Gravitational Wave Memory Effect

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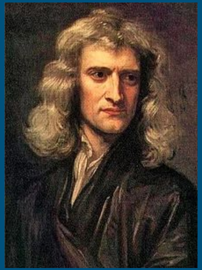
Outline:

- Historical account of Gravitational waves
- Quick survey of Linearised GWs
- Memory Effect : Linear & Non-Linear
- Exact plane wave spacetimes

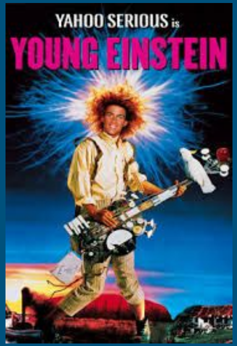


SPACE

TIME

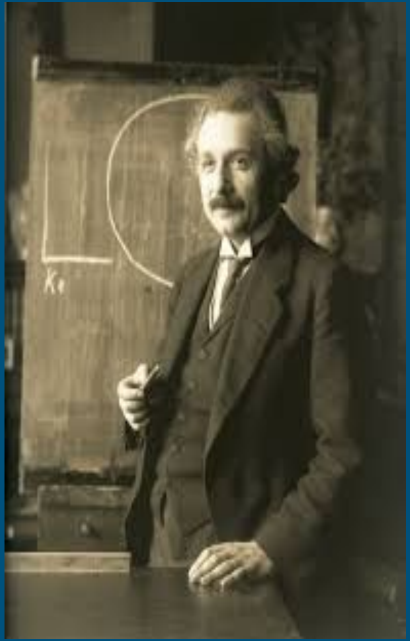
Velocity of light=CONSTANT

SPACETIME



Spacetime+Gravity : Geometry

From doubt...



Einstein

Rosen

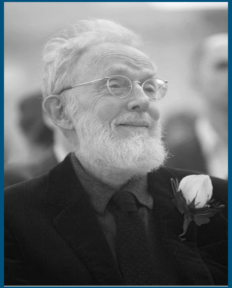
- 1905 : Henri Poincare :
- 1915-16: Einstein linearized gravity
- **Issues :**
  > Plane GWs in full theory?

  > Do full Einstein Equation have solution which can be interpreted a GW?

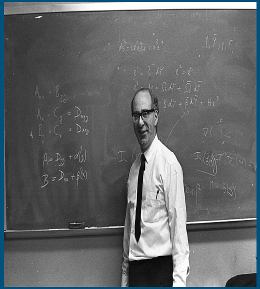
  > Do GWs carry energy?
  >......
- 1937 : Einstein- Rosen metric, first exact solution but with singularities

To belief...

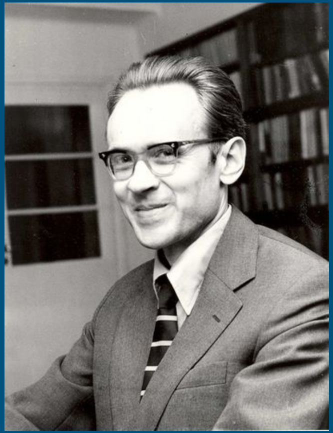
Bondi



Pirani



Robinson



Trautmann

- 1959, Bondi-Pirani-Robinson :
  > the plane wave in the full theory is defined ✓
  > they are solutions to Einstein's Equations. ✓
  > they carry energy in a form of a sandwich wave which affects test particles ✓

- 1958, Andrzej Trautman:
  > Radiation is nonlocal
  > defining GW in full Einstein theory = boundary conditions at infinity

Credit: Green light for GW search, Hill & Nurowksi

Einstein Linearized Theory- Quick Survey

Einstein Field Equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu}$$

{ amount of curvature } { amount of matter }

Weak field approx.(static):

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}$$

Minkowski metric

Metric perturbations

Transverse Traceless gauge

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h_{\alpha\beta}\eta^{\alpha\beta} \rightarrow \Box\bar{h}_{\mu\nu} = 2\kappa T_{\mu\nu} \rightarrow \Box\bar{h}_{\mu\nu} = 0$$

