

Spin-orbit coupling for ultracold atomic gases

(Lecture 1)

Gediminas Juzeliūnas

Institute of Theoretical Physics and Astronomy,
Vilnius University, Vilnius, Lithuania

11:15 — 12:10, Friday, 16 June 2017

57 Cracow School of Theoretical Physics

June 13 -21, 2017

Entanglement and Dynamics

Zakopane, Poland

Vilnius
(Lithuania)
is in north of
EUROPE



Vilnius University



National Center for Physical Sciences



**Institute of Theoretical Physics and
Astronomy (ITPA) of VU**

National Center for Physical Sciences



**Institute of Theoretical Physics and
Astronomy (ITPA) of VU**

Quantum Optics Group @ ITPA, Vilnius University



**R. Juršėnas, J. Ruseckas, T. Andrijauskas, A. Mekys, G. J.,
E. Anisimovas, V. Novichenko, D. Efimov (St. Petersburg
Univ.→ Crakow Univ.) and H. R. Hamed**
Not in the picture: A. Acus and V. Kudriašov, J. Armaitis

Quantum Optics Group @ ITPA, Vilnius University

Research activities:

- Optical control of ultracold atoms
(artificial magnetic field and
spin-orbit coupling for ultracold atoms)
- Cold atoms in (topological) optical lattices
- **Slow** light (two-component, with OAM, ...)
- Floquet engineering
- ...

Quantum Optics Group @ ITPA, Vilnius University

Research activities:

- Optical control of ultracold atoms
(artificial magnetic field and
spin-orbit coupling for ultracold atoms)

This talk



Lecture 1 (Friday, June 16):

***Spin-orbit coupling for
ultracold atomic gases***

Lecture 2 (Sunday, June 18):

Novel topological optical lattices
(synthetic gauge field using semi-synthetic lattices, etc)

Lecture 3 (Tuesday, June 20):

Multicomponent (spinor) slow light

Spin-orbit coupling for ultracold atomic gases

(Lecture 1)

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June 13 -21, 2017

Entanglement and Dynamics

Zakopane, Poland

Collaboration

(artificial magnetism and SOC for ultracold atoms)

- **P. Öhberg & group**, Heriot-Watt Univ., *Edinburgh*
- **M. Fleischhauer & group**, TU Kaiserslautern,
- **L. Santos & group**, Universität Hannover
- **J. Dalibard, F. Gerbier & group**, ENS, Paris
- **I. Spielman, B. Anderson, V. Galitski, D. Campbell, C. Clark and J. Vaishnav**, JQI, NIST/UMD, USA
- **M. Lewenstein & group**, ICFO, ICREA, Barcelona
- **Shih-Chuan Gou, Daw-Wei Wang & groups**, Taiwan
- **Wu-Ming Liu, Qing Sun, An-Chun Ji & groups** (Beijing)
- **Nathan Goldman** (ULB, Brussels)
- ...

OUTLINE

❖ **Background:**

- ❖ *Spin-orbit coupling (SOC) for electrons and ultracold atoms*
- ❖ **Optically-induced SOC for ultracold atoms**
 - ❖ *Various schemes for producing 2D Rashba type SOC*
 - ❖ *2D SOC @ Shanxi University – experimental realization*
- ❖ **SOC in Bilayer system of ultracold atoms**
 - ❖ **Setup for producing 2D Rashba SOC & BEC phases**
- ❖ **Magnetically generated SOC**
- ❖ **Conclusions**

Spin-orbit coupling (SOC)

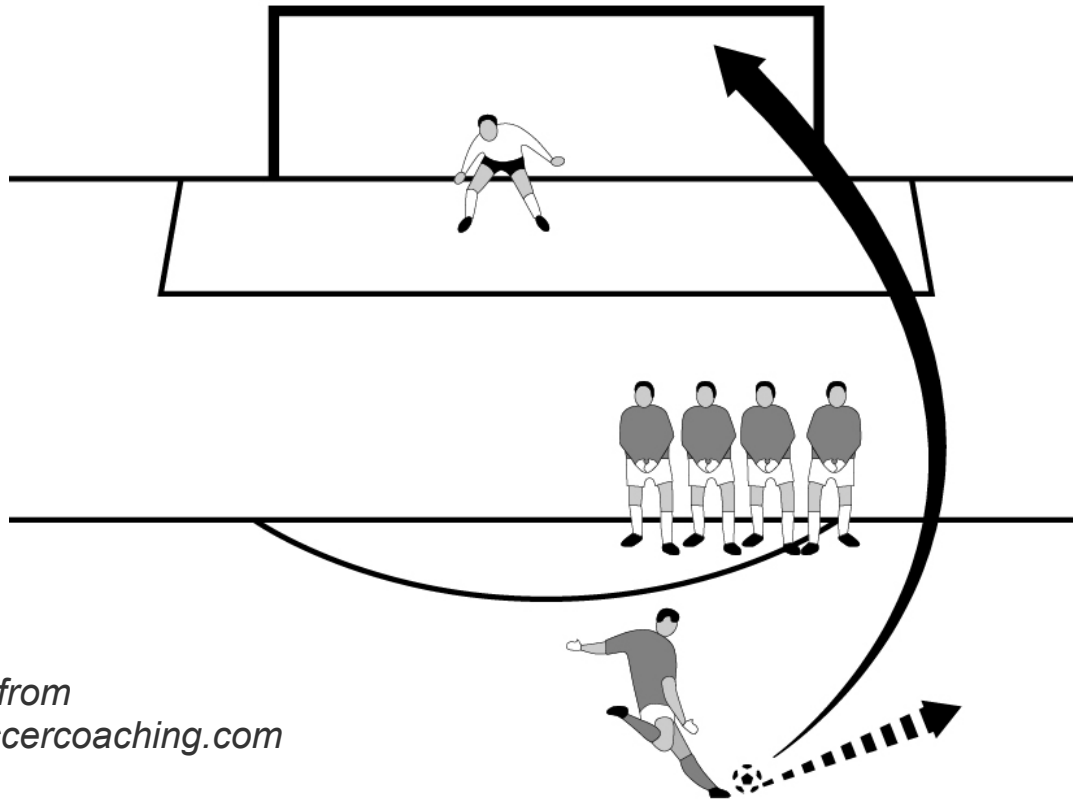
Currently a great deal of interest in **SOC** for **ultracold atoms**

(and also in the **artificial magnetic field** for **ultracold atoms**)

Recent reviews:

- J. Dalibard, F. Gerbier, G. Juzeliūnas and P. Öhberg, Rev. Mod. Phys. 83, 1523 (2011).
- N. Goldman, G. Juzeliūnas, P. Öhberg and I.B. Spielman, Rep. Progr. Phys. 77, 126401 (2014).
- H. Zhai, Rep. Progr. Phys. 78, 026001 (2015)
- N. Goldman, J. C. Budich, P. Zoller, Nat. Phys. 12, 639-645 (2016).

Spin-orbit coupling (SOC)



**SOC in
classical physics**

*Adapted from
bettersoccercoaching.com*

- The curvature of the trajectory is due to the **Magnus force**.
- Rotation of the ball in different directions yields oppositely curved trajectories - a **classical manifestation** of the SOC.

Spin-orbit coupling (SOC)

SOC for electrons

Spin-orbit coupling (SOC)

Electron has a spin



Spin can be coupled to orbital motion - SOC

$$\boldsymbol{\sigma} = \sigma_x \mathbf{e}_x + \sigma_y \mathbf{e}_y + \sigma_z \mathbf{e}_z$$

σ_x , σ_y , σ_z - **Pauli matrices** (describing spin projections)

Spin-orbit coupling (SOC)

For **electrons** appears in an external potential V :

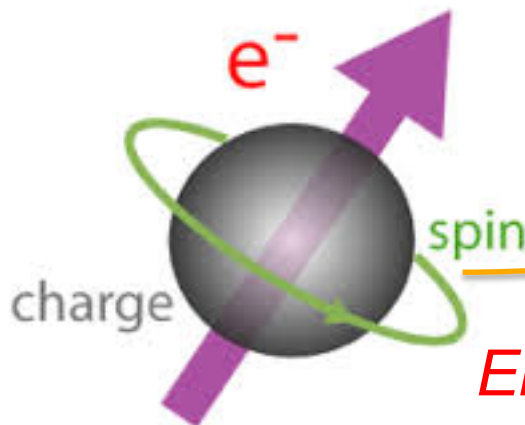
$$H_{soc} = \frac{\hbar}{4m^2c^2} \boldsymbol{\sigma} \cdot (\nabla V \times \mathbf{p})$$

$$\boldsymbol{\sigma} = \sigma_x \mathbf{e}_x + \sigma_y \mathbf{e}_y + \sigma_z \mathbf{e}_z$$

$$\mathbf{p} = -i\hbar\nabla$$

σ_x , σ_y , σ_z - Pauli matrices

Spin is coupled to orbital motion via $\nabla V = -e\mathbf{E}$



Electron motion causes its spin precession

Spin-orbit coupling (SOC)

For **electrons** appears in an external potential V :

$$H_{soc} = \frac{\hbar}{4m^2c^2} \boldsymbol{\sigma} \cdot (\nabla V \times \mathbf{p}) \quad \nabla V = -e\mathbf{E}$$

$$\boldsymbol{\sigma} = \sigma_x \mathbf{e}_x + \sigma_y \mathbf{e}_y + \sigma_z \mathbf{e}_z$$

σ_x , σ_y , σ_z - Pauli matrices

Relativistic effect !!!

Simple interpretation: Zeeman energy:

$$\rightarrow H_{soc} = -\frac{e\hbar}{4m} \boldsymbol{\sigma} \cdot (\mathbf{E} \times \mathbf{v}) / c^2 \quad \mathbf{p} = m\mathbf{v}$$

Electron spin interacts with  **magnetic field** acting on it.

(In moving frame **electric field** transforms into **magnetic field**)

Spin-orbit coupling (SOC)

For **electrons** appears in an external potential V :

$$H_{soc} = \frac{\hbar}{4m^2c^2} \boldsymbol{\sigma} \cdot (\nabla V \times \mathbf{p}) \quad \nabla V = -e\mathbf{E}$$

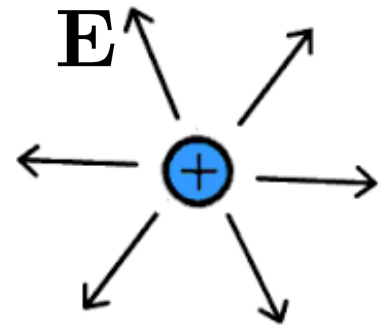
$$\boldsymbol{\sigma} = \sigma_x \mathbf{e}_x + \sigma_y \mathbf{e}_y + \sigma_z \mathbf{e}_z \quad \mathbf{p} = -i\hbar\nabla$$

Centrally symmetric electric field (e.g. in an atom)

$$\rightarrow V = V(r) \quad \nabla V = \frac{\mathbf{r}}{r} \frac{\partial V}{\partial r}$$

$$H_{soc} = \frac{\hbar}{4m^2c^2r} \frac{\partial V}{\partial r} \boldsymbol{\sigma} \cdot \mathbf{L}, \quad \mathbf{L} = \mathbf{r} \times \mathbf{p}$$

↑ ↑ ↑



→ **LS** spin-orbit coupling: Electron spin is coupled to **orbital angular momentum** $\mathbf{L}$ of electron

Spin-orbit coupling (SOC)

For **electrons** appears in an external potential V :

$$H_{soc} = \frac{\hbar}{4m^2c^2} \boldsymbol{\sigma} \cdot (\nabla V \times \mathbf{p})$$

$$\boldsymbol{\sigma} = \sigma_x \mathbf{e}_x + \sigma_y \mathbf{e}_y + \sigma_z \mathbf{e}_z$$

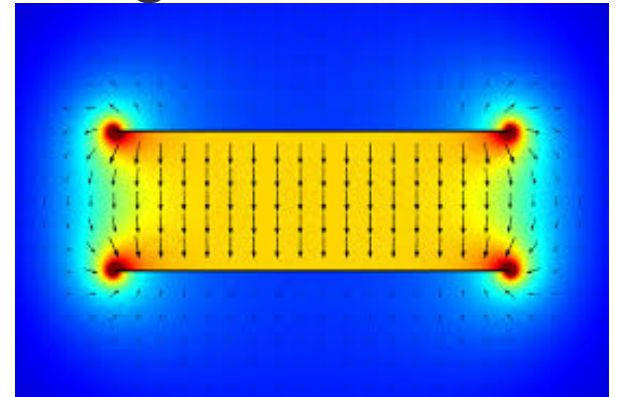
$$\mathbf{p} = -i\hbar\nabla$$

$V = -eEz \rightarrow$ **Homogeneous** $\mathbf{E}$ along z axis $\rightarrow$

$$\nabla V = -eE\mathbf{e}_z = -e\mathbf{E}$$

$$H_{soc} = \frac{e\hbar E}{4m^2c^2} \boldsymbol{\sigma} \times \mathbf{p} \cdot \mathbf{e}_z$$

linear momentum



e.g. 2D electrons in a uniform perpendicular electric field, such as in **asymmetric semiconductor heterostructures**

Spin-orbit coupling (SOC)

For **electrons** appears in an external potential V :

$$H_{soc} = \frac{\hbar}{4m^2c^2} \boldsymbol{\sigma} \cdot (\nabla V \times \mathbf{p})$$

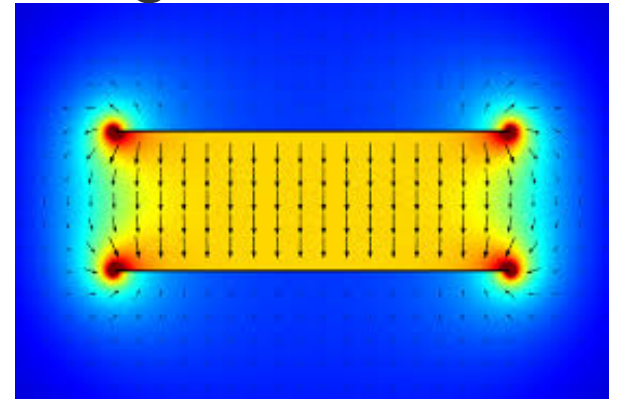
$$\boldsymbol{\sigma} = \sigma_x \mathbf{e}_x + \sigma_y \mathbf{e}_y + \sigma_z \mathbf{e}_z$$

$$\mathbf{p} = -i\hbar\nabla$$

$V = -eEz \rightarrow$ **Homogeneous** $\mathbf{E}$ along z axis $\rightarrow$

$$\nabla V = -eE\mathbf{e}_z = -e\mathbf{E}$$

$$H_{soc} = \frac{e\hbar E}{4m^2c^2} \boldsymbol{\sigma} \times \mathbf{p} \cdot \mathbf{e}_z$$



$\rightarrow$ **Rashba-type** SOC: Electron spin is coupled to its **linear momentum** $\mathbf{p}$ in the xy plane (in a plane perpendicular to $\mathbf{E}$)

Rashba-type SOC

Electron spin is coupled to its **linear momentum** $\mathbf{p}$ in the xy plane.

$$H_{soc} = \frac{e\hbar E}{4m^2c^2} \boldsymbol{\sigma} \times \mathbf{p} \cdot \mathbf{e}_z$$

In terms of components:

$$H_{soc} = \frac{e\hbar E}{4m^2c^2} (\sigma_x p_y - \sigma_y p_x)$$

Relativistic effect !!!

