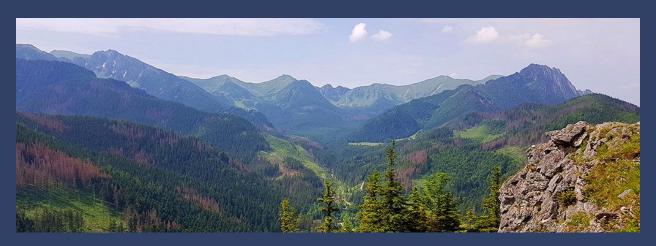
Particle physics aspects of Dark Matter

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Preamble: general properties of the DM particle

The DM particle must:

- be dark (neutral)
- be stable
- account for 26% of the energy content of the Universe:
- be 'cold'
- have a not too large cross section on nucleons: DM direct detection
- not produce too large fluxes of cosmic rays: DM indirect detection
- be able to escape detection at colliders so far
- have not too large self-interactions
- not spoil <u>BBN or CMB....</u>

- ...

Outline

- 1) DM stability
- 2) DM relic density
 - generalities on early Universe hot plasma
 - thermal hot relic
 - thermal cold relic: non-relativistic freeze out
 - freeze-in
 - asymmetric DM
- 3) DM direct detection
- 4) DM indirect detection
- 5) Phenomenology of a few illustrative models
- 6) DM self-interactions (but no time)

Part I DM stability

Dark Matter stability

- DM is around today $\tau_{DM} > \tau_{universe} \simeq 10^{18}\,\mathrm{sec}$
- Given its relic density today one needs in general much larger lifetimes not to produce fluxes of cosmic rays we should have seen already: $\tau_{DM} > \tau_{universe} \simeq 10^{25-28}\,\mathrm{sec}$

unless very light or invisible decay see indirect detection part below

To have a particle with at least those lifetimes is the most constraining property for the general structure of the DM model!

Stability of DM particle: general considerations on decay

if DM decays the coupling causing the decay must be tiny

for example a 2-body decay:
$$\Gamma(DM \to A + B) \sim \frac{1}{8\pi} \, g^2 \, m_{DM}$$
 tree level coupling

$$\tau_{DM} = 1/\Gamma(DM \to A + B) > \tau_{universe} \leftrightarrow g \lesssim 10^{-20} \cdot \sqrt{1 \,\text{GeV}/m_{DM}}$$

$$g \lesssim 10^{-10} \cdot \sqrt{10^{-11} \,\text{eV}/m_{DM}}$$

$$g \lesssim 1 \cdot \sqrt{10^{-31} \,\text{eV}/m_{DM}}$$

- ⇒ to have a long enough lifetime:
 - the coupling vanishes or is very tiny
 - or DM mass very tiny but anyway a tree level coupling of order unity is excluded because $m_{DM}>10^{-22}\,\mathrm{eV}$ is anyway needed to have DM galactic halo: i.e. to have a wavelength smaller than galaxy size
- clearly this suggests a symmetry:
 - -to forbid the decay: absolute DM stability: $g=0\,$
 - -or at least to provide an explanation for a so tiny coupling

2 questions:

do we need a new symmetry beyond the SM for DM stability?

what kind of symmetry could it be?

SM detour: Stable SM particles there is always a deep symmetry reason

- $\underline{\gamma}$: stable because massless (due to unbroken $\underline{U(1)}_{em}$ gauge symmetry)
- lightest $\, {m
 u} \,$: lightest fermion of the SM: stable due to Lorentz invariance
- e^- : stable because lightest particle charged under conserved electric charge due to unbroken $U(1)_{em}$ gauge symmetry
- p : stable due to an accidental symmetry: $U(1)_B$: baryon number conservation

stems from gauge sym. of the SM and charges of particles under them

quantum numbers of SM particles do not allow to write down an interaction which violates baryon number in Lagrangian \Rightarrow accidental symmetry

 $SU(3)_c$ gauge invariance: $\mathcal{L}_{quarks} \propto \bar{q} \dots q$ each time a quark is annihilated another one is created $\implies U(1)_B$ symmetry: $q \to e^{i\phi}q$

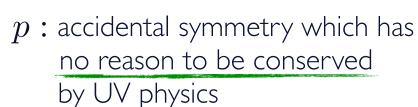
accidental symmetry: $U(1)_B$ not subgroup of $SU(3)_c$

Absolute stability vs approximate stability



 e^-, ν, γ : symmetries expected to be conserved also by any new UV physics



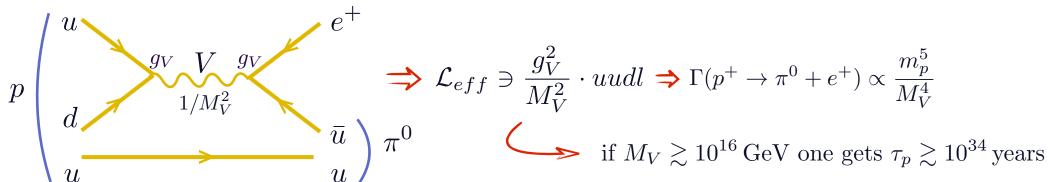




induce higher dimensional operator which leads to proton decay



>> example: grand unification: has gauge boson coupling to a quark and a lepton



natural mechanism to have a very slow decay!

for DM particle stability one could invoke similar mechanisms to the SM or other ones....

2 questions:

do we need a new symmetry beyond the SM for DM stability?

what kind of symmetry could it be?

2 questions:

do we need a new symmetry beyond the SM for DM stability?

yes! except for a simple possibility: a large weak multiplet

what kind of symmetry could it be?

``Minimal dark matter"

a fermion quintuplet under $SU(2)_L$ containing a neutral particle: $\psi_{DM} = \begin{pmatrix} \psi_{DM}^{DM} \\ \psi_{DM}^{0} \\ \psi_{DM}^{0} \\ \psi_{DM}^{0} \end{pmatrix}$

one cannot write down an interaction which would give a too fast decay

unlike for a smaller representation:

$$SU(3)_c \times SU(2)_L \times U(1)$$

- a fermion singlet: $\mathcal{L} \ni Y \bar{L} \psi_{DM} H \Rightarrow \psi_{DM} \to l^- H^+, \nu H^0$ decays expected! not forbidden by $SU(3)_c \times SU(2)_L \times U(1)$
- a fermion doublet: $\psi_{DM} = \begin{pmatrix} \psi_{DM}^0 \\ \psi_{DM}^- \end{pmatrix}$: $\mathcal{L} \ni Y \overline{l_R} \psi_{DM} H \Rightarrow \psi_{DM}^0 \to l^- H^0$ decays expected!
- a fermion triplet: $\psi_{DM} = \begin{pmatrix} \psi_{DM}^+ \\ \psi_{DM}^0 \\ \psi_{DM}^- \end{pmatrix}$ or $\psi_{DM} = \begin{pmatrix} \psi_{DM}^0 \\ \psi_{DM}^- \\ \psi_{DM}^- \end{pmatrix}$: $\mathcal{L} \ni Y \, \overline{L} \psi_{DM} H$ or $\mathcal{L} \ni Y \, \overline{L} \psi_{DM} H^\dagger$ a fermion quadruplet: $\psi_{DM} = \begin{pmatrix} \psi_{DM}^+ \\ \psi_{DM}^0 \\ \psi_{DM}^- \\ \psi_{DM}^- \end{pmatrix}$: $\mathcal{L} \ni \frac{1}{\Lambda} \, \overline{L} \, \psi_{DM} H H$: expected too fast if UV physics below Planck mass

- a fermion quadruplet:
$$\psi_{DM} = \begin{pmatrix} \psi_{DM} \\ \psi_{DM}^0 \\ \psi_{DM}^- \end{pmatrix}$$
 : $\mathcal{L} \ni \frac{1}{\Lambda} \, \overline{L} \, \psi_{DM} H H$: expected too fast if UV physics below Planck mass

a fermion quintuplet under $SU(2)_L$ containing a neutral particle: $\psi_{DM} = \begin{pmatrix} \psi_{DM}^{DM} \\ \psi_{DM}^{D} \\ \psi_{DM}^{-} \\ \psi_{DM}^{-} \end{pmatrix}$

no possible dimension-4 and dimension-5 interactions: accidental symmetry



the exchange of a UV particle could induce only a dim-6 operator:

$$\mathcal{L} \ni \frac{1}{\Lambda^2} \overline{L} \psi_{DM} H H H^{\dagger} \Longrightarrow \Gamma(\psi_{DM}^0 \to L + H) \sim \frac{1}{8\pi} \frac{v^4 m_{DM}}{\Lambda^4}_{m_{DM} = 100 \, \mathrm{GeV}}$$

$$\Rightarrow \tau_{DM} > \tau_{universe} \text{ only if } \Lambda > 3 \cdot 10^{13} \text{ GeV} < m_{Planck}$$
$$\tau_{DM} > (10^{26} \text{ sec}) \text{ only if } \Lambda > 3 \cdot 10^{15} \text{ GeV} < m_{Planck}$$

as long as there is no new physics inducing this operator below these scales: very fine a flux of cosmic rays from this decay could be around the corner but this requires an object as large as a quintuplet

Stability of DM due to a new symmetry beyond the SM

Various possibilities:

- -DM stability due to new unbroken gauge symmetry
- -DM stability due to new broken gauge symmetry
- -DM stability due to accidental symmetry resulting from new gauge symmetry
- -DM stability due to new discrete or global symmetry

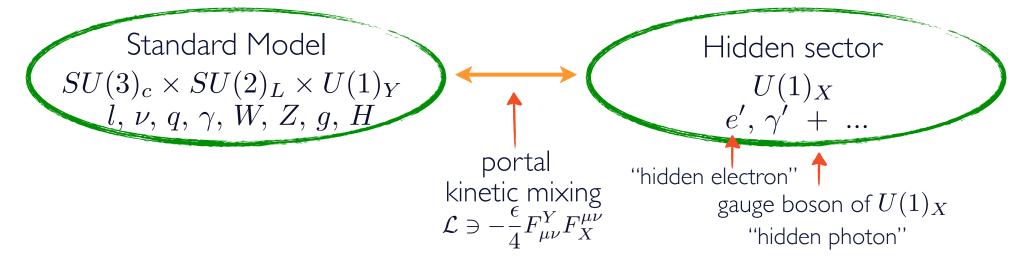
DM stability from new gauge symmetry

simplest example: lightest charged particle under a new U(1): " $U(1)_X$ "

ullet a fermion: a e' which has no charge under SM with SM particles chargeless under $U(1)_X$

Pospelov 07,....

"secluded DM"



If the $U(1)_X$ is unbroken: the e' DM candidate is stable just as the electron: lightest particle charged under a conserved charge

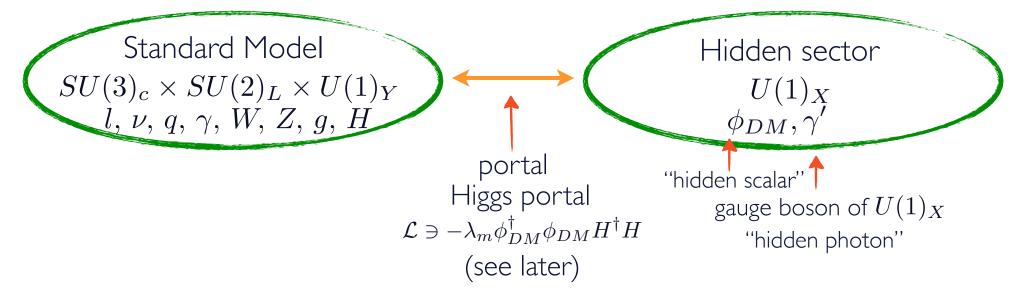
If the $U(1)_X$ is spontaneously broken: still the e' DM candidate is stable because of remnant $Z_2 \in U(1)_X$, because still e' in pairs in $\mathcal L$

DM stability from new gauge symmetry

simplest example: lightest charged particle under a new U(1): " $U(1)_X$ "

ullet a scalar: a ϕ_{DM} which has no charge under SM with SM particles chargeless under $U(1)_X$

"secluded DM"



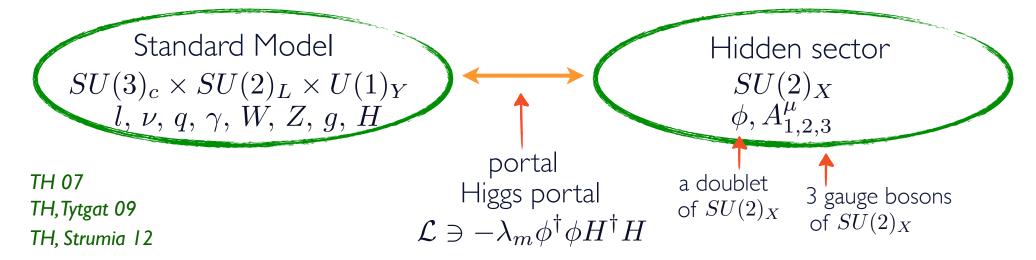
If the $U(1)_X$ is unbroken: the ϕ_{DM} DM candidate is stable just as the electron: lightest particle charged under a conserved charge

If the $U(1)_X$ is spontaneously broken: the ϕ_{DM} could decay if gets a vev for instance or stay stable if no vev but not automatic...

DM stability from accidental symmetry resulting from new gauge symmetry

well known example: conservation of mirror baryon number in a mirror hidden sector

other example: hidden vector DM: it is possible to have gauge boson to be the DM, even a non-abelian one



 \Rightarrow after $SU(2)_X$ sym. breaking: • 3 massive $SU(2)_X$ gauge bosons: stable: DM candidates

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v_{\phi}}{\sqrt{2}} \end{pmatrix}$$

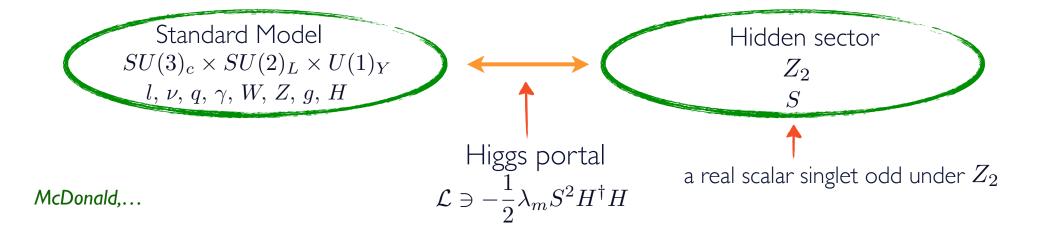
- one real scalar boson
- a remnant $SU(2)_C$ accidental custodial symmetry

 \rightarrow DM = hidden forces!

⇒ accidental symmetry: interesting phenomenology from naturally slow decay

DM stability from discrete symmetry: real scalar singlet

- ullet a real scalar singlet S odd under Z_2 parity: $S \to -S$
- "ad-hoc" symmetry



 \Rightarrow S is stable: the Z_2 symmetry makes sure that all terms involve an even number of S:

$$\mathcal{L} \ni -\frac{1}{2}\mu_S^2 S^2 - \frac{1}{24}\lambda_S S^4 - \frac{1}{2}\lambda_m S^2 H^{\dagger} H$$

 \Rightarrow extremely simple: only 2 relevant parameters: $m_S, \, \lambda_m$

$$m_S^2 = \mu_S^2 + \frac{1}{2}\lambda_m v^2$$

 \Rightarrow more generally from a discrete Z_2 sym. one can stabilize any scalar or fermion SM multiplet (or abelian gauge boson)

DM stability from discrete symmetry: inert doublet

example: a scalar doublet H_2 odd under a Z_2 symmetry: $H_2 \rightarrow -H_2$

$$H_2 = \begin{pmatrix} H^+ \\ \frac{H_0 + iA_0}{\sqrt{2}} \end{pmatrix} \qquad \longleftarrow Y = 1 \neq 0$$

"inert scalar doublet DM"

Deshpande, Ma 78, Barbieri, Hall, Ryshkov 06, Lopez-Honorez, Nezri, Oliver, Tytgat 07, TH, Lin, Lopez-Honorez, Rocher 08

from the most general scalar potential H_0 and A_0 do not have the same mass

$$V = \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4$$
$$+ \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1^{\dagger} H_2|^2 + \frac{\lambda_5}{2} \left[(H_1^{\dagger} H_2)^2 + h.c. \right]$$

 $\implies m_{H_0}^2 - m_{A_0}^2 = \lambda_5 v^2 \implies$ the lightest neutral component is the DM

this is a visible sector DM model: ψ_{DM} has gauge SM interactions (no hidden sector)

DM stability from discrete symmetry: fermion triplet

``wino''

• a fermion triplet under $SU(2)_L$ odd under Z_2 parity: $\psi_{DM} \to -\psi_{DM}$

$$\psi_{DM} = \begin{pmatrix} \psi_{DM}^+ \\ \psi_{DM}^0 \\ \psi_{DM}^- \end{pmatrix}$$

 \longrightarrow this is a visible sector DM model: ψ_{DM} has gauge SM interactions

(no hidden sector)



only interactions it can have in fact

DM stability from discrete symmetry: Susy neutralino

Susy: has many new neutral particles beyond the SM: neutral superpartners;

```
B_Y^{\mu} \leftrightarrow \tilde{B} : "Bino"

W_3^{\mu} \leftrightarrow \tilde{W} : "Wino"

H_u \leftrightarrow \tilde{H}_u : "Higgsino"

H_d \leftrightarrow \tilde{H}_d : "Higgsino"

\nu_{L_i} \leftrightarrow \tilde{\nu}_i : "sneutrinos"

G \leftrightarrow \tilde{G} : "gravitino"
```

if one assume a Z_2 symmetry so that SM particles are even under it and superpartners are odd under it, "R-parity", the lightest superpartner (LSP) is stable

the 4 neutralinos (2 gauginos and 2 Higgsinos) mix: the lightest mass eigenstate, χ , is stable if LSP

R-parity is motivated by proton decay but still totally ad-hoc in MSSM

 \longrightarrow but turns out to be subgroup of $U(1)_{B-L} \Rightarrow$ could derive from gauge symmetry remnant subgroup

Epilogue on DM stability

- DM stability is the most constraining property for the general structure of the DM model!
- DM stability strongly suggests the existence of a new symmetry in Nature!
 even if not absolutely mandatory

perhaps it is the result of new forces in Nature (gauge symmetries) perhaps not if discrete or global symmetry, which is more "ad hoc" although can be directly related to solution of other problems (as neutralino or axion)

whose stability is due to a mixture of several reasons: due to global symmetry and the fact that it is very light and that it's decay occurs at loop level and suppressed by high scale will not be discussed here

Depending on stabilization mechanism several possibilities:

Fermion DM candidate <-> Boson DM candidate: scalar, vector

Visible DM candidate <-> Hidden sector DM candidate

Minimal model of DM <-> DM out of more global model

=> different phenomenologies!

