Particle Acceleration in Astrophysics

2. What can we learn from observations

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Stanford University

Work based on several PhD theses and
collaborations with post doctoral fellows and colleagues

Zakopane-2, 2019

The Problem

Acceleration

Plasma properties

n, T, B, L

Turbulence spectrum

$W(k) \propto k^{-q}$

wave-particle interactions

$D_{\mu\mu}, D_{pp}$

Shock

$u_{\rm sh}$

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{\rm EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}\left[(\tilde{A}(E) - \dot{E}_{\rm L})N\right] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

Radiation

Background density

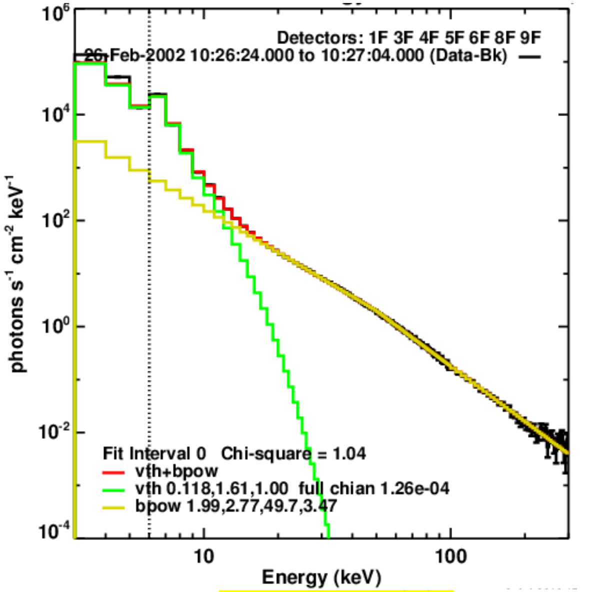
$n(s)$

Radiating electron spectrum

$N_{\rm eff}(E, s)$

$$J(\epsilon, s) = \int_0^\infty vN_{\rm eff}(E, s)\sigma(\epsilon, E, s)dE$$

Observation



$$I(\epsilon) = \frac{J(\epsilon)}{4\pi R^2}$$

The Forward Fitting Method

Forward Fitting

Acceleration

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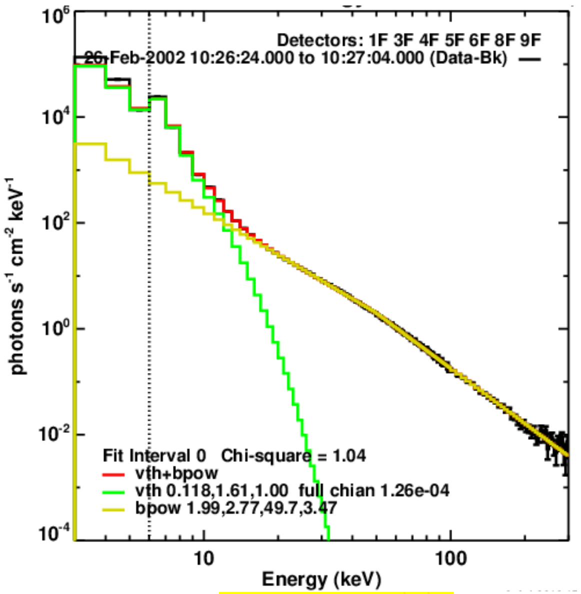

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The Inversion Method

Inverse process

Acceleration

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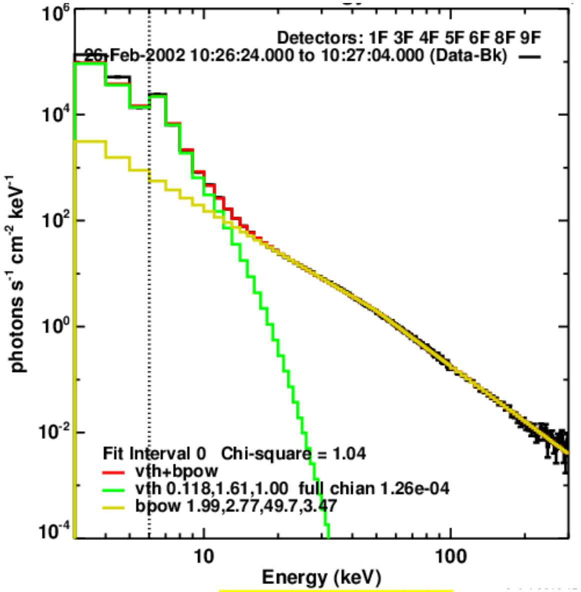

$$I(\epsilon) = \frac{J(\epsilon)}{4\pi R^2}$$

Outline

I. Description of a nonparametric *inversion method*

II. Applications of the Inversion Method

- *A. Cosmic Rays and Supernova Remnant Spectra*
- *B. Solar Flare Hard X-ray emissions*

III. Some Interesting Puzzles of Ion acceleration

- *A. Extreme enhancement of 3He and Heavy ions*
- *B. Electron spectra at Flare sites and observed at Earth*

Back to the Leaky Box Model *(Steady State)*

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \dot{E}_L)N] - \frac{N}{T_{\rm esc}} + \dot{Q} = 0$$

Two unknowns D_{pp} and $D_{\mu\mu}$

Therefore we need two spectral information

1. *Accelerated Particle Flux* $F_{\rm acc}(E) = vN(E)/V$
2. *Escaping Particle Flux* $F_{\rm esc}(E) = N(E)/(T_{\rm esc}S);\ V \sim SL$

These Give the escape time $T_{\rm esc}(E) = \tau_{\rm cross}F_{\rm acc}(E)/F_{\rm esc}(E)$

If there is no *B* convergence from this we can get

$$\tau_{\rm sc} = \tau^2_{\rm cross}/(T_{\rm esc} - \tau_{\rm cross}) \quad \text{and} \quad A_{\rm sh} \propto \tau_{\rm sc}^{-1} \text{ and } D_{\mu\mu}$$

Then integration of the kinetic equation over *E* gives

$$D_{\rm EE} = E\left[\xi - \frac{d\ln N}{d\ln E}\right]^{-1}\left[\dot{E}_{\rm L} - A_{\rm sh} + \frac{1}{N}\int_E^\infty \left(\frac{N}{T_{\rm esc}} - \dot{Q}\right)dE\right] = v^2 D_{pp}$$

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$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \dot{E}_L)N] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

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These Give the escape time $T_{\rm esc}(E) = \tau_{\rm cross}F_{\rm acc}(E)/F_{\rm esc}(E)$

from which we get $\tau_{\rm sc} = \tau^2_{\rm cross}/(T_{\rm esc} - \tau_{\rm cross})$ and $A_{\rm sh} \propto \tau_{\rm sc}^{-1}$

Then integration of the kinetic equation over *E* gives

$$D_{\rm EE} = E\left[\xi - \frac{d\ln N}{d\ln E}\right]^{-1}\left[\dot{E}_{\rm L} - A_{\rm sh} + \frac{1}{N}\int_E^\infty\left(\frac{N}{T_{\rm esc}} - \dot{Q}\right)dE\right] \longrightarrow \begin{matrix} A_{\rm SA} = \xi D_{EE}/E \\ \tau_{\rm ac} = E/(A_{\rm SA} + A_{\rm sh}) \end{matrix}$$

Transport and Spectrum of Escaping Particles

An Important Special Case

$$\frac{\partial N(E)}{\partial t} = -\frac{\partial}{\partial E}(\dot{E}_{\mathrm{L}}^{\mathrm{tra}} N(E)) - \frac{N(E)}{T_{\mathrm{esc}}^{\mathrm{tra}}} + \dot{Q}^{\mathrm{tra}}(E) = 0, \quad \dot{Q}^{\mathrm{tra}}(E) = \frac{N^{\mathrm{ac}}(E)}{T_{\mathrm{esc}}^{\mathrm{ac}}}$$

$$N_{\mathrm{eff}}(E) = \frac{1}{\dot{E}_L^{\mathrm{tra}}(E)} \int_E^{\infty} dE' \; \dot{Q}^{\mathrm{tra}}(E') \; \exp\left[-\int_E^{E'} \frac{dE''}{E''} \; \frac{\tau_L^{\mathrm{tra}}(E'')}{T_{\mathrm{esc}}^{\mathrm{tra}}(E'')} \right]$$

If particles lose all their energy in the transport region then

$$N_{\mathrm{eff}}(E)\dot{E}_L^{\mathrm{tra}}(E) = \int_E^{\infty} dE' \; \frac{N_{\mathrm{acc}}}{T_{\mathrm{esc}}^{\mathrm{ac}}(E')}$$

Transport and Spectrum of Escaping Particles

An Important Special Case

$$N_{\rm eff}(E)\dot{E}_L^{\rm tra}(E) = \int_E^\infty dE' \, \frac{N_{\rm acc}}{T_{\rm esc}^{\rm ac}(E')}$$

$$T_{\rm esc}^{\rm acc}(E) = \tau_L^{\rm tra}\left[\frac{N_{\rm acc}}{N_{\rm eff}}\right][\delta_{\rm eff}(E)+\delta_L(E)-1]^{-1}$$

Recall that $D_{\rm EE} = E\left[\xi - \frac{d\ln N}{d\ln E}\right]^{-1}\left[\dot{E}_{\rm L} - A_{\rm sh} + \frac{1}{N}\int_E^\infty \left(\frac{N}{T_{\rm esc}} - \dot{Q}\right) dE\right]$

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From These we can get the effective acceleration rate

$$A_{eff} \equiv A_{sh} + (1+\delta_{acc}/\xi)A_{SA} = E[\tau_{L,\rm acc}^{-1} + \tau_{L,\rm tra}{}^{-1}(N_{eff}/N_{acc})]$$

The Inversion Method

For radiating sources the first step is to obtain particle spectrum fromphoton spectrum

Inverse process

Acceleration

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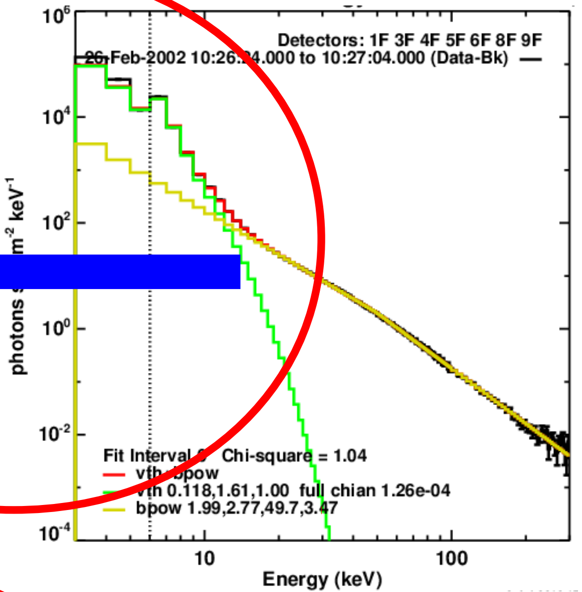


$$I(\epsilon) = \frac{J(\epsilon)}{4\pi R^2}$$

The Inversion Method

For radiating sources the first step is to obtain particle spectrum fromphoton spectrum

We need to invert this to obtain $N_{\text{eff}}(E)$

$$J(\epsilon, s) = \int_0^\infty vN_{\text{eff}}(E, s)\sigma(\epsilon, E, s)dE$$

There are two methods that work well when the cross section is a simple function of photon and particle energies like bremsstrahlung and synchrotron. IC more complex but approximation can be used satisfying this requirements.

1. Generalized Inversion method by Piana et al.

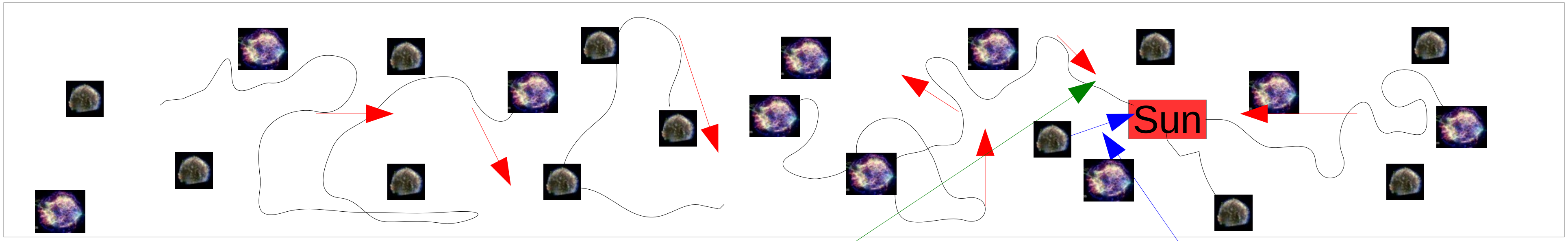
2. The Johns-Lin method of matrix inversion

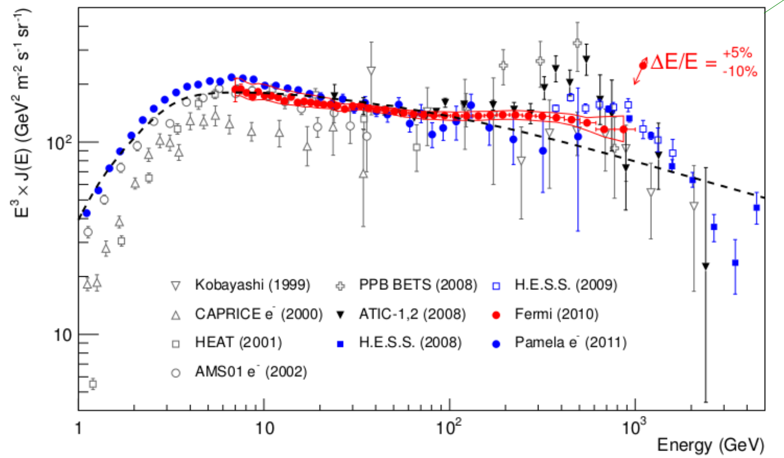
II-A. Application to Acceleration of Electrons in Supernova Remnants

Using Observed Radiation and Cosmic Ray Spectra

Supernova Remnant and Cosmic Ray Observations

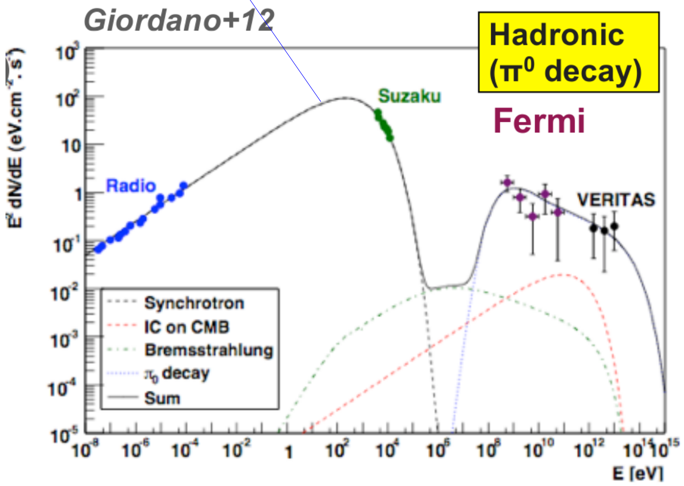
A. Toy Model





Cosmic-rays Observed near Earth This gives

$$N_{\mathrm{eff}}(E) \propto E^{-3-\varepsilon} e^{-E/E_c}$$

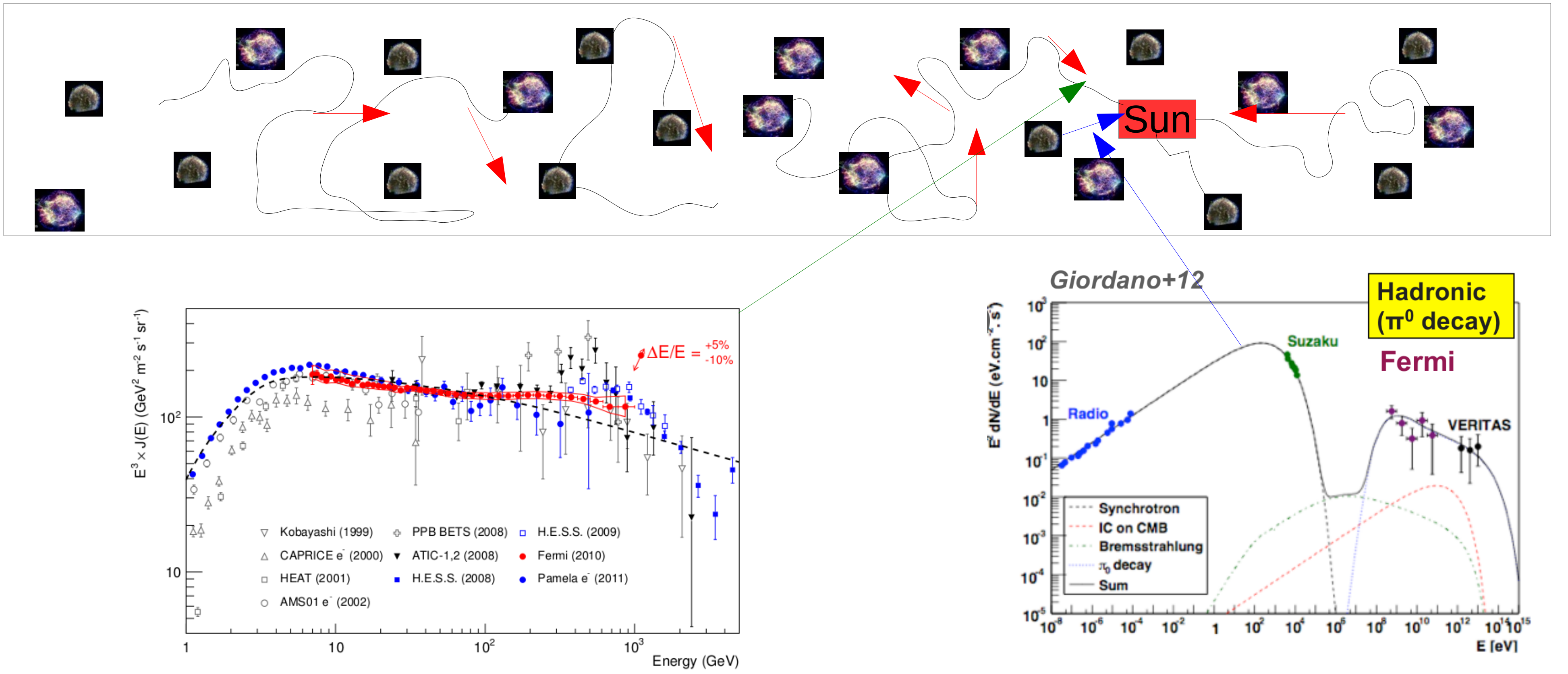


SNR radiation Observed near Earth gives

$$N_{\mathrm{acc}}(E) \propto E^{-\alpha_1} \exp\left[-(E/E_{\max})\right]^{\alpha_2}$$

Supernova Remnant and Cosmic Ray Observations

A. Toy Model



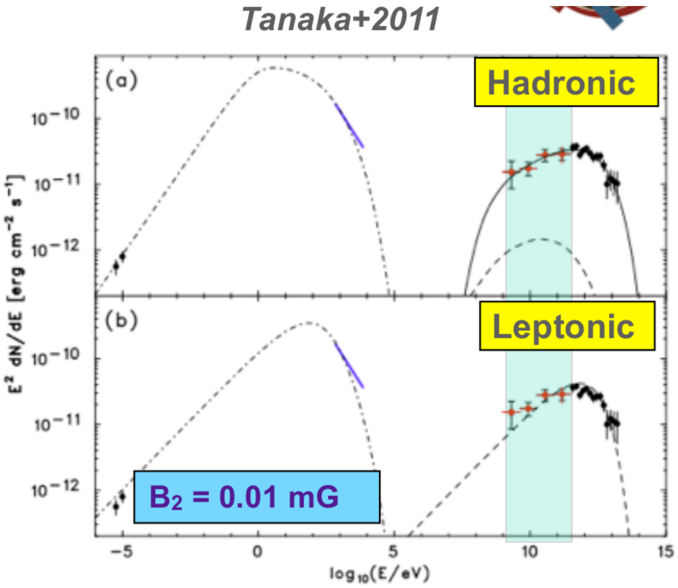
Cosmic-rays Observed near Earth This gives

SNR radiation Observed near Earth gives

Spectra of Accelerated Electrons

From Radio and X-ray (and gamma-ray?) Observations

Tanaka+2011



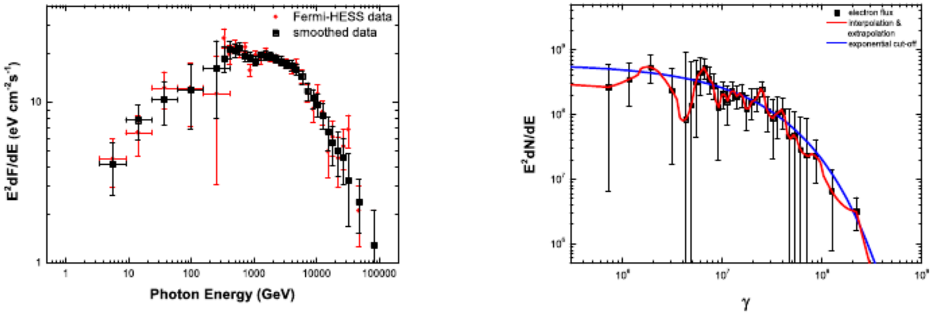


Fig. 1.— Left: the observed (circle) and smoothed (square) GeV and TeV γ-ray fluxes for RX J1713.7-3946. (Abdo et al. 2011, Aharonian et al. 2006). Right: the derived electron distribution with error bars. The red solid line represents the inter- and extrapolated electron distribution, which is used to calculate the synchrotron and IC radiation spectra in Figure 2. The blue solid line shows an analytical function, which can also reproduce the observed radiation spectrum.