$p_y \rightarrow p_x$, $p_x \rightarrow -p_y$ (spatial rotation around z)

$$\rightarrow H_{soc} = \frac{e\hbar E}{4m^2c^2} (\sigma_x p_x + \sigma_y p_y)$$

Rashba-type SOC

SOC for electrons in solids

$$H_{soc} = \frac{e\hbar E}{4m^2c^2} (\sigma_x p_x + \sigma_y p_y)$$

SOC due to electric field
(internal or external) →

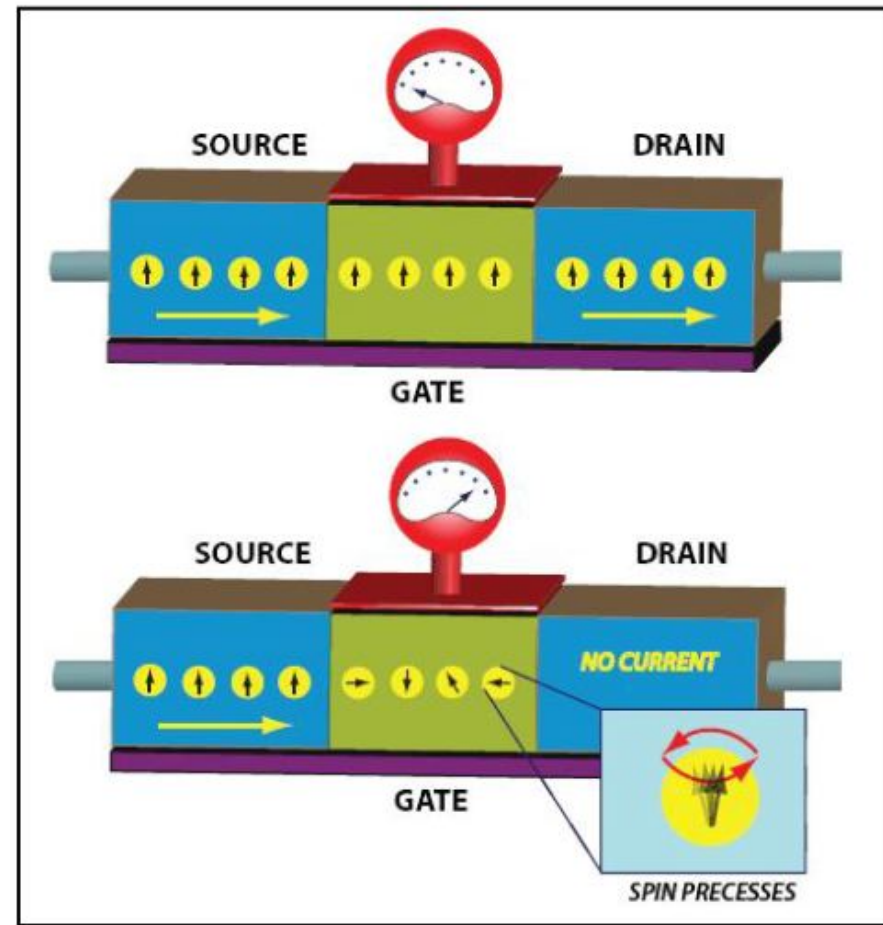
Electron spin field transistor,

Proposal: S. Datta and B. Das
(Appl. Phys. Lett. 1990)

Gate voltage controls
SO coupling,
i.e. **spin precession**,
i.e. **spin current**

P. Chuang et al, Nature
Nanotechnology, 10, 35 (2015)

THE DATTA-DAS TRANSISTOR



Rashba-type SOC

$$H_{soc} = \frac{e\hbar E}{4m^2 c^2} (\sigma_x p_x + \sigma_y p_y)$$

■ Rashba-Dresselhaus Hamiltonian:

$$H = \frac{(\mathbf{p} - \mathbf{A})^2}{2m} = \frac{p^2}{2m} - \frac{\mathbf{p} \bullet \mathbf{A}}{m} + const \quad \mathbf{p} = -i\hbar \nabla$$

■ Vector potential $\mathbf{A}$ – 2x2 matrix-vector:

$$\mathbf{A} = \chi (\alpha_x \sigma_x \mathbf{e}_x + \alpha_y \sigma_y \mathbf{e}_y) \quad [A_x, A_y] \neq 0 \quad (\text{non-Abelian } \mathbf{A})$$

■ σ_x , σ_y – Pauli matrices (if α_x & α_y non-zero)

■ α_x , α_y – dimensionless asymmetry parameters

Rashba-Dresselhaus SO coupling

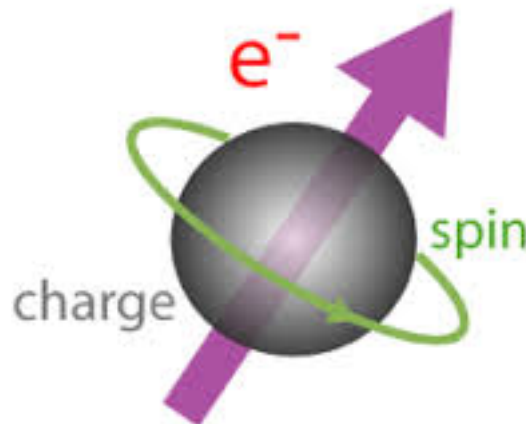
$$H = \frac{(\mathbf{p} - \mathbf{A})^2}{2m} = \frac{p^2}{2m} + \frac{\mathbf{p} \cdot \mathbf{A}}{m} + \text{const} \quad \mathbf{p} = -i\hbar\nabla$$

- Vector potential $\mathbf{A}$ – 2x2 matrix:

$$\mathbf{A} = \chi(\alpha_x \sigma_x \mathbf{e}_x + \alpha_y \sigma_y \mathbf{e}_y)$$

- σ_x , σ_y - **Pauli matrices**;

- H acts on a **two-component spinor**: $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$



**Spin-up &
Spin-down** states
of electron

Rashba-Dresselhaus SO coupling

$$H = \frac{(\mathbf{p} - \mathbf{A})^2}{2m} = \frac{p^2}{2m} + \frac{\mathbf{p} \cdot \mathbf{A}}{2m} + const$$

$$\mathbf{p} = -i\hbar\nabla$$

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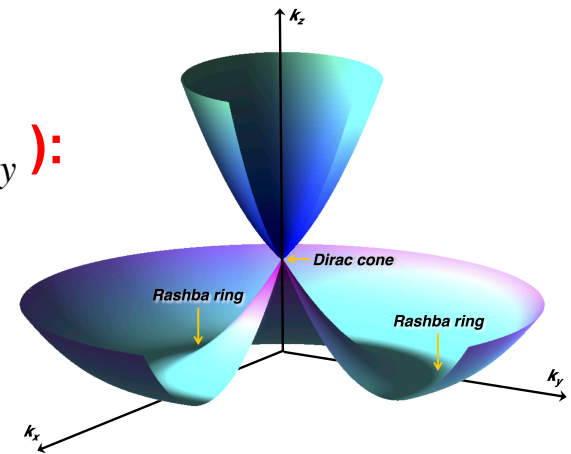
- H acts on a **two-component spinor**:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

❖ **Position independent Hamiltonian**

❖ **Dispersion for the Rashba 2D SOC ($\alpha_x = \alpha_y$):**

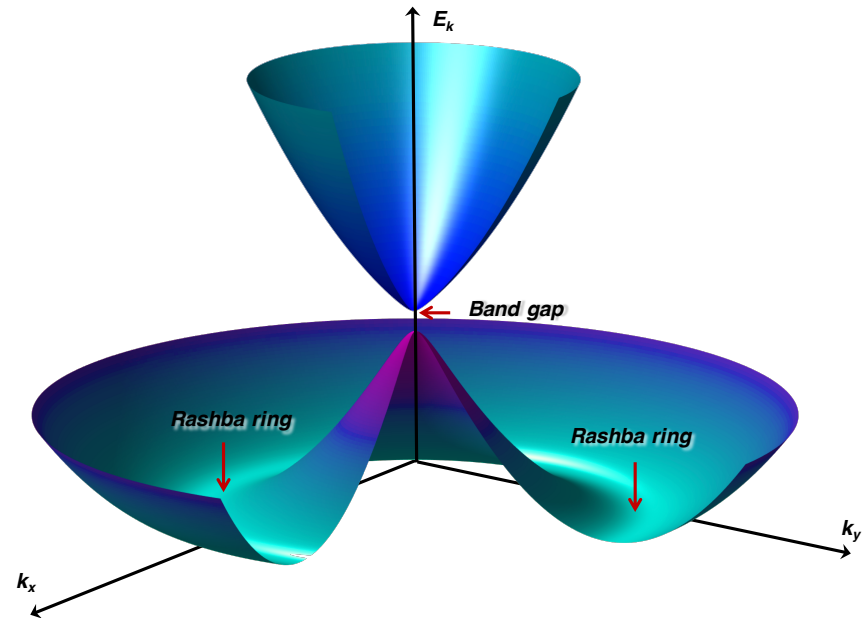
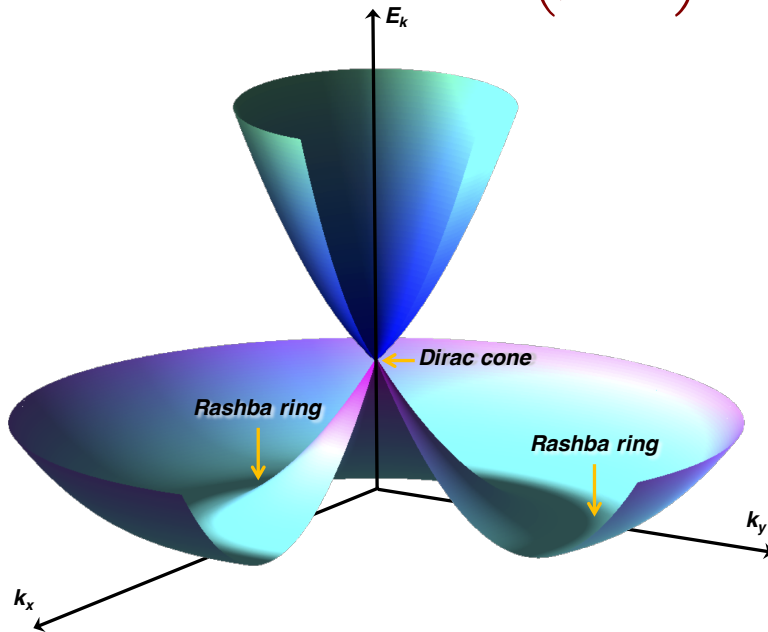
- ❖ Dirac cone around band intersection;
- ❖ Lower band - Mexican hat type potential
- ❖ Energy minimum – Rashba ring



Rashba dispersion with Zeeman term $b\sigma_z$

$$H = \frac{p^2}{2m} + \frac{\mathbf{p} \cdot \mathbf{A}}{2m} + b\sigma_z \quad \mathbf{A} = \chi\alpha(\sigma_x \mathbf{e}_x + \sigma_y \mathbf{e}_y) \quad \mathbf{p} = \hbar \mathbf{k}$$

Zero Zeeman term ($b=0$): Non-zero Zeeman term ($b \neq 0$):



- ❖ **Dirac cone** around band intersection for $b=0$;
- ❖ **Band gap** for $b \neq 0$ (non-zero Berry phase around $k=0$): topological fermions
- ❖ Lower band - **Mexican hat** type potential – unusual BEC at **Rashba ring**

How to produce Rashba SOC for cold atoms?

Ways to produce Rashba-type SOC for ultracold atoms

Ways to produce Rashba-type SOC for ultracold atoms

A proposal:

M. Ericsson and E. Sjöqvist Phys. Rev. A **65**, 013607 (2001) :

Aharonov-Casher [PRL 53,319, (1984)] Hamiltonian

$$\mathcal{H}_{AC} = \frac{1}{2m} \left(\mathbf{p} - \frac{1}{c^2} \boldsymbol{\mu} \times \boldsymbol{\mathcal{E}} \right)^2 \quad \boldsymbol{\mu} = \mu \boldsymbol{\sigma}$$

Atomic spin interacts with **magnetic field** acting on it.

(In moving frame **electric field** transforms into **magnetic field**)

Relativistic effect !!!

Extremely **weak** for slowly moving atoms

→ **Other methods are needed**

Ways to produce Rashba-type SOC for ultracold atoms (using optical lattices)

VOLUME 92, NUMBER 15

PHYSICAL REVIEW LETTERS

week ending
16 APRIL 2004

Spin-Orbit Coupling and Berry Phase with Ultracold Atoms in 2D Optical Lattices

Artem M. Dudarev,^{1,2,*} Roberto B. Diener,^{3,1} Iacopo Carusotto,^{4,5} and Qian Niu¹

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We show how spin-orbit coupling and Berry phase can appear in two-dimensional optical lattices by coupling atoms' internal degrees of freedom to radiation. The Rashba Hamiltonian, a standard description of spin-orbit coupling for two-dimensional electrons, is obtained for the atoms under certain circumstances. We discuss the possibility of observing associated phenomena, such as the anomalous Hall and spin Hall effects, with cold atoms in optical lattices.

Ways to produce Rashba-type SOC for ultracold atoms (using optical lattices)

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In the subspace spanned by these two eigenvectors the effective Hamiltonian once v_1 and v_2 are taken into account has the form

$$H_{\Gamma}(\mathbf{k}) = \epsilon_- I + h_0 \sigma_z - \gamma(k_y \sigma_x + k_x \sigma_y), \quad (12)$$

where σ_i are Pauli matrices and I is the unit matrix. This reduces to the standard form of the Rashba Hamiltonian

(Dirac-type Hamiltonian – p^2 term is missing)

Ways to produce Rashba-type SOC for ultracold atoms (using optical lattices)

(Dirac-type Hamiltonian)

Realization of Two-Dimensional Spin-orbit Coupling for Bose-Einstein Condensates

Zhan Wu, Long Zhang, Wei Sun, Xiao-Tian Xu, Bao-Zong Wang, Si-Con Ji, Youjin Deng, Shuai Chen, Xiong-Jun Liu, Jian-Wei Pan, Science **354**, 83-88 (2016).

(Dirac-type Hamiltonian – p^2 term is missing)

Ways to produce Rashba-type SOC for ultracold atoms

without a lattice (continuous model)

Light-induced geometric potentials
(for adiabatic motion of atoms in a pair of degenerate internal states)

Ways to produce Rashba-type SOC for ultracold atoms

Light-induced geometric potentials

(for adiabatic motion of atoms in a pair of degenerate internal states)

linear momentum

spin

Aim:
$$H = \frac{(\mathbf{p} - \mathbf{A})^2}{2m} = \frac{p^2}{2m} + \frac{\mathbf{p} \cdot \mathbf{A}}{2m} + \text{const}$$

$$\mathbf{p} = -i\hbar\nabla$$

- Geometric vector potential $\mathbf{A}$ – 2x2 matrix:

$$\mathbf{A} = \chi(\alpha_x \sigma_x \mathbf{e}_x + \alpha_y \sigma_y \mathbf{e}_y)$$

- σ_x , σ_y - **Pauli matrices**;

- H acts on a **two-component spinor**: $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$

Two degenerate or almost degenerate atomic internal (**quasi-spin**) states are needed.

Geometric potentials

- Emerge in various areas of physics (molecular, condensed matter physics etc.)
- First considered by Mead, Berry, Wilczek and Zee and others in the 80's (**initially in the context of molecular physics**).
- More recently – in the context of motion of cold atoms affected by laser fields
(**Currently: a lot of activities**)
- **Recent reviews:**
- J.Dalibard, F. Gerbier, G. Juzeliūnas and P. Öhberg, **Rev. Mod. Phys.** **83**, 1523 (2011).
- N. Goldman, G. Juzeliūnas, P. Öhberg and I.B. Spielman **Rep. Progr. Phys.** **77**, 126401 (2014).

Geometric potentials

- Advantage of ultracold atoms: possibilities to control and shape gauge potentials by choosing proper laser fields.
- **Vector potential** can be made **matrix valued**.
- H acts on a **two-component spinor**:
$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$
- **Rashba-type spin-orbit coupling (SOC)**
- For ultracold atoms SOC is usually due to the atomic recoil (large energy for ultracold atoms).
- → Such a SOC can effectively be much stronger than SOC for electrons in solids.

Geometric potentials

Atomic dynamics taking into account both **internal degrees of freedom** and also **centre of mass** motion.

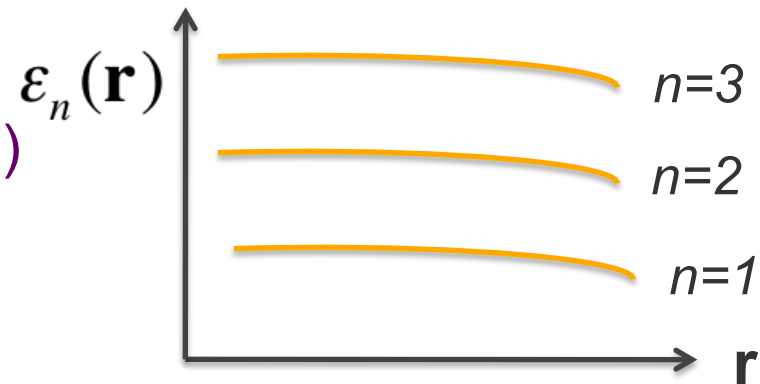
$H_{at}(r)$ - Hamiltonian for atomic internal degrees of freedom
(includes **r**-dependent atom-light coupling)

For fixed **r** :

$$H_{at}(r)|\chi_n(\mathbf{r})\rangle = \varepsilon_n(\mathbf{r})|\chi_n(\mathbf{r})\rangle$$

$|\chi_n(\mathbf{r})\rangle$ – n -th eigenstate (“**dressed**” state): **r**-dependent

$\varepsilon_n(\mathbf{r})$ – corresponding eigenenergy: **r**-dependent



Additionally $T = p^2 / 2m$

→ Full state vector $|\Phi\rangle = \sum_{n=1}^N |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r}, t)$

$\Psi_n(\mathbf{r}, t)$ – wave-function of the atomic **centre of mass** motion in the n -th atomic internal “**dressed**” state

Non-degenerate state with $n=1$

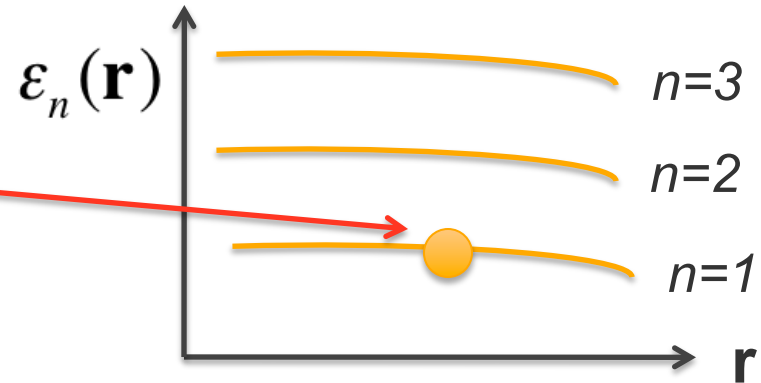
Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r}, t)$$