A wide variety of DM models!

Illustrative examples:

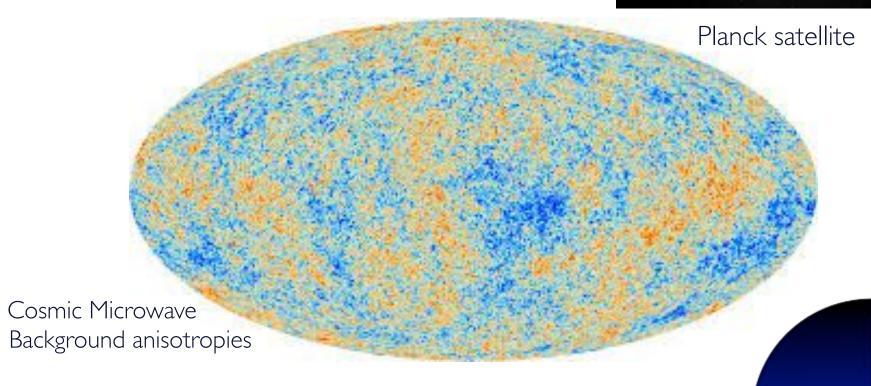
- ullet A real scalar singlet odd under a Z_2 : the simplest DM model
- ullet A scalar doublet odd under a Z_2 : ``inert scalar doublet'' DM model
- ullet A fermion triplet odd under a Z_2 : "Wino DM model" if Majorana
- A fermion quintuplet stable in an accidental way only on the basis of SM symmetries
- ullet A hidden fermion or scalar charged under a new $U(1)_X$ gauge symmetry
- ullet Hidden gauge bosons of a new $SU(2)_X$ gauge symmetry accidentally stable
- The MSSM neutralino stable due to R-parity
- Other possibilities not covered here: axion (strong CP problem),
 Kaluza-Klein DM (from extra dimensions)
 gravitino (in Susy),

Part 2 DM relic density

DM relic density: $\Omega_{DM}=26\%$

$$\Omega_{DM} \equiv \frac{\rho_{DM}}{\rho_{crit}}\Big|_{today} \Omega_B \equiv \frac{\rho_B}{\rho_{crit}}\Big|_{today}$$





$$\Omega_{DM}h^2 = 0.1199 \pm 0.0027 \qquad \Omega_{DM} \simeq (26 \pm 1)\%$$

$$\Omega_B h^2 = 0.02205 \pm 0.00028 \qquad \Omega_B \simeq (4.9 \pm 0.2)\%$$

$$h = 0.673 \pm 0.012$$

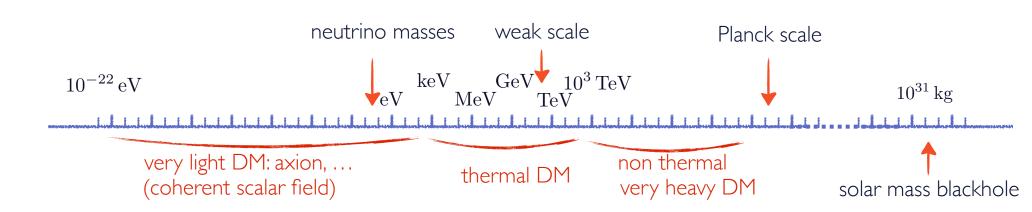
$$\Omega_{rad}h^2 = 4.31 \cdot 10^{-5} \leftarrow 2.7 \text{ K black body CMB radiation}$$

$$\Omega_{DM} \simeq (26 \pm 1)\%$$

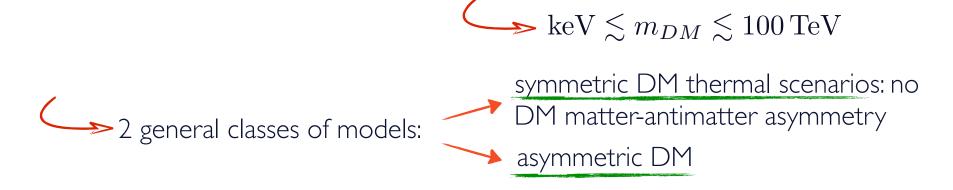
$$\Omega_B \simeq (4.9 \pm 0.2)\%$$

DM relic density: $\Omega_{DM}=26\%$

can be obtained for m_{DM} all the way from $\sim 10^{-22}\,\mathrm{eV}$ to $\sim 10^{31}\,\mathrm{kg}$ to have wavelength black hole with smaller than galactic size \sim solar mass



in the following we will consider the thermal DM scenarios



Generalities on early Universe hot thermal bath

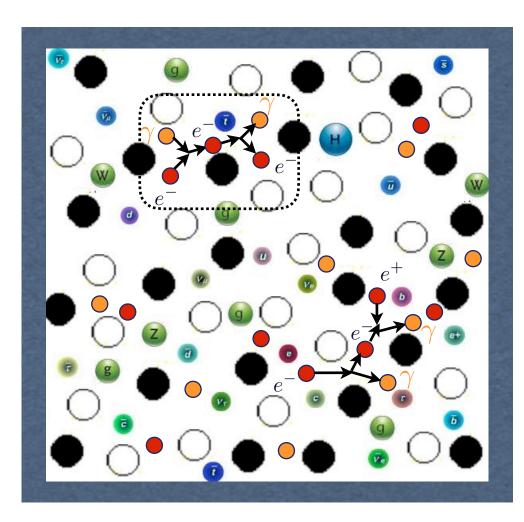
we consider the « radiation domination » epoch when all SM particles were forming a hot thermal soup: plasma

History of the Universe n 🕥 Key: W, Z bosons M photon Particle Data Group, LBNL, @ 2000. Supported by DOE and NSF

 $T \gtrsim 1 \, \mathrm{eV}$

Generalities on early Universe hot thermal bath

→ hot plasma:



Example with e^- and γ :

2 relevant processes

$$e^{\pm} + \gamma \rightarrow e^{\pm} + \gamma$$

$$e^+ + e^- \leftrightarrow \gamma + \gamma$$

if many $e^- + \gamma \rightarrow e^- + \gamma$ processes:

 \Rightarrow e^{-} and γ equilibrate their kinetic energy:

 e^- and γ are in "kinetic equilibrium"

probability that e^- has a given energy is given by a Fermi-Dirac distribution characterized by a temperature T

~averaged $e^-{\rm kinetic}$ energy

and similarly for γ given by a

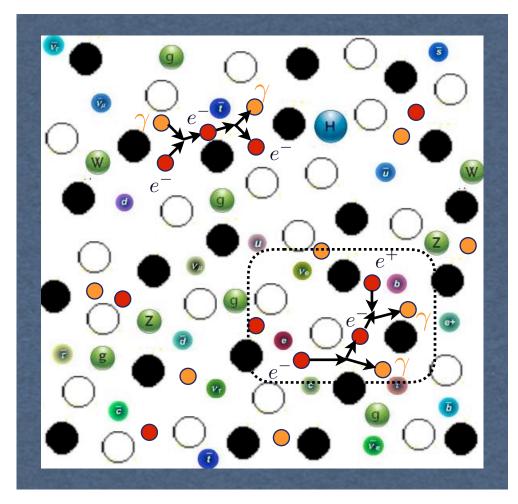
Bose-Einstein distribution characterized

by same temperature: $T_{e^-} = T_{e^+} = T_{\gamma} \equiv T$

 \sim averaged γ kinetic energy

Generalities on early Universe hot thermal bath

⇒ hot plasma:



Example with e^- and γ :

2 relevant processes

$$e^{\pm} + \gamma \rightarrow e^{\pm} + \gamma$$

 $e^{+} + e^{-} \leftrightarrow \gamma + \gamma$

if many $e^+ + e^- \leftrightarrow \gamma + \gamma$ processes:

 \Rightarrow e^{\pm} and γ equilibrate their numbers

$$n_{e^-} + n_{e^+} \leftrightarrow n_{\gamma}$$

 e^{\pm} and γ are in "chemical equilibrium"

in this case not only the energy distribution is known but also its normalization: how many particles have a given energy

$$\Rightarrow f_{FD}^{e^{\pm}} = \frac{1}{e^{E_{e^{\pm}}/T} + 1} \Rightarrow n_{e^{\pm}} = g_{e^{\pm}} \int \frac{d^3 p_{e^{\pm}}}{(2\pi)^3} f_{FD}^{e^{\pm}}$$

$$\rho_{e^{\pm}} = g_{e^{\pm}} \int \frac{d^3 p_{e^{\pm}}}{(2\pi)^3} f_{FD}^{e^{\pm}} \cdot E_{e^{\pm}}$$

assuming here no e^+ - e^- asymmetry: $n_{e^+}=n_{e^-}$ $g_{\gamma}=g_{e^-}=g_{e^+}=2$

$$\Rightarrow f_{BE}^{\gamma} = \frac{1}{e^{E_{\gamma}/T} - 1} \Rightarrow n_{\gamma} = g_{\gamma} \int \frac{d^{3}p_{\gamma}}{(2\pi)^{3}} f_{BE}^{\gamma} = \frac{\zeta(3)}{\pi^{2}} g_{\gamma} T^{3} \quad \rho_{\gamma} = g_{\gamma} \int \frac{d^{3}p_{\gamma}}{(2\pi)^{3}} f_{BE}^{\gamma} \cdot E_{\gamma} = \frac{\pi^{2}}{30} g_{\gamma} T^{4}$$

Relativistic and non relativistic thermal equilibrium regimes

$$n_{e^{\pm}} = g_{e^{\pm}} \int \frac{d^{3}p_{e^{\pm}}}{(2\pi)^{3}} f_{FD}^{e^{\pm}} = \frac{\frac{3}{4} \frac{\zeta(3)}{\pi^{2}} g_{e^{\pm}} T^{3} \propto 1/V \qquad (T >> m_{e}) \quad \text{relativistic } e^{-}}{g_{e^{\pm}} \left(\frac{m_{e} T}{2\pi}\right)^{3/2} e^{-m_{e}/T}} \qquad (T << m_{e}) \quad \text{non-relativistic } e^{-}$$

If no interactions or relativistic:

$$n_{e^-} = \frac{const}{V}$$

to see the variation of n_{e-} due to interactions we look at "comoving number density":

$$n_{e-} \cdot V$$
 or $Y_{e^-} = n_{e^-}/s$

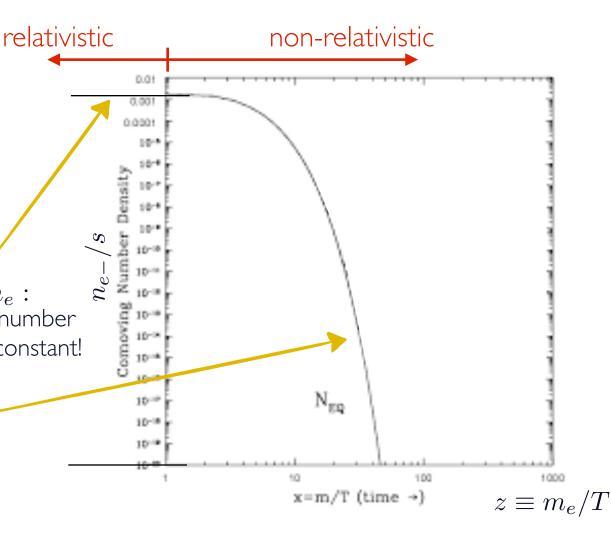
entropy density is conserved:

$$s = \frac{2\pi^2}{45} g_*^s \, T^3 \propto 1/V$$

 $T>m_e$: comoving number density is constant!

 $T < m_e$:

comoving number density is exponentially suppressed: Boltzmann suppression $\propto e^{-m_e/T}$



Boltzmann suppression in non-relativistic regime

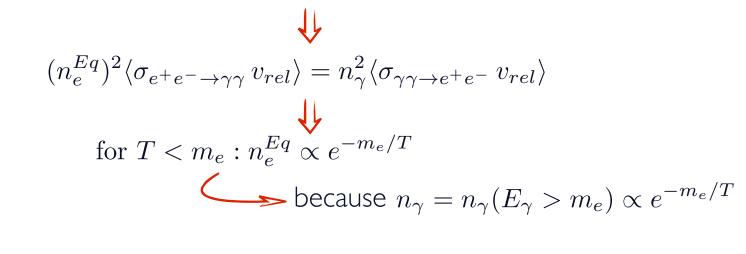
first assume no expansion: temperature is constant



number of particles is constant



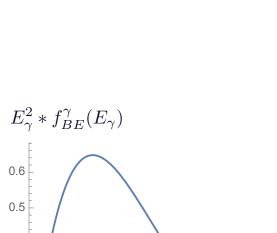
same number of interactions in both directions



tail of thermal distribution $\propto e^{-m_e/T}$



as T decreases with expansion $n_{e^-}^{Eq}\,$ more and more exponentially suppressed



0.4

0.3

0.2

0.1

More generalities on early Universe thermodynamics: radiation energy density

For a relativistic fermion particle:

$$\rho_f = g_f \int \frac{d^3 p_f}{(2\pi)^3} f_{FD}^f \cdot E_f = \frac{\pi^2}{30} \frac{7}{8} g_f T_f^4$$

For a relativistic boson particle:

$$\rho_b = g_b \int \frac{d^3 p_b}{(2\pi)^3} f_{BE}^b \cdot E_b = \frac{\pi^2}{30} g_b T_b^4$$

For a plasma with several species with same temperature

if kinetic equilibrium

$$\rho_{rad}^{Tot} = \frac{\pi^2}{30} g_* T^4 \qquad g_* = \sum_{b_i} g_{b_i} + \frac{7}{8} \sum_{f_i} g_{f_i}$$

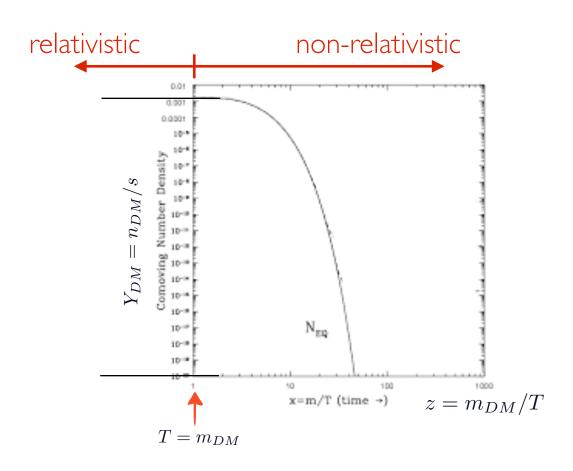
$$\implies H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G\rho}{3}} \sim 1.7\sqrt{g_*} \frac{T^2}{m_{Planck}}$$

a =Universe scale factor

DM thermal equilibrium comoving number density

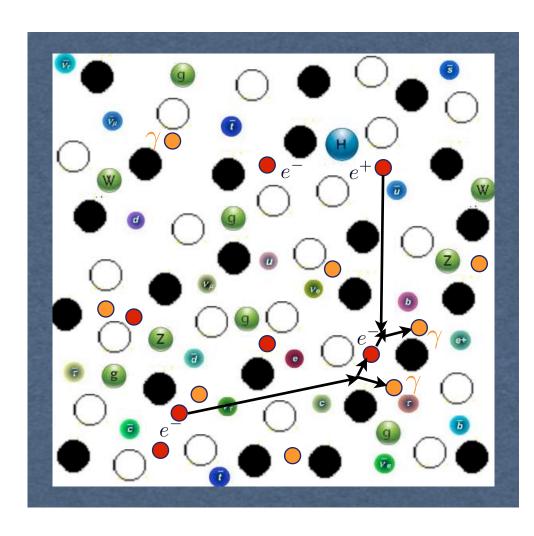
Comoving DM number density is constant when relativistic and becomes Boltzmann suppressed when becoming non-relativistic when $T < m_{DM}$ in the same way as e^- if DM is in kinetic equilibrium and chemical equilibrium

$$\bigcirc DM SM \leftrightarrow DM SM \longrightarrow DM DM \leftrightarrow SM SM$$