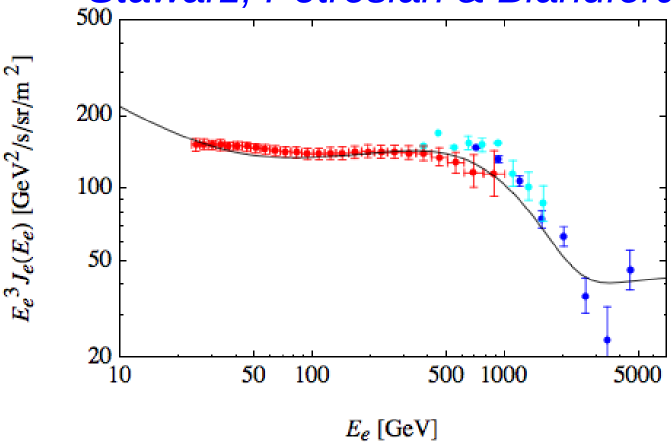
Hui, Li et al. 2010

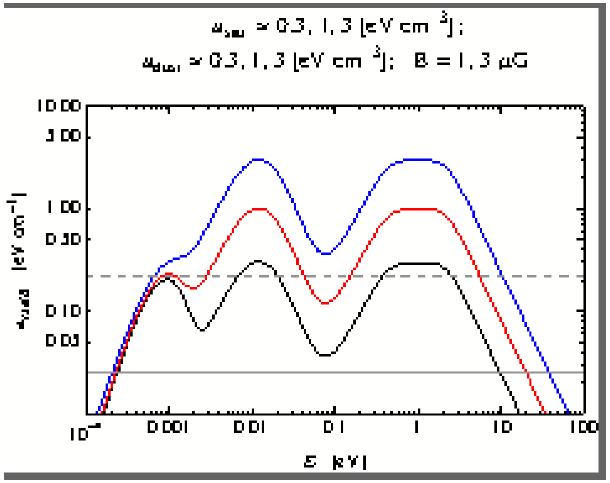
$\alpha_1 \sim 2; \ \ \alpha_2 \sim 0.6$ Lazendic et al. 2004

Effective Spectra of Escaping Electrons
Two Interpretation of the Bump

Klein-Nishina Effect in IC Losses

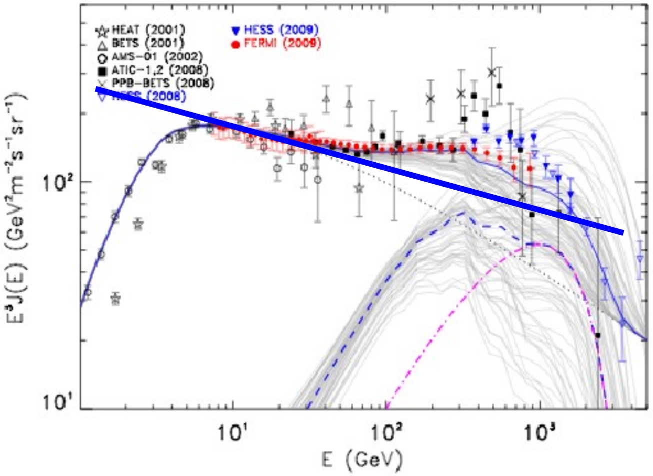
Stawarz, Petrosian & Blandford

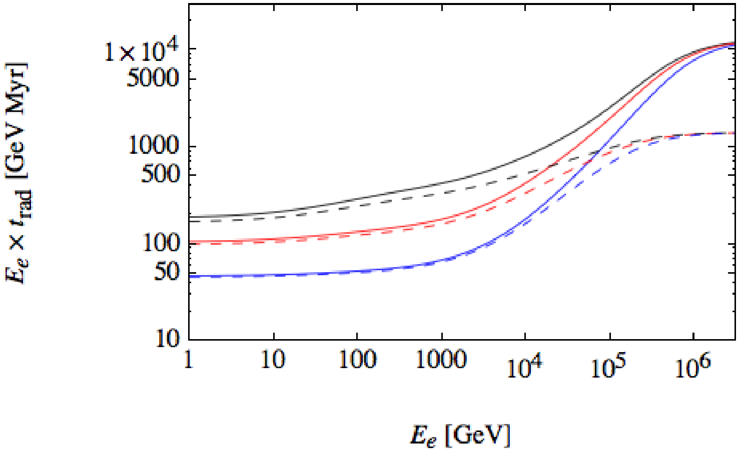




Nearby Pulsar

Fermi Collabor. Abdo et al.





Zakopane-2, 2019

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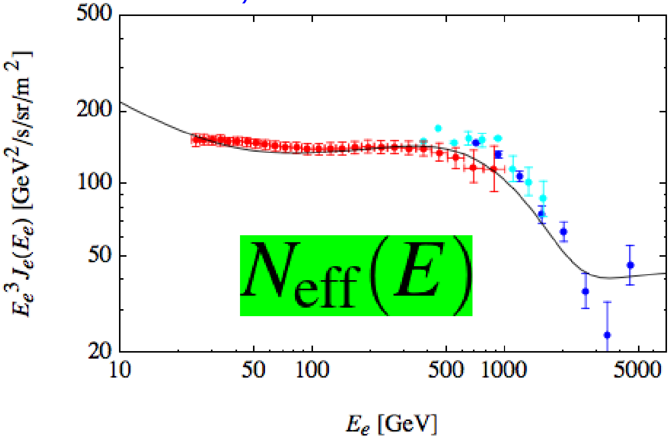
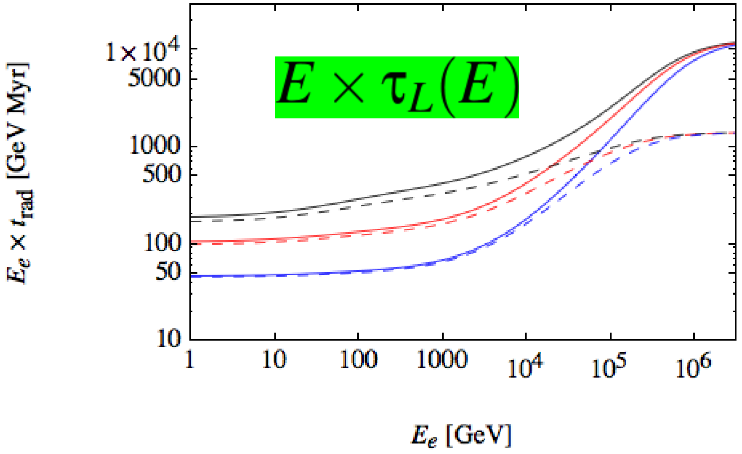


$N_{\rm eff}(E)$



$E \times \tau_L(E)$

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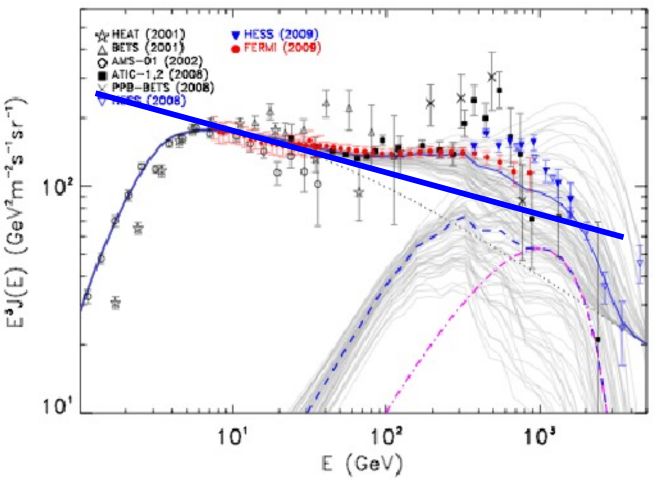


$$N_{\rm eff}(E) \propto E^{-3-\varepsilon} e^{-E/E_c}$$

$$E \times \tau_L(E) \sim {\rm Const.}$$

Escape Time: *Two Methods*

A. $$T_{\rm esc}^{\rm acc}(E) = \tau_L^{\rm tra}\left[\frac{N_{\rm acc}}{N_{\rm eff}}\right][\delta_{\rm eff}(E) + \delta_L(E) - 1]^{-1}$$

$$N_{\rm acc}^{\rm gal} = \bar{N}_{\rm acc}^{\rm snr} \times \dot{n}_{\rm snr} \times \tau_{\rm active}$$

$$N_{\rm eff}^{\rm gal} = (4\pi J_{\rm CR,e}/v)V_{\rm CR,e}^{\rm gal}$$

B. $$T_{\rm esc}(E) = \tau_{\rm cross}F_{\rm acc}(E)/F_{\rm esc}(E)$$

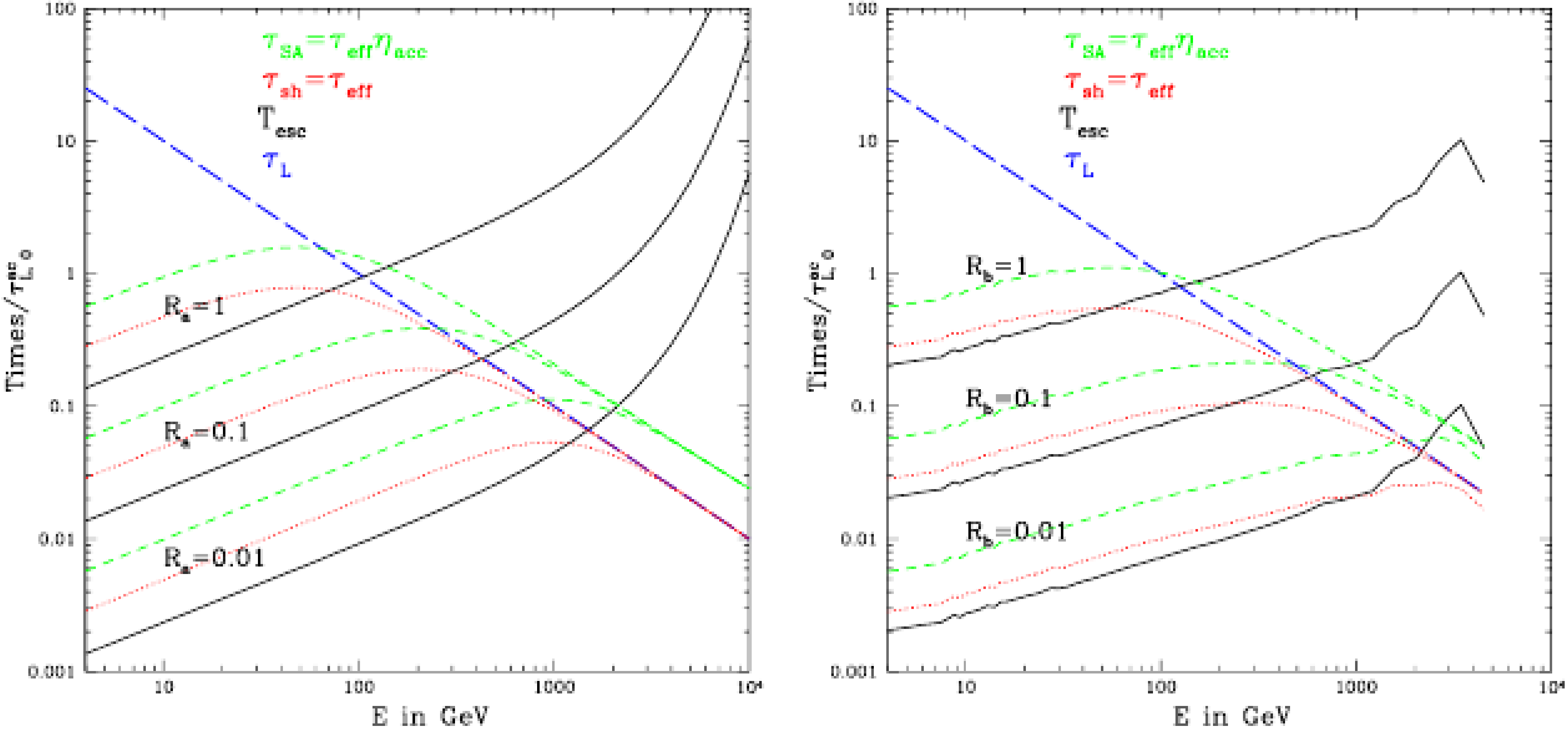
$$F_{\rm acc} = vN_{\rm acc}^{\rm snr} \quad \text{and} \quad F_{\rm esc} = \dot{Q}_{\rm inj}^{\rm gal}$$

Injected Q obtained using e.g. Galprop

Fitting to the obseved CR spectrum

$$A_{eff} \equiv A_{sh} + (1+\delta_{acc}/\xi)A_{SA} = E[\tau_{L,\rm acc}^{-1} + \tau_{L,\rm tra}{}^{-1}(N_{eff}/N_{acc})]$$

RESULTS From Two Methods



RESULTS: *Scatering Time*

Two methods of determining the coefficient $D_{\mu\mu}$

or the scattering time $\tau_{\rm sc} = \langle (1-\mu^2)^2/\bar{D}_{\mu\mu} \rangle$

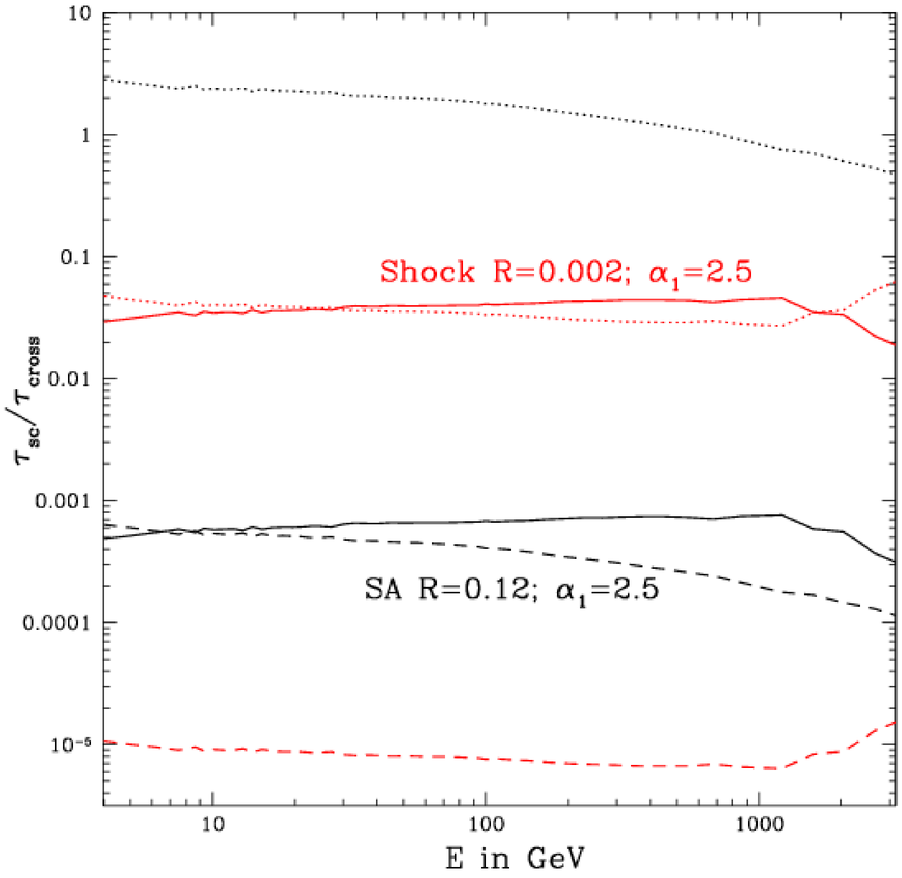
$$\tau_{\rm sc} = \tau_{\rm cross}^2/(T_{\rm esc} - \tau_{\rm cross}) \quad \text{and} \quad A_{\rm sh} \propto \tau_{\rm sc}^{-1}$$

$$\tau_{\rm sc} = \tau_{\rm cross}^2/T_{\rm esc} \quad \text{and} \quad \tau_{\rm sc} \sim \beta_{\rm eff}^2 \tau_{\rm ac}$$

$$\beta_{\rm eff}^2 = \beta_{\rm sh}^2 + \beta_{\rm A}^2/\eta_{\rm acc}$$

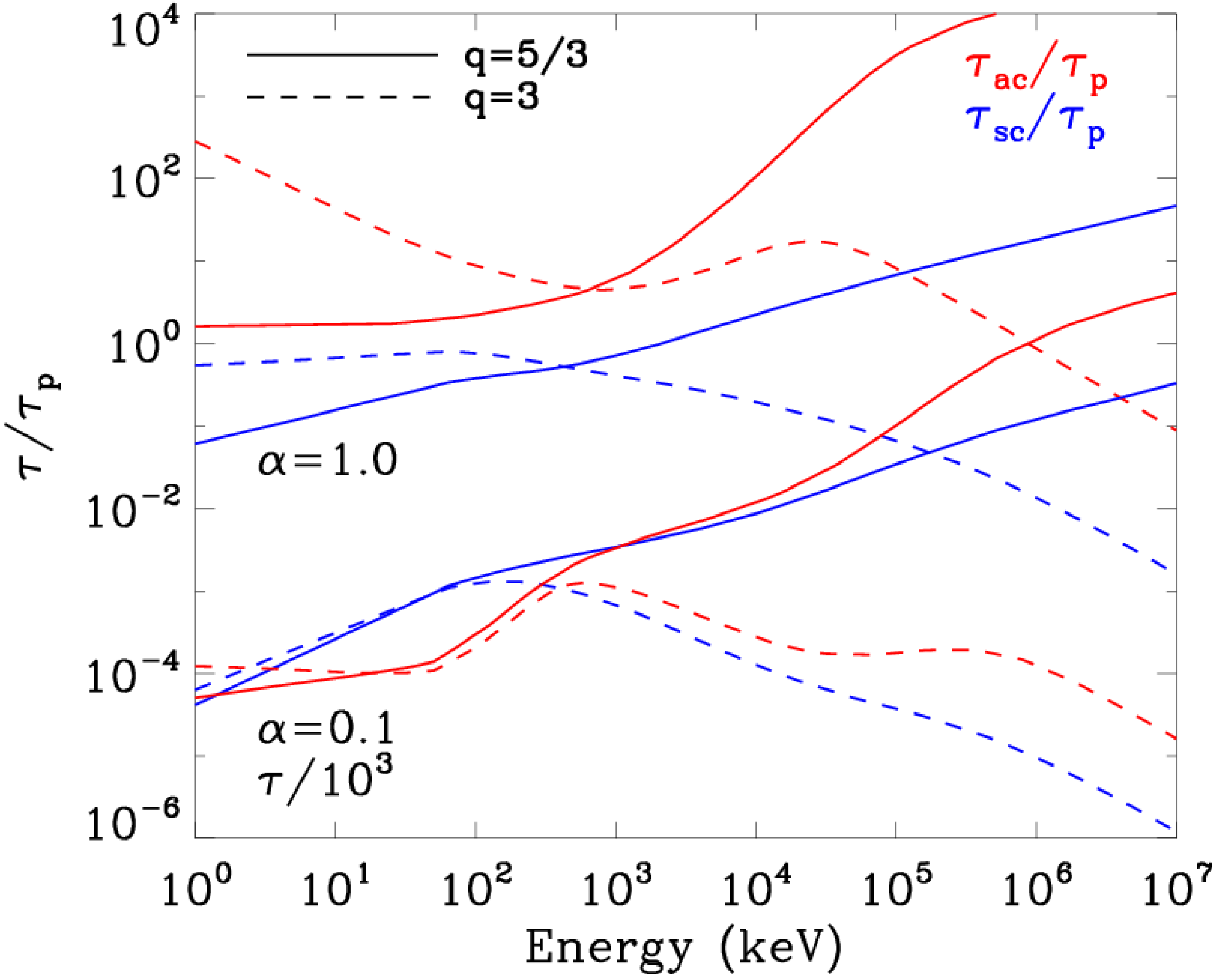
RESULTS: *Scattering Time*

Solid Curves from Escape time
Dashed (SA) and Dotted (Shock) from Acceleration times