Full state-vector

$$|\Phi\rangle = \sum_{n=1}^N |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r}, t)$$

$\Psi_n(\mathbf{r}, t)$ – wave-function of the atomic centre of mass motion in the n -th atomic internal “dressed” state

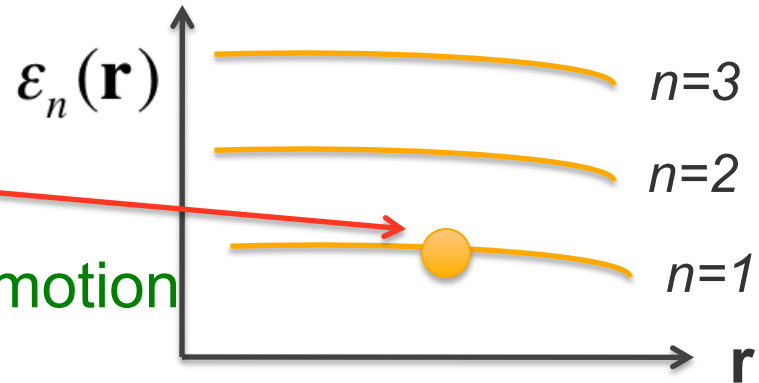


Non-degenerate state with $n=1$

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r}, t)$$

Equation of the atomic c. m. motion
in the internal state $|\chi_1(\mathbf{r})\rangle$



$$i\hbar \partial_t \Psi_1(\mathbf{r}, t) = H \Psi_1(\mathbf{r}, t)$$

$$\hat{H} = \frac{(\mathbf{p} - \mathbf{A}_{11})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$$

$$\mathbf{p} = -i\hbar \nabla$$

$$\mathbf{A}_{11} = i\hbar \langle \chi_1(\mathbf{r}) | \nabla \chi_1(\mathbf{r}) \rangle$$

Effective Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ appears (due to the position-dependence of the **atomic internal “dressed” state** $|\chi_1(\mathbf{r})\rangle$)

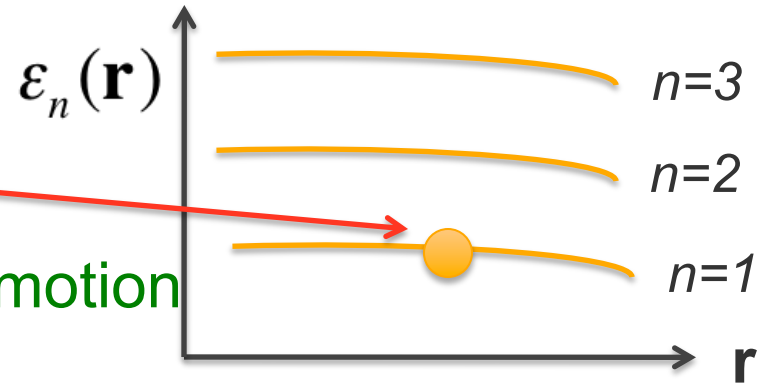
R. Dum and M. Olshanii, **PRL** **76**,1788 (1996).

Non-degenerate state with $n=1$

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r}, t)$$

Equation of the atomic c. m. motion
in the internal state $|\chi_1(\mathbf{r})\rangle$



$$i\hbar \partial_t \Psi_1(\mathbf{r}, t) = H \Psi_1(\mathbf{r}, t)$$

$$\hat{H} = \frac{(\mathbf{p} - \mathbf{A}_{11})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$$

$$\mathbf{p} = -i\hbar \nabla$$

$$\mathbf{A}_{11} = i\hbar \langle \chi_1(\mathbf{r}) | \nabla \chi_1(\mathbf{r}) \rangle$$

Effective Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$

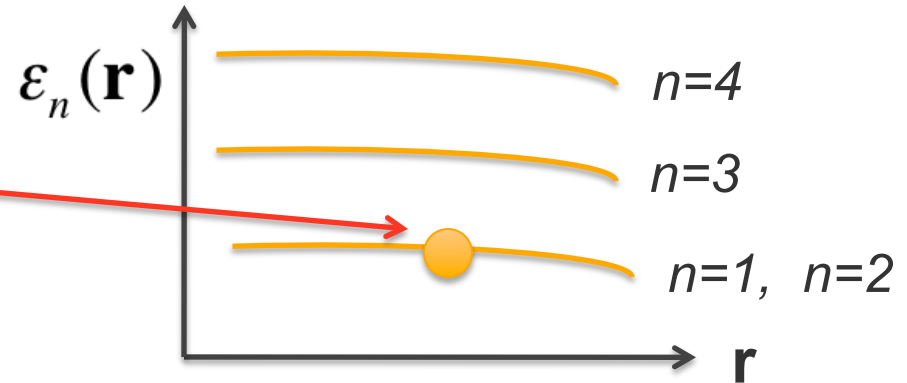
$\mathbf{B} = \nabla \times \mathbf{A}$ - effective magnetic field (non-trivial situation if $\mathbf{B} \neq 0$)

G.Juzeliūnas, and P. Öhberg, **PRL** **93**, 033602 (2004) & subsequent work.

Degenerate states with $n=1$ and $n=2$

Adiabatic approximation:

$$|\Phi\rangle \approx \sum_{n=1}^2 |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r}, t)$$



$\Psi_n(\mathbf{r}, t)$ – wave-function of the atomic centre of mass motion in the n -th atomic internal “dressed” state ($n=1, 2$)

$$\Psi(\mathbf{r}, t) = \begin{pmatrix} \Psi_1(\mathbf{r}, t) \\ \Psi_2(\mathbf{r}, t) \end{pmatrix}$$

-two-component atomic wave-function
(spinor wave-function)
→ **Quasi-spin 1/2**

Degenerate states with $n=1$ and $n=2$

Adiabatic approximation:

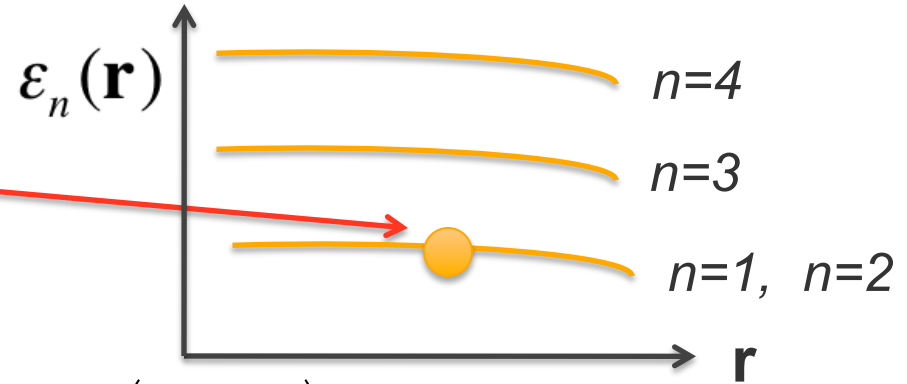
$$|\Phi\rangle \approx \sum_{n=1}^2 |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r}, t)$$

Equation of motion:

$$\hat{H} = \frac{(\mathbf{p} - \mathbf{A})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$$

$$\Psi(\mathbf{r}, t) = \begin{pmatrix} \Psi_1(\mathbf{r}, t) \\ \Psi_2(\mathbf{r}, t) \end{pmatrix}$$

- two-comp. atomic wave-function



$$\mathbf{A}_{ij} = i\hbar \langle \chi_i(\mathbf{r}) | \nabla \chi_j(\mathbf{r}) \rangle, \quad (i, j = 1, 2)$$

2x2 matrix

- effective vector potential

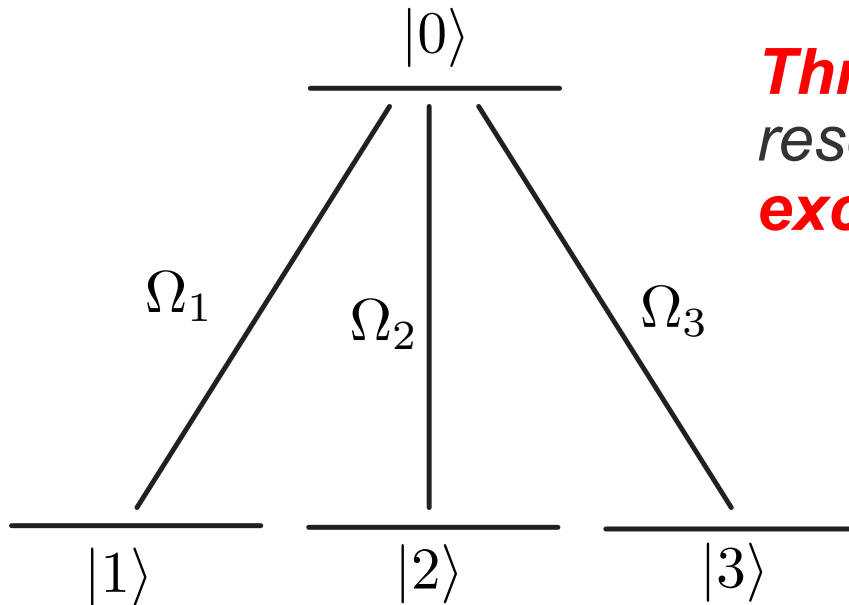
$\mathbf{A}$ appears due to position-dependence of $|\chi_n(\mathbf{r})\rangle$



Spin-orbit coupling is formed

Two degenerate atomic “**dressed**” states are needed

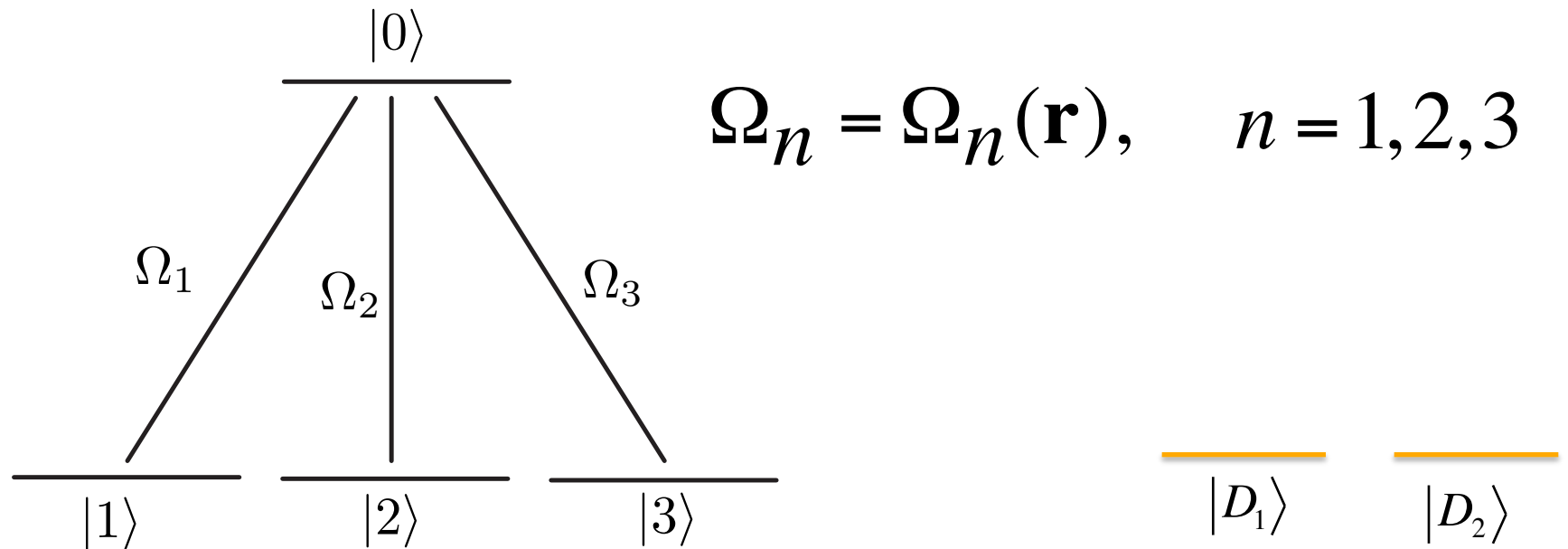
Tripod configuration



Three atomic **ground states** are resonantly coupled to the same **excited state** $|0\rangle$ by laser fields

- M.A. Ol' shanii, V.G. Minogin, Quant. Optics 3, 317 (1991)
- R. G. Unanyan, M. Fleischhauer, B. W. Shore, and K. Bergmann, Opt. Commun. **155**, 144 (1998)
- **Two** degenerate dressed states (**dark states**)
- (Superposition of atomic ground states **immune** to atom-light coupling)

(Non-Abelian) light-induced vector potential



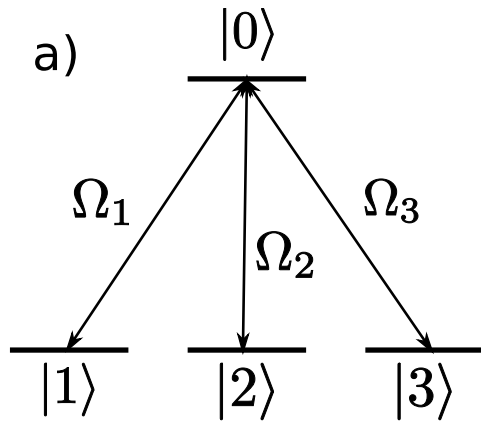
for centre of mass motion of dark-state atoms:
(Due to the **spatial dependence of the dark states**)

$$\mathbf{A}_{n,m} = i\hbar \langle D_n | \nabla D_m \rangle \quad \blacktriangleleft \text{(two dark states)}$$

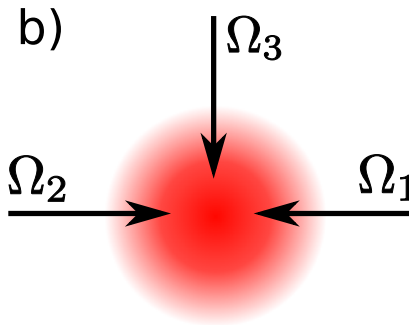
J. Ruseckas, G. Juzeliūnas and P. Öhberg, and M. Fleischhauer, Phys. Rev. Letters 95, 010404 (2005).

A - effective vector potential - 2x2 matrix $\rightarrow$ SOC

Three plane wave setup



$$\Omega_n = \Omega_n(\mathbf{r}),$$



$$\Omega_n = \Omega_n^{(0)} e^{i\mathbf{k}_n \cdot \mathbf{r}}, \quad n = 1, 2, 3$$

$$\Psi(\mathbf{r}, t) = \begin{pmatrix} \Psi_1(\mathbf{r}, t) \\ \Psi_2(\mathbf{r}, t) \end{pmatrix}$$

T. D. Stanescu,, C. Zhang, and V. Galitski,
Phys. Rev. Lett 99, 110403 (2007);
A. Jacob, P. Öhberg, G. Juzeliūnas, and L.
Santos, Appl. Phys. B 89, 439 (2007);
G. Juzeliūnas, J. Ruseckas, M. Lindberg, L.
Santos, and P. Öhberg, Phys. Rev. A 77,
011802(R) (2008).

