Quantum annealing with ultracold atoms in multimode resonators

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Zakopane, June 15, 2017

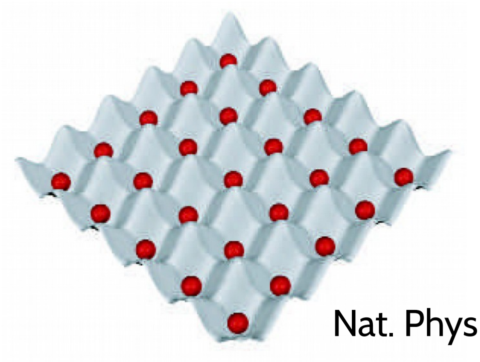
FWF
Der Wissenschaftsfonds.

Outline

- Cold atoms in cavity-generated dynamical optical potentials
- Self-optimizing classical dynamics in multi-mode cavities
- Quantum annealing

Optical lattices

Optical lattices



Nat. Phys. 1, 23

Optical lattices

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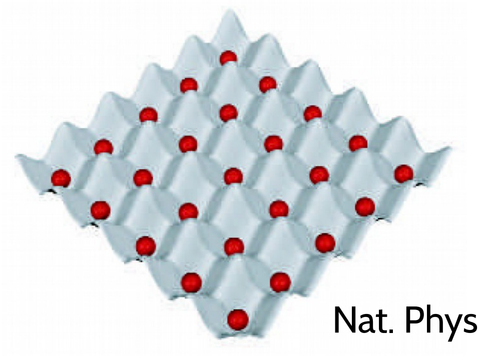

Nat. Phys. 1, 23

$$V(x) = \hbar \frac{|\Omega(x)|^2}{\Delta_a}$$

Optical lattices

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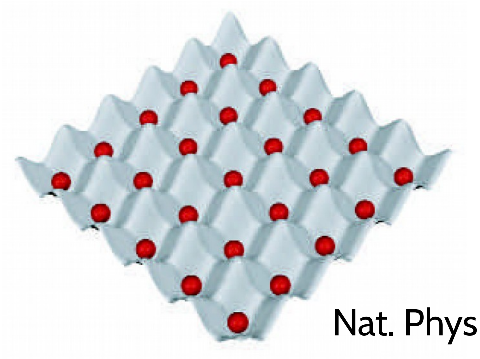

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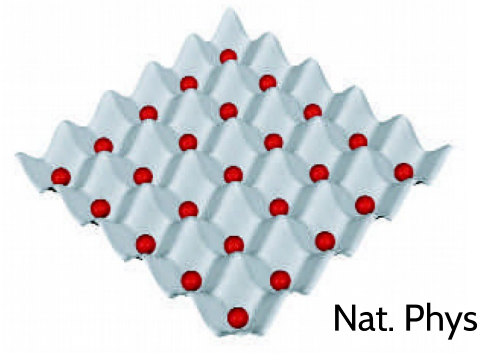
Potential imposed by lasers

Optical lattices

Optical lattices



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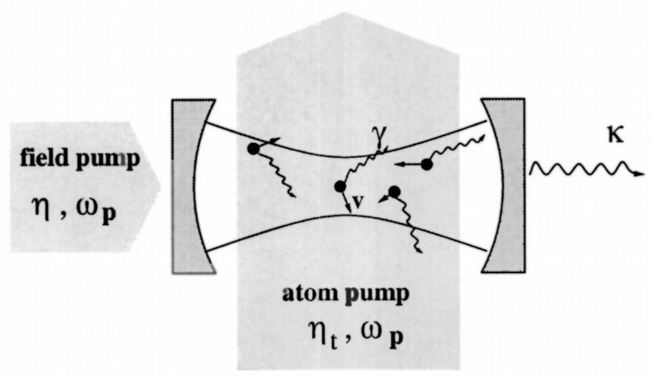
Dynamical optical lattices in cavities



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Potential imposed by lasers

Ritsch et al., Rev. Mod. Phys., 85.2 (2013)

Optical lattices

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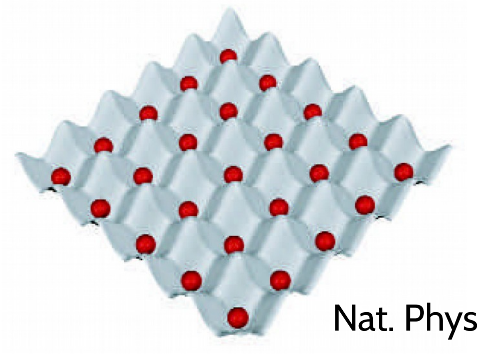

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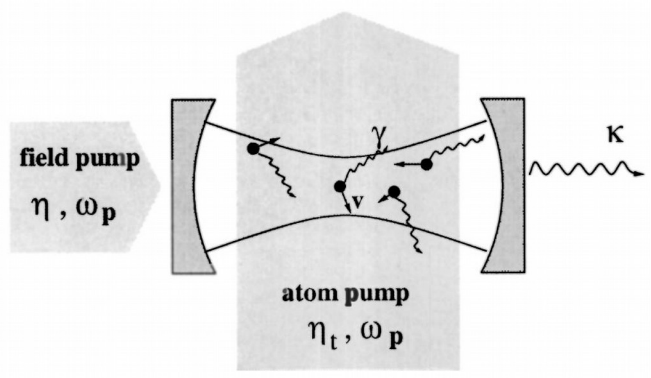

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Potential imposed by lasers

Atoms create own trapping potential

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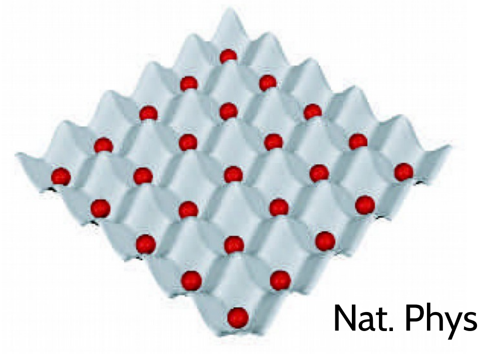

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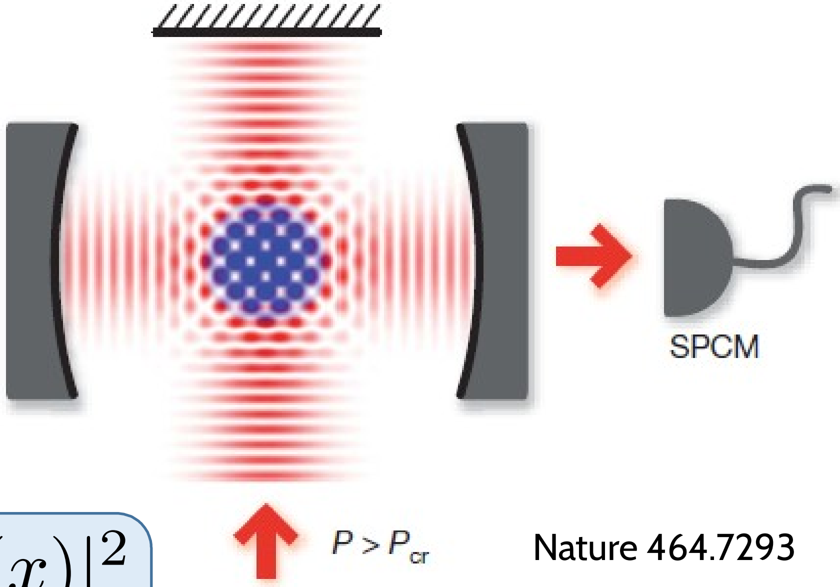

Nature 464.7293

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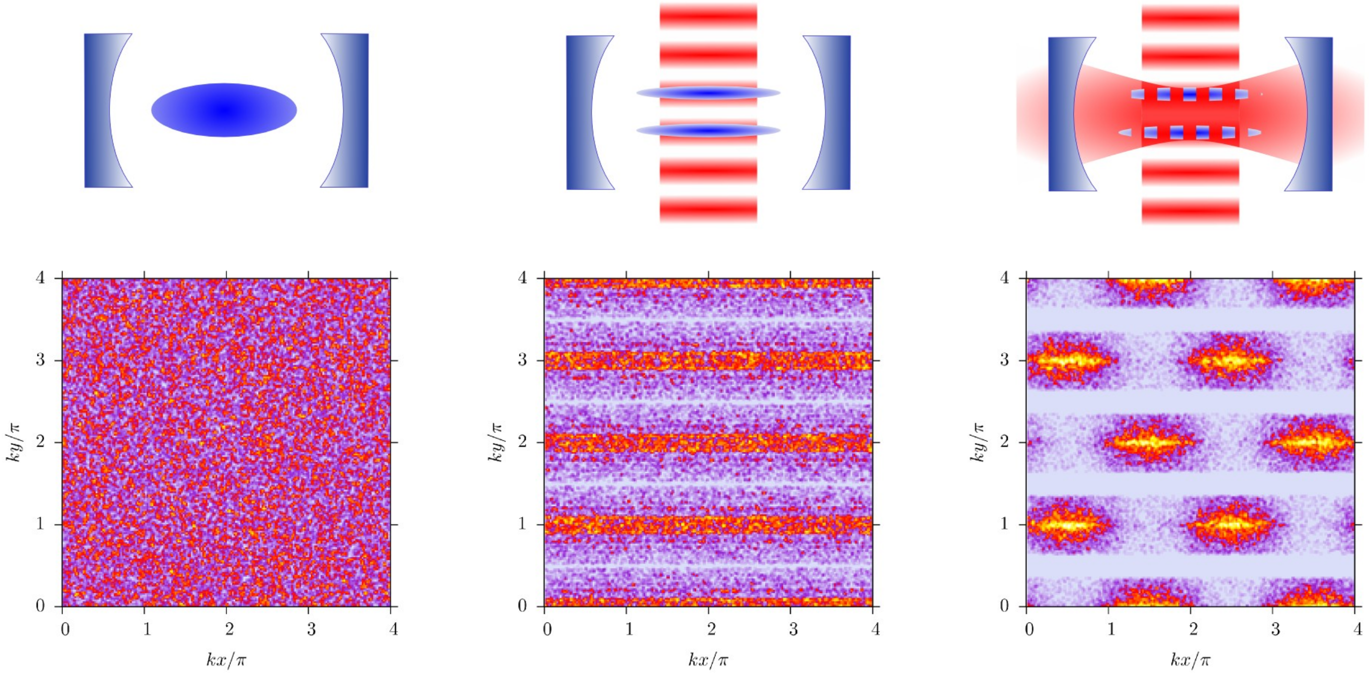
Self-organization



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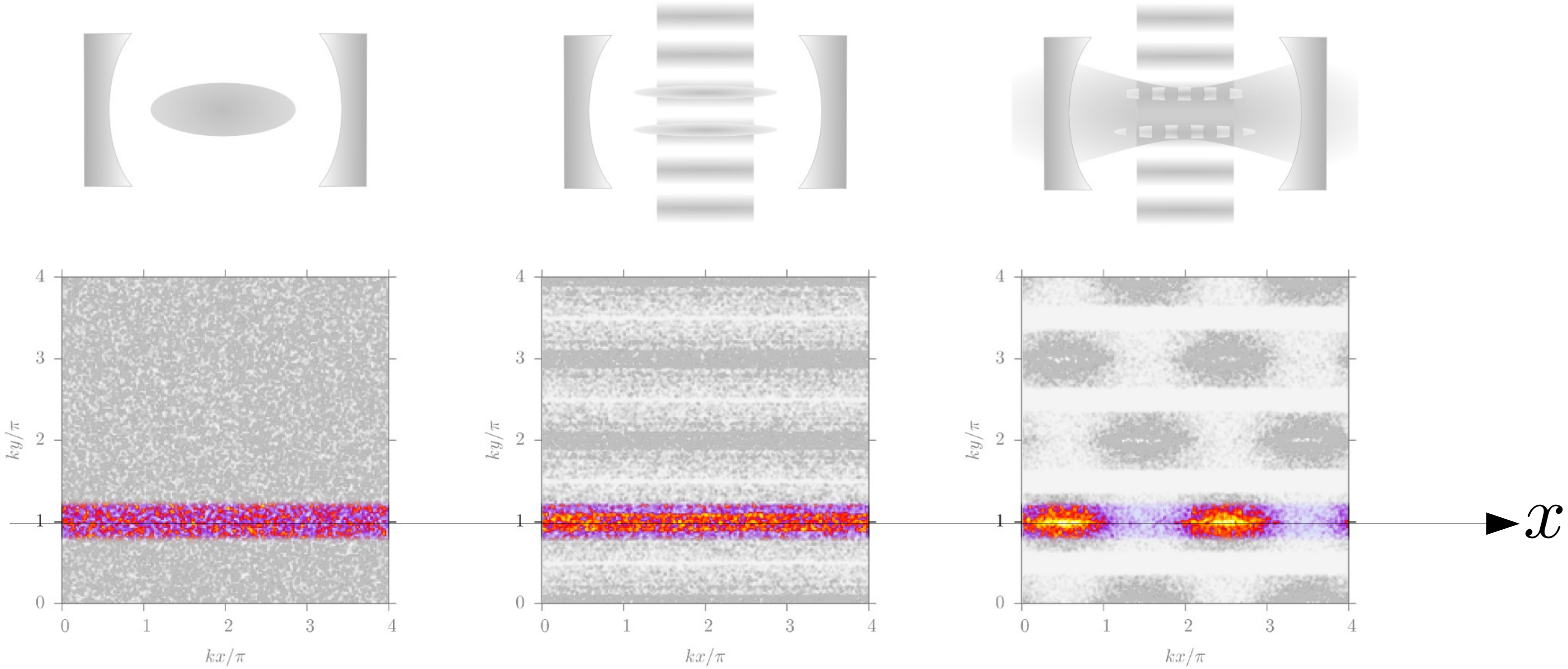

