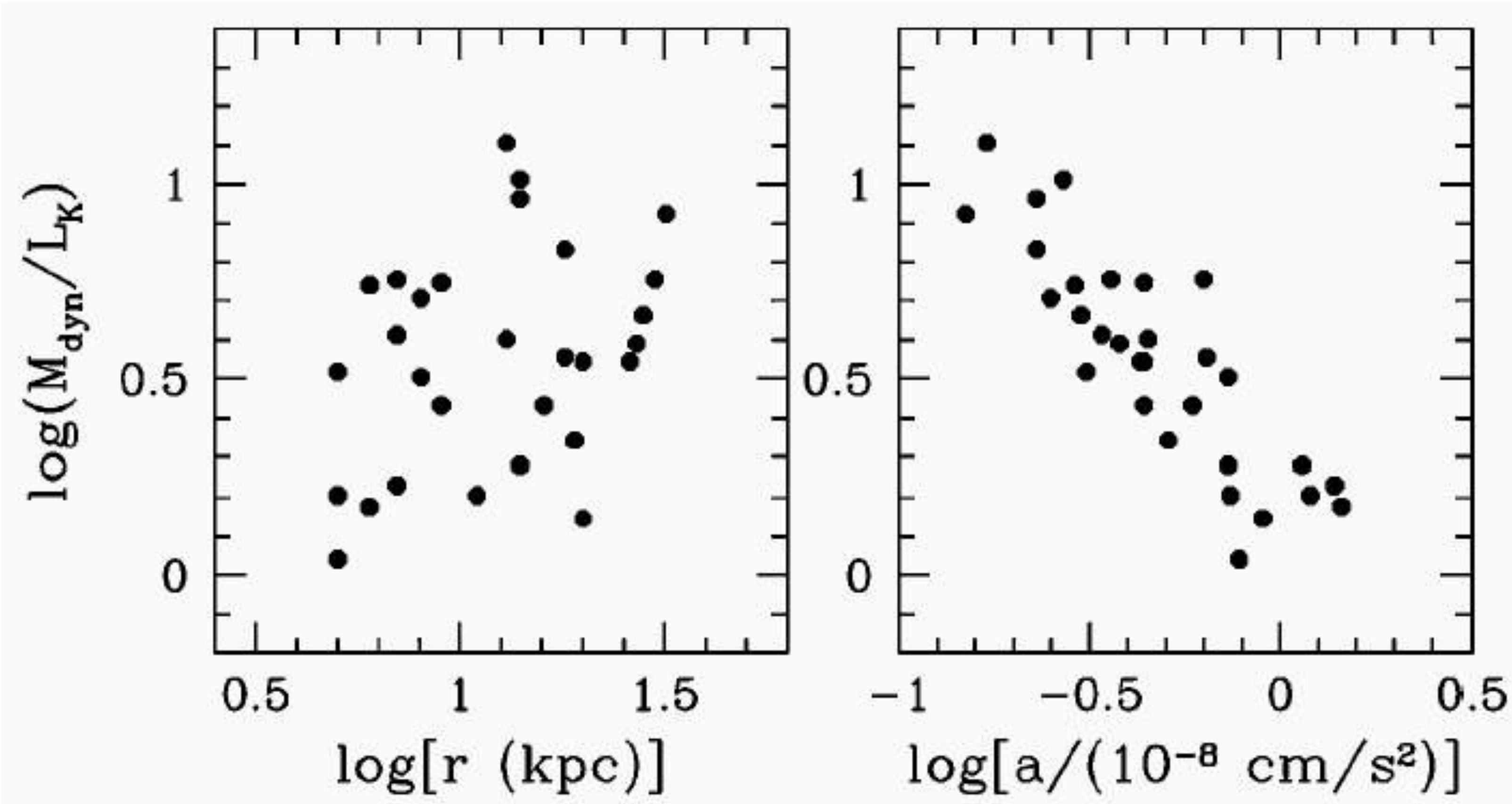
2nd January 1860: "*Gentlemen, I Give You the Planet Vulcan*" French mathematician Urbain Le Verrier announces the discovery of a new planet between Mercury and the Sun, to members of the Académie des Sciences in Paris (following up on his earlier prediction of Neptune in 1856).

Some astronomers even see Vulcan in the evening sky!



But the precession of Mercury is *not* due to a dark planet ...
but because Newton is superseded by Einstein

Dark matter is required only where the test particle acceleration is below $a_0 \sim 10^{-8}$ cm/s^2 - it is *not* a scale-dependent effect



What if Newton's law is modified in weak fields?

$$F_N \to \sqrt{\frac{GM}{r^2} a_0}$$

Milgrom, ApJ **270**:365,1983

Bekenstein—Milgrom Equation

Suppose $\mathbf{F} = -\nabla\phi$ where

$$\nabla^2\phi_{\mathrm{N}} = 4\pi G\rho \quad \rightarrow \quad \nabla\cdot[\mu(|\nabla\phi|/a_0)\nabla\phi] = 4\pi G\rho$$

where

$$\mu(x) \rightarrow \begin{cases} 1 & \text{for } x \gg 1 \\ x & \text{for } x \ll 1 \end{cases}$$

Then

$$0 = \nabla\cdot[\mu(|\nabla\phi|/a_0)\nabla\phi - \nabla\phi_{\mathrm{N}}]$$

implies

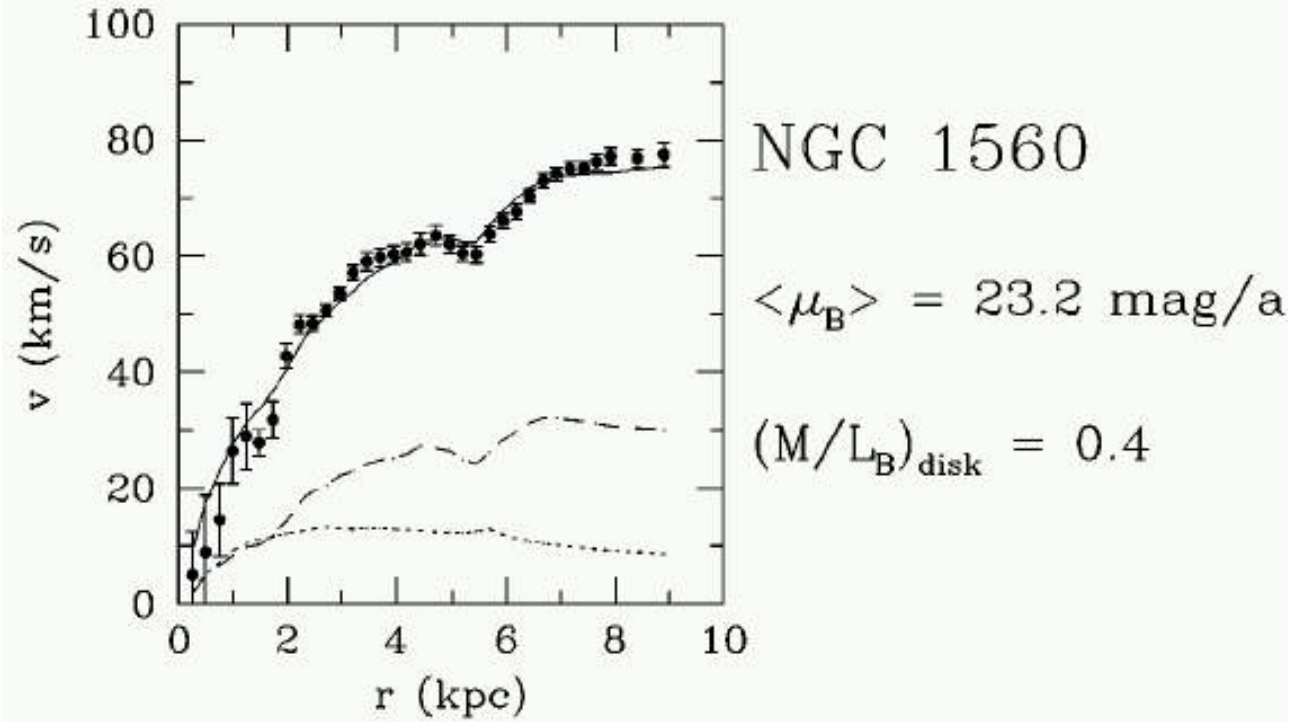
$$\mu(|\nabla\phi|/a_0)\nabla\phi = \nabla\phi_{\mathrm{N}} + \nabla\times\mathbf{A}$$