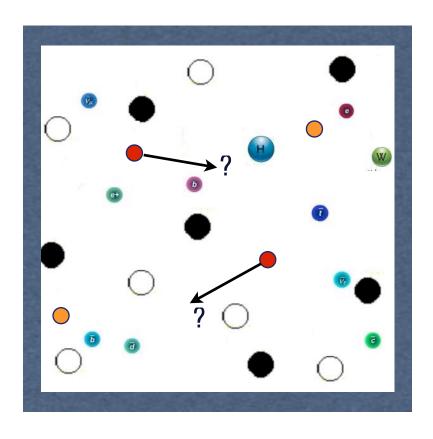
No expansion : no thermal decoupling

If no expansion: e^+ and e^- or 2 DM particles will always finish by encountering \Rightarrow will still equilibrate numbers and energies \Rightarrow no decoupling



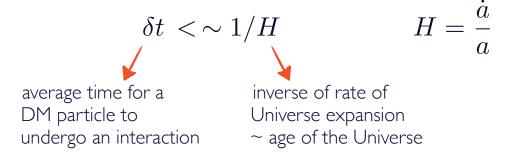
DM thermal decoupling due to expansion

Clearly DM cannot remain in thermal equilibrium for ever: if it has not already decoupled when it was relativistic it will anyway when it is non-relativistic: as Universe expands the DM number density becomes more and more exponentially suppressed: at some point too few DM particles for them to annihilate \Rightarrow the annihilation $DM DM \leftrightarrow SM SM$ process doesn't occur anymore and n_{DM}/s freezes





particles couple as long as:



Thermal decoupling condition

Particle decouples when:
$$\delta t > \sim 1/H$$
 \longleftrightarrow $\Gamma \equiv 1/\delta t < H$

For a decay:
$$\delta t = 1/\Gamma_D$$
 decay width

For an annihilation $i + j \rightarrow k + l$

$$\sigma(i+j\to k+l) = \frac{\text{number of transition a single i particle undergoes per unit time}}{\text{incoming flux of j particles}}$$

$$= \frac{\text{number of transition a single i particle undergoes per unit time}}{\text{number of j particles crossing a unit surface per unit time}}$$

$$= \frac{\text{number of transition a single i particle undergoes per unit time}}{n_j \cdot v_{rel}}$$
 relative velocity between i and j

interaction rate

 $\Gamma_i = 1/\delta t_i = \text{number of transition a single i particle undergoes per unit time} = n_j \langle \sigma_{ij \to k l} \cdot v_{rel} \rangle$

average over i and j momentum distribution

rate of Universe expansion

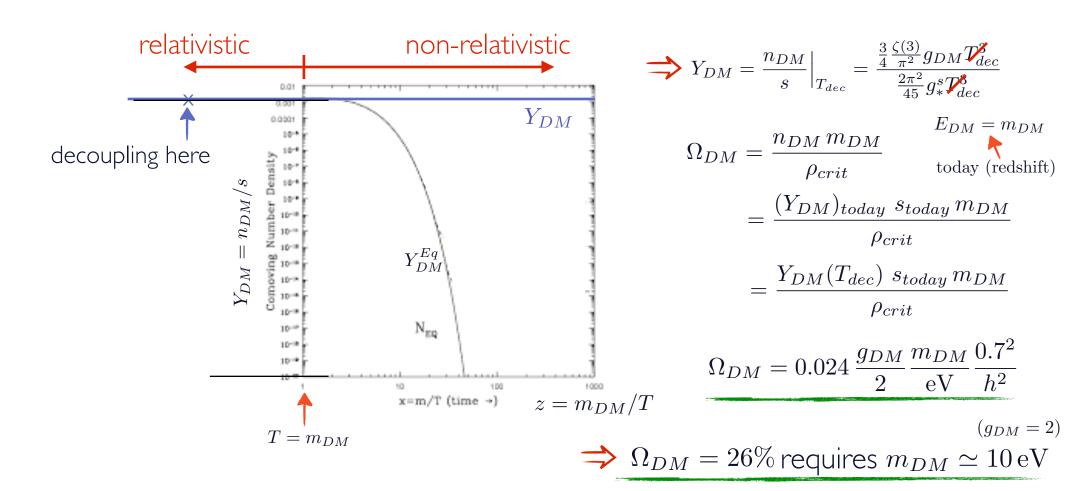
$$\Rightarrow \Gamma_i = n_j \langle \sigma_{ij \to k \, l} \, v_{rel} \rangle \qquad \longleftarrow \neq \gamma_{ij \to k \, l} \equiv n_i n_j \langle \sigma_{ij \to k \, l} \, v_{rel} \rangle$$

= number of transitions per unit time per unit volume

Relativistic DM thermal decoupling: "hot relic"

if
$$\Gamma_{DM} = n_{DM}^{Eq} \langle \sigma_{DM \, DM \to SM \, SM} \, v_{rel} \rangle < H$$
 occurs when $T > m_{DM}$:

$$\frac{n_{DM}}{s}\Big|_{T_{today}} = \frac{n_{DM}}{s}\Big|_{T_{dec}} = \text{constant number it has when relativistic}$$



Why a hot DM relic points towards eV scale?

because for a hot relic
$$\frac{n_{DM}}{n_{\gamma}}\Big|_{T_{dec}} \sim 1 \implies \frac{n_{DM}}{n_{\gamma}}\Big|_{today} \sim 1$$

but each DM particle today has much more energy than each γ today: $E_{DM} \simeq m_{DM}$ $E_{\gamma} \simeq T_{today}$ $\sim 10^{-3} \, {\rm eV}$

 \Rightarrow to have Ω_{DM} not larger than 26% today we need DM to be very light!

more precisely: numerically it is an experimental fact that today:

$$\Omega_{rad} = 9.6 \cdot 10^{-5}$$
 $\rho_{rad}^{Tot} = \frac{\pi^2}{30} g_* T^4$
 $\Omega_{DM} = 26\%$
 $T_{\gamma} = 2.7 \,\mathrm{K} \sim 10^{-3} \,\mathrm{eV}$

$$\Rightarrow \frac{\Omega_{DM}}{\Omega_{rad}} \sim 3000 \sim \frac{E_{DM}}{E_{\gamma}} \Big|_{today} \sim \frac{m_{DM}}{10^{-3} \, \text{eV}} \Rightarrow m_{DM} \sim \mathcal{O}(10 \, \text{eV})$$

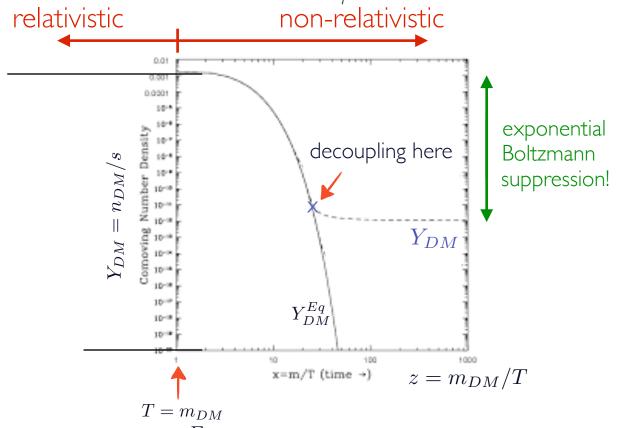
Why a hot DM relic points towards eV scale?

 \Rightarrow unless $m_{DM}\lesssim \mathcal{O}(10\,\mathrm{eV})$ the relic density constraint requires a mechanism which gives: $\frac{n_{DM}}{n_{\gamma}}<<1$

this is similar to the baryon case $\Omega_B = 5\%$ and $m_p \simeq 1 \, GeV \rightarrow \left. \frac{n_B}{n_\gamma} \right|_{today} \sim 10^{-9}$

Non-relativistic DM thermal decoupling: "cold relic"

 $\Gamma_{DM}=n_{DM}^{Eq}\left\langle \sigma_{DM\;DM\to SM\;SM}\;v_{rel}
ight
angle < H$ occurs when $T< m_{DM}$: gives nothing but what we need: $\frac{n_{DM}}{n_{\gamma}}<<1$



 $\Rightarrow \text{ for } T>T_{dec} : n_{DM}=n_{DM}^{Eq}$ for $T< T_{dec} :$ no more annihilation at all: $n_{DM}/s=const$

$$\Rightarrow \frac{n_{DM}}{s}\Big|_{today} = \frac{n_{DM}}{s}\Big|_{T_{dec}} = \frac{n_{DM}^{Eq}}{s}\Big|_{T_{dec}} \Rightarrow \text{ all we just need to know is } T_{dec}$$

Non-relativistic DM thermal decoupling: "cold relic"

$$T_{dec}$$
 is solution of: $\left. \frac{\Gamma}{H} \right|_{T_{dec}} = \frac{n_{DM}^{Eq} \langle \sigma_{DM\,DM o SM\,SM\,v_{rel}}
angle}{H} \Big|_{T_{dec}} = 1$

$$z_{dec} \equiv \frac{m_{DM}}{T_{dec}} = \ln[0.038 \, \frac{g_{DM}}{\sqrt{g_*}} \, m_{DM} \, m_{Planck} \langle \sigma_{DMDM \to SMSM} \, v_{rel} \rangle]$$

$$\frac{n_{DM}^{Eq}}{s}\Big|_{T_{dec}}$$

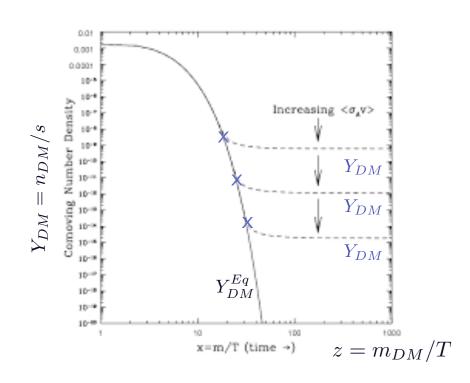
$$\frac{n_{DM}^{Eq}}{s}\Big|_{today}$$

$$\Omega_{DM} = \text{const'} \frac{z_{dec}}{\langle \sigma_{DMDM \to SMSM} v_{rel} \rangle} \simeq \text{const''} \frac{1}{\langle \sigma_{DMDM \to SMSM} v_{rel} \rangle}$$

relic density depends only on annihilation cross section

Non-relativistic DM thermal decoupling: "cold relic"

the larger is the annihilation cross section the longer DM will remain in thermal equilibrium, the smaller will be the equilibrium number density when DM decouples, the smaller will be Ω_{DM}



the relic density fixes the value of the cross section to a value

basically independent of m_{DM} $\Omega_{DM} = \mathrm{const'} \frac{z_{dec}}{\langle \sigma_{DMDM \to SMSM} \, v_{rel} \rangle} \simeq \mathrm{const''} \frac{1}{\langle \sigma_{DMDM \to SMSM} \, v_{rel} \rangle}$

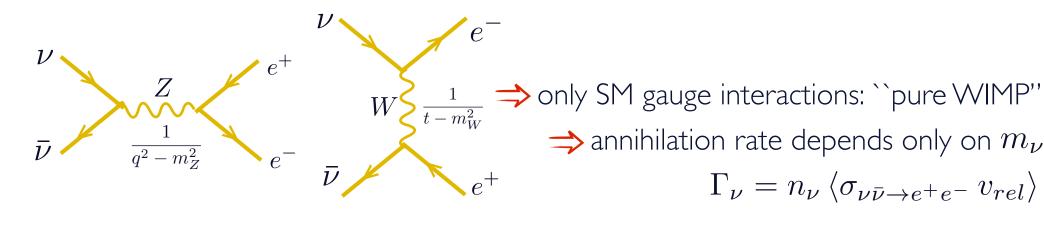
$$\Rightarrow \Omega_{DM} = 26\% \leftrightarrow \langle \sigma_{DMDM \to SMSM} v_{rel} \rangle \simeq 3 \cdot 10^{-26} \text{cm}^3/\text{sec} \simeq 10^{-9} \,\text{GeV}^{-2} \simeq 1 \,\text{pb}$$

$$\Rightarrow$$
 if $\langle \sigma_{DMDM \to SMSM} \, v_{rel} \rangle \propto \frac{g^4}{m_{DM}^2}$ and $g \sim 1 \sim g_{EW}$ one needs $m_{DM} \sim 1 \, {\rm TeV}$

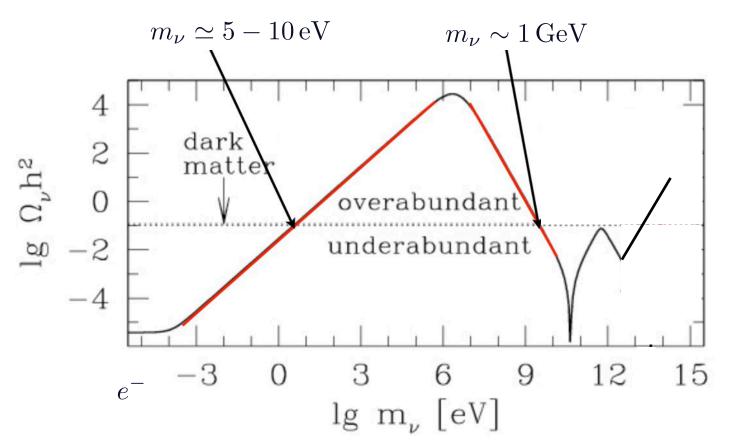
$$\sim$$
 $z_{dec} \simeq 22$

"WIMP miracle"

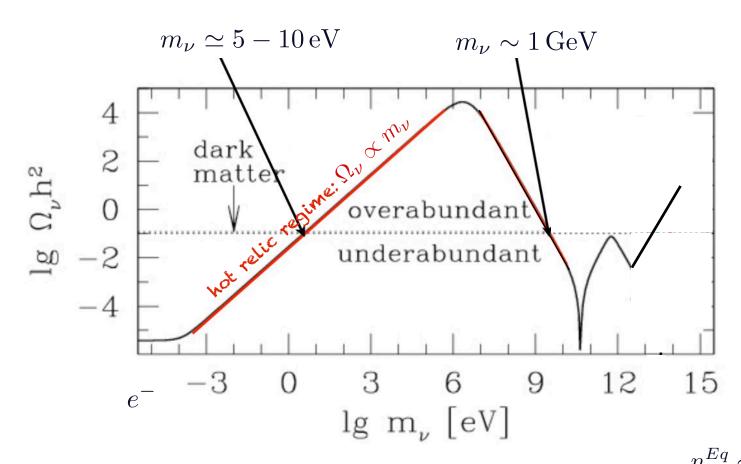
The example of a Dirac neutrino with mass $m_ u$



 $g_{\nu}=4$



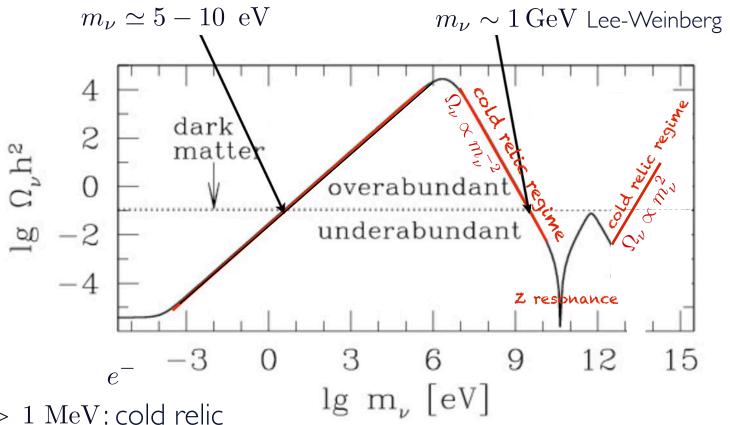
The example of a Dirac neutrino with mass m_{ν} : hot relic regime



we consider
$$m_{\nu} << m_{Z,W}$$
 and assume $T_{dec} > m_{\nu}$:
$$\begin{array}{c} n_{\nu}^{Eq} \sim g_{\nu} T^{3} \\ \langle \sigma_{\nu\bar{\nu}\rightarrow e^{+}e^{-}} v_{rel} \rangle \sim \alpha_{W}^{2} \frac{T^{2}}{m_{Z,W}^{4}} \\ \frac{\Gamma_{\nu}}{H} \Big|_{T_{dec}} = \frac{n_{\nu}^{Eq} \left\langle \sigma_{\nu\bar{\nu}\rightarrow e^{+}e^{-}} v_{rel} \right\rangle}{1.7 \sqrt{g_{*}} \frac{T^{2}}{m_{Planck}}} \Big|_{T_{dec}} \sim \frac{g_{\nu} \alpha_{W}^{2} \frac{T_{dec}^{5}}{m_{W,Z}^{4}}}{\sqrt{g^{*}} \frac{T_{dec}^{2}}{m_{Planck}}} = 1 \quad \Longrightarrow T_{dec} \simeq 1 \, \mathrm{MeV} \end{aligned}$$

