Many–body localization due to random interactions

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Contents

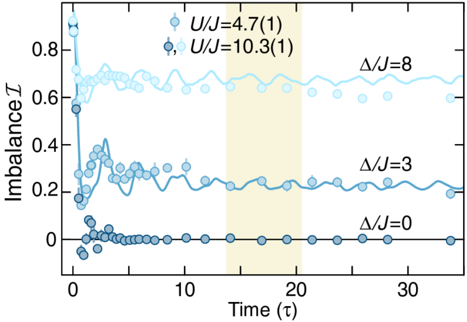
MBL in optical lattice

D. Basko, et. al., Ann. Phys. (NY) 321, 1126 (2006) – theoretical prediction of MBL

M. Schreiber et. al., Science **349**, *842 (2015)*:
Interacting fermions in quasi–random optical lattice:

$$H = -J\sum_{i,\sigma}\left(c^{\dagger}_{i,\sigma}c_{i+1,\sigma} + h.c.\right) + \Delta\sum_{i,\sigma}\cos(2\pi\beta i + \phi)c^{\dagger}_{i,\sigma}c_{i,\sigma} + U\sum_{i} n_{i,\uparrow}n_{i,\downarrow}$$

Time evolution of initial charge–density wave state:



- non–zero stationary imbalance – ergodicity breaking
- lack of thermalization – ETH is false
- the system is many–body localized
- fully localized single–particle spectrum

Bosons in optical lattice

Consider the Bose–Hubard model

$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \mu \sum_i \hat{n}_i$$

- Optical speckle field $\longrightarrow$ random on–site potential

$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i \mu_i \hat{n}_i$$

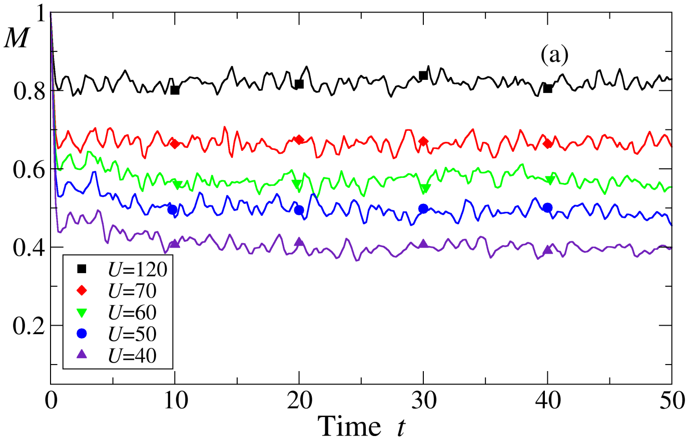
$\mu_i \in [-W/2, \ W/2]$

- Atom chip providing spatially random magnetic field $\longrightarrow$ random interactions
H. Gimperlein et. al., Phys. Rev. Lett. **95**, *170401 (2005)*

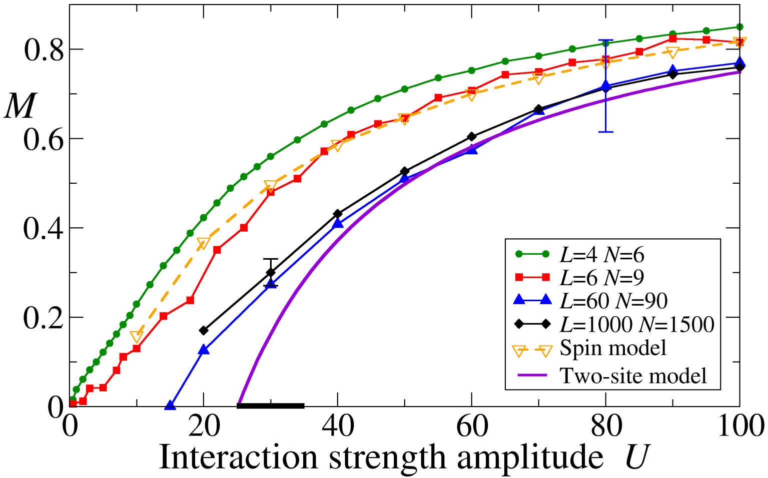
$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \frac{U_i}{2} \hat{n}_i(\hat{n}_i - 1) + \sum_i \mu \hat{n}_i$$

