Remote synchronization: detailed account of a peculiar pattern-formation mechanism

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Context

A “rewarding” experiment about relationship(s) between structural connectivity and synchronization in an electronic network
What is remote synchronization?

Synchronised

Non-synchronised

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Remote synchronization from mismatches

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Remote synchronization in star networks

A. Bergner,1,3 M. Frasca,2 G. Sciuto,2 A. Buscarino,2 E. J. Ngamga,3 L. Fortuna,2 and J. Kurths3,4,5
Remote synchronization as morphogenesis

Image credit: Wikipedia, others
Remote synchronization in brain networks?

Sensory-motor network: directly wired

Default-mode network: emergent

No direct anatomical link to posterior areas. Remotely synchronized?

Image source: Rosazza & Minati, 2011; © Springer Verlag
A simple, reconfigurable non-linear network
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Oscillator equations

\[
\begin{align*}
\frac{dv_1}{dt} &= \Gamma \left( 2\pi F_1(G_4v_4 + G_5v_5 - v_1), v_1 \right) \\
\frac{dv_2}{dt} &= \Gamma \left( 2\pi F_2(G_1v_6 - v_2), v_2 \right) \\
\frac{dv_3}{dt} &= \Gamma \left( K_1v_6, v_3 \right) \\
\frac{dv_4}{dt} &= \Gamma \left( 2\pi F_3(G_2v_2 + G_3v_3 - v_4), v_4 \right) \\
\frac{dv_5}{dt} &= \Gamma \left( K_2v_2, v_5 \right) \\
\frac{dv_6}{dt} &= \Gamma \left( 2\pi F_1(G_6v_1 + G_i v_i + G_e v_e - v_6), v_6 \right)
\end{align*}
\]

\[
\Gamma(x, y) = R(x) H(V_s - y) - R(-x) H(V_s + y)
\]
Applications in versatile pattern generation

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Image source: Minati et al., IEEE Access 2018. © IEEE
Phase vs. amplitude synchronization

Phase coherence

\[ r_{ij} = \left| \langle e^{i\varphi_i(t)} - e^{i\varphi_j(t)} \rangle \right| \]

Instantaneous amplitude (envelope)

\[ v_i(t) + i\hat{v}_i(t) = A_i(t)e^{i\varphi_i(t)} \]

where \( \hat{v}_i \) is the Hilbert transform of \( v_i(t) \)

\[ \hat{v}_i(t) = \frac{1}{\pi} \text{p.v.} \left[ \int_{-\infty}^{\infty} \frac{v_i(\tau)}{t-\tau} d\tau \right] \]

and where p.v. denotes the Cauchy principal value of the integral\(^{18}\).

Maximum cross-correlation or mutual information

\[ C_{XY}(\tau) = \frac{k_{XY}(\tau)}{\sqrt{\sigma_X^2 \sigma_Y^2}} \]

\[ N_{XY}(d) = \frac{I_{XY}(d)}{\sqrt{H_X H_Y}} \]
Numerical simulations reveal three regimes

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Image source: Minati et al., CHAOS 2015. © AIP
Numerical simulations reveal three regimes:

- **a:** \( G_6 = 0.196, \ G_7 = -1.365 \)
- **b:** \( G_6 = 0.096, \ G_7 = -1.53 \)

Broadband chaos

Quasi-periodicity

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Numerical simulations reveal three regimes

- **a:** $G_6 = 0.196$, $G_7 = -1.365$
- **b:** $G_6 = 0.096$, $G_7 = -1.53$
- **c:** $G_6 = 0.188$, $G_7 = -1.14$

**Broadband chaos**

**Quasi-periodicity**

**Narrowband chaos**

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Numerical simulations reveal three regimes
Numerical simulations reveal three regimes

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Image source: Minati et al., CHAOS 2018. © AIP
Effect of parametric mismatches

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Experimental implementation
Experimental implementation

Upon permission and courtesy of Anadigm, Inc.
Experimental implementation

The Configurable Analog Module (CAM)
Experimental implementation

- **GainHalf**
  - Half-cycle

- **GainHold**
  - Inverting only

- **GainInv**
  - Continuous Time

- **SumInv**
  - Up to three inputs

- **SumDiff (SumHalf)**
  - Up to four inputs
  - Add or subtract since input branches can be inverting or non-inverting

- **RectifierFilter**
  - Full Wave/Half Wave
  - Inverting/non-inverting

- **RectifierHalf**
  - Full Wave/Half Wave
  - Inverting/non-inverting

- **RectifierHold**
  - Half Wave Inverting only

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Experimental implementation

- **FilterBilinear – One pole**
  - Low Pass/High Pass/All Pass

- **FilterBiquad – Two poles**
  - Low Pass/High Pass/Band Pass/Band Stop
  - Automatically chooses from multiple circuit topologies