 \Rightarrow if $m_{\nu} \lesssim 1 \,\mathrm{MeV}$ the ν is a hot relic and $\Omega_{\nu} \propto m_{\nu} \Rightarrow \Omega_{\nu} = 26\%$ for $m_{\nu} \sim 5 - 10 \text{ eV}$

The example of a Dirac neutrino with mass m_{ν} : cold relic regime



if
$$m_{\nu} > 1 \; \mathrm{MeV}$$
: cold relic

$$\Rightarrow z_{dec} \equiv \frac{m_{\nu}}{T_{dec}} = \ln[0.038 \frac{g_{\nu}}{\sqrt{g_*}} m_{\nu} m_{Planck} \langle \sigma_{\nu\bar{\nu}\to e^+e^-} v_{rel} \rangle] \sim 22$$

$$\Omega_{\nu} \simeq \text{const} \frac{1}{\langle \sigma_{\nu\bar{\nu}\to e^+e^-} v_{rel} \rangle} \propto 1/m_{\nu}^2 \text{ if } m_{\nu} << m_{W,Z} \leftarrow \langle \sigma_{\nu\bar{\nu}\to e^+e^-} v_{rel} \rangle \propto \frac{m_{\nu}^2}{m_{W,Z}^4}$$

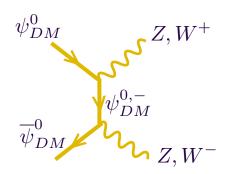
$$\propto m_{\nu}^2 \text{ if } m_{\nu} >> m_{W,Z} \leftarrow \langle \sigma_{\nu\bar{\nu}\to e^+e^-} v_{rel} \rangle \propto \frac{1}{m_{\nu}^2}$$

> Thermal decoupling way highly depends on mass of mediator in annihilation

Other example: the fermion triplet example: a pure WIMP

``wino''

another true "WIMP" is for example a fermion triplet (see above): $\psi_{DM} = \begin{pmatrix} \psi_{DM}^+ \\ \psi_{DM}^0 \\ \psi_{DM}^- \end{pmatrix}$ only EW interactions



$$\implies m_{DM} \simeq 3.0 \text{ TeV}$$

Maximum mass allowed for a thermal cold relic

thermal freezeout requires: $\langle \sigma_{DMDM \to SMSM} \, v_{rel} \rangle \simeq 3 \cdot 10^{-26} \, \mathrm{cm}^3/\mathrm{sec}$

but unitarity of S matrix requires:
$$\sigma_{DMDM \to SMSM} \leq \frac{\pi(2J+1)}{\vec{p}_{DM}^2}$$

$$\vec{p}_{DM}^2 = p_{DM}^2 - m_{DM}^2 = m_{DM}^2 v_{rel}^2 / 4$$

$$\Rightarrow$$
 for $J=0$: $\sigma_{DMDM\to SMSM}\,v_{rel} \leq \frac{4\pi}{m_{DM}^2v_{rel}} \Rightarrow \underline{m_{DM}} \leq 110\,\mathrm{TeV}$

$$v_{rel}^2 \simeq \frac{6 \, T_{dec}}{m_{DM}} \sim \frac{6}{22}$$

Minimum mass allowed for a thermal relic: Cold DM constraint

 \triangleright DM remains relativistic as long as $T \gtrsim m_{DM}$



the lighter it is, the more distance it will have done



the less small structure will develop

 $T \sim eV$



in practice by the time matter begins to dominate the Universe, we need anisotropies at scales smaller than supercluster scale!



otherwise less smaller structures (i.e. galaxies) will form and galaxies will form only much later



galaxies younger than superclusters

"free streaming length"

however this will be not the case if $m_{DM} \lesssim 1 \, \mathrm{keV}$ because in this case one can calculate that the comoving distance that DM would have done is larger than supercluster comoving size— erase anisotropies at smaller distance

Cold/Warm/Hot DM

 $\Rightarrow m_{DM} \gtrsim 1 \, \mathrm{keV}$: DM must be cold!

 $\Rightarrow m_{DM} >> 1 \,\mathrm{keV}$: DM is cold

 $m_{DM} << 1 \,\mathrm{keV}$: DM is hot

 $m_{DM} \sim 1 \, \mathrm{keV}$: DM is warm

DM is a cold relic: decouple non-relativistic

DM is a hot relic: decouple relativistic

excluded because for example a ν with $m_{\nu} = 30 \, \mathrm{keV}$ is cold but a hot relic! gives $\Omega_{\nu} >>> 26\%$

in practice all hot relics excluded because if cold they overclose the Universe!

for example a u with $m_
u \sim 10\,\mathrm{eV}$ is hot and a hot relic!

excluded because hot

HOT COLD WARM

Accurate calculation of the DM relic density: Boltzmann equations

equation giving the variation of the DM number density per unit time

if no interactions:
$$n_{DM} = \frac{\mathrm{const}}{V} \propto \frac{const}{a^3}$$

$$\Rightarrow \frac{dn_{DM}}{dt} \equiv \dot{n}_{DM} = -3\frac{\dot{a}}{a} \cdot \frac{const}{a^3} = -3H n_{DM}$$

$$\Rightarrow \dot{n}_{DM} + 3H n_{DM} = 0$$

if interactions: $DMDM \leftrightarrow SMSM$

$$\Gamma_{DMDM \to SMSM} = n_{DM} \langle \sigma_{DMDM \to SMSM} v_{rel} \rangle$$

= number of $DMDM \rightarrow SMSM$ transitions a single DM part. has per unit time

$$\gamma_{DMDM \to SMSM} = n_{DM}^2 \langle \sigma_{DMDM \to SMSM} v_{rel} \rangle$$

= number of $DMDM \rightarrow SMSM$ transitions per unit time per unit volume

number of $SMSM \rightarrow DMDM$ reactions number of $DMDM \rightarrow SMSM$ reactions per unit time per unit volume

per unit time per unit volume

Accurate calculation of the DM relic density: Boltzmann equations

7 steps ahead:

$$\dot{n}_{DM} + 3H \, n_{DM} = \int d^{3}\tilde{p}_{DM_{1}} d^{3}\tilde{p}_{DM_{2}} d^{3}\tilde{p}_{SM_{1}} d^{3}\tilde{p}_{SM_{2}} (2\pi)^{4} \delta^{4} (p_{DM_{1}} + p_{DM_{2}} - p_{SM_{1}} - p_{SM_{2}})$$

$$\cdot \left[f_{SM_{1}} f_{SM_{2}} |\mathcal{M}_{SM_{1}SM_{2} \to DM_{1}DM_{2}}|^{2} - f_{DM_{1}} f_{DM_{2}} |\mathcal{M}_{DM_{1}DM_{2} \to SM_{1}SM_{2}}|^{2} \right] \cdot 2$$

- (2) SM particles are in thermal equilibrium $\rightarrow f_{SM_{1,2}} = f_{SM_{1,2}}^{Eq}$
- (3) Maxwell Boltzmann statistic approximation $\to f^{Eq} = \frac{1}{e^{E/T} \pm 1} \simeq e^{-E/T}$ $f_{SM_1}^{Eq} f_{SM_2}^{Eq} = e^{-(E_{SM_1} + E_{SM_2})} = e^{-(E_{DM_1} + E_{DM_2})} = f_{DM_1}^{Eq} f_{DM_2}^{Eq}$

(4)
$$|\mathcal{M}_{SM_1SM_2 \to DM_1DM_2}|^2 = |\mathcal{M}_{DM_1DM_2 \to SM_1SM_2}|^2 \equiv |\mathcal{M}|^2$$

CP conservation assumed here

Accurate calculation of the DM relic density: Boltzmann equations

(5)
$$\dot{n}_{DM} + 3H n_{DM} =$$

$$\int d^3 \tilde{p}_{DM_1} d^3 \tilde{p}_{DM_2} d^3 \tilde{p}_{SM_1} d^3 \tilde{p}_{SM_2} (2\pi)^4 \delta^4 (p_{DM_1} + p_{DM_2} - p_{SM_1} - p_{SM_2}) f_{DM_1}^{Eq} f_{DM_2}^{Eq} |\mathcal{M}|^2 \quad \leftarrow \equiv \gamma_{DM_1 DM_2 \to SM_1 SM_2}^{Eq}$$

$$\cdot 2 \cdot \left(1 - \frac{f_{DM_1} f_{DM_2}}{f_{DM_2}^{Eq}} \right)$$

kinetic equilibrium of all particles
$$\rightarrow \frac{f_{DM_1}f_{DM_2}}{f_{DM_1}^{Eq}f_{DM_2}^{Eq}} = \frac{n_{DM}^2}{n_{DM}^{Eq\,2}}$$

(6)
$$n_{DM} \rightarrow Y_{DM} = \frac{n_{DM}}{s}$$

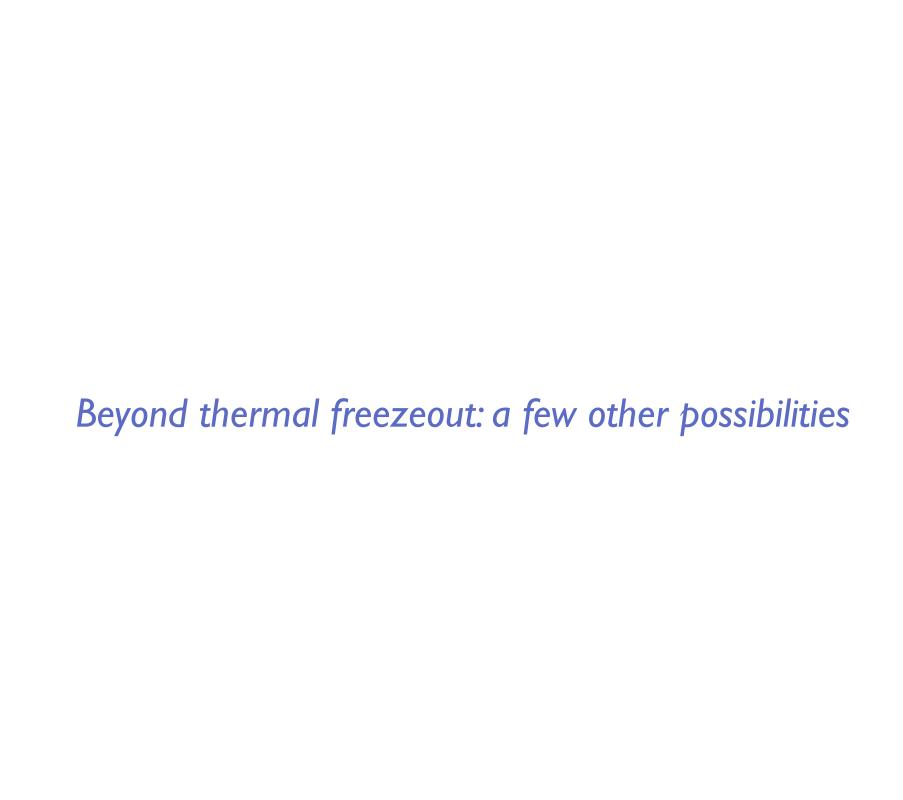
$$s \propto T^3 \propto t^{-3/2} \quad \frac{ds}{dt} = -3Hs$$

$$\dot{Y}_{DM} = \frac{\dot{n}_{DM}}{s} + 3H \frac{n_{DM}}{s}$$

(7)
$$t \to z = m_{DM}/T$$
 radiation epoch: $H = \frac{1}{2t}$ \Longrightarrow $\frac{dz}{dt} = zH$

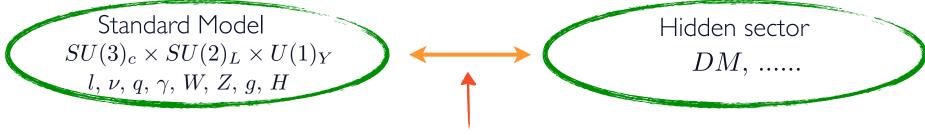
$$\Rightarrow \left| szH(z) \frac{dY_{DM}}{dz} = 2 \cdot \gamma_{DMDM \to SMSM}^{Eq} \cdot \left(1 - \frac{Y_{DM}^2}{Y_{DM}^{Eq2}} \right) \right|$$

 \Rightarrow integrating this equation over z one finds $Y_{DM}|_{today}$ shows that instantaneous decoupling approximation above is good



Freezein

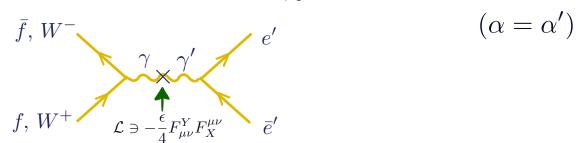
if DM belongs to a hidden sector it might be that it has never thermalized with the SM thermal bath, if the portal small



small portal interaction: $\Gamma_{portal} < H$ always

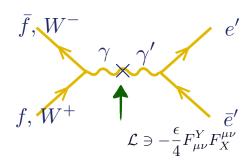
for example with a Higgs portal, $\mathcal{L} \ni -\lambda_m \phi^\dagger \phi H^\dagger H$, both sectors do not thermalize with each other if : $\lambda_m \lesssim 10^{-6} \leftarrow m_\phi \sim TeV$ $H^\dagger H \leftrightarrow \phi^\dagger \phi$

for example with a kinetic mixing portal, $\mathcal{L}\ni -\frac{\epsilon}{4}F^Y_{\mu\nu}F^{\mu\nu}_X$, both sectors do not thermalize with each other if : $\epsilon\lesssim 10^{-6}$ \leftarrow $m_{e'}\sim TeV$ $(\alpha=\alpha')$



Freezein

 \Rightarrow in this case, even if no thermalization, the SM thermal bath can still produce very slowly (out-of-equilibrium) pairs of DM particles to get the right amount of DM: freeze-in remember the n_{DM}/n_{γ} we need to get is much



$$\frac{dn_{DM}}{dt} = n_{SM_i}^2(T) \langle \sigma_{SM_iSM_i \to DMDM} \, v \rangle \, 2$$

$$\frac{dY_{DM}}{dT} = \frac{n_{SM_i}^2(T) \langle \sigma_{SM_iSM_i \to DMDM} \, v \rangle}{TH(T)s(T)} \propto 1/T^2 \qquad \qquad Y_{DM} \equiv \frac{n_{DM}}{s}$$

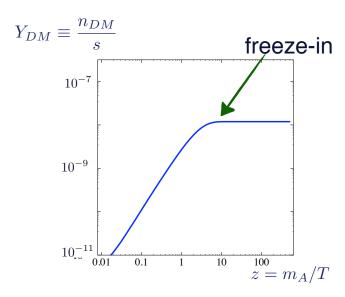
$$\downarrow \qquad \qquad T > m_{DM}$$

$$Y_{DM} \propto 1/T \; \text{down to} \; T \sim m_{DM} \qquad \qquad \text{where} \; n_{SM_i}^{eq} \; \text{becomes} \; \text{Boltzmann suppressed}$$

→ DM production IR dominated as for freezeout

smaller than the relativistic thermal value: $n_{DM}/n_{\gamma}\sim 1$: no need for DM to necessarily thermalize

Mc Donald 02'
Hall, Jedamzik,
March-Russell, West 09',
Yaguna 11',
Frigerio, TH, Masso 11',...



