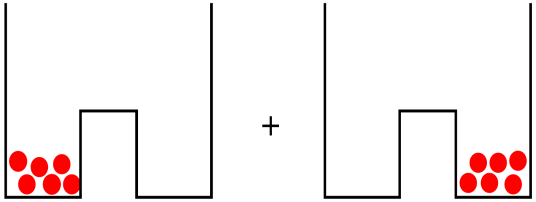
INSTITUT für PHYSIK
UNIVERSITÄT AUGSBURG

Scaling behavior of dynamical entanglement and localization

Klaus Ziegler

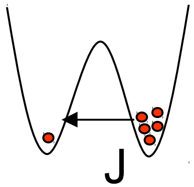
in collaboration with

Doron Cohen (Beer-Sheva)
Slava Yukalov (Dubna)

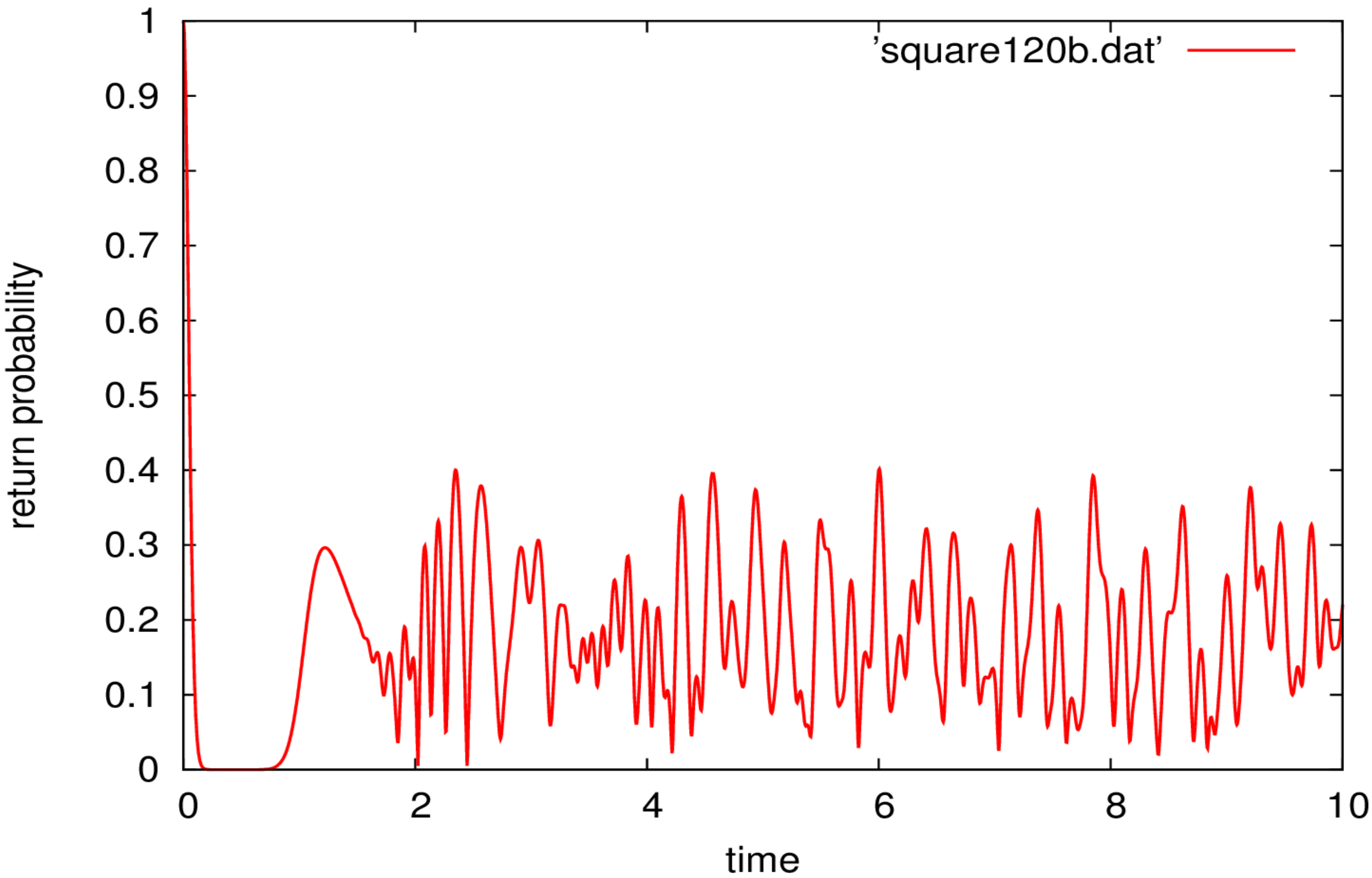


JSF Julian Schwinger Foundation

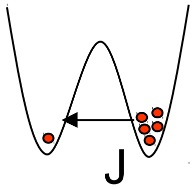
Cracow School of Theor. Physics: Entanglement and Dynamics
Zakopane, June 16, 2017