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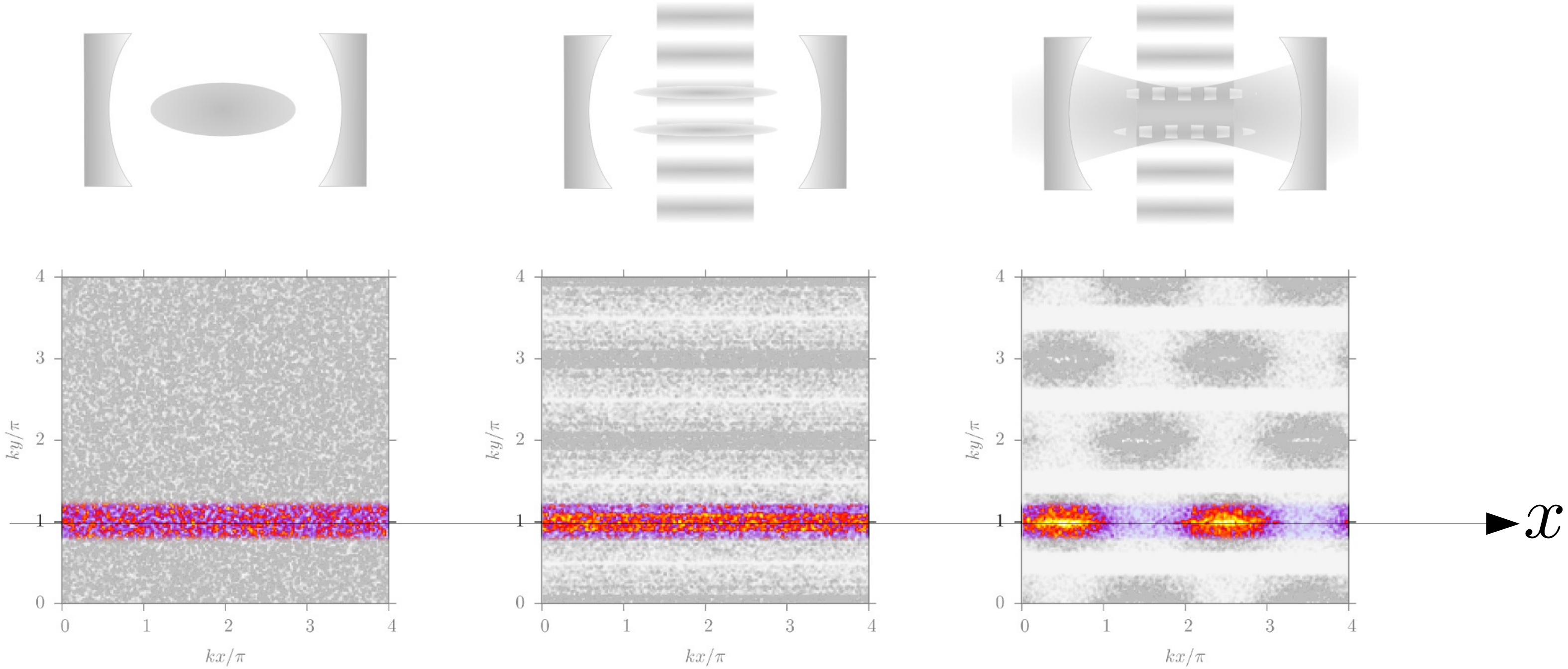
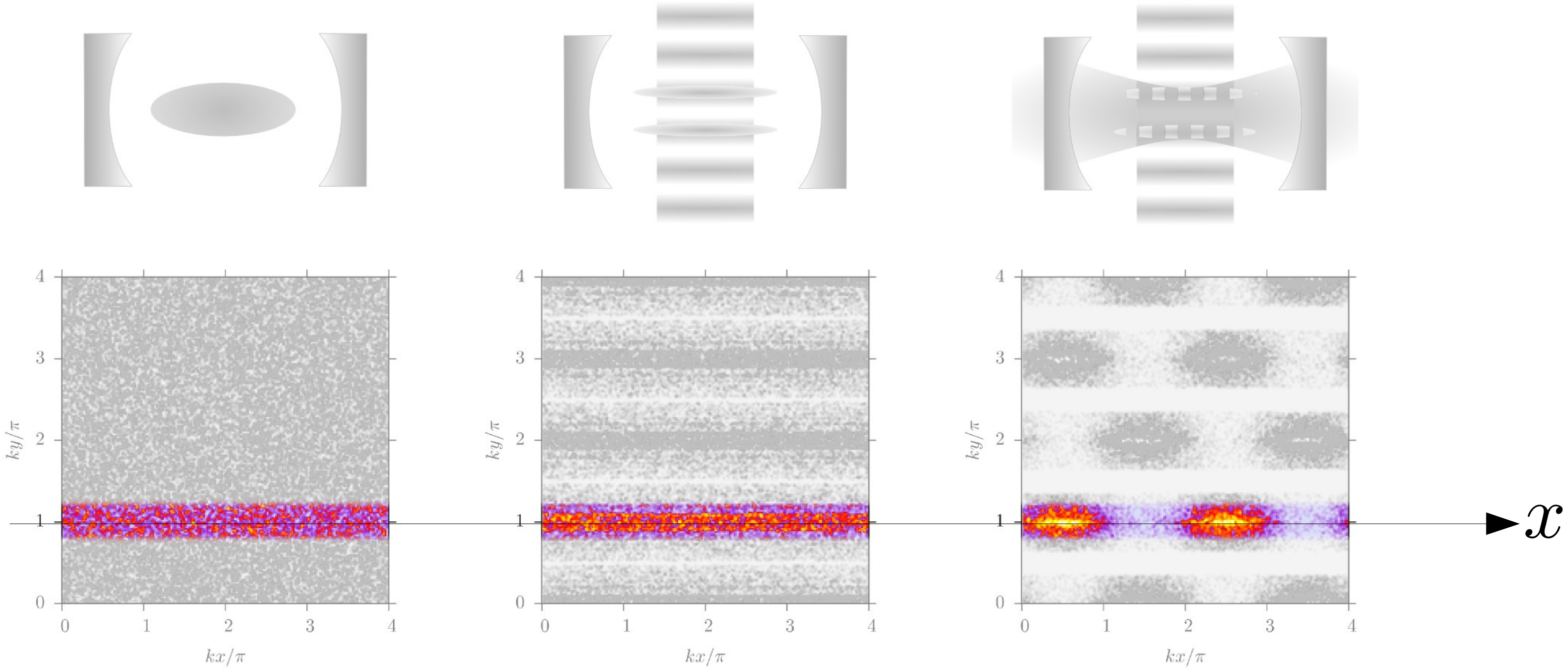


$$V(x,\alpha) = \hbar \frac{|\Omega(x,\alpha)|^2}{\Delta_a}$$

Self-organization



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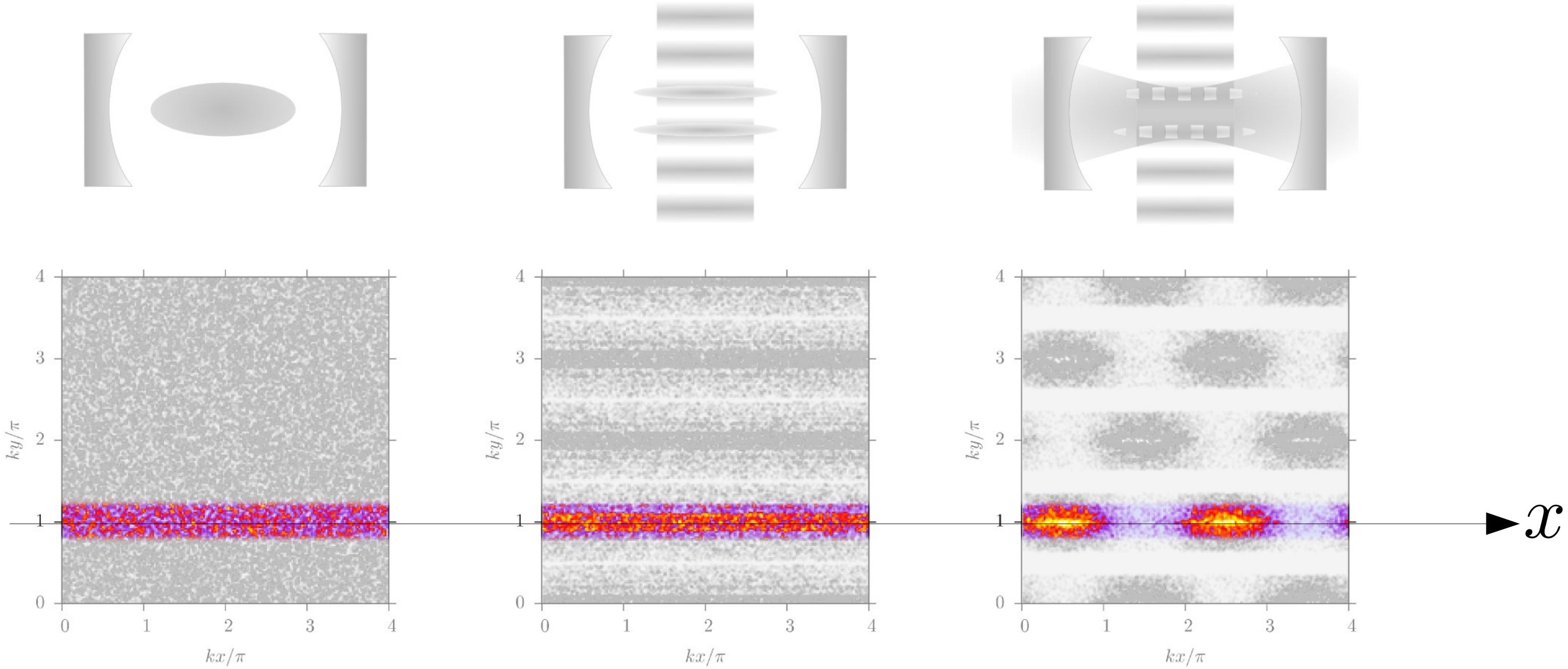
$$\Omega(x,\alpha) = g\alpha\cos(kx) + \Omega_0$$

Self-organization



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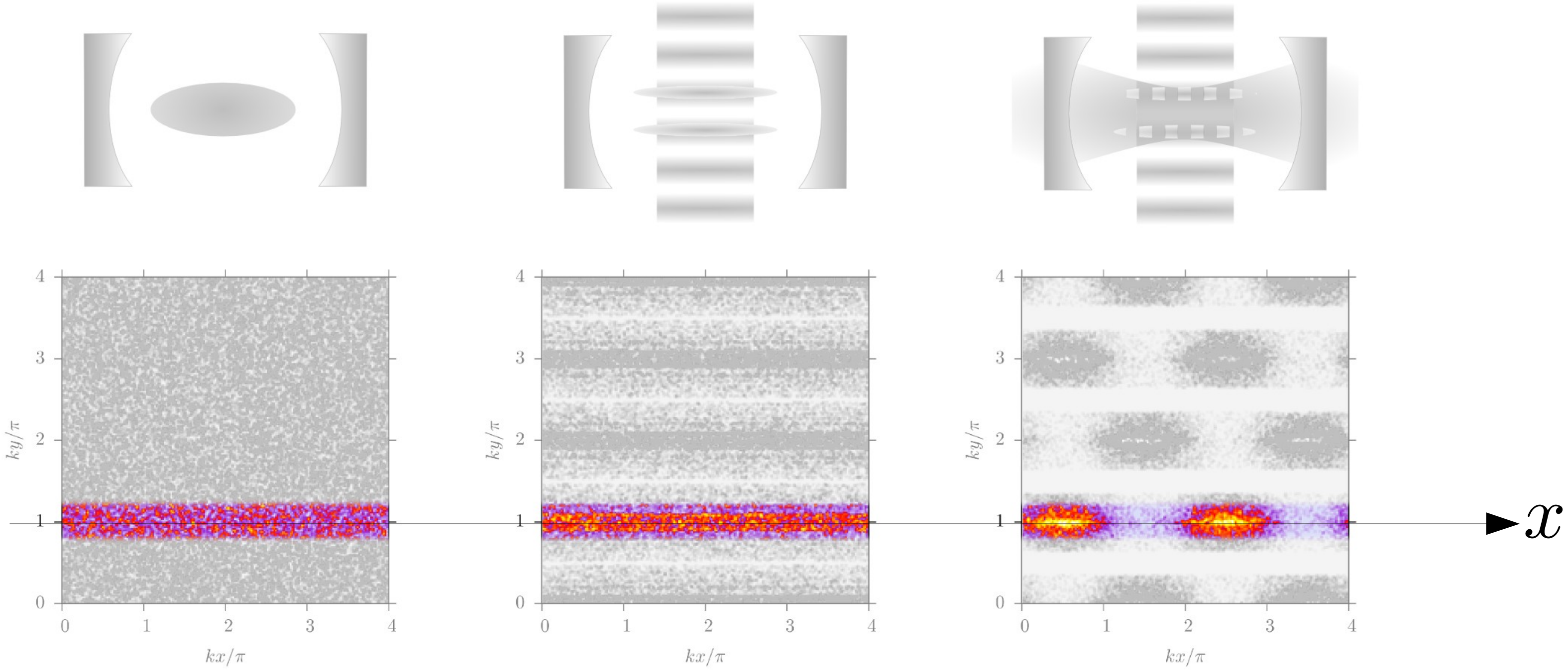
$$= \hbar(U_0|\alpha|^2\cos^2(kx) + \eta(\alpha+\alpha^*)\cos(kx) + \Omega_0^2)$$

Self-organization



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Collective potential

Dynamical potential:

$$V(x,\alpha) \approx \hbar\eta(\alpha+\alpha^*)\cos(kx)$$

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$$\alpha = \frac{\eta}{\Delta_c + i\kappa}\sum_j \cos(kx_j)$$