→ Spin-Orbit coupling of the Rashba-Dresselhaus type

in the limit of large Rabi frequencies $\Omega_1^{(0)} = \Omega_2^{(0)}$ & $\Omega_3^{(0)}$

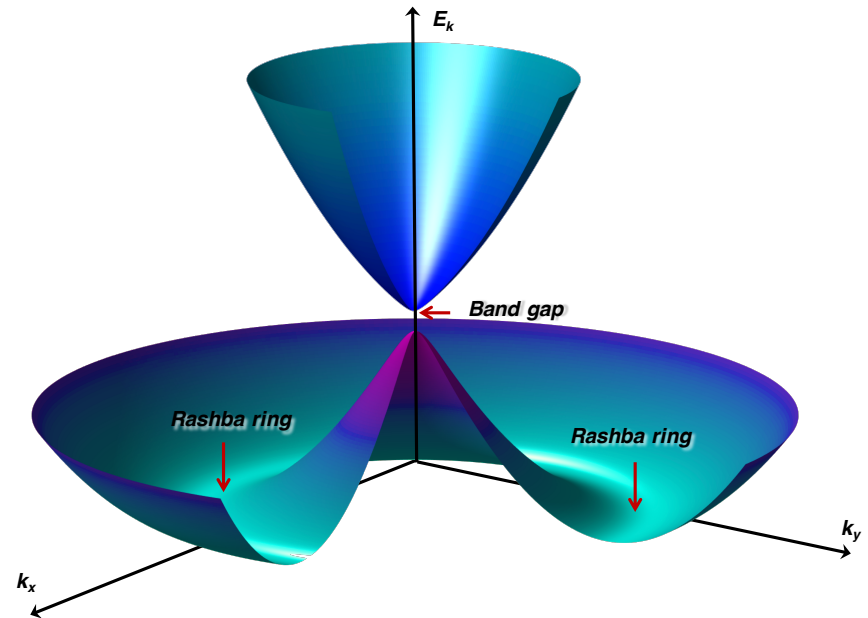
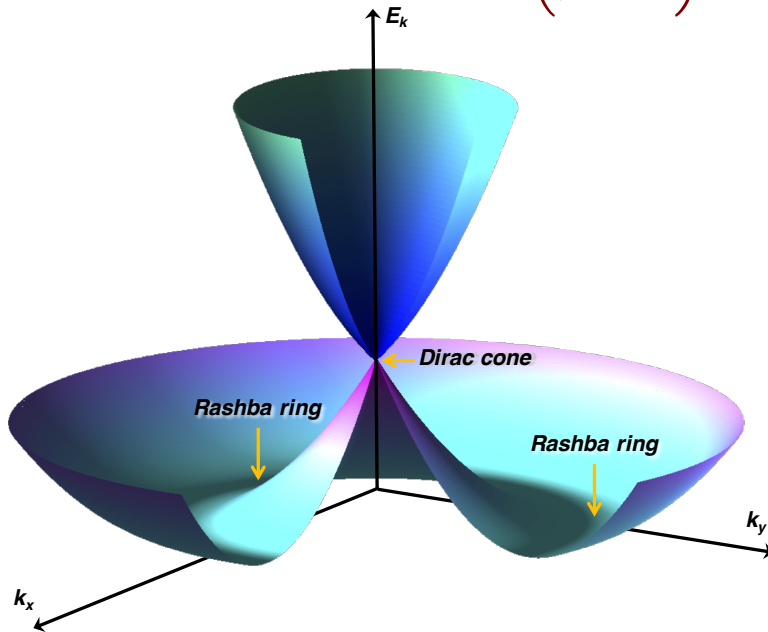
Additionally,

Zeeman term is controllable by the ratio $\Omega_2^{(0)} / \Omega_3^{(0)}$

Rashba dispersion with Zeeman term $b\sigma_z$

$$H = \frac{p^2}{2m} + \frac{\mathbf{p} \cdot \mathbf{A}}{2m} + b\sigma_z \quad \mathbf{A} = \chi\alpha(\sigma_x \mathbf{e}_x + \sigma_y \mathbf{e}_y) \quad \mathbf{p} = \hbar \mathbf{k}$$

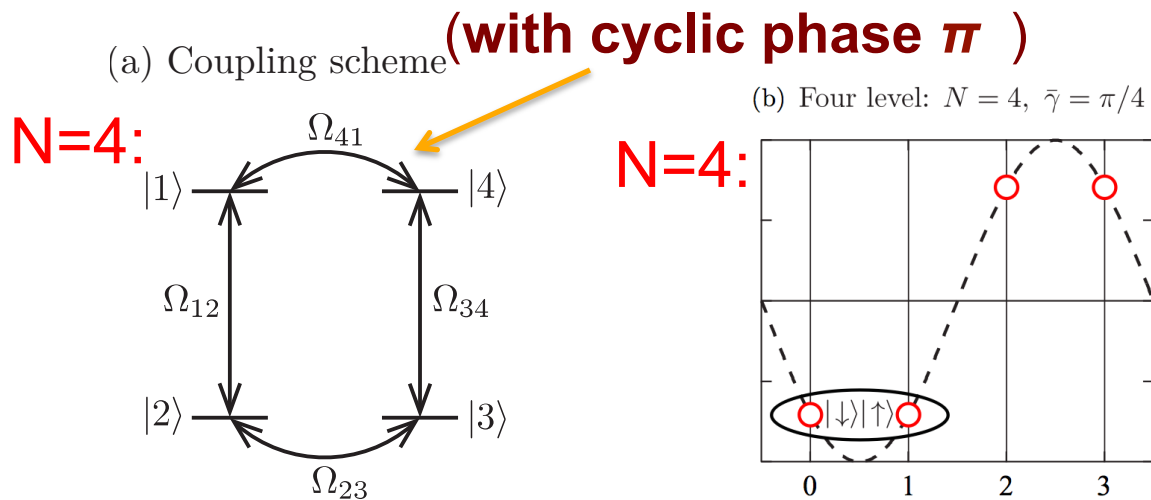
Zero Zeeman term ($b=0$): Non-zero Zeeman term ($b \neq 0$):



- ❖ **Dirac cone** around band intersection for $b=0$;
- ❖ **Band gap** for $b \neq 0$: non-zero Berry phase around $k=0$; topological fermions
- ❖ Lower band - **Mexican hat** type potential – unusual BEC at **Rashba ring**:

[see a review by H. Zhai, Rep. Progr. Phys. **78**, 026001 (2015)]

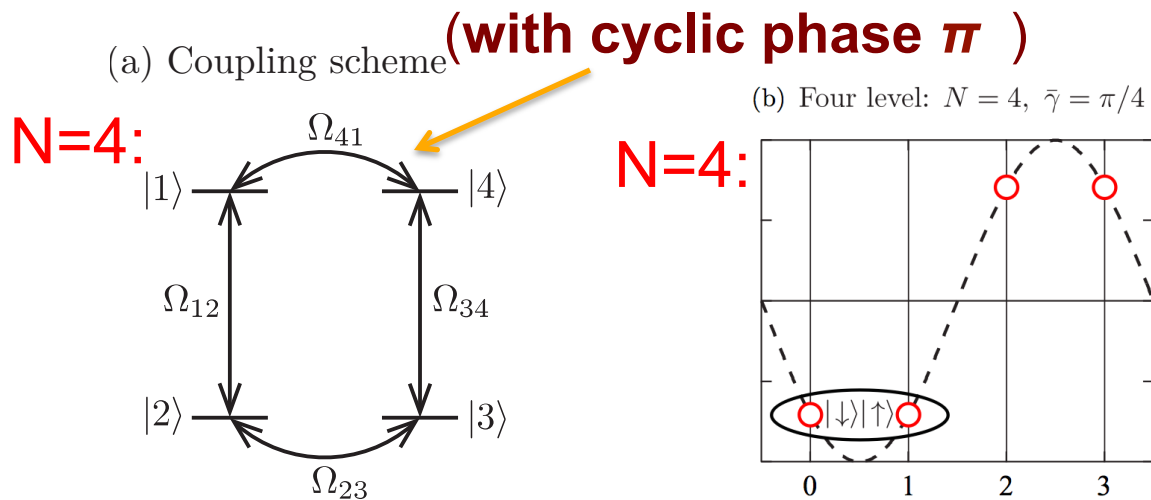
- Drawback of the tripod scheme: degenerate dark states are not the ground atomic dressed states $\rightarrow$ collision-induced losses
- Ring coupling setup overcomes this drawback:



N atomic internal states coupled in a cyclic manner **with a cyclic phase π**

- (π flux in synthetic dimension).

- Drawback of the tripod scheme: degenerate dark states are not the ground atomic dressed states $\rightarrow$ collision-induced losses
- Ring coupling setup overcomes this drawback:



N atomic internal states coupled in a cyclic manner **with a cyclic phase π**

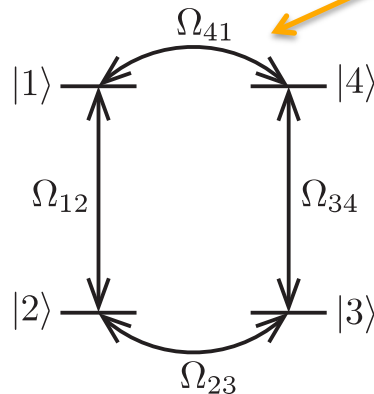
- **Two degenerate** internal ground dressed states
- $\rightarrow$ **Spin-orbit coupling** for ground-state manifold

- Laser fields represent counter-propagating plane waves:

$$\Omega_{j,j+1} = \Omega e^{i\mathbf{k}_j \cdot \mathbf{r} + i\phi_j}$$

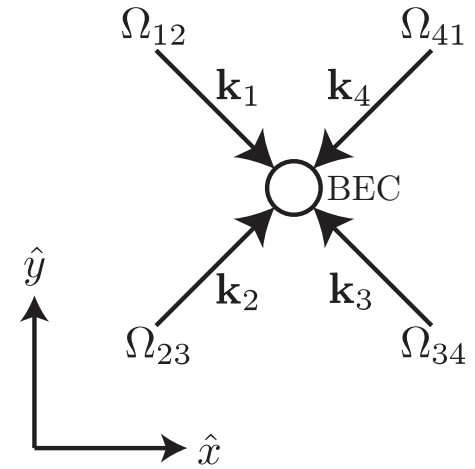
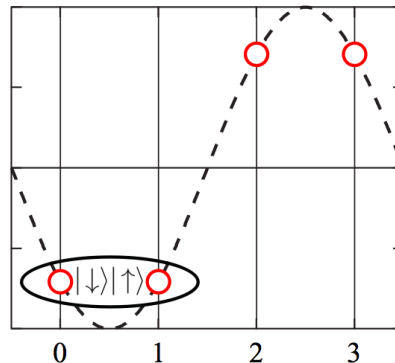
N=4:

(a) Coupling scheme



(with cyclic phase: $\bar{\phi}$)

(b) Four level: $N = 4$, $\bar{\gamma} = \pi/4$



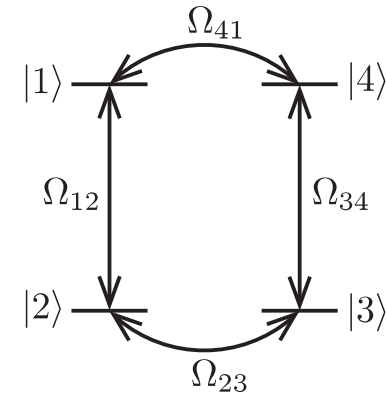
Zeeman term if

$$\bar{\phi} \neq \pi$$

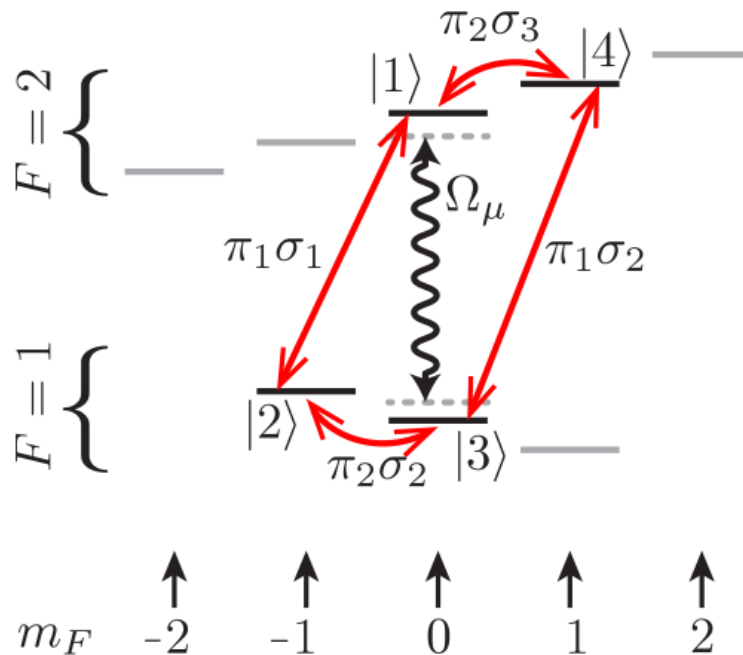
→ Ring coupling setup with **N=4** produces 2D Rashba-Dresselhaus SO coupling for cold atoms (also with **N=3**)

- Possible implementation of the ring coupling setup using the Raman transitions (N=4):

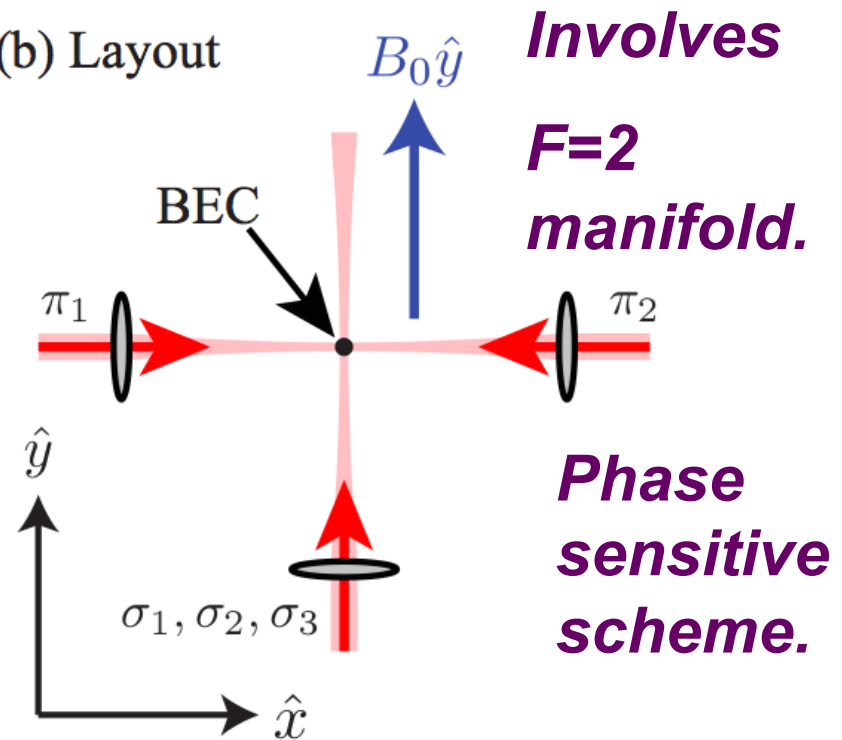
(a) Coupling scheme



(a) Coupling diagram



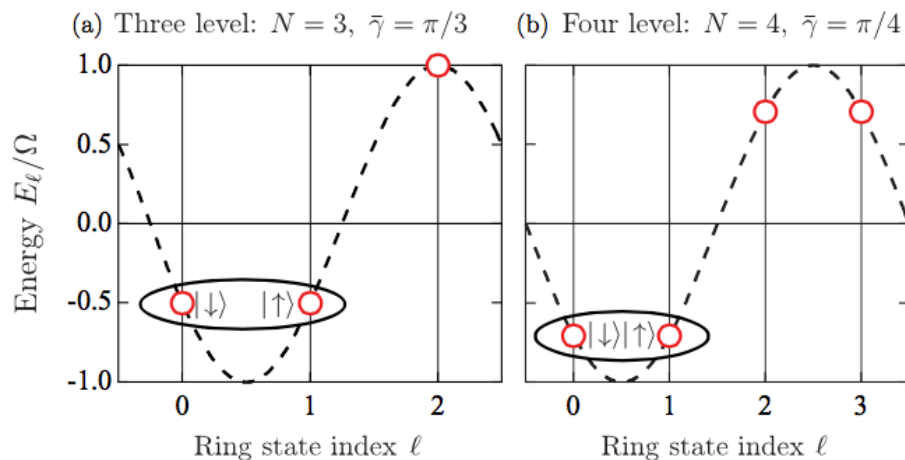
(b) Layout



■ Implementation using far detuned tripod (N=3)

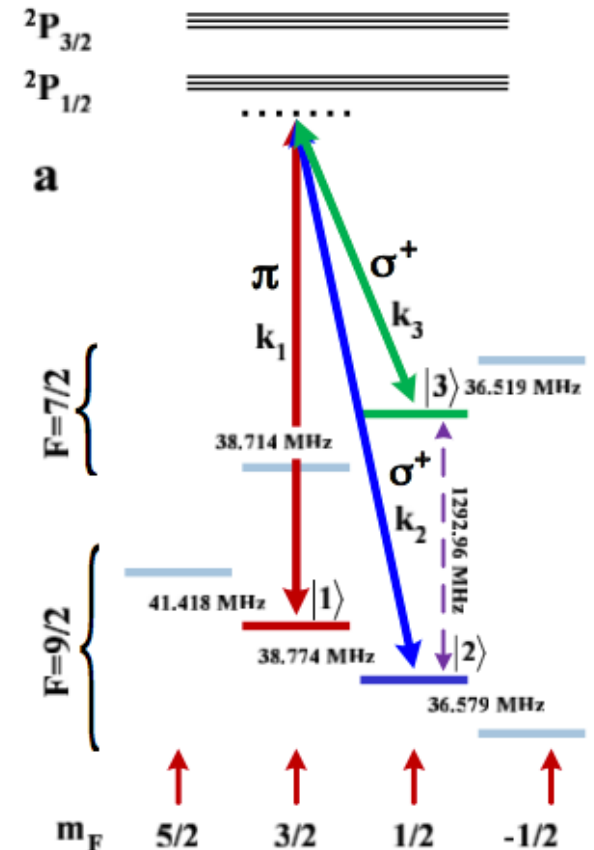
Ring coupling setup with $\bar{\phi} = \pi$

Double degenerate ground state
for N=3 and N=4



D. L. Campbell, G. Juzeliūnas and I. B. Spielman,
PRA 84, 025602 (2011)

→ Rashba SOC in both cases,
but N=4 better converges
to the Rashba ring



L.Huang et al, Nature Phys. 12, 540 (2016)

Dirac cone observed, not Rashba ring
(group of Jing Zhang, Shanxi University)

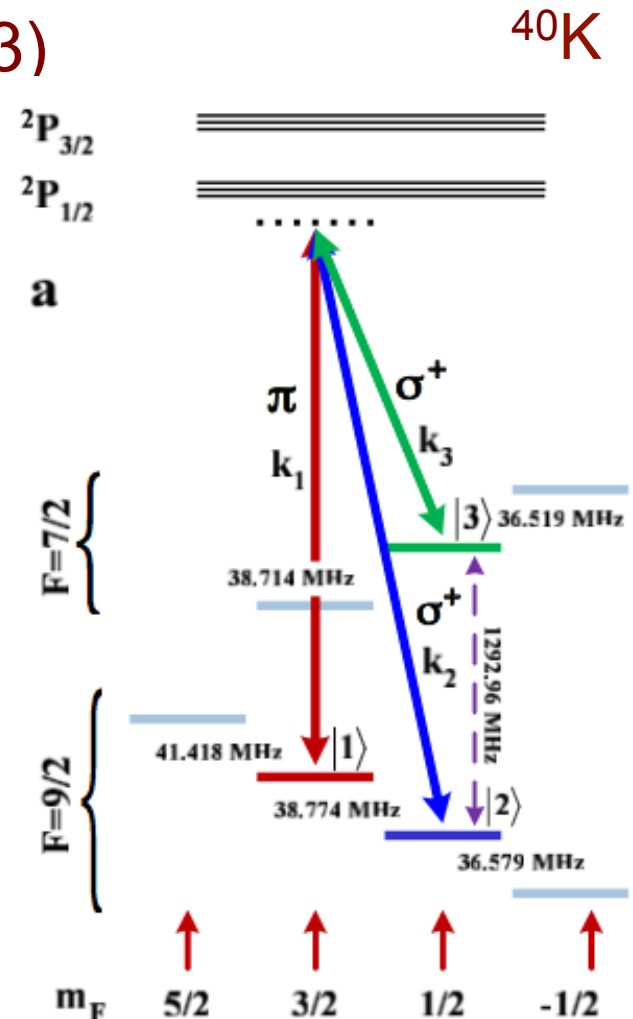
- Implementation of a ring coupling setup using far detuned tripod (N=3) with $\bar{\phi} = \pi$.

