*57 Cracow School of Theoretical Physics: Entanglement and Dynamics*

# Magnetic phases of spin-1 lattice gases with random interactions

Kenneth McAlpine

Queen's University Belfast

Centre for Theoretical Atomic, Molecular and Optical Physics
CTAMOP
Queen's University Belfast

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S

Department for Employment and Learning
www.delni.gov.uk

# Outline

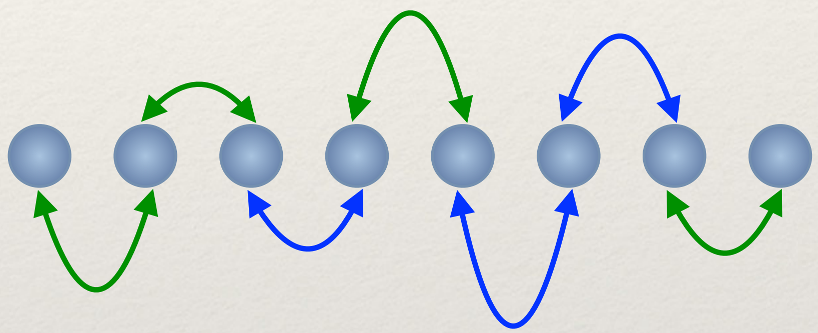
- Motivation
- Bilinear-Biquadratic model
- Homogeneous phase diagram
- Results
- Conclusions and open questions

# Motivation

- How to introduce disorder in ultracold atoms
  - Disordered potential:
    - Extra speckle potentials
    - Optical superlattices of incommensurate frequencies
    - Holographic masks
    - Different atomic species acting as impurities
  - Disordered interaction terms:
    - Magnetic Feshbach resonances
    - Optical Feshbach resonances
- The spin-1 Bilinear-Biquadratic chain has been widely studied and its possible phases when no disorder is present are well known

# Overview

- Method:
  - Introduce randomness into self-interaction coefficients of the Bilinear-Biquadratic model
  - Allow coefficients to take values on either side of a first order phase transition
  - Density matrix renormalisation-group (DMRG)
  - Investigate disorder averaged order parameters
  - Study entanglement entropy distributions



- Findings:
  - Found an intermediate phase between the disordered ferromagnetic and the disordered dimer phases
  - Proposed a modified Edwards - Anderson order parameter as the order parameter to characterise the intermediate phase
  - Entanglement entropy distribution changes from broad in the intermediate phase to peaks in the disordered dimer phase

# Bilinear-Biquadratic model

Effective Bose-Hubbard Hamiltonian for Spin-1 Bosons is:

$$H = \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right) + \frac{U_2}{2}\sum_i\left(\mathbf{S}_i^2 - 2n_i\right) - \mu\sum_i n_i$$
$$- t\sum_{i,\sigma}\left(a^\dagger_{i,\sigma}a_{i+1,\sigma} + a^\dagger_{i+1,\sigma}a_{i,\sigma}\right)$$

With filling per site 1 and $t \ll U_0$, this reduces to:

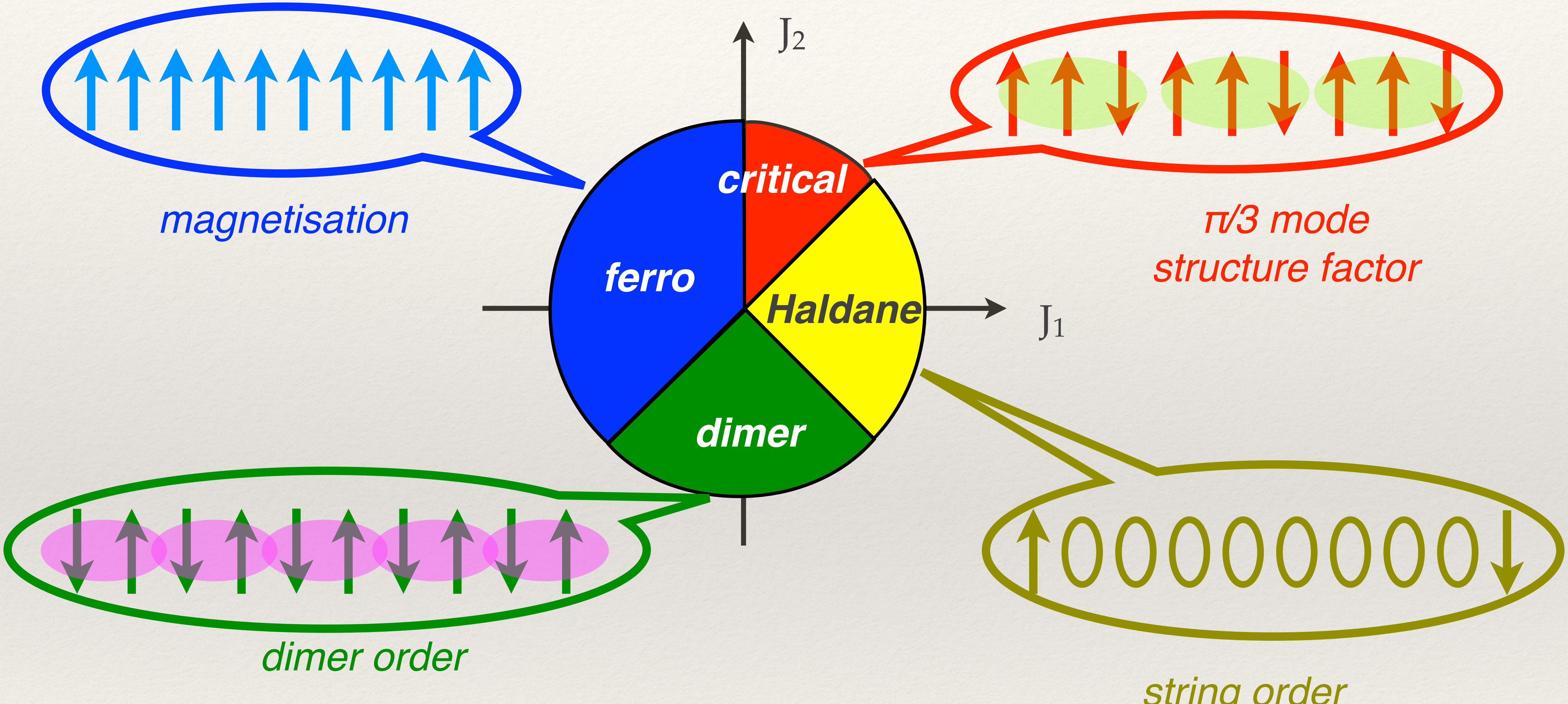
$$H = \sum_i^{L-1} J_{1_i}\left(\mathbf{S}_i \cdot \mathbf{S}_{i+1}\right) + \sum_i^{L-1} J_{2_i}\left(\mathbf{S}_i \cdot \mathbf{S}_{i+1}\right)^2$$

$$J_{1_i} = f(U_0, U_{2_i}, U_{2_{i+1}}) \qquad J_{2_i} = g(U_0, U_{2_i}, U_{2_{i+1}})$$

The Bilinear-Biquadratic Hamiltonian.