Collective potential

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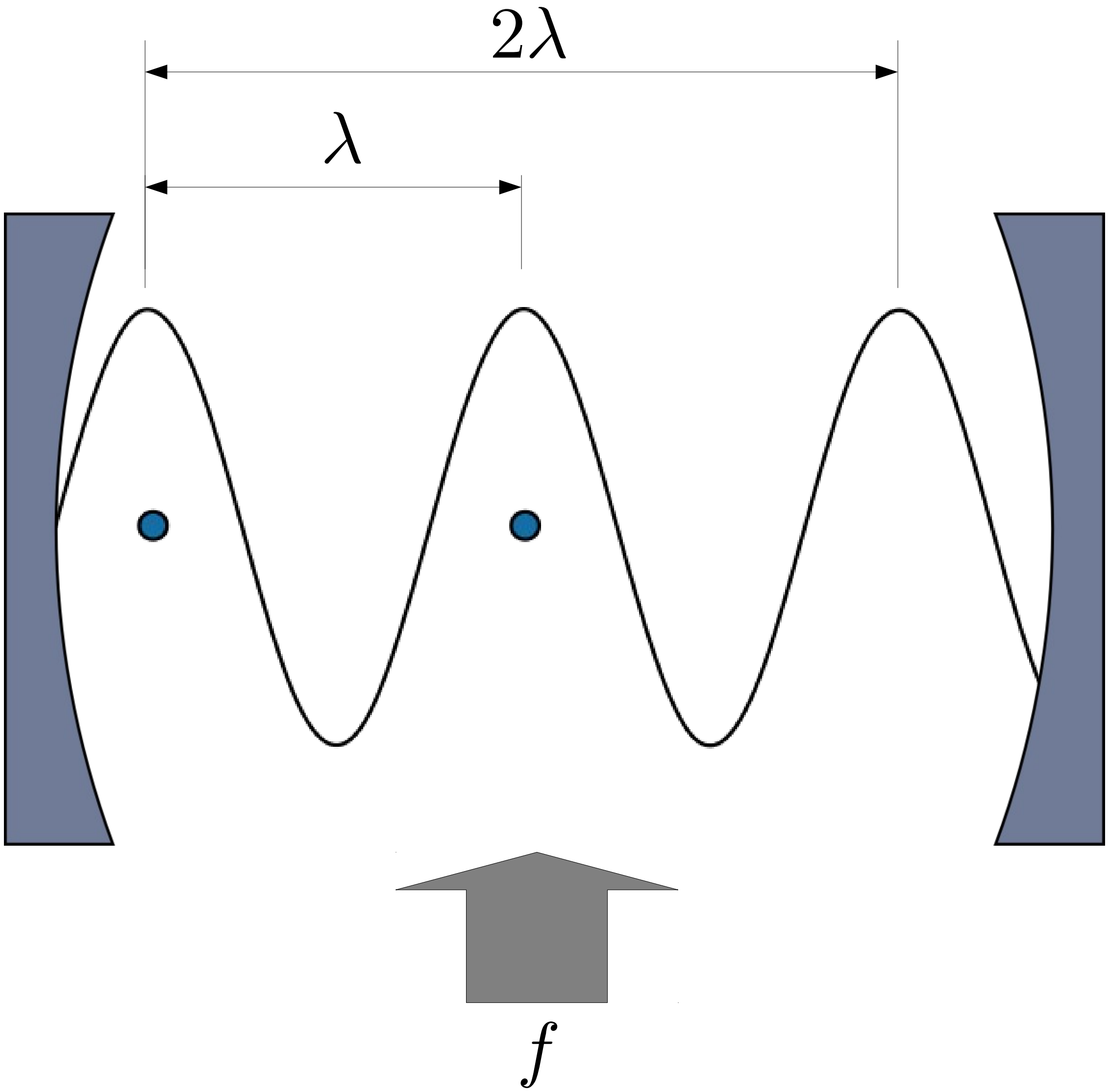
Collective potential:

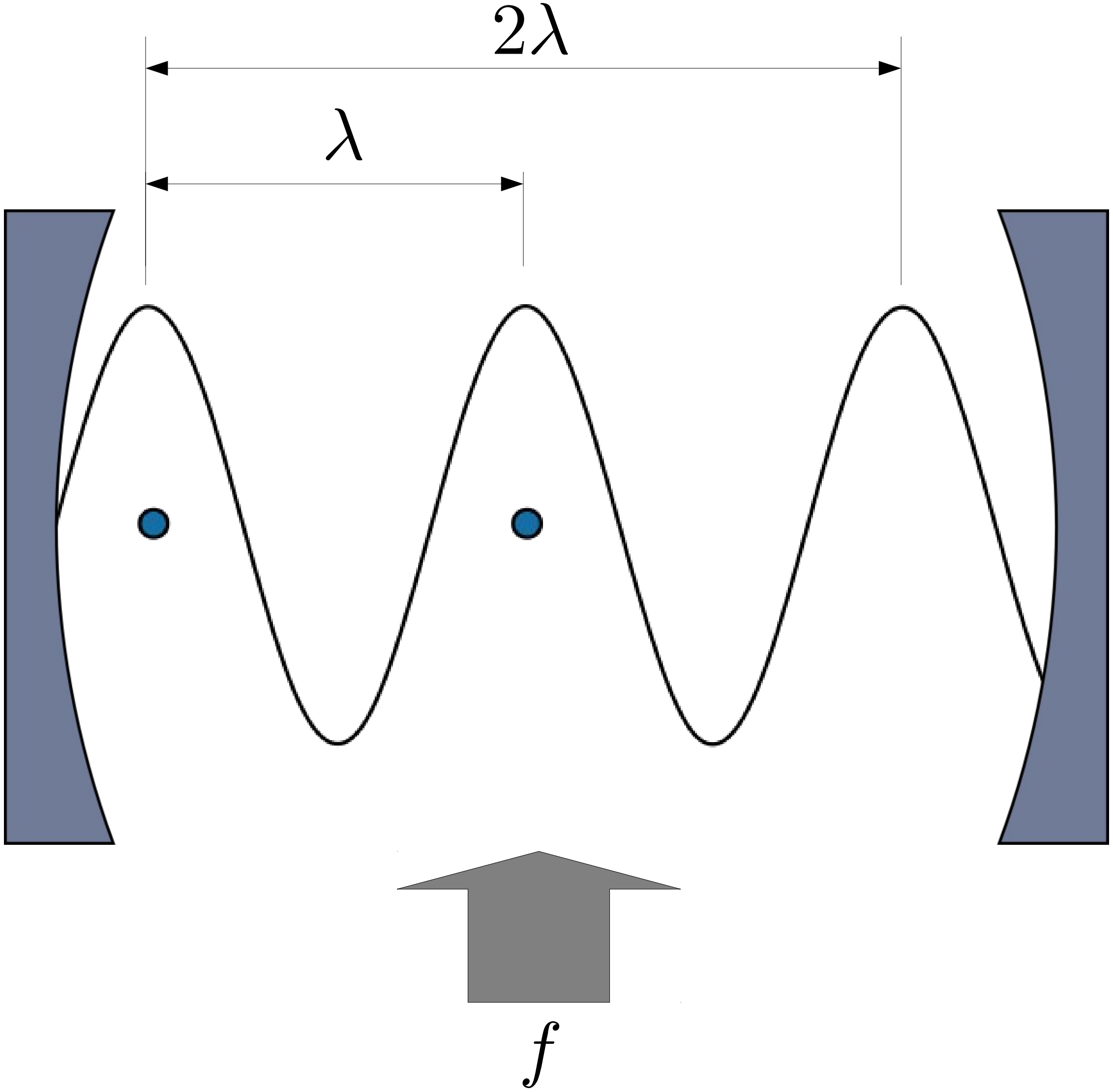
$$\boxed{V(x_1,...,x_N) = -f\left(\sum_{i=1}^{N}\cos(kx_i)\right)^2}$$

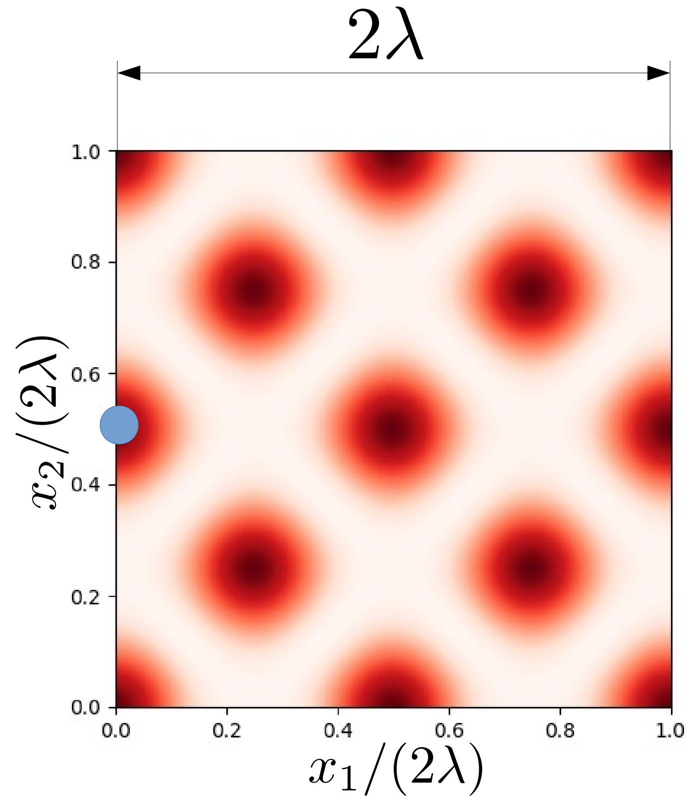
$$f = -\hbar\frac{\eta^2\Delta_c}{\Delta_c^2+\kappa^2}$$