Wave Equation

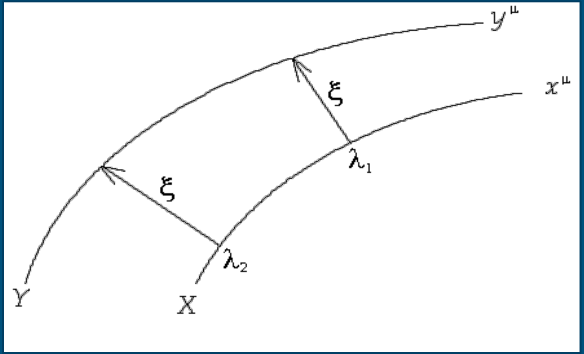
Analogy with Electromagnetism (CREDIT: Gravity,James Hartle)

	Linearized Gravitation	Electromagnetism
Basic "potentials"	Linearized metric perturbation $h_{\alpha\beta}(x)$	Vector and scalar potentials $(\Phi(t,\vec{x}),\ \vec{A}(t,\vec{x}))$
Field quantities	Linearized Riemann curvature $\delta R_{\alpha\beta\gamma\delta}(x)$	Electric and magnetic fields $\vec{E}(t,\vec{x}),\ \vec{B}(t,\vec{x})$
Gauge transformation leading to new potentials but the same fields	$h_{\alpha\beta} \to h_{\alpha\beta} - \partial_\alpha \xi_\beta - \partial_\beta \xi_\alpha$	$\vec{A} \to \vec{A} + \nabla\Lambda$ $\Phi \to \Phi - \partial\Lambda/\partial t$
Example of a gauge condition	Lorentz gauge $\partial_\beta h_\alpha^\beta - \frac{1}{2}\partial_\alpha h_\beta^\beta = 0$	Lorentz condition $\vec{\nabla}\cdot\vec{A} + \partial\Phi/\partial t = 0$
Field equations simplified by the gauge condition	$\Box h_{\alpha\beta} = 0$	Maxwell's equations $\Box\vec{A} = 0$ $\Box\Phi = 0$

Einstein Linearized Theory- Quick Survey

$$\Box \bar{h}_{\mu\nu} = 0$$

Tidal deformation → Geodesic deviation



$$\delta x_j = \frac{1}{2} h_{jk}^{\mathrm{TT}} x_0^k \quad \text{or} \quad h \approx \frac{\Delta L}{L}$$