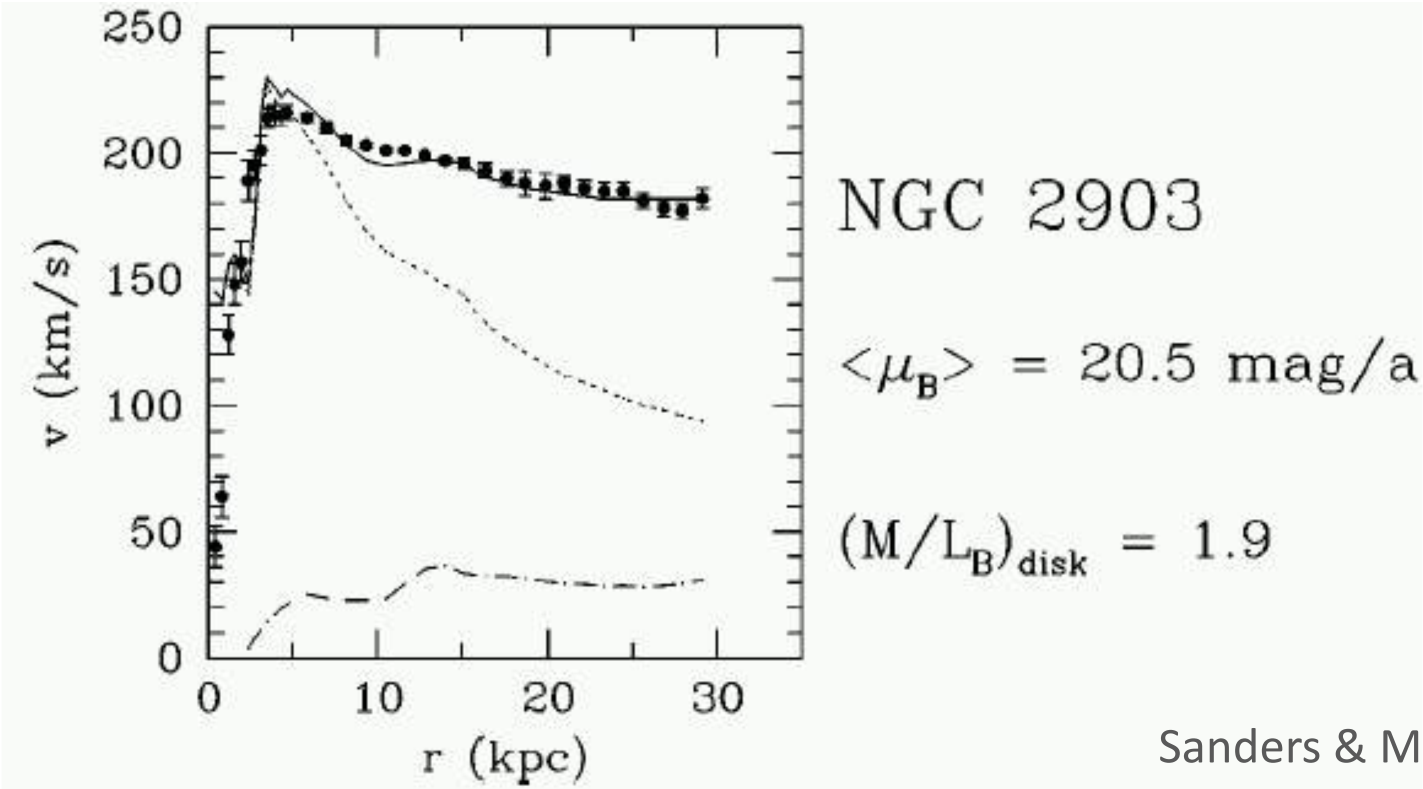
so when $\mathbf{A} \simeq 0$ and $|\nabla\phi| \ll 1$

$$g_{r\to\infty} \rightarrow -\sqrt{MGa_0}\frac{\vec{r}}{r^2} + \mathcal{O}\left(\frac{1}{r^2}\right), \frac{|\nabla\phi|^2}{a_0} = |\nabla\phi_{\mathrm{N}}|$$

Milgrom [arXiv:0912.2678]

