Time crystal behavior of excited eigenstates

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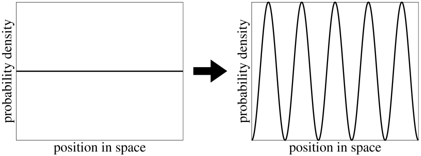
Space Crystal

$$\hat{H} = \sum_j \frac{\hat{\vec{p}}_j^{\,2}}{2m_j} + \sum_{i<j} V(\vec{r}_i - \vec{r}_j)$$

invariant under translation of all particles by the same vector in space and so are the eigenstates.

Spontaneous symmetry breaking
$\Longrightarrow$ Space crystal formation

ground state $t = const$

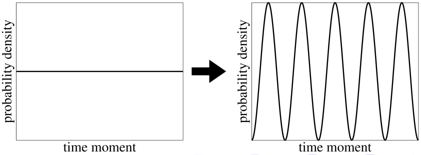


Time Crystal

- The time-independent hamiltonian is also invariant under time translations.
- Analogy to the case of space crystal formation:

Spontaneous breaking of continuous time translation symmetry
$\Longrightarrow$ Time crystal formation

ground state $\vec{r} = const$



Discrete Time Crystals

Theoretical prediction:

K. Sacha, PRA **91**, 033617 (2015).

V. Khemani et al., PRL **116**, 250401 (2016).

D. V. Else et al., PRL **117**, 090402 (2016).

Experiments:

J. Zhang et al., Nature **543**, 217 (2017).

S. Choi et al., Nature **543**, 221 (2017).

Mean-field approximation - SSB (CM localization)

Mean-field approximation $\Longrightarrow$ the Gross-Pitaevski equation (all particles occupy the same single-particle state)

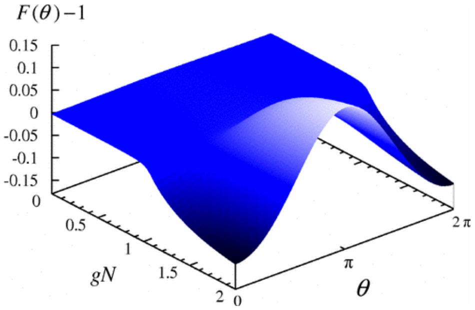
$$\left(-\frac{1}{2}\partial_x^2 + g_0(N-1)|\phi_0|^2\right)\phi_0 = \mu\phi_0$$

- GPE solution does depend only on $g_0(N-1)$.
- In thermodynamic limit we consider $N \to \infty$, $g_0 \to 0$ $g_0(N-1) = \text{const.}$
- The weaker interactions we have the longer lifetime we expect.
- **Phase transition:**

  When $g_0(N-1) < -\pi^2$

  particles want to keep each other

  GS: uniform solution $\Longrightarrow$ **non-uniform** solution



Fano factor: $F = \dfrac{\langle n^2(\theta)\rangle - \langle n(\theta)\rangle^2}{n(\theta)}$

R. Kanamoto, H. Saito, M. Ueda, Phys. Rev. A **73**, 033611 (2016).

Wilczek's idea of Time Crystal formation

N bosons on a 1D Aharonov-Bohm ring ($\alpha \leftrightarrow$ magnetic flux, $g_0 < 0$ and $m = \hbar = 1$).

$$H = \sum_{j=1}^{N} \frac{(p_j - \alpha)^2}{2} + \frac{g_0}{2} \sum_{i \neq j} \delta(x_i - x_j),$$

Switching to the CM coordinates (P - CM momentum, $\widetilde{H} \leftrightarrow$ relative d.o.f.)

$$H = \frac{(P - N\alpha)^2}{2N} + \widetilde{H}(\widetilde{x}_i, \widetilde{p}_i), \qquad P = P_j = 2\pi j, \qquad GS: \frac{\partial H}{\partial P_j} = 2\pi \frac{j}{N} - \alpha \approx 0.$$

SSB $\Longrightarrow$ CM localization

The CM probability current corresponds to ($P_N = 2\pi N$)

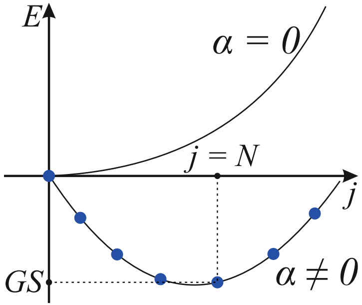
$$\dot{X}_{CM} \leftrightarrow \frac{\partial H}{\partial P_N} = 2\pi - \alpha \neq 0$$

In the limit $N \to \infty$ for all α exists j for which

$$\frac{\partial H}{\partial P_j} = 2\pi \frac{j}{N} - \alpha = 0$$

The eigenstate corresponding to $P_N = 2\pi N$ is **not** the ground state!

CM would not move in ground state!!!