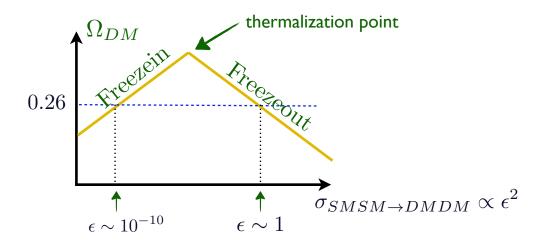
production depends only on interactions at $T\sim m_{DM}$ and not on physics at higher scales but unlike freezeout it depends on the DM number initial conditions

$$\Omega_{DM} = \Omega_{DM}^{"end \ of \ inflation"} + \Omega_{DM}^{freezein}$$

Freezein

$$Y_{DM} \sim \frac{n_{SM}^2 \langle \sigma_{SMSM \to e'\bar{e}'} \, v_{rel} \rangle}{s} \Big|_{T \sim m_{DM}} \cdot \frac{1}{H(T \sim m_{DM})} \propto \epsilon^2$$
 number of DM particles created per unit time per unit volume over entropy age of the Universe

 \Rightarrow $\Omega_{DM}=26\%$ requires a tiny coupling: $\epsilon\sim 10^{-10}$



Mc Donald 02'
Hall, Jedamzik,
March-Russell, West 09',
Yaguna 11',
Frigerio, TH, Masso 11',...

 \Rightarrow so far we got 3 ways to get Ω_{DM} to be only 26% despite of the fact that DM is matter:

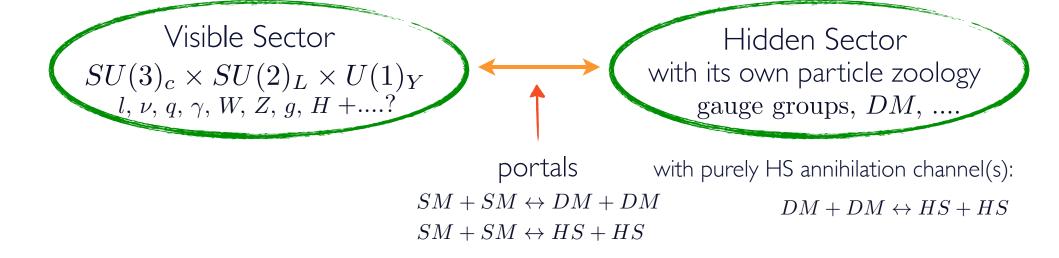
or $n_{DM} \sim n_{\gamma} \Rightarrow$ DM must be very light but excluded (hot)

or $n_{DM} << n_{\gamma} \Rightarrow$ DM must be heavier (cold)

from freezeout (Boltzmann suppression)

or freezein (tiny coupling suppression)

Going more general: general hidden sector structure

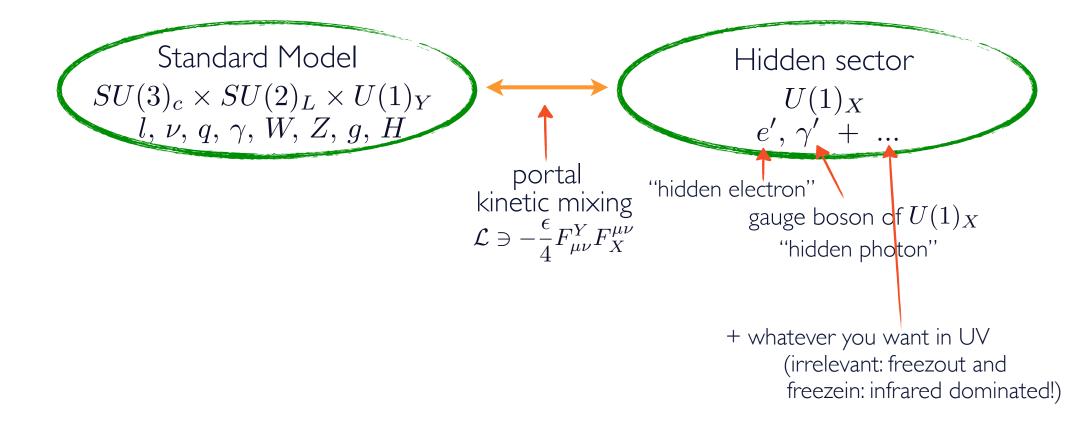


⇒ DM can annihilate in the HS sector and/or to SM particles

A prototype Hidden Sector model: hidden photon + hidden electron

the QED' model already considered above: $\mathcal{L} = \mathcal{L}_{SM} + \bar{\psi}'(i\mathcal{D}' - m_{\psi})\psi'$

a e^\prime charged under a new $U(1)_X$ with no charge under SM with SM particles chargeless under $U(1)_X$



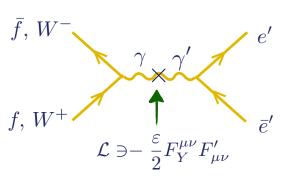
A prototype Hidden Sector model: hidden photon + hidden electron

$$\mathcal{L} = \mathcal{L}_{SM} + \bar{\psi}'(i\mathcal{D}' - m_{\psi})\psi' - \frac{1}{4}F_{\mu\nu}^{Y}F_{X}^{\mu\nu}$$

3 parameters: $m_{DM}, \, \alpha', \, \epsilon$

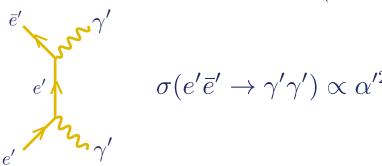
DM electric charge : $\kappa = \sqrt{\alpha'/\alpha \epsilon}$

Portal processes:



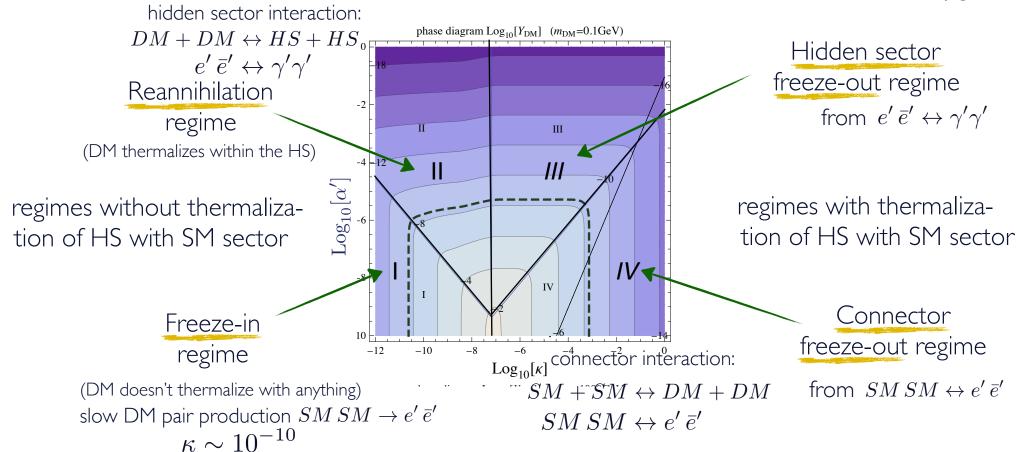
e' $\sigma(SMSM \to e'\bar{e}') \propto \alpha^2 \kappa^2$

Hidden sector process:



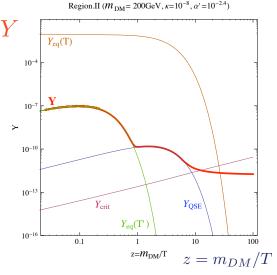
Visible sector/Hidden sector/Connector structure: 5 basic ways to get the observed relic density

Chu, T.H., Tytgat 11



Visible sector/Hidden sector/Connector structure: 5 basic ways to get the observed relic density

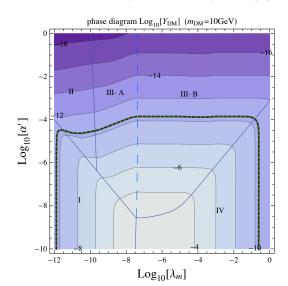
Regime II is new: reannihilation: the connector $SM SM \leftrightarrow e' \bar{e}'$ process is out of equilibrium



but the HS $e' \bar{e}' \leftrightarrow \gamma' \gamma'$ process is in equilibrium \Rightarrow hidden sector has its own temperature $T' \neq T$

freezeout in HS with $T' \neq T$ and with still at same time slow $SM \, SM \to e' \, \bar{e}' \, \, {\rm DM} \,$ production

NB: with (massive) Higgs portal: a fifth regime: Dark Freezeout:



freezeout in HS with $T' \neq T$ and at this time no more slow SM $SM \rightarrow e'$ \bar{e}' DM production more details in Chu,T.H.,Tytgat 11

Asymmetric DM

above we have assumed no DM matter-antimatter asymmetry but there could be one

2 asymmetric relics do exist already in Nature: protons and electrons!

The proton asymmetric relic example

at $T\sim m_p$: protons number density becomes to be Boltzmann suppressed from $p\bar p\leftrightarrow\gamma\gamma$ $p\bar p\leftrightarrow\pi\pi$

. .

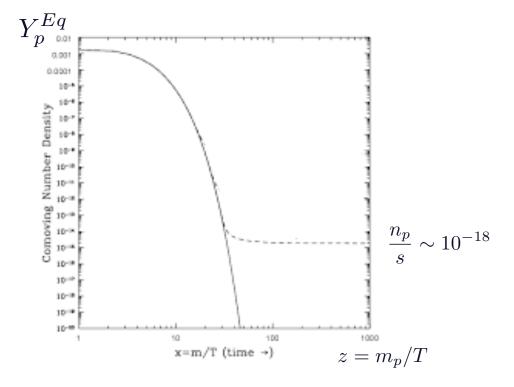
if the number of p was at this time the same than the number of \bar{p} one would have a symmetric freezeout just as DM above but driven by a strong process rather than a weak size one \Rightarrow very strong suppression:

$$\frac{n_p}{s} = \frac{n_{\bar{p}}}{s} \propto \frac{1}{\langle \sigma_{p\bar{p}\to\pi\pi} \cdot v_{rel} \rangle} \sim 10^{-18}$$

"annihilation catastrophy"

we would not be here to talk about!

but we know there are many more protons today: $\frac{n_p}{s}\sim 10^{-10}$ $\Omega_B\sim 5\%$ $\frac{n_{\bar p}}{s}\sim 0$

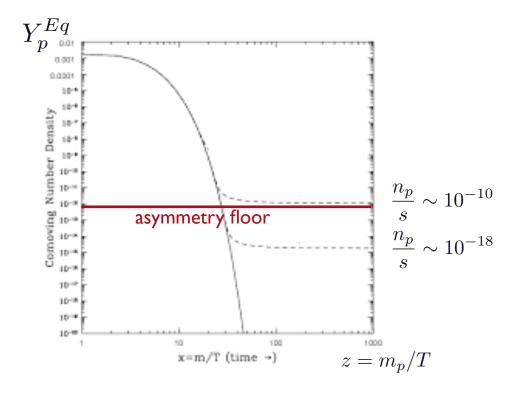


The proton asymmetric relic example

at $T\sim m_p$: protons number density becomes to be Boltzmann suppressed from $p\, \bar p o \pi\, \pi$

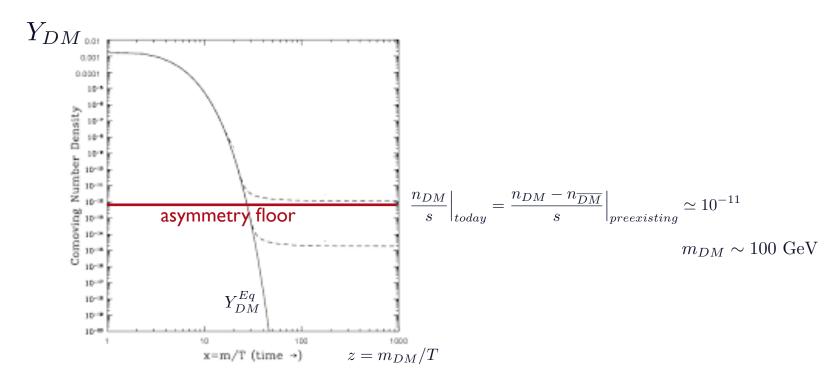
if instead at this time $n_p>n_{\bar p}$ then the efficient $p\,\bar p\to\pi\,\pi$ process cannot annihilate so many p because once it will have annihilated (almost) all $\bar p$ still we will be left with $n_p=n_{\bar p}=n_{\bar p}=n$

a
$$p$$
 population: $\frac{n_p}{s}\Big|_{today} = \frac{n_p - n_{\bar{p}}}{s}\Big|_{preexisting} \simeq 10^{-10}$



Asymmetric DM: same story as baryons

(in simplest version)



- we still need a DM annihilation to put DM in equilibrium and to Boltzmann suppressed it at $T \lesssim m_{DM}$ the annihilation cross section must be larger than for symmetric freezeout to get rid of the symmetric component
- we need a preexisting asymmetry:

$$\left. \frac{n_{DM} - n_{\overline{DM}}}{s} \right|_{preexisting} \simeq 10^{-11}$$

 $m_{DM} \sim 100 \text{ GeV}$

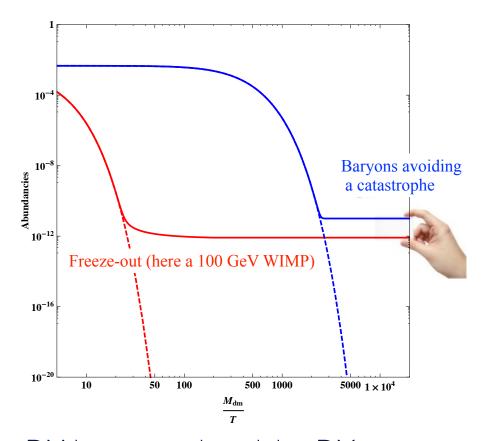
created before from other interactions

the relic density is not anymore determined by the physics at m_{DM} energy scale but by the physics at the origin of the asymmetry \implies becomes a UV problem!

we loose a lot in predictivity and testability % symmetric freezeout!

$\Omega_B \leftrightarrow \Omega_{DM}$ similarity \Rightarrow asymmetric DM?

No explanation for such a similarity along the symmetric thermal freezeout scenario:



Suggests that maybe DM is asymmetric and that DM asymmetry is related to the baryon asymmetry

$\Omega_B \leftrightarrow \Omega_{DM}$ similarity \Rightarrow asymmetric DM?

Common creation of both asymmetries from a same process: "Co-genesis"



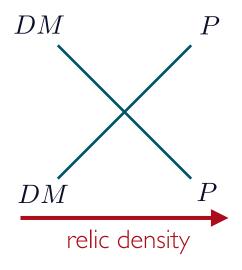
$$\Rightarrow \frac{n_{DM} - n_{\overline{DM}}}{s} = \frac{n_q - n_{\overline{q}}}{s} = \frac{1}{3} \frac{n_p - n_{\overline{p}}}{s} \Rightarrow m_{DM} \sim 3 \frac{\Omega_{DM}}{\Omega_B} \cdot m_p \sim 15 \,\text{GeV}$$

- \Rightarrow we trade the $\Omega_{DM}\leftrightarrow\Omega_{B}$ coincidence for a $m_{DM}\leftrightarrow m_{p}$ mass coincidence we should have seen already this DM particle in many cases
 - need to complicate the model, ... but remains a possibility

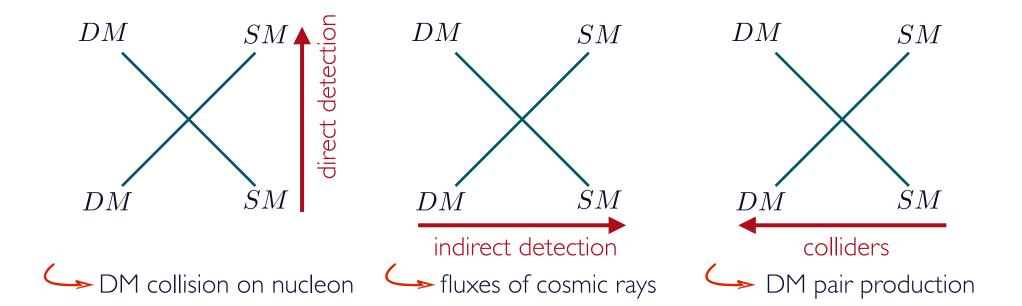
DM particle phenomenology

3 main ways to probe the DM particle

Symmetric freezeout as well as asymmetric DM scenarios require an annihilation process



If annihilation is into SM particles: 3 main ways to probe DM as a particle:



DM direct detection

Flux of DM particles crossing the earth

 $ho_{DM} \simeq 0.3\,\mathrm{GeV/cm^{-3}}$ \longleftarrow simulations of DM halo formation fitting the observations $v_{DM} \simeq 220\,\mathrm{km/sec}$ \longleftarrow orbit velocity of Sun in galaxy with distribution of velocity around: Maxwellian: $f(v_{DM}) = \frac{1}{\sqrt{2\pi\sigma}} e^{-v_{DM}^2/2\sigma^2}$ $\sigma \simeq 270\,\mathrm{km/sec}$

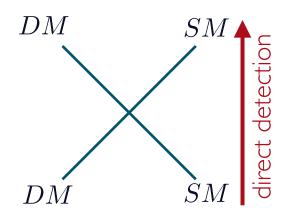
$$\implies$$
 DM particle flux: $\mathcal{O}(10^5\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1})\cdot\left(\frac{100\,\mathrm{GeV}}{m_{DM}}\right)$