**A. Imambekov et al.**
Phys. Rev. A 68, 063602 (2003)

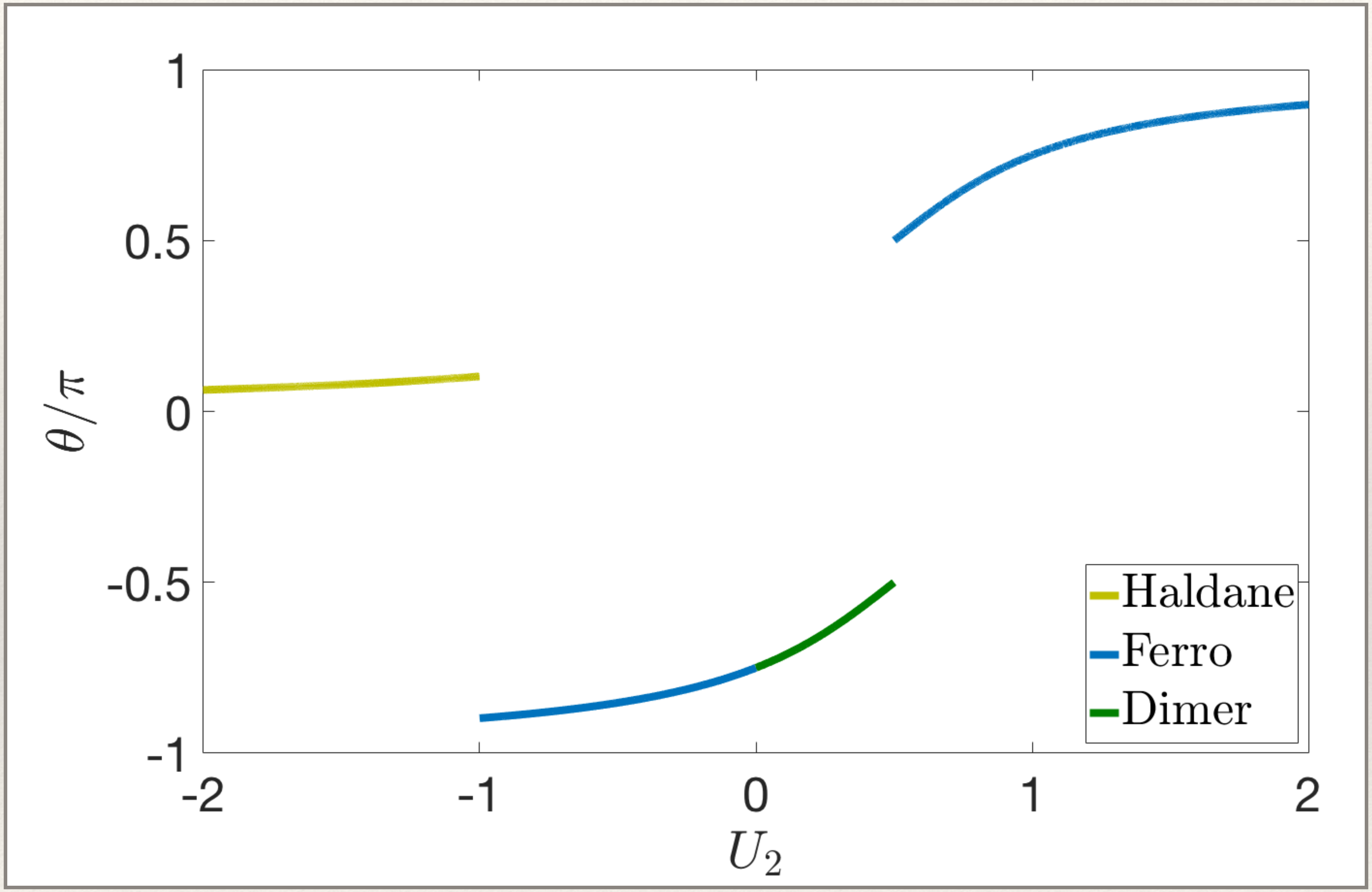
# Homogeneous phase diagram




$$D = |H(i) - H(i+1)|$$

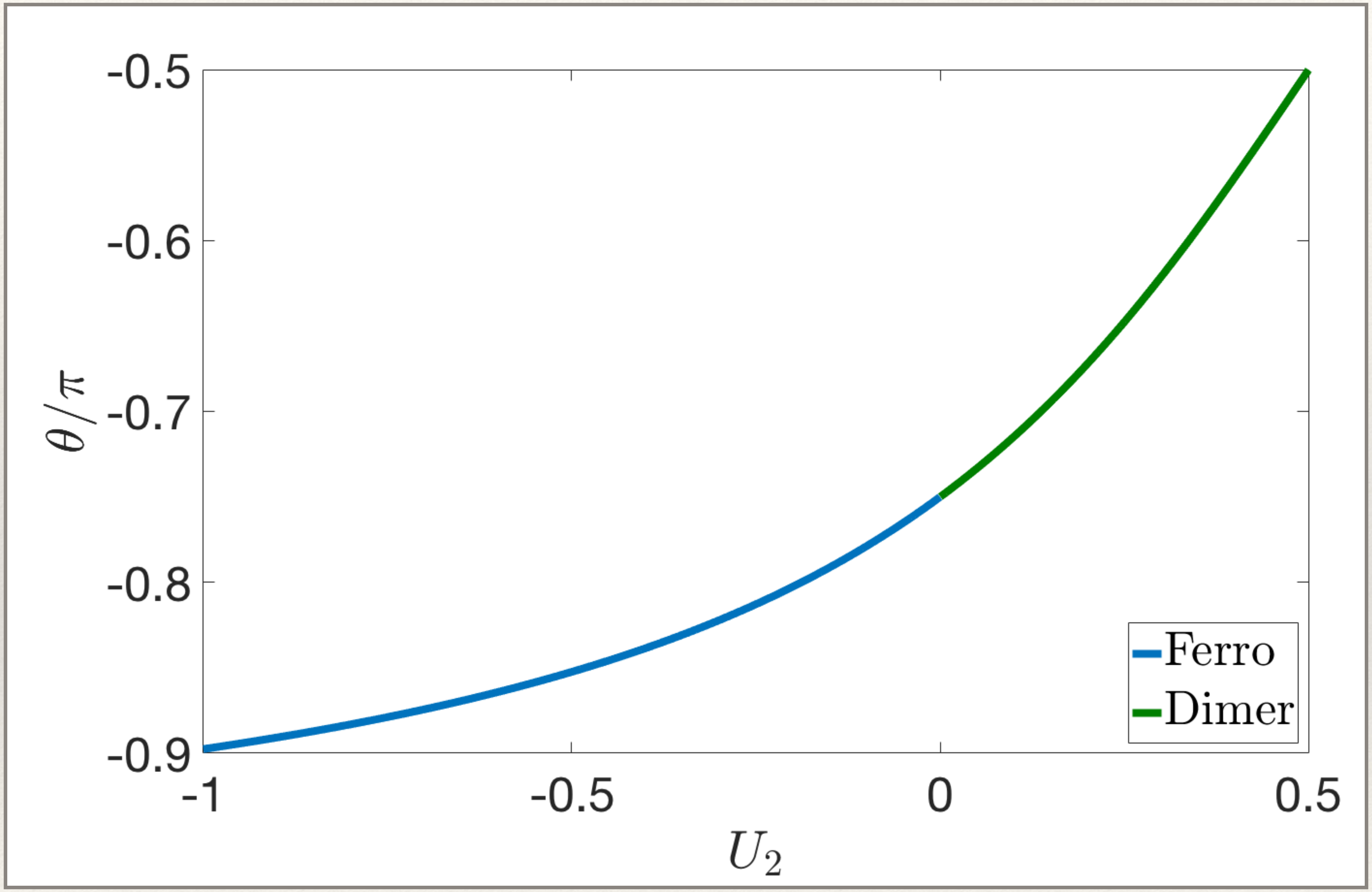
$$J_1 = \cos\theta \qquad J_2 = \sin\theta$$

# Coupling relation




$$\theta = \arctan\left(\frac{J_2}{J_1}\right) - \pi$$

# Coupling relation

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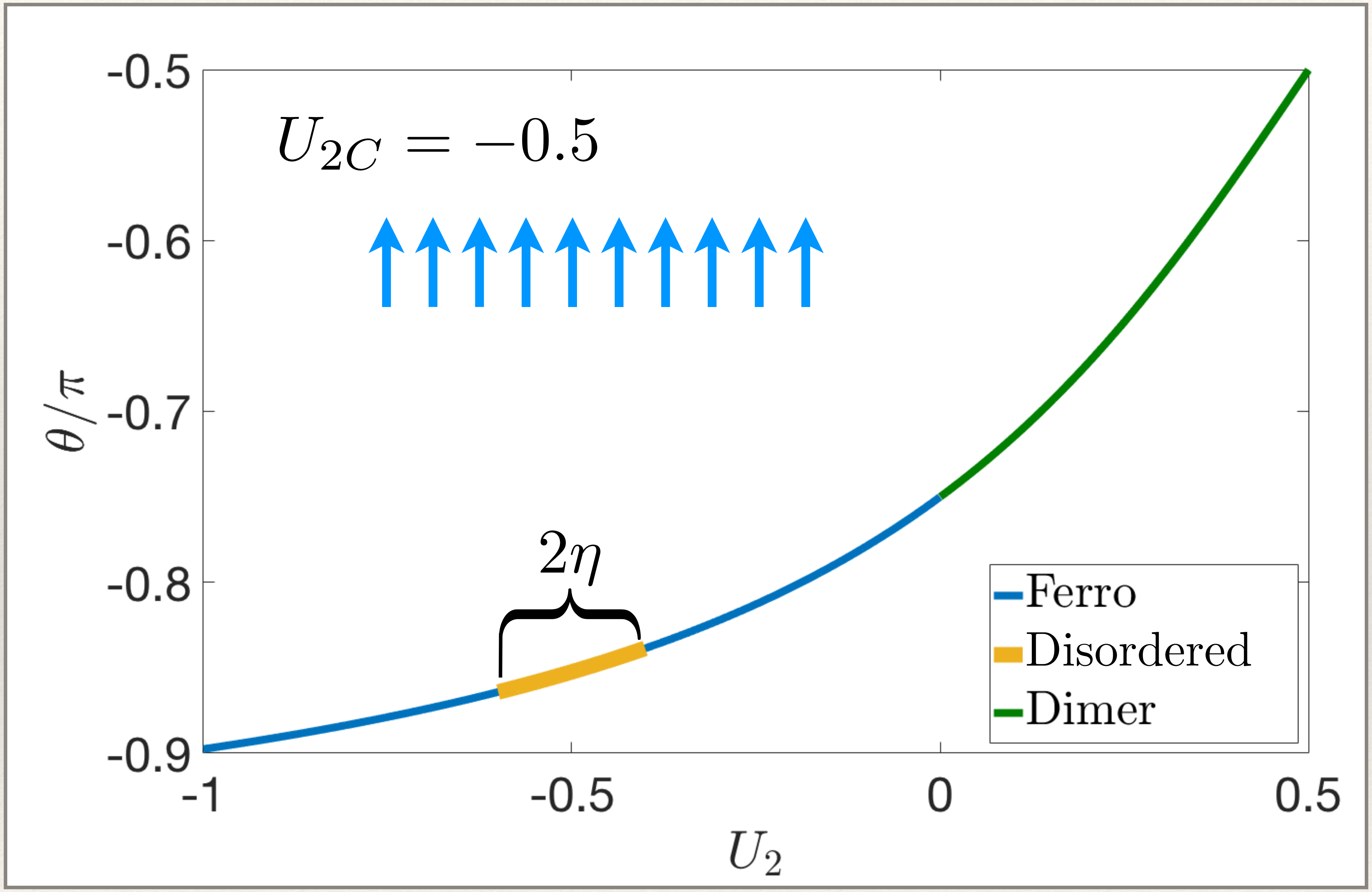
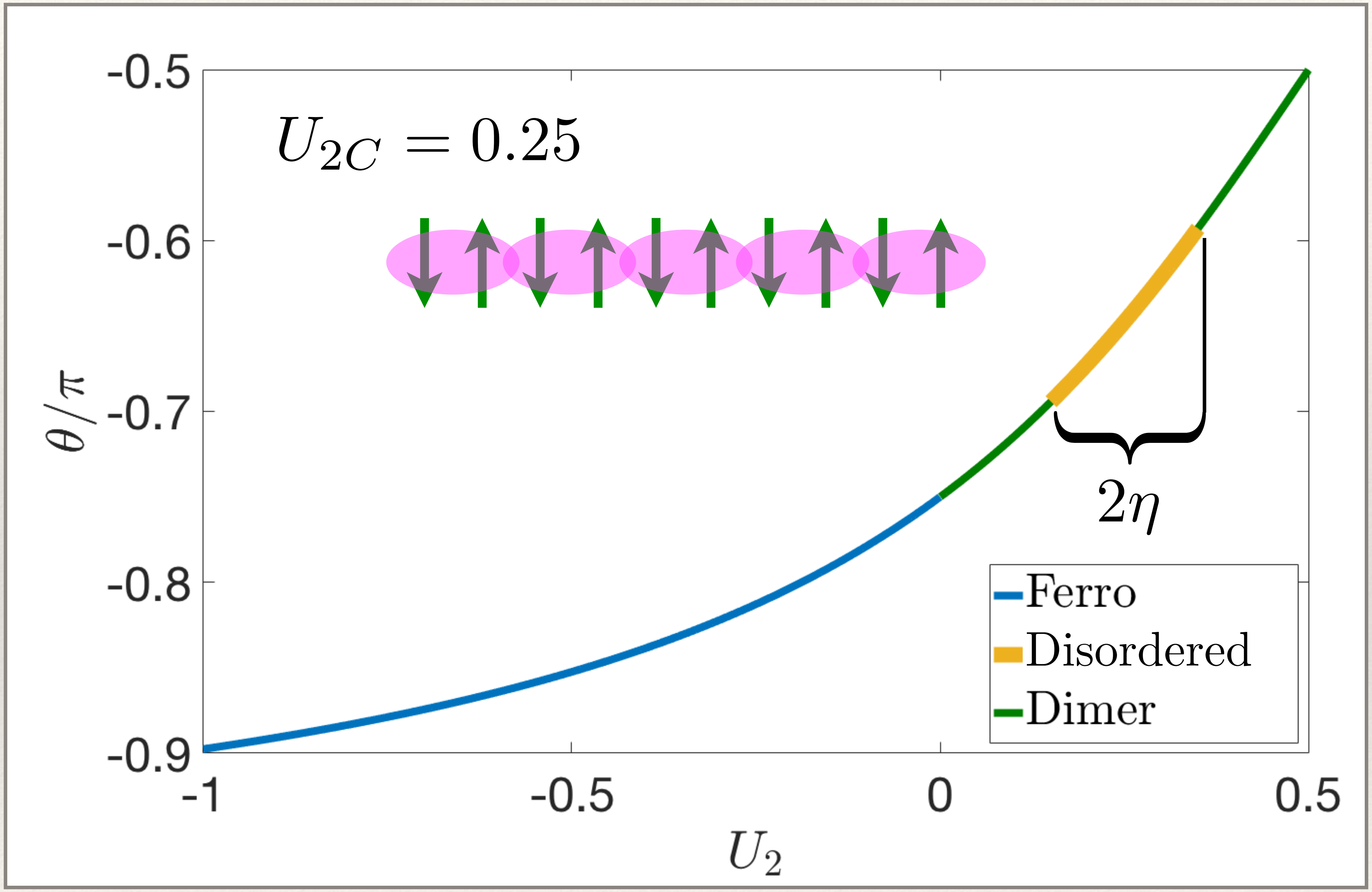