MOND fits *all* galactic rotation curves with $a_0=1.2\text{x}10^{-8}$ cm s^{-2}



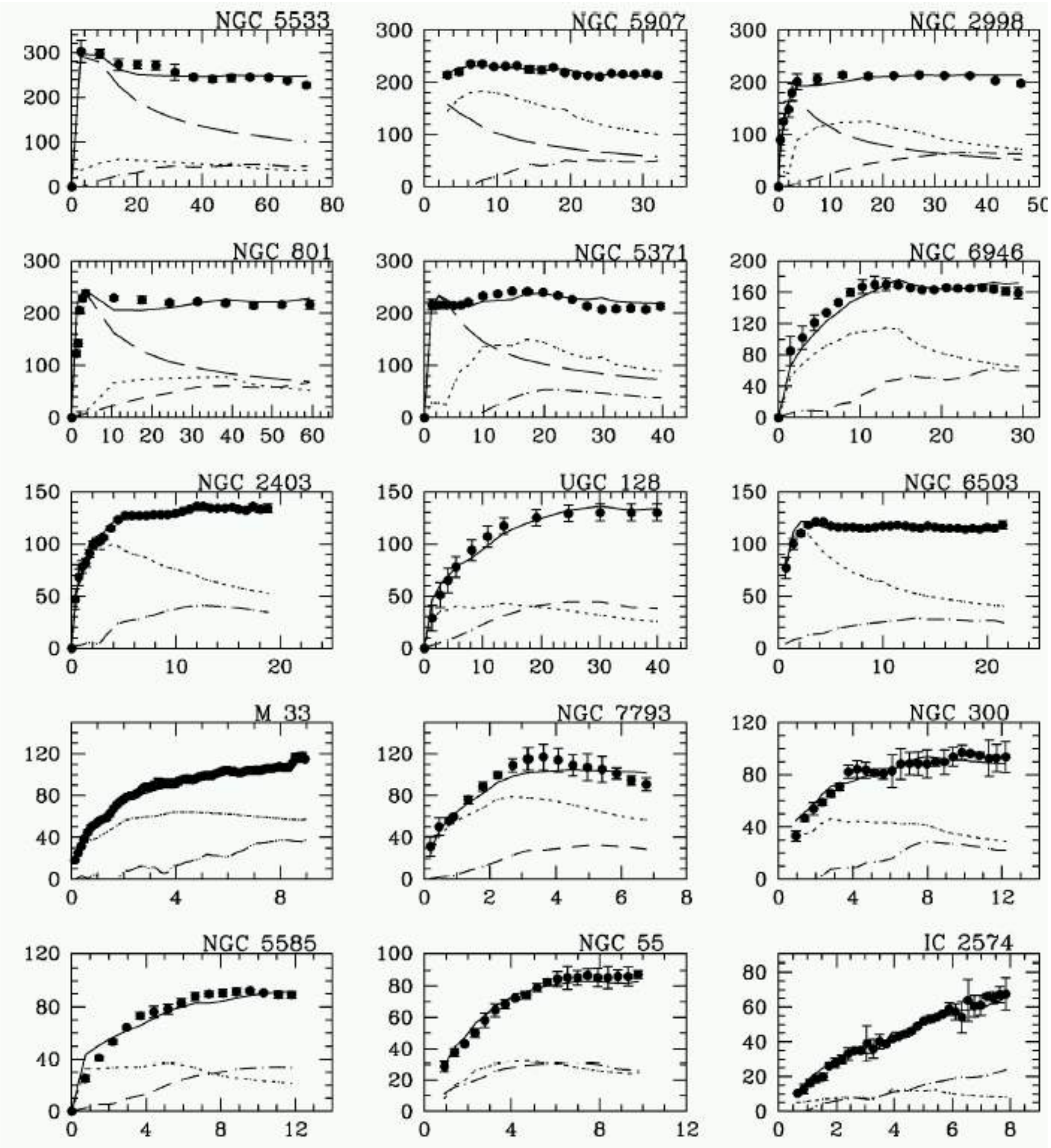


Features in the baryonic disc have *counterparts* in the rotation curve

Sanders & McGaugh , ARAA **40**:263,2002

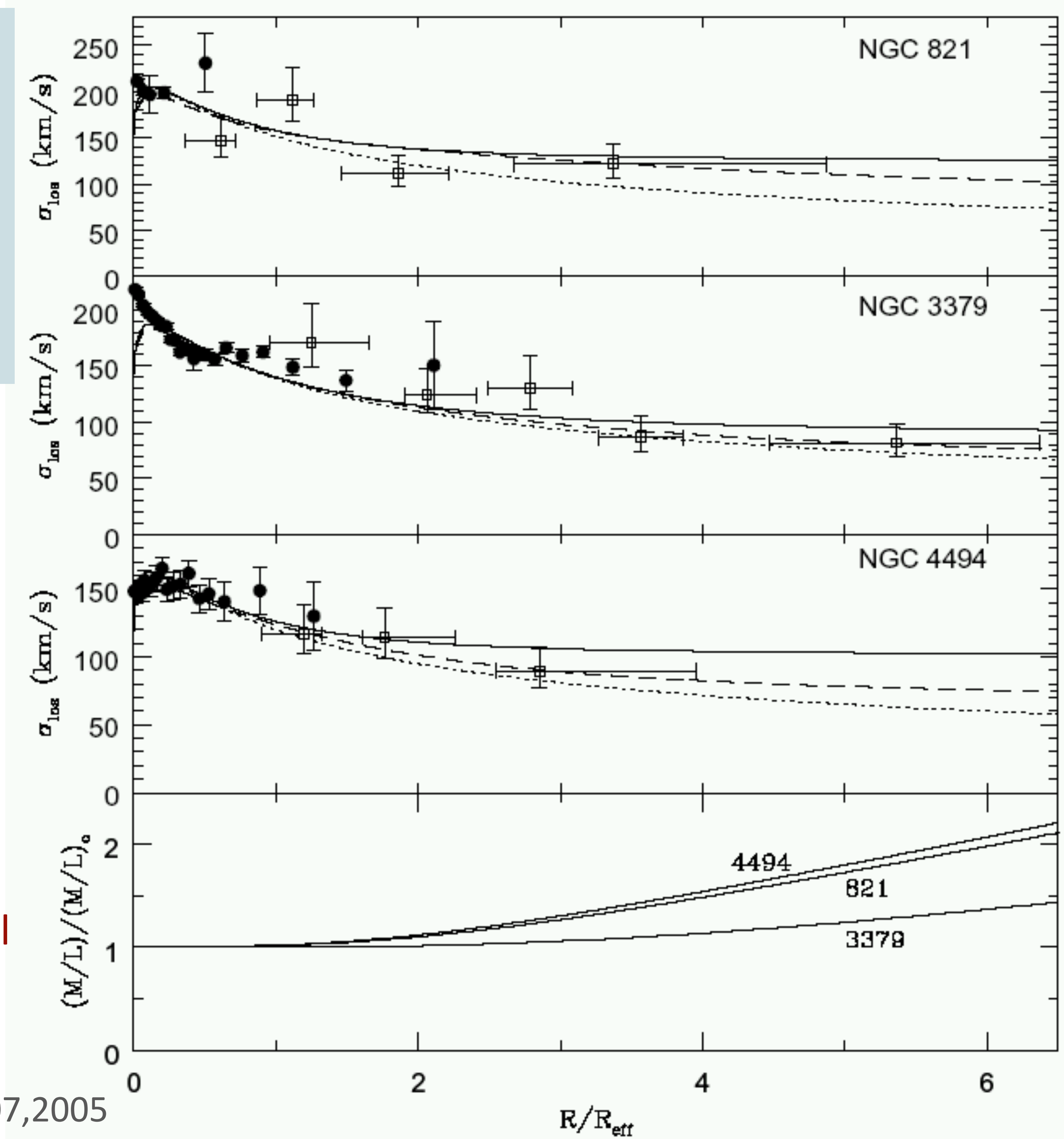
A huge variety of rotation curves is well fitted by MOND

… with *fewer* parameters than is required by the dark matter model



Sanders & Verheijen, ApJ **503**:97,1998

Moreover some giant ellipticals *do* exhibit a Keplerian fall-off of the velocity dispersion, just as MOND *predicts*



Data:
Romanowsky *et al*, Science **301**:1696,2003

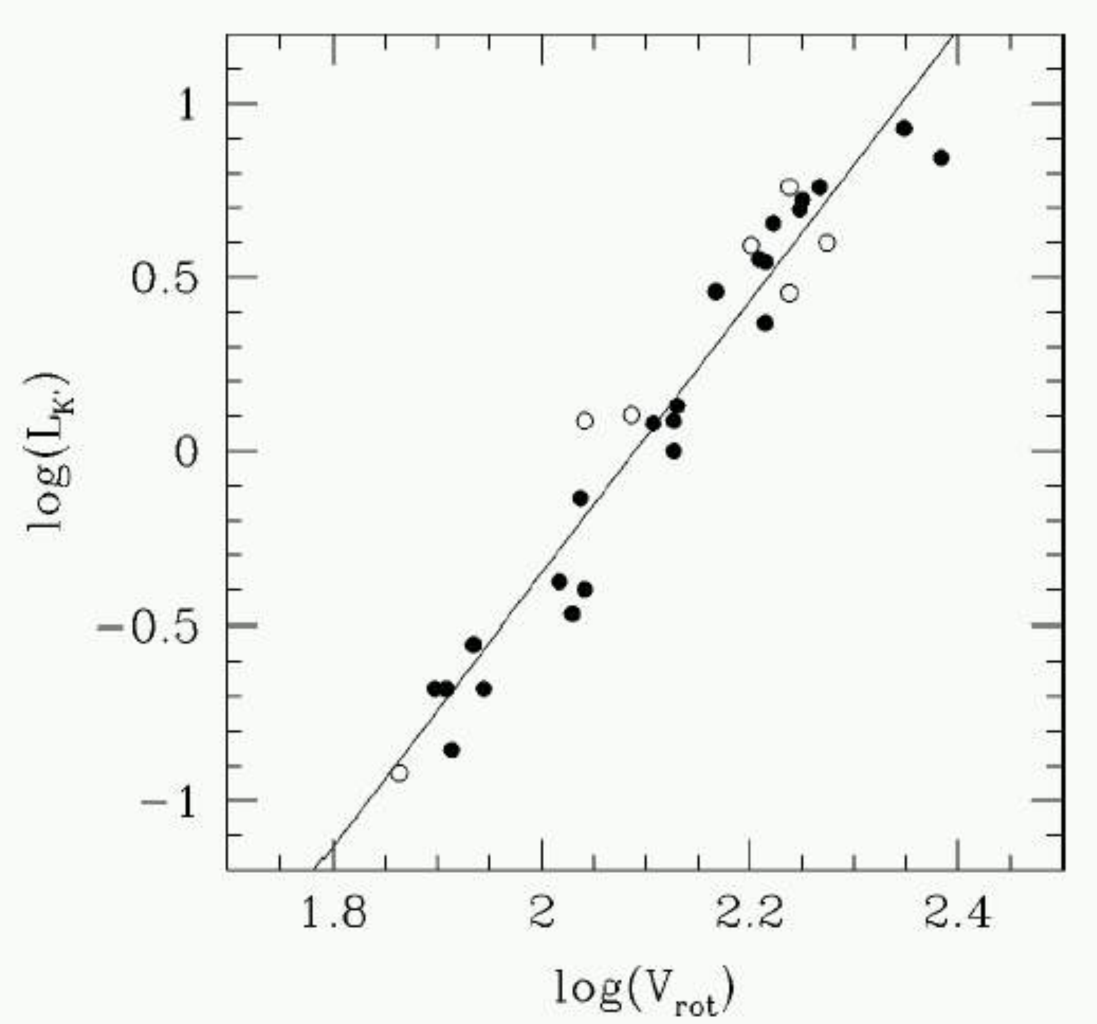
Models:
Milgrom & Sanders ApJ **599**:L25,2003

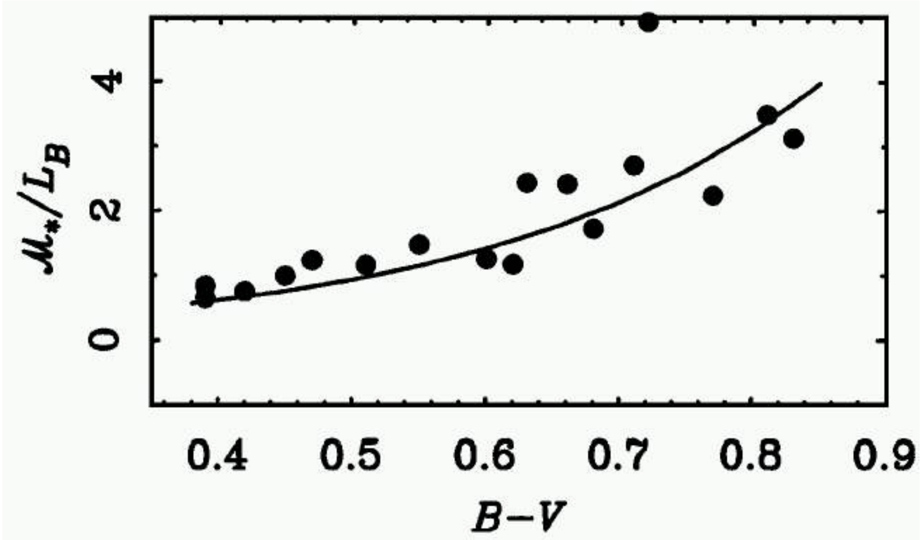
This can be explained in a dark matter model only if stellar orbits are *very* elliptical