Collective potential



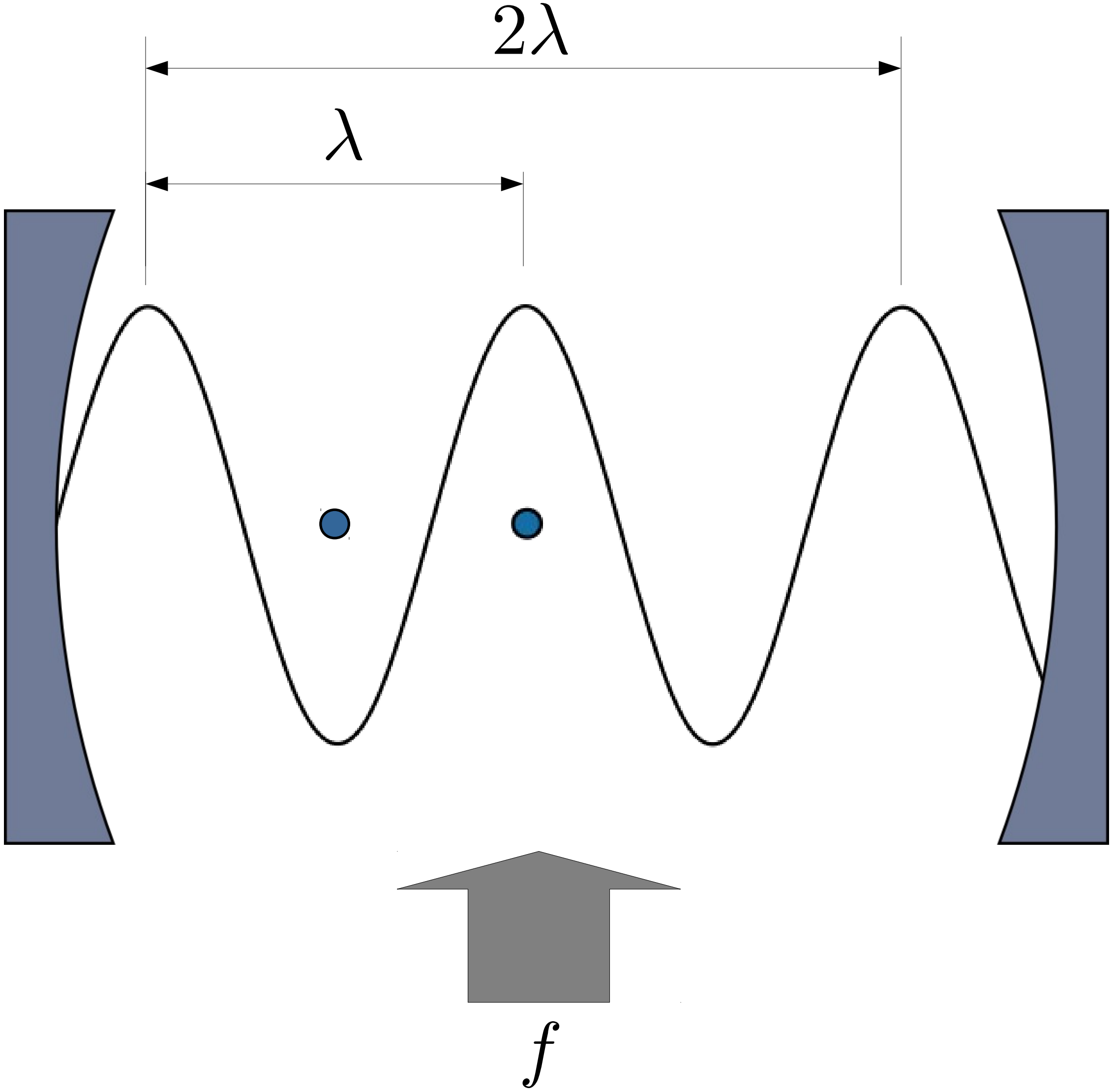
Collective potential

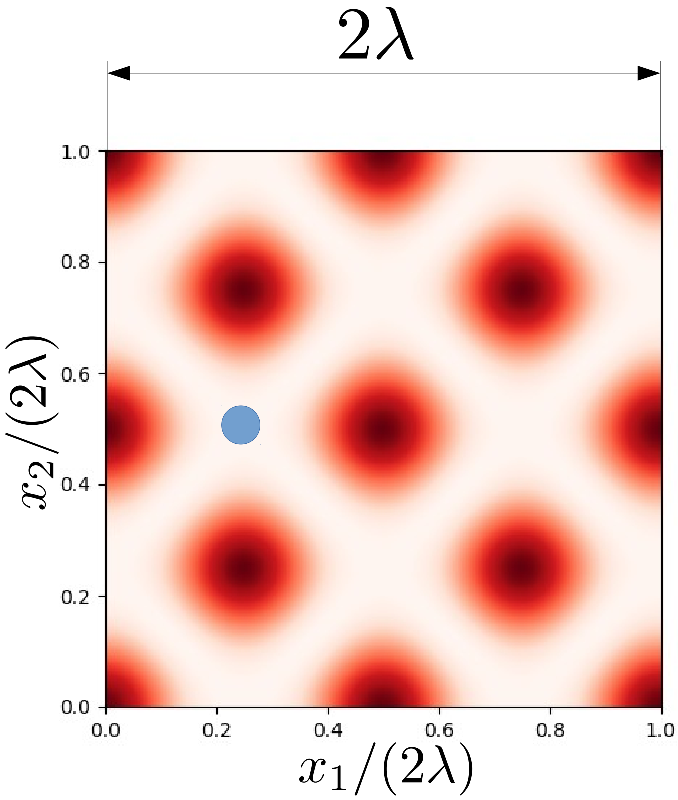




$$V(x_1, x_2) = -f \cdot \left(\sum_{j=1}^{2} \cos(k x_j) \right)^2$$

Collective potential





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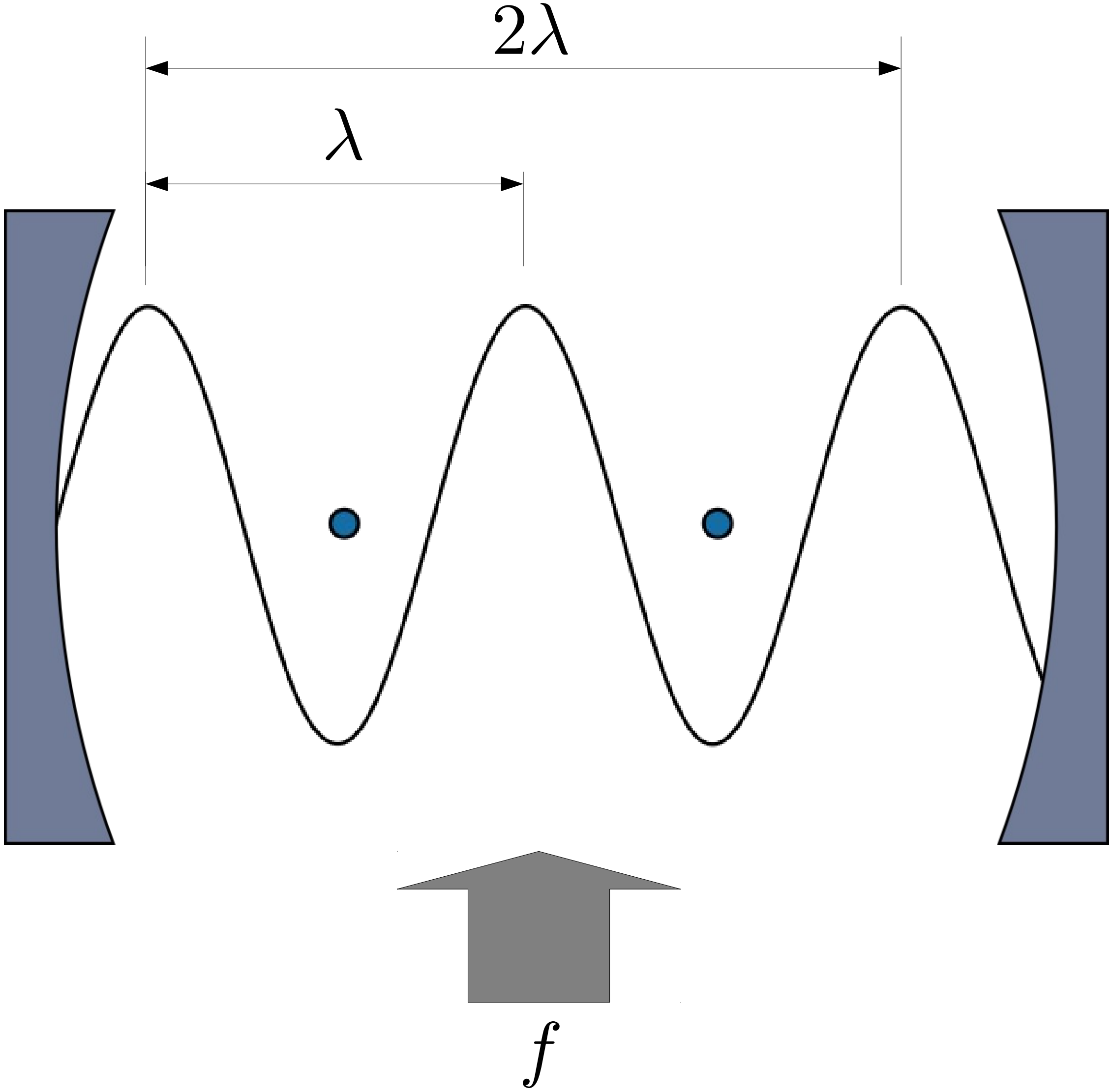

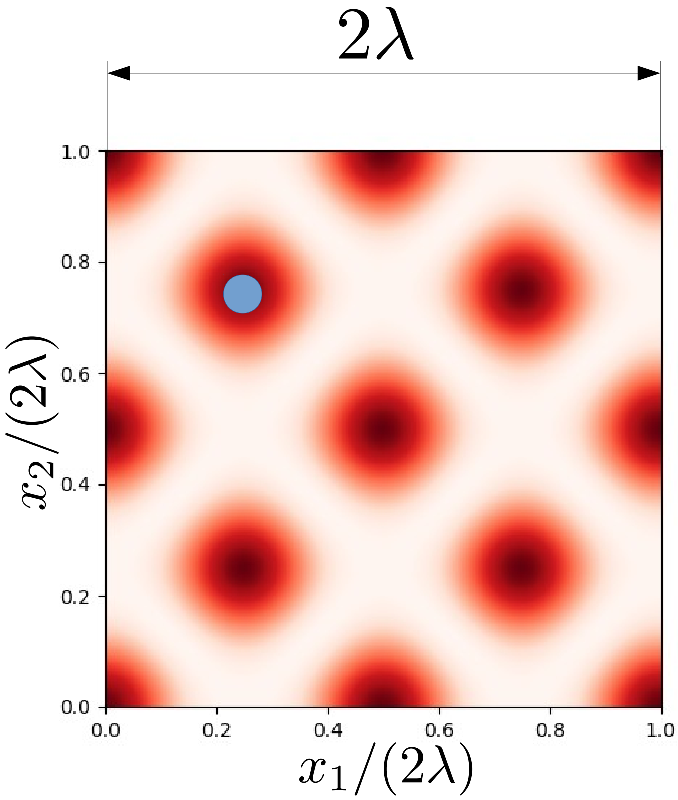

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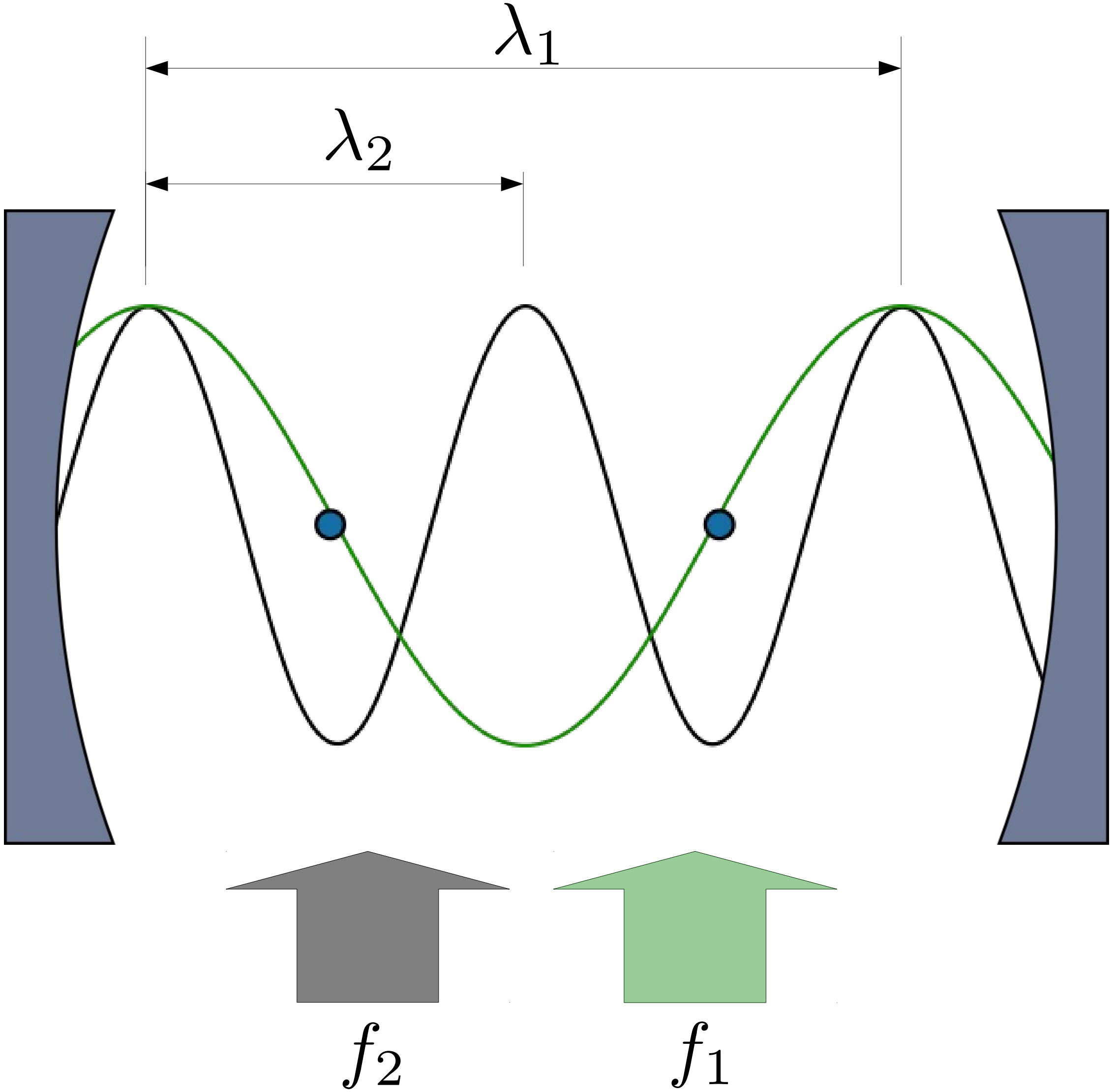
Multi-color pump



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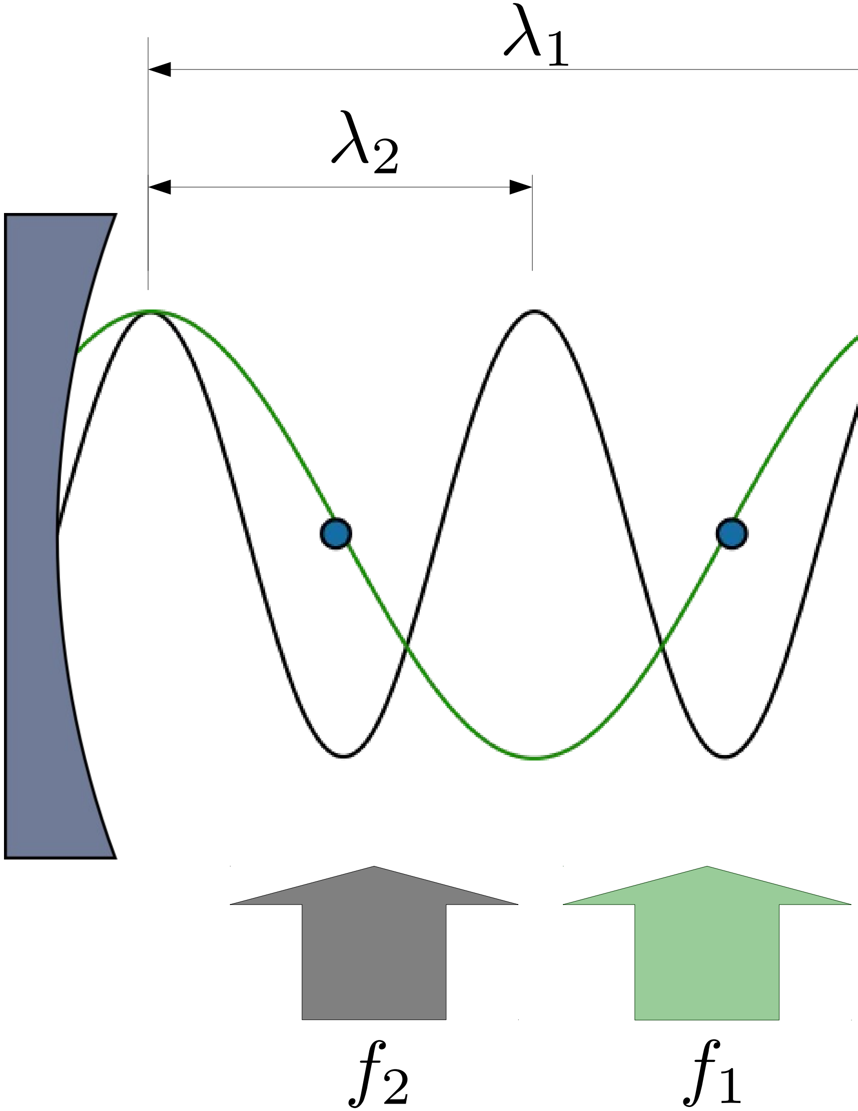
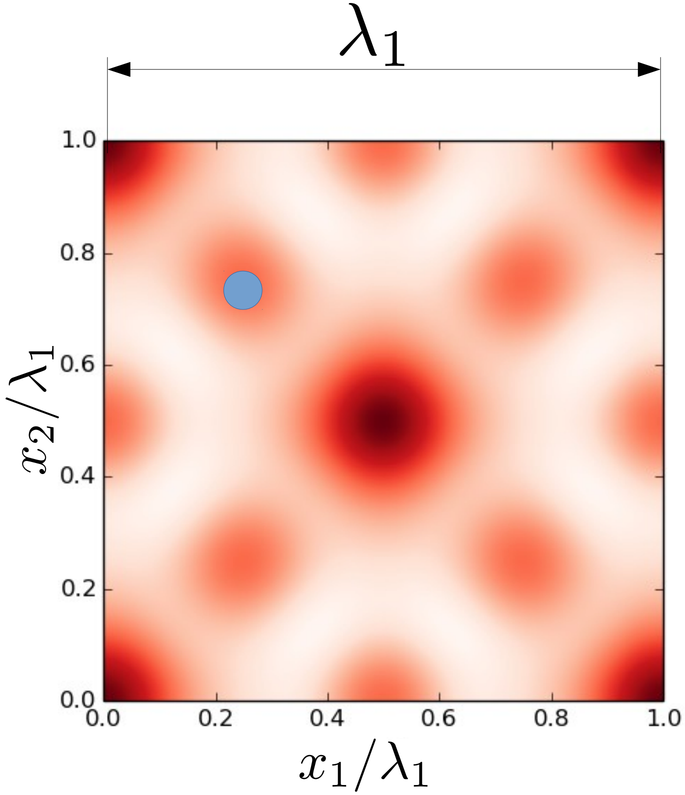
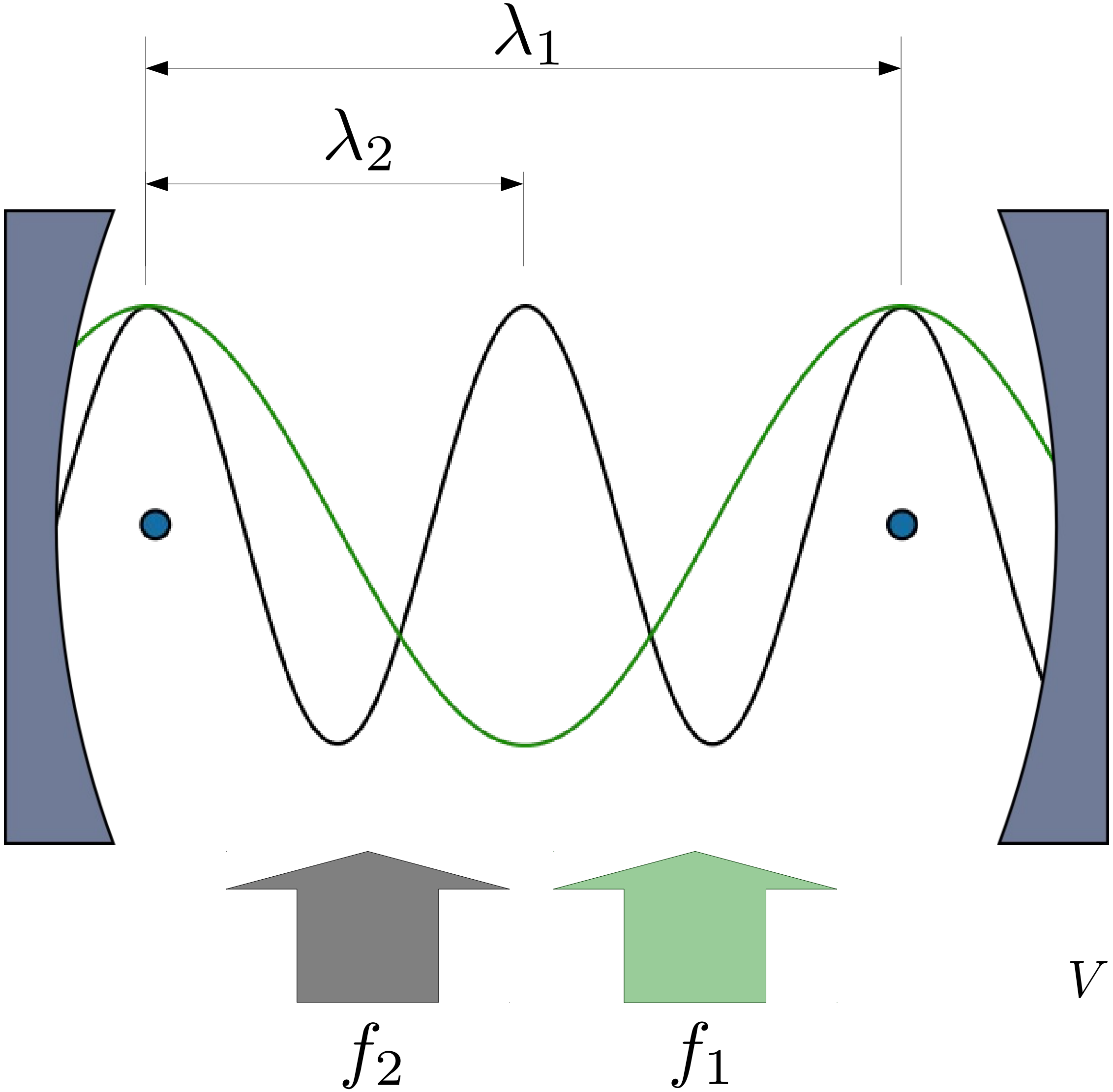
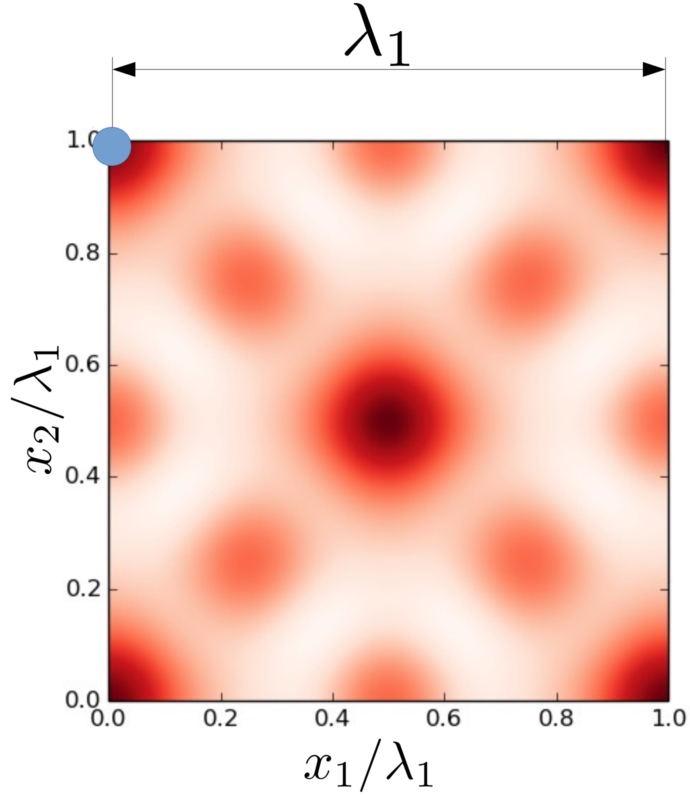