$$U_{2i} = U_{2C} + \eta\,(2\zeta_i - 1)$$

# Coupling relation

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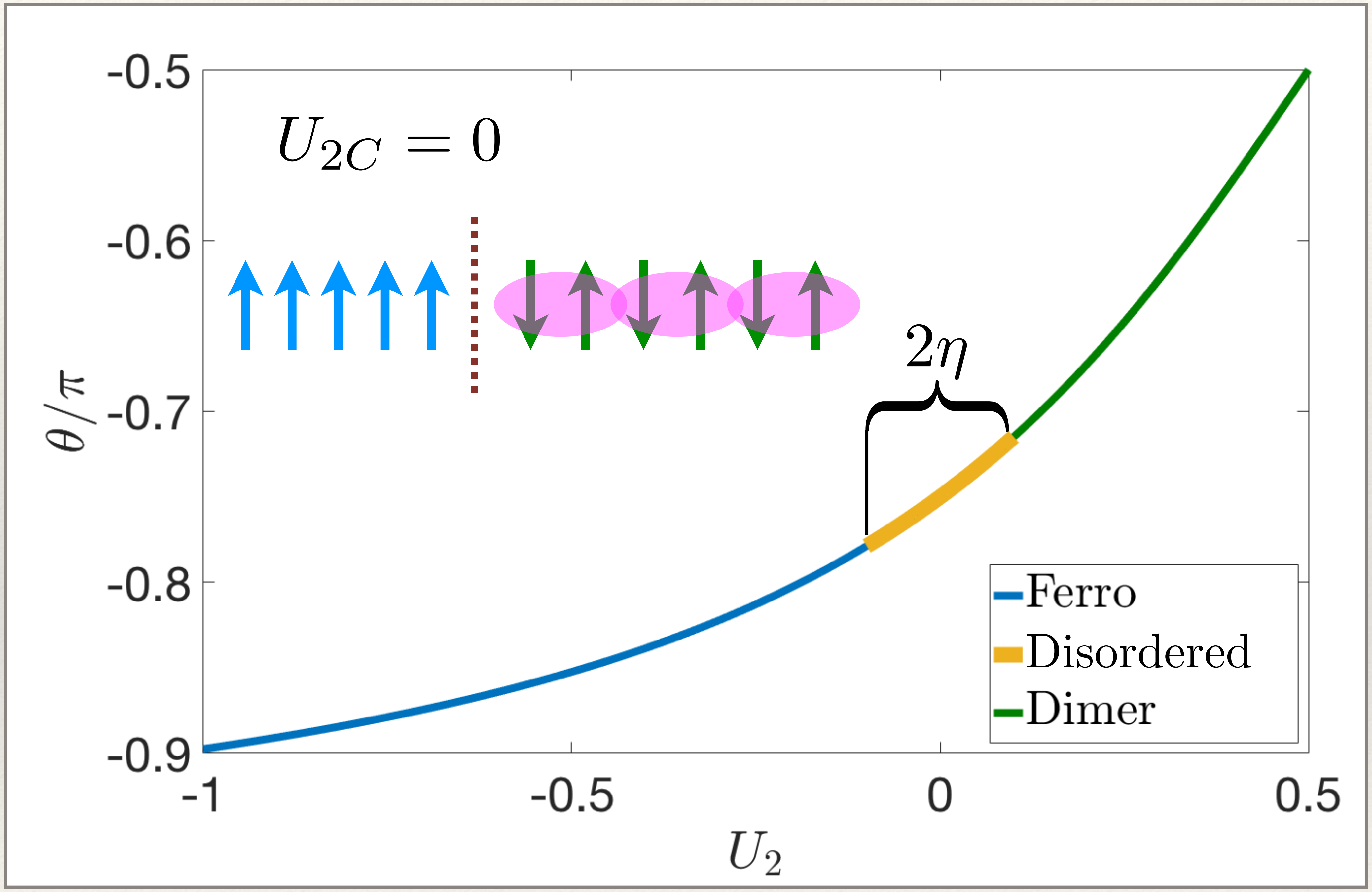


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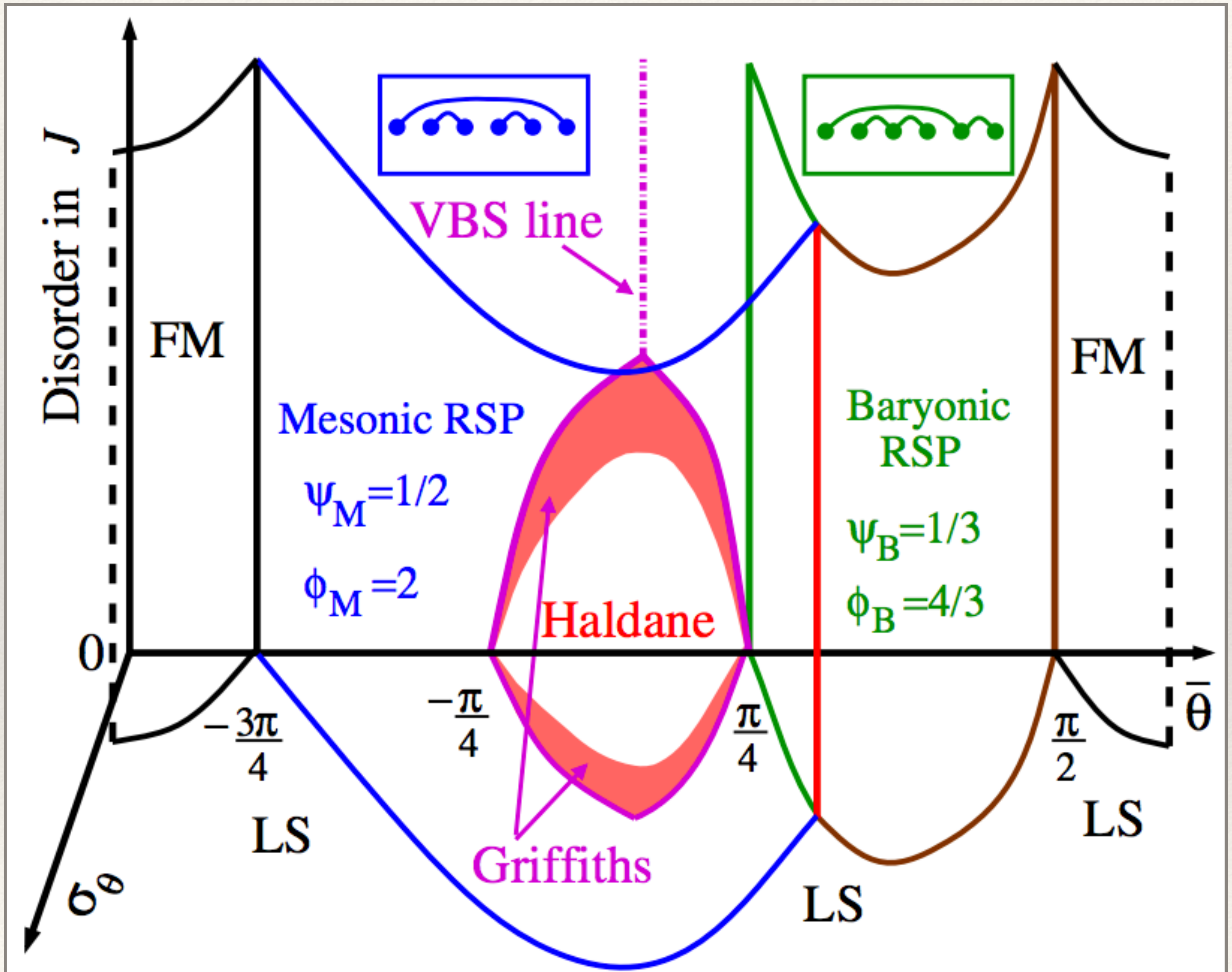
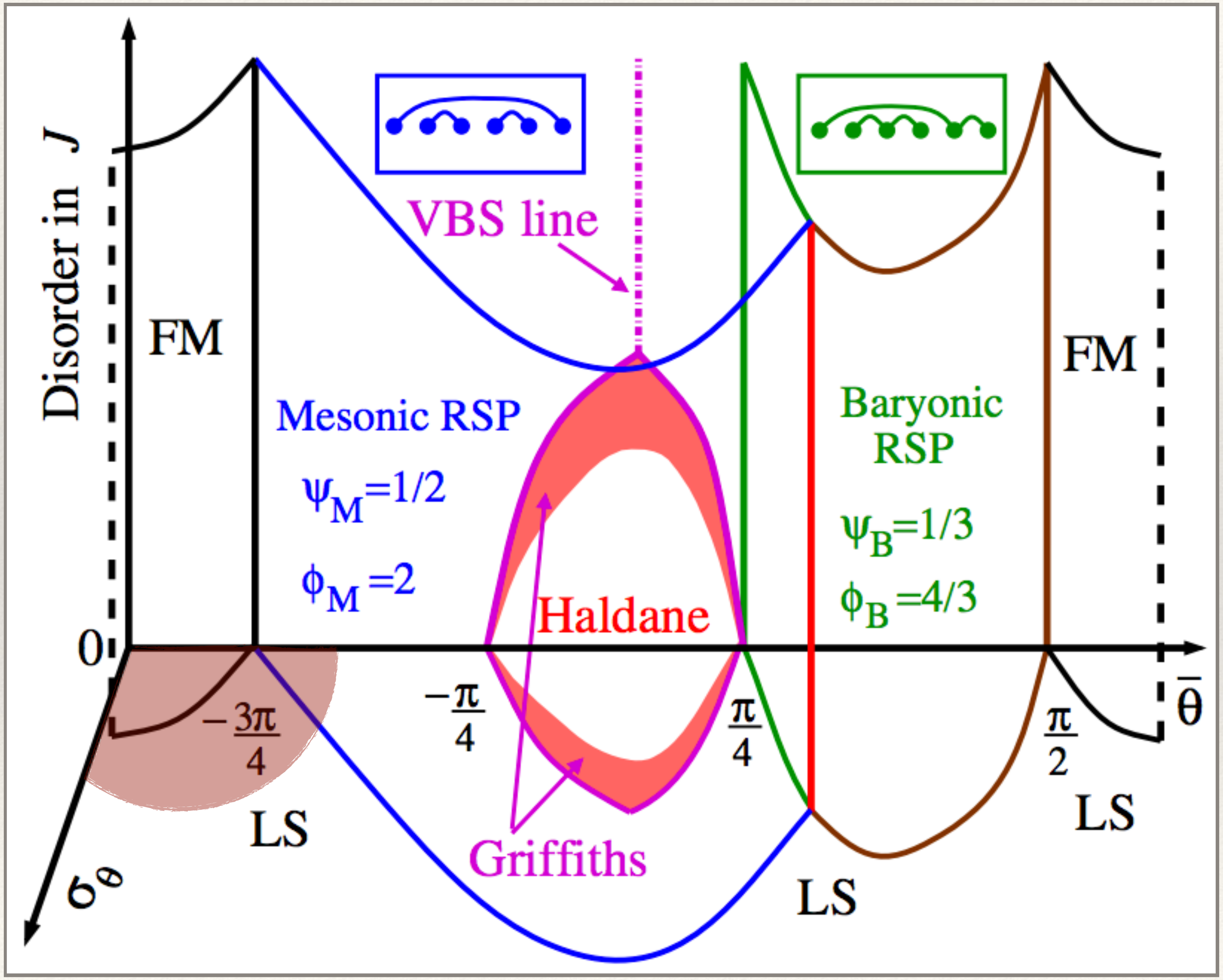
# Strong-disorder renormalisation-group