Dekel *et al,* Nature **437**:707,2005

Simply squaring the MOND ansatz $F_N \to \sqrt{\frac{GM}{r^2} a_0}$ *predicts* a known astronomical law

$$\frac{v^4}{r^2} = \frac{GM}{r^2} a_0 \quad \Rightarrow \quad M \propto v^4 \quad \left(\text{Tully–Fisher if } \frac{M}{L} = \text{const}\right)$$



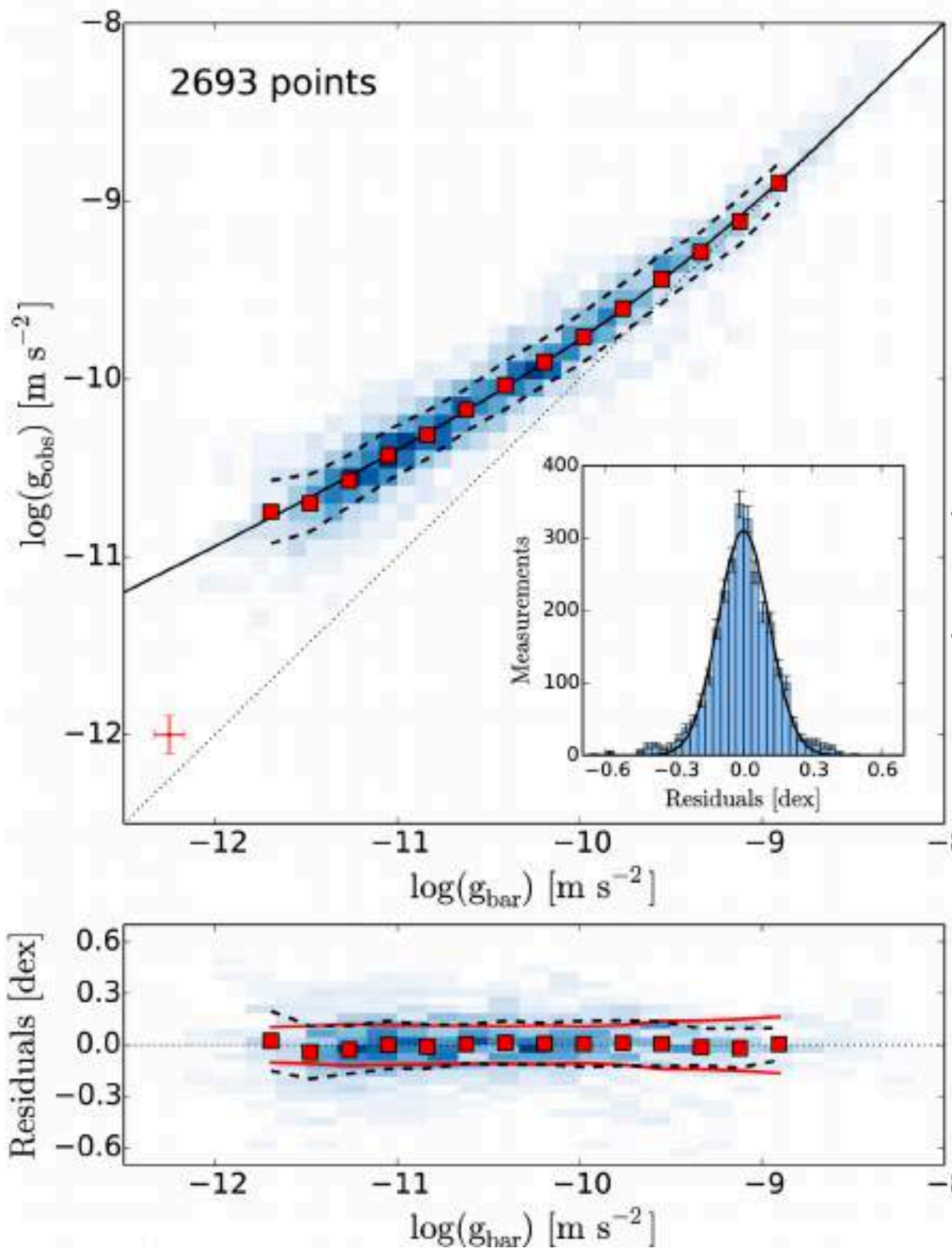


… the fitted value of the only free parameter (M/L) agrees very well with population synthesis models

Sanders & Verheijen, ApJ **503**:97,1998

This is an impressive correlation for which dark matter has *no* simple explanation

Recently this has gained new prominence as the 'MDAR relationship'



McGaugh *et al*, PRL **117**:201101, 2016

153 disk galaxies (SPARC set) measured at multiple radii

$$g_{\rm obs} = \mathcal{F}(g_{\rm bar}) = \frac{g_{\rm bar}}{1 - e^{-\sqrt{g_{\rm bar}/g_\dagger}}}$$

$$g_\dagger = (1.2 \pm 0.24) \times 10^{-10} {\rm m\ s}^{-2}$$

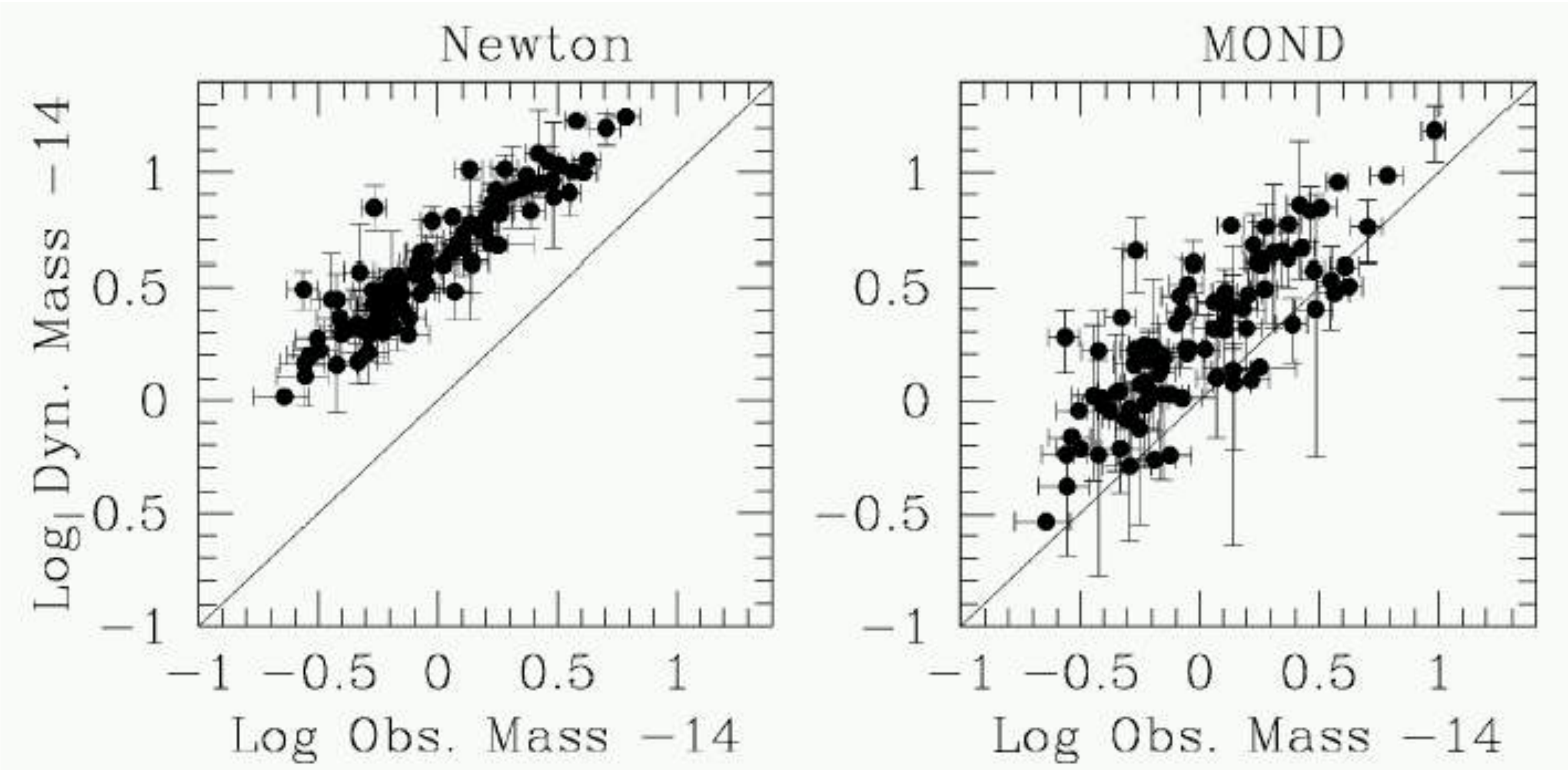
The functional form is *not* unique, e.g.

