Anderson localization of a Rydberg electron along a classical orbit

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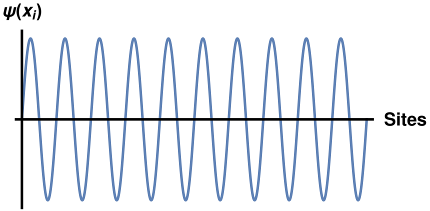
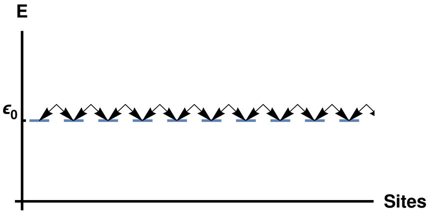
Zakopane, 16 VI 2017

"Absence of Diffusion in Certain Random Lattices" - P. Anderson, 1958

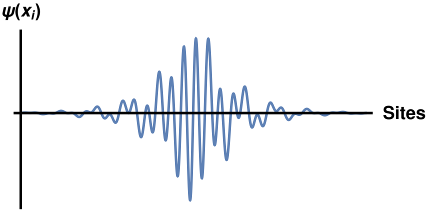
1D example: $H = t \sum_{\langle i,j \rangle} c_j^\dagger c_i + \sum_i \epsilon_i c_i^\dagger c_i \ , \ \epsilon_i \in [\epsilon_0 - \Delta, \epsilon_0 + \Delta]$(random)

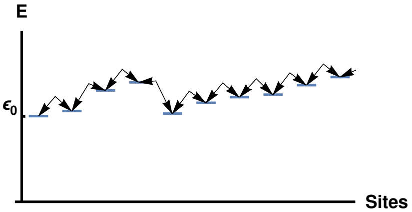
Extended Bloch waves





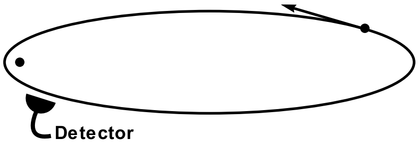
Localized

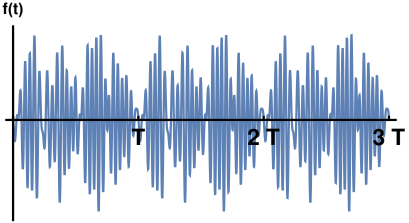




Can we expect Anderson Localization signatures in time?

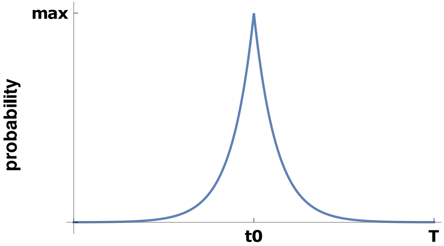
Rydberg atom in a fluctuating microwave field - 1D AL





$$H_0 = \frac{p^2}{2} - \frac{1}{z} \quad , \quad z \geq 0$$

$$H_1 = Fzf(t) \quad , \quad f(t+T) = f(t)$$



Integrable 1D system: $(z, p) \to (\theta, I)$

$$H_0(z,p) \longrightarrow H_0(I) \implies I = const, \quad \theta = \frac{\partial H_0(I)}{\partial I} t + \theta_0.$$

Add time periodic perturbation:

$$H_1 = f(t)\, z \longrightarrow H_1 = \sum_n \sum_k g_n(I) f_k \left(e^{i(n\theta + k\omega t)} + e^{i(n\theta - k\omega t)} \right).$$

Around the resonance condition:

$$\omega = \frac{\partial H_0(I_r)}{\partial I_r},$$

We can look at system in rotating frame:

$$\Theta = \theta - \omega t$$

where for small perturbation new angle Θ is varying slowly:

$$\dot{\Theta} = \frac{\partial H(I)}{\partial I} - \omega \approx 0$$

A. Buchleitner, D. Delande, J. Zakrzewski *Physics Reports 368 (2002)*

Small perturbation:

$$P = I - I_r \approx 0$$

We can restric H_0 to lowest order:

$$H_0 \approx \frac{P^2}{2} \frac{\partial^2 H_0(I_r)}{\partial I_r{}^2} = \frac{P^2}{2m_{eff}}$$