V. L. Quito et al.
PRL 115, 167201 (2015)

# Strong-disorder renormalisation-group

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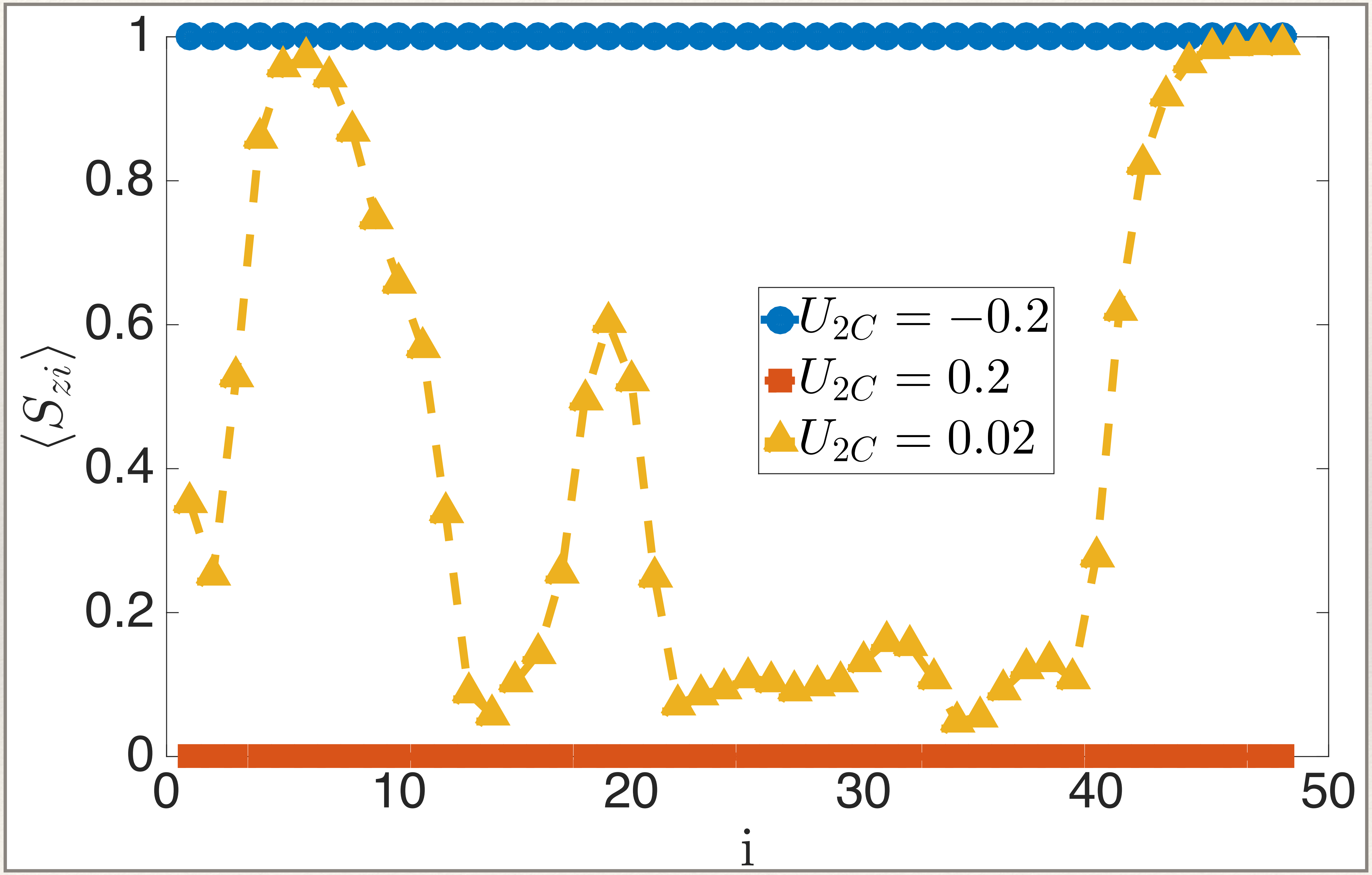
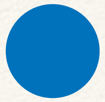
PRL 115, 167201
(2015)