h : dimensionless gravitational strain

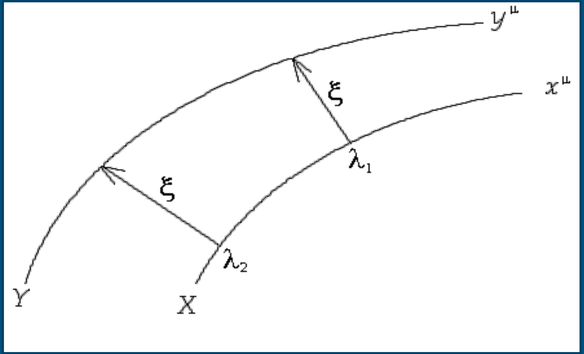
$$\Delta L/L \sim 10^{-21}$$

Einstein Linearized Theory- Quick Survey

$$\Box \bar{h}_{\mu\nu} = 0$$

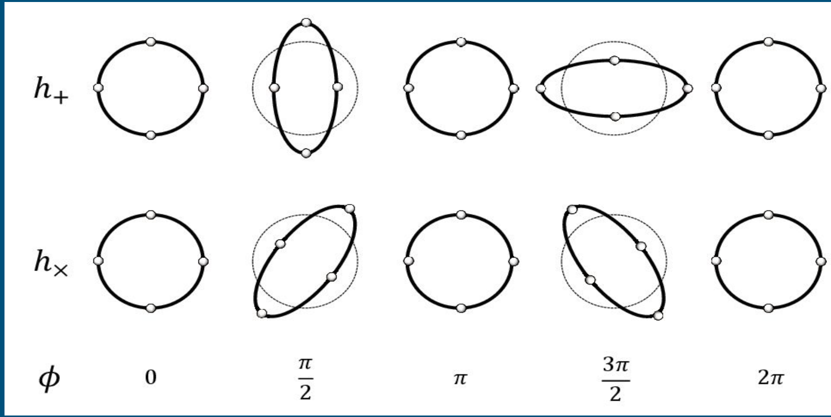
Tidal deformation → Geodesic deviation

$$\Delta L/L \sim 10^{-21}$$



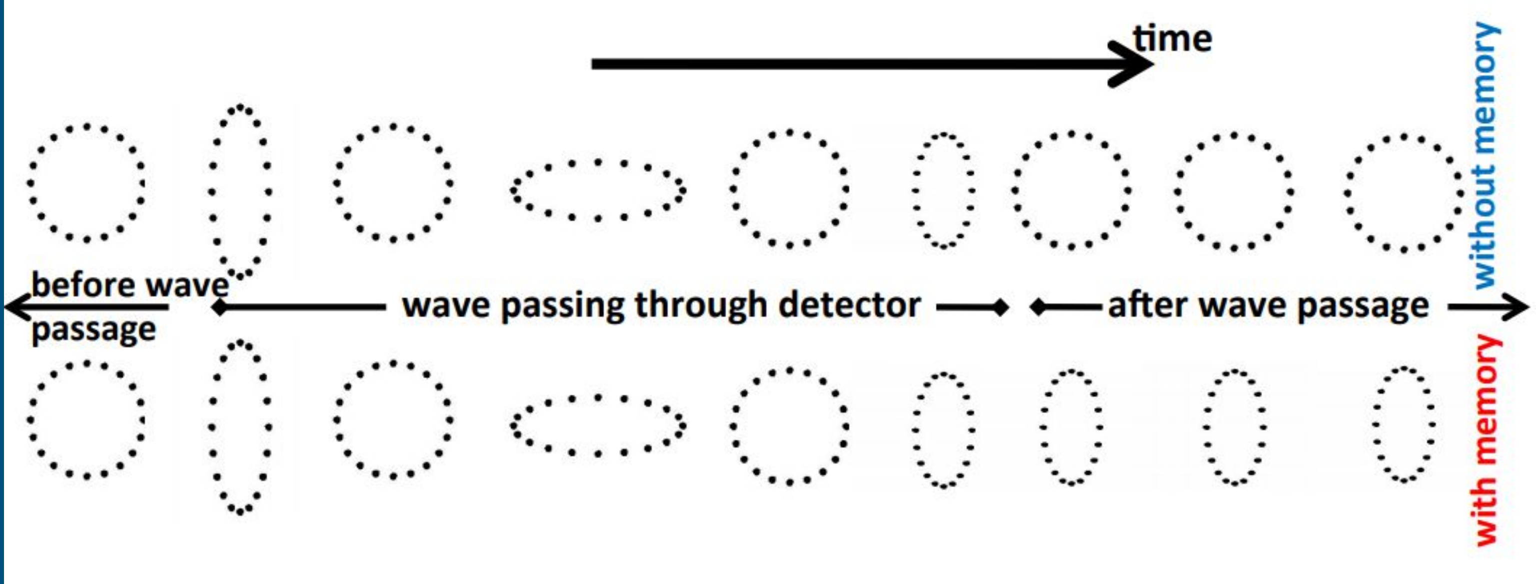
$$\tilde{h}^{\mu\nu} = A^{\mu\nu} e^{i k_\alpha x^\alpha}$$