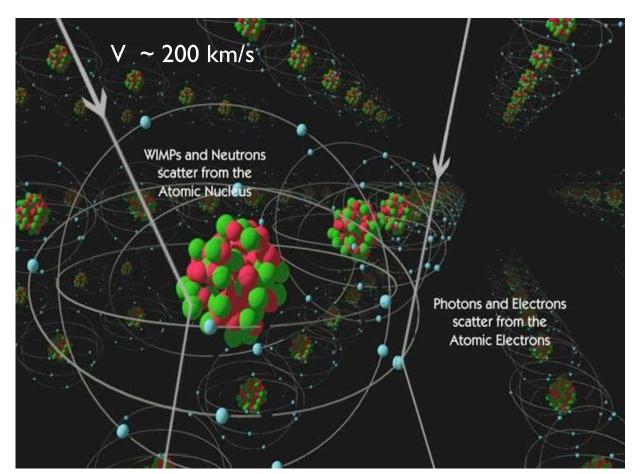
 10^5 more than μ flux

 10^5 less than total solar ν flux

100 less than $E_{\nu} > {
m MeV}$ solar flux

Search for a DM-nuclei or DM-electron scattering: direct detection





Search for a recoil of a nuclei or electron from DM hit: direct detection

Event rate (on nuclei):
$$\frac{dR}{dE_R} = \frac{\rho_{DM}}{m_{DM}} \frac{1}{m_N} \int_{v_{min}}^{\infty} v_{DM} f(v_{DM}) \frac{d\sigma}{dE_R} \, d^3v_{DM}$$
 nucleus recoil energy rate per mass of nuclei cross section
$$v \cos \theta = \sqrt{\frac{m_N E_R}{\mu^2}}$$

$$\mu = \frac{m_N m_{DM}}{m_N + m_{DM}}$$

for a given v_{DM} there is a kinematic upper bound: $E_R < \mu^2 v_{DM}^2/m_N \sim 0 - 100 \, \mathrm{keV}$

 \Rightarrow exponential fall-off of $f(v_{DM})$ for large v_{DM} gives an exponential fall-off for large E_R

$$\frac{dR}{dE_R} \sim \left(\frac{dR}{dE_R}\right)_{E_R=0} F^2(E_R) \, e^{-E_R/E_c}$$
 nuclear form factor

 \Rightarrow need for detectors with sensitivity to low $E_R \sim \text{few keV}$ and low noise

Search for a recoil of a nuclei or electron from DM hit: direct detection

 $\frac{d\sigma}{dE_{B}}$: depending on the DM candidate the cross section is:

spin-independent:

$$\sigma_{N-DM} \propto A^2$$



couples coherently to nucleons in nuclei



much better sensitivity, especially for large A (even if suppression of form factor for large E_R is larger for large A)



applies to scalar DM or fermion DM with vector coupling, ...

spin-dependent:

$$\sigma_{N-DM} \propto J(J+1)$$

need for nuclei with a spin, e.g. with odd number of nucleons coupling only to spin of the last nucleon

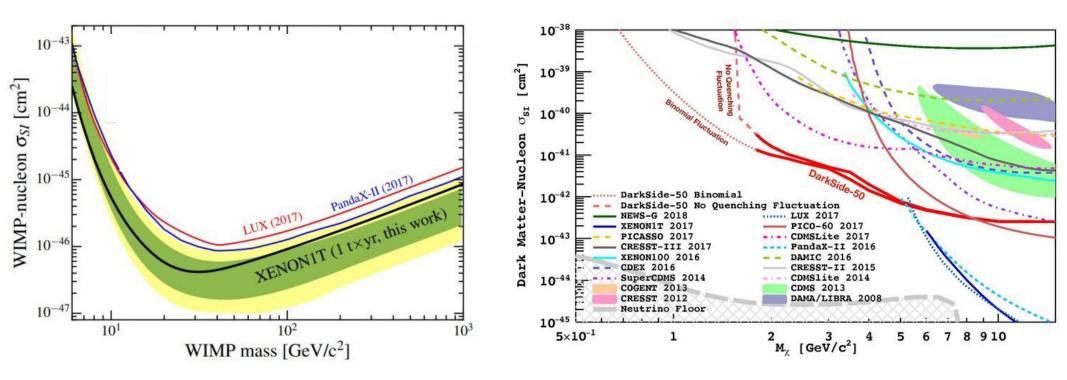


applies to fermion DM with axial vector coupling, ...

Spin independent direct detection

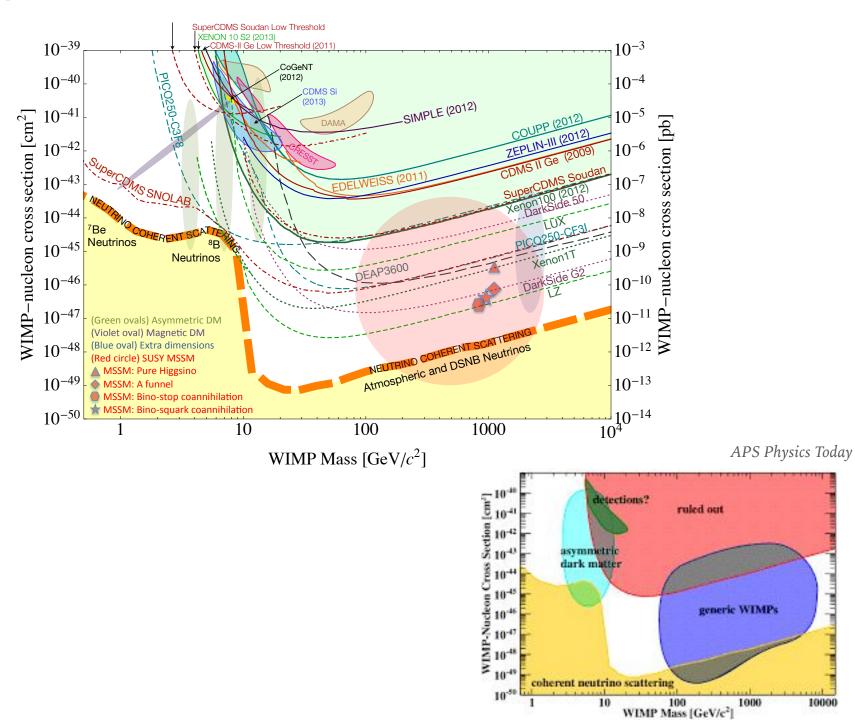
$$\frac{d\sigma}{dE_R} = \frac{1}{v^2} \frac{m_N \sigma_n^0}{2\mu^2} \frac{[Zf_p + (A-Z)f_n]^2}{f_n^2} F^2(E_R)$$
 DM-neutron cross section coupling of proton to the mediator nuclear form factor coupling of neutron to mediator

Xenon IT (2018): best limit for $m_{DM} > a \text{ few GeV}$

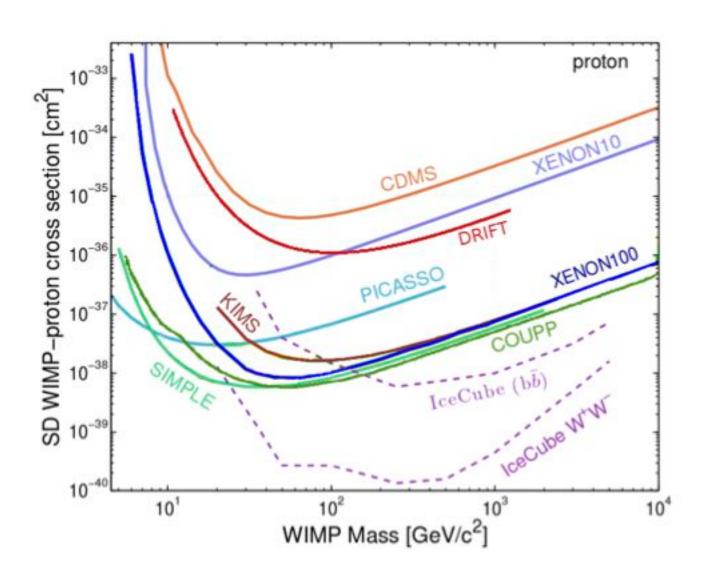


arXiv: 1802

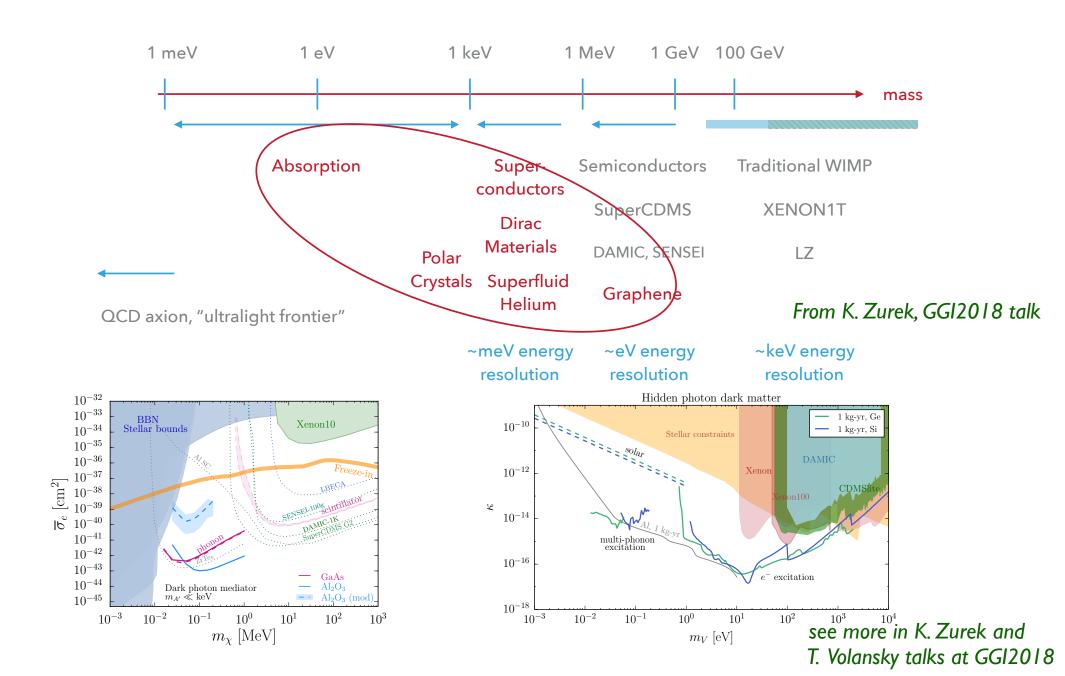
Spin independent direct detection: neutrino floor



Spin dependent direct detection

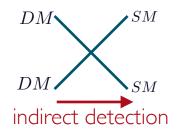


A new trend for the future: direct detection of sub-GeV DM



Part 4 DM indirect detection

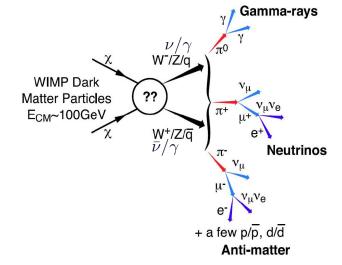
DM indirect detection: a huge field!





- 1) Gamma-rays:
 - radiation from charged particles produced by DM: diffuse flux
 - or created directly at loop level (DM is neutral): monochromatic flux
- 2) anti-protons
- 3) electron and positron
- 4) neutrinos
- 5) anti-nuclei
- 6) effect on synchrotron radiation flux
- 7) heat deposited by DM products on CMBR,





DM indirect detection: regions of production

annihilation: many more in dense DM region: galactic center and dwarf galaxies number of annihilation $\propto n_{DM}^2$

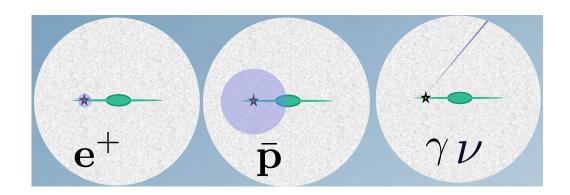
decay: also many from less dense DM region

number of decays $\propto n_{DM}$

 γ , ν : flux, energy spectra and direction basically unaffected during propagation \Rightarrow points to the source and the many ones produced in the galactic center reach us!

 e^{\pm} : very local: magnetic field + absorption

 $ar{p}$: less local but still the ones from galactic center do not reach us much



diffuse flux: astrophysical backgrounds!

 \Rightarrow

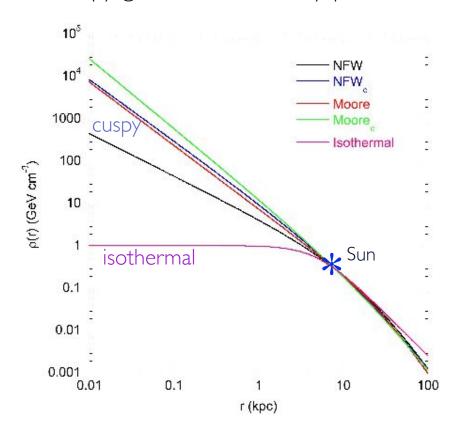
provides interesting upper limits but interpretation of excesses in general difficult

Pamela, Fermi, Integral, AMS,... excesses

monochromatic γ and ν : no astrophysical backgrounds + flux, energy spectra and and direction unaffected!

Uncertainty on the DM density profile towards the galactic center

Simulation of DM galactic halo formation predicts somewhat cuspy galactic DM density profile:



Observations give some indications for a somewhat more "cored" profile ("isothermal") profile but not precise at all so far

Calculation of the flux on earth: example of γ -rays

$$\frac{d\Phi_{\gamma}}{dE_{\gamma}} = \frac{1}{4\pi} \frac{dN_{\gamma}}{dE_{\gamma}} \frac{\sigma v}{m_{\chi}^{2}} \int_{V_{ann}} \frac{1}{r^{2}} \frac{\rho_{DM}^{2}}{2} d^{3}x \qquad \text{integration over galactic DM halo profile}$$
 flux on earth
$$\frac{d\Phi_{\gamma}}{dE_{\gamma}d\Omega} = r_{0}\rho_{s}^{2} \frac{dN_{\gamma}}{dE_{\gamma}} \frac{\sigma v}{8\pi m_{\chi}^{2}} J(\Theta)$$
 for detectors with good directional resolution
$$J(\Theta) = \int_{I=\pi} \frac{ds}{r_{0}} \left(\frac{\rho_{DM}(r(s,\Theta))}{\rho_{s}}\right)^{2} \qquad \text{J-factor}$$

$$\Rightarrow$$
 for monochromatic photons: $\frac{dN_{\gamma}}{dE_{\gamma}}=2\delta(E_{\gamma}-m_{DM})$

for \bar{p} and e^{\pm} : much more complicated: propagation effects

Part 5 Phenomenology of example scenarios and models (briefly)

3 different phenomenological approaches

Effective operators: most model independent approach

Explicit DM-SM mediator setups

Explicit DM models

Effective operators and explicit mediators

Effective operator approach

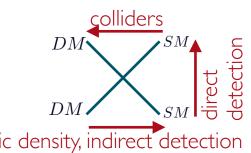
examples: vector and axial operators

$$\mathcal{O} = \frac{1}{\Lambda^2} \bar{\psi}_{DM} \gamma_{\mu} \psi_{DM} \, \bar{q} \, \gamma^{\mu} q$$

10

WIMP mass m, [GeV]

spin-independent direct detect.

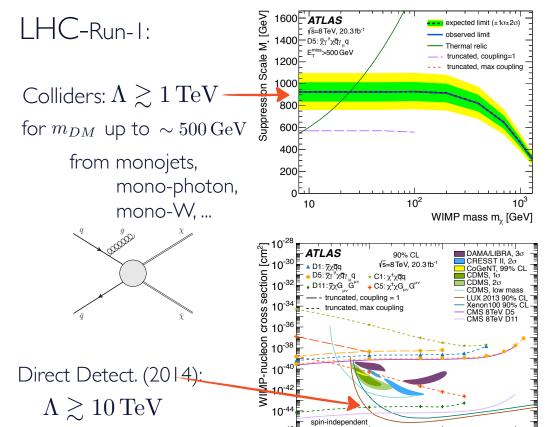


relic density, indirect detection

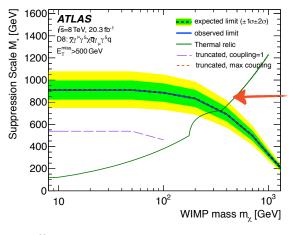
induced by heavier particles whose mass and coupling are not specified

$$\mathcal{O} = \frac{1}{\Lambda^2} \bar{\psi}_{DM} \gamma_{\mu} \gamma_5 \psi_{DM} \, \bar{q} \, \gamma^{\mu} \gamma_5 q$$

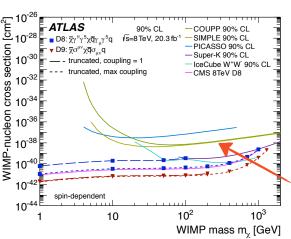
spin-dependent direct detect.



for $10 \, {\rm GeV} \gtrsim m_{DM} \gtrsim 1 \, {\rm TeV}$



Colliders: $\Lambda \gtrsim 1\,\mathrm{TeV}$ for m_{DM} up to $\sim 500 \, \mathrm{GeV}$



Direct Detect. (2014): $\Lambda \gtrsim 600 \, \mathrm{GeV}$ $10 \, {\rm GeV} \gtrsim m_{DM} \gtrsim 1 \, {\rm TeV}$

Effective operator approach

examples: vector and axial operators

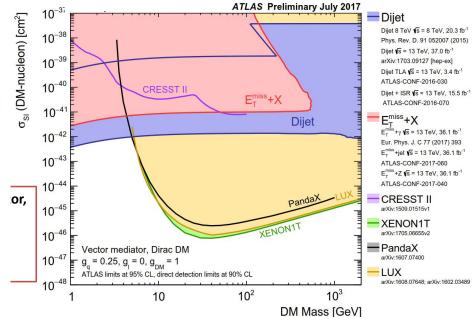
$$\mathcal{O} = \frac{1}{\Lambda^2} \bar{\psi}_{DM} \gamma_{\mu} \psi_{DM} \, \bar{q} \, \gamma^{\mu} q$$

spin-independent direct detect.