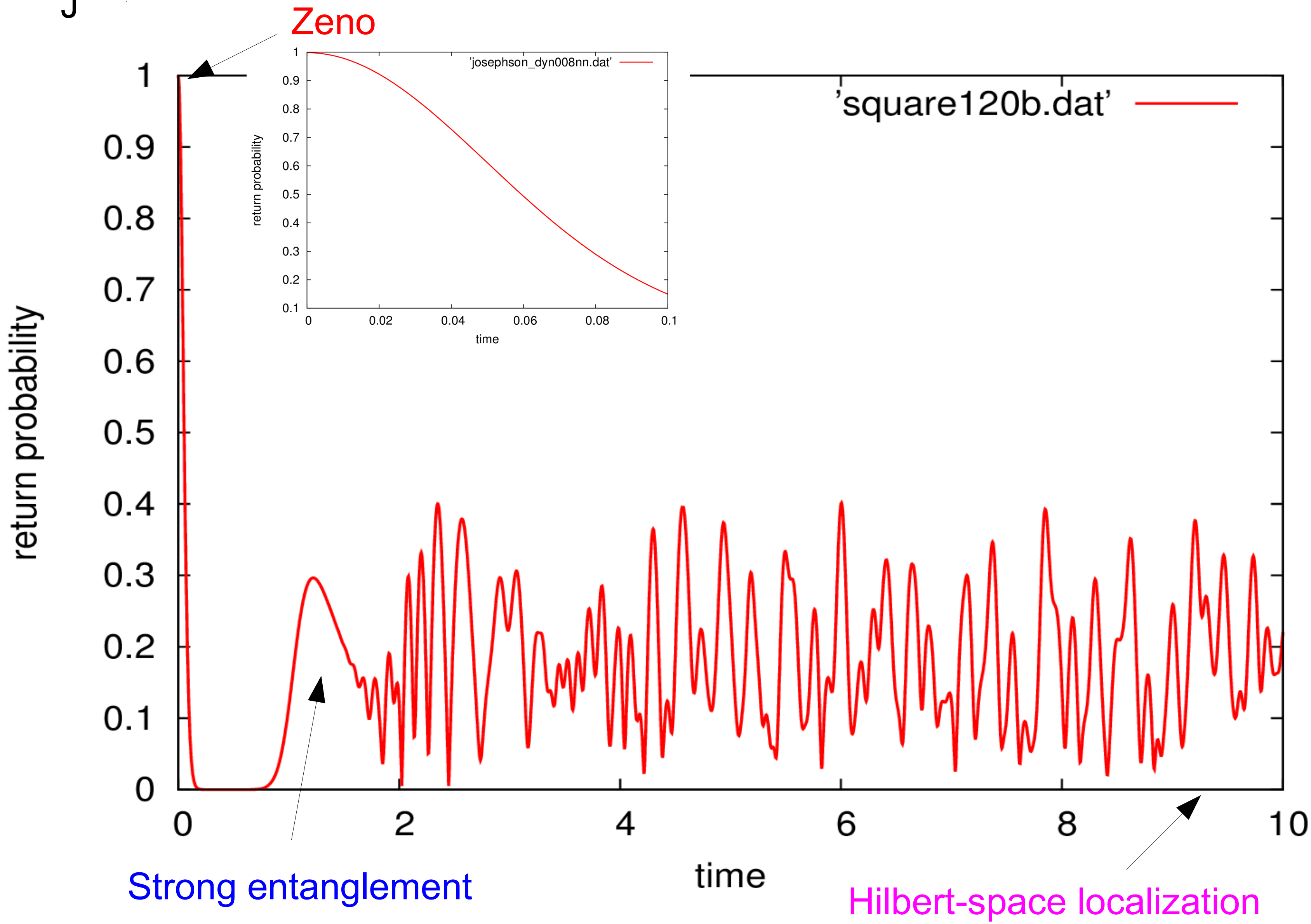


Unitary evolution of 120 interacting bosons

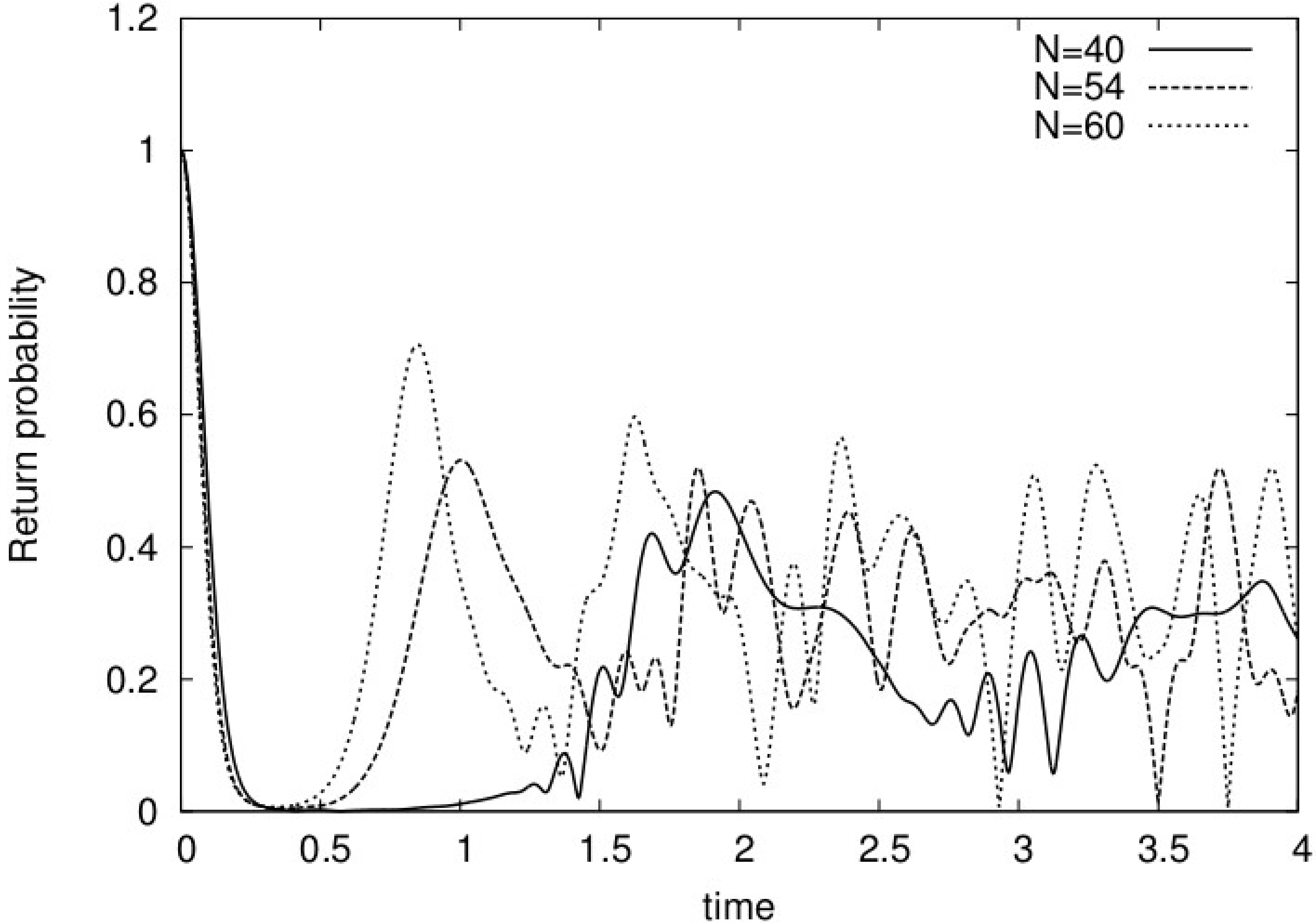


Unitary evolution of 120 interacting bosons





Quantum dynamics: interacting particles



Outline

- Bose-Hubbard model

1. Dynamical entanglement

- Characteristic quantities

- Scaling with the boson number N

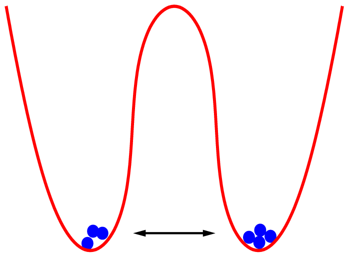
2. Hilbert-space localization (HSL)

- Characteristic quantities

- Scaling with the boson number N

Simple model: bosons in a double well

Simple system: bosonic Josephson tunneling junction

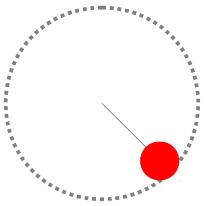


$$|N-n,n\rangle \;\equiv\; |N-n\rangle\otimes|n\rangle$$

Two-site Bose-Hubbard model

$$\mathcal{H}_{\mathrm{BH}} = \frac{U}{2}\sum_{j=1}^{2} a_j^\dagger a_j^\dagger a_j a_j - \frac{J}{2}\left(a_2^\dagger a_1 + \mathrm{h.c.}\right)$$

pendelum analog:



E=T+V

Characteristic parameter (from mean-field approximation)

$$u = \frac{NU}{J}$$

Josephson junction

$$\{|k, N-k\rangle\}_{k=0,\dots,N}, \qquad |k, N-k\rangle \equiv |k\rangle \otimes |N-k\rangle$$

$$H_{2M} = -\frac{E_J}{N}(c_l^\dagger c_r + c_r^\dagger c_l) + H_1, \qquad H_1 = \frac{E_c}{2}n^2, \quad n = \frac{1}{2}(c_l^\dagger c_l - c_r^\dagger c_r)$$

spin representation

$$J^x = \frac{1}{2}(c_l^\dagger c_r + c_r^\dagger c_l), \qquad J^y = \frac{-i}{2}(c_l^\dagger c_r - c_r^\dagger c_l), \qquad J^z = \frac{1}{2}(c_l^\dagger c_l - c_r^\dagger c_r)$$

spin Hamiltonian

$$H_{2M} = \frac{E_c}{2}(J^z)^2 - \frac{2E_J}{N}J^x$$

average spin and fluctuations

$$\bar{J}_z = \langle\Psi_t|J_z|\Psi_t\rangle = \frac{N}{2}(|C_0|^2 - |C_1|^2), \quad \Delta J_z = \sqrt{\langle\Psi_t|J_z^2|\Psi_t\rangle - \langle\Psi_t|J_z|\Psi_t\rangle^2}$$