Scattering And Acceleration Times

Interaction with Parallel Waves

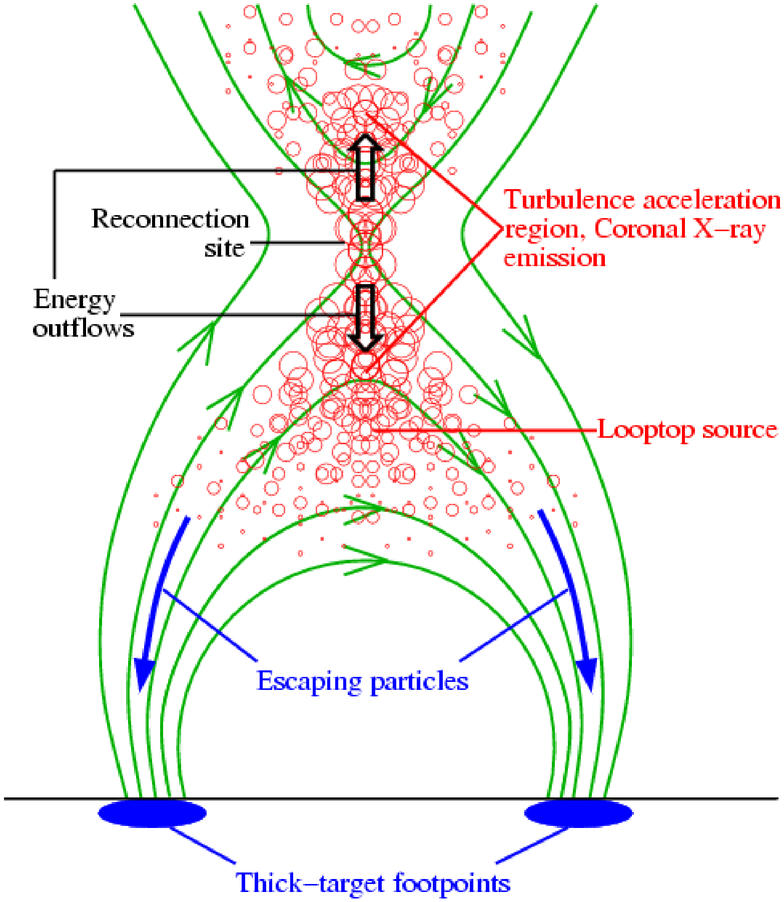


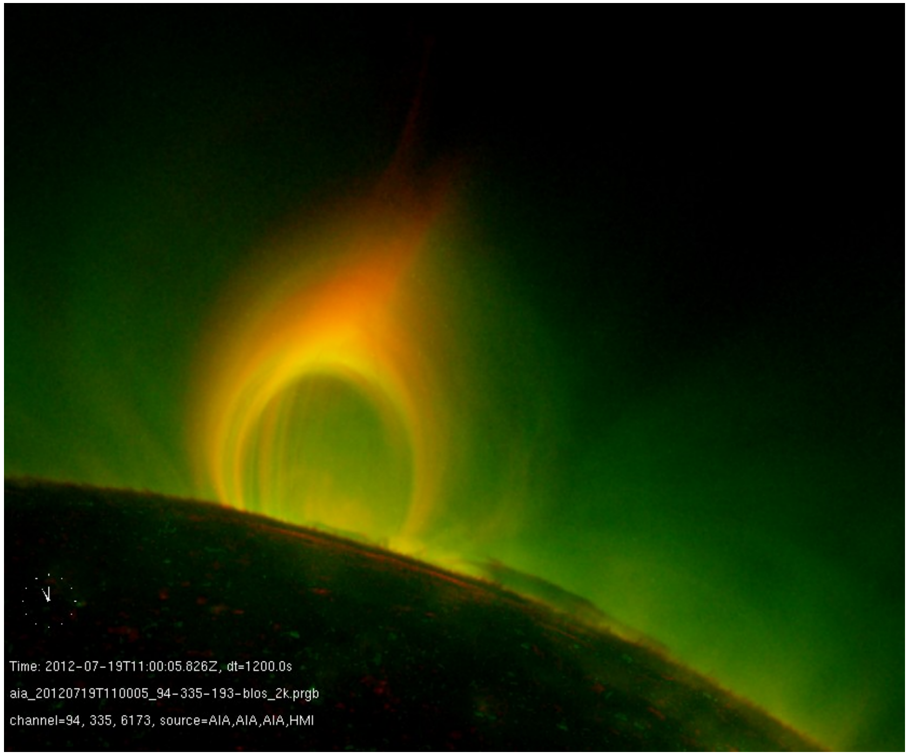
Pryadko and Petrosian 1997

II-B. Application to Acceleration of Electrons in Solar Flares

Using Observed hard X-ray Spectra from Loop top and Foot points of Flare Loops

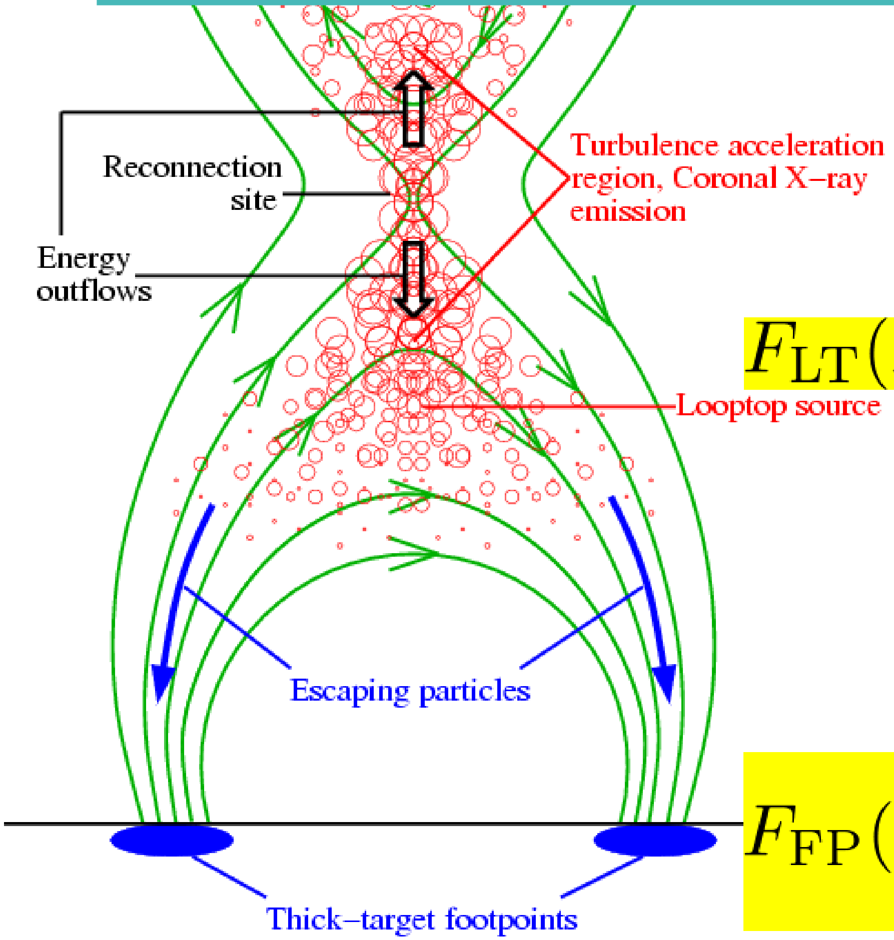
Cartoon and Observation of a Flare Loop





The Basic Solar Flare Model

Relating Electrons and Photons



See Petrosian & Chen (2010)
ApJ Letters, 2010, 712, 131

$$F_{\rm LT}(E) = v(E)N(E)$$

Connected by Escaping Process

$$F_0(E) = F_{\rm LT}(E)(\tau_{\rm cross}/T_{\rm esc})$$

$$F_{\rm FP}(E) = vN_{\rm FP} = \frac{v(E)}{\dot{E}_{\rm L}(n)} \int_E^\infty \frac{N(E')}{T_{\rm esc}(E')} {\rm d}E'$$

HXR: $$\left\{ \begin{array}{c} I_{\rm LT}(\epsilon) \\ I_{\rm FP}(\epsilon) \end{array} \right\} = \frac{nV}{4\pi R^2} \int_\epsilon^\infty \left\{ \begin{array}{c} F_{\rm LT}(E) \\ F_{\rm FP}(E) \end{array} \right\} \sigma(\epsilon, E) dE$$

Results From Inversion of the Kinetic Equation

$$D_{\rm EE} = E\left[\zeta - \frac{d\ln N}{d\ln E}\right]^{-1}\left[\dot{E}_{\rm L} + \frac{1}{N}\int_E^{\infty}\left(\frac{N}{T_{\rm esc}} - \dot{Q}\right)dE\right]$$

$$N(E) = F_{\rm LT}(E)/v$$

$$F_{\rm FP} = \frac{v}{\dot{E}_{\rm L}}\int_E^{\infty}\frac{N(E')}{T_{\rm esc}(E')}{\rm d}E'$$

From these we derive Energy Dependence of the diffusion rates

$$D_{\rm EE} = \frac{E^2}{\tau_{\rm L}}\left[\frac{F_{\rm FP}}{F_{\rm LT}} + 1\right]\left[\delta_{\rm LT}(E) + \frac{2\gamma}{\gamma+1}\right]^{-1}$$

$$\langle D_{\mu\mu}(E)\rangle = \tau_{\rm scat}^{-1} = \frac{\tau_L}{\tau_{\rm cross}^2}\frac{F_{\rm LT}}{F_{\rm FP}}\left[\delta_{\rm FP}(E) + \frac{2}{\gamma^2+\gamma}\right]^{-1}$$

Both Coefficients Depend only on Observables

(Petrosian & Chen, 2010 ApJ L, 712, 131)

The Basic Model:

Relating Electrons and Photons

Bremsstrahlung Hard X-ray Emission

$$I_i(\epsilon) = \frac{1}{4\pi R^2}\int_\epsilon^\infty X_i(E)\sigma(\epsilon, E)dE, \text{ with } X_{\rm LT,FP} = \bar{n}_{\rm LT,FP}F_{\rm LT,FP}(E)$$

Applying our inversion relations we get

$$T_{\rm esc}(E) = \tau_{\rm L}^{\rm LT}\left[\frac{X_{\rm LT}}{X_{\rm FP}}\right]\left[\delta_{\rm eff}(E) + \frac{2}{\gamma^2+\gamma}\right]^{-1}$$

and

$$D_{\rm EE} = \frac{E^2}{\tau_{\rm L}^{\rm LT}}\left[\frac{X_{\rm FP}}{X_{\rm LT}}+1\right]\left[\delta_{\rm LT}(E) + \frac{2\gamma}{\gamma+1}\right]^{-1}$$

or

$$A_{\rm sh} = \frac{E}{E_L^{\rm LT}}\left[\frac{X_{\rm LT}}{X_{\rm FP}}+1\right]$$

Petrosian & Chen (2013)
See recent astro-ph posting

Regularized Inversion of Photon Images to Electron Images

$$I(x,y;\epsilon) = \frac{a^2}{4\pi R^2}\int_{E=\epsilon}^{\infty} N(x,y)\overline{F}(x,y;E)Q(\epsilon,E)\,dE \quad J(x,y;q)\,dq = \int_x\int_y\int_{\epsilon=q}^{\infty} D(q,\epsilon)I(x,y;\epsilon)\,d\epsilon\,dx\,dy$$

RHESSI produces count visibility, Fourier component of the source

$$V(u,v;q) = \mathcal{F}^2(J(x,y;q)) \equiv \int_x\int_y J(x,y;q)e^{2\pi i(ux+vy)}\,dx\,dy$$

Defining electron flux visibility spectrum and count cross section

$$W(u,v;E) = a^2\int_x\int_y N(x,y)\overline{F}(x,y;E)e^{2\pi i(ux+vy)}\,dx\,dy \qquad K(q,E)\,dq = \int_{\epsilon=q}^{\infty} D(q,\epsilon)Q(\epsilon,E)\,d\epsilon$$

We get $V(u,v;q) = \frac{1}{4\pi R^2}\int_q^{\infty} W(u,v;E)K(q,E)\,dE$

Regularized inversion produced *smoothed* electron flux visibility spectrum

$$\left\|\boldsymbol{V}_{[u,v]} - \boldsymbol{K}\cdot\boldsymbol{W}_{[u,v]}\right\|^2 + \lambda_{[u,v]}\left\|\boldsymbol{W}_{[u,v]}\right\|^2 = \text{minimum}$$

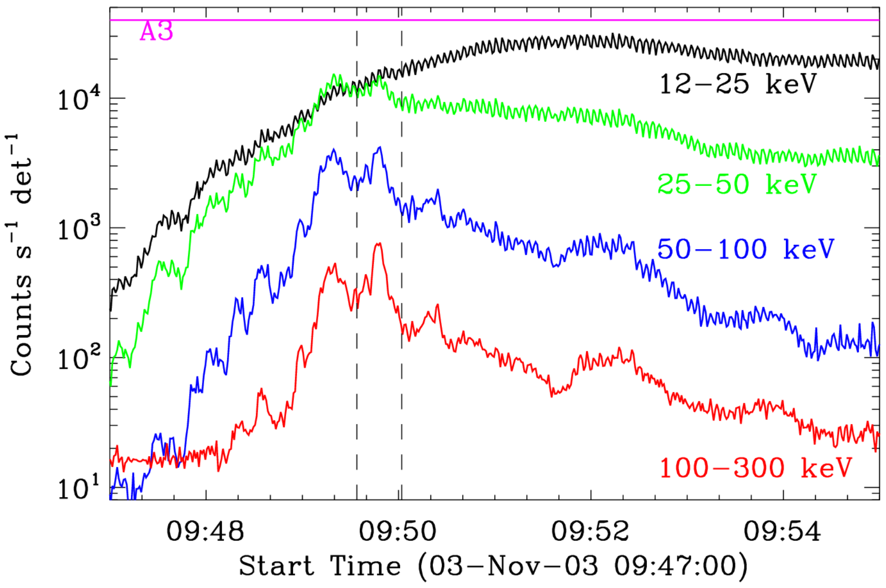
Fourier Transform Gives $N(x,y)\overline{F}(x,y;E) = \frac{1}{a^2}\int_u\int_v W(u,v;E)e^{-2\pi i(ux+vy)}\,du\,dv$

Piana et al. 2007

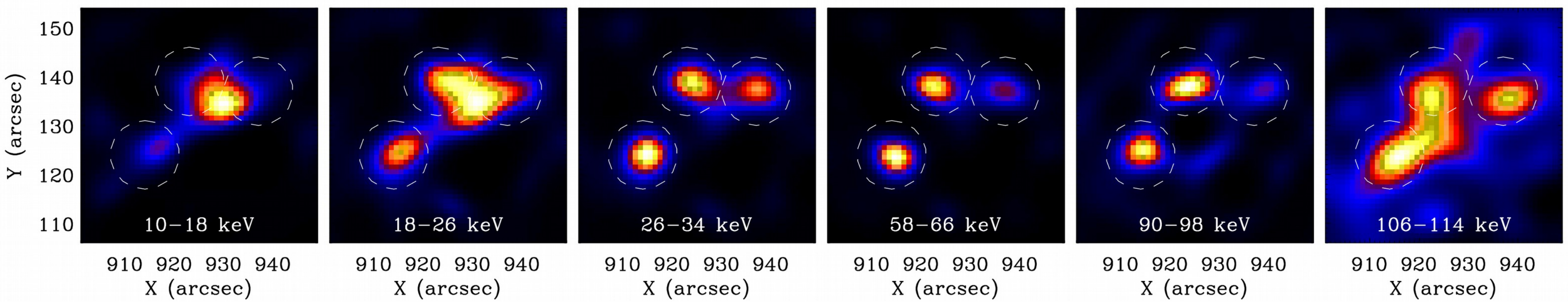
2003 Nov 3 Flare (X3.9 class)

LT source detected up to 100-150 keV

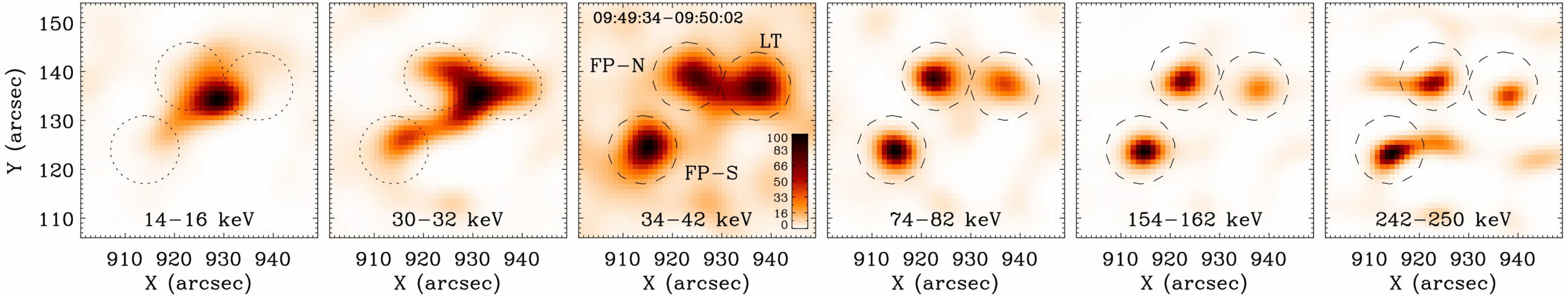
(Chen & Petrosian 2013)