L.Huang et al, Nature Phys. 12, 540 (2016)

Dirac cone observed, **not a Rashba ring**
(group of Jing Zhang, Shanxi University, China)

$|1\rangle$, $|2\rangle$ and $|3\rangle$ are
coupled in a cyclic manner
via virtual excited states

Involves different F manifolds



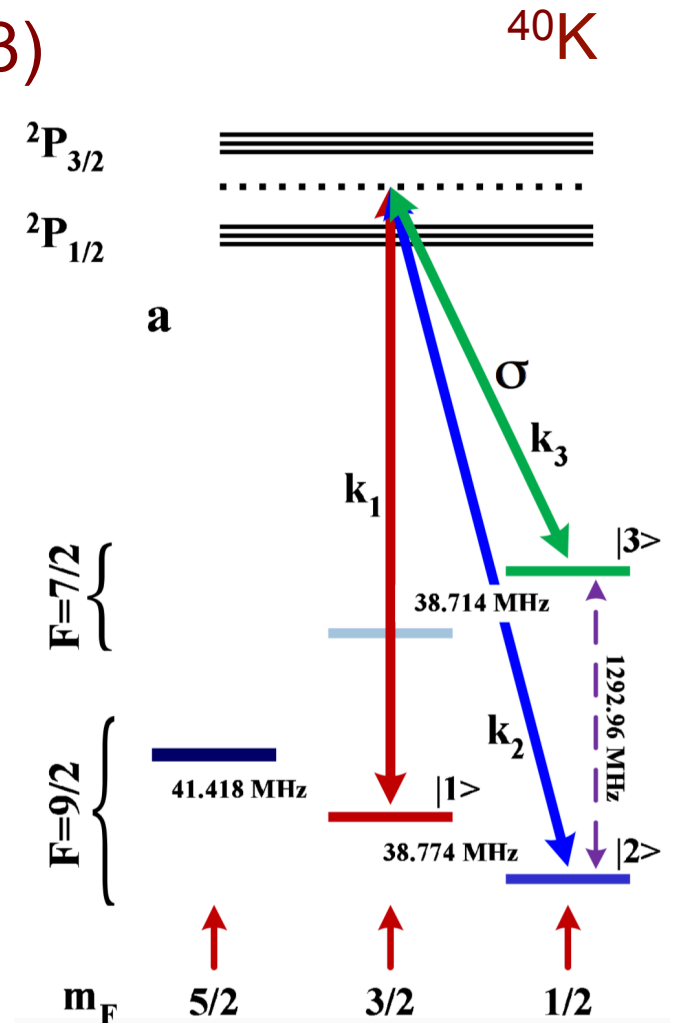
- Implementation of a ring coupling setup using far detuned tripod (N=3) with $\bar{\phi} \neq \pi$

Z. Meng et al, PRL 117, 235304 (2016)

Opening a gap at the Dirac cone
observed, **not a Rashba ring**

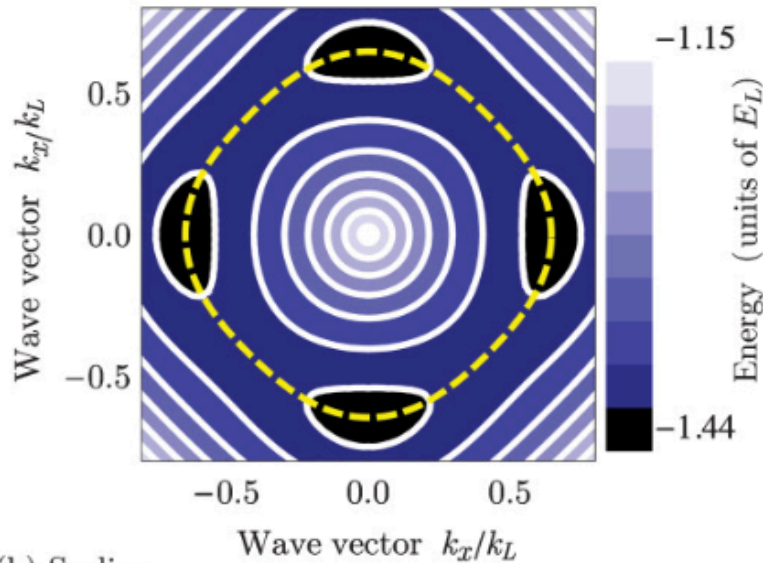
(group of Jing Zhang, Shanxi University, China)

Controlling $\bar{\phi}$ and hence the gap
by laser polarisations

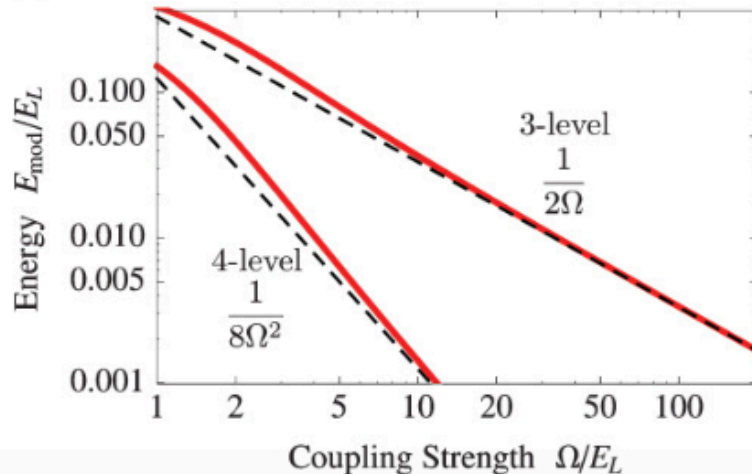


N=4 better covers to the Rashba ring (compared to N=3):

(a) 4-Level dispersion at $3E_L$ coupling

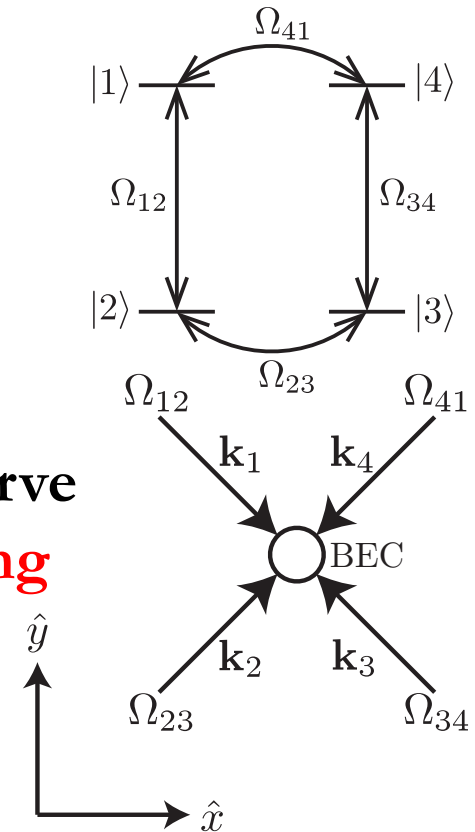


(b) Scaling



N=4

(a) Coupling scheme



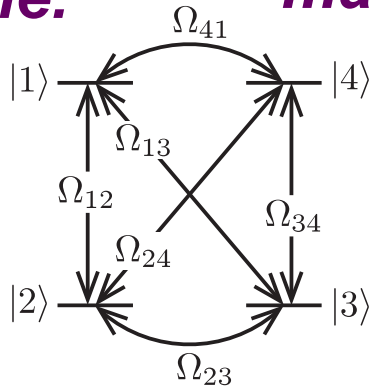
N=4 is **better suited** to observe the **Rashba ring**

D. L. Campbell, G. Juzeliūnas and I. B. Spielman, PRA 84, 025602 (2011)

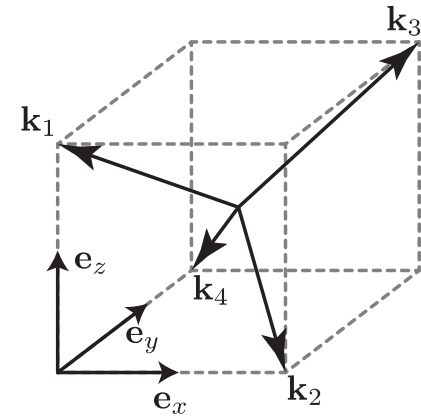
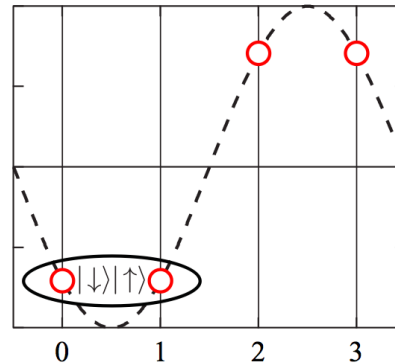
- Laser fields represent plane waves with wave-vectors forming a tetrahedron:

Phase sensitive scheme.

Specific scheme involving $F=2$ manifold



(b) Four level: $N = 4$, $\bar{\gamma} = \pi/4$



(d) Spectrum

Minimum of atomic energy - the sphere (in the momentum space)

Modified ring coupling setup produces **3D (Weil) SOC**:

$$H = p^2 + \alpha(p_x \sigma_x + p_y \sigma_y + p_z \sigma_z)$$

Rashba 3D SOC

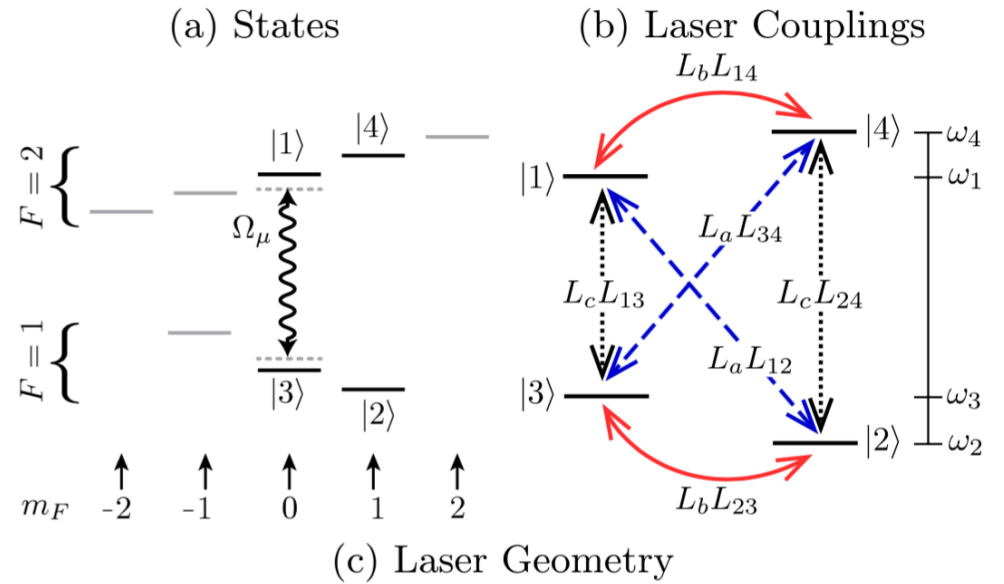


FIG. 2 (color online). Four hyperfine states $|F, m_F\rangle$ of ^{87}Rb are coupled using nine lasers. The quantization axis is set by a Zeeman field along the $\hat{z}$ axis. The couplings are produced in pairs. (a) The four states in the tetrahedral coupling are mapped to physical states according to $|1\rangle = |2, 0\rangle$, $|2\rangle = |1, +1\rangle$, $|3\rangle = |1, 0\rangle$, $|4\rangle = |2, +1\rangle$. (b) The frequencies of the three sets of lasers are given by $\{\omega_a, \omega_a + \delta_{12}, \omega_a + \delta_{34}\}$ (dashed blue), $\{\omega_b, \omega_b + \delta_{13}, \omega_b + \delta_{24}\}$ (dotted black), and $\{\omega_c, \omega_c + \delta_{14}, \omega_c + \delta_{23}\}$ (solid red), where $\delta_{ij} = \omega_i - \omega_j$ is the frequency difference between the states $|i\rangle$ and $|j\rangle$ in the rotating frame. (c) The geometry of the nine laser beams. $-\mathbf{k}_{12} = -\mathbf{k}_{13} = \mathbf{k}_{23} = \mathbf{k}_{34} = k_L \hat{e}_-$, $\mathbf{k}_a = \mathbf{k}_{23} = -\mathbf{k}_{14} = k_L \hat{z}$ and $\mathbf{k}_b = -\mathbf{k}_c = k_L \hat{e}_+$. The unit vectors $\hat{e}_\pm = \pm \frac{1}{\sqrt{2}}(\hat{x} \pm \hat{y})$. For a complete description of the laser parameters, see the Supplemental Material [21].

B. Anderson, G. Juzeliūnas, I. B. Spielman, and V. Galitski, Phys. Rev. Lett. 108, 235301 (2012).

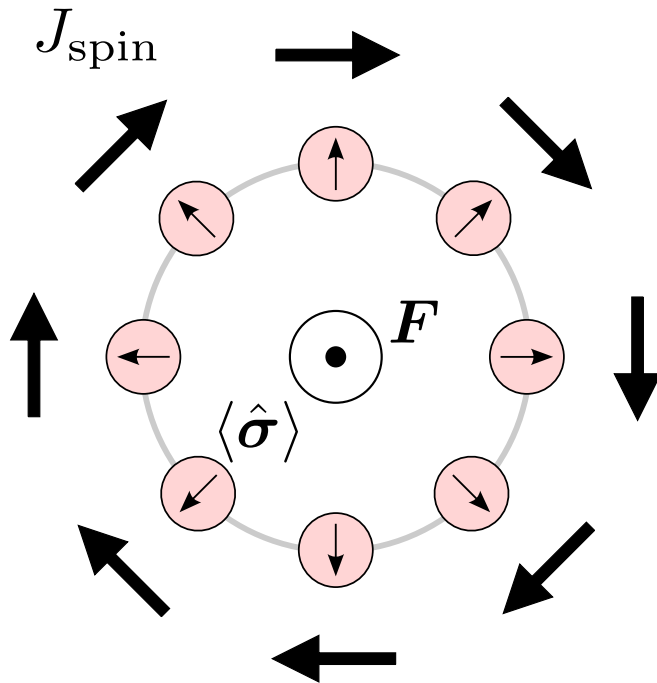
3D (Weil) SOC:

$$H = p^2 + \alpha(p_x \sigma_x + p_y \sigma_y + p_z \sigma_z)$$

SOC for all directions of atomic motion $\rightarrow$

Omnidirectional spin Hall effect in Weyl SOC atomic gas:

J. Armaitis, J. Ruseckas and G. Juzeliūnas, PRA 95, 033635 (2017)



The induced spin current flows in a direction perpendicular both to the driving force and also to the spin that the current carries.