$$V(x_1,x_2) = -\sum_m f_m \left(\sum_{j=1}^{2} \cos(k_m x_j)\right)^2$$

Multi-color pump





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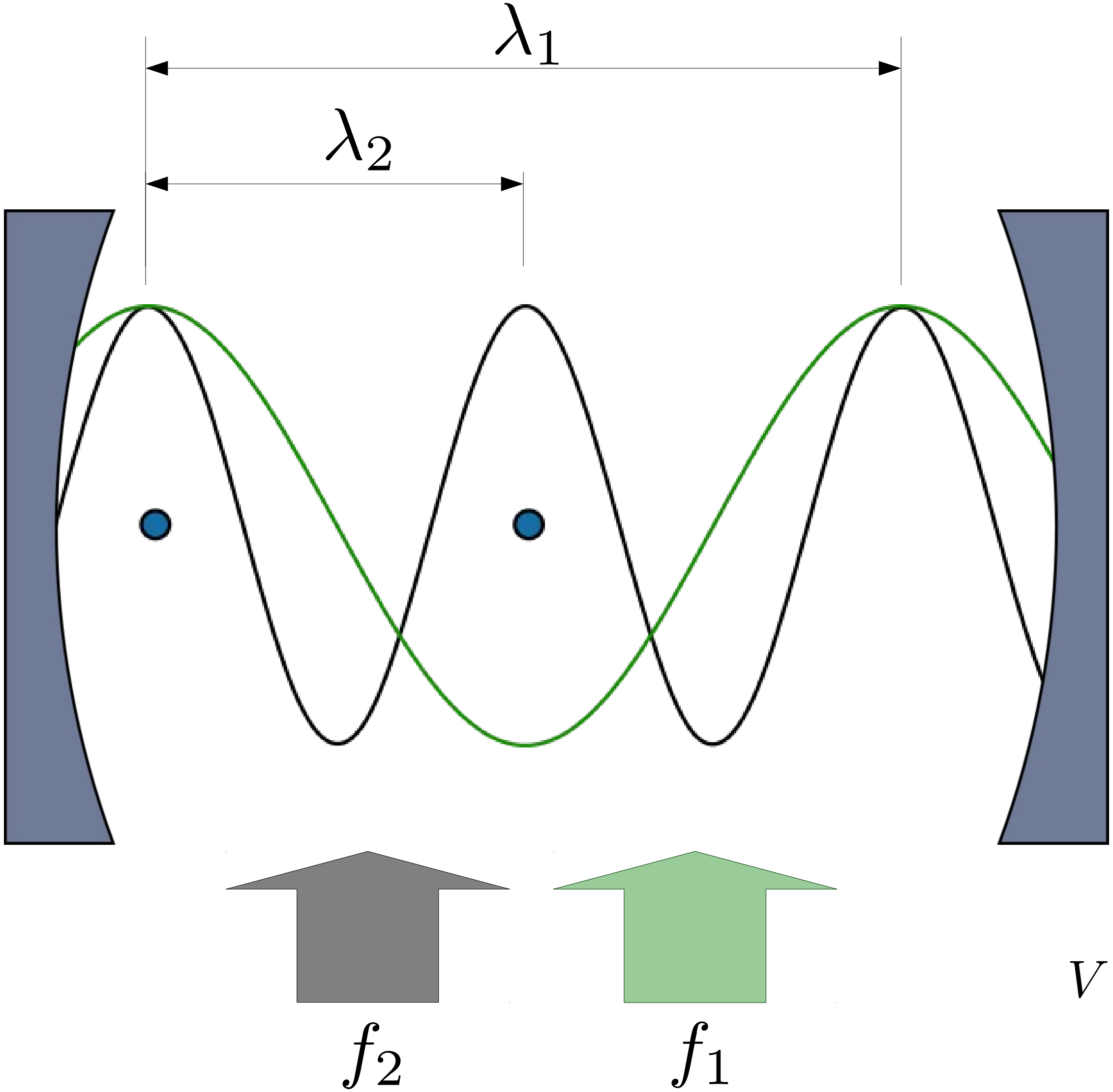

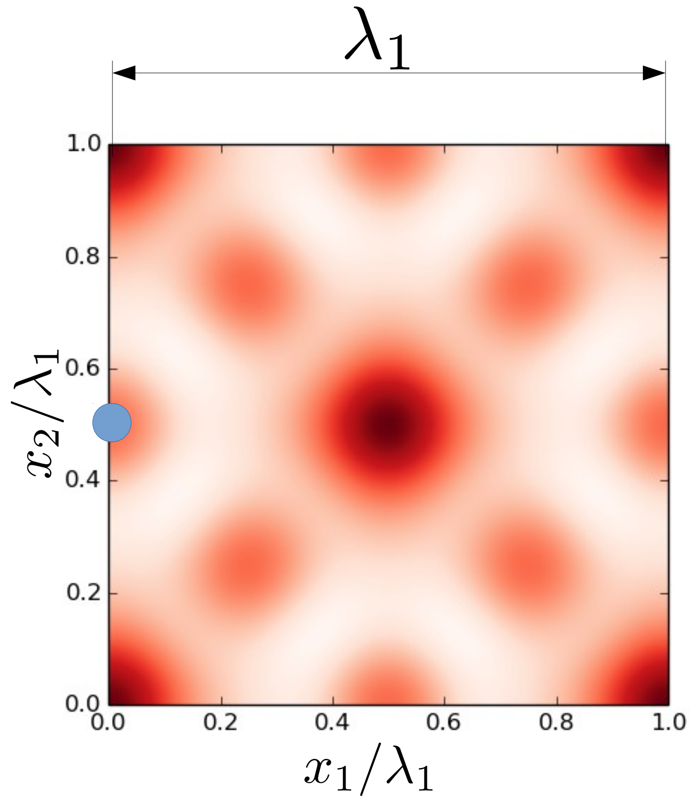

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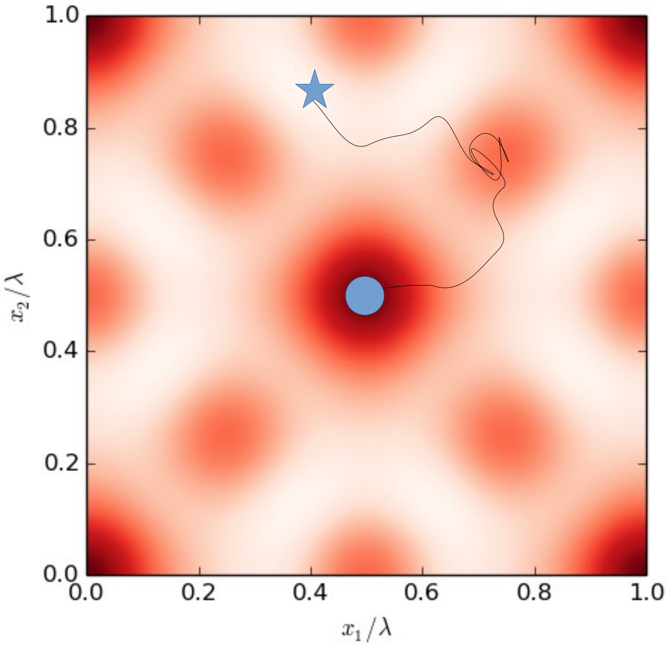
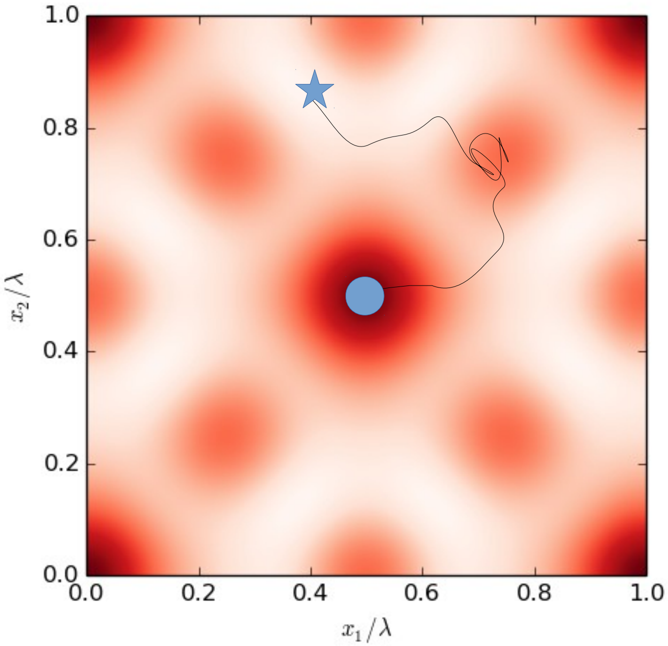
Self-optimizing dynamics

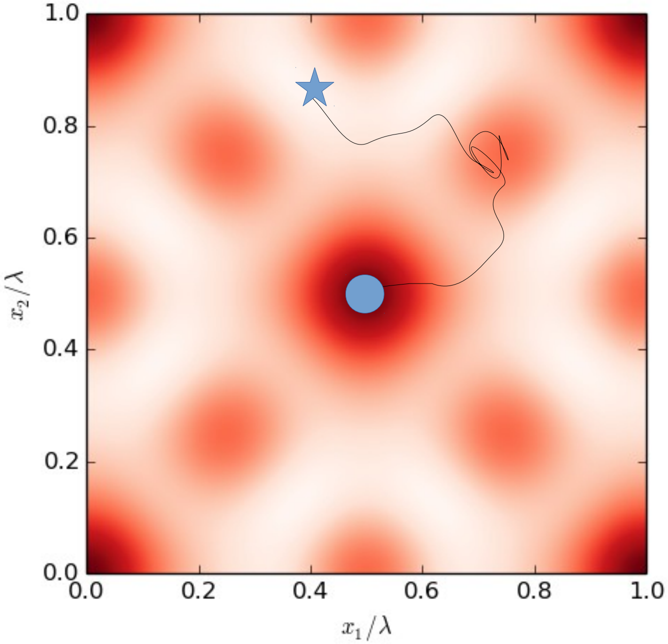


Self-optimizing dynamics

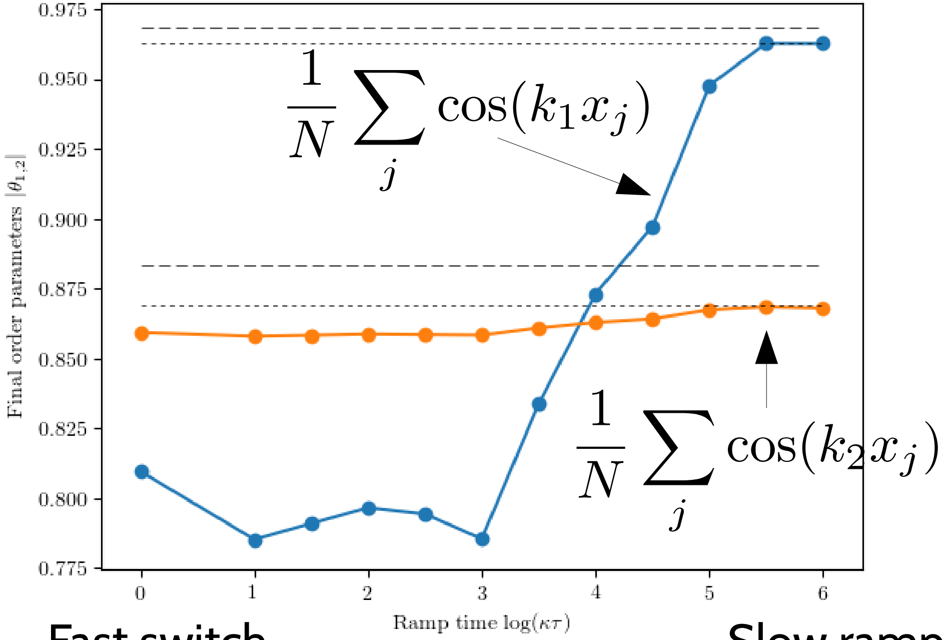


Does the system dynamically find a global minimum?