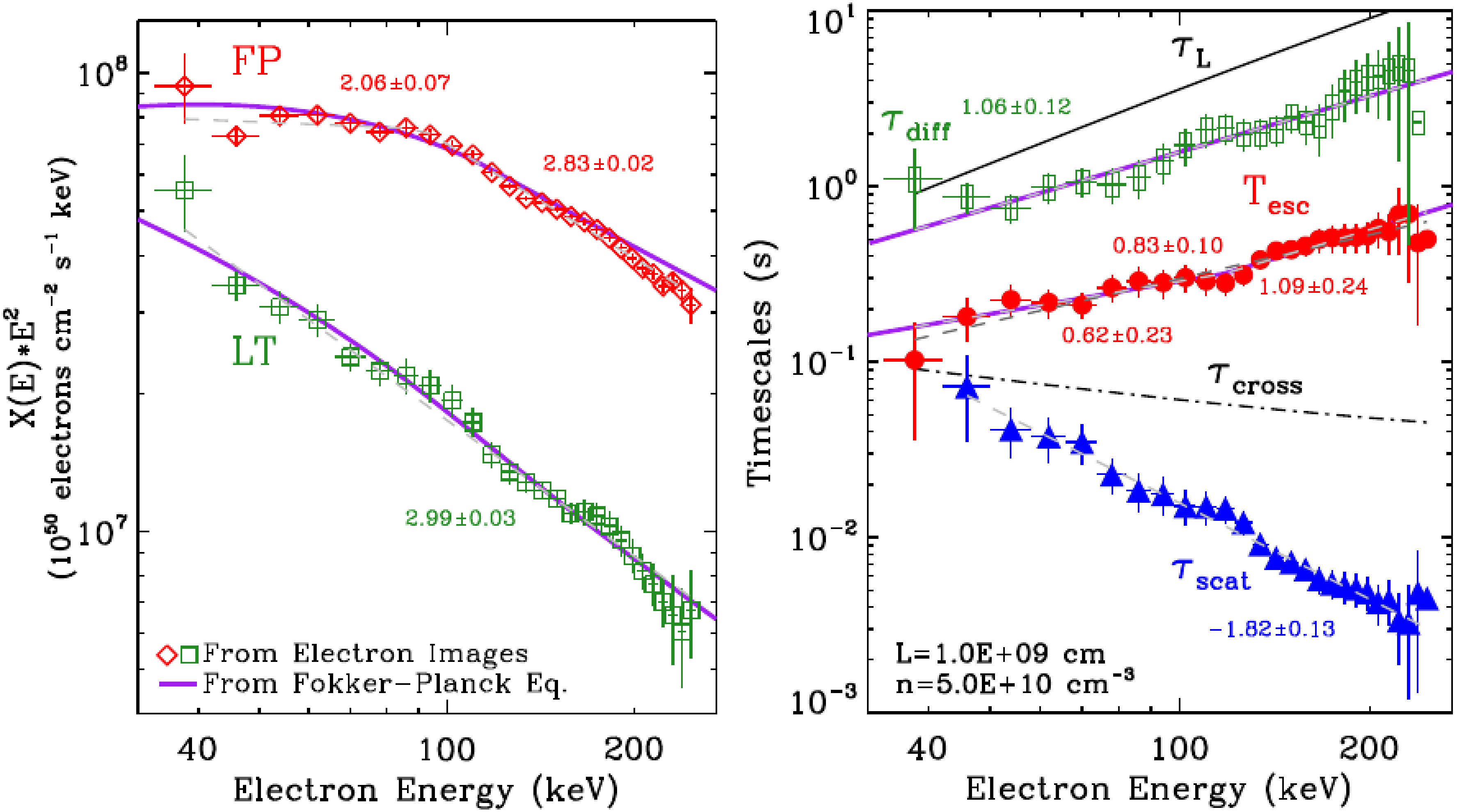
HXR images by MEM_NJIT



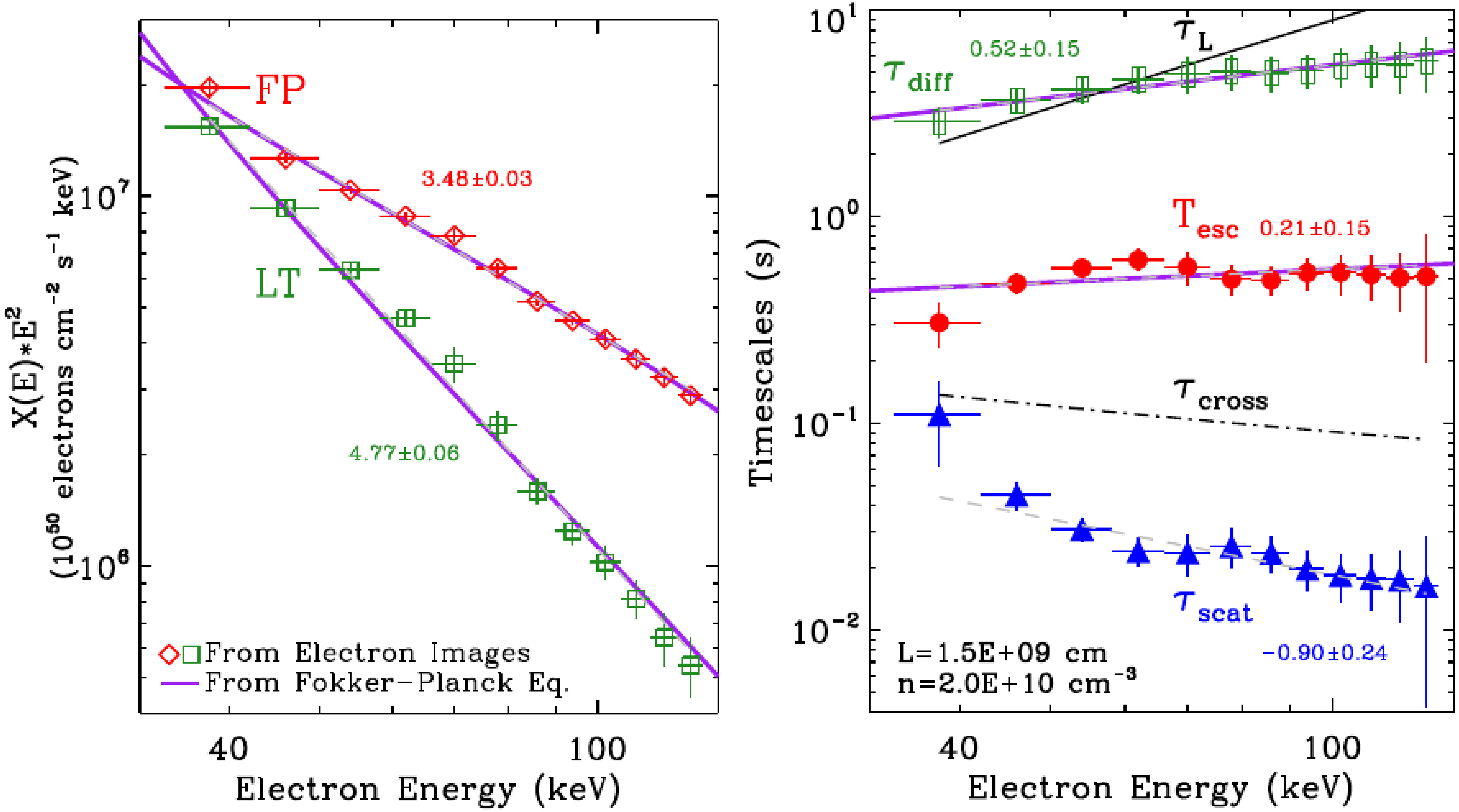
Electron flux images by MEM_NJIT



2003 November 3 Flare X3.9



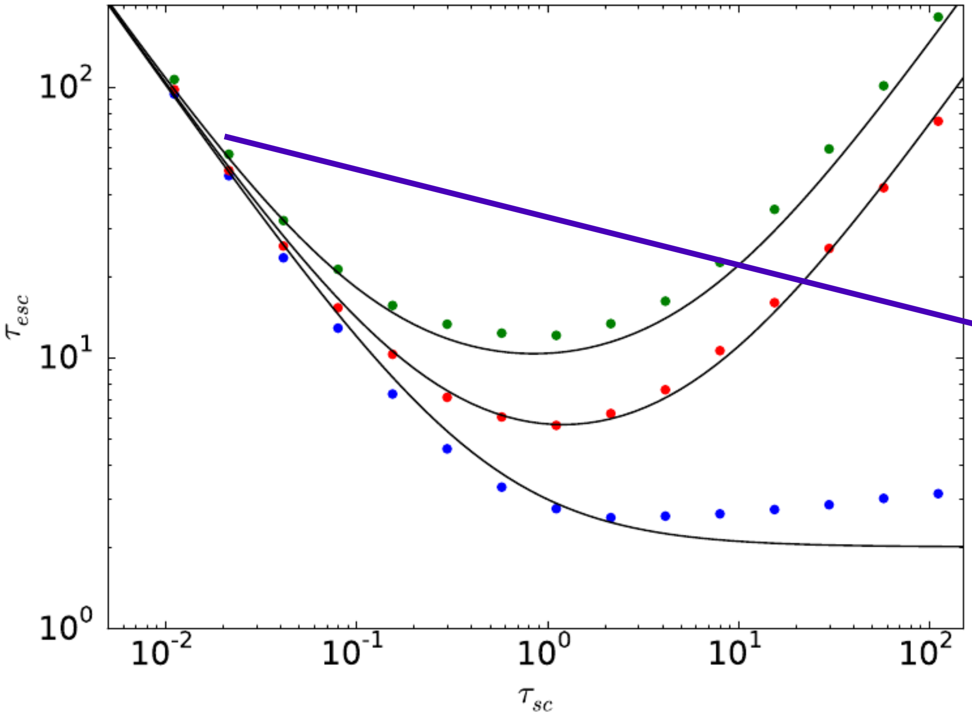
2005 September 8 Flare M2.1



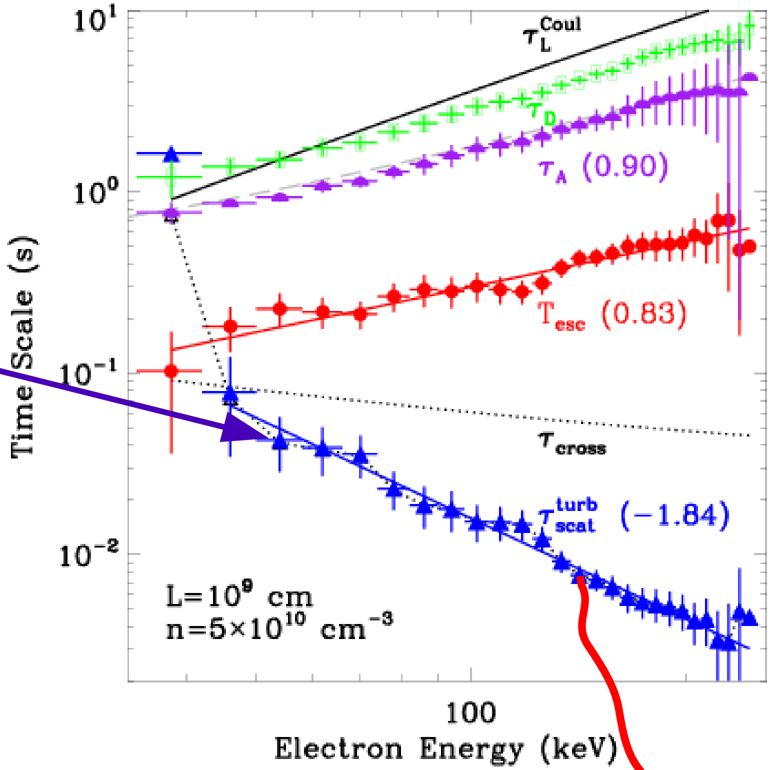
Escape and Scattering Times

Theory and Empirical Determinations

Effenberger and VP 2017



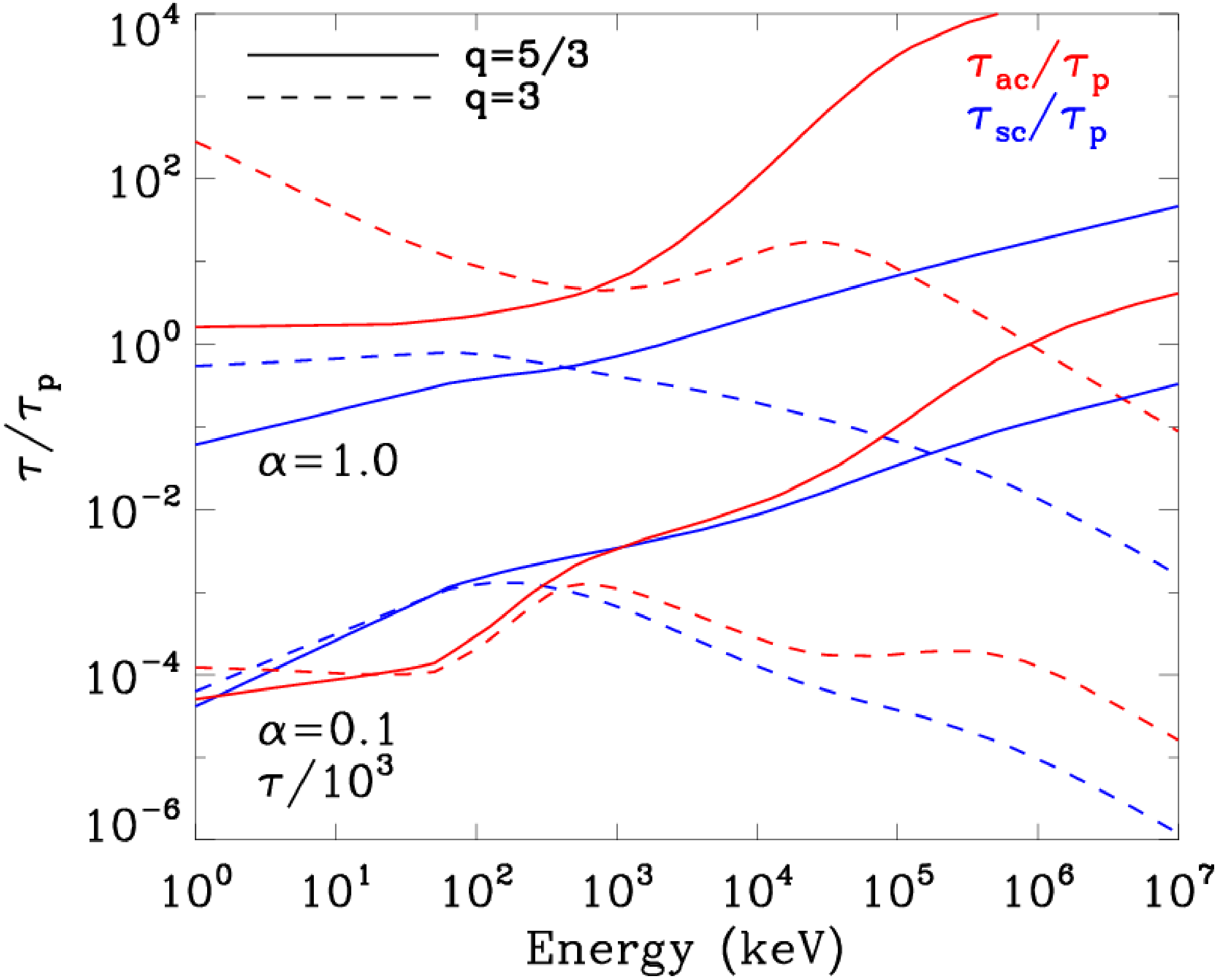
Chen & VP 2013



$$T_{\rm esc} = \tau_{\rm cross}\begin{cases} 1 & \text{if } \tau_{\rm sc} \gg \tau_{\rm cross}, \text{ Free stream} \\ \propto \tau_{\rm sc}/\tau_{\rm cross} & \text{if } \tau_{\rm sc} \gg \tau_{\rm cross}, \text{ Converging field} \\ \tau_{\rm cross}/\tau_{\rm sc} & \text{if } \tau_{\rm sc} \ll \tau_{\rm cross}, \text{ Strong diffusion} \end{cases}$$