$$\mathcal{F}(y) = [1 + \sqrt{(1 + 4/y)}]/2$$

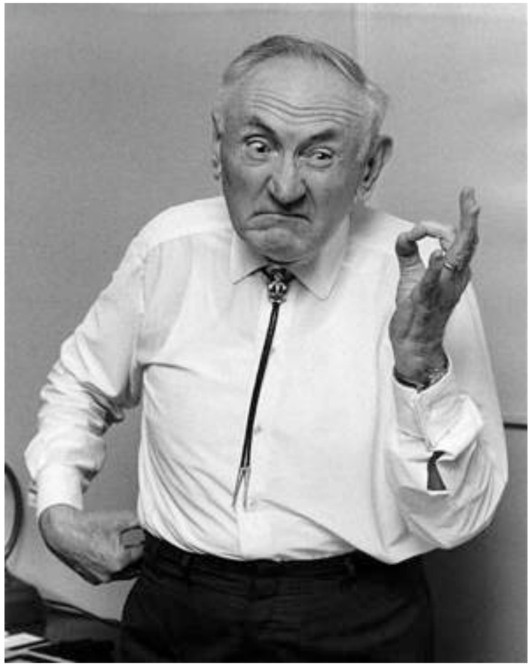
which follows from $\mu(x) = x/(1+x)$, fits just as well (Milgrom, 1609.06642)

Can CDM simulations *including* baryon physics predict this curve?

However MOND *fails* on the scale of clusters of galaxies



The "missing mass" cannot be accounted for entirely
by invoking MOND … **dark matter *is* required**
(thus vindicating the original proposal of Zwicky)



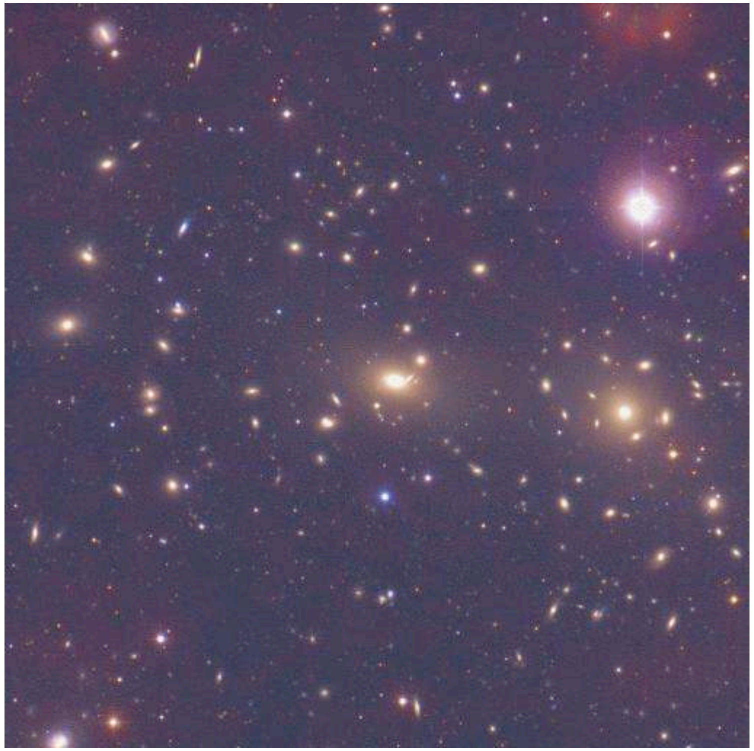
Fritz Zwicky (1933) measured the velocity dispersion in the Coma cluster to be as high as 1000 km/s $\Rightarrow$ M/L ~ O(100) $M_\odot/L_\odot$

"... If this overdensity is confirmed we would arrive at the astonishing conclusion that **dark matter** is present (in Coma) with a much greater density than luminous matter"

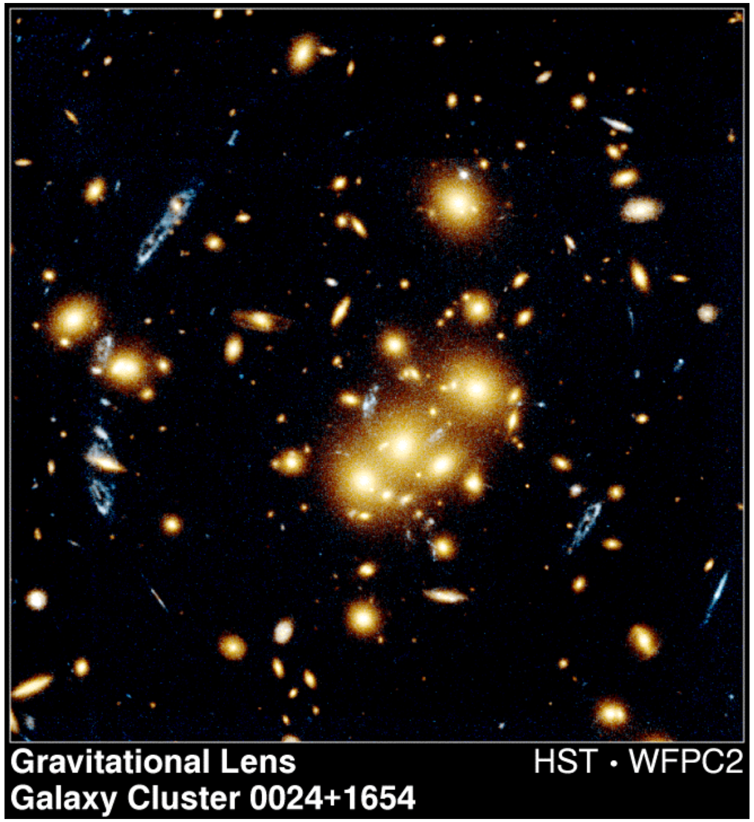
Virial Theorem: $\langle V \rangle + 2\langle K \rangle = 0$

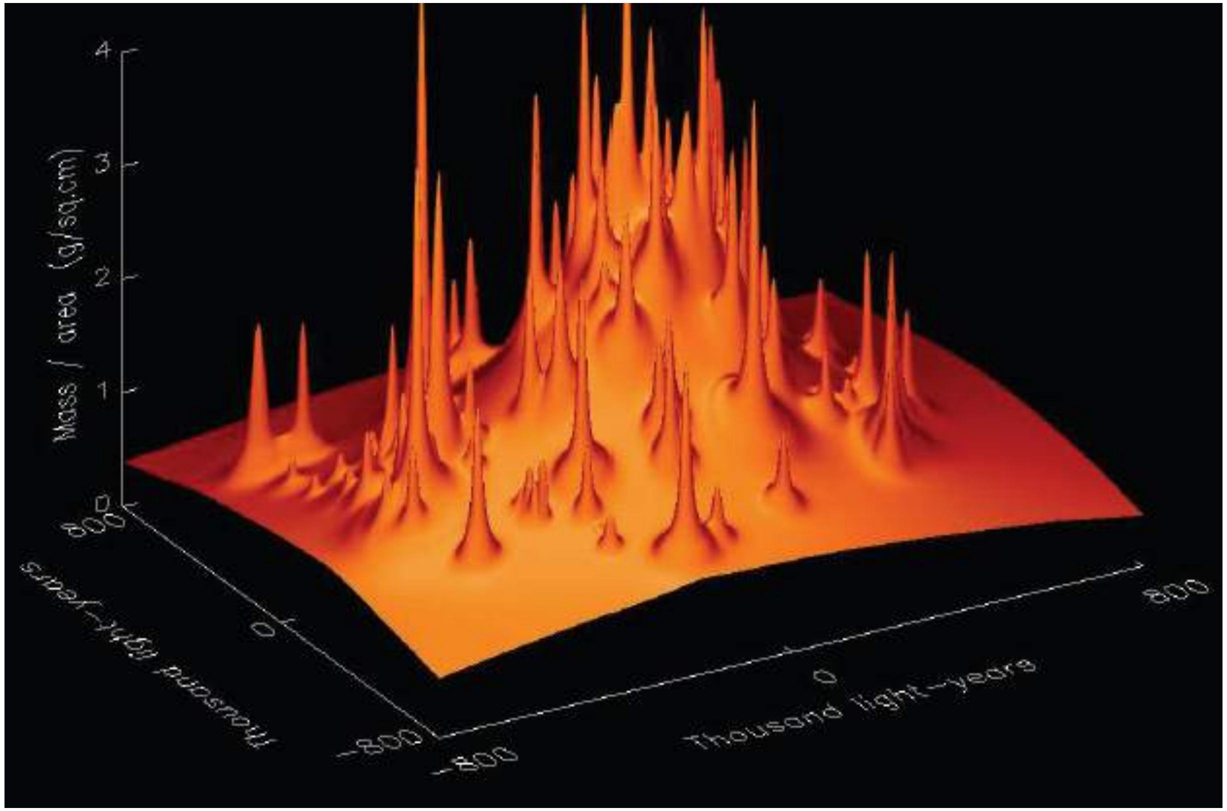
$$V = -\frac{N^2}{2} G_N \frac{\langle m^2 \rangle}{\langle r \rangle}, \quad K = N \frac{\langle m v^2 \rangle}{2}$$

$$M = N\langle m \rangle \sim \frac{2\langle r \rangle \langle v^2 \rangle}{G_N} \gg \sum m_{\mathrm{galaxies}}$$