Average out fast variables:

$$H_1 = \sum_n \sum_k g_k(I) f_k \left(e^{i(n(\Theta+\omega t)+k\omega t)} + e^{i(n(\Theta+\omega t)-k\omega t)} \right) \approx \sum_k g_k(I) f_{-k} e^{ik\Theta}.$$

In total we get:

$$H_{eff} = \langle H \rangle_t = \frac{P^2}{2m_{eff}} \; + \; F \sum_k g_k f_{-k} e^{ik\Theta}. \quad (1)$$

$$\text{Kinetic} \; + \; \text{Potential V}(\Theta) \quad (2)$$

$$H_{eff} = \frac{P^2}{2m_{eff}} + F\sum_k g_k f_{-k} e^{ik\Theta} \tag{3}$$

m_{eff} is given by H_0:

$$m_{eff}^{-1} = \frac{\partial^2 H_0(I_r)}{\partial {I_r}^2},$$

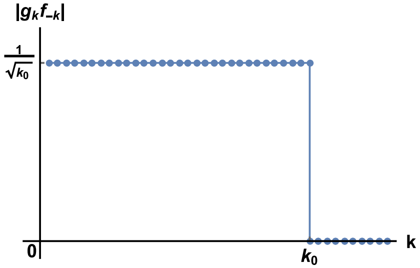
g_n's are given by interaction structure in H_1:

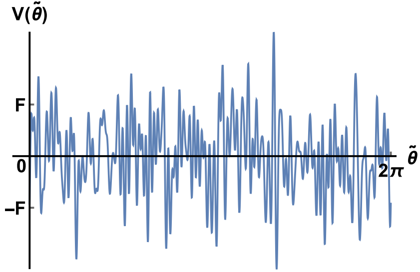
$$z = \sum_k g_k e^{ik\theta}$$

But experimentator gets full control over f_k's:

$$f(t) = \sum_k f_k e^{ik\omega t}$$

$$H_{eff} = \frac{P^2}{2m_{eff}} + F\sum_k g_k f_{-k} e^{ik\Theta} = \frac{P^2}{2m_{eff}} + V(\Theta) \quad (4)$$





We ask experimentator for:

- resonant state (ω),
- perturbative regime (F),
- fluctuating driving (f_k's).

Quantum description

First option: Quantization of the classical H_{eff}

$$P \rightarrow \hat{P} \quad (5)$$

$$\Theta \rightarrow \hat{\Theta} \quad (6)$$

$$\hat{H}_{eff} = \frac{\hat{P}^2}{2m_{eff}} + F \sum_k g_k f_{-k} e^{ik\hat{\Theta}}$$

Anderson localized states if localization length smaller than 2π.

Second option: Effective Hamiltonian perturbative expansion directly in quantum description:

$$U = e^{-i\hat{n}\omega t} \tag{7}$$

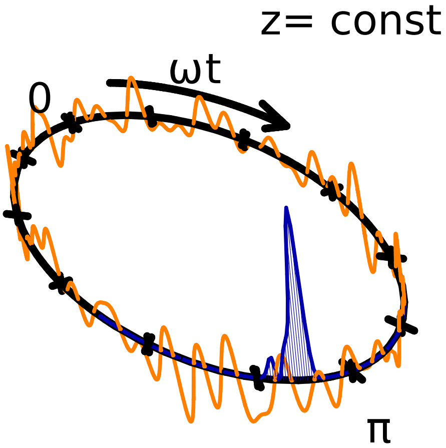
$$\langle n'|H_{eff}|n\rangle = -\frac{1}{2\hat{n}^2} - \omega\hat{n} + F\langle n'|\hat{z}|n\rangle f_{n'-n} \tag{8}$$

Close to resonance we can identify semiclassical Hamiltonian:

$$\hat{P} \approx (\hat{n} - \omega^{-\frac{1}{3}}) \qquad \langle n'|\hat{z}|n\rangle \approx \langle|\omega^{-\frac{1}{3}} + (n'-n)|\hat{z}|\omega^{-\frac{1}{3}}\rangle = g_{n'-n} \tag{9}$$

$$\hat{H}_{eff} = \frac{\hat{P}^2}{2m_{eff}} + F\sum_k g_k f_{-k} e^{ik\hat{\Theta}}$$

Laboratory frame: $(E, z, t) \to (n, \Theta) \approx (I, \theta - \omega t)$.