Self-optimizing dynamics



N = 100 particles



Fast switch

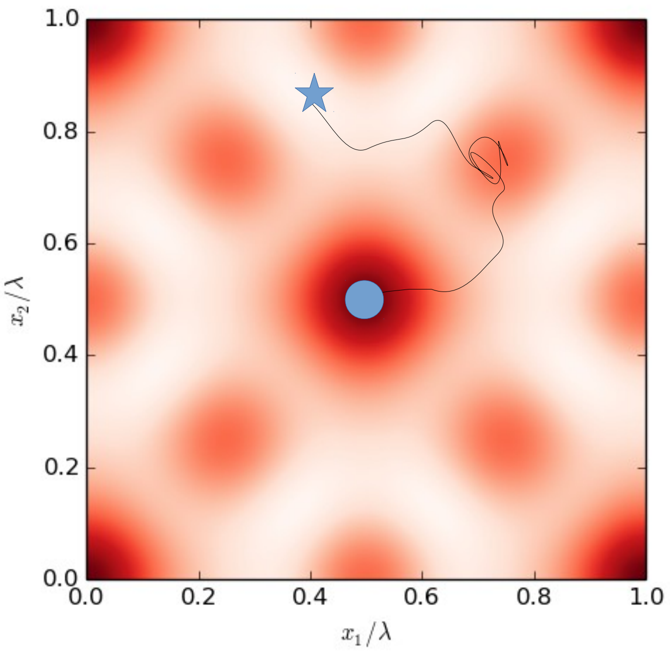
Slow ramp

Does the system dynamically find a global minimum?

Optimization methods

Simulated annealing

- Thermal fluctuations

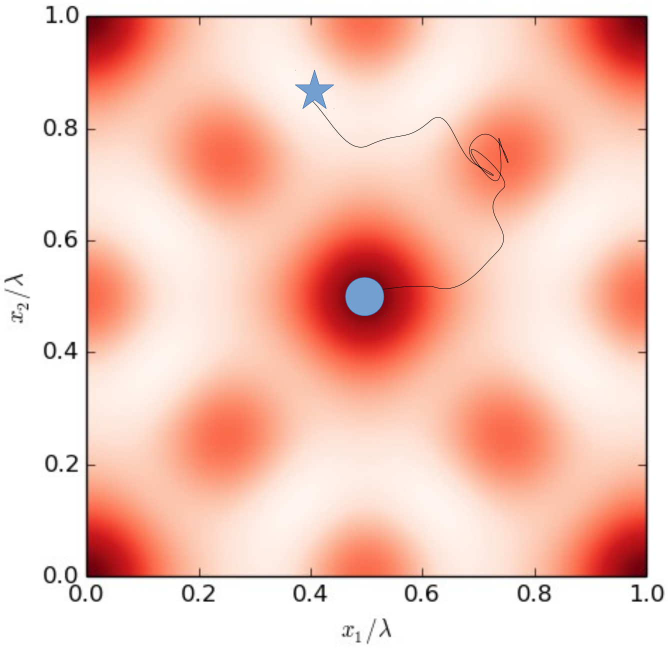


Kirkpatrick et al., Science 220.4598 (1983)

Optimization methods

Simulated annealing

- Thermal fluctuations



Quantum annealing

- Quantum fluctuations

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Kadowaki, Nishimori., PRE, 58.5 (1998)

Optimization methods

Simulated annealing

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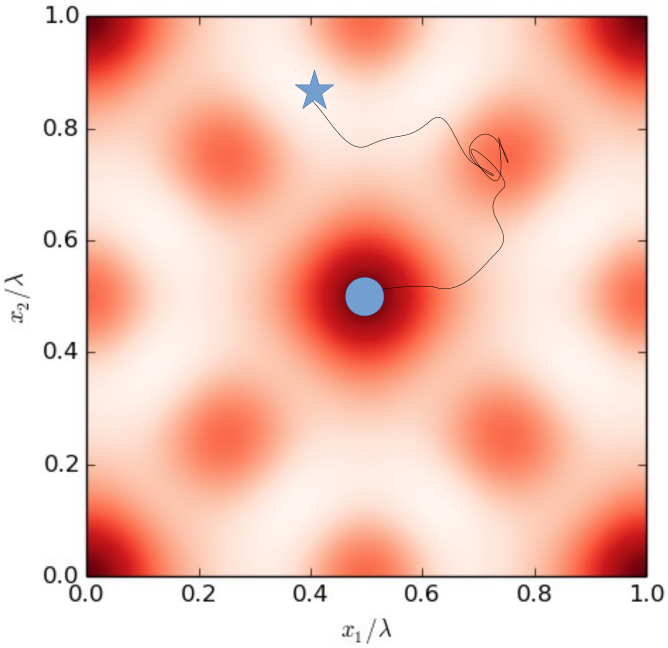

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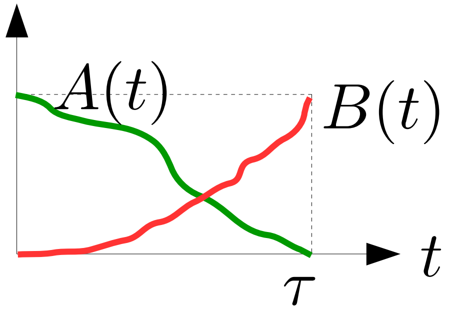
Quantum annealing

- Quantum fluctuations

$$H(t) = A(t)H_{\text{kin}} + B(t)H_{\text{prob}}$$



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Simulated annealing

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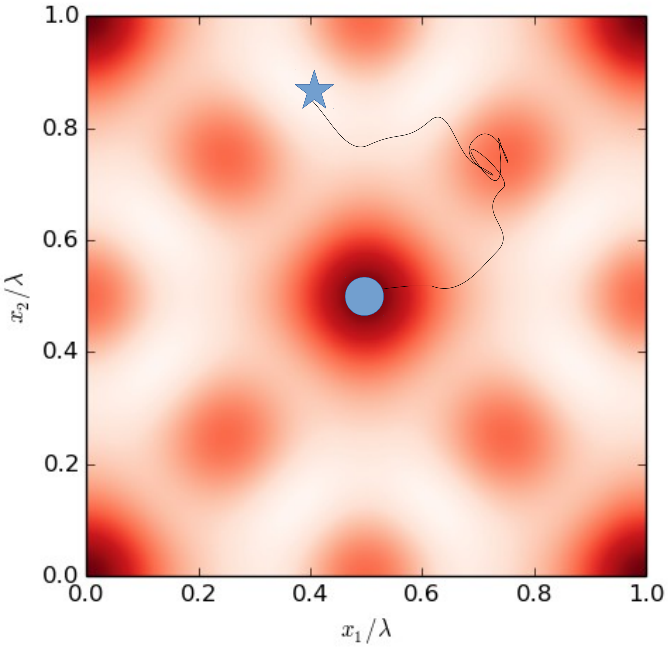

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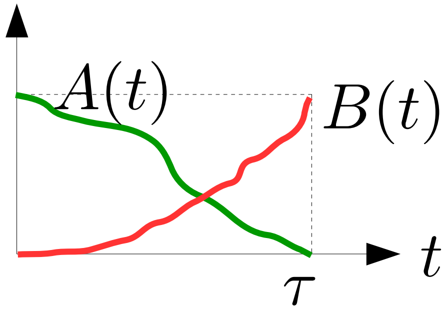
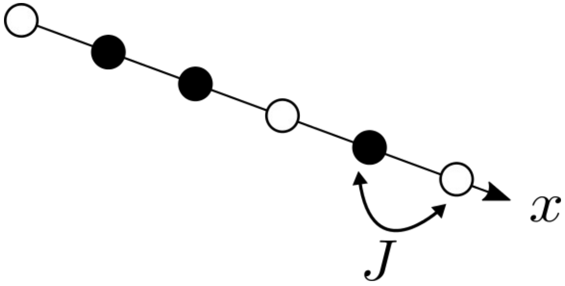


$$H_{\mathrm{prob}} = -\sum_{i,j} A_{ij}\sigma_i^z\sigma_j^z$$

$$H_{\mathrm{kin}} = -\sum_{i} \sigma_i^x$$

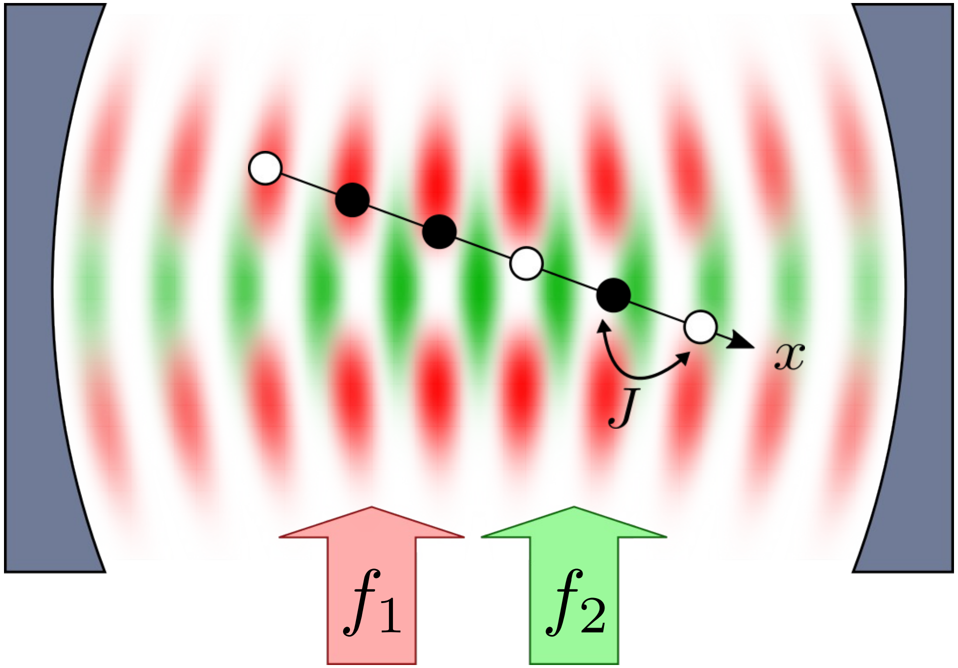
Kadowaki, Nishimori., PRE, 58.5 (1998)

Optical lattice in cavity



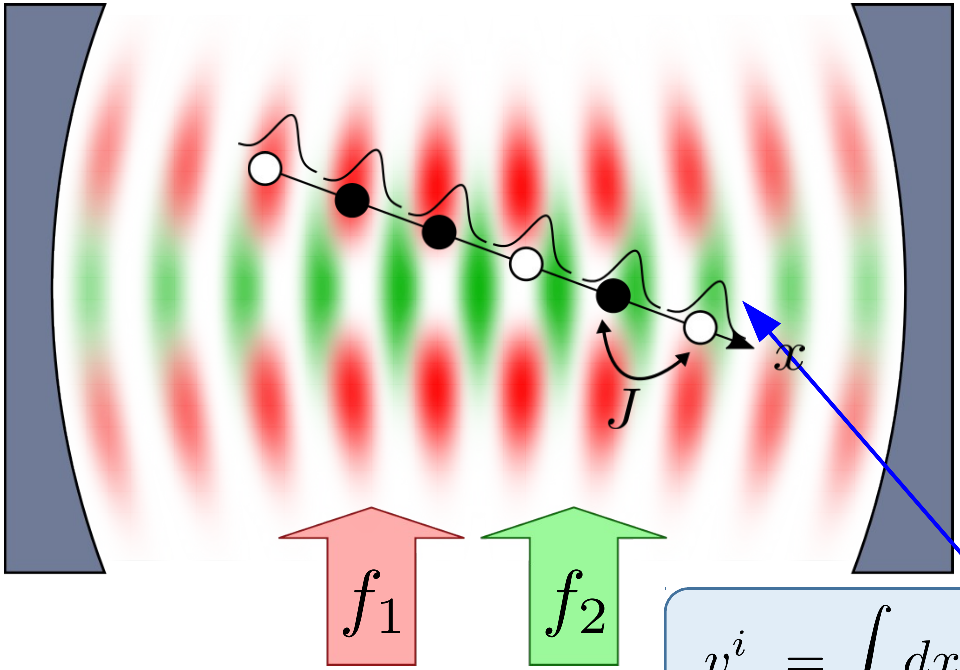
$$H = -J\sum_i (b^\dagger_{i+1} b_i + b^\dagger_i b_{i+1}) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i + 1)$$

Optical lattice in cavity



$$H = -J\sum_i (b^\dagger_{i+1}b_i + b^\dagger_i b_{i+1}) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i + 1)$$
$$-\sum_m f_m \left|\sum_i v^i_m \hat{n}_i\right|^2$$