# Local correlations with random interactions



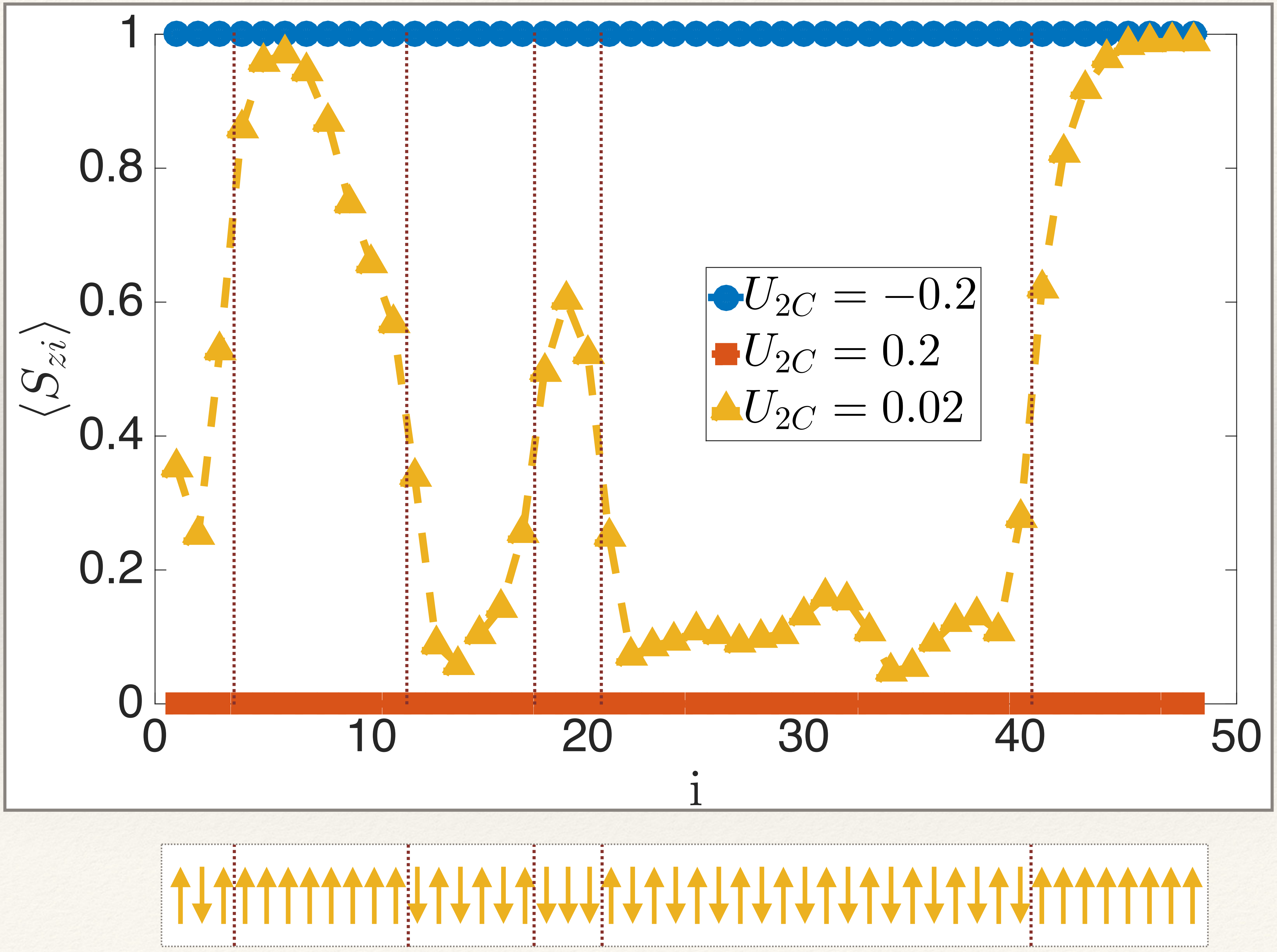



Deep Ferromagnetic

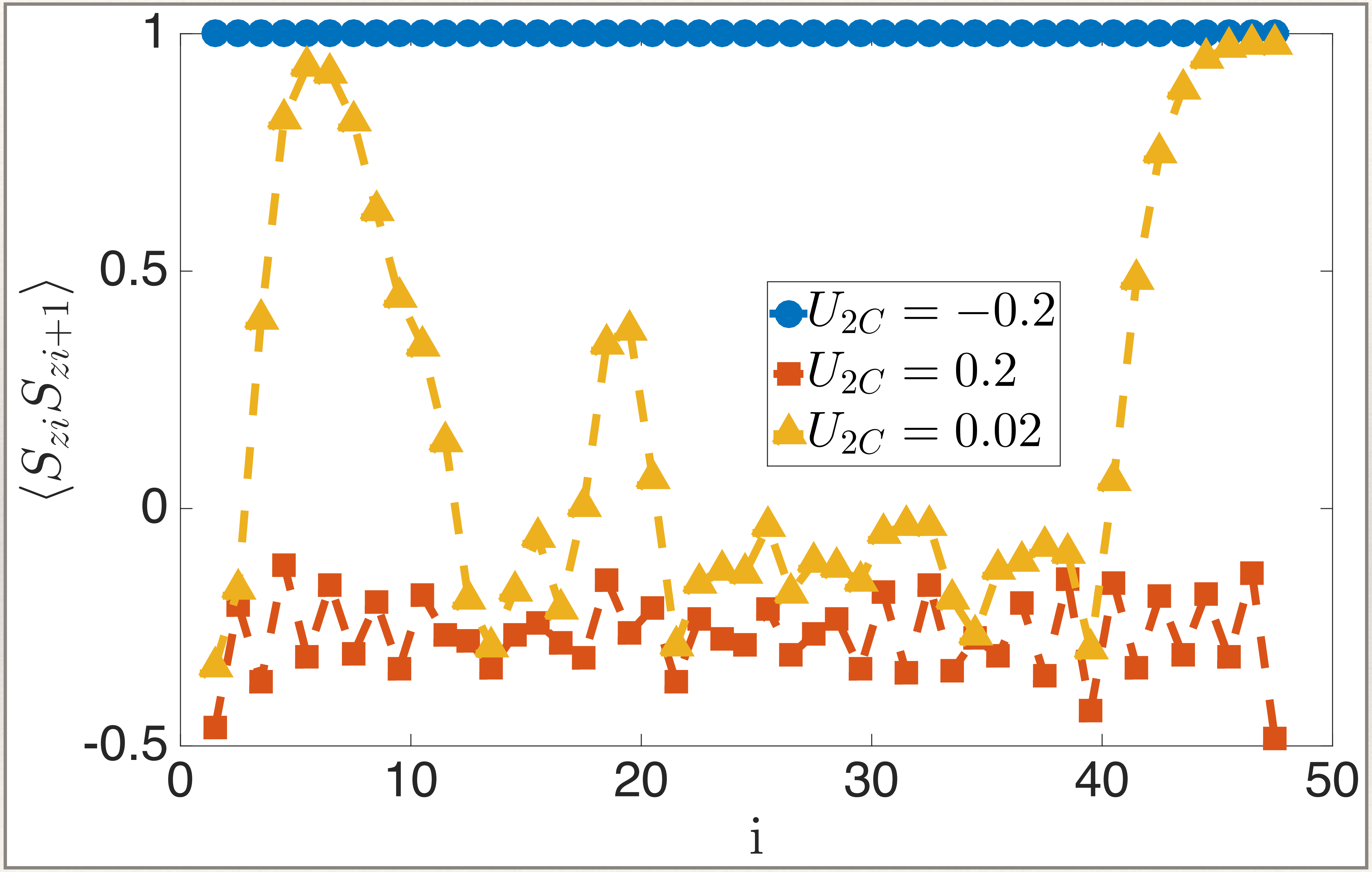
Deep Dimer

Near Phase Transition

# Local correlations with random interactions



# Nearest neighbour correlations with random interactions

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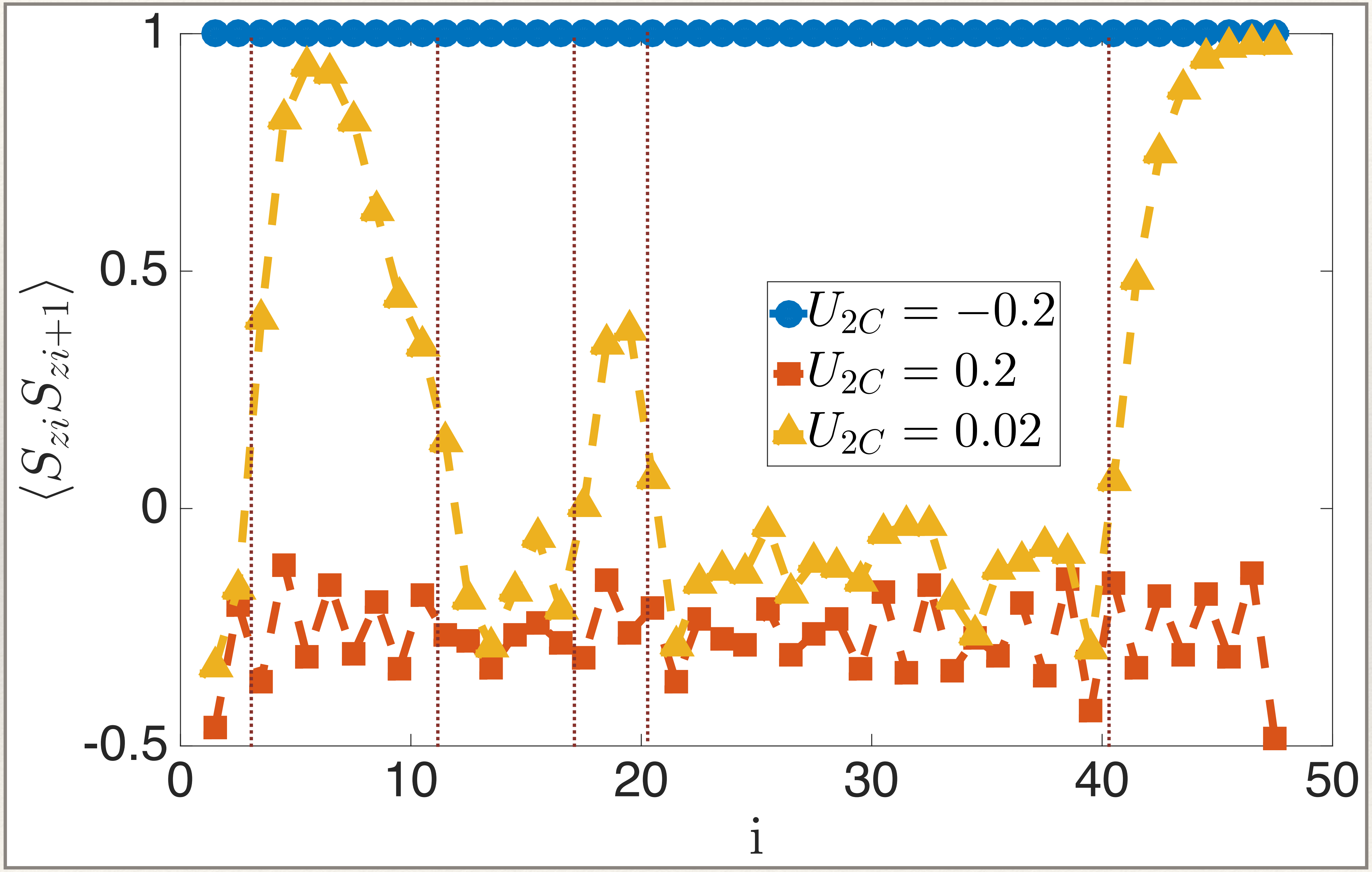
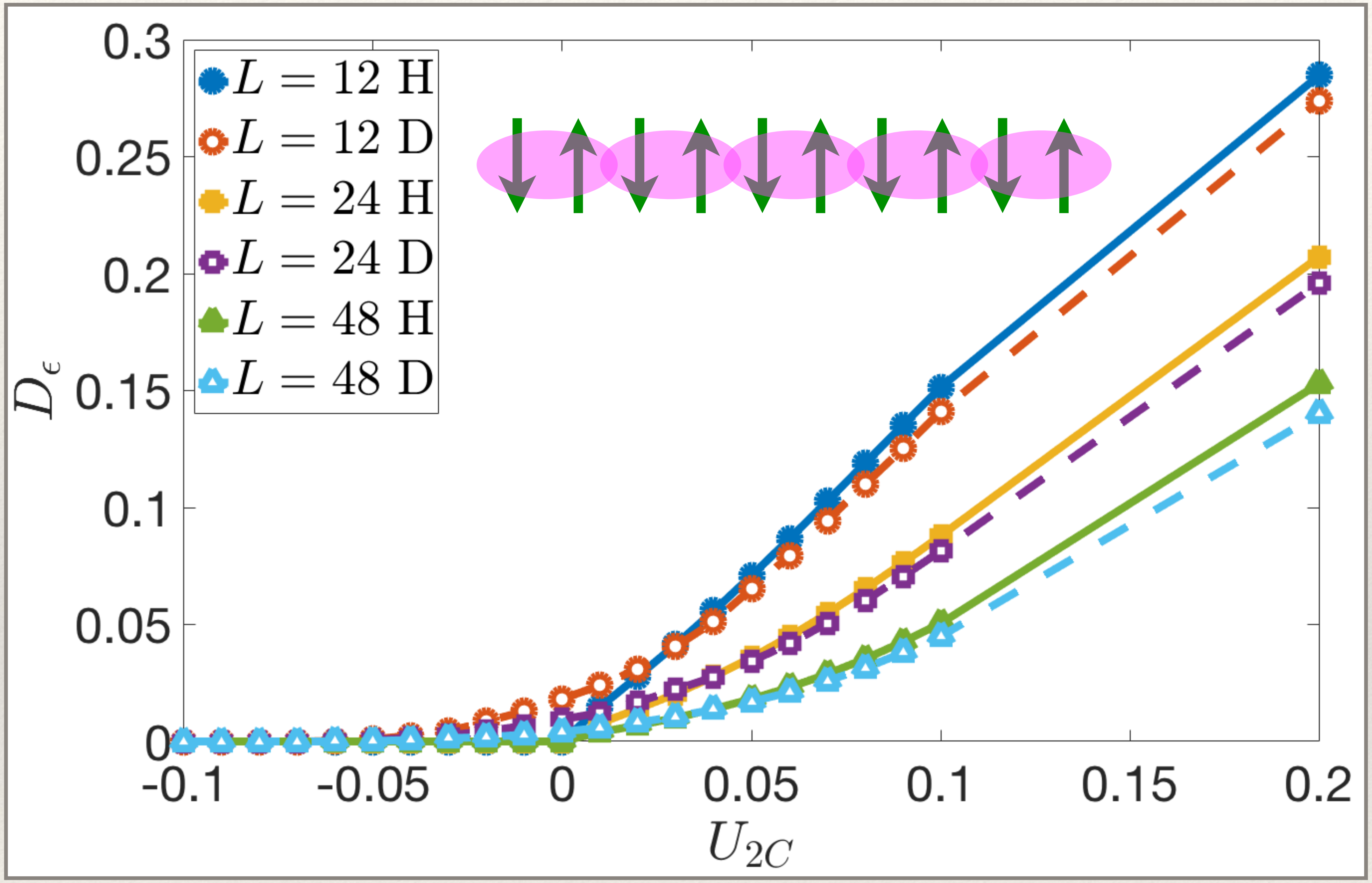
# Nearest neighbour correlations with random interactions