Scattering And Acceleration Times

Interaction with Parallel Waves



Pryadko and Petrosian 1997

So How Do We Fix It

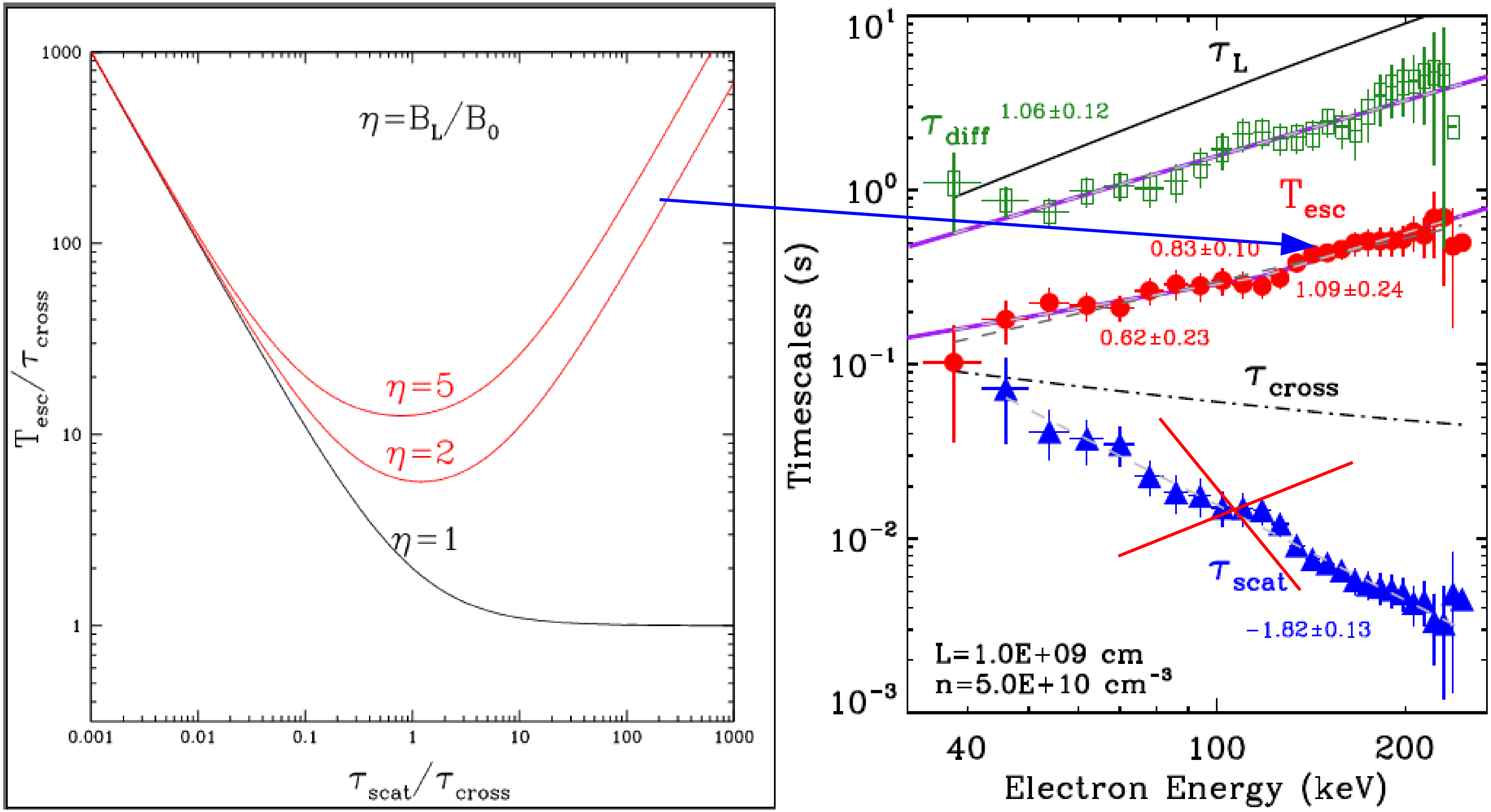
Possible Solutions

Weak Diffusion and

Converging Field Lines at the Loop top

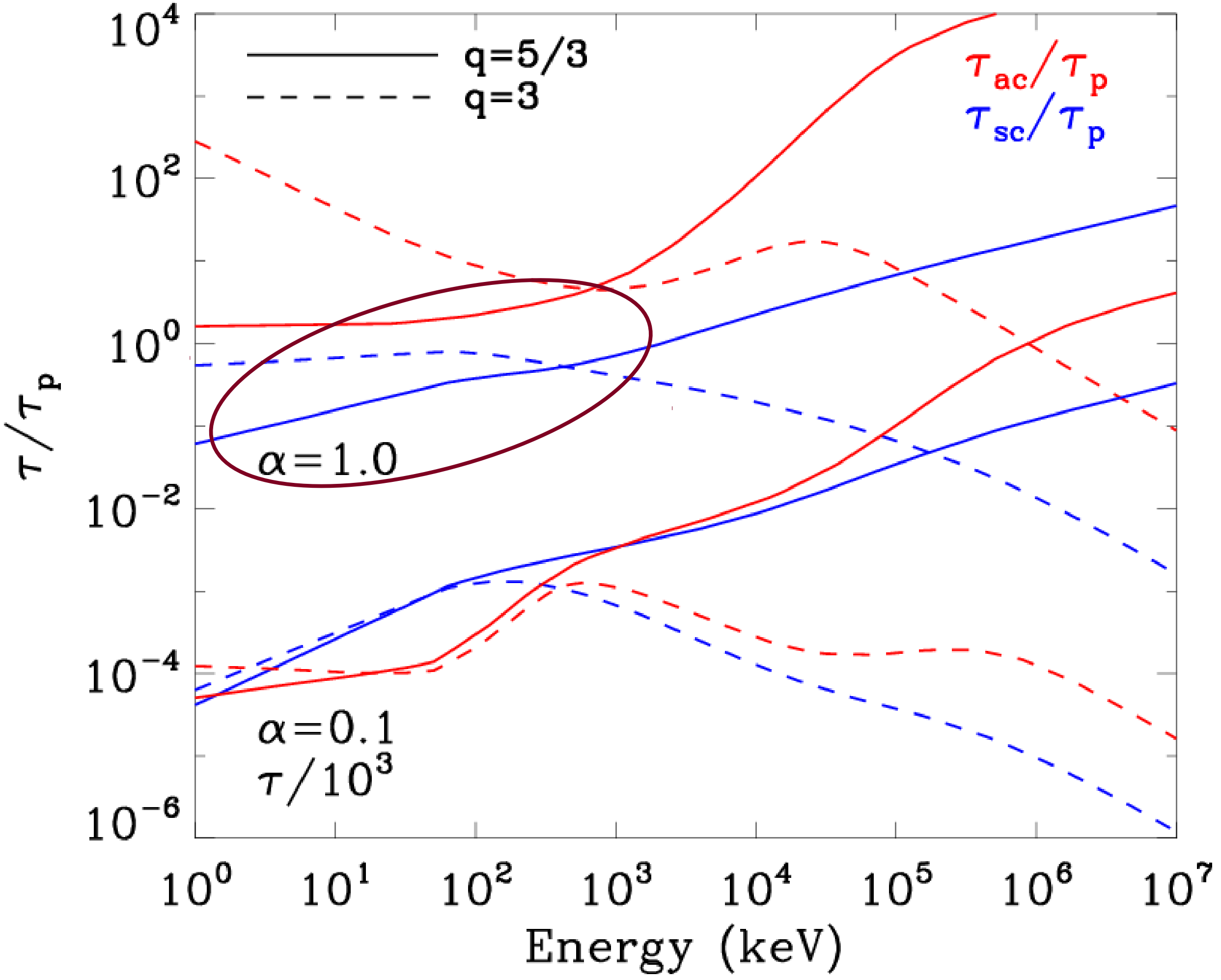
Escape time (determined by scatterings into the loss cone) *will increase with energy*

Trapped by a Magnetic Bottle



Scattering And Acceleration Times

Interaction with Parallel Waves



Pryadko and Petrosian 1997

SUMMARY and CONCLUSIONS

1. For sources where observations give the *Accelerated and Escaping Particle Spectra* we can determine the basic acceleration parameters given the plasma characteristics using the newly developed *inversion technique.*

2. The results from application to supernova remnants and cosmic ray observations are very promising.

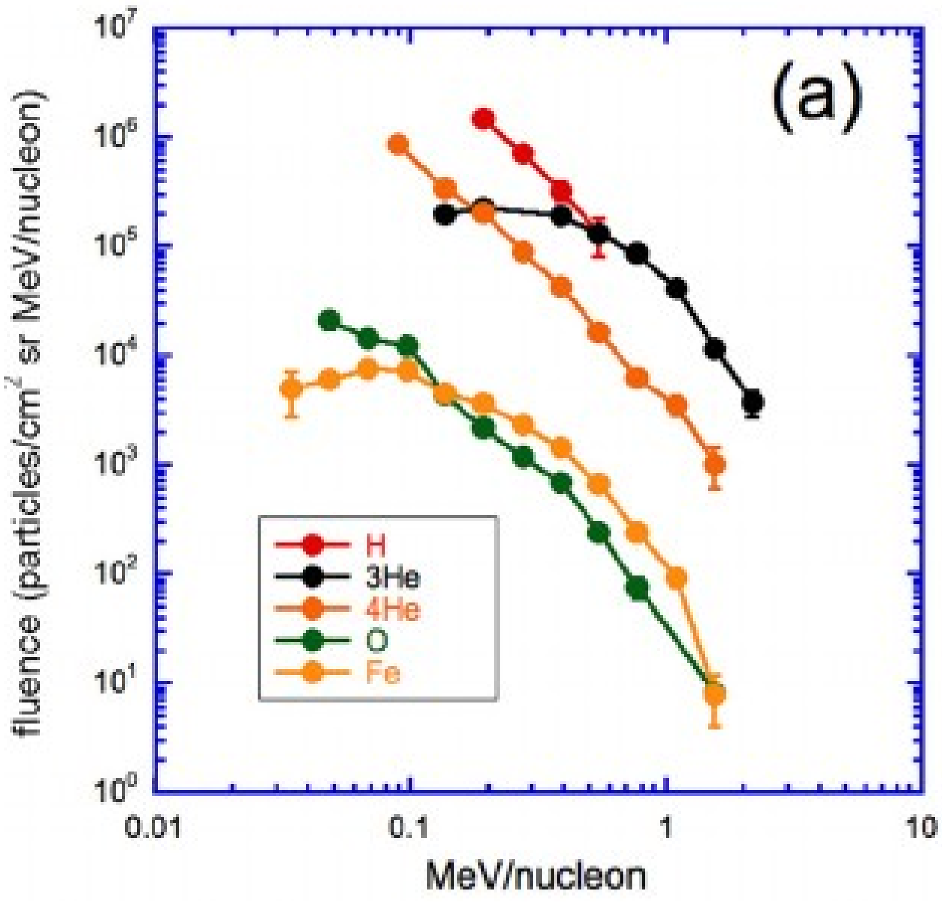
3. Application to *RHESSI* Imaging spectroscopic data show some new results.

III. Some Interesting Puzzle on Acceleration of Ions

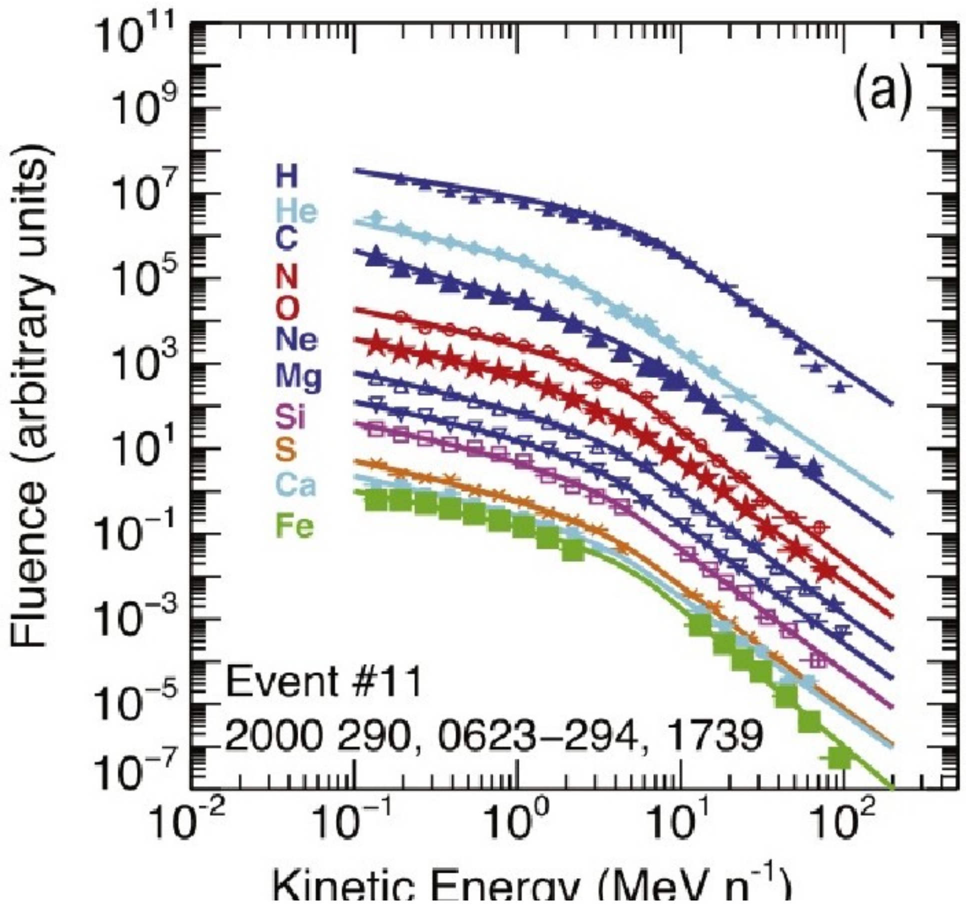
Extreme enhancement of 3He and heavy ions

•2. SEP-Ion Spectra and 3He Enrichment

“Impulsive” Events Mason et al. 2016

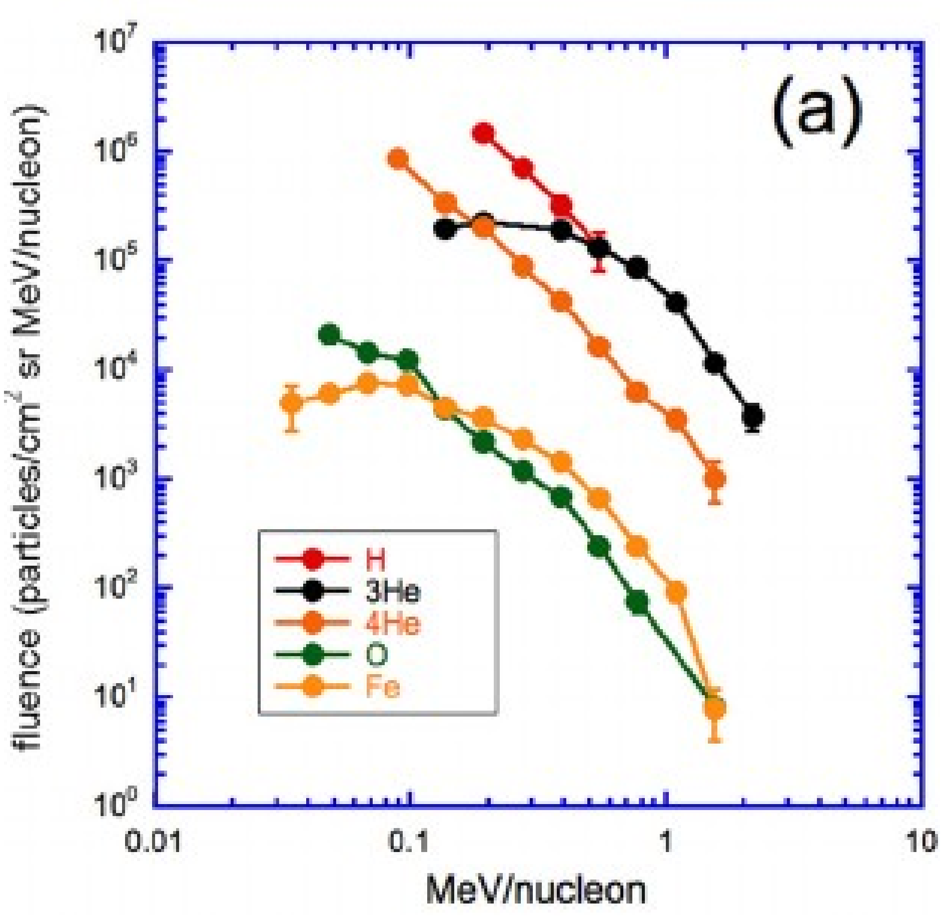


“Gradual” Events Desi t al. 2015

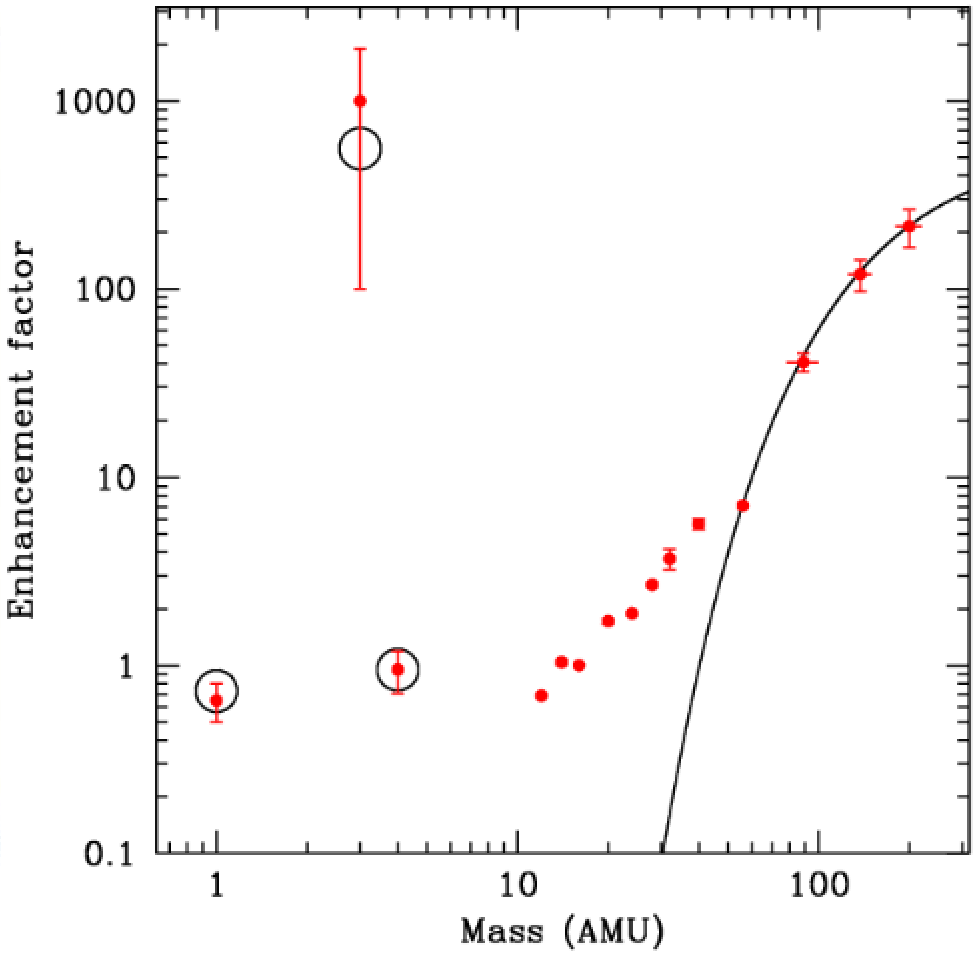


•2. SEP *ION* Enrichments

"Impulsive" Events Mason et al. 2016

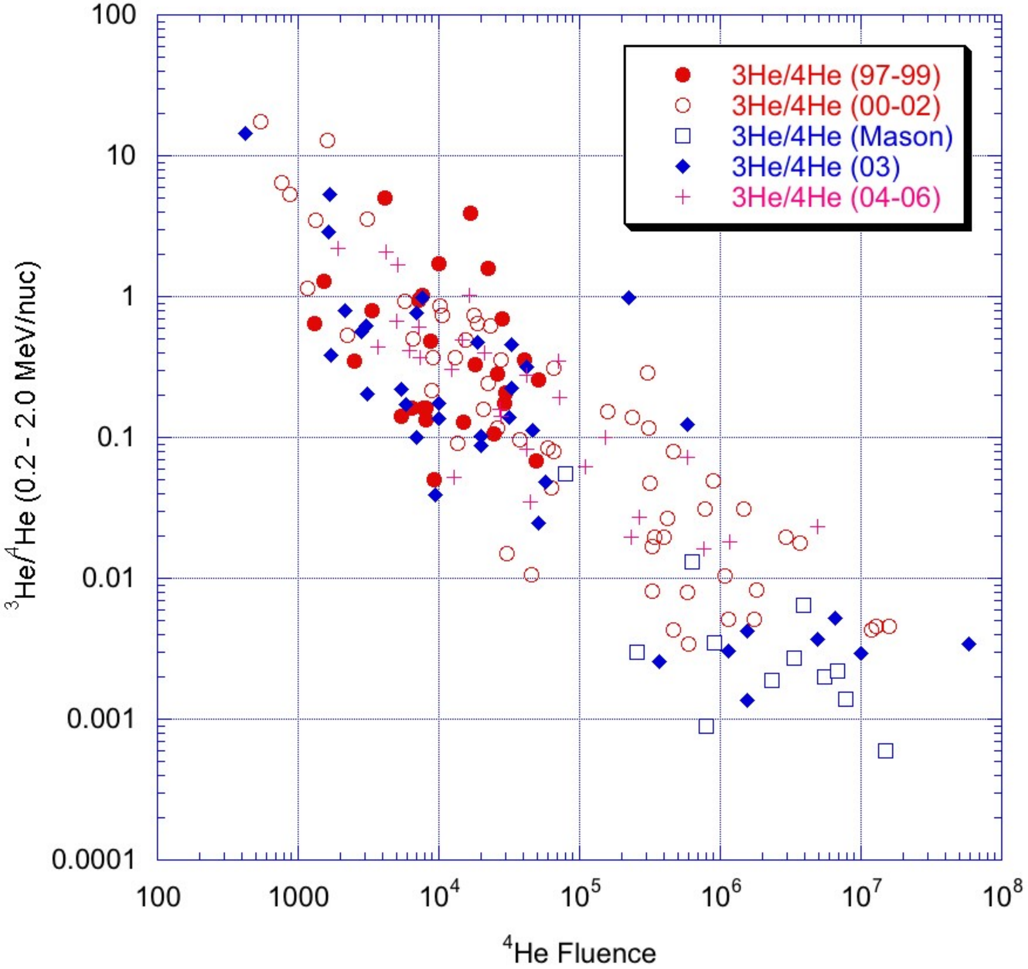


Reames et al.