 $\mathcal{O} = \frac{1}{\Lambda^2} \bar{\psi}_{DM} \gamma_{\mu} \gamma_5 \psi_{DM} \, \bar{q} \, \gamma^{\mu} \gamma_5 q$

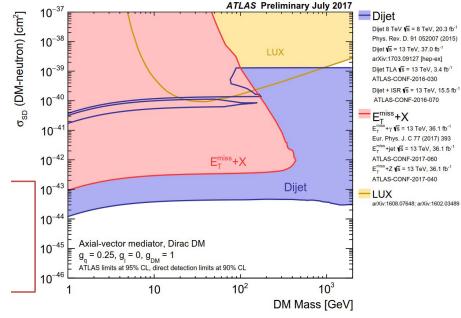
spin-dependent direct detect.

LHC-Run-II:



LHC: $\Lambda \gtrsim 3 \text{ TeV}$

Xenon1T: $\Lambda \gtrsim 25 \text{ TeV}$



LHC: $\Lambda \gtrsim 7 \text{ TeV}$

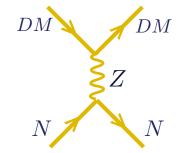
NB: for operators with 2 DM and 2 leptons colliders very competitive % direct detection

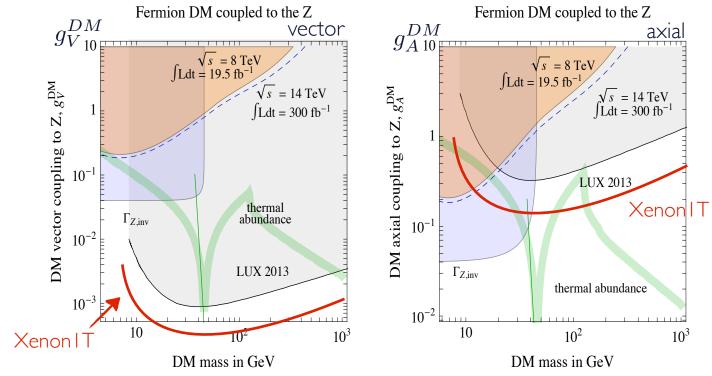
Explicit mediator approach: Z mediator for fermion DM

e.g. assuming DM/SM specific mediator with given coupling and masses:

Z mediator: fermion DM: vector and axial DM coupling to the Z

$$\mathcal{L} \ni -Z_{\mu} \frac{g}{\cos \theta_W} \bar{\psi}_{DM} (g_V^{DM} + g_A^{DM} \gamma_5) \gamma^{\mu} \psi_{DM}$$





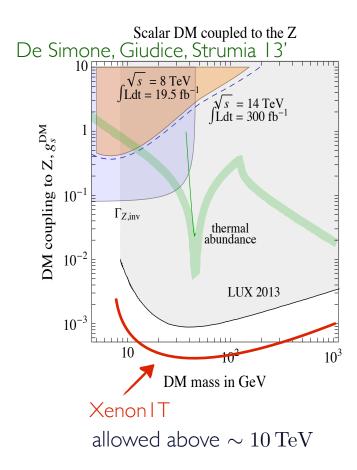
DM candidates which have hypercharge of $\mathcal{O}(1)$: totally excluded by direct detection except for special cases such as fermion axial coupling case above 10 TeV

DM candidates with vanishing hypercharge but still small coupling to Z through mixing: coupling to Z must be small and relic density allows only candidates above $\sim 2~{\rm TeV}~$ and $\sim 150~{\rm GeV}$ respectiv.

Explicit mediator approach: Z mediator for scalar DM

$$\mathcal{L} \ni -Z_{\mu} \frac{g}{\cos \theta_{W}} g_{\phi} \left[\phi_{DM}^{*} \partial^{\mu} \phi_{DM} - \partial^{\mu} \phi_{DM}^{*} \phi_{DM} \right]$$

similar to fermion DM vector case



similar to fermion DM with vector coupling \Rightarrow totally excluded for "standard" Z couplings



except for a specific case: inert doublet (see below)

Explicit mediator approach: Higgs boson mediator: scalar DM

• Scalar DM: Higgs portal interaction: $\mathcal{L} \ni \lambda_{DM} H^{\dagger} H \phi_{DM}^* \phi_{DM}$ "Higgs portal" Scalar DM coupled to the Higgs λ_{DM} $\sqrt{s} = 8 \text{ TeV}$ DM coupling to Higgs, λ_{DM} $\sqrt{s} = 14 \text{ TeV}$ $\int Ldt = 300 \text{ fb}^{-1}$ LUX 2013 abundance $\mathsf{Xenon}\mathsf{IT}$ De Simone, Giudice, Strumia 13' 10^{-2} 10 10^{2} m_{DM}^{N} (GeV) DM mass in GeV

(except around h resonance)

N.B.: Xenon IT probes it up to $\sim 1\,\mathrm{TeV}$ for $\lambda_{DM}\sim 1$

excluded below $\sim 600~{\rm GeV}$

Explicit mediator approach: Higgs boson mediator: fermion DM

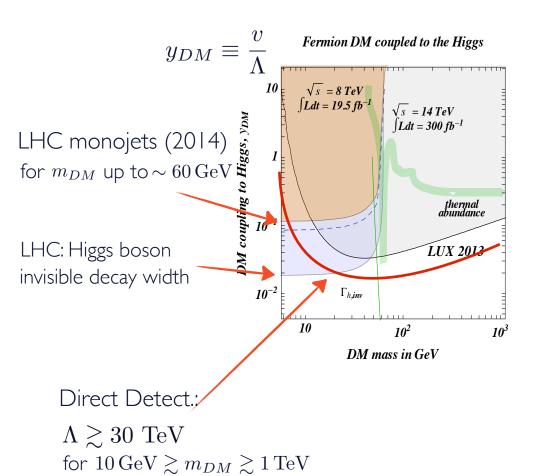
• Fermion DM: lowest gauge invariant interaction: dim-5

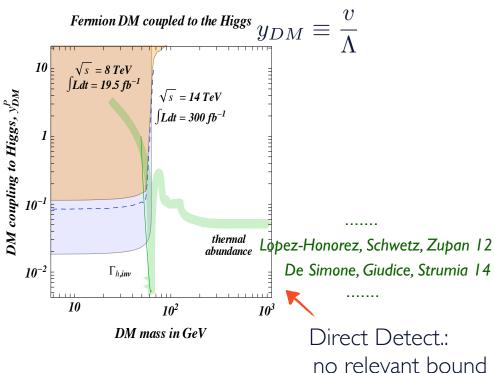
$$\mathcal{O} = \frac{1}{\Lambda} H^{\dagger} H \; \bar{\psi}_{DM} \psi_{DM}$$

spin-independent direct detect.

$$\mathcal{O} = \frac{1}{\Lambda} H^{\dagger} H \, \bar{\psi}_{DM} i \gamma_5 \psi_{DM}$$

spin-dependent direct detect.





Explicit models (briefly)

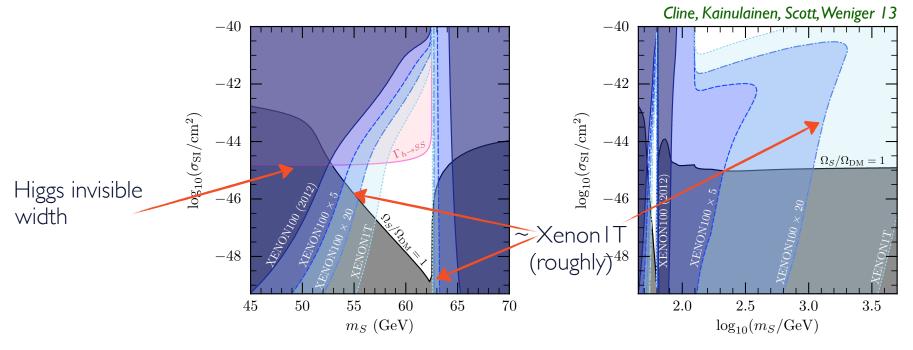
The simplest example: real scalar singlet DM

 \longrightarrow a real singlet S odd under Z_2 parity: $S \to -S$

$$\mathcal{L} \ni -\frac{1}{2}\mu_S^2 S^2 - \frac{1}{24}\lambda_S S^4 - \frac{1}{2}\lambda_{hs} H^{\dagger} H S^2$$

$$m_S^2 = \mu_S^2 + \frac{1}{2}\lambda_{hs}v^2$$

For m_S fixed, λ_{hs} can be fixed by $\Omega_{DM} \simeq 26\%$ constraint



Xenon IT direct detection requires: $m_{DM} \gtrsim 800 \; \mathrm{GeV}$

Future: CTA should probe m_{DM} up to $5\,\mathrm{TeV}$

GAMBIT collaboration 18

or 55 GeV $\lesssim m_{DM} \lesssim 63$ GeV

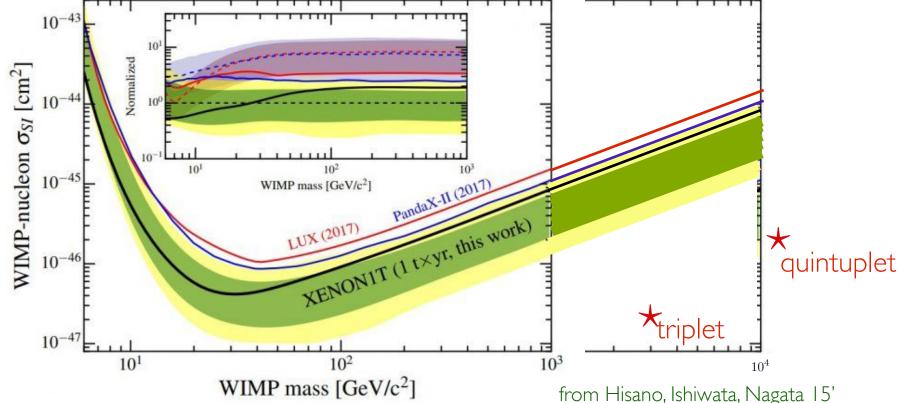
Dwarf galaxies γ -ray flux requires: $m_{DM} \gtrsim 50 \, \mathrm{GeV}$

⇒ shows how a model is can be already largely constrained when it depends on only very few parameters

$SU(2)_L$ multiplet DM: should we have seen already it in direct detection experiments if thermal?

other examples of models with very few parameters

let's take the examples above with Y=0 to avoid ruled-out Z exchange e.g. a Y=0 fermion triplet, quintuplet, ... `minimal dark matter'' have only gauge interactions with SM fields: too high for LHC relic density totally fixed by value of $m_{DM} \Rightarrow m_{DM} \simeq 3.0 \; {\rm TeV}$ $m_{DM} \simeq 11 \; {\rm TeV}$



multi-TeV domain still very open for direct detection

Mitridate, Redi, Smirnov, Strumia 17'

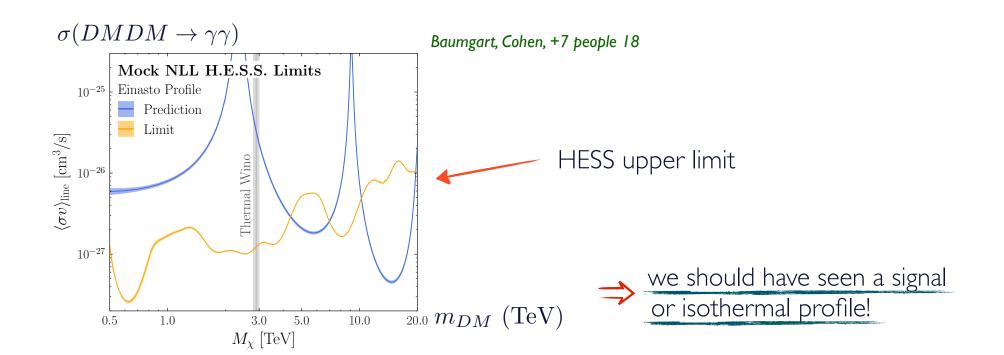
$SU(2)_L$ multiplet DM: should we have seen already it in indirect detection experiments if thermal?

 \longrightarrow let's consider the $SU(2)_L$ triplet DM example again: "wino" $m_{DM} \simeq 3.0~{
m TeV}$

have only gauge interactions with SM fields: indirect detection fully predicted

Hisano et al. 03-09

Indirect detection very efficient here!! \longrightarrow production of γ -line is Sommerfeld enhanced



$SU(2)_L$ multiplet DM: more freedom for scalar multiplet: inert scalar doublet example Deshpande, Ma 78, Barbieri, I

Deshpande, Ma 78, Barbieri, Hall, Ryshkov 06, Lopez-Honorez, Nezri, Oliver, Tytgat 07, TH, Lin, Lopez-Honorez, Rocher 08

a scalar doublet H_2 odd under a Z_2 symmetry: $H_2 o -H_2$

$$H_2 = \begin{pmatrix} H^+ \\ \frac{H_0 + iA_0}{\sqrt{2}} \end{pmatrix} \qquad \longleftarrow Y = 1 \neq 0$$

not only gauge interactions: additional scalar interactions

$$V = \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4$$
$$+ \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1^{\dagger} H_2|^2 + \frac{\lambda_5}{2} \left[(H_1^{\dagger} H_2)^2 + h.c. \right]$$

additional annihilation channels mediated by Higgs boson, etc

moreover H_0 and A_0 do not have the same mass

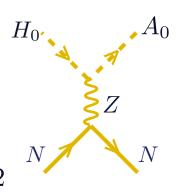
$$\implies m_{H_0}^2 - m_{A_0}^2 = \lambda_5 v^2 \implies$$
 the lightest neutral component is the DM

if splitting larger than $\sim 100~{
m keV}$: no direct detection through Z exchange even if Y
eq 0!

 \longrightarrow DM is highly non-relativistic today: $E_{kin} \lesssim 100 \text{ keV}$

direct detection signal expected well below present Xenon IT sensitivity! but relevant indirect detection and collider constraints

still quite open for $m_{DM} \gtrsim 520 \, {
m GeV}$ and possibilities for $m_{DM} \sim m_h/2$



the lightest neutralino is in general a mixture of Bino, Higgsinos and Wino

if it is the lightest Susy particle (LSP): DM candidate

the direct constraints on the neutralino are mild: m_χ as light as ~few tens of GeV still allowed the partners and interactions entering in its annihilation are nevertheless constrained

• a pure bino neutralino:

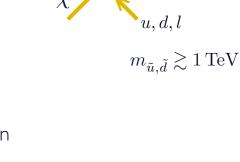
> annihilation through squarks and leptons:

but $\Omega_{DM}=26\%$ requires squarks and sleptons lighter than allowed experimentally

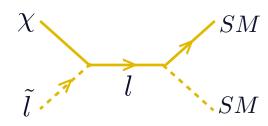
$$m_{\tilde{l}} \lesssim 100 \text{ GeV}$$

need for other channels having larger annihilation cross section: co-annihilation channels or channels close to a resonance for instance if slepton not more than ~10% heavier than Bino it is still around in

thermal bath when Bino is about to decouple \Rightarrow co-annihilation dominates the Bino decoupling



 $\tilde{u}, \tilde{d}, \tilde{l}$



The MSSM neutralino

a pure Wino neutralino:

annihilation through gauge interactions: $m_{DM} \simeq 3.0 \; {
m TeV}$

was not much considered as attractive because sets the Susy scale quite high

 \Rightarrow excluded by γ -line search or isothermal profile!