# Dimer order parameter

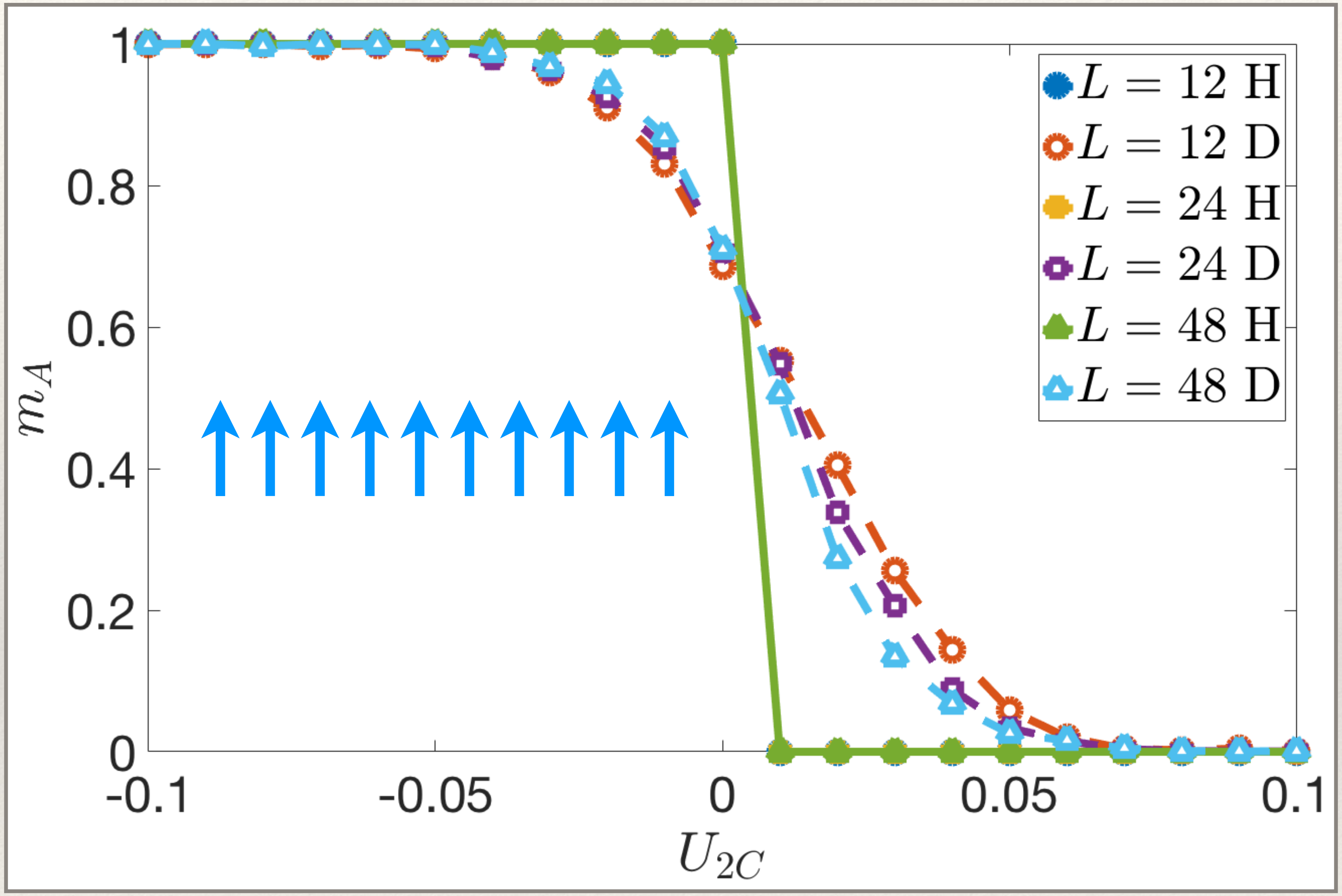



$$D_\epsilon = -\frac{1}{L}\sum_{mn} \sin\left[\frac{\pi}{2}(m+n)\right] G_z(m,n)$$

$$G_z(m,n) \equiv \langle \mathbf{S}_{z_m} \mathbf{S}_{z_n} \rangle - \langle \mathbf{S}_{z_m} \rangle \langle \mathbf{S}_{z_n} \rangle$$

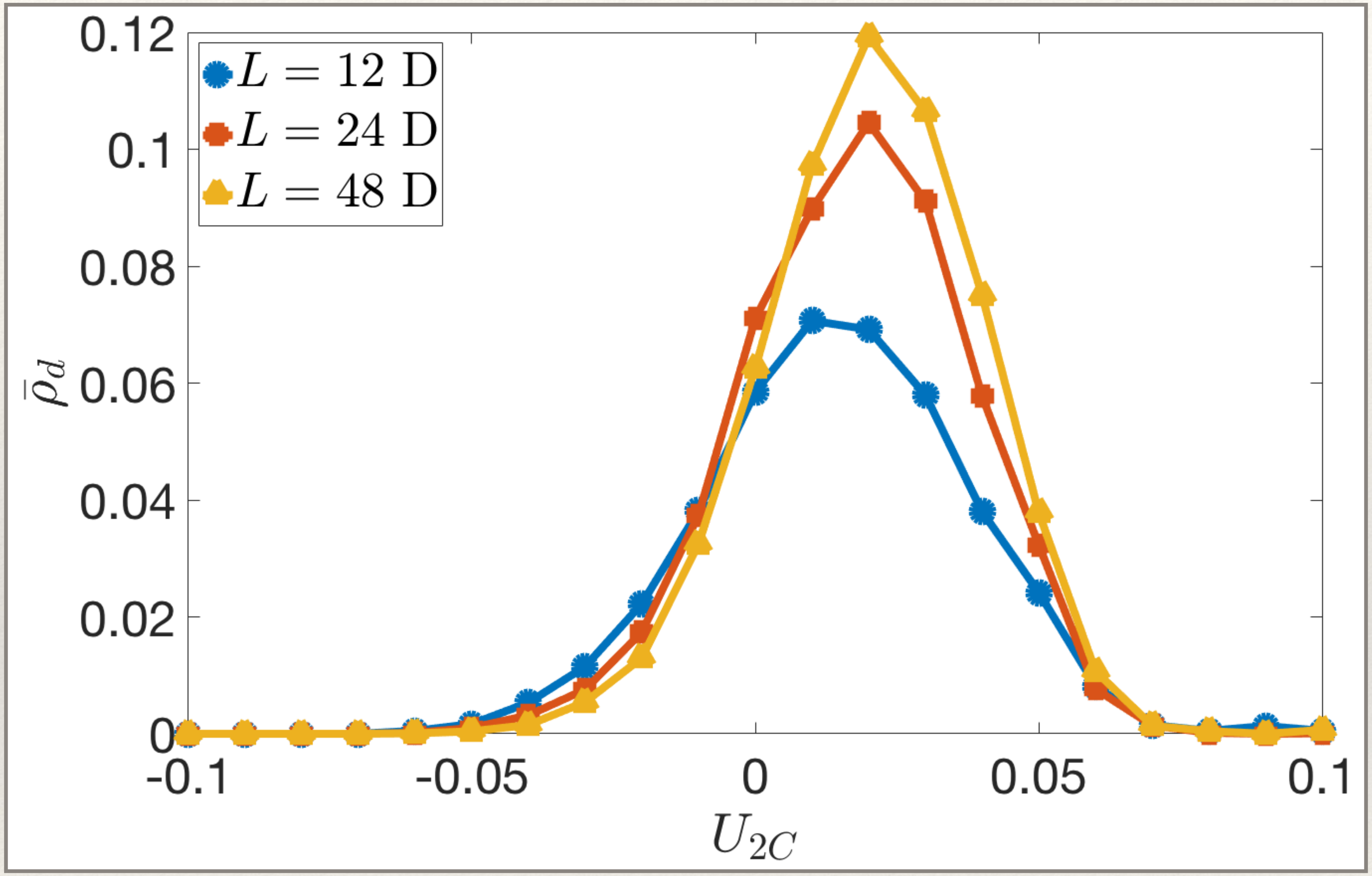
**G. De Chiara et al.**
Phys. Rev. A 83, 021604 (2011)