He3, He4 Fluence Ratios

Not bimodal: *gradual variation with acceleration rate*

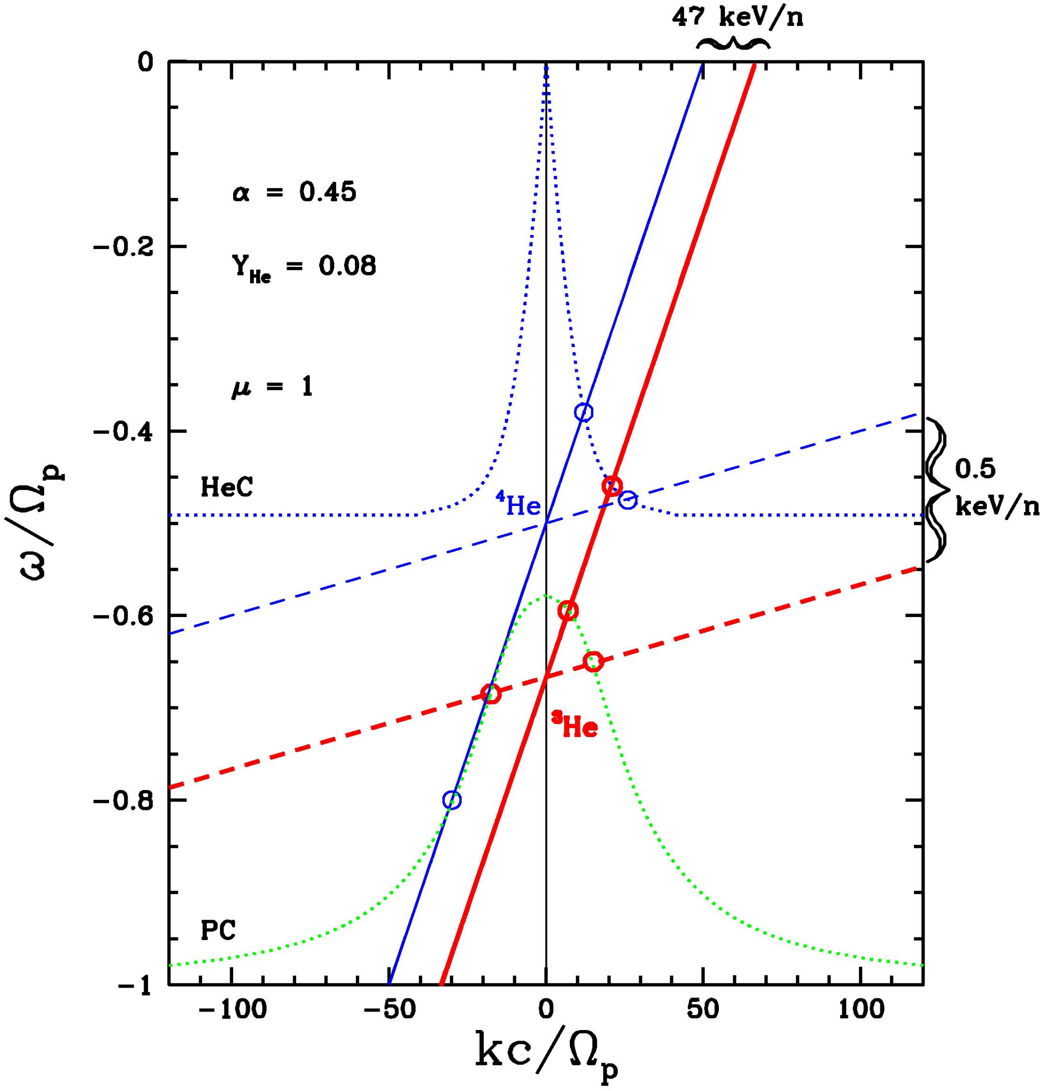


Observations

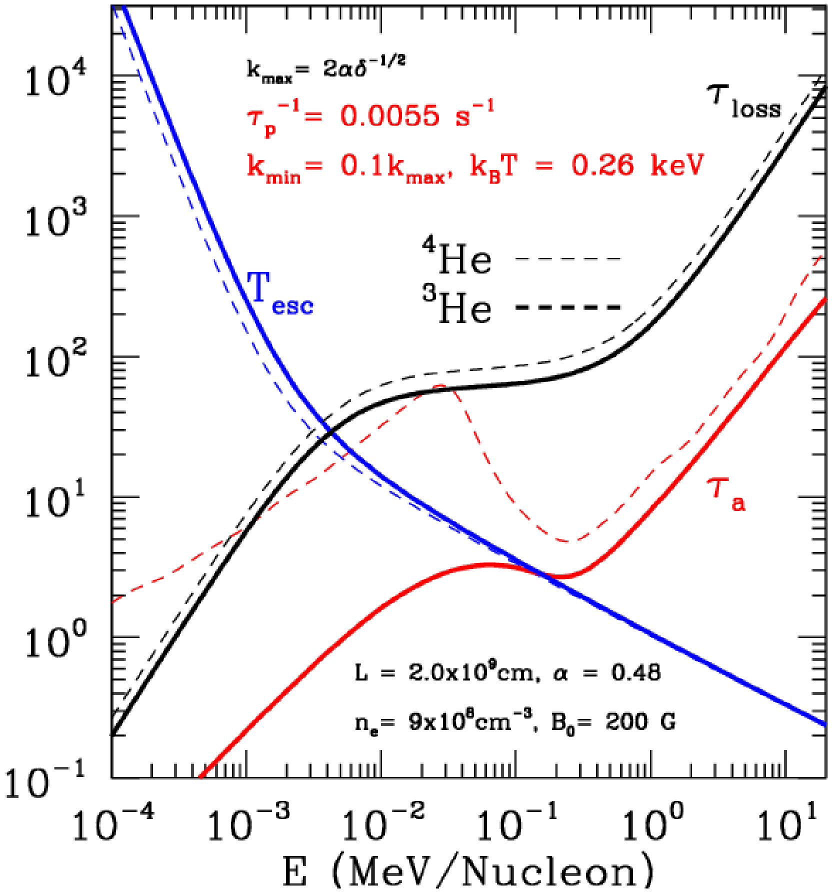
Zakopane-2, 2019

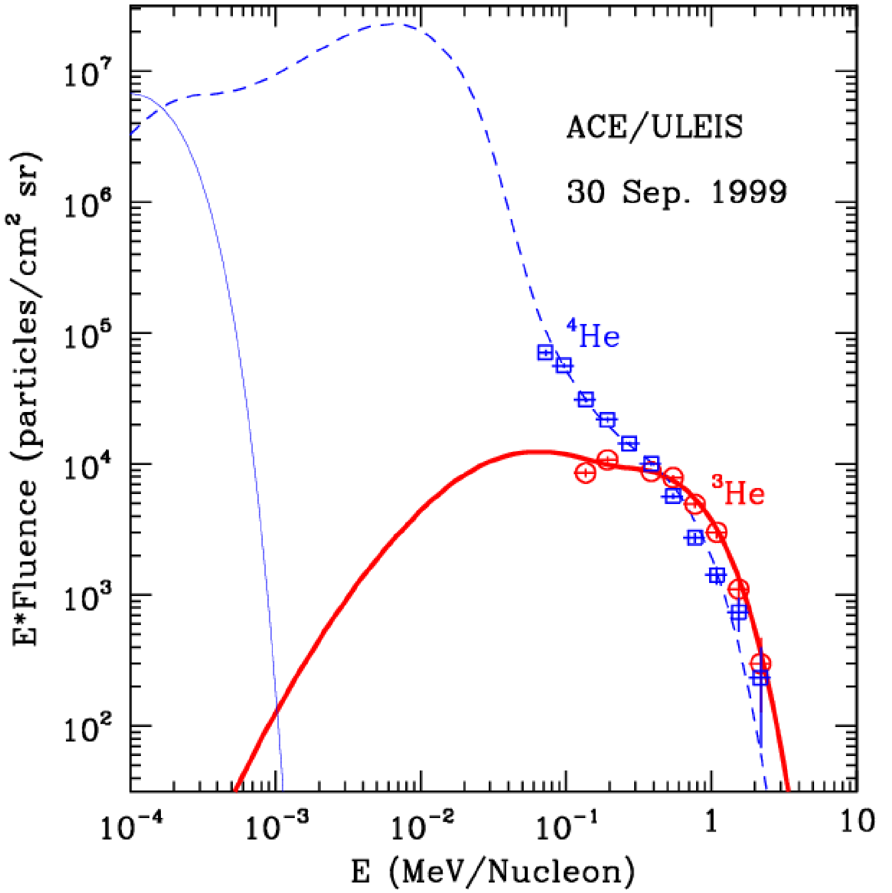
Resonant Wave-Particle Interactions
4He and 3He

$$\omega = \mu v k + \frac{\Omega_i}{\gamma}.$$



B. He3, He4 Abundances and Spectra

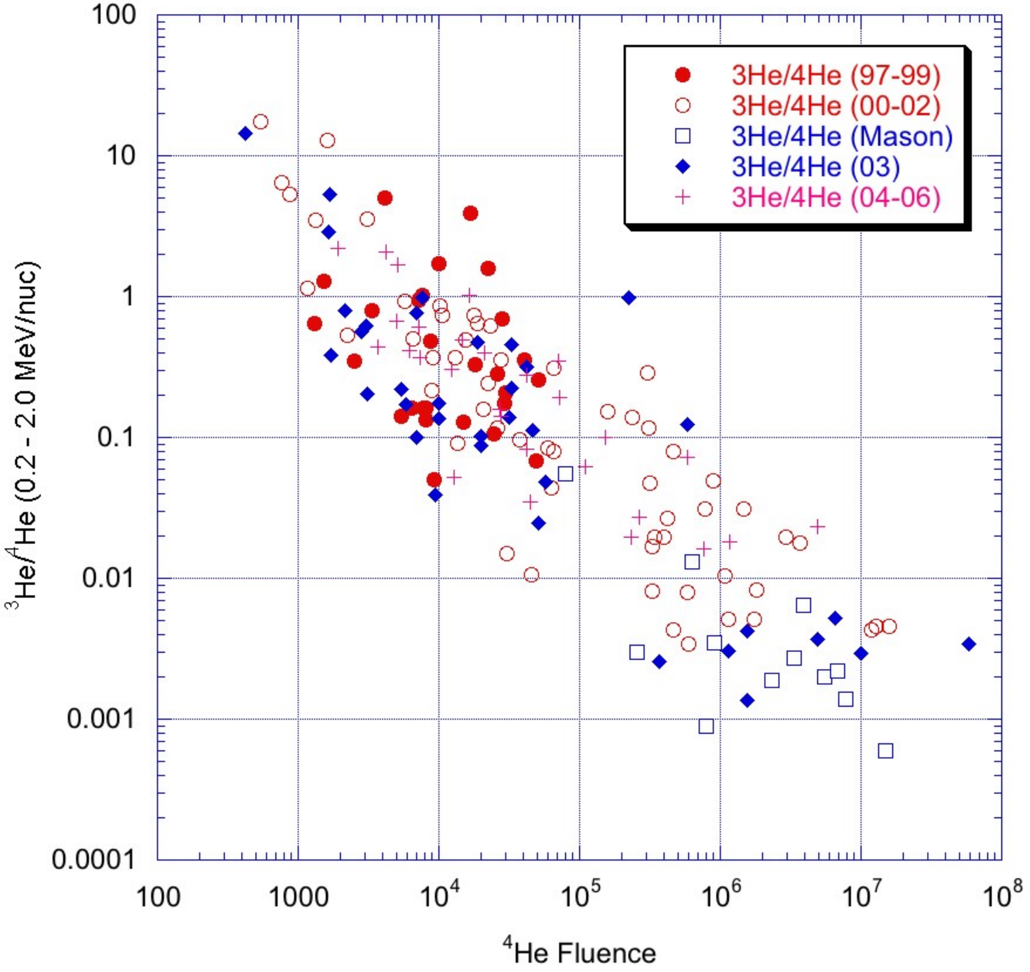




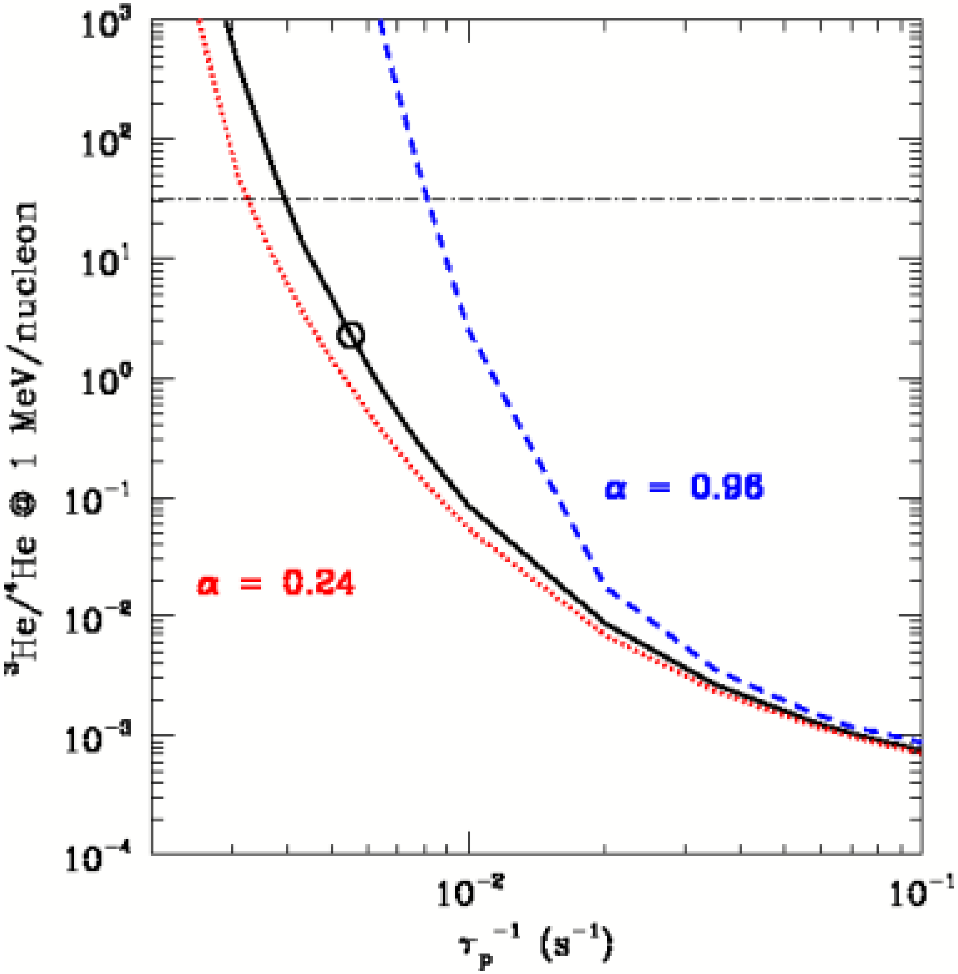
(Liu, Petrosian and Mason, 2004 ApJ)

He3, He4 Fluence Ratios

Not bimodal: *gradual variation with acceleration rate*



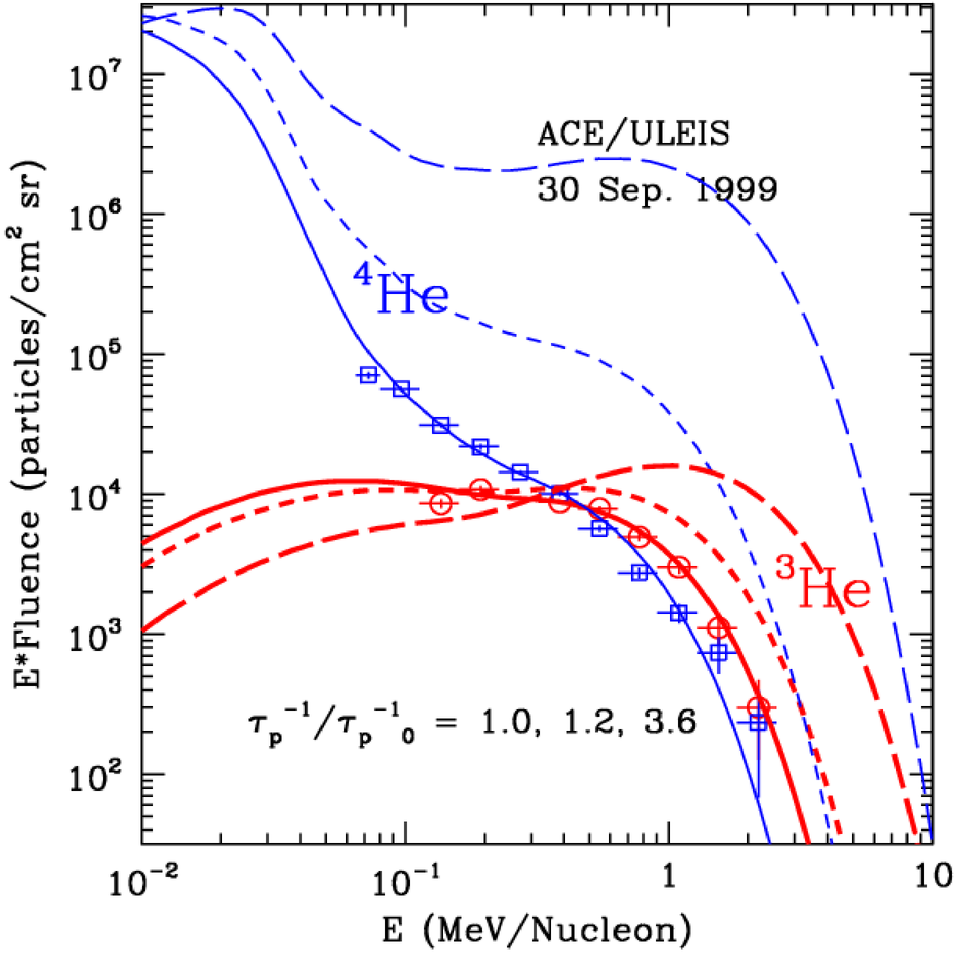
Observations

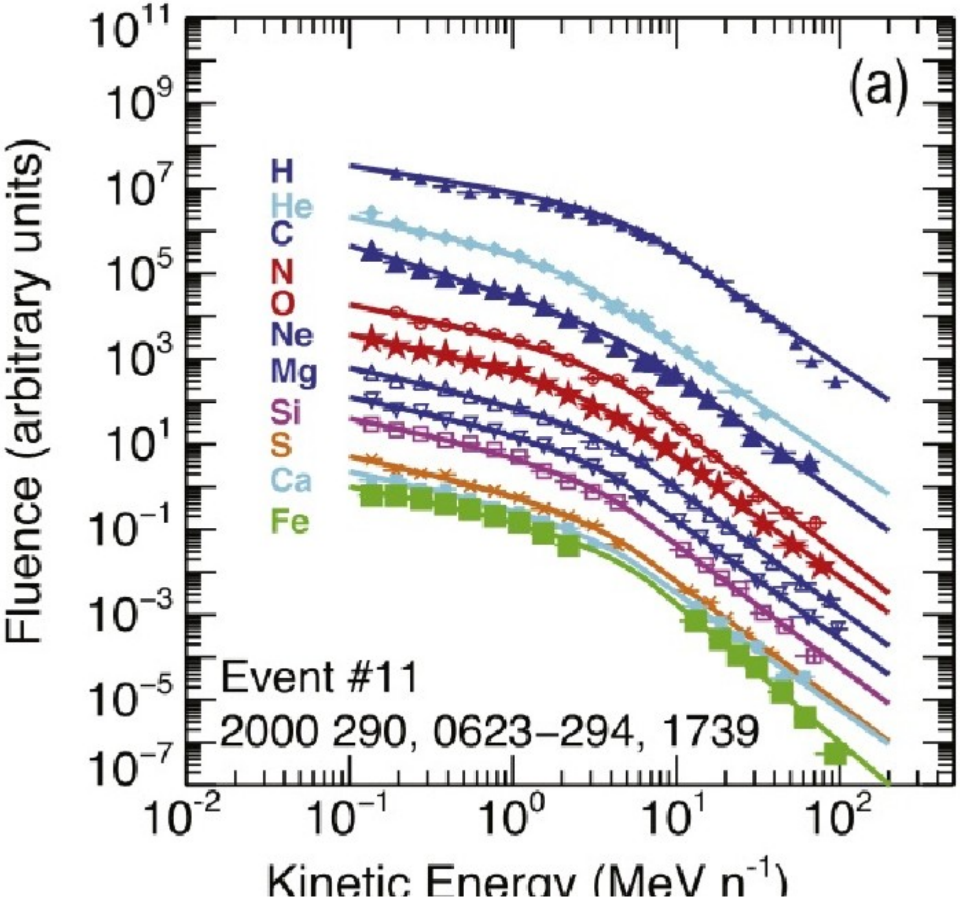


Model Results

Flare accelerated He4 Spectra

Do not agree with observed gradual events





Gradual or Delayed Events

Re-acceleration at the CME shock

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \cancel{\dot{E}_L})N] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