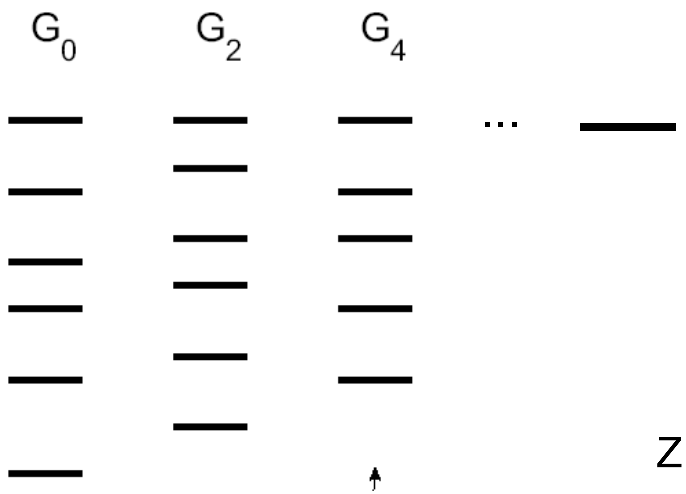
Directed random walk in Hilbert space

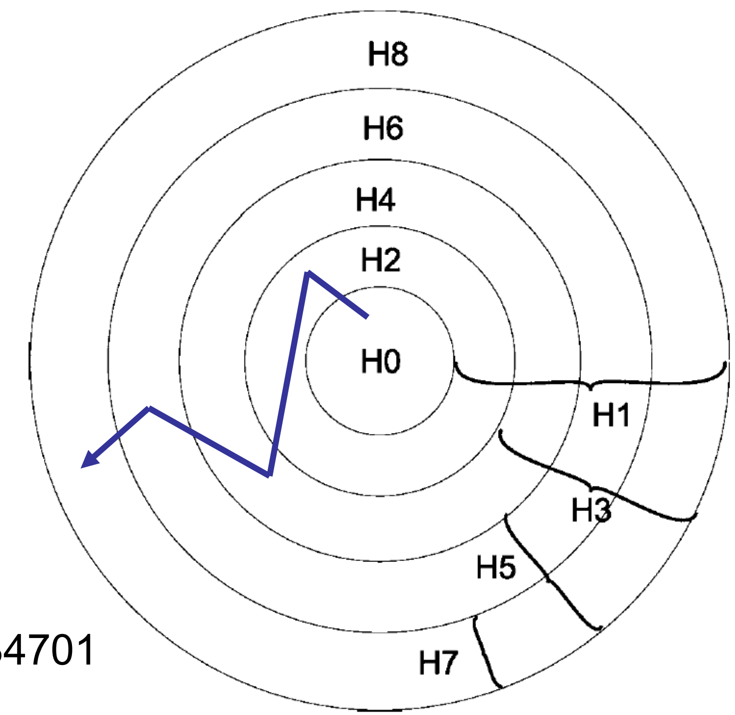
Resolvent:

$$\langle \Psi_1 | (E - i\epsilon - H)^{-1} | \Psi_0 \rangle$$

Recursive projection method: Inverted Russian doll

$$G_j(z) = \left[z_j - h_j G_{j+1}(z) h_j^T \right]_{2j}^{-1}$$





Z,PRA 81,034701

Mesoscopic entanglement: N00N state

$$|\psi_t\rangle = \sum_{j=0}^{N} c_j |N-j, j\rangle$$

$$|N00N\rangle = \left(|N,0\rangle + e^{i\phi N}|0,N\rangle\right)/\sqrt{2}$$

$$\langle N00N|\Psi_t\rangle = \frac{c_0 + e^{-i\phi N} c_N}{\sqrt{2}}$$

$$a_l|N00N\rangle = \frac{\sqrt{N}}{\sqrt{2}}|N-1,0\rangle$$

Interferometry!

$$A = |N,0\rangle\langle 0,N| + |0,N\rangle\langle N,0|$$

$$A^2|\Psi_t\rangle = c_0|0,N\rangle + c_N|N,0\rangle$$

$$\langle N00N|A|N00N\rangle = \cos\phi$$

$$\langle\Psi_t|A|\Psi_t\rangle = 2Re(c_0^* c_N)$$

$$\langle\Psi_t|A^2|\Psi_t\rangle = |c_0|^2 + |c_N|^2$$

Characterization of N00N entanglement

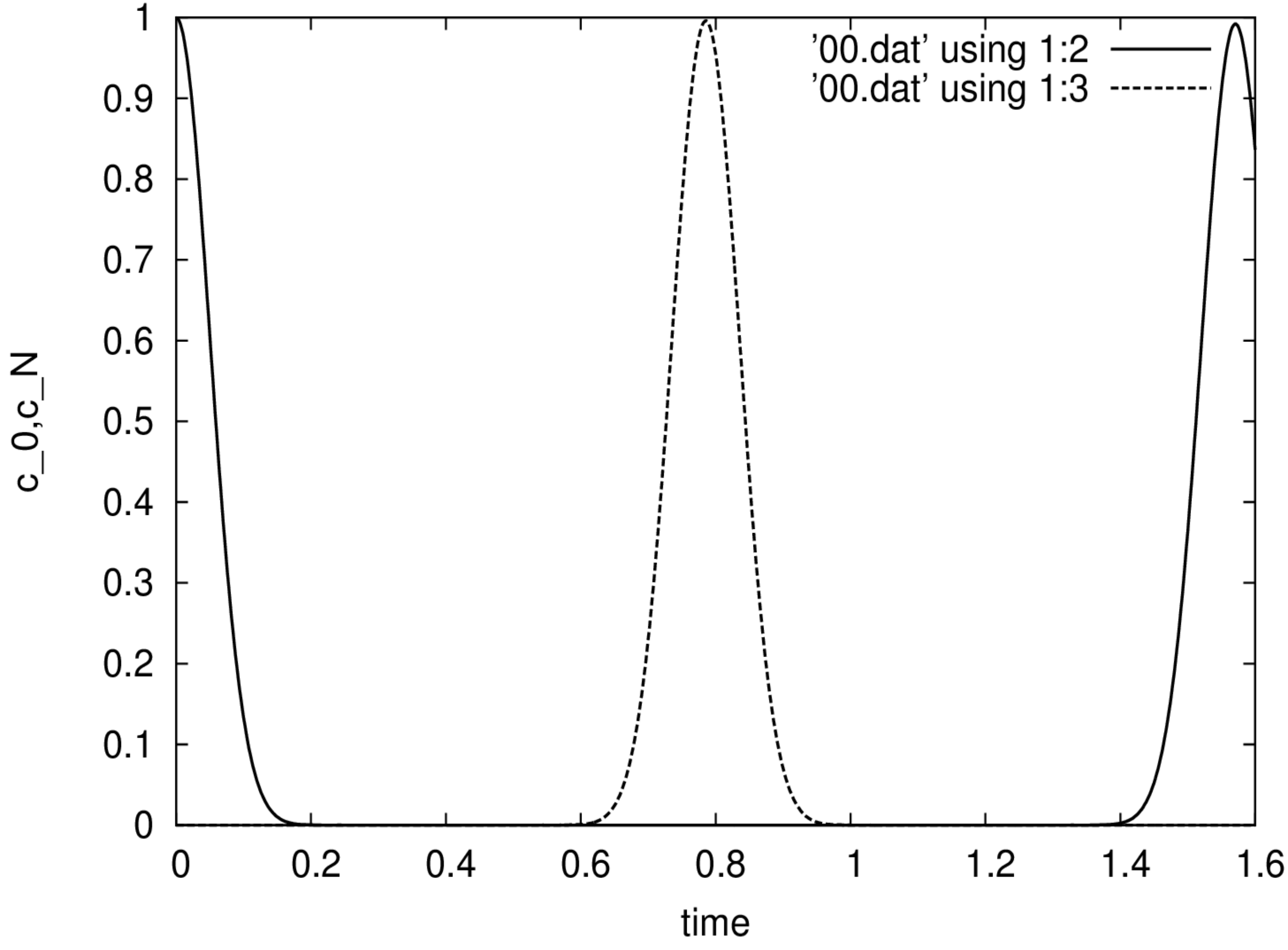
Husimi-Q function (fidelity)