# Average magnetisation



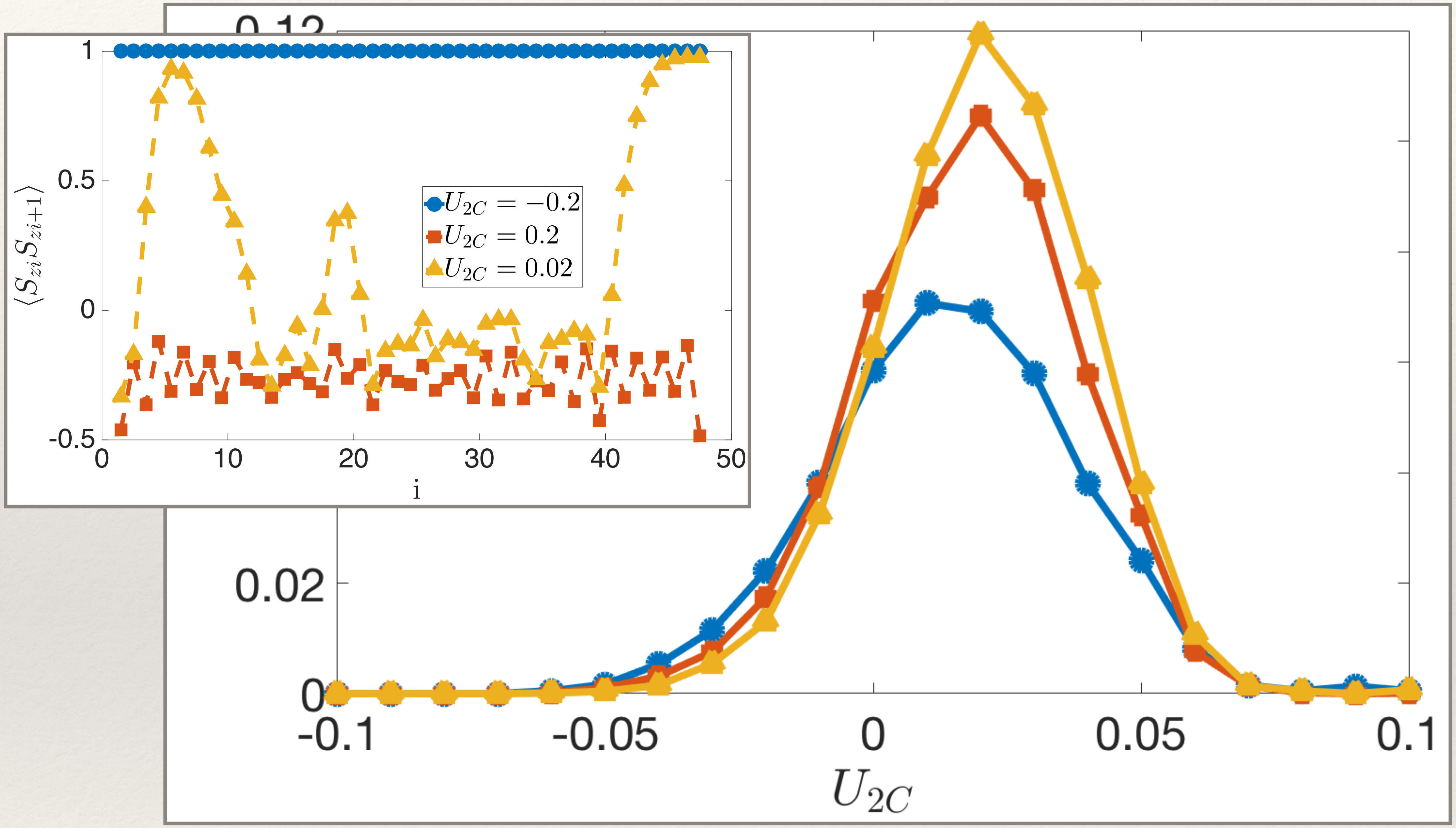

$$m_A = \frac{1}{L} \sum_i [\langle S_{zi} \rangle]_D$$

# Domain walls



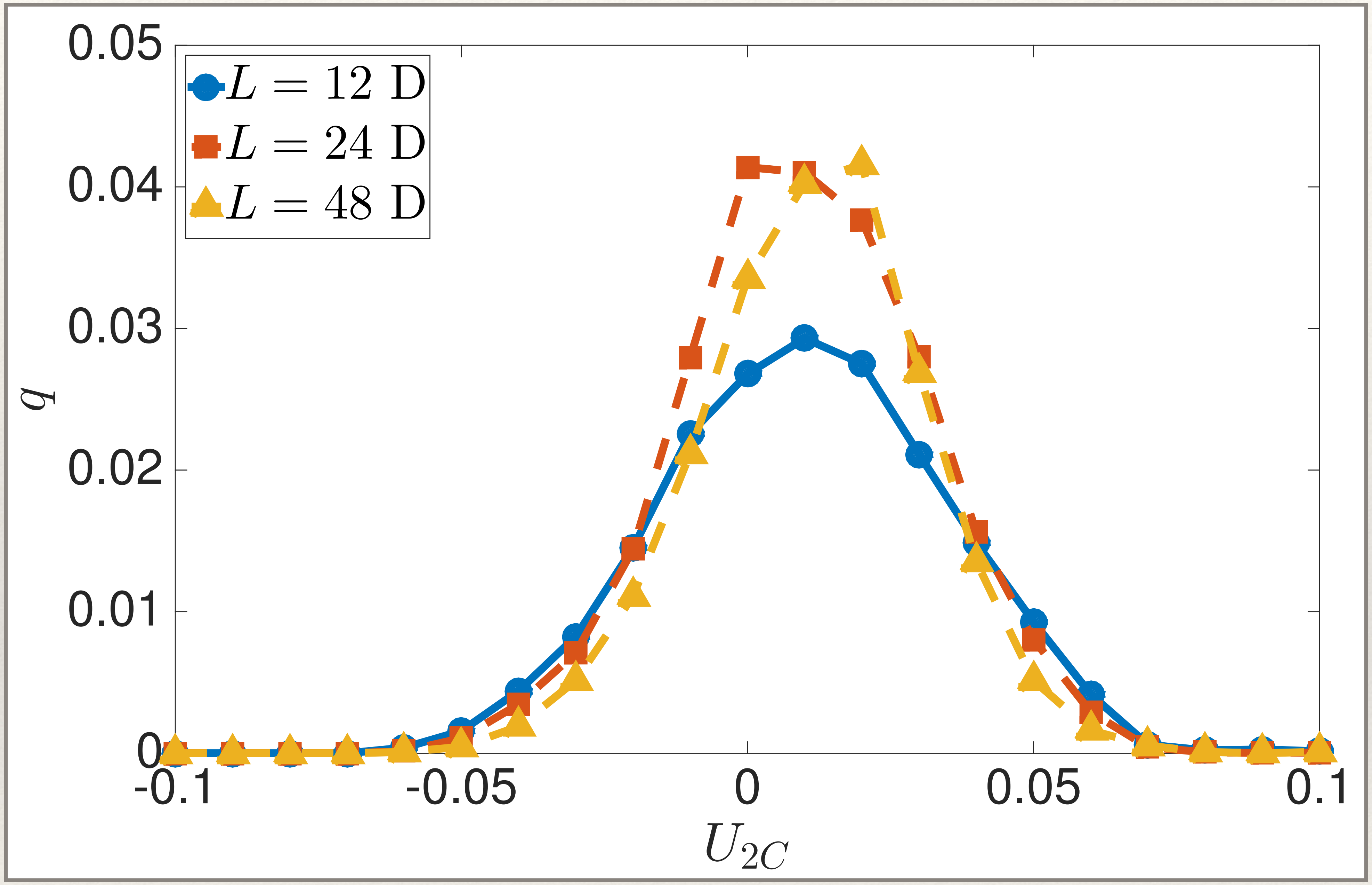

Domain walls are identified by counting the sign changes occurring in the nearest neighbour correlations.

# Domain walls



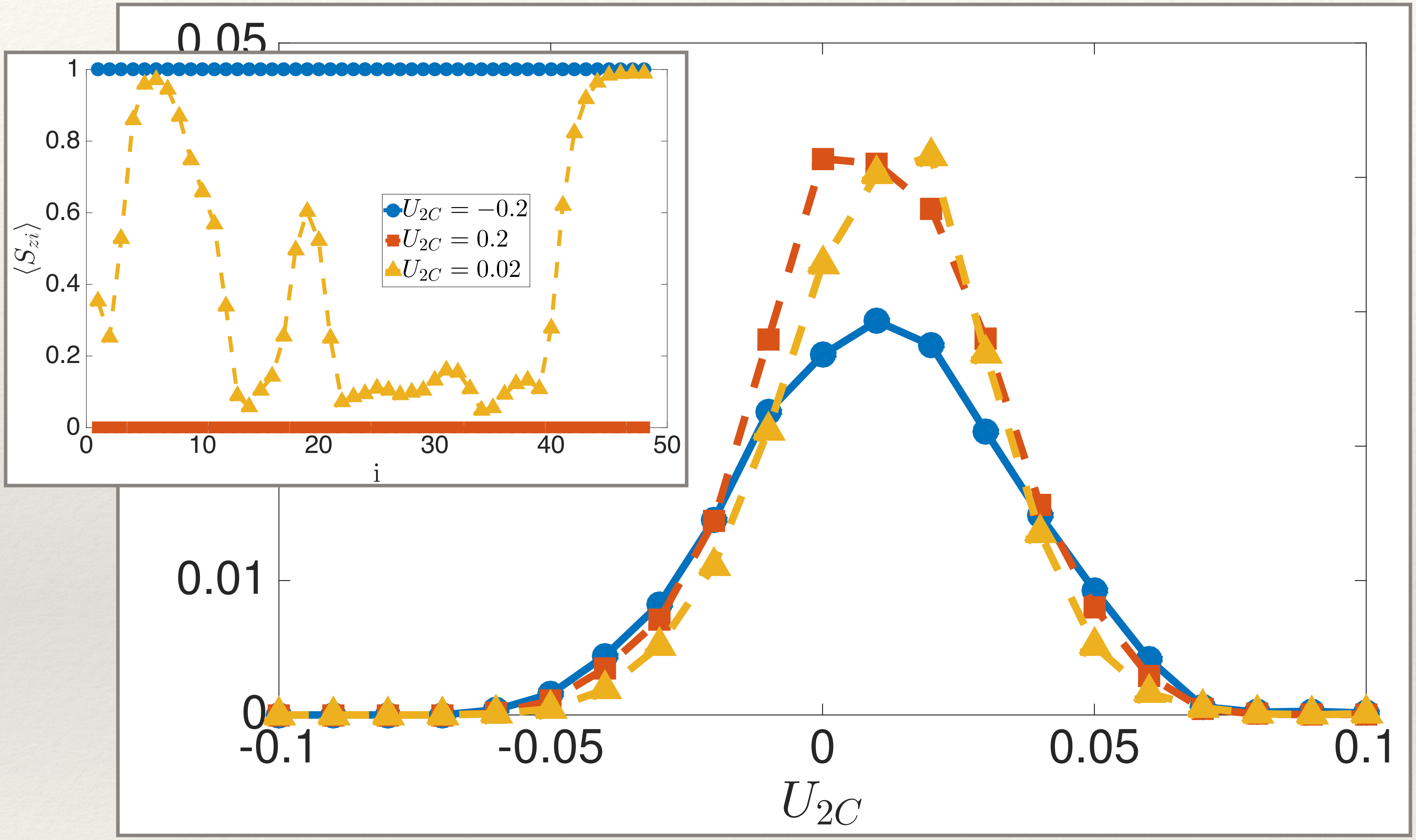

Domain walls are identified by counting the sign changes occurring in the nearest neighbour correlations.

# Modified Edwards - Anderson order parameter



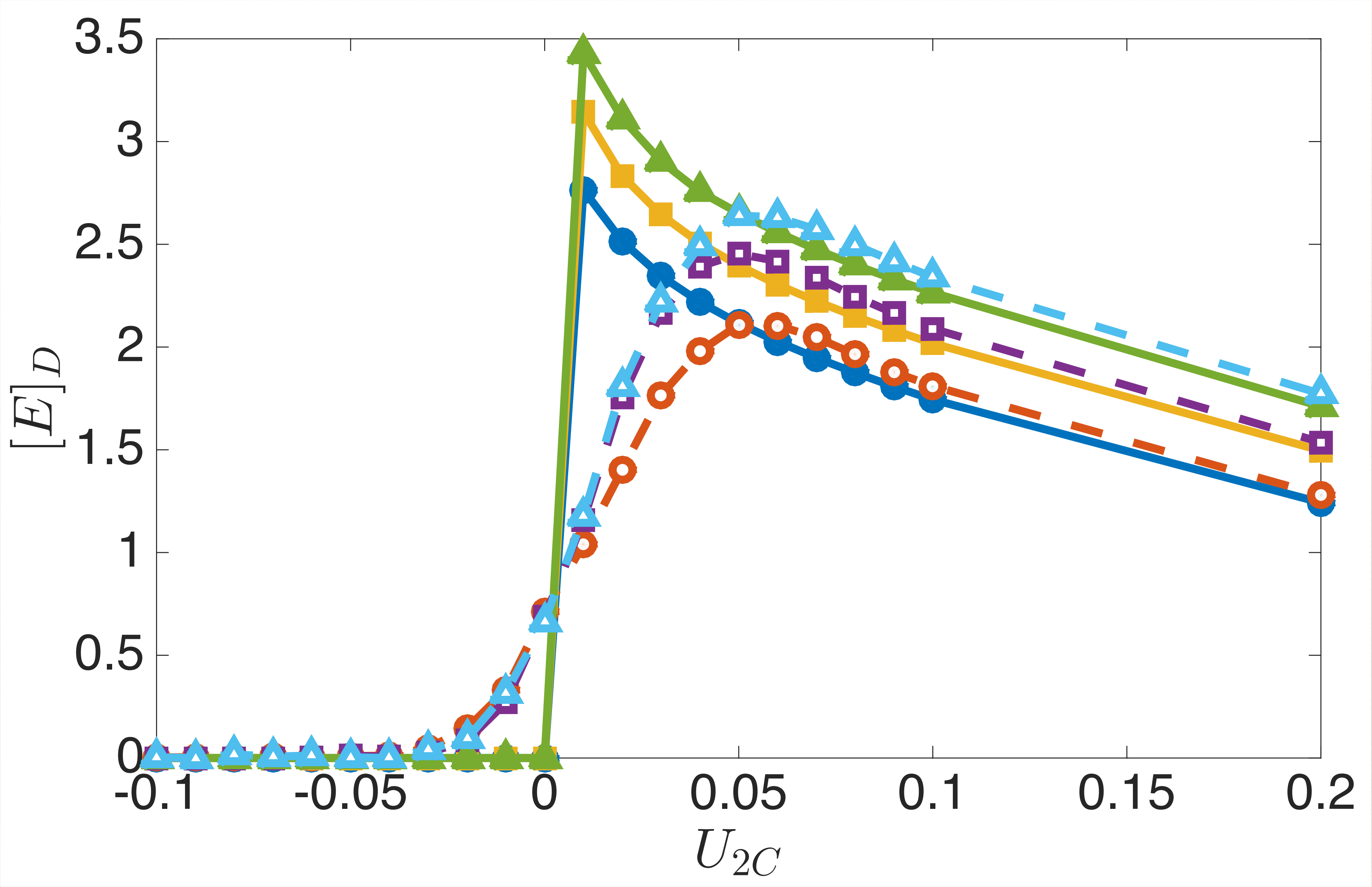

$$q = \frac{1}{L} \sum_i \left[\langle S_{zi} \rangle^2\right]_D - \left[\langle S_{zi} \rangle\right]_D^2$$