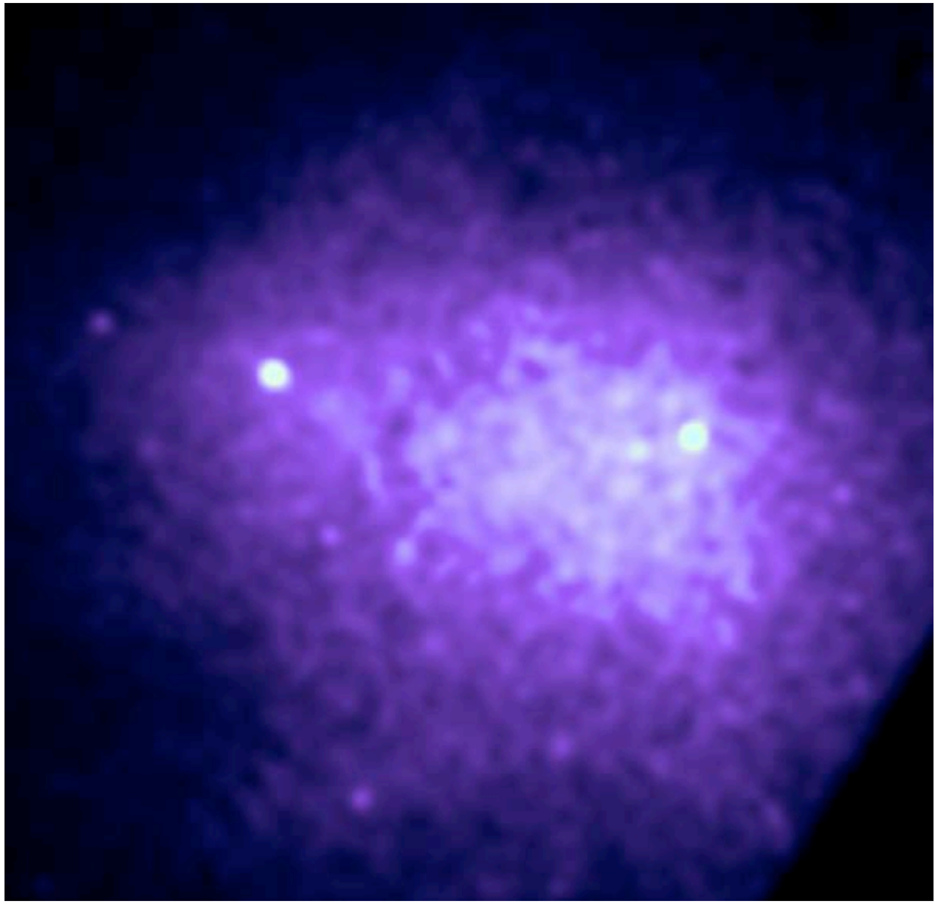
Further evidence comes from observations of **gravitational lensing** of distant sources by a foreground cluster ... enabling the potential to be reconstructed





This reveals that the gravitational mass is dominated by an extended smooth distribution of dark matter

The gravitating mass can also be obtained from X-ray observations of the hot gas in the cluster

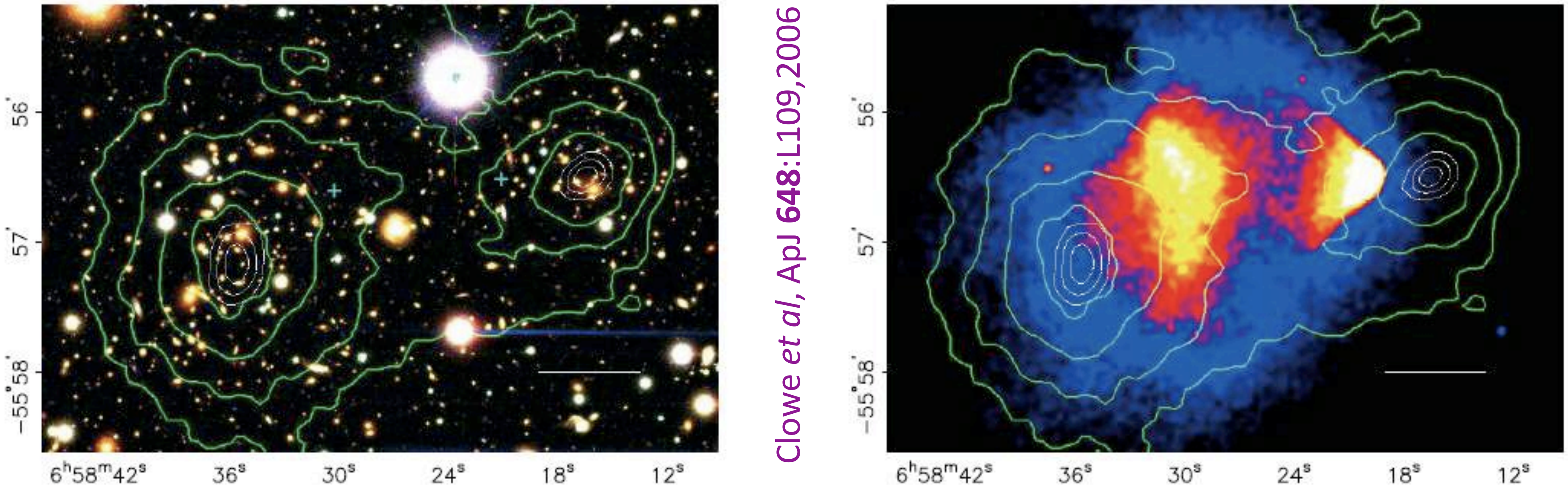


… assuming it is in thermal equilibrium:

$$\frac{1}{\rho_{\rm gas}}\frac{{\rm d}P_{\rm gas}}{{\rm d}r} = \frac{G_{\rm N}M(<r)}{r^2}$$

The Chandra picture of the 'bullet cluster' shows that the X-ray emitting baryonic matter is displaced from the galaxies and the dark matter (inferred through gravitational lensing)

… for many this is convincing evidence of dark matter



Clowe *et al*, ApJ **648**:L109,2006

FIG. 1.—*Left panel*: Color image from the Magellan images of the merging cluster 1E 0657−558, with the white bar indicating 200 kpc at the distance of the cluster. *Right panel*: 500 ks *Chandra* image of the cluster. Shown in green contours in both panels are the weak-lensing κ reconstructions, with the outer contour levels at $\kappa = 0.16$ and increasing in steps of 0.07. The white contours show the errors on the positions of the κ peaks and correspond to 68.3%, 95.5%, and 99.7% confidence levels. The blue plus signs show the locations of the centers used to measure the masses of the plasma clouds in Table 2.

However this is nothing new … it was known *already* that MOND fails (by a factor of ~2) to explain the 'missing matter' in galaxy clusters

(Moreover the rather high relative velocity of the merging clusters is a puzzle in the ΛCDM cosmology as well – only ~0.1 such systems are expected (Kraljic & Sarkar, JCAP **04**:050,2015)