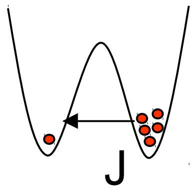
$$Q_t = \frac{1}{2\pi}\left(\langle\Psi_t|A^2|\Psi_t\rangle + \langle\Psi_t|A|\Psi_t\rangle\right)$$

$$P_e = 2\max_t |c_0 c_N|$$

U=0: very weak N00N entanglement

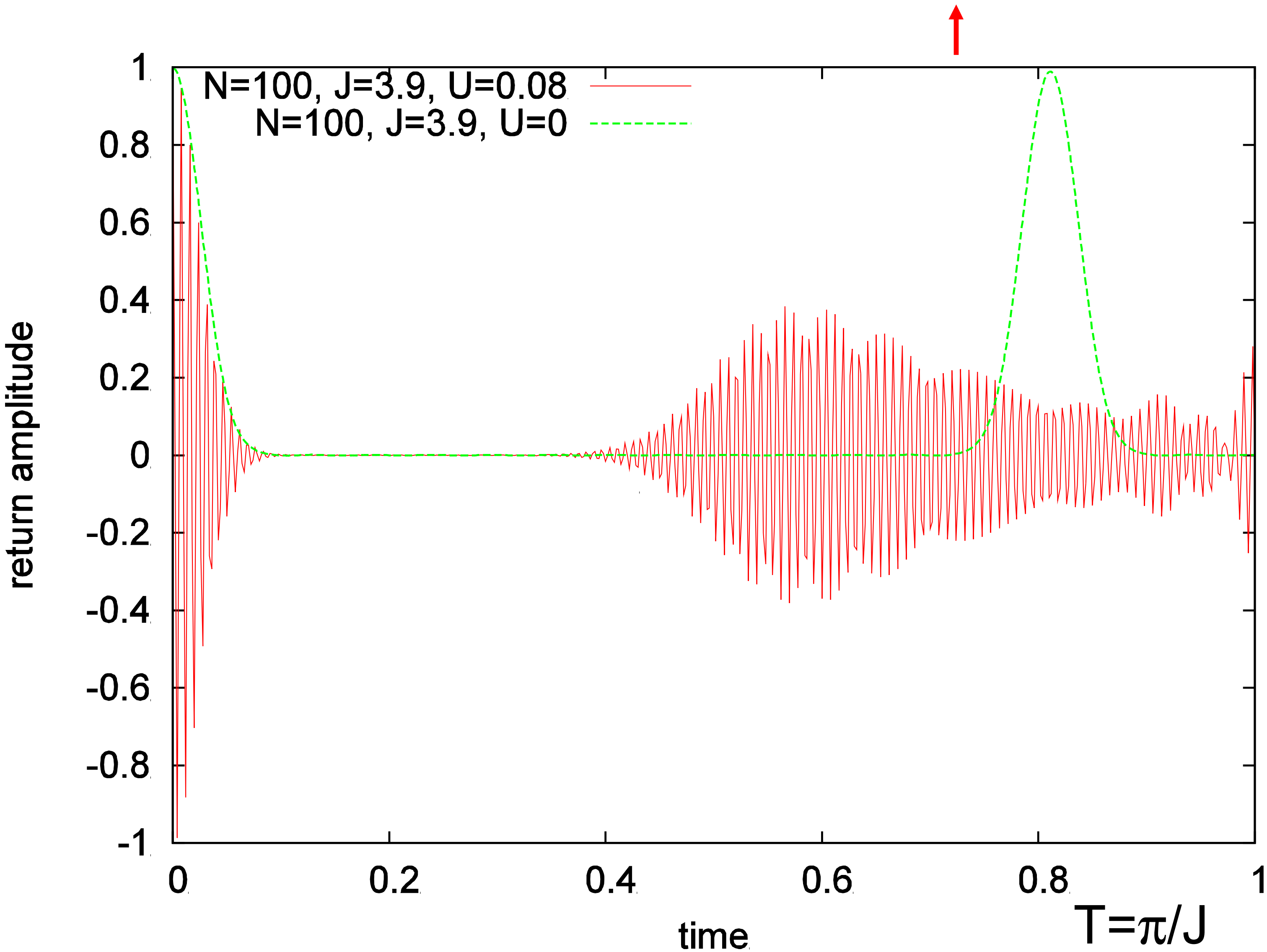
$$P_{N00N}|\Psi_t\rangle = c_0(t)|N,0\rangle + c_N(t)|0,N\rangle$$