Schemes for creating 2D SOC -
complex atom-light coupling
with many atomic states;
Have been only very recently
implemented experimentally

for a far detuned tripod
(ring coupling scheme with $N=3$)

[Nature Phys. 12, 540 (2016) & PRL 117, 235304 (2016)]

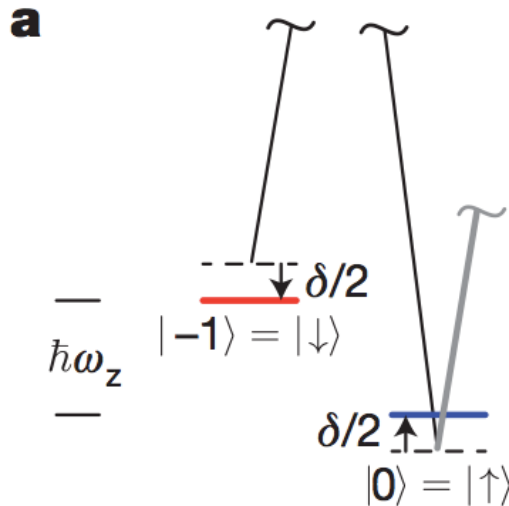
Experimental implementation of 1D SOC

Y.-J. Lin, K. Jimenez-Garcia and I. B. Spielman, Nature 471, 83 (2011).
J.-Y. Zhang et al., Phys. Rev. Lett. 109, 115301 (2012).
P. J. Wang et al, Phys. Rev. Lett. 109, 095301 (2012).
L. W. Cheuk et al., Phys. Rev. Lett. 109, 095302 (2012).
C. Qu et al, Phys. Rev. A 88, 021604(R) (2013).
etc.

Using a simpler scheme involving **Raman dressing** (with **recoil**)
of **two** atomic **internal ground states** (belonging to the same F):
J. Higbie and D. M. Stamper-Kurn, Phys. Rev. Lett. 88, 090401(2002).
X.-J. Liu, M. F. Borunda, X. Liu and J. Sinova, Phys. Rev. Lett. 102,
046402 (2009.)

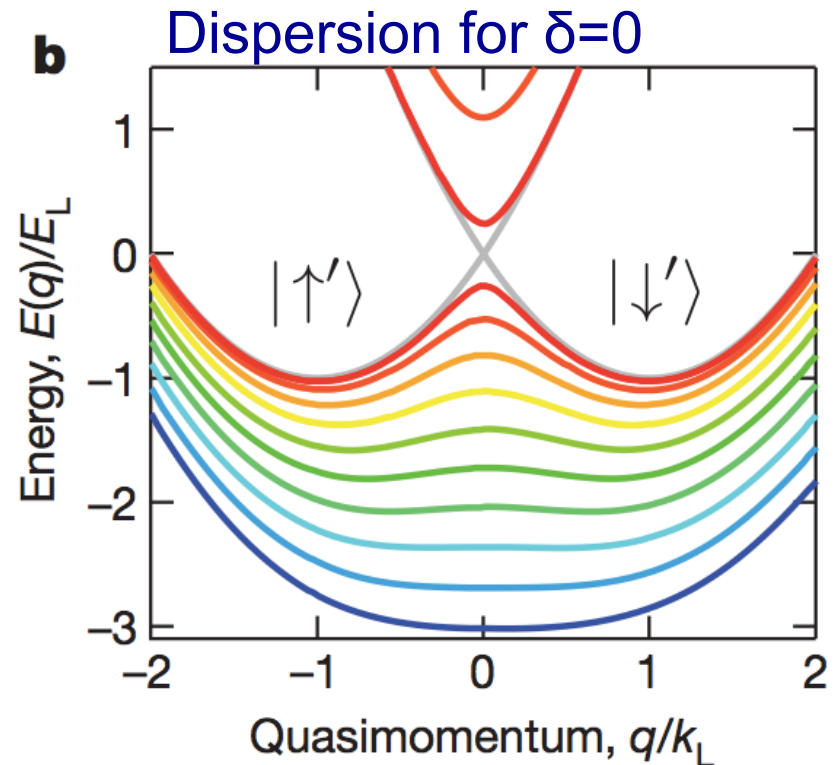
Spin-orbit coupling using a 2 hyperfine levels coupled by Raman lasers with recoil

Y.-J. Lin, K. Jimenez-Garcia and I. B. Spielman, Nature 471, 83 (2011).



$$H = \frac{(p - \chi\sigma_y)^2}{2m} + \frac{\Omega}{2}\sigma_z$$

Abelian (1D) vector potential
(but non-trivial situation if $\Omega \neq 0$)



SOC only in the direction of
Raman recoil $\rightarrow$

$$H = \frac{p^2}{2m} - \frac{\chi p \sigma_y}{m} + \frac{\Omega}{2}\sigma_z + \text{const} \quad \textbf{(1D SOC – Not Rashba SOC)}$$

Rashba-type SOC using a bilayer system

*S.-W. Su et al, PRA **93**, 053630 (2016) (TW, Beijing, Vilnius) - bosons*

*L.-L. Wang et al, PRA **95**, 053628 (2017) (Beijing, Vilnius) - fermions*

2D SOC using only two atomic internal states

Rashba-type SOC using a bilayer system

*S.-W. Su et al, PRA **93**, 053630 (2016) (TW, Beijing, Vilnius) - bosons*

*L.-L. Wang et al, PRA **95**, 053628 (2017) (Beijing, Vilnius) - fermions*

2D SOC using only two atomic internal states

Lecture 2 (Sunday, June 18):

Novel topological optical lattices

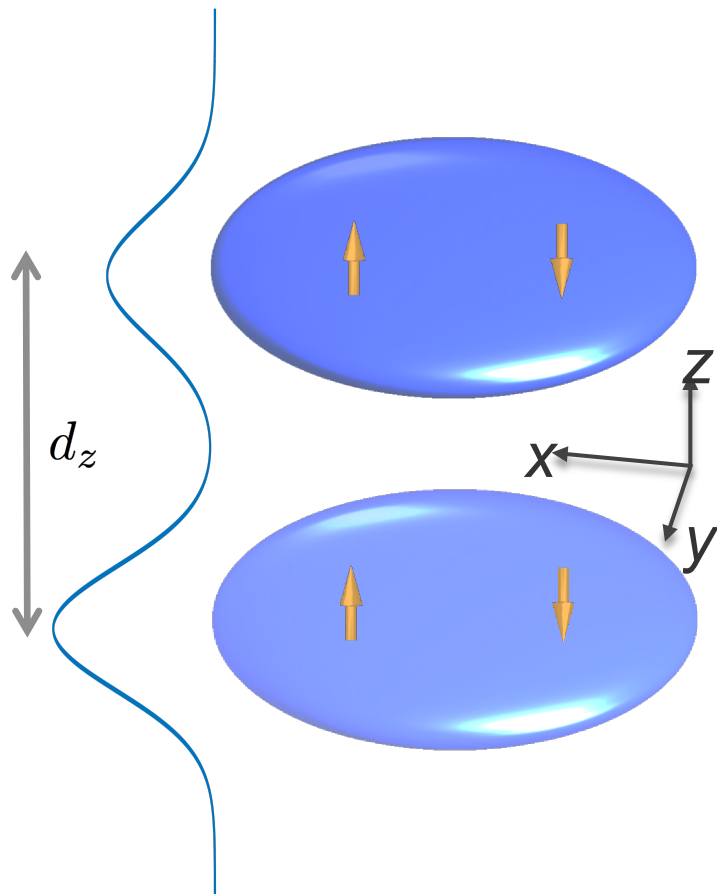
Rashba-type SOC using a bilayer system

S.-W. Su et al, PRA **93**, 053630 (2016) (TW, Beijing, Vilnius)

only **two** atomic internal states

Setup: double well potential: provides an extra degree of freedom by the layer index $\rightarrow$

Four combined spin-layer states: $|spin, layer\rangle$



$|\uparrow, 1\rangle$   $|\downarrow, 1\rangle$

$|\uparrow, 2\rangle$   $|\downarrow, 2\rangle$

serve as the atomic states in the N=4 ring coupling scheme

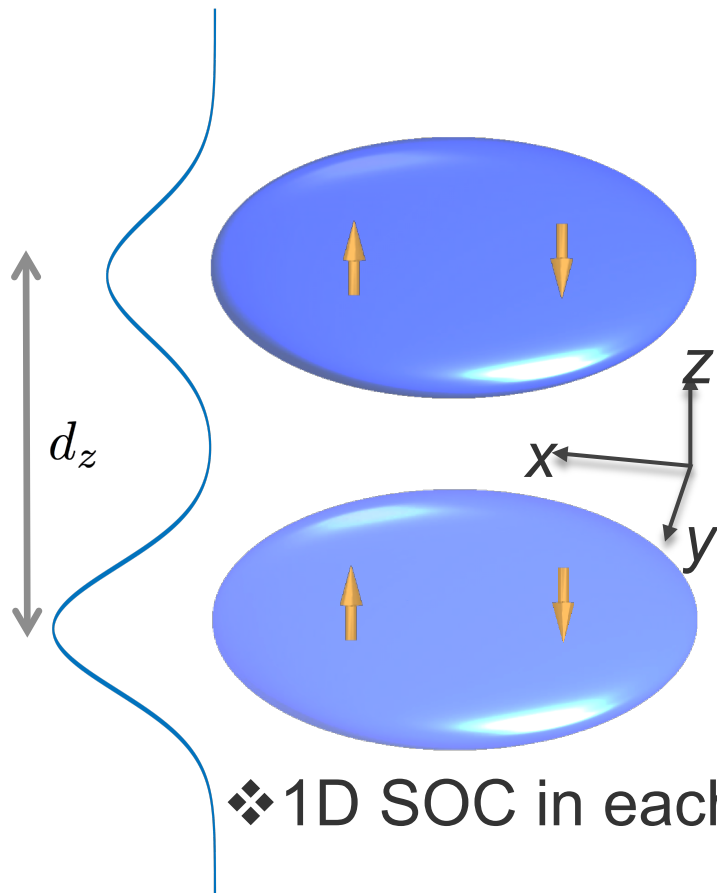
Rashba-type SOC using a bilayer system

S.-W. Su et al, PRA **93**, 053630 (2016) (TW, Beijing, Vilnius)

only **two** atomic internal states

Setup: double well potential: provides an extra degree of freedom by the layer index $\rightarrow$

Four combined spin-layer states: $|spin, layer\rangle$



$\rightarrow$ $|\uparrow, 1\rangle \xrightarrow{\Omega e^{ik(x-y)+i\phi}} |\downarrow, 1\rangle$

$|\uparrow, 2\rangle \xrightarrow{\Omega e^{ik(x-y)-i\phi}} |\downarrow, 2\rangle$

serve as the atomic states in the N=4 ring coupling scheme

❖ 1D SOC in each layer (recoil in the **same direction x-y**)

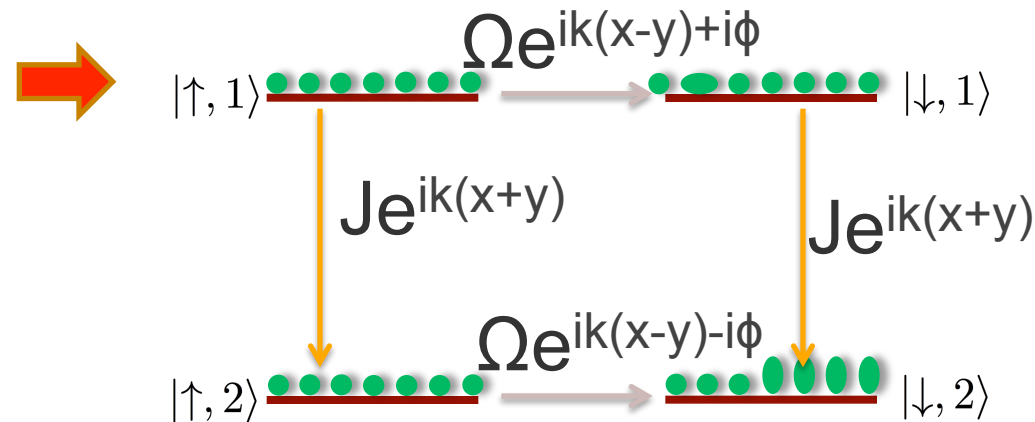
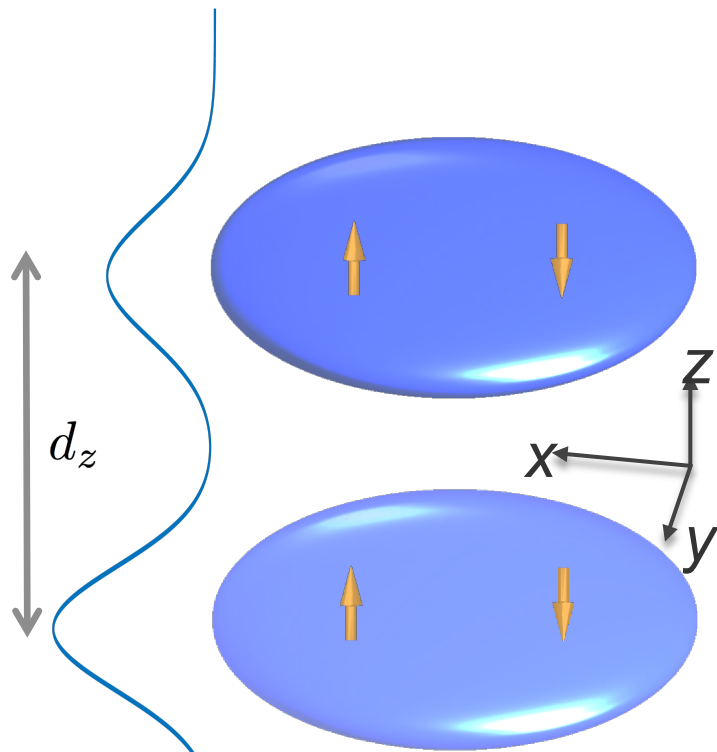
Rashba-type SOC using a bilayer system

S.-W. Su et al, PRA **93**, 053630 (2016) (TW, Beijing, Vilnius)

only **two** atomic internal states

Setup: double well potential: provides an extra degree of freedom by the layer index $\rightarrow$

Four combined spin-layer states: $|spin, layer\rangle$



serve as atomic states in the N=4 ring coupling scheme

- ❖ 1D SOC in each layer (recoil in the **same direction $x-y$**)
- ❖ Interlayer laser-assisted interlayer tunnel. (**recoil in $x+y$**)

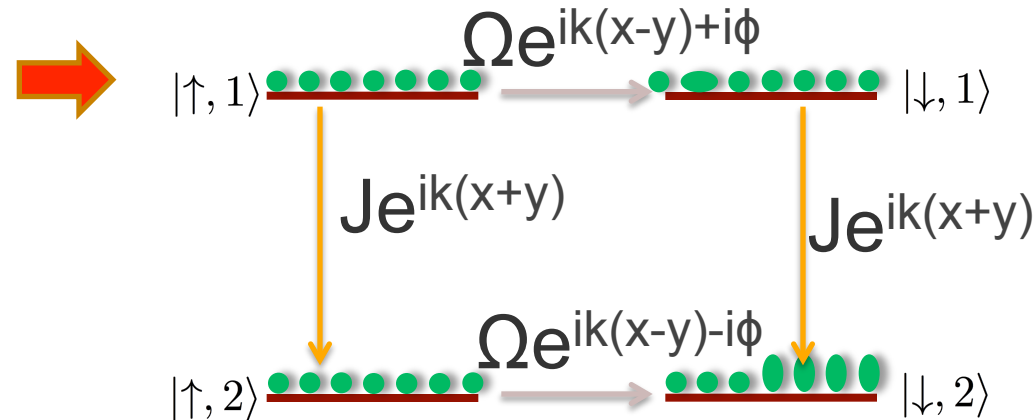
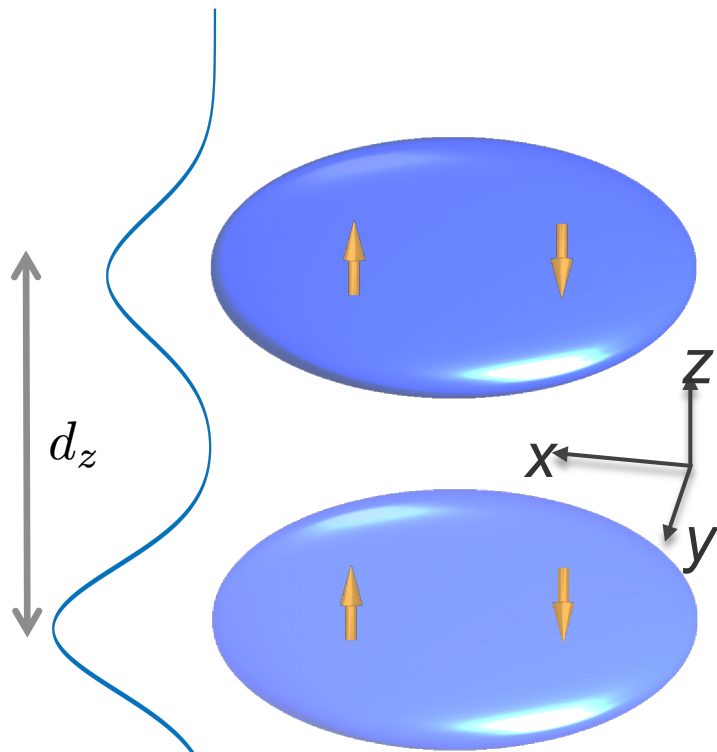
Rashba-type SOC using a bilayer system

S.-W. Su et al, PRA **93**, 053630 (2016) (TW, Beijing, Vilnius)

only **two** atomic internal states

Setup: double well potential: provides an extra degree of freedom by the layer index $\rightarrow$

Four combined spin-layer states: $|spin, layer\rangle$



serve as atomic states in the **N=4 ring coupling scheme**

- ❖ 1D SOC in each layer (recoil in the **same direction $x-y$**)
- ❖ Interlayer laser-assisted interlayer tunnel. (**recoil in $x+y$**)

2ϕ phase shift for the Ω term in different layers: $2\phi=\pi$

Other relatively simple schemes to
create 2D or 3D SOC?