$U_i \in [0, \ U]$, delocalized single–particle spectrum

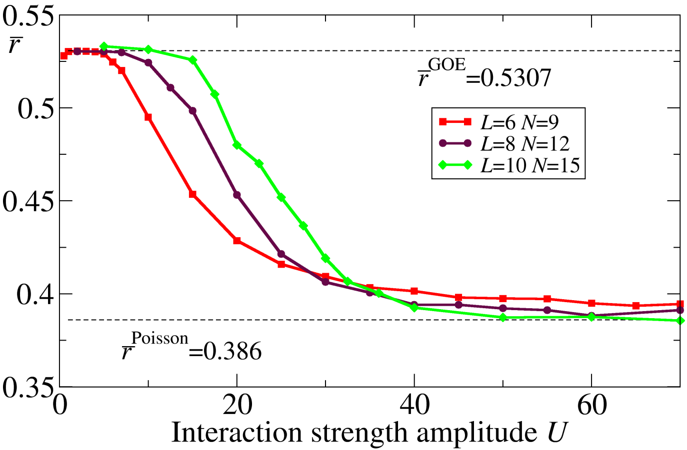
time evolution of the density wave state $|DW\rangle = |2121...\rangle$
define: $M = 3(N_e - N_o)/(N_e + N_o)$



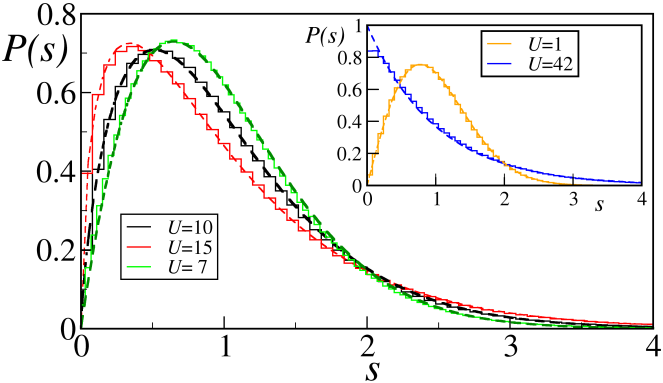
Stationary value of magnetization



Average ratio of adjacent energy gaps:
$r_n = \min[\delta^E_n, \delta^E_{n-1}] / \max[\delta^E_n, \delta^E_{n-1}]$ with $\delta^E_n = E_n - E_{n-1}$