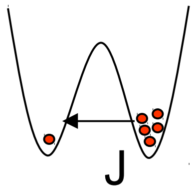


BHM: return to the initial well

$$P_{N00N}|\Psi_t\rangle = c_0(t)|N,0\rangle + c_N(t)|0,N\rangle$$

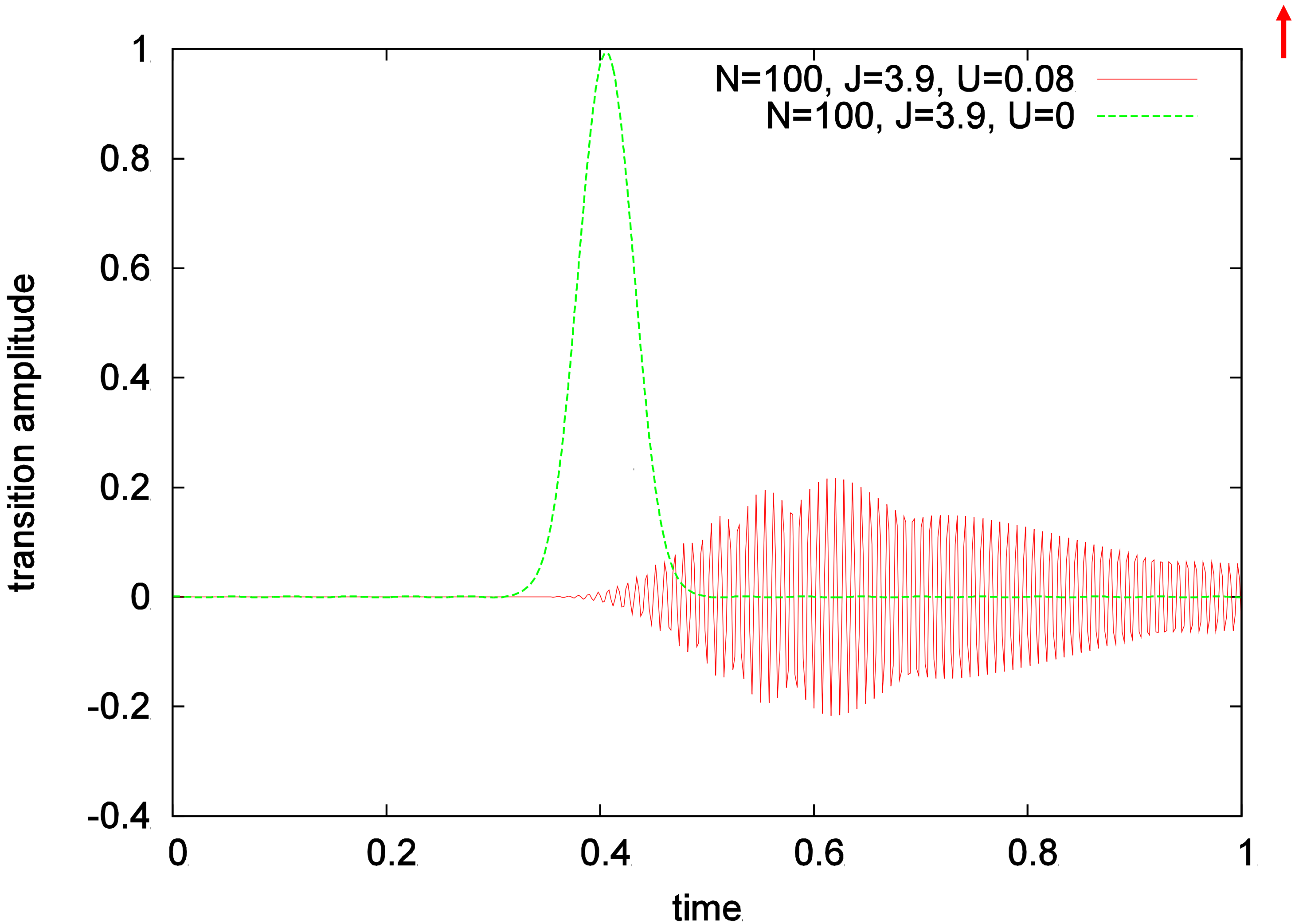


T=π/J



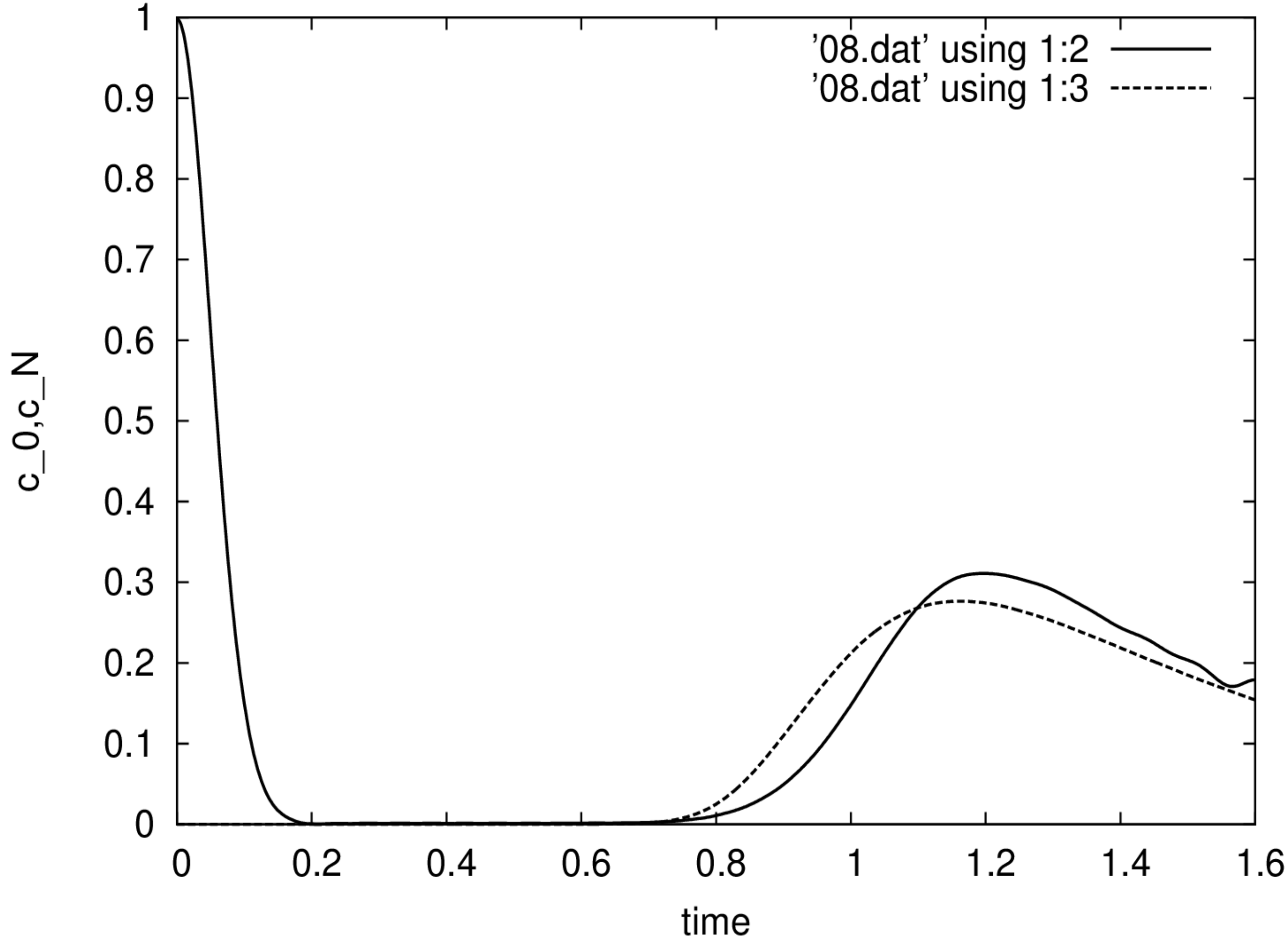
BHM: transition between the other well