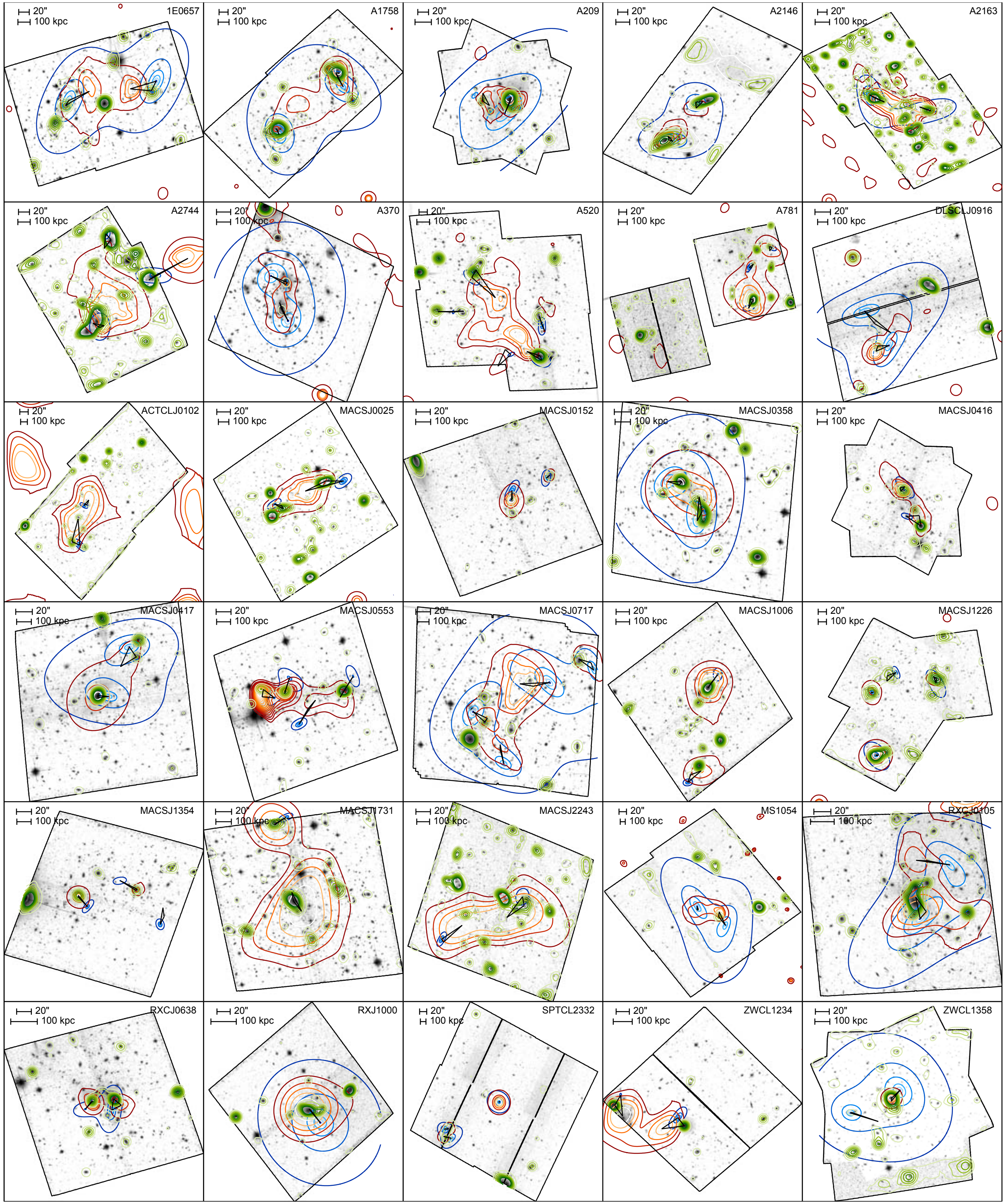
Colliding Clusters

There have been several studies on constraining DM self-interactions via the observation of merging galaxy clusters

Through statistical analysis of a large number of gravitationally lensed clusters in the Chandra catalogue, the DM self-interaction is bounded as:

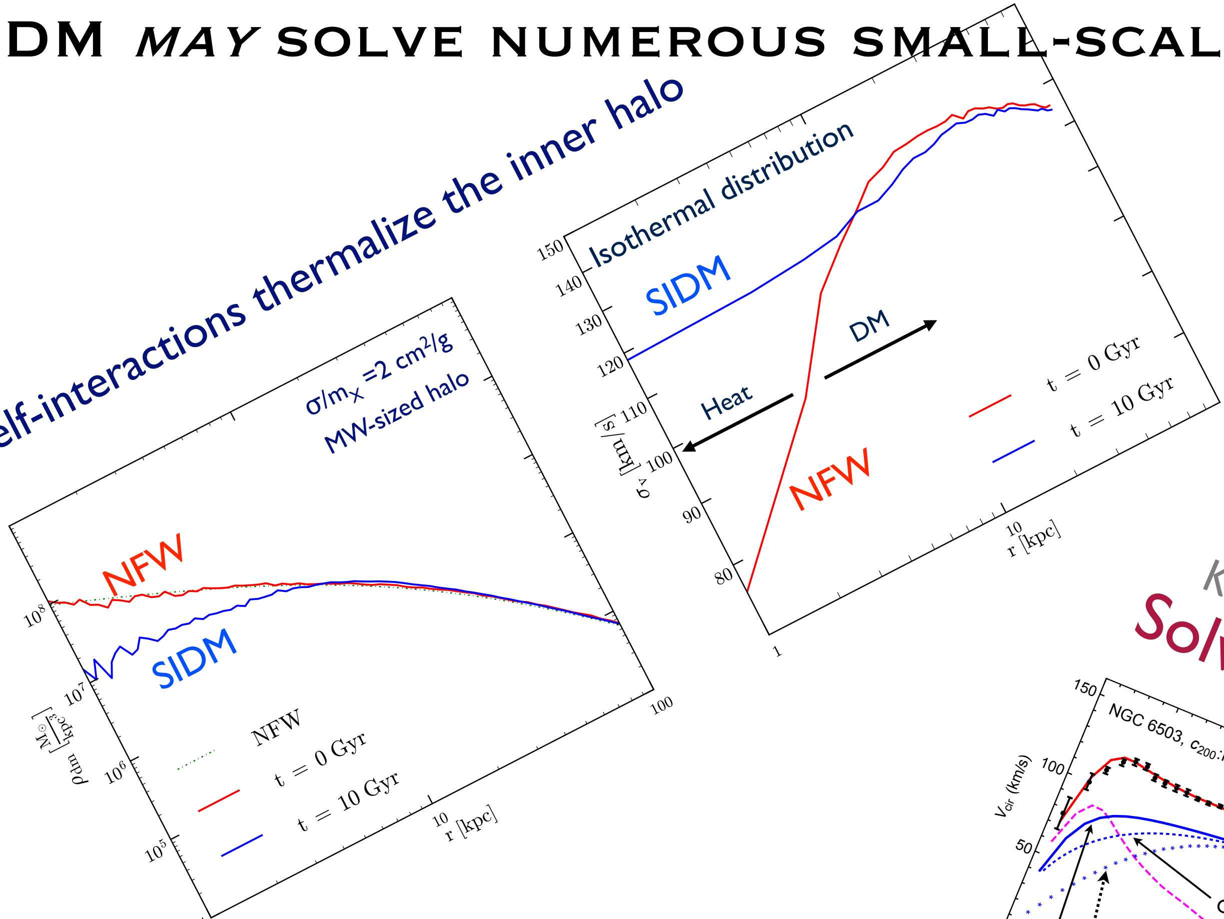
$$\sigma/m_\chi \lesssim 1\ \mathrm{cm}^2/\mathrm{g}$$

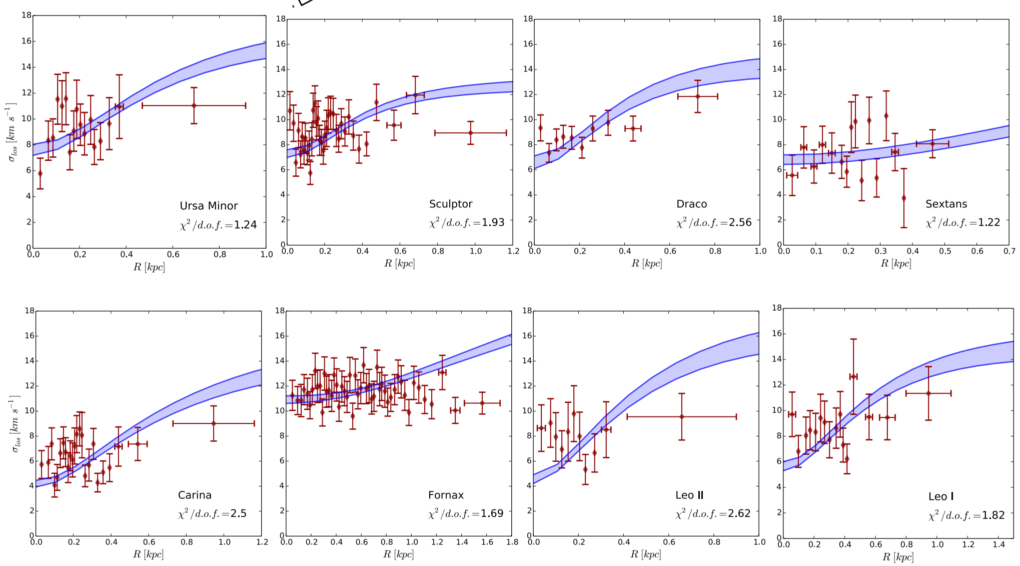
Harvey *et al*, Science **347**:6229,2015