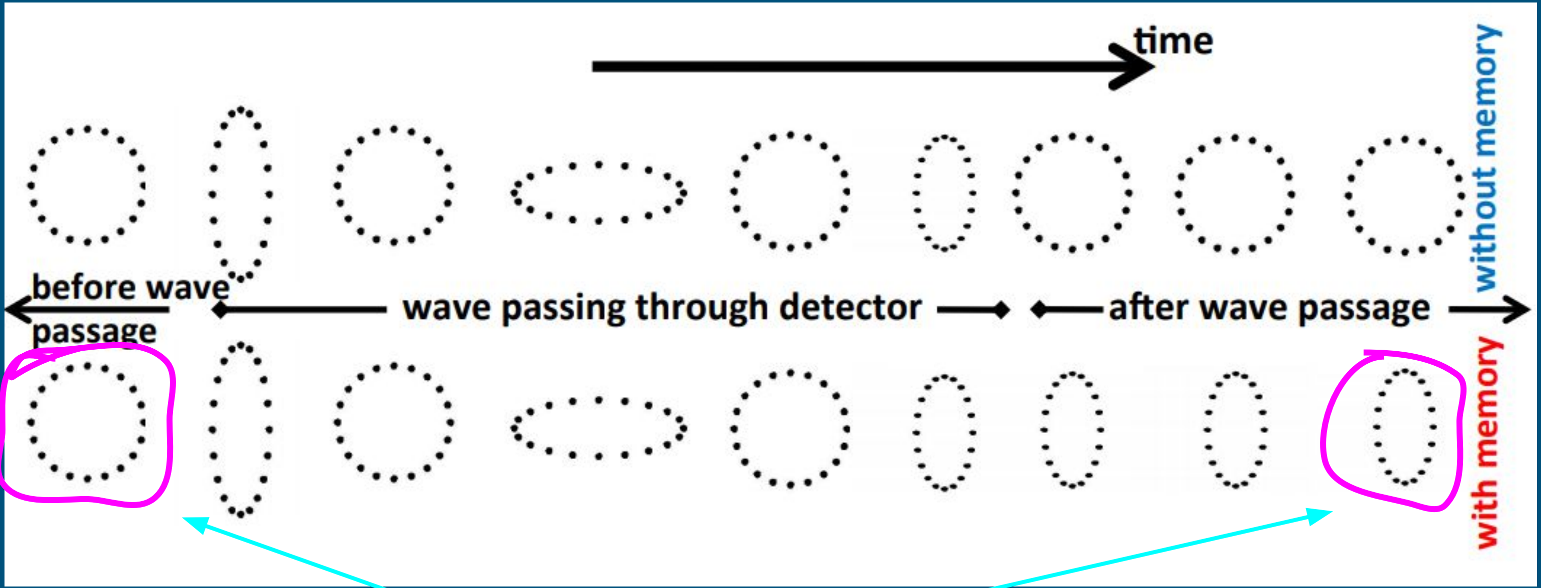
$$A^{\mu\nu} = h_+ \epsilon_+^{\mu\nu} + h_\times \epsilon_\times^{\mu\nu}$$



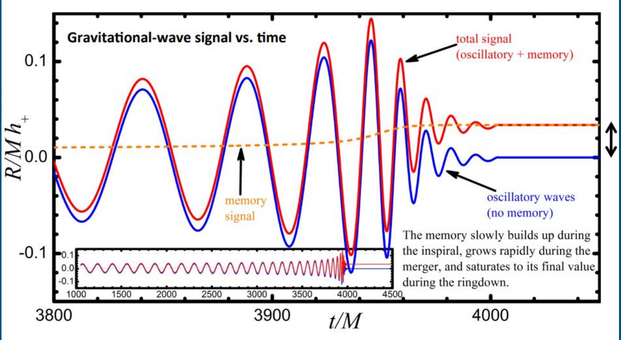
Memory Effect :

Permanent change in the configuration of spacetime after GW has passed.