$$P_{N00N}|\Psi_t\rangle = c_0(t)|N,0\rangle + c_N(t)|0,N\rangle$$

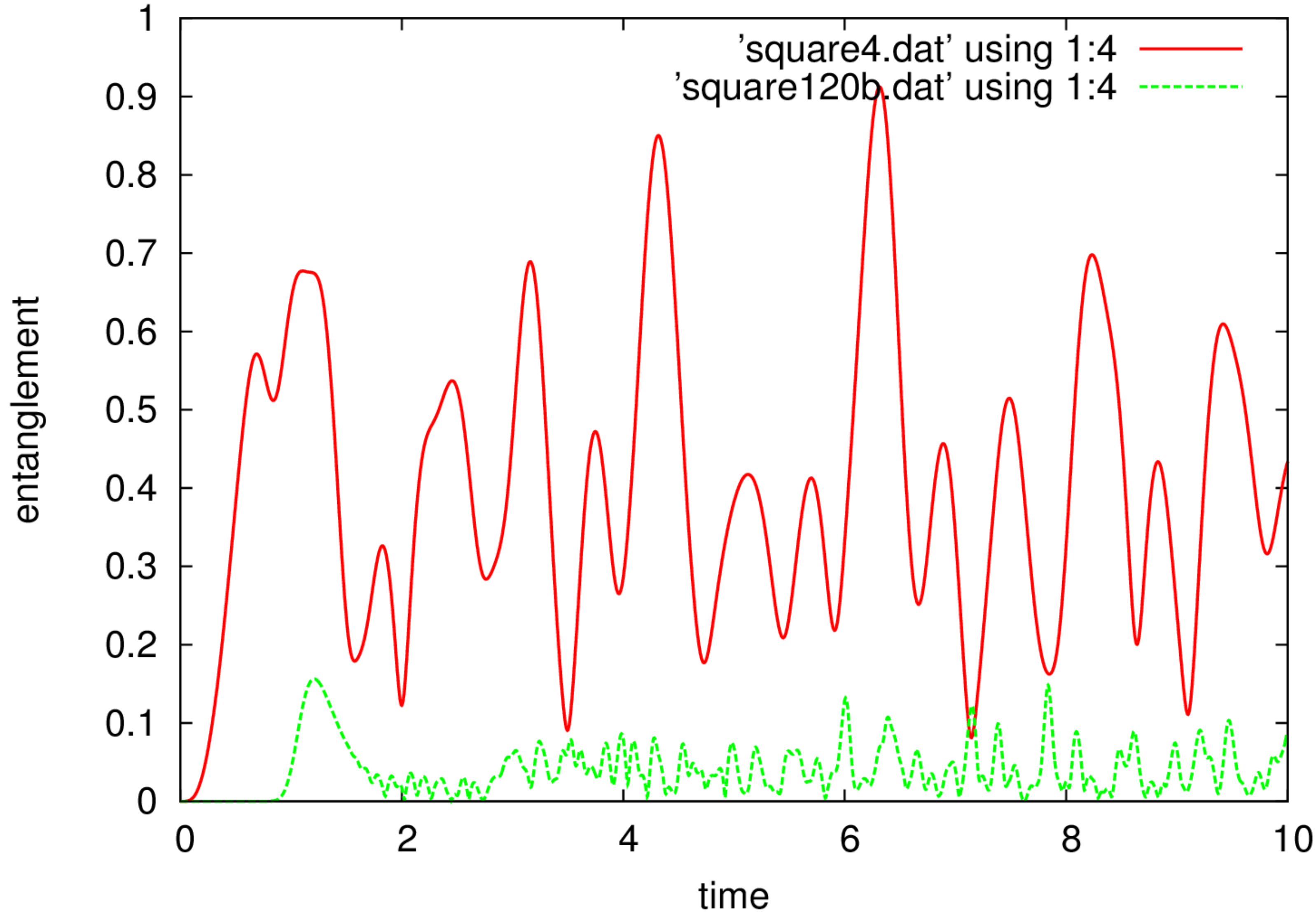


Optimal N00N entanglement

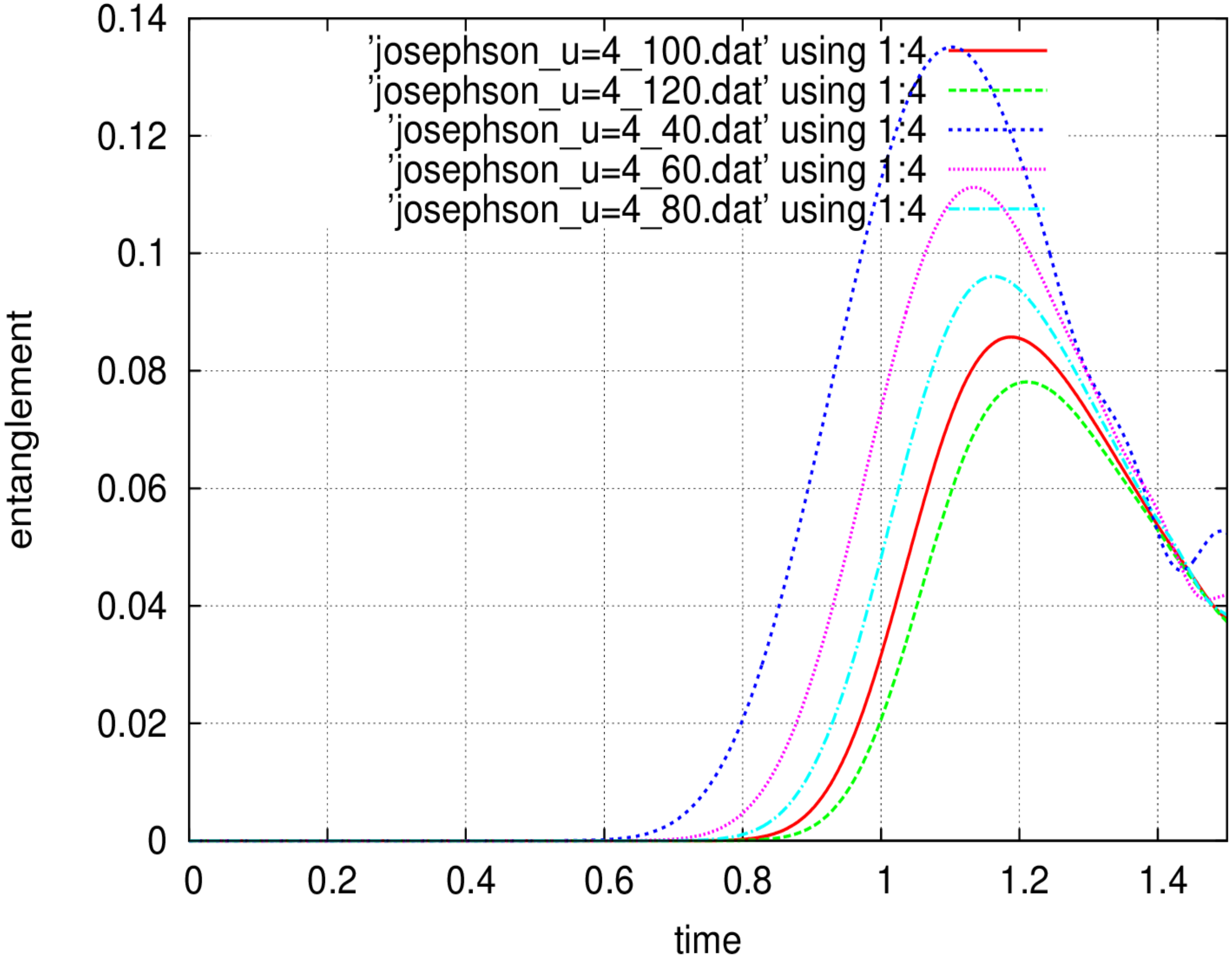
$$P_{N00N}|\Psi_t\rangle = c_0(t)|N,0\rangle + c_N(t)|0,N\rangle$$