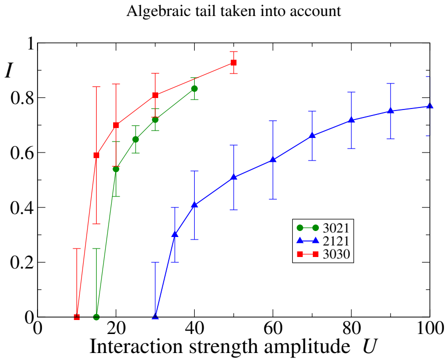
Level spacing distribution

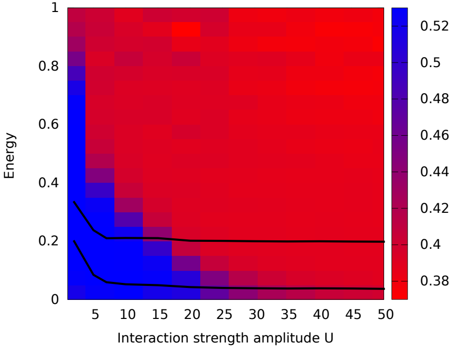


- $U = 1$: $P(s) \propto s\mathrm{e}^{-\frac{\pi}{4}s^2}$ – GOE statistics
- $U = 7$: $P(s) \propto s^{\beta}\mathrm{e}^{-C_2 s^{2-\gamma}}$ – M. Serbyn, J.E. Moore, PRB 93, 041424 (2016)
- $U = 10, 15$: $P(s) \propto s^{\beta}\mathrm{e}^{-(\beta+1)s}$ – Semi–Poisson
- $U = 42$: $P(s) \propto \mathrm{e}^{-s}$ – Poisson statistics

Mobility edge

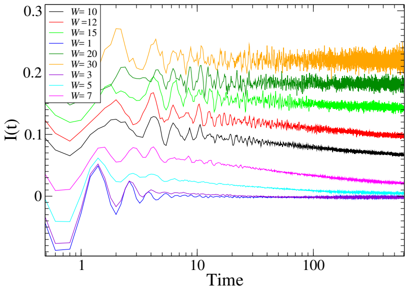
Localization transition depends strongly on the energy density

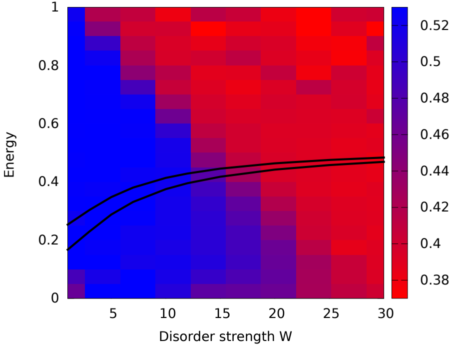




Random chemical potential

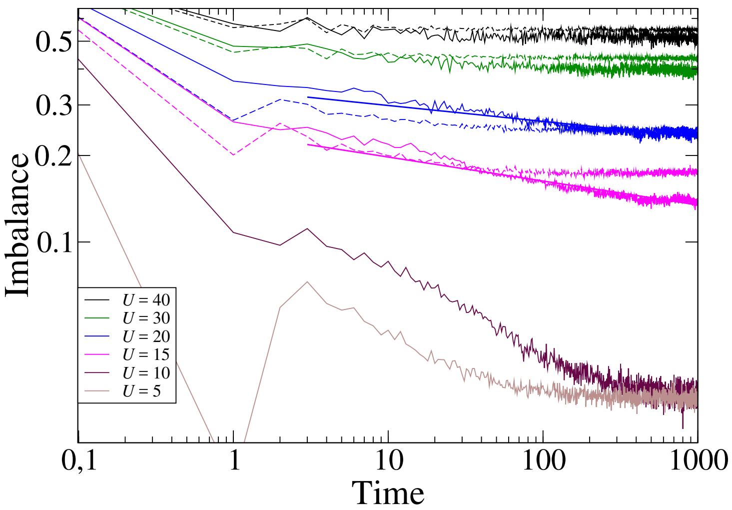
$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i \mu_i \hat{n}_i$$





Finite size effects

$L = 8\ N = 12$ vs $L = 6\ N = 9$



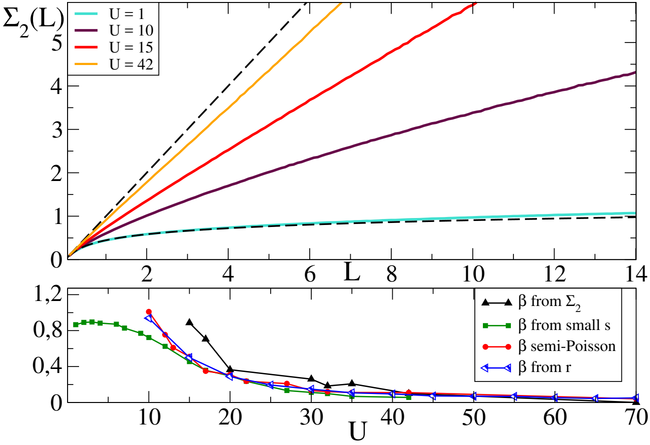
Conlcusion

- random interactions - MBL occurs solely due to the interactions in the system with extended single–particle states
- the models are experimentally accesible
- bosons allow for much larger density variations than fermions
- more details about random interactions case in PRA **95**, 021601(R), 2017