Optical lattice in cavity



$$v_m^i = \int dx\, w^2(x - x_i) u_{p,m}(x) u^*_{c,m}(x)$$

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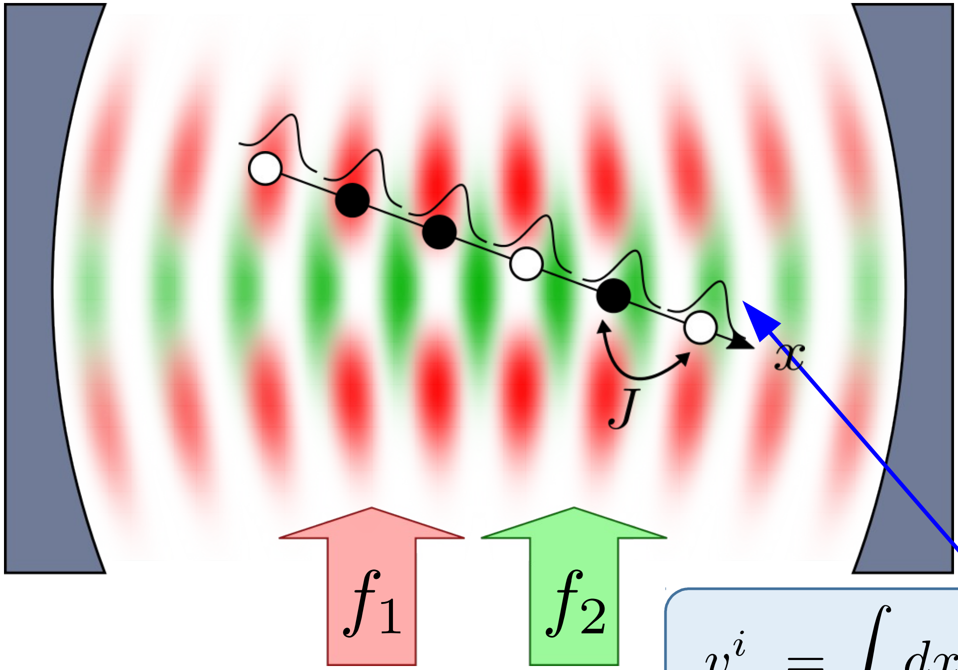
Optical lattice in cavity



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C Maschler, H Ritsch, PRL 95, 260501 (2005)
I Mekhov, H Ritsch, Laser physics 19.4 (2009)
R Landig et al., Nature (2016)

All-to-all interactions

$$H_{\text{int}} = -\sum_m f_m \left| \sum_i v_m^i \hat{n}_i \right|^2$$

All-to-all interactions

$$H_{\text{int}} = -\sum_{m} f_m \left| \sum_{i} v_m^i \hat{n}_i \right|^2 = -\sum_{i,j} \sum_{m} f_m v_m^i (v_m^j)^* \hat{n}_i \hat{n}_j$$

All-to-all interactions

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$$= \zeta \cdot A_{ij}$$

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Light fields → Interaction matrix

$$A_{ij} = \frac{1}{\zeta} \sum_m f_m \mathrm{Re}(v_m^i (v_m^j)^*)$$

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Easy to change

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Easy to change

Fixed by geometry

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Interaction matrix → Light fields

$$f_m(A) = \zeta \sum_n (G^{-1})_{mn} \langle V_n, A \rangle$$

$$G_{mn} = \langle V_m, V_n \rangle$$

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VT, S Krämer, H Ritsch, PRA 95, 032310 (2017)

Quantum annealing

$$H = -J \sum_i (b_{i+1}^\dagger b_i + b_i^\dagger b_{i+1}) + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i + 1) - \zeta(t) \sum_{i,j} A_{ij} \hat{n}_i \hat{n}_j$$

Quantum annealing

$$\begin{aligned} H = &- J\sum_i (b^\dagger_{i+1} b_i + b^\dagger_i b_{i+1}) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i + 1) \\ &- \zeta(t)\sum_{i,j} A_{ij}\hat{n}_i\hat{n}_j \end{aligned}$$

$$\zeta(t) = ||\sum_m f_m(t) V_m||$$

Quantum annealing

$$H = -J\sum_i (b^\dagger_{i+1} b_i + b^\dagger_i b_{i+1}) + \underset{\gg J, \zeta}{\boxed{\frac{U}{2}}} \sum_i \hat{n}_i(\hat{n}_i + 1)$$

$$- \zeta(t) \sum_{i,j} A_{ij} \hat{n}_i \hat{n}_j$$

$$\boxed{\begin{aligned} b_i &\rightarrow \sigma_i \\ \hat{n}_i &\rightarrow (\sigma_i^z + 1)/2 \end{aligned}}$$

$$\zeta(t) = ||\sum_m f_m(t) V_m||$$

Quantum annealing

$$H = - J \sum_i (\sigma^\dagger_{i+1}\sigma_i + \sigma^\dagger_i \sigma_{i+1}) - \frac{\zeta(t)}{4}\left(\sum_{i,j} A_{ij}\sigma^z_i\sigma^z_j + \sum_i \left[2\sum_j A_{ij}\right]\sigma^z_i\right) .$$

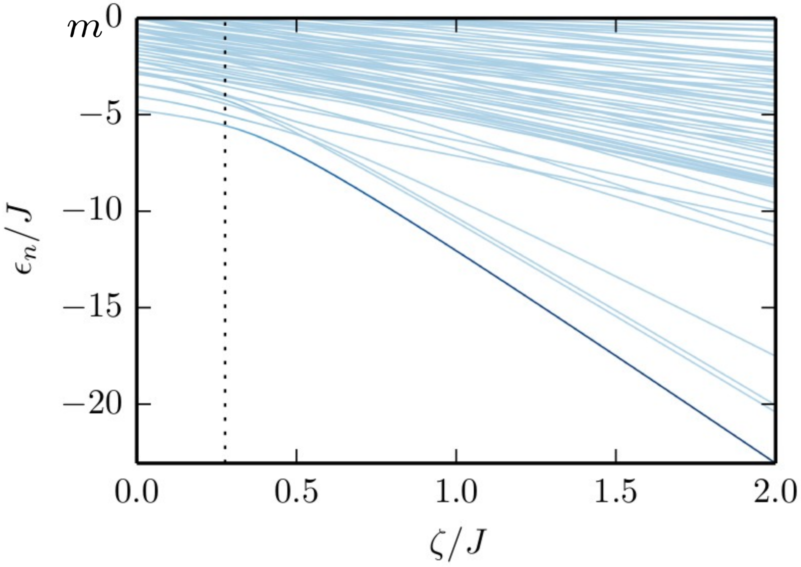
$$\zeta(t) = || \sum_m f_m(t) V_m ||$$

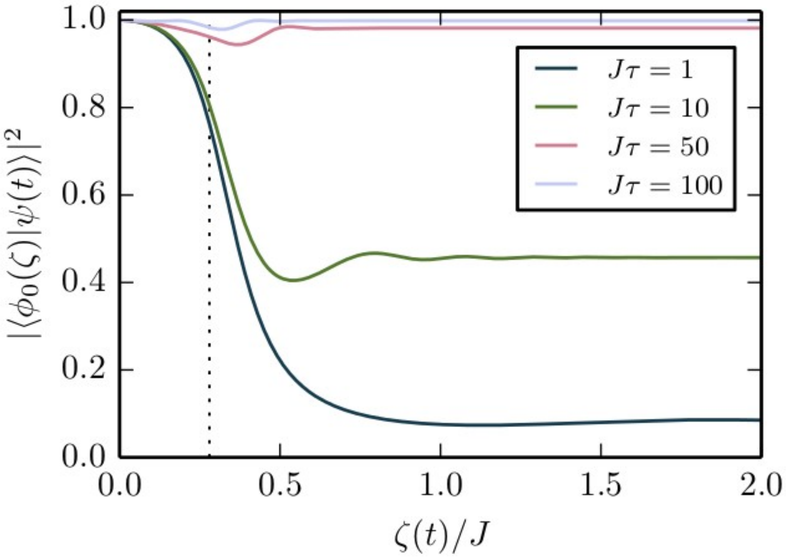
Quantum annealing

$$H = -J\sum_i(\sigma^\dagger_{i+1}\sigma_i + \sigma^\dagger_i\sigma_{i+1})$$

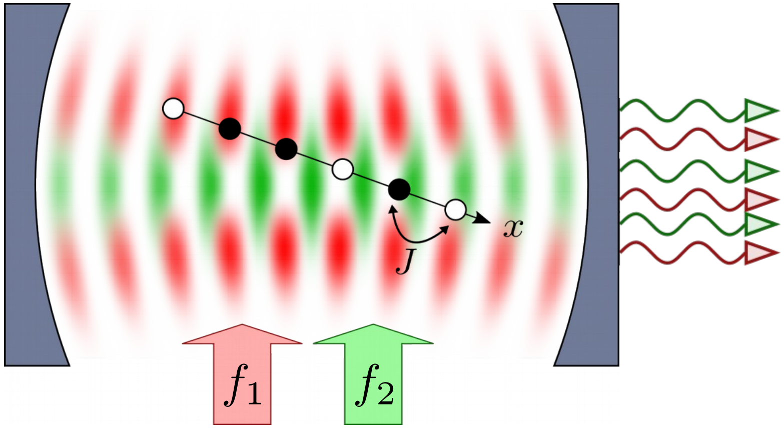
$$-\frac{\zeta(t)}{4}\left(\sum_{i,j} A_{ij}\sigma^z_i\sigma^z_j + \sum_i\left[2\sum_j A_{ij}\right]\sigma^z_i\right).$$

$\zeta(t) = ||\sum_m f_m(t)V_m||$





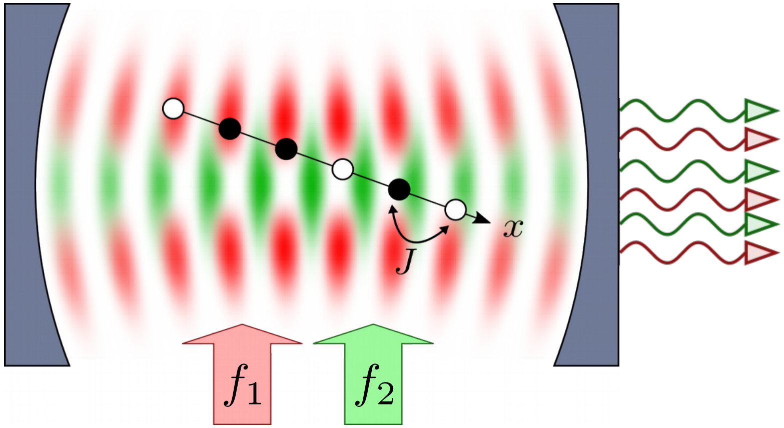
Open system dynamics



$$\dot{\rho} = \mathcal{L}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_m \kappa \left(2a_m\rho a_m^\dagger - a_m^\dagger a_m\rho - \rho a_m^\dagger a_m\right)$$

$$\begin{aligned} H =& H_{\mathrm{BH}} - \hbar\sum_m \Delta_{c,m} a_m^\dagger a_m \\ &+ \hbar\sum_m \eta_m \sum_i ((v_m^i)^* a_m + v_m^i a_m^\dagger)\hat{n}_i \end{aligned}$$

Open system dynamics



$$\dot{\rho} = \mathcal{L}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_m \kappa \left(2a_m\rho a_m^\dagger - a_m^\dagger a_m\rho - \rho a_m^\dagger a_m\right)$$

$$H = H_{\mathrm{BH}} - \hbar\sum_m \Delta_{c,m} a_m^\dagger a_m$$

$$+ \hbar\sum_m \eta_m \sum_i ((v_m^i)^* a_m + v_m^i a_m^\dagger)\hat{n}_i$$

Combine coherent sweep with cavity cooling!

Conclusion

- 2 main ingredients:
  - Dynamical cavity field → global interaction
  - Many modes → tailor interaction matrix

- Annealing dynamics by pump intensity ramp

- States can be measured via cavity output field

Thank you for your attention!

Semi-classical model (1D)

Semi-classical approximation yields stochastic differential equations:

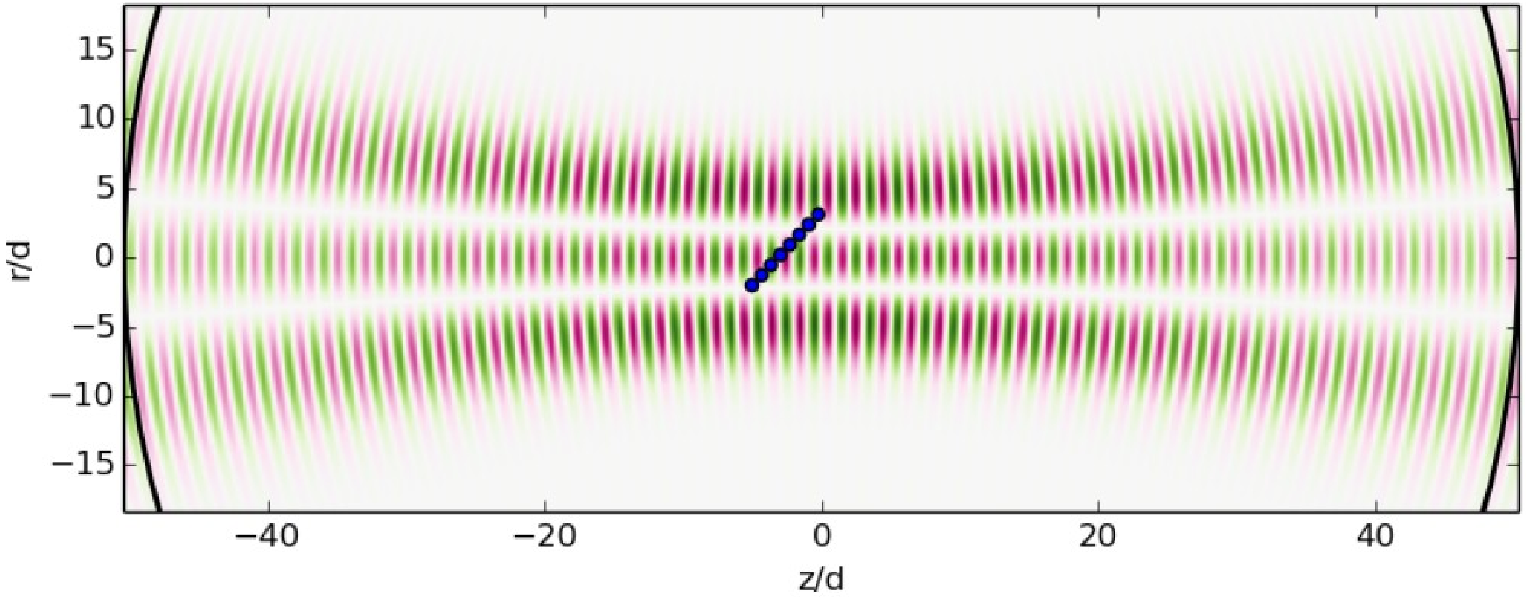
$$\dot{x}_j = \frac{p_j}{m}$$

$$\dot{p}_j = -\hbar \sum_n k_n \left(U_{0,n} |\alpha_n|^2 \sin(2k_n x_j) + \eta_n (\alpha_n + \alpha_n^*) \cos(k_n x_j) \right)$$

$$\dot{\alpha}_n = i \left(\delta_{c,n} - U_{0,n} \sum_j \sin^2(k_n x_j) \right) \alpha_n - \kappa_n \alpha_n - i\eta_n \sum_j \sin(k_n x_j) + \xi$$

with $U_{0,n} = g_n^2/\delta_a$, $\eta_n = g_n \tilde{\eta}_n/\delta_a$.