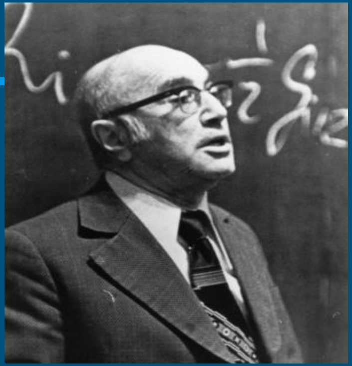




Change in the early and final values of GW polarisations



Ripples leave behind Memories ➡ LINEAR

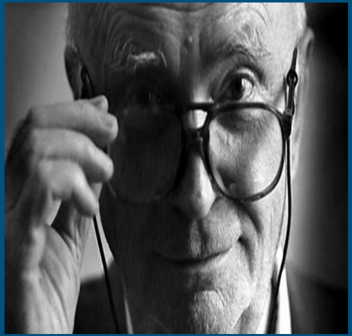


Zeldovich

Polnarev

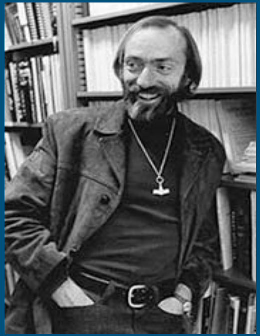
★ changes in the initial and final values of the masses and velocities of the components of a gravitating system

★ Two Types: Displacement and velocity



Braginsky

Grishchuk



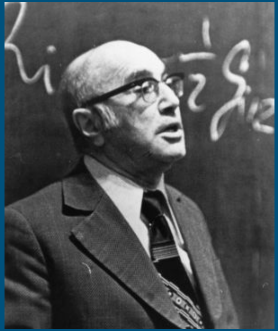
Thorne

How they occur ?

- ➔ Hyperbolic orbits
- ➔ Asymmetric supernovae explosion
- ➔ GRB

Ripples leave behind Memories ➡ LINEAR

- solve EFE for space-space part of metric $\Box \bar{h}_{jk} = -16\pi T_{jk}$



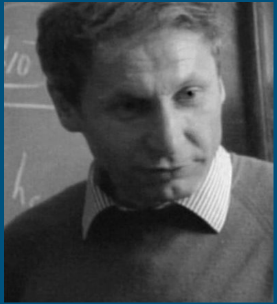
Zeldovich

Polnarev

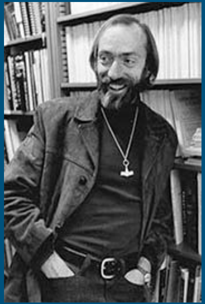
Unbound system

$$\Delta h^{\mathrm{TT}}_{jk} = \Delta \sum_{A=1}^{N} \frac{4M_A}{R\sqrt{1-v_A^2}} \left[\frac{v_A^j v_A^k}{1 - \boldsymbol{v}_A \cdot \boldsymbol{N}} \right]^{\mathrm{TT}}$$

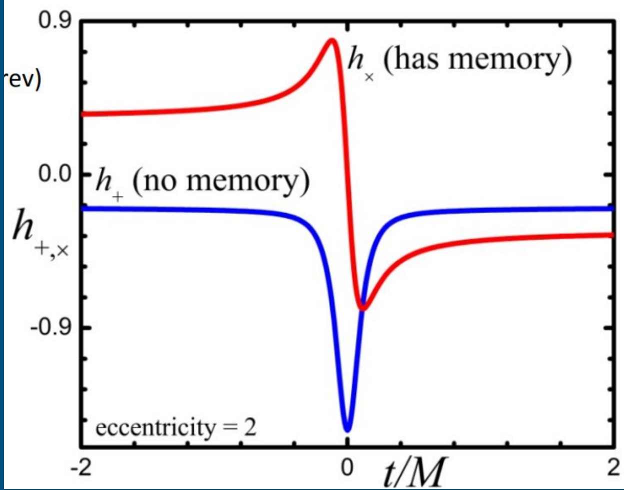
Braginsky



Grishchuk

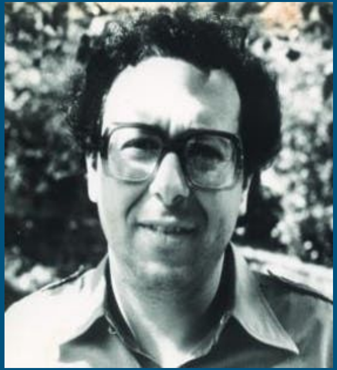


Thorne

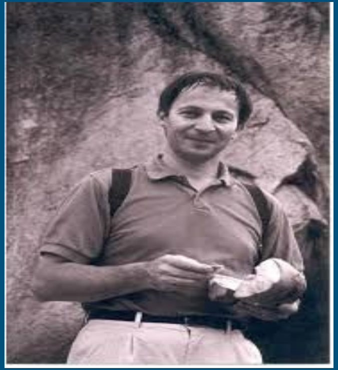


Favata et al.