# Modified Edwards - Anderson order parameter




$$q = \frac{1}{L} \sum_i \left[ \langle S_{zi} \rangle^2 \right]_D - \left[ \langle S_{zi} \rangle \right]_D^2$$

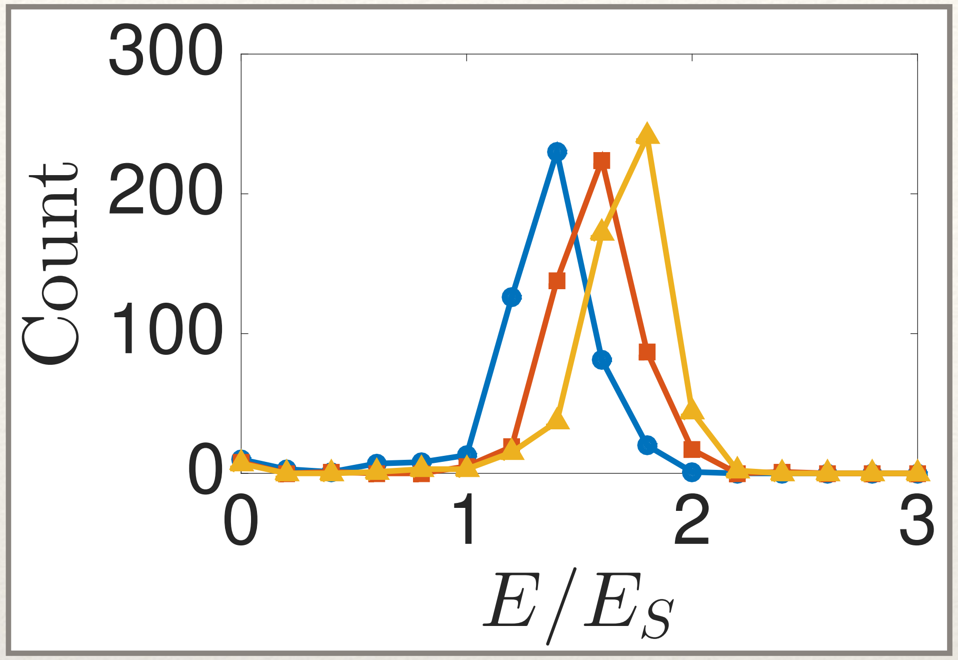
# Von Neumann entropy



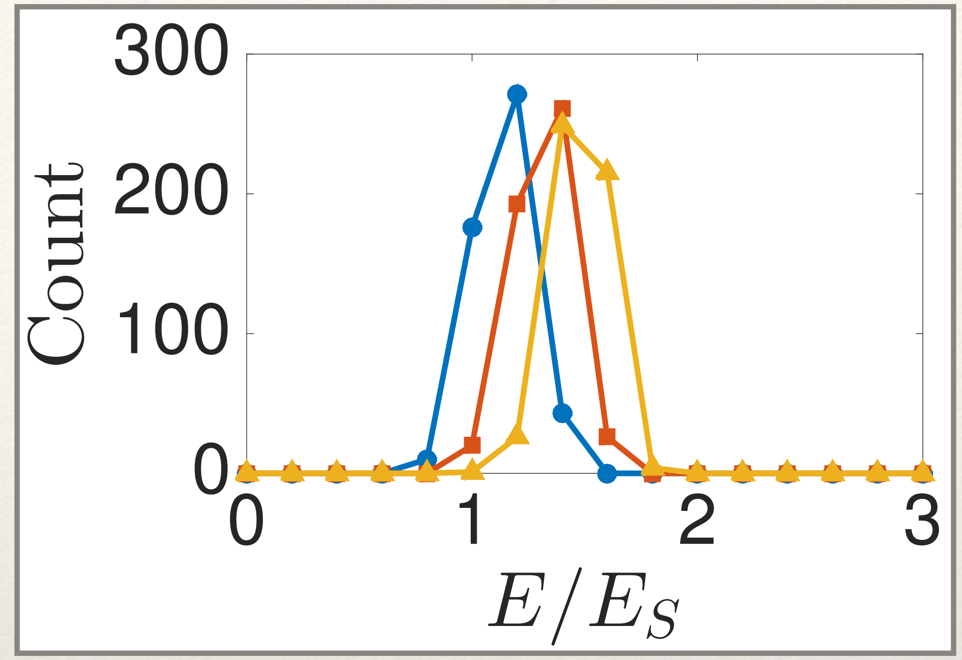

$$E = -\mathrm{Tr}\rho_l \log_2 \rho_l$$

$$\rho_l = \mathrm{Tr}_{L-l}|\psi_G\rangle\langle\psi_G|$$

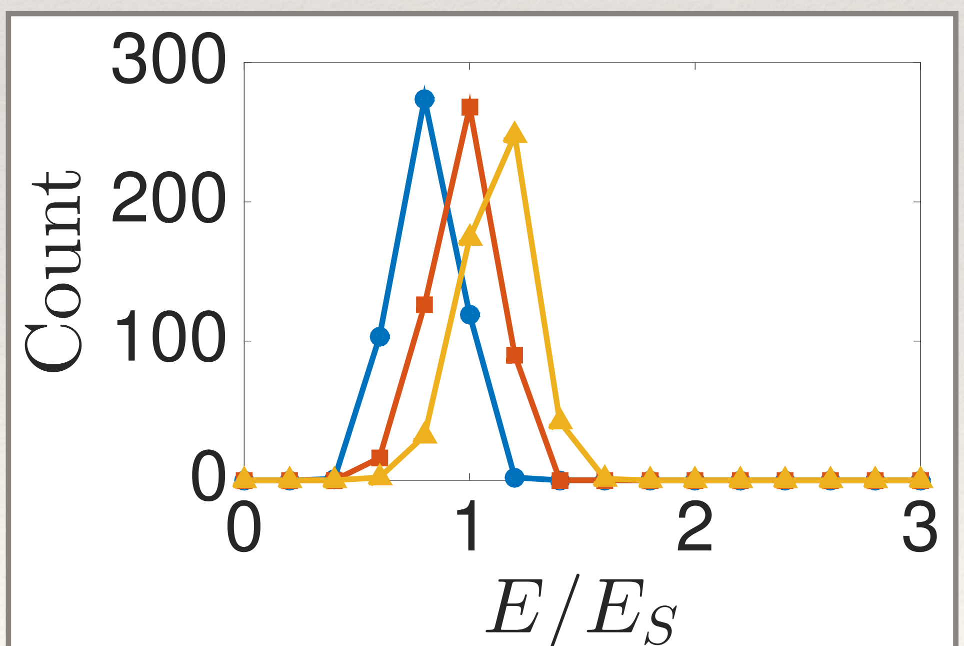
# Entropy distribution in dimer phase



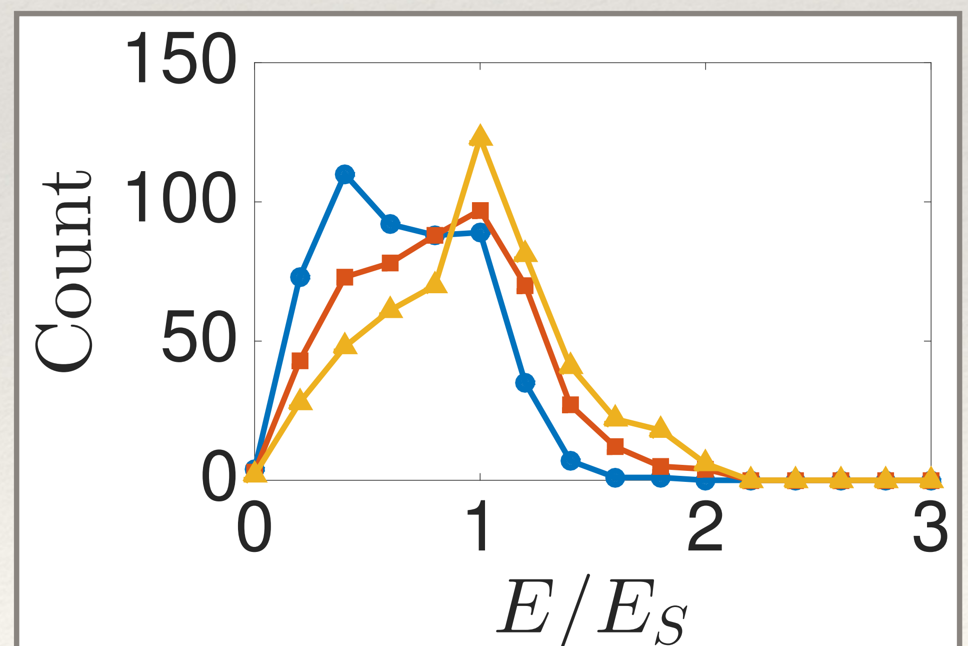

$U_{2C} = 0.05$




$U_{2C} = 0.1$




$U_{2C} = 0.2$




$U_{2C} = 0.39$

# Conclusions and open questions

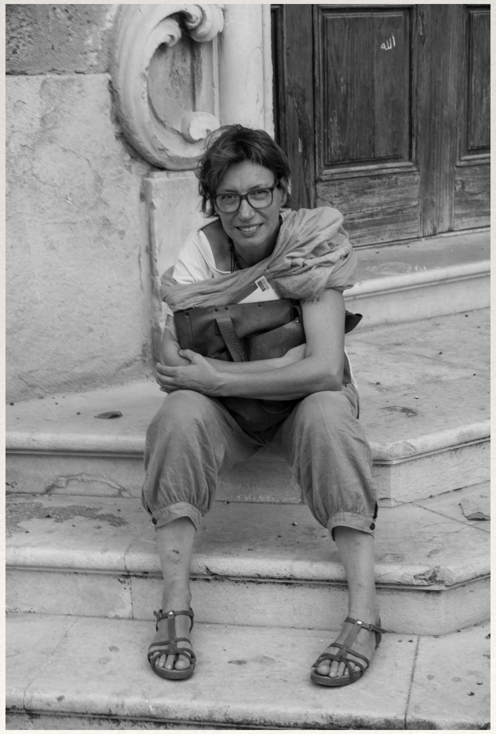
- Found evidence of an intermediate phase between the disordered ferromagnetic and disordered dimer phases
- Characterised by a finite EA order parameter
- Disordered dimer phase moves towards random singlet phase (RSP) as disorder increases
- Does phase scale with disorder?
- How does the intermediate phase react to the presence of uniaxial field?

# Collaborators

**Gabriele De Chiara**
Queen's University Belfast

**Simone Paganelli**
University of L'Aquila

**Sergio Ciuchi**
University of L'Aquila



**Anna Sanpera**
Autonomous
University of Barcelona