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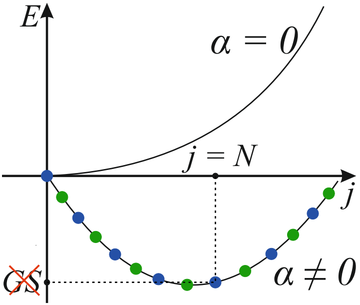

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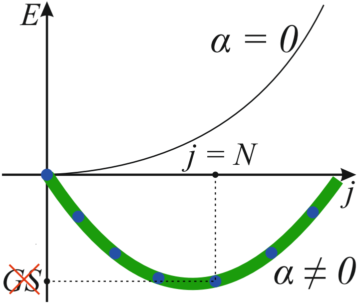
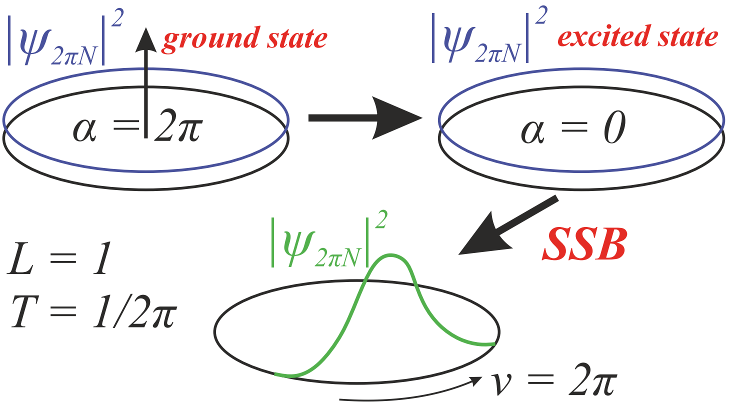


Idea of experiment:

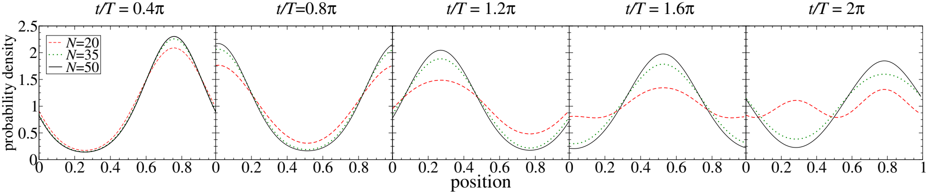


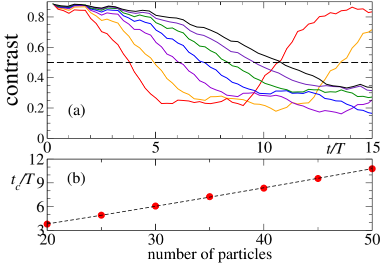
TIME CRYSTAL BEHAVIOR OF EXCITED EIGENSTATE!

Numerical simulations: exact diagonalization + evolution

We consider time evolution of density-density correlation function asssuming that the first particle was measured in time moment $t = 0$ at the position $x_1 = 0.5$ ($P = P_N$, $\alpha = 0$)

$$\rho_2(x,t) \propto \left\langle \psi_0^{P_N} \middle| \hat{\psi}^\dagger(x,t)\hat{\psi}(x,t)\hat{\psi}^\dagger(x_1,0)\hat{\psi}(x_1,0) \middle| \psi_0^{P_N} \right\rangle, \quad T = \frac{1}{2\pi}, \quad g_0(N-1) = -15.$$





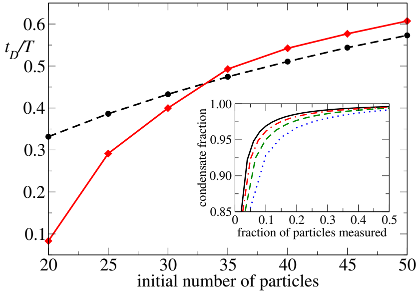
$$\text{contrast} = \frac{\max(\rho_2) - \min(\rho_2)}{\max(\rho_2) + \min(\rho_2)}$$

- The time evolution reveals smearing of ρ_2 in time.
- The lifetime grows linearly with N.
- Gaussian-like spread

$$\sigma(t) \propto \sqrt{\frac{\sigma^4 + \sigma^2 t^2}{\sigma^4 N^2}} \rightarrow \frac{t}{N}.$$

Numerical simulations: time evolution of CM distribution

- Measurement of 20% fraction of particles $\Longrightarrow$ CM is well localized.
- Time evolution of many-body CM distribution.
- t_D - time after the width of the CM distribution becomes significantly wider than the initial width ($\sigma^{CM}_{M-B} \approx \sigma_{GPE}/2$)



- Using Central Limit Theorem we may estimate the evolution of CM distribution

$$\sigma(t) \propto \sqrt{\frac{(\sigma/\sqrt{N})^4 + (\sigma/\sqrt{N})^2 t^2}{(\sigma/\sqrt{N})^4 N^2}} \to \frac{t}{\sqrt{N}}$$

- The many-body symmetry broken state evolution converges to the CLT predictions

Recapitulation

- Spontaneous breaking of a continuous time translation symmetry to the discrete symmetry in the time crystal model introduced by Frank Wilczek
- Spontaneous rotation of a non-uniform density can not be observed for large number of particles if the system is prepared in the ground state.
- Excited eigenstate, although the initial single particle density is uniform and does not display any motion, measurement of the position of a single particle reveals a rotation of the remaining particle cloud.
- The spontaneous rotation can be observed in ultra-cold atomic gases!

AS, J. Zakrzewski, K. Sacha, *Time crystal behavior of excited eigenstates*, arXiv:1702.05006 (2017)