Noon entanglement for N=4 & N=120

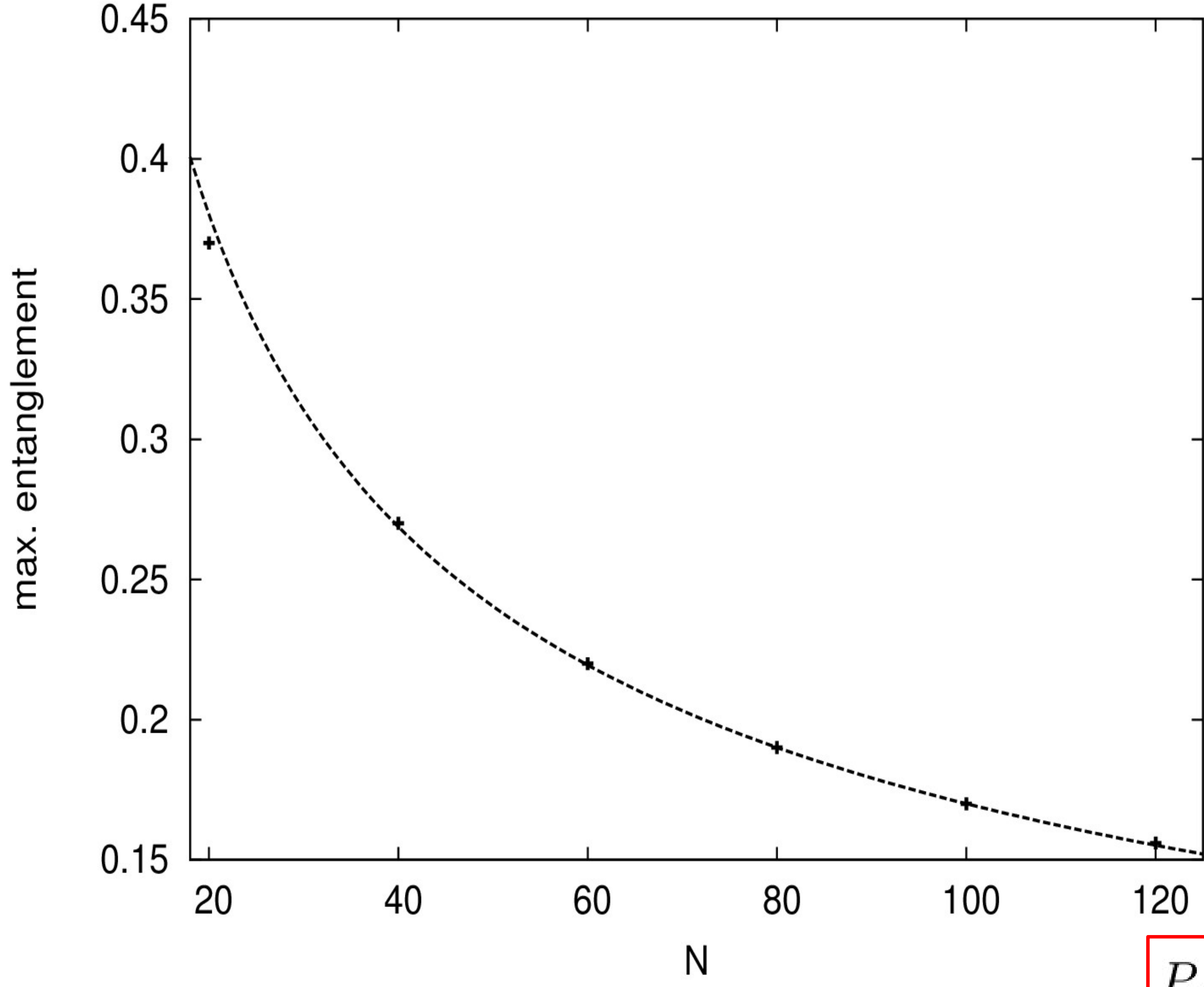


Scaling with number of bosons N



fixed u=NU/J

Scaling with number of bosons N



$$P_e \sim 1.7 N^{-1/2}$$

Summery:
Scaling of N00N entanglement

noninteracting system $(U = 0)$

$$c_0 = \cos^N(Jt/2),\ c_N = (-i)^N \sin^N(Jt/2)$$

$$Q_t = \frac{1}{2\pi}\left[\cos^{2N}(Jt/2) + \sin^{2N}(Jt/2) + \cos(N\pi/2)\frac{\sin^N(Jt)}{2^{N-1}}\right]$$

$$P_e = 2^{-(N-1)}$$

interacting system $(U \neq 0)$

$$P_e \sim 1.7N^{-1/2}$$

Characterization of HS localization

Inverse participation ratio:

return probability

$$\mathcal{N}_\infty^{-1} = \lim_{\epsilon\to 0}\epsilon\int_0^\infty |\langle\psi|e^{-i\mathcal{H}t}|\psi\rangle|^2 e^{-\epsilon t}dt$$

>0 localized

=0 delocalized

density of states

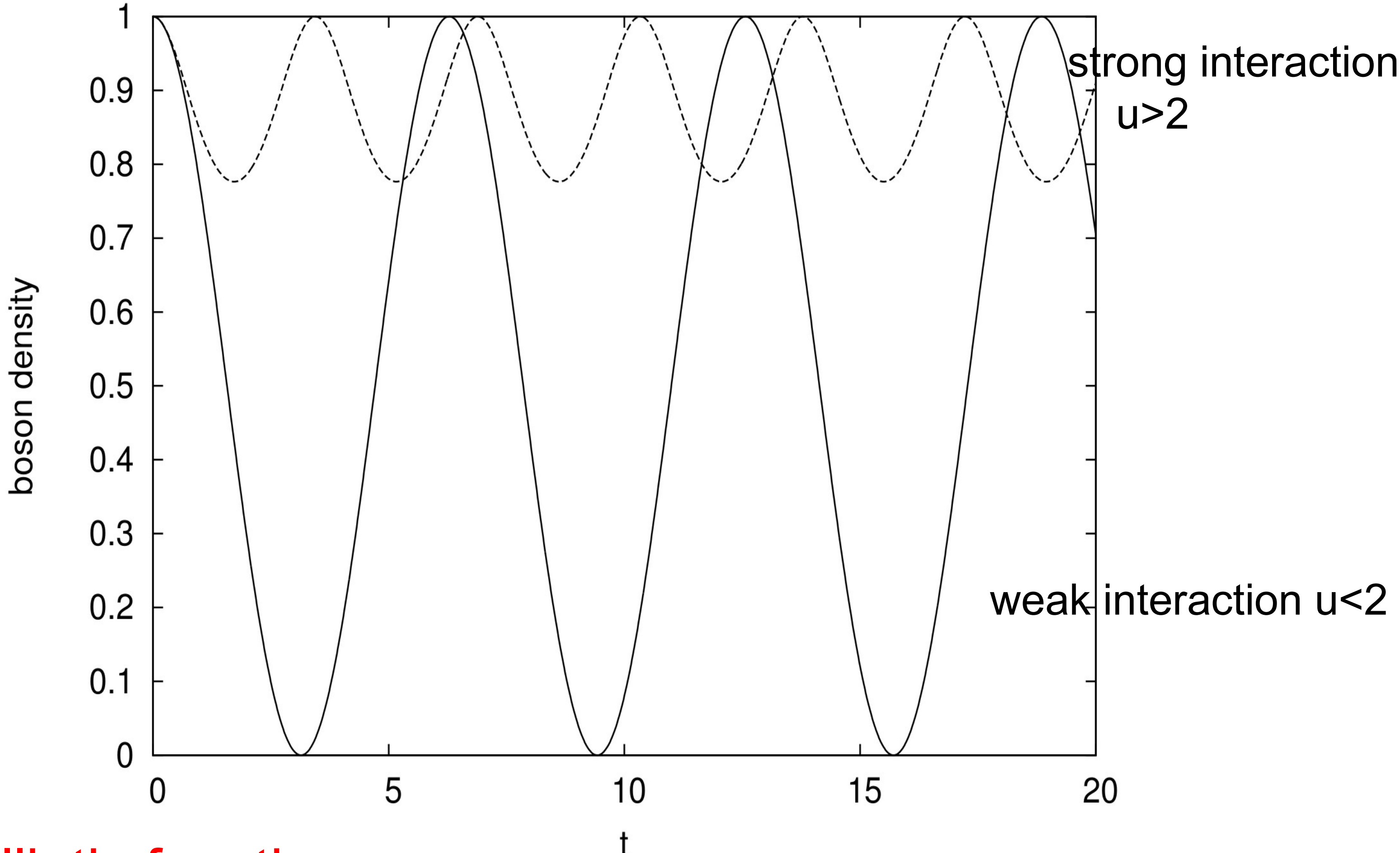
$$\varrho_\epsilon(\omega) = \frac{1}{\pi}\mathrm{Im}\langle\psi|(\omega - i\epsilon - \mathcal{H})^{-1}|\psi\rangle$$

$$\mathcal{N}_\infty^{-1} = 2\pi\lim_{\epsilon\to 0}\epsilon\int[\varrho_\epsilon(\omega)]^2 d\omega$$

Cohen, Yukalov and Z., arxiv:1511.04667 (PRA93)

Localization in Hilbert space

Mean-field approximation: Gross-Pitaevskii equation



Jacobi elliptic functions

Full quantum dynamics: scaling behavior of HS localization

noninteracting bosons

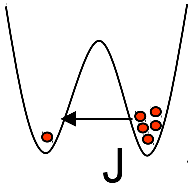
$$\mathcal{N}_\infty^{-1} \sim \frac{\sqrt{2}}{\sqrt{\pi N}}$$