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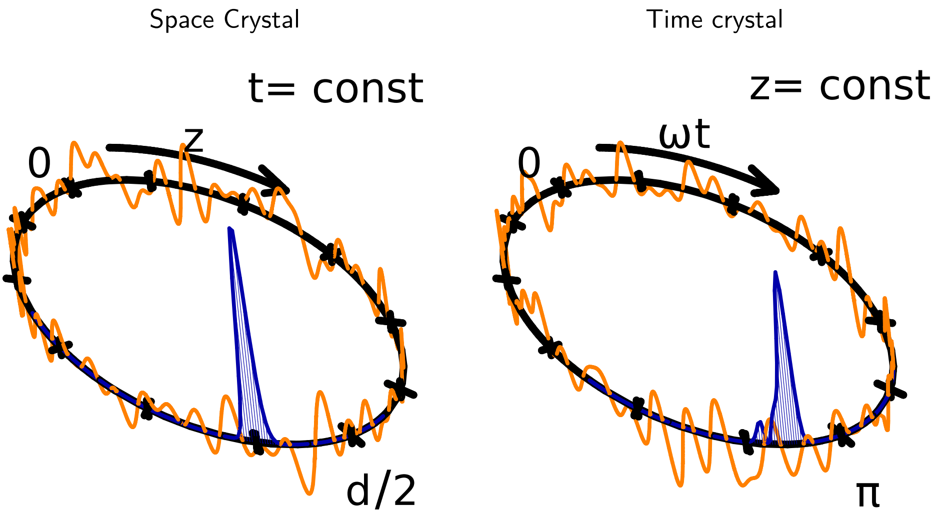
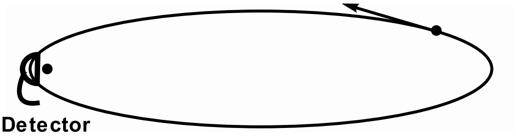
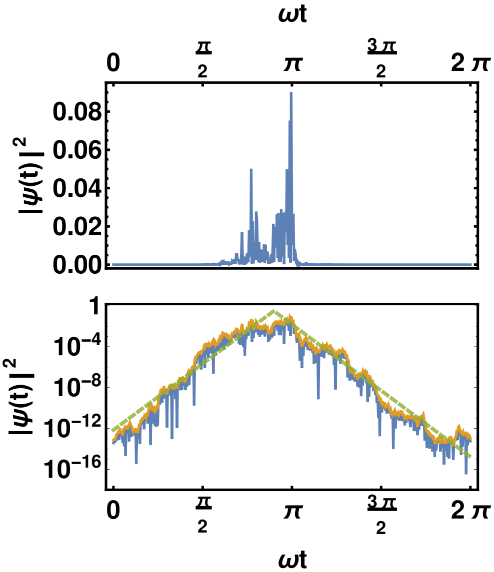


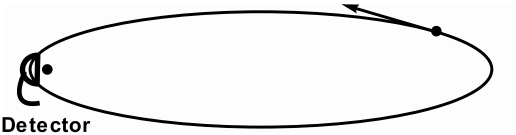
$n_0 = 10^6$





Detector has fixed position.
Point closest to nucleus (for all figures).
Setup could be the same as:

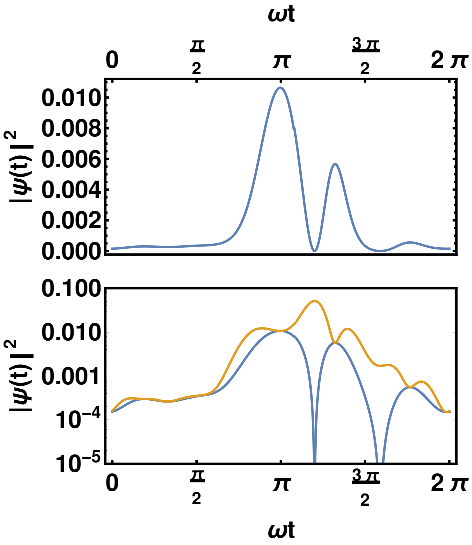
Nondispersing Wave Packets
H. Maeda and T. F. Gallagher
Phys. Rev. Lett. 92(2004)
$n_0 \approx 70$



Detector has fixed position.
Point closest to nucleus (for all figures).