• a pure Higssino neutralino:

annihilation through gauge interactions are too fast unless it is heavy (as Wino) $m_{DM} \simeq 1 \text{ TeV}$

can escape Z exchange direct detection constraint despite it has $Y \neq 0$ because the Z couples to 2 different neutral Higgsino component which can have mass splitting forbidding kinematically the Z exchange

as "inert scalar doublet DM" above

was not much considered as attractive because sets the Susy scale quite high and not obtained as LSP in many Susy breaking framework

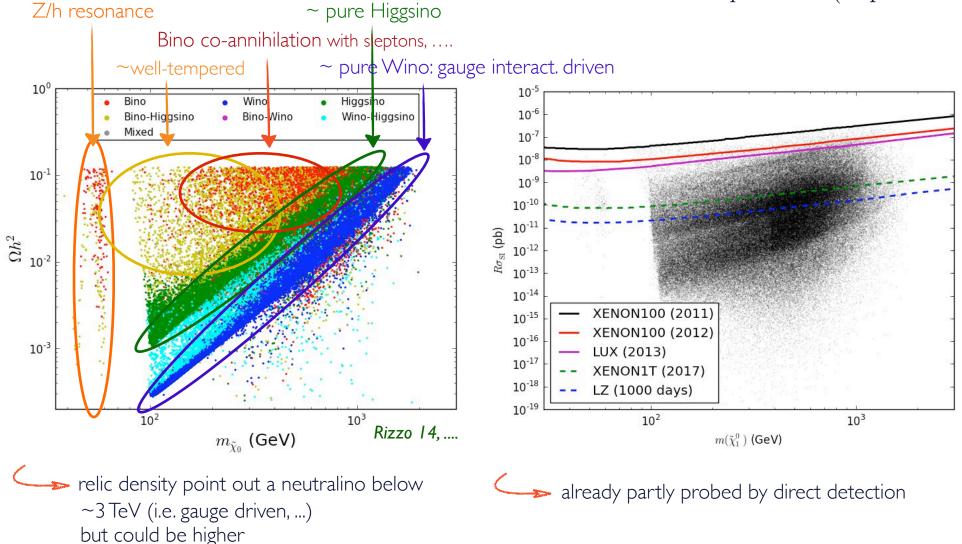
Higgs boson mass measured at LHC requires typically a large stop mass which indirectly typically requires a large Higgsino mass which fits with the mass a Higgsino must have if DM

a mixed neutralino:

offers more possibility playing around (as "well-tempered neutralino")

The MSSM neutralino

pMSSM (19 parameters)



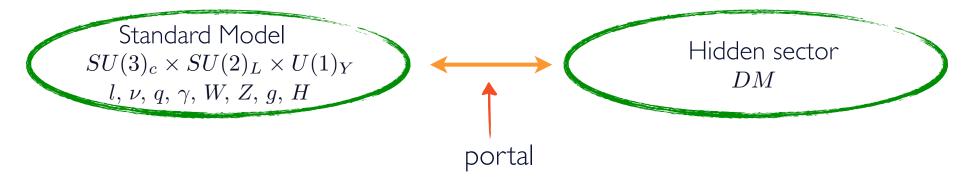
still many possibilities to get the relic density in itself

but much more difficult if one adds some naturalness considerations into the game,

Low scale susy maybe does not exist but DM does!!!

 $m_{\tilde{g}} \gtrsim 1 \, \text{TeV} \quad m_{\tilde{u},\tilde{d}} \gtrsim 1 \, \text{TeV}$

Hidden sector DM



Testability all depends on size and mass of portal and on whether DM communicates directly to visible sector through portal

example: Higgs portal: $\mathcal{L} \ni -\lambda_m \phi_{DM}^\dagger \phi_{DM} H^\dagger H$ or $\mathcal{L} \ni -\lambda_m \phi^\dagger \phi H^\dagger H$



in both cases invisible Higgs decay width constraints if HS particles light enough

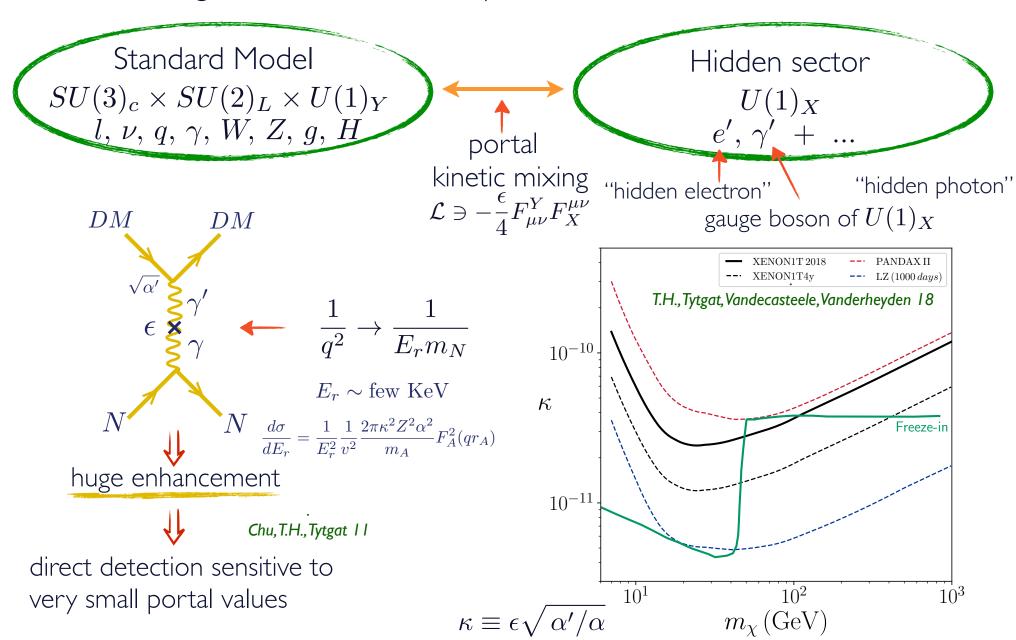
$$Br(h \to \phi_{DM}\phi_{DM}, \phi\phi) \le 20\%$$

for massive connector the upper bounds on connector coupling are typically of order $\sim 1-10^{-2}$

however for light connector the bounds can be much more stringent!

Hidden sector DM: direct detection is already testing the freezein regime for light mediator

Let's consider again the hidden electron/photon QED' model above:



Phenomenology trends....

Various thermal models become at last to be really tested experimentally or even excluded:

- all models which allow a kinematically allowed Z exchange with "standard" Z coupling are excluded by direct detection expts (except pure axial case, ...),
- h exchange begins to be seriously probed by direct detection experiments
- fermion thermal candidates which have only gauge interactions: triplet (Wino), quintuplet pure electroweak multi-TeV models: not excluded by LHC because true WMPS are often out of energy reach for LHC but excluded by indirect detection: \(^1\)-lines (except for isothermal DM halo profile)

unlike scalar multiplets: more freedom due to possible scalar quartic couplings, mass splittings,...

- some models with very few parameters: example: real scalar singlet (still allowed around h resonance and for high mass)
- very global models with many constraints on partner particles entering the DM annihilation, direct detection, ...: example: MSSM: allows still a lot of possbilities from DM point of view, especially beyond TeV but much more squeezed if one adds external considerations (hierarchy problem,....)

Phenomenology trends....

as soon as we go away from some of these global models (not much favored anymore by LHC data), ..., and away from some of the very minimal visible models, thermal (and beyond) DM candidates are still largely allowed:

- many visible sector DM models
- hidden sector models: new trend! <- DM could be the tip of an all hidden sector world!

with clear possibilities of future signatures:

- new generation of direct detection experiments: at high mass
 at low masses (new!)
 with even possibilities to test the freezein scenario
- new generation of indirect detection (CTA, ...), especially for still quite open multi-TeV range, including from still unexplored high energy neutrinos,

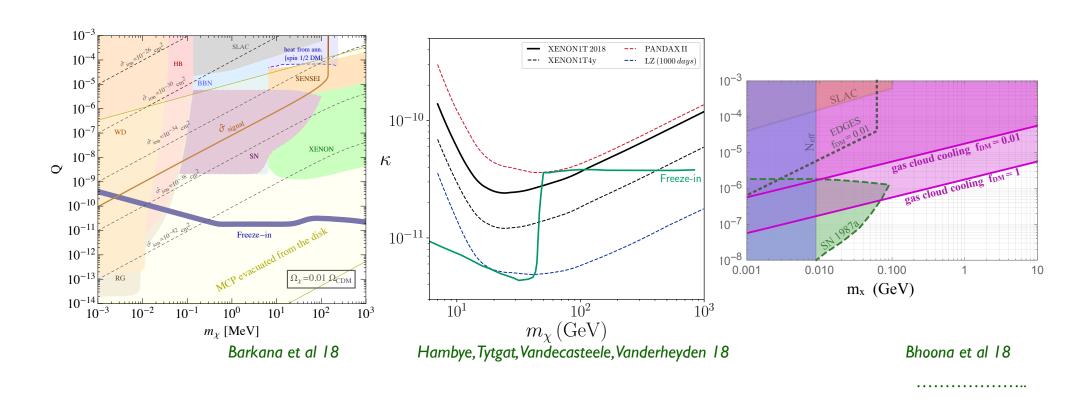
Thank you

Many thanks to All the Organizers for this very nice and interesting school!!!

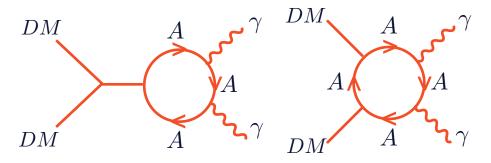
Dark Matter must be dark

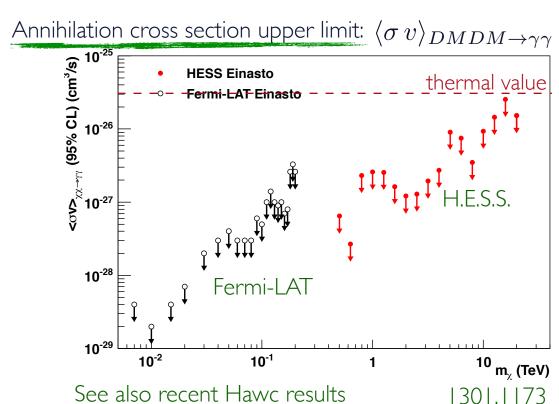
A non electrically neutral DM particle would « shine » unless:

- it forms neutral bound states, but basically excluded (ionized population, annihilation into 2 photons, ...)
- its electric charge is tiny: strong constraints but not excluded:



Search for γ -lines: DM smoking gun

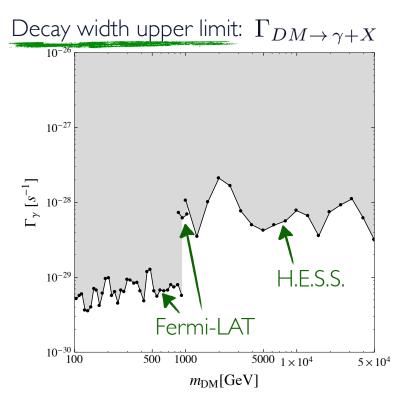




Sensitivity to cross sections better than thermal value!

 of order what could be expected given the loop suppression

 Sensitivity to γ-line cross sections 2-3 orders of magnitude better than to diffuse γ cross sections

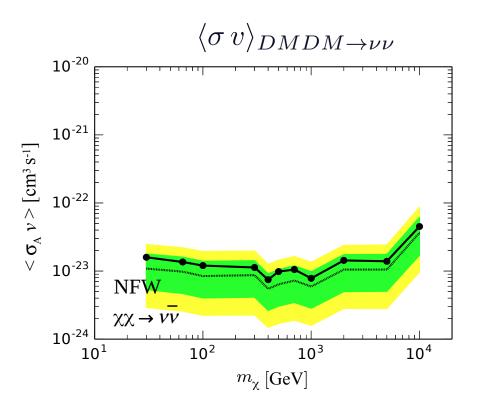


Sensitivity to $10^{(28-29)}$ sec lifetimes!

Search for V-lines: the other DM smoking gun

from DM annihilation or decay

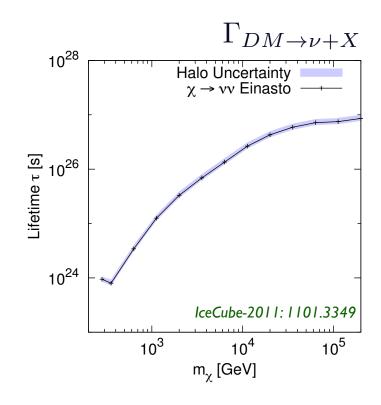
Annihilation cross section upper limit:



IceCube 1606.00209 (2011-2012)

Still far from thermal value but large improvements to be expected

Decay width upper limit:



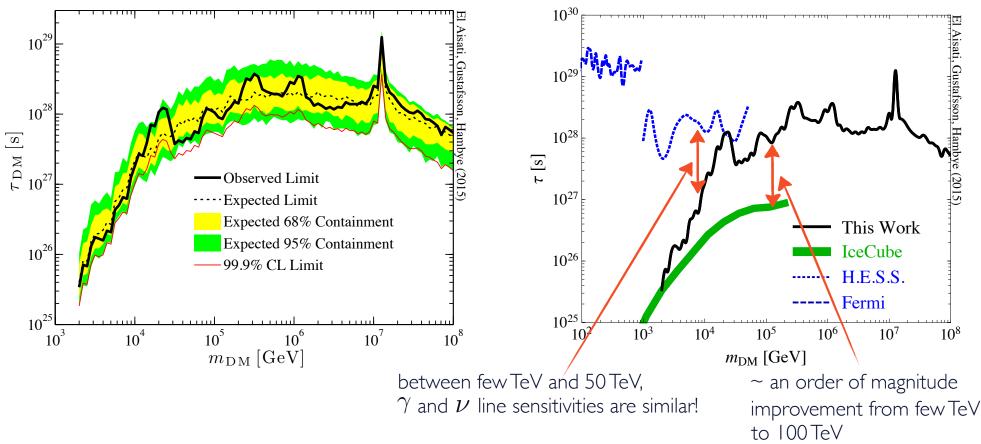
Above 100 TeV there are other limits: Rott, Kohri, Park , 14' Esmaili, Kang, Serpico 14'

Search for V-lines: the other DM smoking gun

from DM annihilation or decay

using a 2010-2012 public IceCube data sample: for DM decay: $\Gamma_{DM o
u+X}$

dedicated line search using Fermi-LAT statistical techniques, El Aisati, Gustafsson, TH 15'



> neutrino are coming into the game and also for an annihilation....

monochromatic neutrino production not loop suppressed, Sommerfeld boost, ...

Non-relativistic DM thermal decoupling: "cold relic"

$$T_{dec}$$
 is given by: $\left. \frac{\Gamma}{H} \right|_{T_{dec}} = \frac{n_{DM}^{Eq} \langle \sigma_{DM\,DM o SM\,SM\,V_{rel}}
angle}{H} \right|_{T_{dec}} = 1$

$$n_{DM}^{Eq} = g_{DM} \left(\frac{m_{DM} T}{2\pi}\right)^{3/2} e^{-m_{DM}/T}$$

 $g_* = \text{number of relativistic degrees of freedom in thermal bath}$

$$H = \sqrt{\frac{8\pi G\rho}{3}} \sim 1.7 \sqrt{g_*} \frac{T^2}{m_{Planck}}$$

$$\Rightarrow z_{dec} \equiv \frac{m_{DM}}{T_{dec}} = \ln[0.038 \frac{g_{DM}}{\sqrt{g_*}} m_{DM} m_{Planck} \langle \sigma_{DMDM \to SMSM} v_{rel} \rangle]$$

$$Y_{DM}|_{today} = \frac{n_{DM}}{s} \Big|_{today} = \frac{n_{DM}}{s} \Big|_{T_{dec}} = \frac{n_{DM}^{Eq}}{s} \Big|_{T_{dec}} = \frac{H(T_{dec})}{\langle \sigma_{DMDM \to SMSM} \, v_{rel} \rangle} \frac{1}{s(T_{dec})}$$

$$= \frac{1.7\pi^2}{4} \frac{\sqrt{g_*}}{g_{DM}} \frac{z_{dec}}{m_{Planck} m_{DM}} \frac{1}{\langle \sigma_{DMDM \to SMSM} \, v_{rel} \rangle}$$

$$= \cot \frac{1}{m_{DM}} \frac{z_{dec}}{\langle \sigma_{DMDM \to SMSM} \, v_{rel} \rangle}$$

$$\Rightarrow \Omega_{DM} = \frac{(n_{DM})_{today} \, m_{DM}}{\rho_{crit}} = \frac{(Y_{DM})_{today} \, s_{today} \, m_{DM}}{\rho_{crit}} = \text{const'} \frac{z_{dec}}{\langle \sigma_{DMDM \to SMSM} \, v_{rel} \rangle} \simeq \text{const''} \frac{1}{\langle \sigma_{DMDM \to SMSM} \, v_{rel} \rangle}$$