interacting bosons

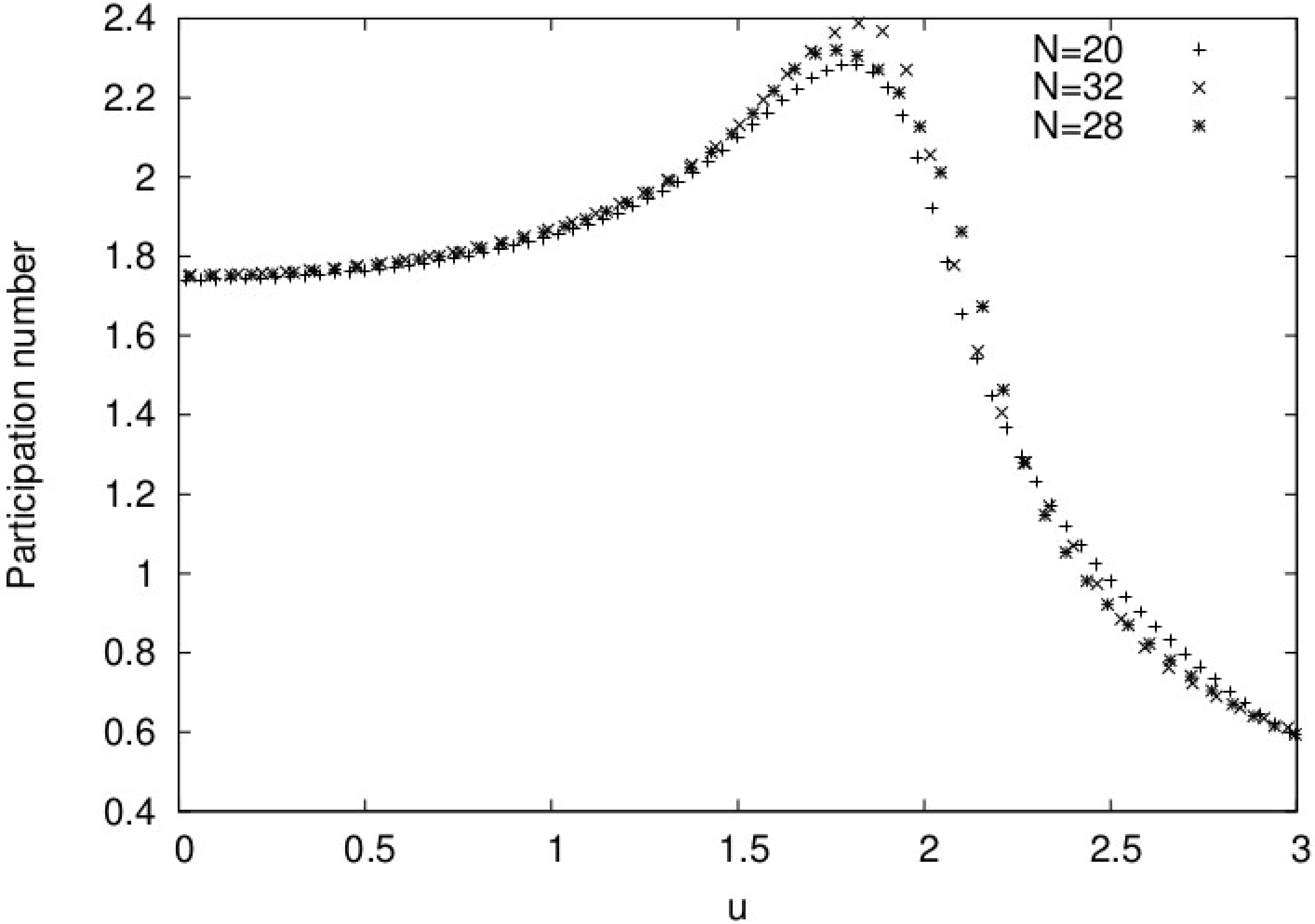
$$\mathcal{N}_\infty(N,u) \sim 1 \quad \text{for} \quad u \sim \infty$$

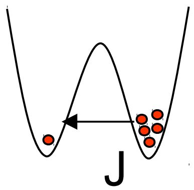
$$\mathcal{N}_\infty(N,u) \quad \approx \quad (N+1)^\alpha \; f(u)$$

$\alpha = 0.5 ... 1$

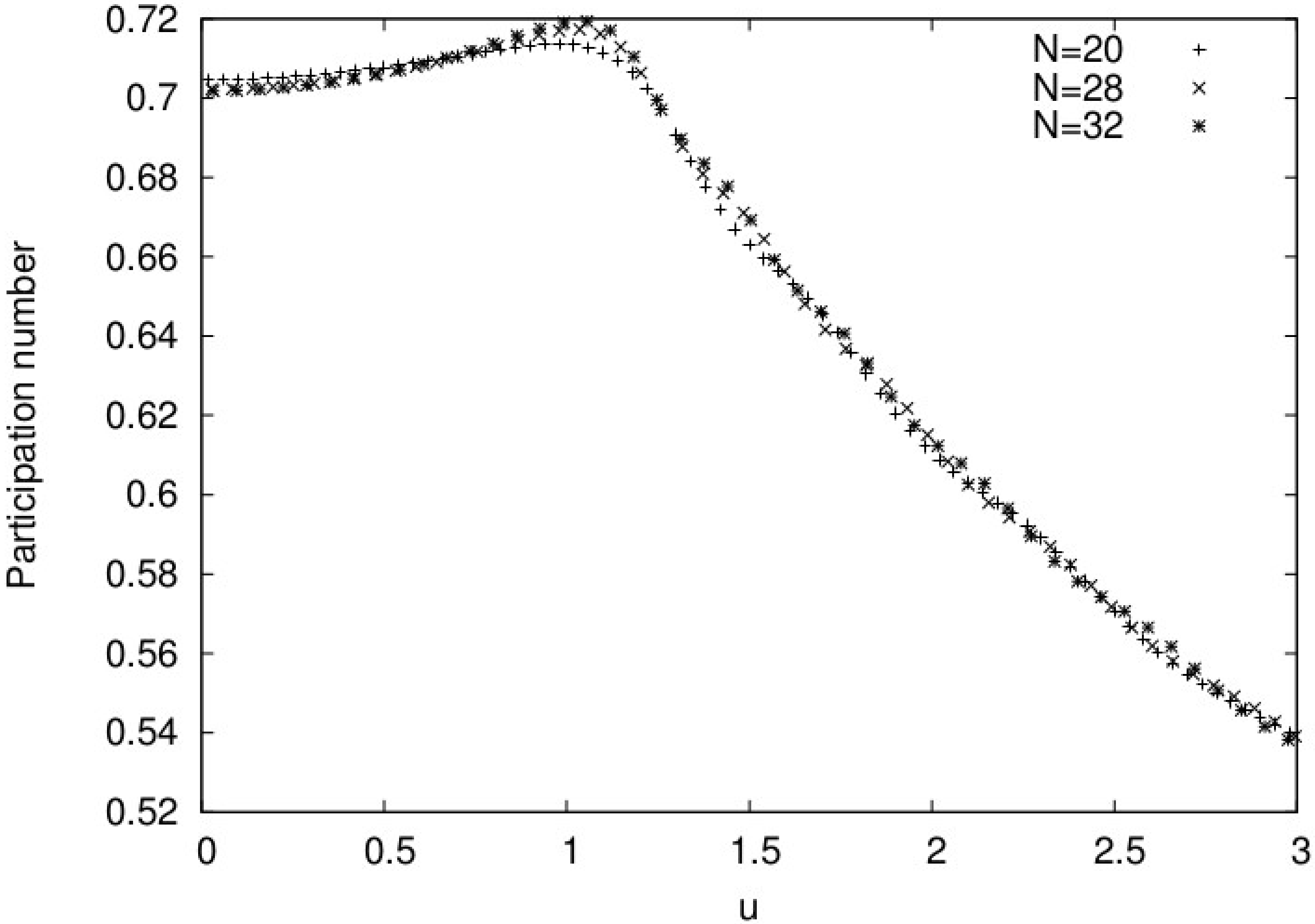


Scaling function f(u) for initial state |N,0>





Scaling function f(u) for initial state |N/2,N/2>



Conclusions dynamical entanglement

- dynamical entanglement
- Hilbert-space localization
- “correlation physics”

are a consequence of the competition of tunneling & interaction

dynamical entanglement is achievable for large number of bosonic atoms in interacting systems

However, this requires

- intermediate time scales
- fine tuning of model parameters interaction vs. tunneling

Conclusions HSL

Participation number obeys finite-size scaling

$$\mathcal{N}_\infty(N,u) \approx (N+1)^\alpha \ f(u)$$

which depends on the initial state:

$\alpha=1/2$ (|N,0> initial) $\alpha=0.84$ (|N/2,N/2> initial)

In contrast to the mean-field approximation the full quantum dynamics does not have a transition to HSL as a function of u

Thank you