Diffusion

Accel. Loss

Escape

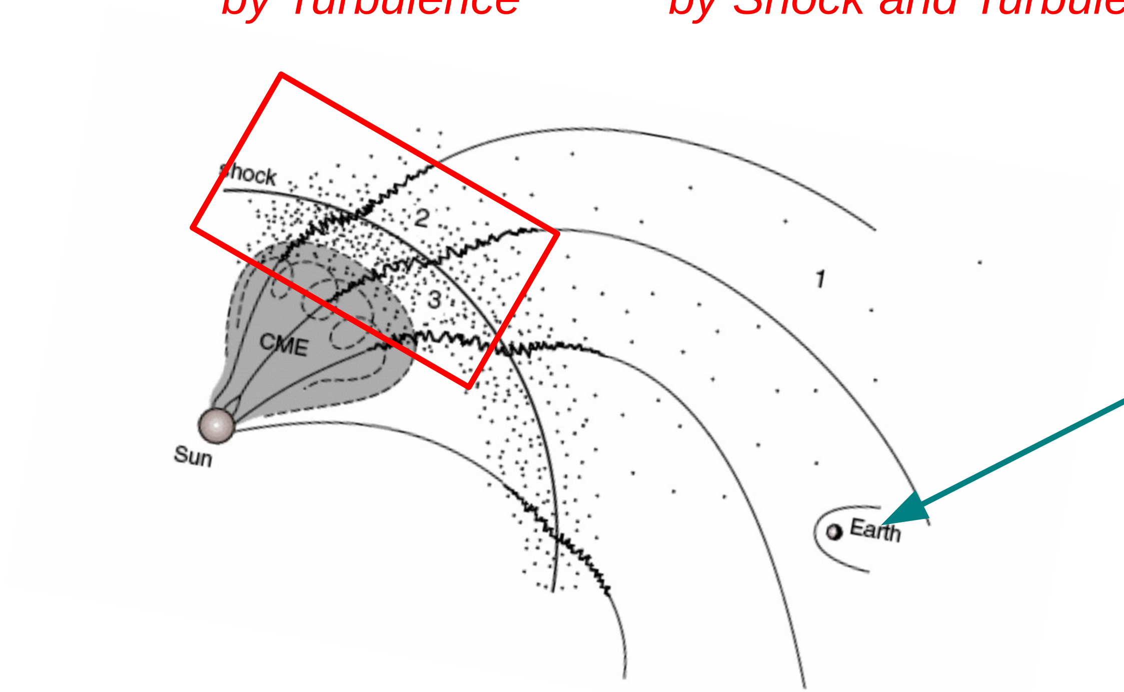
Source

by Turbulence

by Shock and Turbulence

Flare Accelerated Particles

Observed SEPs



Gradual or Delayed Events

Re-acceleration at the CME shock

$$\partial N/\partial t = -\partial(A_{\rm sh}N)/\partial E - N/T_{\rm esc} + \dot{Q} = 0$$

Solution with Source term flare accelerated electrons

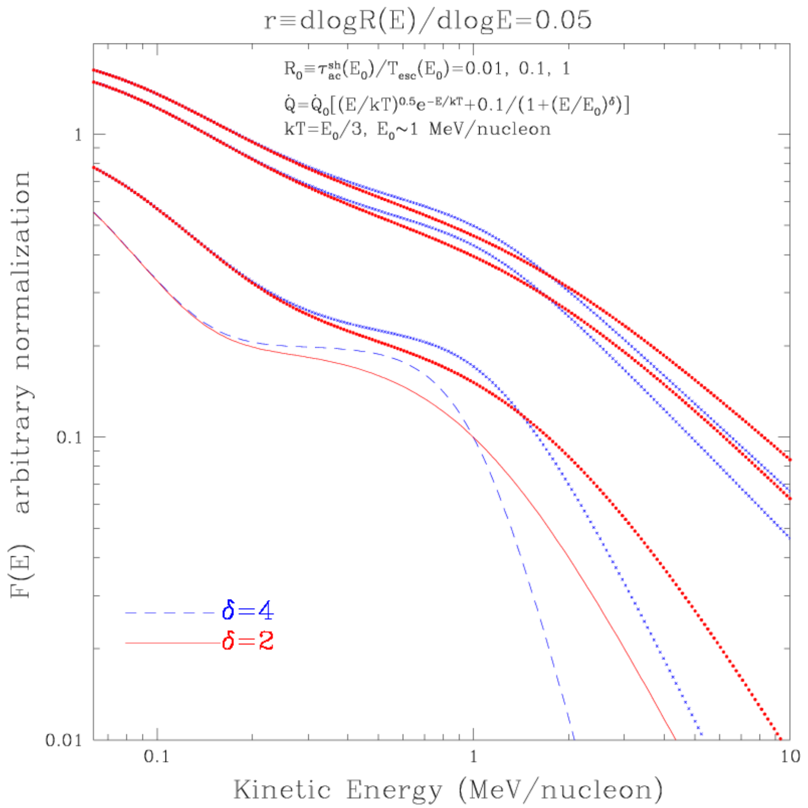
$$F(E) = N(E)/T_{\rm esc} = (\tau_{\rm ac}^{\rm sh}/T_{\rm esc}) \int_0^E \dot{Q} dE'/E$$

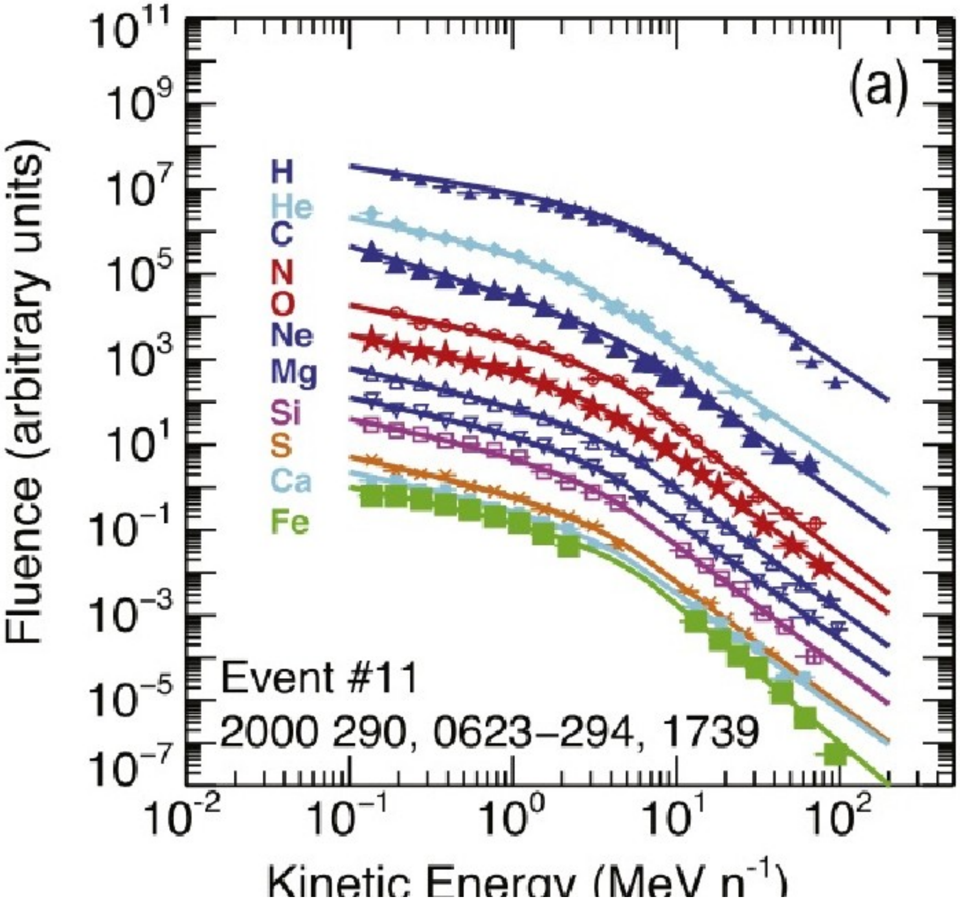
$$R(E) \equiv \tau_{\rm ac}^{\rm sh}/T_{\rm esc} = R_0 E^r$$

BUT Flare accelerated He4 Spectra

after re-acceleration at the CME-shock agree

$$F(E) = \frac{R(E)}{E} e^{-\eta} \int_0^E e^{\eta'} \dot{Q}(E')dE'; \quad \frac{d\eta}{dE} = \frac{R(E)}{E}$$

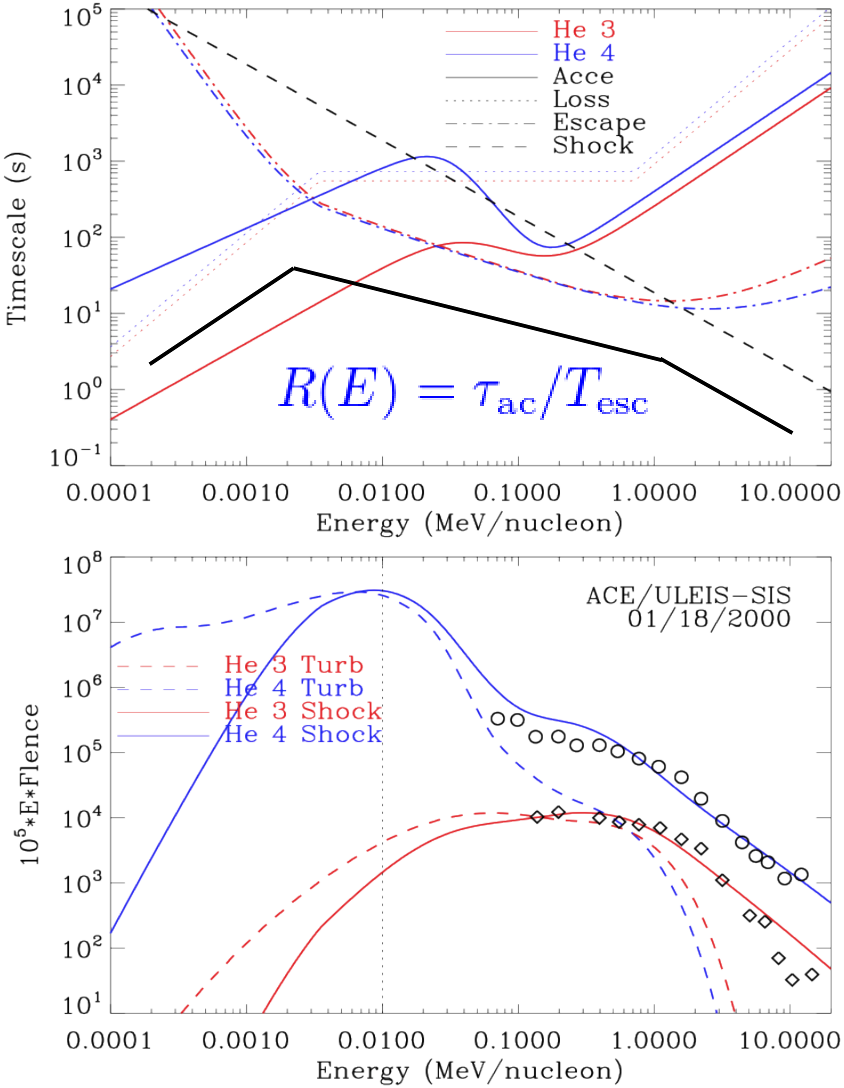




Numerical treatment of re-Acceleration

Re-acceleration timescales

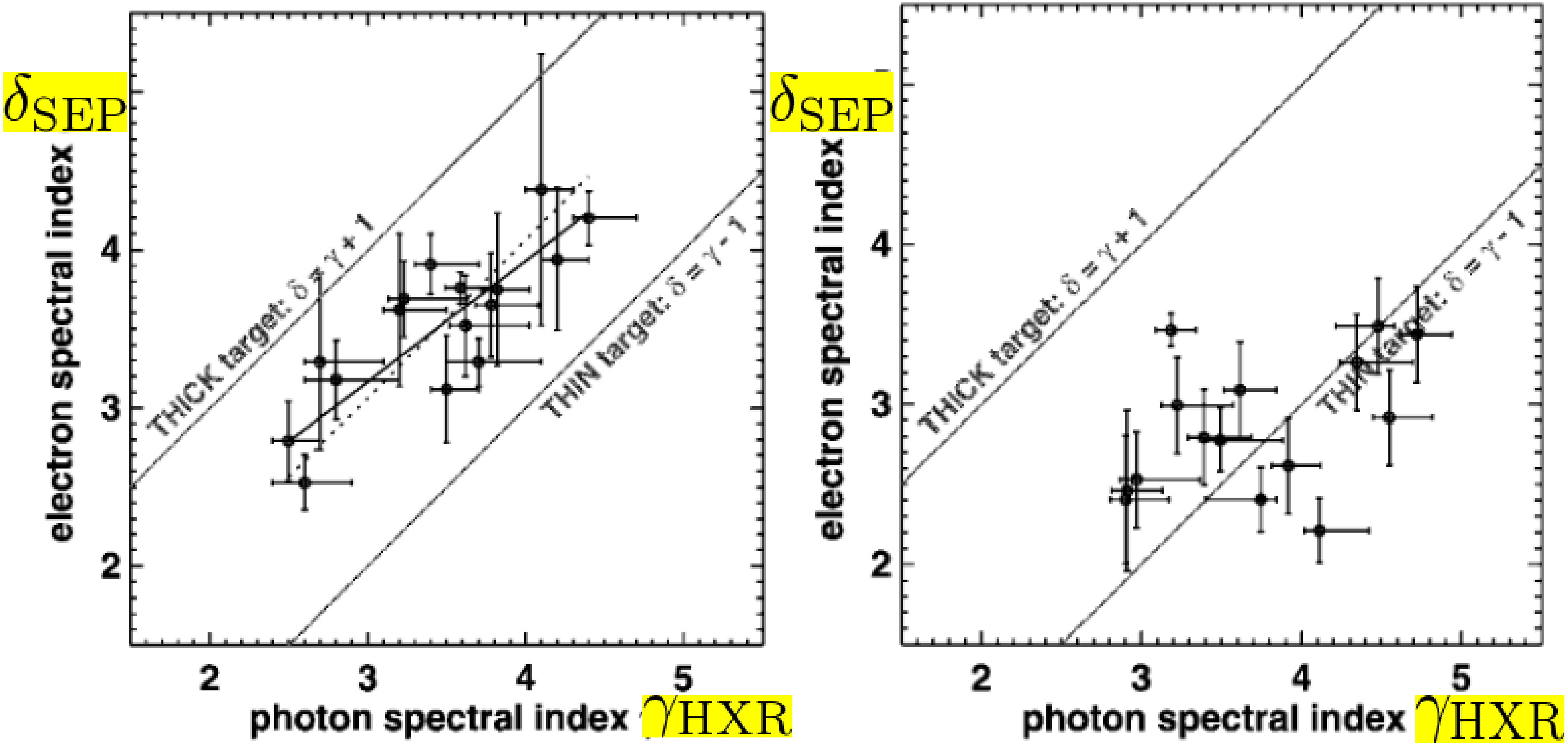
$$\tau_{\mathrm{ac}}^{\mathrm{sh}};\ \tau_{\mathrm{diff}};\ T_{\mathrm{esc}};\ \cancel{\tau_{\mathrm{loss}}}$$



A. Electrons

1. SEP and HXR *Electron* Spectra

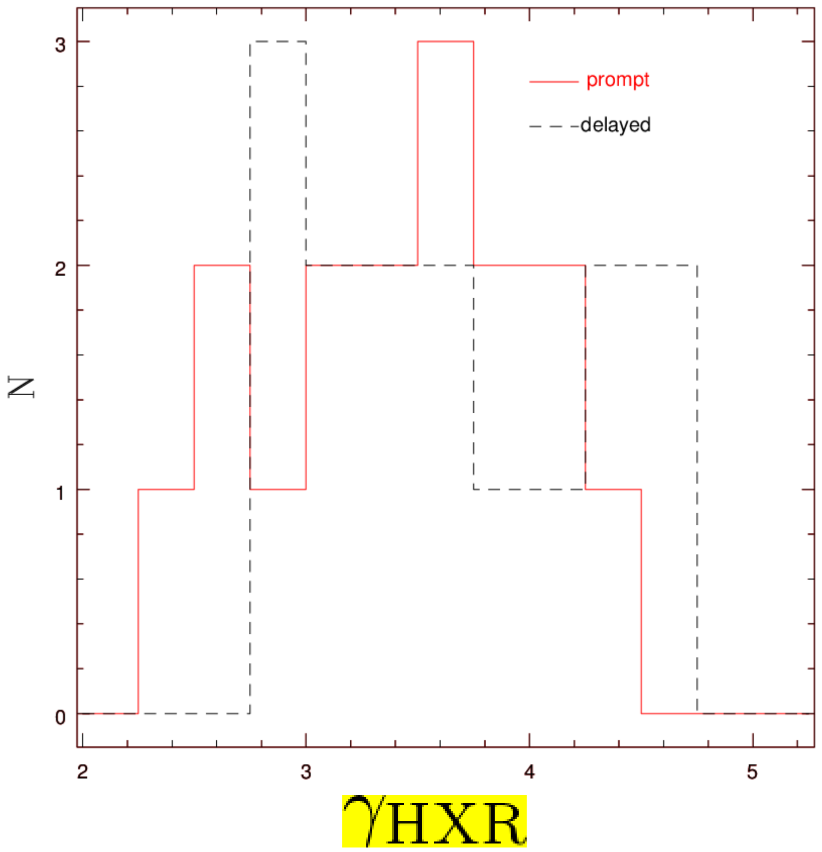
“Impulsive; Prompt” *“Gradual; Delayed” Events*