Ripples leave behind Memories → NON-LINEAR



Christodoulou



Blanchet

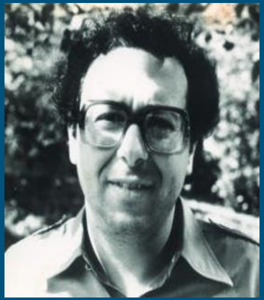
Damour

- ★ due to change in the mass of a binary caused by the emission of GWs
- ★ GWs produced by GWs

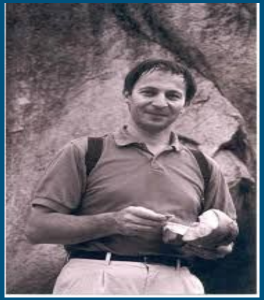
$$\Box \bar{h}_{\mu\nu} = -16\pi (T_{\mu\nu} + \tau_{\mu\nu}[h,h])$$

→ Nonlinear piece of Einstein's equations

Ripples leave behind Memories ➡ NON-LINEAR



Christodoulou



Blanchet

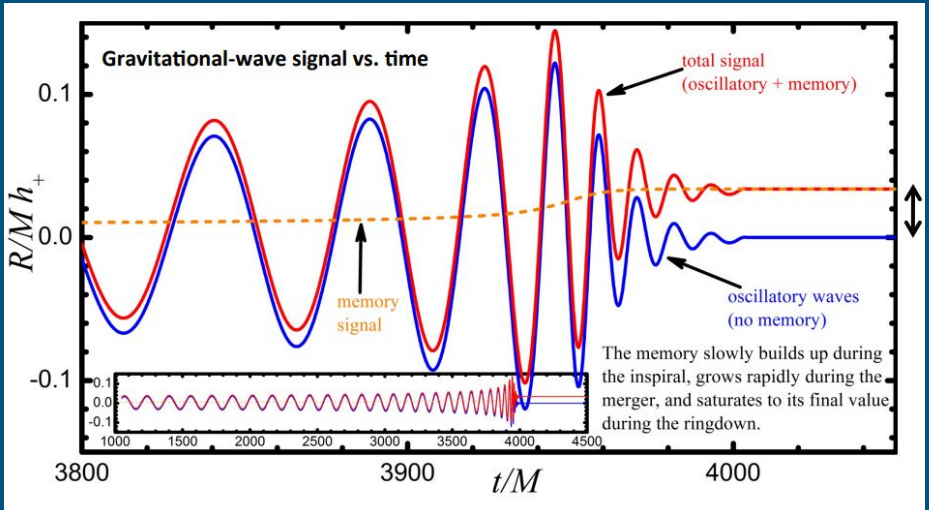
Damour

★ due to change in the mass of a binary caused by the emission of GWs

★ GWs produced by GWs

- Enters at leading order in Post-Newtonian expansion
- Distinctly visible in waveform

Favata et al.

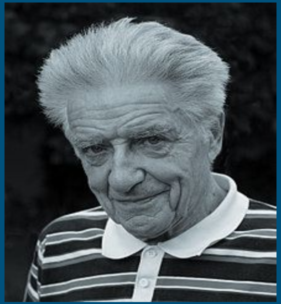


Exact plane wave spacetimes

pp-wave spacetimes

- exact solutions of Einstein's field equation
- model radiation moving at the speed of light

Ehlers



Kundt

Brinkmann Coordinates : belonging to the family of pp-wave metrics

$$ds^2 = H(u,x,y)du^2 + 2dudv + dx^2 + dy^2$$

Easy to interpret : coordinate free definition

Exact plane wave spacetimes

pp-wave spacetimes

Plane Wave Spacetime

$$ds^2 = H(u,x,y)du^2 + 2dudv + dx^2 + dy^2$$

$H(u,x,y)$ → quadratic

Zhang , Duval, Gibbons : 'The Memory Effect for Plane Gravitational Waves'