Optical lattice in a cavity



Example: 8 sites, 4 atoms, 36 modes.

Figure: TEM20 mode for $n = 100$.

Hopfield associative memory

Dynamics from update rule:

$$s_i \leftarrow \begin{cases} +1 & \text{if } \sum_j W_{ij} s_j \geq 0, \\ -1 & \text{otherwise.} \end{cases}$$

Energy function:

$$E(\boldsymbol{s}) = -\sum_{i<j} W_{ij} s_i s_j$$

Memory: $M = \{\boldsymbol{w}_p\}_{p=1,\ldots,P}$

Hebbian learning rule: $W_{ij} = \frac{1}{P} \sum_p w_p^i w_p^j$

Hopfield network

Memory: $M = \{\boldsymbol{w}_p\}_{p=1,\ldots,P}$

Recall pattern: $\boldsymbol{\chi}$

Interaction matrix:

$$A_{ij} = \frac{1}{P}\sum_p w_p^i w_p^j + \nu\chi_i\delta_{ij} = W_{ij} + \nu\chi_i\delta_{ij}$$

Hamiltonian:

$$H = -J\sum_i (\sigma_{i+1}^\dagger \sigma_i + \sigma_i^\dagger \sigma_{i+1}) - \frac{\zeta}{2}\left(\sum_{i<j} W_{ij}\sigma_i^z\sigma_j^z + \nu\sum_i \chi_i\sigma_i^z\right)$$

J. J. Hopfield, Proc. Natl. Acad. Sci. USA **79**, 2554 (1982)

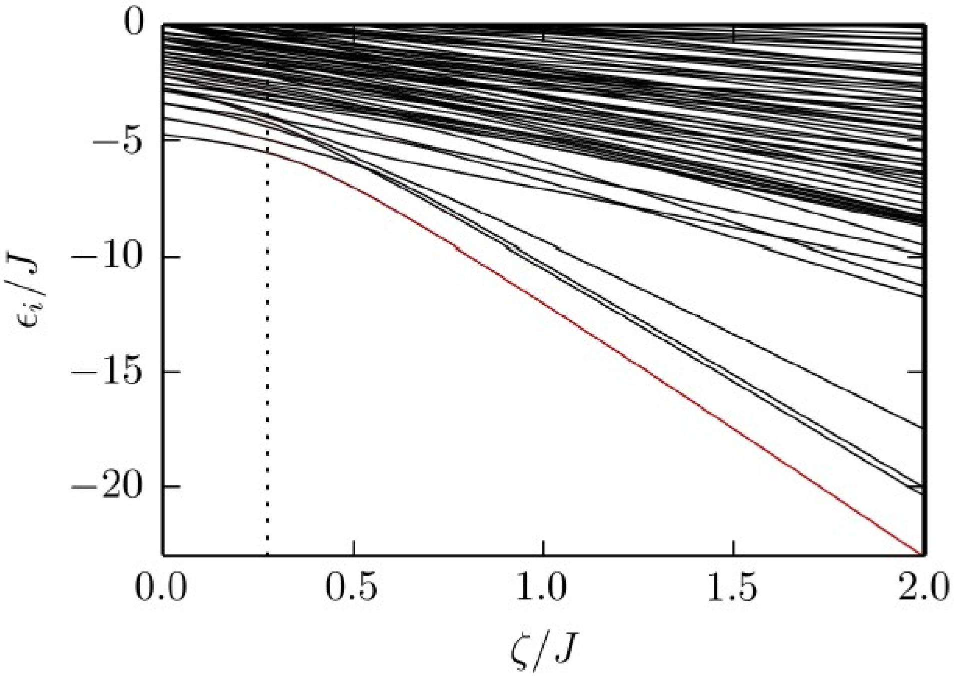
S. Santra, O. Shehab, R. Balu, arXiv:1602.08149v1 [quant-ph] (2016)

Recall of χ_1

Memory:
$\boldsymbol{w}_1 = (1, 1, 0, 0, 1, 0, 1, 0)$
$\boldsymbol{w}_2 = (1, 1, 0, 1, 1, 0, 0, 0)$

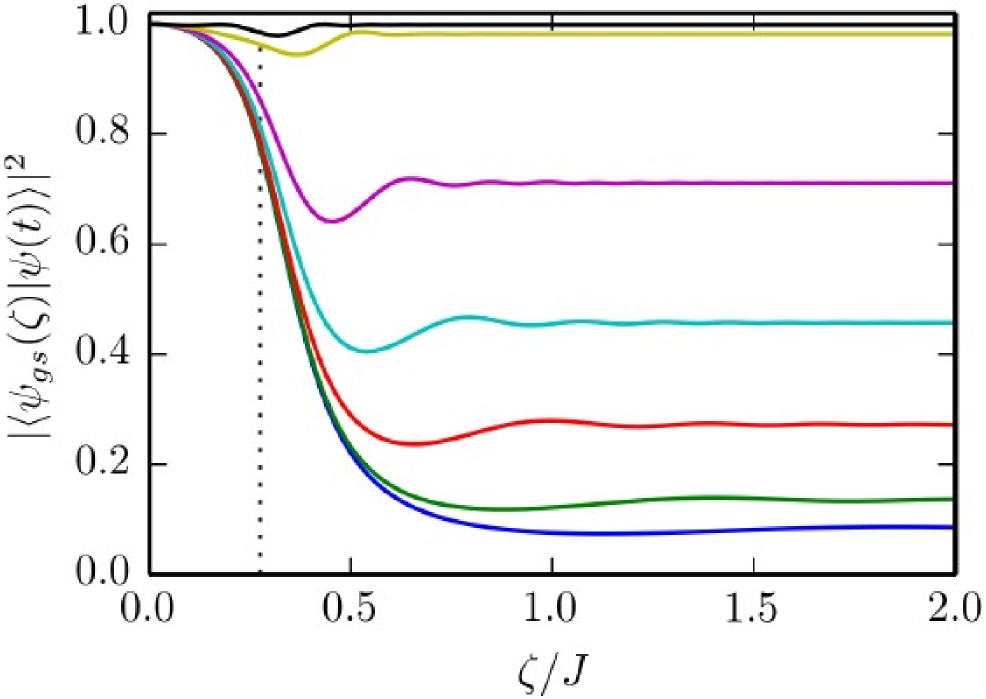
Input state:
$\boldsymbol{\chi}_1 = (1, 1, 1, 0, 0, 0, 1, 0)$

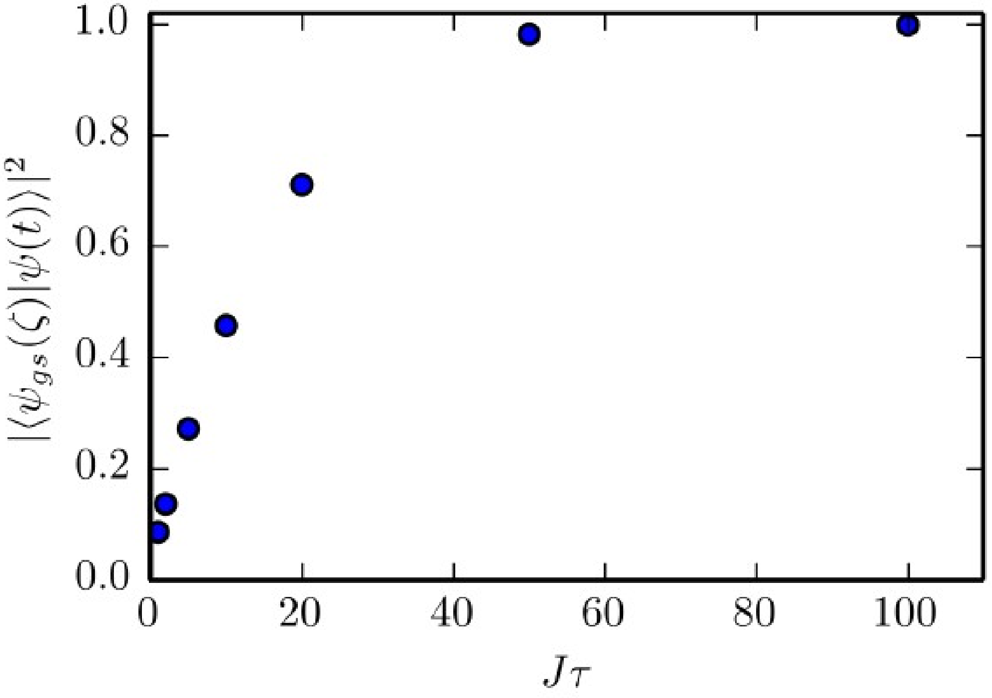
Bias strength:
$\nu = 0.7$



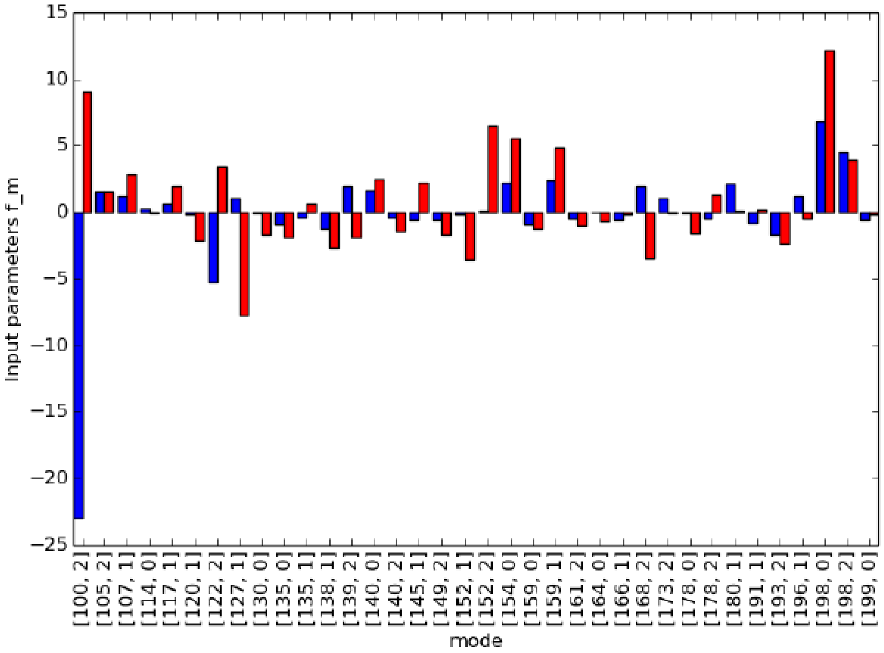
Minimum gap $\Delta = 0.56J$ at $\zeta = 0.28J$.

Recall of χ_1





Input parameters



$$f_m = -\hbar\delta_{c,m}\eta_m^2/(\delta_{c,m}^2 + \kappa_m^2).$$

Blue: Recall $\boldsymbol{\chi}_1$, Red: Recall $\boldsymbol{\chi}_2$

$A_{ij} = W_{ij} + \nu\chi_i\delta_{ij}$

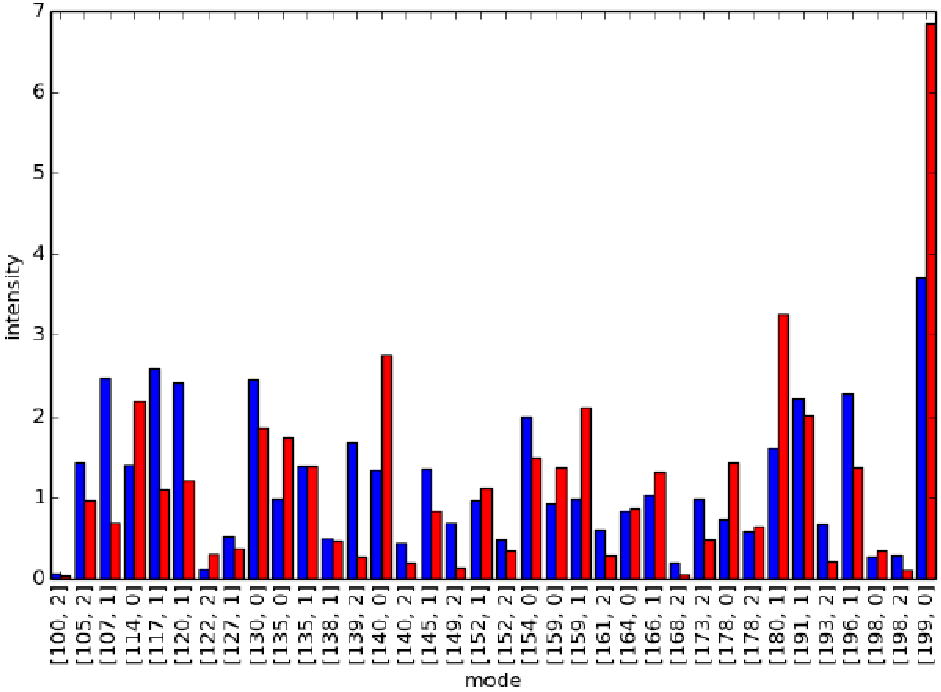
Cavity output

Field:

$$\langle a_m \rangle = \frac{\eta_m}{\delta_{c,m} + i\kappa_m} \sum_i v_m^i \langle \hat{n}_i \rangle .$$

Intensity:

$$\langle a_m^\dagger a_m \rangle = \frac{\eta_m^2}{\delta_{c,m}^2 + \kappa_m^2} \sum_{i,j} V_m^{ij} \langle \hat{n}_i \hat{n}_j \rangle .$$



Blue: $\boldsymbol{w}_1$, Red: $\boldsymbol{w}_2$