Number variance

$\mathcal{N}(L)$ - number of energy levels in interval $(E, E+L)$,
$\mathrm{Var}\left[\mathcal{N}(L)\right]$ fulfills

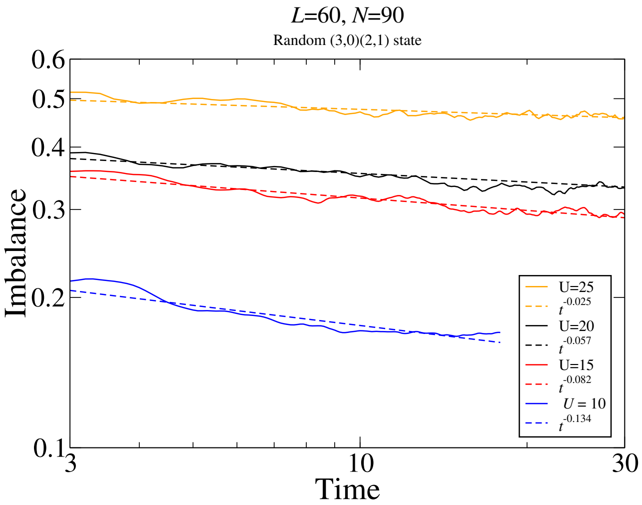
$$\Sigma_2(L) = L - 2\int_0^L dr(L-r)Y_2(r)$$



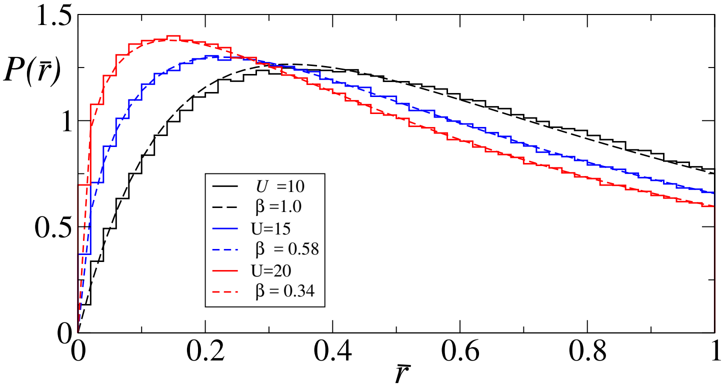
Algebraic decay of magnetization

Close to transition – Griffths regime

$$M(t) \propto t^{-1/z} \quad (1)$$



$\bar{r}$ distribution in the crossover region fitted with distribution for Semi–Poisson statistics from Y.Atas et. al., J. Phys. A: Math. Theor. 46, 355204 (2013)



Approximate models in MBL phase

1. low–energy subspace with $\hat{n}_i = 1, 2$; the XX Heisenberg spin chain in random magnetic field

$$H = J\sum_i \left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+\right) + \sum_i h_i S_i^z,$$

$\Leftrightarrow$ Anderson localization non–interacting fermions, physically different – entanglement entropy saturates

2. two–site model: in basis $|21\rangle$, $|12\rangle$, the Hamiltonian reads

$$H = \begin{bmatrix} U_1 & -2J \\ -2J & U_2 \end{bmatrix}, \Rightarrow$$

diagonalize, average over time oscillations and disorder, consider transfer of $n_i = 2$ into both directions to get $M = 1 - 8\pi J/U$.

Stationary value of magnetization