SIDM *MAY* SOLVE NUMEROUS SMALL-SCALE PROBLEMS OF sCDM

- Self-interactions thermalize the inner halo



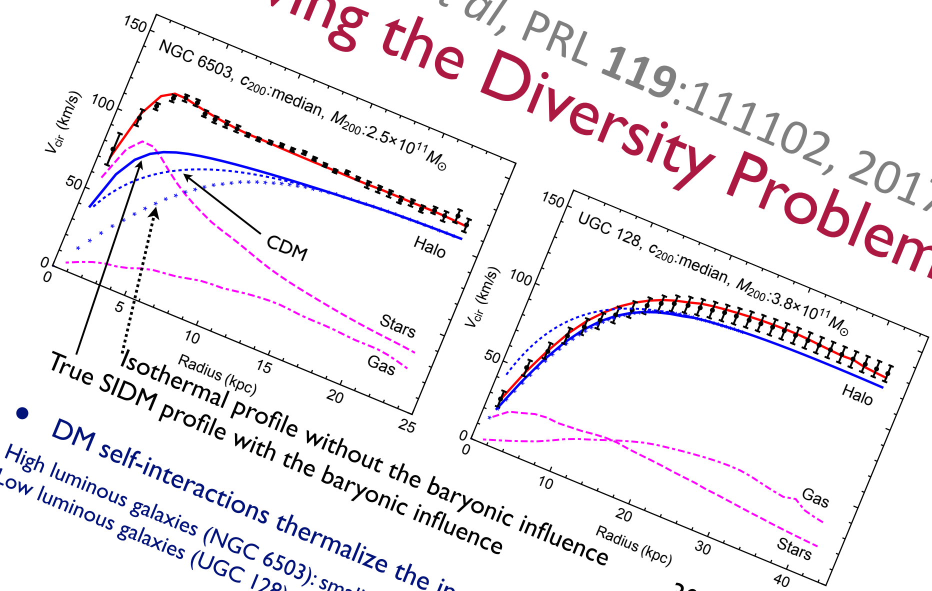


Valli & Yu, Nature Astron. **2**:907, 2018

... but the role of baryons must first be understood (this may account for many of the discrepancies with collisionless dark matter)

Kamada *et al*, PRL **119**:111102, 2017

Solving the Diversity Problem



Isothermal profile without the baryonic influence

True SIDM profile with the baryonic influence

30 galaxies

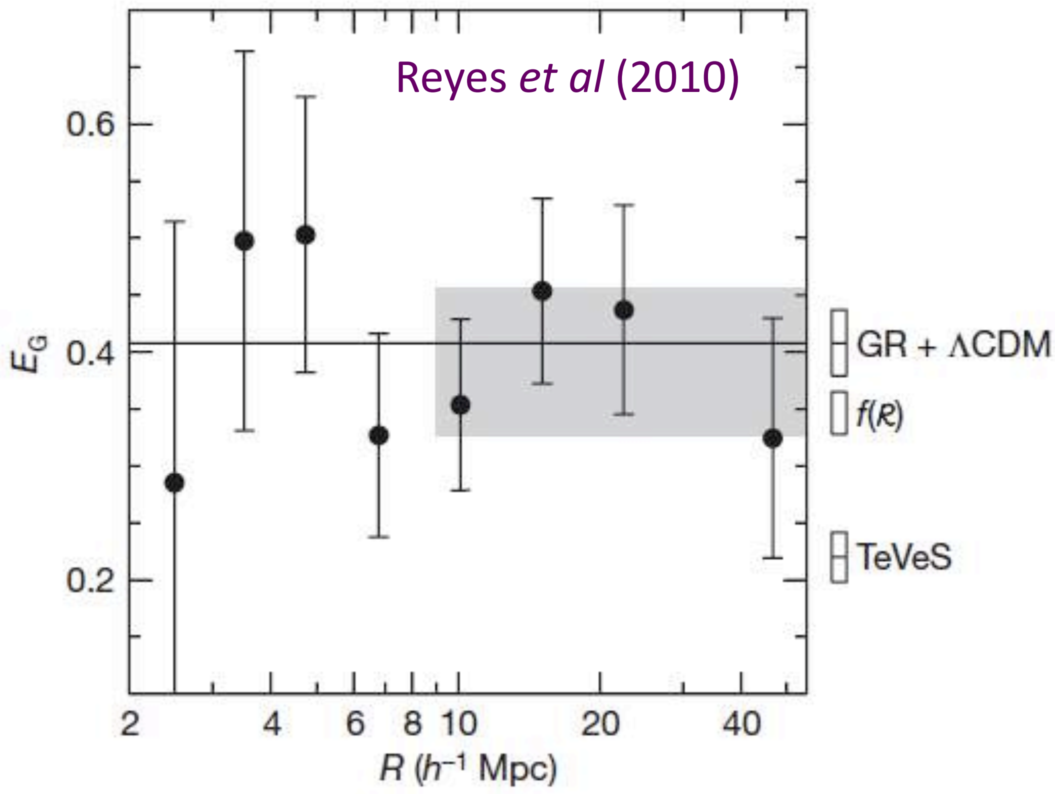
V_{max}~25-300 km/s

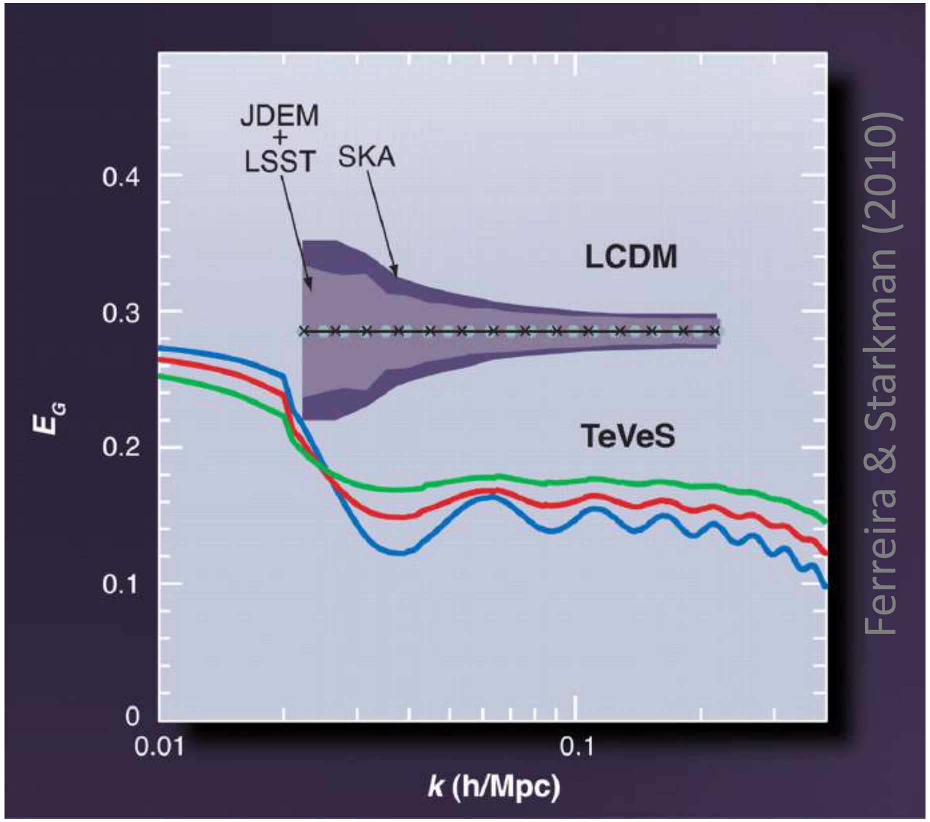
- DM self-interactions thermalize the inner halo together with baryons

High luminous galaxies (NGC 6503): small and dense core

Low luminous galaxies (UGC 128): large and shallow core

Although *new* gravitational physics (underlying MOND) can in principle account for the growth of cosmological structure, there will always be an *observable* distinction ('gravitational slip') between GR and modifications





This is measurable via weak lensing (shearing of galaxy shapes) and its cross-correlation with the galaxy density field … with forthcoming telescopes

Does dark matter exist?

Modified Newtonian Dynamics (MOND) accounts *better* for galactic rotation curves than does dark matter - moreover it predicts the observed correlation between luminosity and rotation velocity: $L \sim v_{rot}^4$ ("Tully-Fisher relation")

… however MOND *fails* on the scale of galaxy clusters and cannot naturally explain the segregation of 'bright' and 'dark' matter seen in merging clusters (e.g. 1E 0657-558)

MOND is *not* a physical theory – relativistic covariant theories that yield MOND exist (e.g. 'TeVeS' by Bekenstein, Phys.Rev.D**70**:083509,2004) but they do *not* provide as satisfactory an understanding of CMB anisotropies and structure formation, as has the standard (cold) dark matter cosmology

… nevertheless you may like to keep an open mind until dark matter is actually detected (non-gravitationally)!