Plane Gravitational waves
Brinkmann coordinates

$$g = \delta_{ij}\, dX^i dX^j + 2dU\,dV + K_{ij}(U)X^i X^j\, dU^2$$

Profile of wave

$$K_{ij}(U)X^i X^j = \frac{1}{2}\mathcal{A}(U)\Big((X^1)^2 - (X^2)^2\Big)$$

Wave produced by gravitational collapse modelled as sandwich wave,

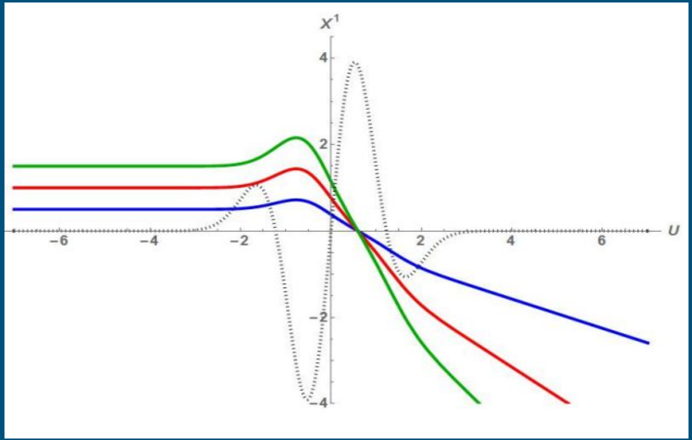
$$\mathcal{A}(U) = \frac{1}{2}\frac{d^3(e^{-U^2})}{dU^3}$$

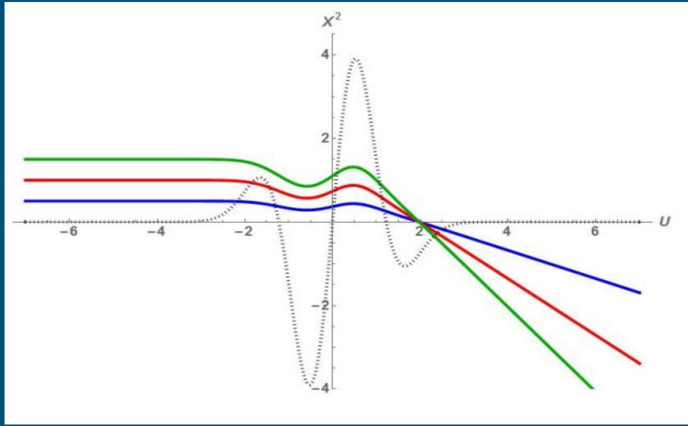
$$\frac{d^2\mathbf{X}}{dU^2} - \frac{1}{2}\mathrm{diag}(\mathcal{A}, -\mathcal{A})\,\mathbf{X} = 0$$

Zhang , Duval, Gibbons : 'The Memory Effect for Plane Gravitational Waves'

Geodesics in Brinkmann coordinates for particles initially at rest

$$\frac{d^2\mathbf{X}}{dU^2} - \frac{1}{2}\mathrm{diag}(\mathcal{A}, -\mathcal{A})\,\mathbf{X} = 0$$





Conclusion

- ➔ Look into different gravitational wave solutions
- ➔ Analyze exact solutions of EFEs
- ➔ How this ‘memory’ looks like in different exact solutions

Prospects : build up memory, stacking signals and get lucky in terms of inclination angle

BackUp:

$$ds^2 = H(u,x,y)du^2 + 2dudv + dx^2 + dy^2$$

$$H(u,x,y) = a(u)\,(x^2 - y^2) + 2\,b(u)\,xy + c(u)\,(x^2 + y^2)$$

- a,b describes wave profile ot two GW polarisations modes.
- c=0, we have vacuum plane waves=plane GWS!