- **Differentiator**
  - Output voltage slews – see documentation

- **Integrator**
  - Optional reset

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Experimental implementation

- **Multiplier**
  - Uses SAR (Input Y is quantized)
  - Subject to internal reference voltage error
  - Optional sample and hold on input X to equalize sampling time of two inputs (uses chip resources)

- **Comparator**
  - Single/Dual Input
  - Variable Reference

- **Hold – Sample and hold**

- **OscillatorSine**
  - Subject to internal reference voltage error

- **Voltage (+/- 3 VDC)**
  - Subject to internal reference voltage error

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Experimental implementation

Continuous-value, discrete-time

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Experimental implementation

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From: A. Buscarino et al., A Concise Guide to Chaotic Electronic Circuits, 73, SpringerBriefs in Applied Sciences and Technology
Experimental implementation

A soft of “Chimera”: an analog plug-in system for digital computer

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Remote sync. close to quasi-periodicity

Correlation dimension ($D_2$)

Remote synchronization ($\eta$)

$$\eta[r_{nm}] = \frac{\sum_{n=1}^{32} \sum_{m=1}^{32} \Theta[H(r-r')]_{nm}}{\sum_{n=1}^{32} \sum_{m=1}^{32} H(r_{nm}-r')}$$

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Experimental data - basics

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Spectrogram

Phase sync.

Amplitude sync.

Reminiscent of a diffraction pattern?

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Non-stationarity

Adjacent time-windows…
Non-stationarity

...reminiscent of observations in resting-state functional MRI

Experimental data – focus on spectrum

Two concomitant spectral relations:

1) \( f_B = f_H - f_C = f_C - f_L \)
2) \( f_H = f_L + f_C \) \( \rightarrow f_L = f_B \)

Reminiscent of classic AM modulation!

Lower sideband and baseband overlap!
From synchronization to causality

- Regression of the present of the target on its own past:
  \[ e_{j|j,n} = y_{j,n} - \mathbb{E}[y_{j,n} | y_{j,n}^-] \quad \rightarrow \quad \lambda_{j|j} = \mathbb{E}[e_{j|j,n}^2] \]

- Regression of the present of the target on its past and the past of the source:
  \[ e_{j|ji,n} = y_{j,n} - \mathbb{E}[y_{j,n} | y_{j,n}^-, y_{i,n}^-] \quad \rightarrow \quad \lambda_{j|ji} = \mathbb{E}[e_{j|ji,n}^2] \]

Granger causality (GC)

\[ F_{i \rightarrow j} = \ln \frac{\lambda_{j|j}}{\lambda_{j|ji}} \]

Transfer Entropy (TE)

\[ T_{i \rightarrow j} = \frac{1}{2} \ln \frac{\lambda_{j|j}}{\lambda_{j|ji}} \]


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Courtesy: Luca Faes
Mutual information and causality

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Effect of “lesioning” by noise injection

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Ring size and auxiliary system simulations

![Graphs showing synchronization metrics for different ring sizes and auxiliary systems.](image source: Minati et al., CHAOS 2018. © AIP)
Propagation of external perturbations
Simplified chain model

1) An open network is considered in the form of a chain.
2) Two dynamical equations are removed.
3) \( \Gamma(x,y) \) is removed for all voltages except \( v_3 \).
4) The parameters are set identically across all nodes.

\[
\begin{align*}
\frac{dv_1}{dt} &= 2\pi F(G_4 v_4 - v_1) \\
\frac{dv_2}{dt} &= 2\pi F(G_1 v_o - v_2) \\
\frac{dv_3}{dt} &= \Gamma(Kv_o, v_3) \\
\frac{dv_4}{dt} &= 2\pi F(G_2 v_2 + G_3 v_3 - v_4)
\end{align*}
\]
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Simplified chain model

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Demodulation and interference
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Demodulation and interference

![Graphs showing demodulation and interference patterns.](Image source: Minati et al., CHAOS 2018. © AIP)
Instancing filters at specific points of chain

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Revised Granger model: baseband + sideband

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Image source: Minati et al., CHAOS 2018. © AIP
Small-world features, nonetheless...

\[ S^{WS} = \gamma_g^{WS} / \lambda_g = \left( C_g^{WS} L_{rand} \right) / \left( C_{rand}^{WS} L_g \right) \]
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Small-world features, nonetheless...

Small-worldness in the brain (and not only) is an efficient trade-off!
Conclusions

1) A complex mechanism of pattern generation was demonstrated.

2) Is this just “apparent” remoteness? Central importance of measure choice…

3) To what systems may such mechanism apply? Broadband vs. narrowband chaos, spectral relationships
Thank you for your attention

References:

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