Magnetically-induced SOC

B.M. Anderson, I.B. Spielman and G. Juzeliūnas,
Phys. Rev. Lett. **111**, 125301 (2013).

Z.F. Xu, L. You and M. Ueda, Phys. Rev. A **87** 063634 (2013).

A **relatively simple** scheme to create 2D or 3D SOC
using inhomogeneous pulsed magnetic field

(**no light beams, no complex atom-light coupling**)

Magnetically-induced SOC

Atom in inhomogeneous time-dependent
magnetic field $\mathbf{B}=\mathbf{B}(\mathbf{r},t)$: $\mathbf{F}$ - Spin operator

$$H_{\mathbf{B}}(\mathbf{r}, t) = \boldsymbol{\Omega}(\mathbf{r}, t) \cdot \mathbf{F} \qquad \boldsymbol{\Omega}(\mathbf{r}, t) = g_F \mu_B \mathbf{B}$$

Alternating magnetic pulses:

$$\boldsymbol{\Omega}_x = -\beta(t) k_{\text{eff}} y \mathbf{e}_x \qquad 0 \leq t < \tau$$

$$\boldsymbol{\Omega}_y = \beta(t - \tau) k_{\text{eff}} x \mathbf{e}_y \qquad \tau \leq t < 2\tau$$

$$\omega_{\text{recoil}} \tau \ll 1$$

$\boldsymbol{\Omega}_x$ & $\boldsymbol{\Omega}_y$ - yield spin-dependent forces along y & x :

Magnetically-induced SOC

Atom in inhomogeneous time-dependent magnetic field $\mathbf{B}=\mathbf{B}(\mathbf{r},t)$: $\mathbf{F}$ - Spin operator

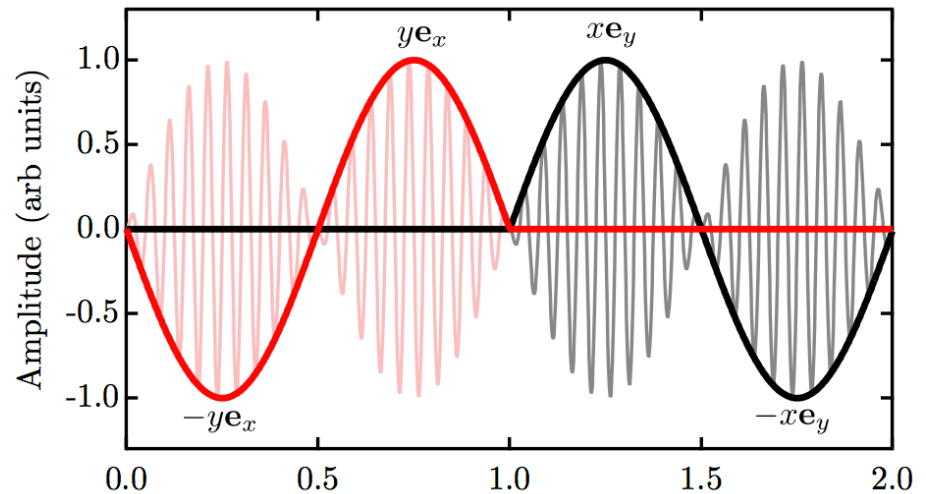
$$H_{\mathbf{B}}(\mathbf{r}, t) = \mathbf{\Omega}(\mathbf{r}, t) \cdot \mathbf{F}$$

$$\mathbf{\Omega}(\mathbf{r}, t) = g_F \mu_B \mathbf{B}$$

Alternating magnetic pulses:

$$\mathbf{\Omega}_x = -\beta(t) k_{\text{eff}} y \mathbf{e}_x$$

$$\mathbf{\Omega}_y = \beta(t - \tau) k_{\text{eff}} x \mathbf{e}_y$$



$\mathbf{\Omega}_x$ & $\mathbf{\Omega}_y$ - yield spin-dependent forces along y & x :

Magnetically-induced SOC

Atom in inhomogeneous time-dependent magnetic field $\mathbf{B}=\mathbf{B}(\mathbf{r},t)$: $\mathbf{F}$ - Spin operator

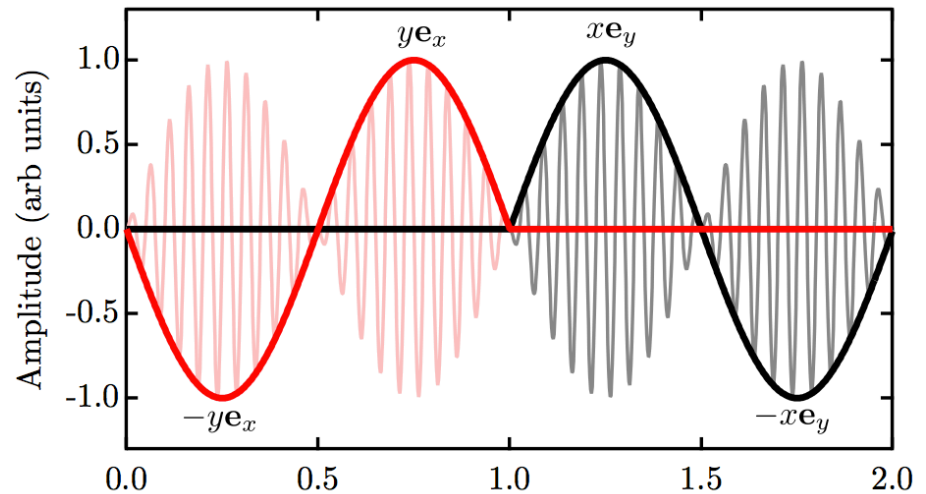
$$H_{\mathbf{B}}(\mathbf{r}, t) = \mathbf{\Omega}(\mathbf{r}, t) \cdot \mathbf{F}$$

$$\mathbf{\Omega}(\mathbf{r}, t) = g_F \mu_B \mathbf{B}$$

Alternating magnetic pulses:

$$\mathbf{\Omega}_x = -\beta(t) k_{\text{eff}} y \mathbf{e}_x$$

$$\mathbf{\Omega}_y = \beta(t - \tau) k_{\text{eff}} x \mathbf{e}_y$$



$\mathbf{\Omega}_x$ & $\mathbf{\Omega}_y$ - yield spin-dependent forces along y & x :

1D SOC in y & x directions after each pulse

Magnetically-induced SOC

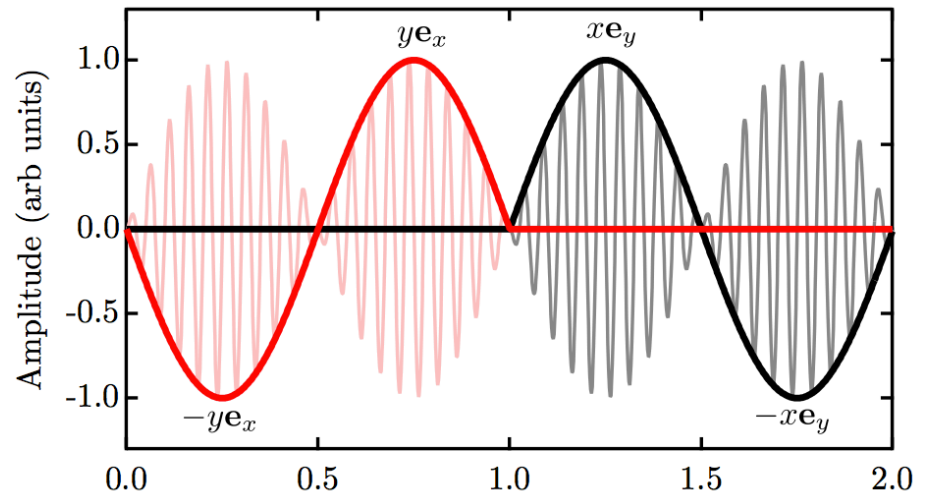
Atom in inhomogeneous time-dependent magnetic field $\mathbf{B}=\mathbf{B}(\mathbf{r},t)$: $\mathbf{F}$ - Spin operator

$$H_{\mathbf{B}}(\mathbf{r}, t) = \mathbf{\Omega}(\mathbf{r}, t) \cdot \mathbf{F} \quad \mathbf{\Omega}(\mathbf{r}, t) = g_F \mu_B \mathbf{B}$$

Alternating magnetic pulses:

$$\mathbf{\Omega}_x = -\beta(t) k_{\text{eff}} y \mathbf{e}_x$$

Pulse 1:



$\mathbf{\Omega}_x$ - yields spin-dependent force along y :

1D SOC in y directions during first pulse

Magnetically-induced SOC

Atom in inhomogeneous time-dependent magnetic field $\mathbf{B}=\mathbf{B}(\mathbf{r},t)$: $\mathbf{F}$ - Spin operator

$$H_{\mathbf{B}}(\mathbf{r}, t) = \mathbf{\Omega}(\mathbf{r}, t) \cdot \mathbf{F} \quad \mathbf{\Omega}(\mathbf{r}, t) = g_F \mu_B \mathbf{B}$$

Alternating magnetic pulses:

$$\mathbf{\Omega}_x = -\beta(t) k_{\text{eff}} y \mathbf{e}_x$$

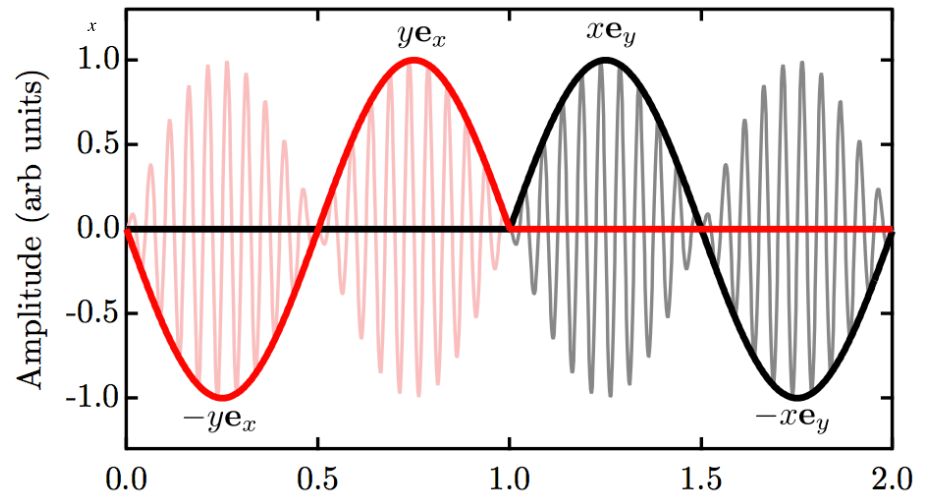
Pulse 1:

$$H_1 = (\mathbf{p} - \alpha F_x \mathbf{e}_y)^2 \quad \dot{\alpha} = \beta k_{\text{eff}}$$

$$H_1 = p^2 - 2\alpha p_y F_x + \alpha^2 F_x^2$$

$\mathbf{\Omega}_x$ - yields spin-dependent force along y :

1D SOC in y direction during first pulse



Magnetically-induced SOC

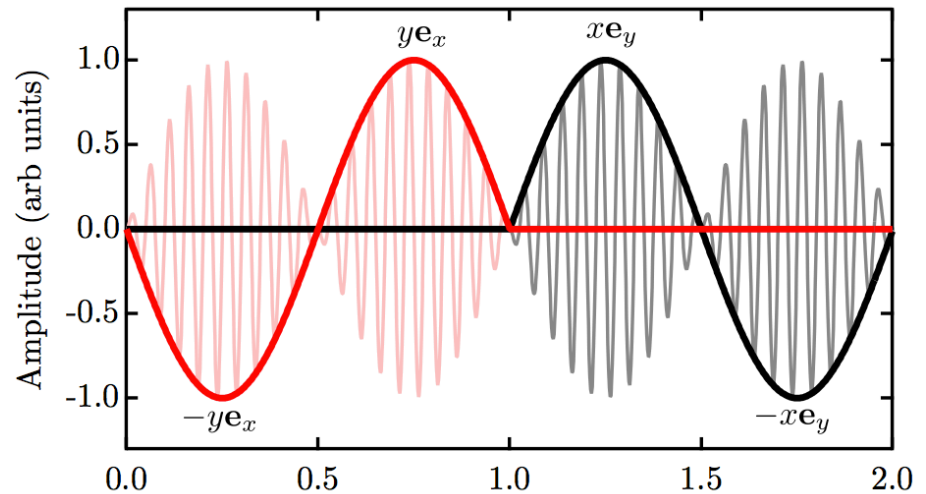
Atom in inhomogeneous time-dependent magnetic field $\mathbf{B}=\mathbf{B}(\mathbf{r},t)$: $\mathbf{F}$ - Spin operator

$$H_{\mathbf{B}}(\mathbf{r}, t) = \mathbf{\Omega}(\mathbf{r}, t) \cdot \mathbf{F} \quad \mathbf{\Omega}(\mathbf{r}, t) = g_F \mu_B \mathbf{B}$$

Alternating magnetic pulses:

$$\mathbf{\Omega}_y = \beta(t - \tau) k_{\text{eff}} x \mathbf{e}_y$$

Pulse 2:



$\mathbf{\Omega}_y$ - yields spin-dependent force along x :

1D SOC in x direction during second pulse

Magnetically-induced SOC

Atom in inhomogeneous time-dependent magnetic field $\mathbf{B}=\mathbf{B}(\mathbf{r},t)$: $\mathbf{F}$ - Spin operator

$$H_{\mathbf{B}}(\mathbf{r}, t) = \mathbf{\Omega}(\mathbf{r}, t) \cdot \mathbf{F} \quad \mathbf{\Omega}(\mathbf{r}, t) = g_F \mu_B \mathbf{B}$$

Alternating magnetic pulses:

$$\mathbf{\Omega}_y = \beta (t - \tau) k_{\text{eff}} x \mathbf{e}_y$$

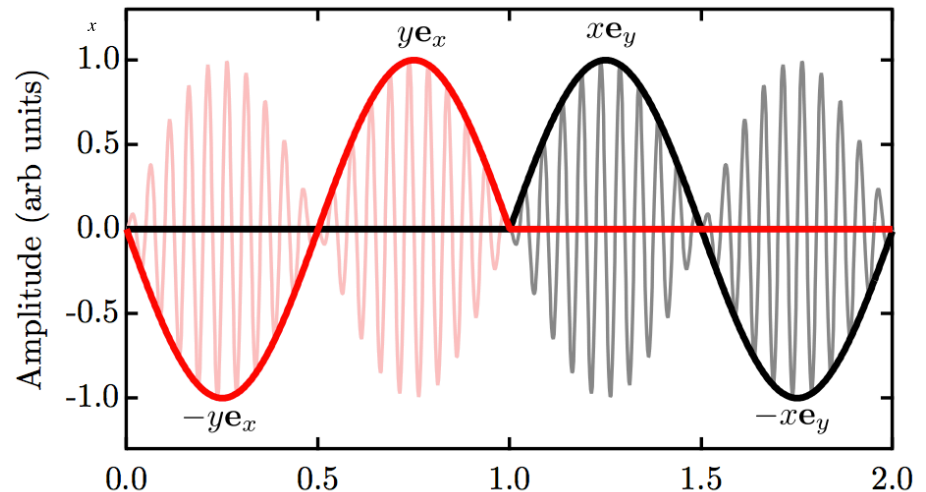
Pulse 2:

$$H_2 = \left(\mathbf{p} + \alpha F_y \mathbf{e}_x \right)^2 \quad \dot{\alpha} = \beta k_{\text{eff}}$$

$$H_2 = p^2 + 2\alpha p_x F_y + \alpha^2 F_y^2$$

$\mathbf{\Omega}_y$ - yields spin-dependent force along x:

1D SOC in x direction during second pulse



Magnetically-induced SOC

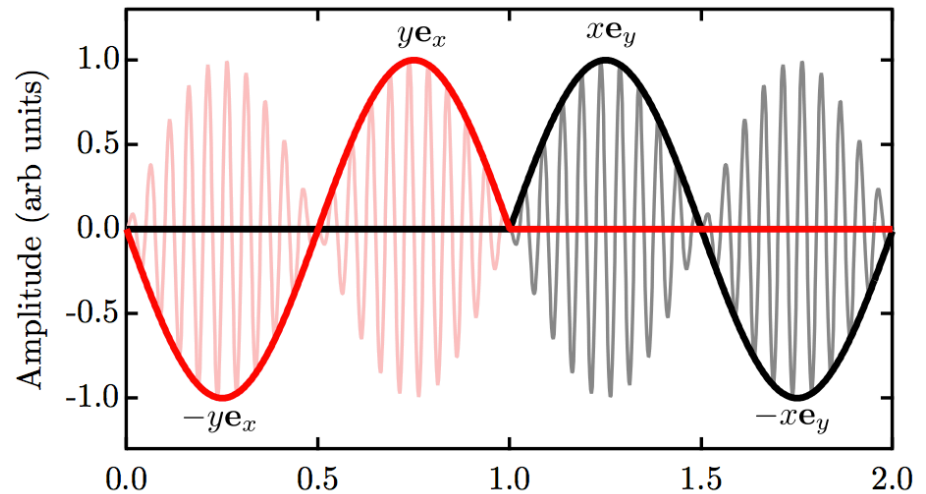
Atom in inhomogeneous time-dependent magnetic field $\mathbf{B}=\mathbf{B}(\mathbf{r},t)$: $\mathbf{F}$ - Spin operator

$$H_{\mathbf{B}}(\mathbf{r}, t) = \mathbf{\Omega}(\mathbf{r}, t) \cdot \mathbf{F} \quad \mathbf{\Omega}(\mathbf{r}, t) = g_F \mu_B \mathbf{B}$$

Alternating magnetic pulses:

$$\mathbf{\Omega}_x = -\beta(t) k_{\text{eff}} y \mathbf{e}_x$$

$$\mathbf{\Omega}_y = \beta(t - \tau) k_{\text{eff}} x \mathbf{e}_y$$



1D SOC in y & x directions after each pulse

2D Rashba SOC after time averaging (**small τ** : $\omega_{\text{recoil}} \tau \ll 1$)

$$H_1 = p^2 - 2\alpha p_y F_x + \alpha^2 F_x^2$$

$$H_2 = p^2 + 2\alpha p_x F_y + \alpha^2 F_y^2$$

Time-averaged Hamiltonian:

H_{2D} - **2D Rashba-type Hamiltonian (small τ):**

$$H_{2D} = \frac{p^2}{2m} - k_{\text{eff}} \frac{p_y F_x - p_x F_y}{2m} + c_2 k_{\text{eff}}^2 \frac{F_x^2 + F_y^2}{4m}$$

Zero-order Floquet Hamiltonian

(Terminology of a talk by Nathan Goldman yesterday)

→ Formation of 2D SOC

$$H_1 = p^2 - 2\alpha p_y F_x + \alpha^2 F_x^2$$

$$H_2 = p^2 + 2\alpha p_x F_y + \alpha^2 F_y^2$$

Time-averaged Hamiltonian:

H_{2D} - **2D Rashba-type Hamiltonian (small τ):**

$$H_{2D} = \frac{p^2}{2m} - k_{\text{eff}} \frac{p_y F_x - p_x F_y}{2m} + c_2 k_{\text{eff}}^2 \frac{F_x^2 + F_y^2}{4m}$$

2D spin-orbit coupling

**Additional quadratic Zeeman term
(for spins with $F > 1/2$)**

$$H_1 = p^2 - 2\alpha p_y F_x + \alpha^2 F_x^2$$

$$H_2 = p^2 + 2\alpha p_x F_y + \alpha^2 F_y^2$$

Time-averaged Hamiltonian:

H_{2D} - **2D Rashba-type Hamiltonian (small τ):**

$$H_{2D} = \frac{p^2}{2m} - k_{\text{eff}} \frac{p_y F_x - p_x F_y}{2m} + c_2 k_{\text{eff}}^2 \frac{F_x^2 + F_y^2}{4m}$$

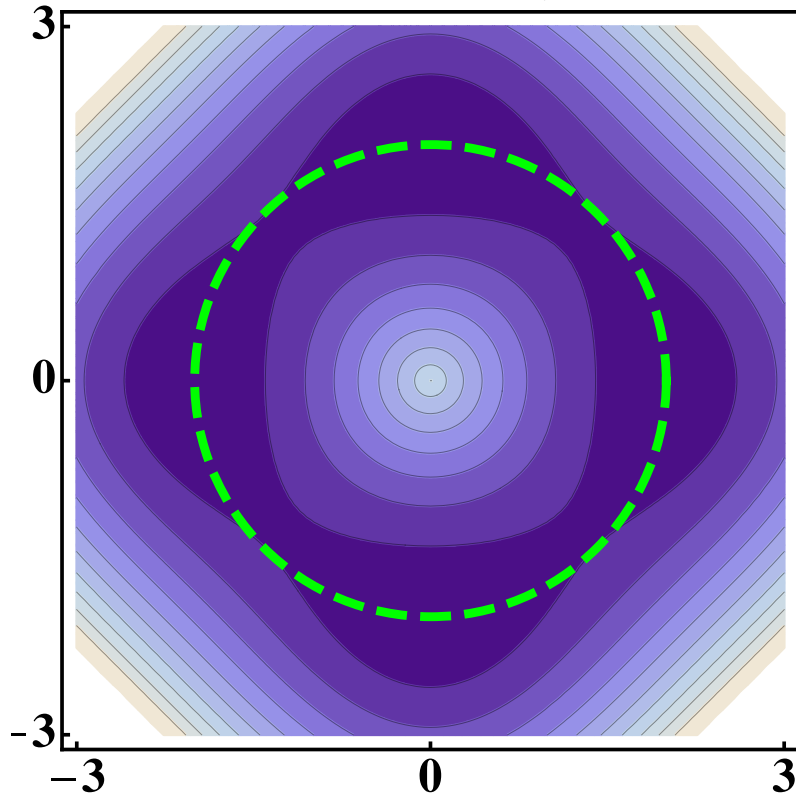
Exact Floquet treatment for spin 1/2

(for finite τ):

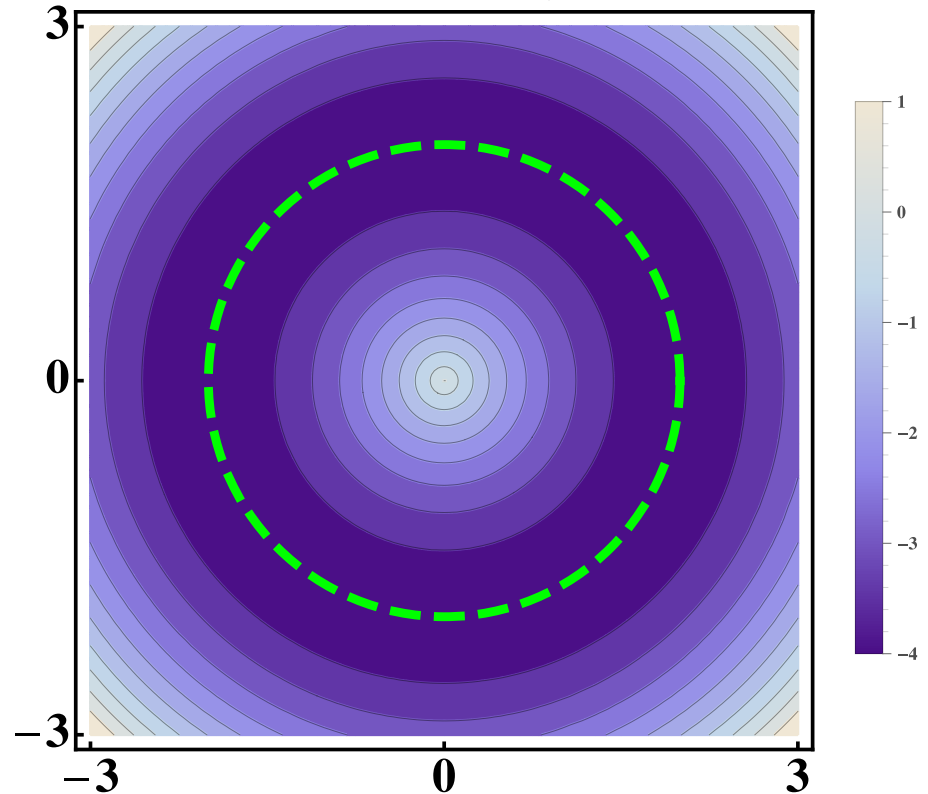


Exact Floquet treatment (for finite τ):

(a) Spectrum for $\frac{\hbar k_{\text{eff}}^2}{2m} \tau = 0.25$

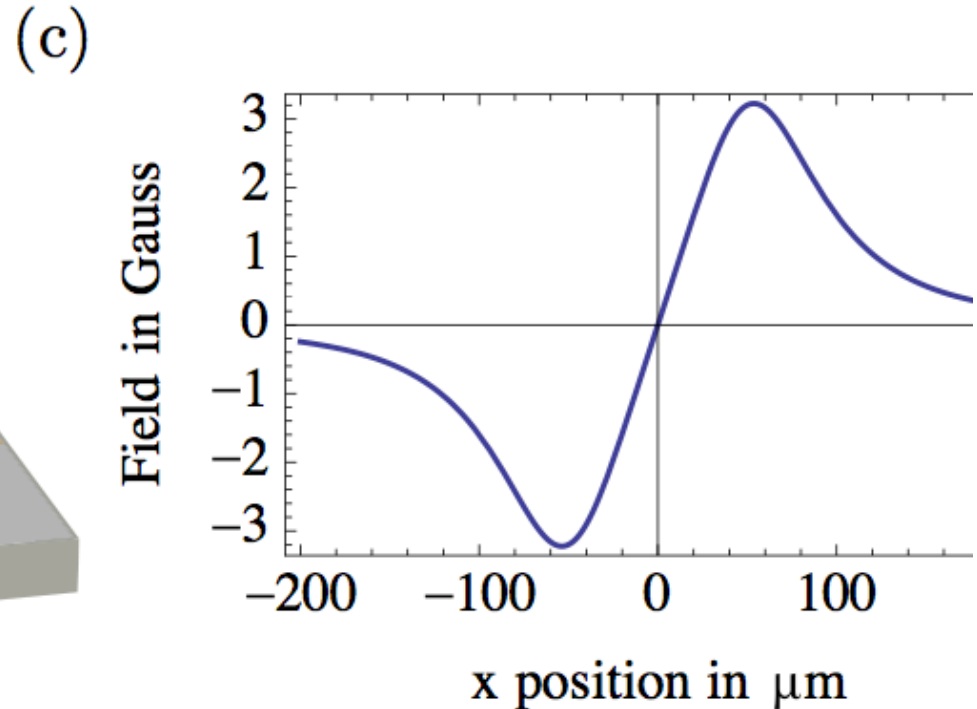
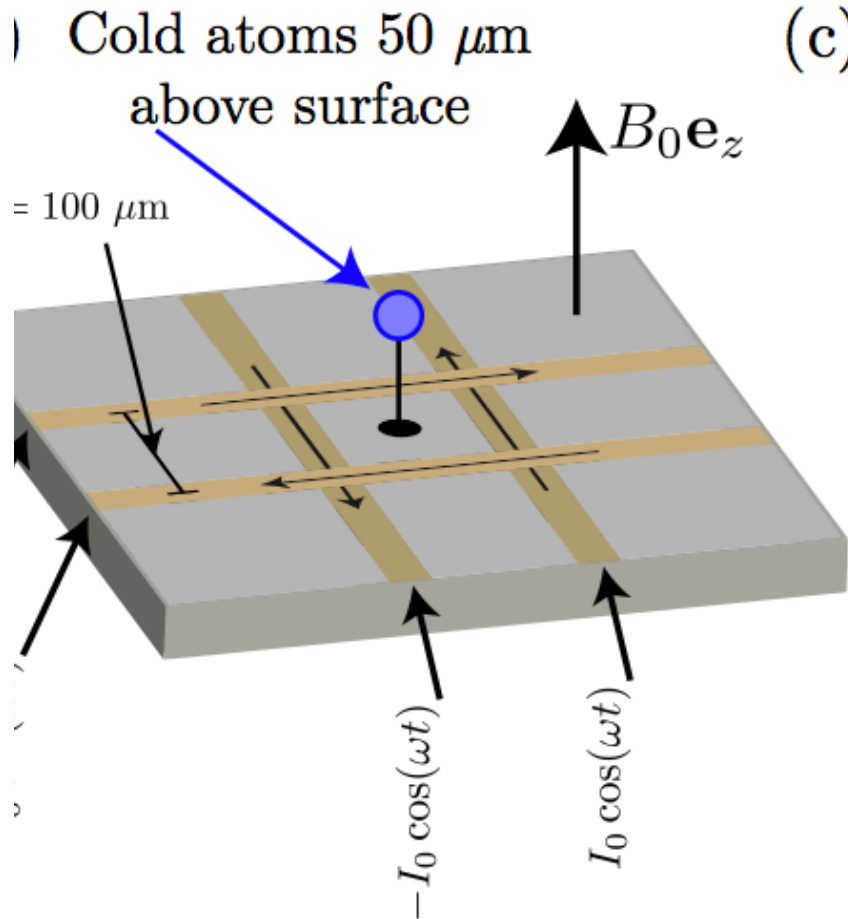


(b) Spectrum for $\frac{\hbar k_{\text{eff}}^2}{2m} \tau = 0.025$

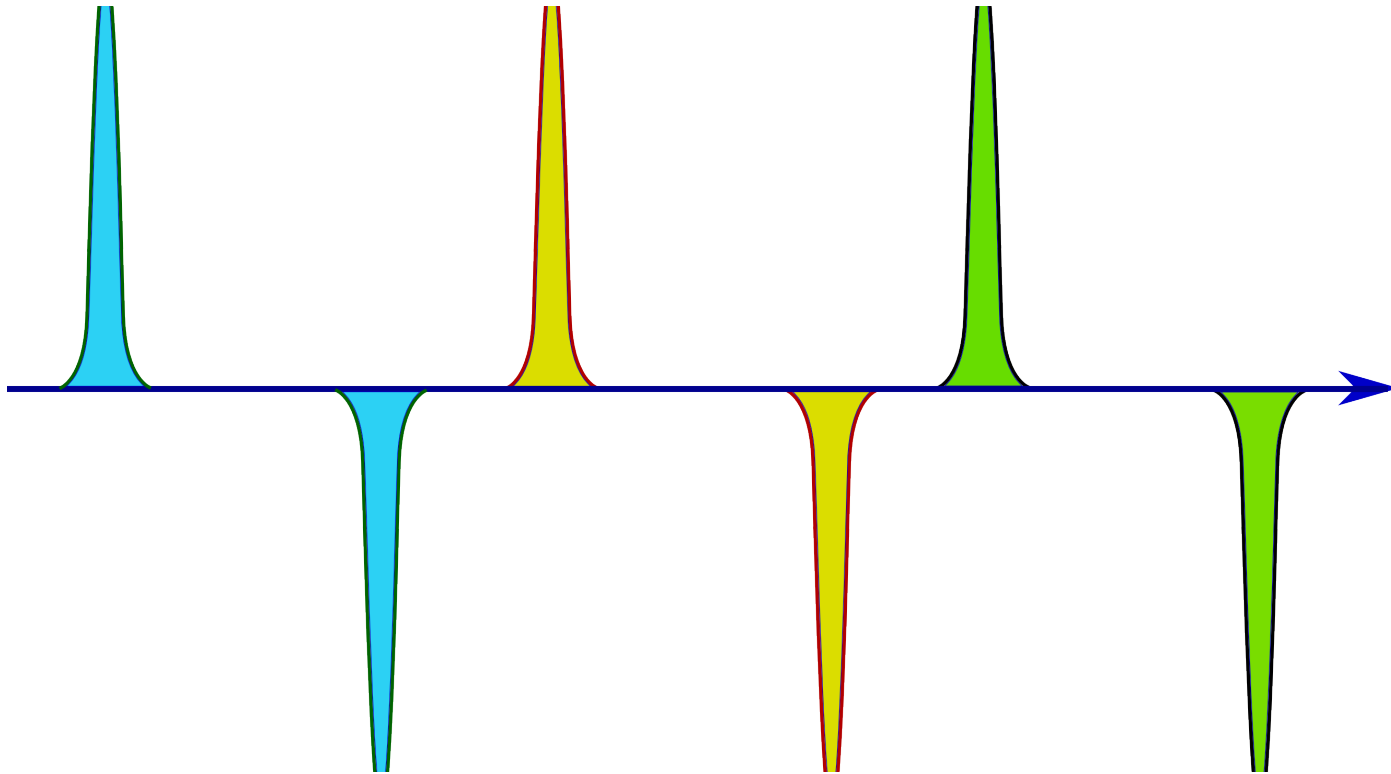


Converges to the Rashba ring for small τ : $\omega_{\text{rec}} \tau \ll 1$

Proposed experimental setup:



3D SOC: (by taking the third magnetic pulse)



$$H_{3D} = \frac{\mathbf{p}^2}{2m} - v \frac{2}{\hbar} (p_x F_y + p_y F_z + p_z F_x)$$

Minimum of atomic energy - the **sphere (Rashba sphere)**
(in the momentum space)

Conclusions

- Light induced gauge potentials can simulate **Rashba-Dresselhaus** SOC for ultracold atoms (2D SOC or even 3D SOC!).
 - Yet these are complex schemes involving many atomic states
- Up to now experimentally a simpler **1D SOC** (with 2 atomic states) & the first two experiments on 2D SOC (coupling of 3 atomic states using a far detuned tripod scheme)
- **An alternative scheme:** Rashba type SOC using a **bilayer** system:
- **Another alternative scheme:** Magnetically induced 2D&3D Rashba SOC

Thank you!