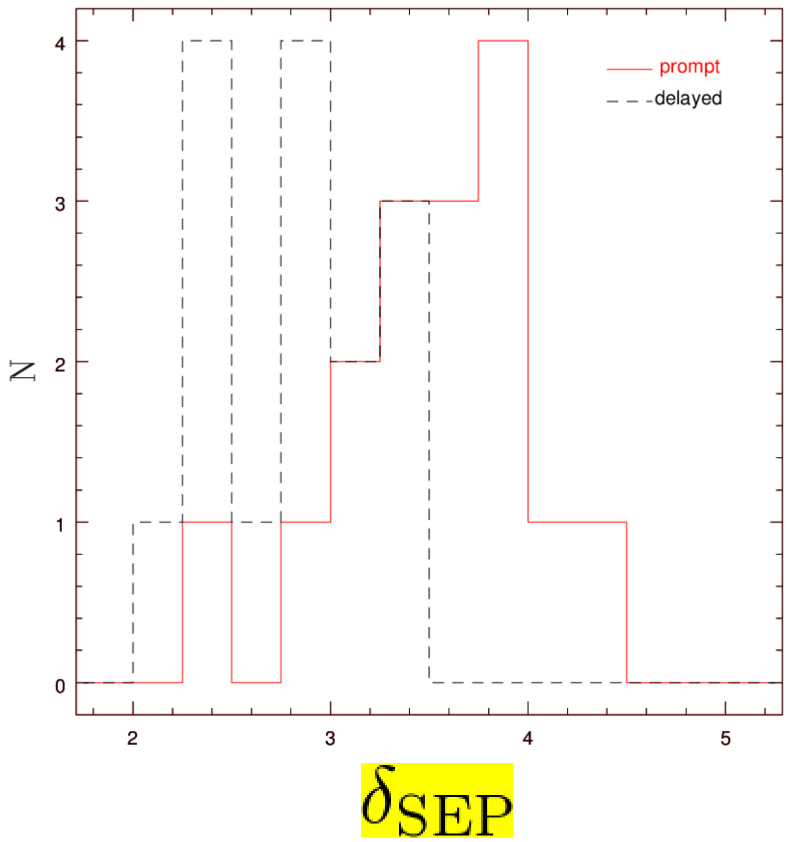


Krucker et al. 2007

• 1. SEP and HXR *Electron* Spectra

Distributions of *“Impulsive; Prompt”* and *“Gradual; Delayed”*

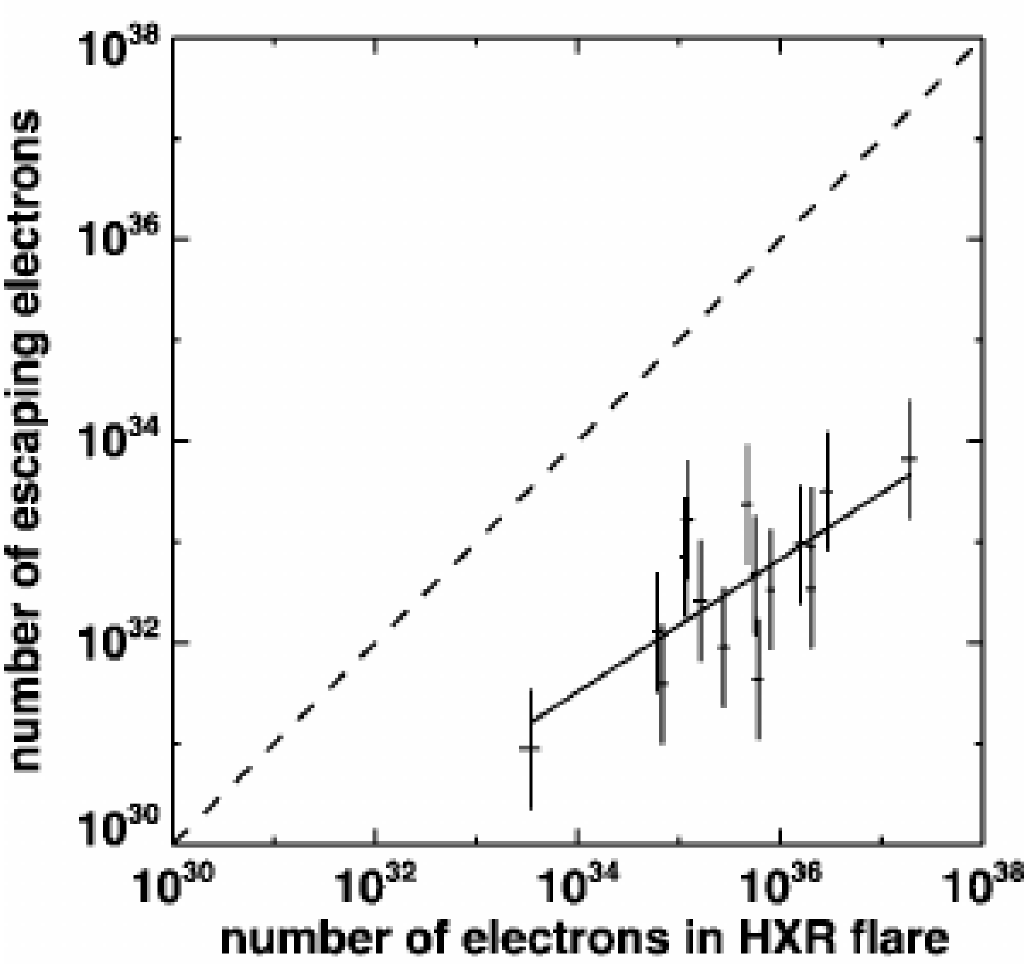




VP 2016

• 1. SEP and HXR *Electron* Numbers

Correlations of *“Impulsive; Prompt”* Events only



Krucker et al. 2007

• Impulsive or Prompt Events

• *Acceleration by Turbulence at the Flare Site*

$$N(E) \propto E^{-\delta}$$

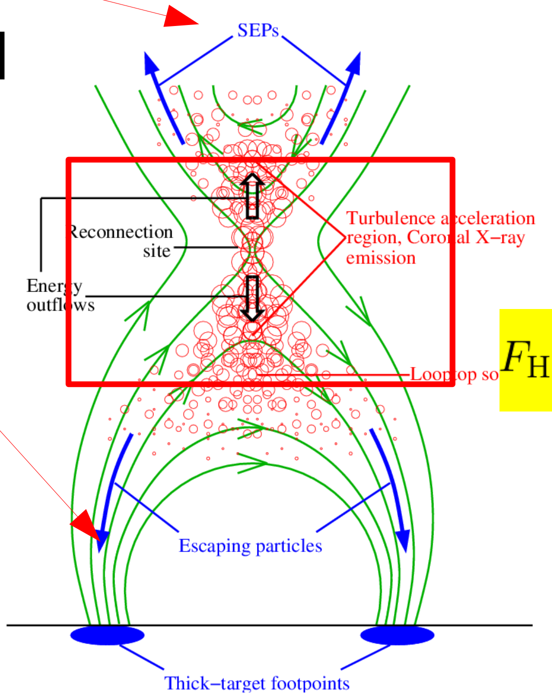
- Escape up: SEPs

$$T^u_{\rm esc} \sim L/v \propto E^{\alpha_u}, \ \alpha_u = -d\ln v/d\ln E$$

$$F_{\rm SEP}(E) \propto E^{-\delta-\alpha_u}$$

$$\delta_{\rm SEP} = \delta + \alpha_u$$

- Escape down: HXI



$$T^d_{\rm esc} \propto E^{\alpha_d}$$

$$F_d(E) \propto E^{-\delta-\alpha_d}$$

$$F_{\rm HXR}(E) = vN_{\rm eff}(E) \propto \frac{v}{\dot{E}_L}\int_E^\infty N(E)E^{-\alpha_d}dE$$

$$\delta_{F_{\rm HXR}} = \delta + \alpha_d - 1 - 2\alpha_u$$

$$\gamma_{\rm HXR} = \delta + \alpha_d - 1$$

$$\delta_{\rm SEP} = \gamma_{\rm HXR} + 1 + \alpha_u - \alpha_d$$

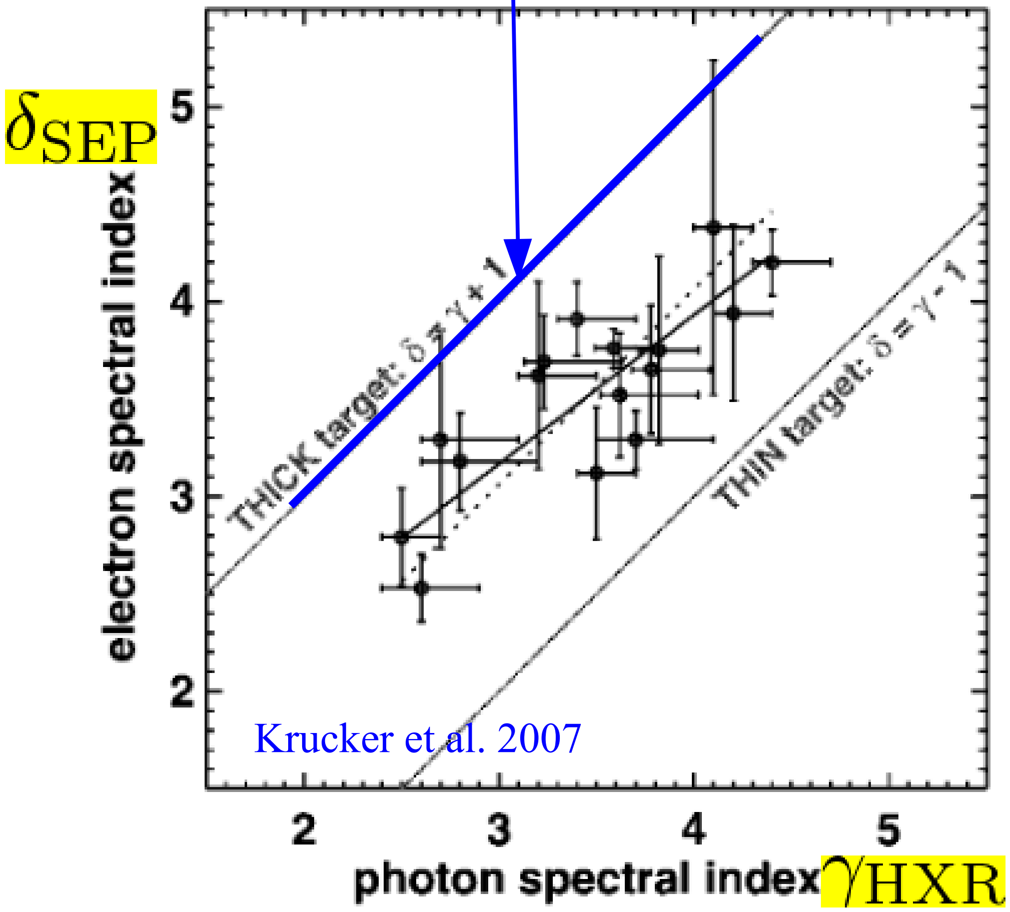
• Impulsive or Prompt Events

• *Acceleration by Turbulence **only** at the Flare Site*

$$\delta_{\mathrm{SEP}} = \gamma_{\mathrm{HXR}} + 1 + \alpha_u - \alpha_d$$

Strong diffusion

$\alpha_u = \alpha_d$



Krucker et al. 2007

• Impulsive or Prompt Events

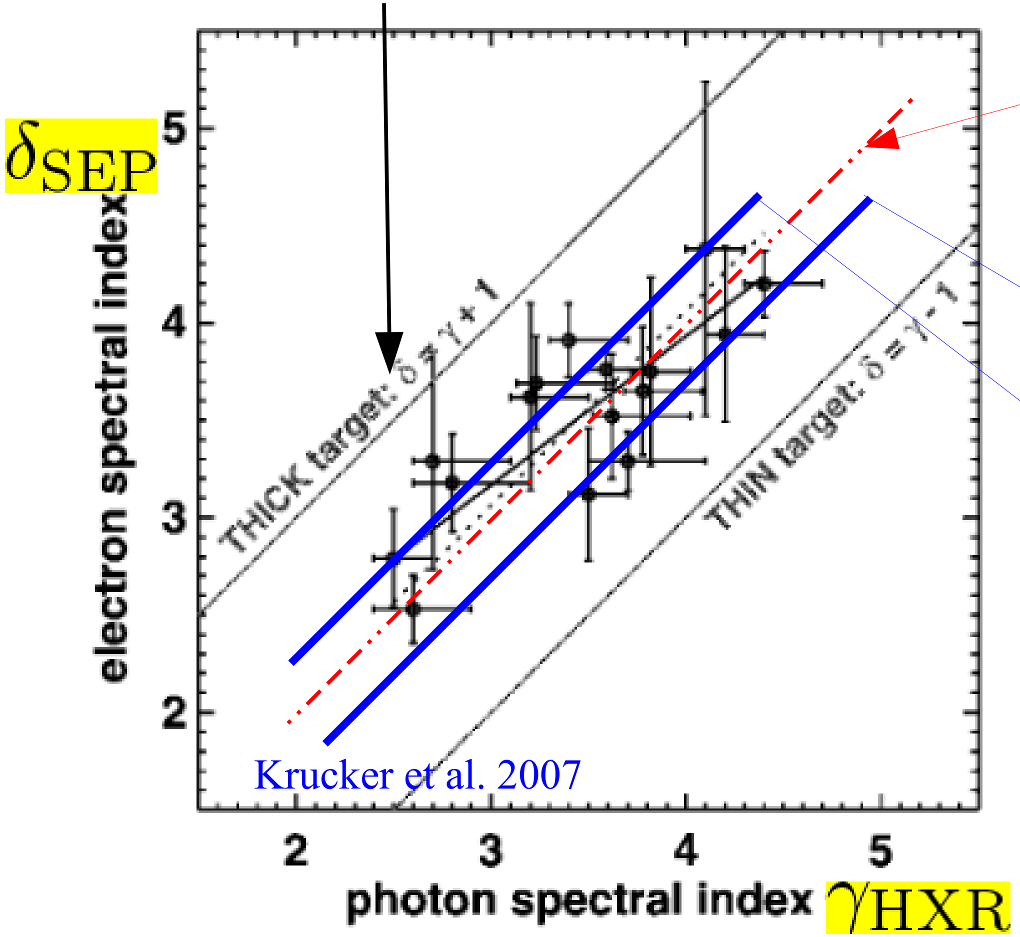
- *Acceleration by Turbulence* ***only*** *at the Flare Site*

$$\alpha_d = \gamma_{\rm HXR} - \delta_{\rm SEP} + \alpha_u + 1$$

Weak diffusion

$T_{\rm esc,u} \sim L/v \propto E^{-0.5}$ and $T_{\rm esc,d} \propto \tau_{sc}$

$\alpha_u \sim -0.4$, $\alpha_d \sim 0.6$; $\delta_{\rm SEP} \sim \gamma_{\rm HXR}$



Krucker et al. 2007

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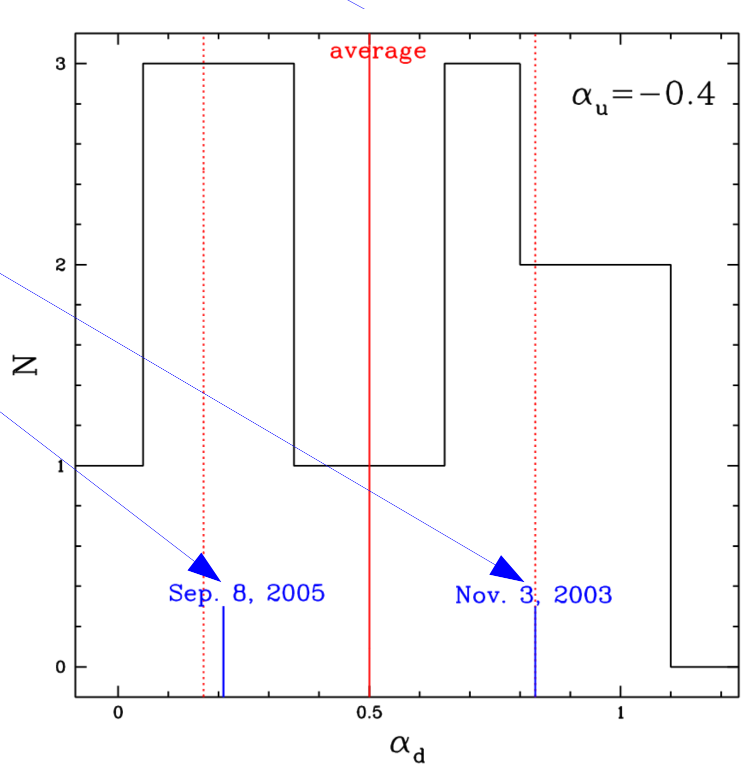


• Impulsive or Prompt Events

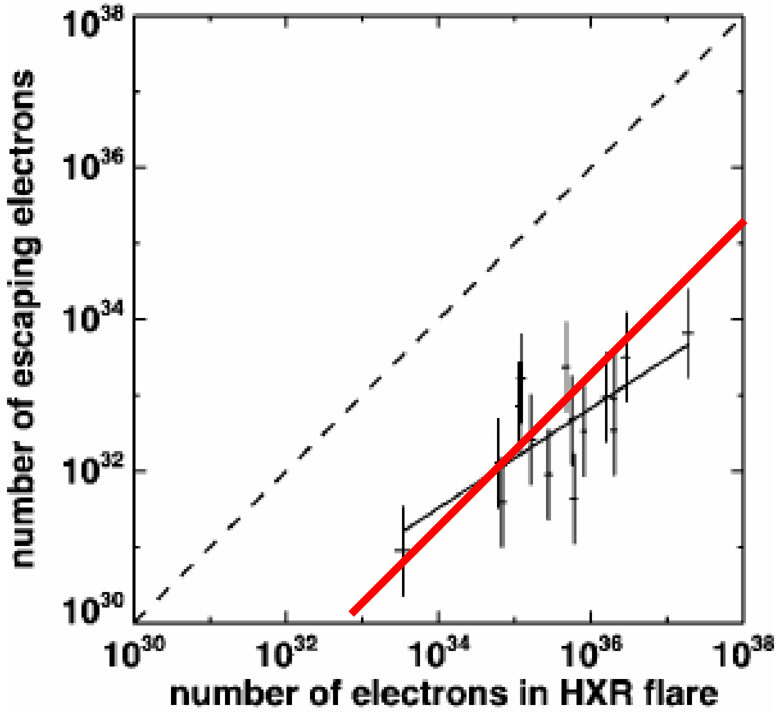
• *Acceleration by Turbulence **only** at the Flare Site*

Relative numbers of RPP and SEP *(electrons)*

$$N_{\rm SEP} = \left(\frac{4\pi}{\Omega}\right)\left(\frac{c\Delta T}{L_u}\right)\int_{E_0}^{\infty}\beta(E)N(E)dE$$

$$N_{\rm HXR} = \frac{\Delta Tc}{L_d}\frac{(E_0)}{T^d_{\rm esc}(E_0)}\frac{E_0^{\alpha_d-1}}{\beta(E_0)}\int_{E_0}^{\infty}\frac{N(E')}{E^{\prime,\alpha_d}}dE'\int_{E_0}^{E'}\beta^2 dE$$

$$R_N = \frac{N_{\rm HXR}}{N_{\rm SEP}} \sim \left(\frac{\Omega}{4\pi}\right)\left(\frac{\tau_{\rm coul}}{T^d_{\rm esc}}\right)g(\delta) \sim 100(10^{10}\,{\rm cm}^{-3}/n)$$



Krucker et al. 2007

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Solution with Source term flare accelerated electrons

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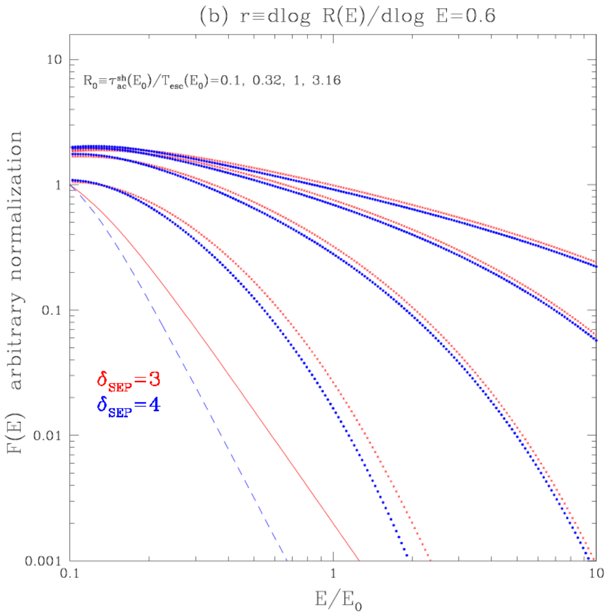
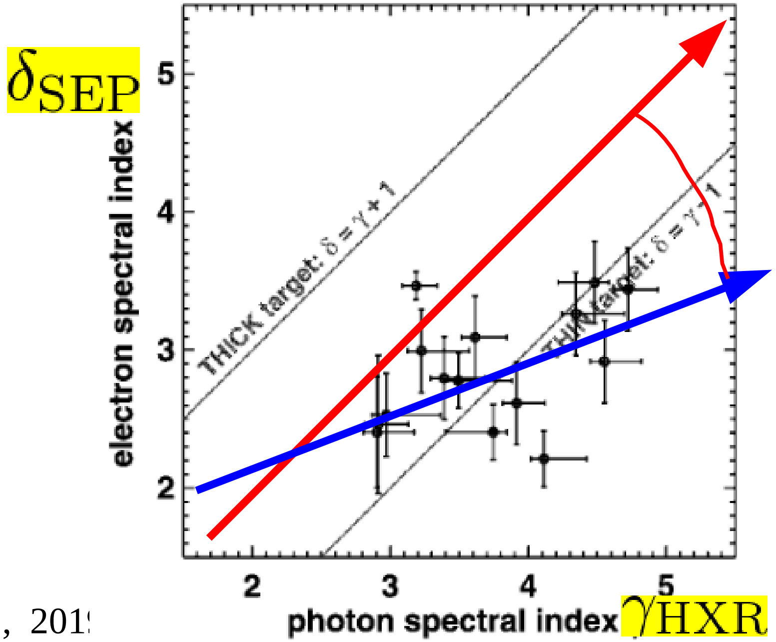
$$R(E) \equiv \tau_{\rm ac}^{\rm sh}/T_{\rm esc} = R_0E^r$$





SUMMARY and CONCLUSIONS 2

1. Inversion methods can determine the Fokker-Planck Coefficients directly and non parametrically from observed spectra.

2. Both Shock and Stochastic Acceleration Driven by Turbulence Can Account For Many High Energy Astrophysical Phenomena.