Setup could be the same as:
Creating and Transporting Trojan Wave Packets
B. Wyker, S. Ye, F.B. Dunning, S. Yoshida, C.O. Reinhold, J Burgdorfer
Phys. Rev. Lett. 108(2012)
$n_0 \approx 300$

$$n_0 = 300$$



3D

$$H = \frac{p^2}{2} - \frac{1}{|\vec{r}|} + Fzf(t)$$

Kepler ellipse:

- J - orbit radius,
- θ - position of electron on the orbit,
- $L \approx$ shape of the elliptical orbit,
- ψ -angle between semi-major axis and field polarization.

1D model is the limit with $L \to 0$ and $\psi = 0$.

How to force system into stable point for those parameters?:

$$H = \frac{p^2}{2} - \frac{1}{|\vec{r}|} + Fz\cos(\omega t) + F_s\ z$$

K. Sacha, J. Zakrzewski, and D. Delande,
Europ. Phys. J. D 1, 231 (2002)

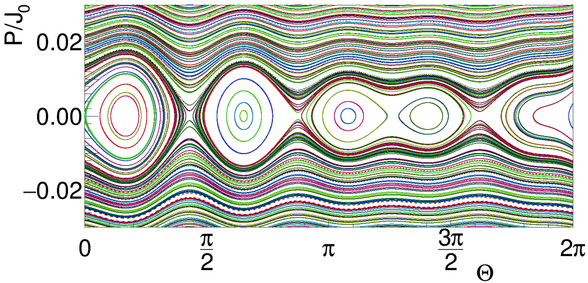
$$H = \frac{p^2}{2} - \frac{1}{|\vec{r}\,|} + Fzf(t) + F_s\ z$$

Kepler ellipse:

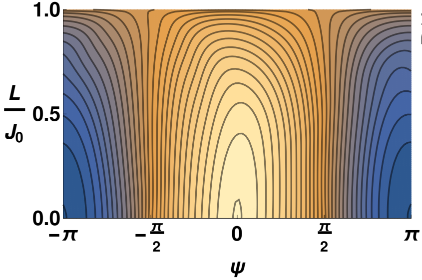
- J - orbit radius,
- θ - position of electron on the orbit,
- $L \approx$ shape of the elliptical orbit,
- ψ -angle between semi-major axis and field polarization.

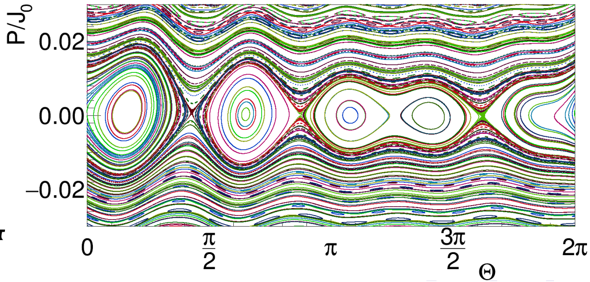
$$H_{eff} = \frac{P^2}{2m_{eff}} + F\sum_{k\neq 0} U_k(L,\Psi) f_{-k} e^{ik\Theta} + F_s V(L,\Psi)$$

New parameters L, Ψ must be well defined or the Anderson Localization will not occur.
In AL language "disorder is not quenched".


P/J0
0.02
0.00
–0.02
0
π/2
π
3π/2
2π
Θ

Let's go 3D.

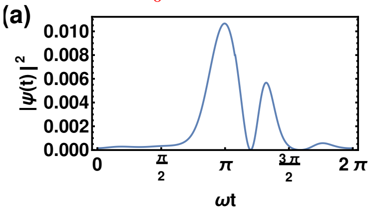

1.0
L/J0
0.5
0.0
–π
–π/2
0
π/2
π
ψ

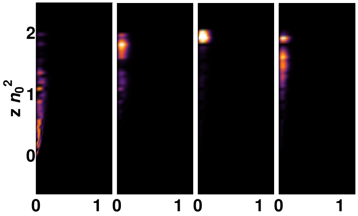

P/J0
0.02
0.00
–0.02
0
π/2
π
3π/2
2π
Θ

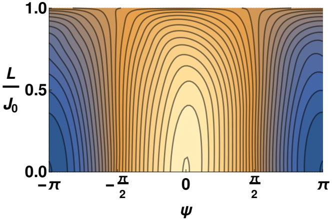
Anderson localization in the time domain 3D

$$H = \frac{\mathbf{p}^2}{2} - \frac{1}{r} - F_s z + F z f(t),$$

$n_0 = 300$



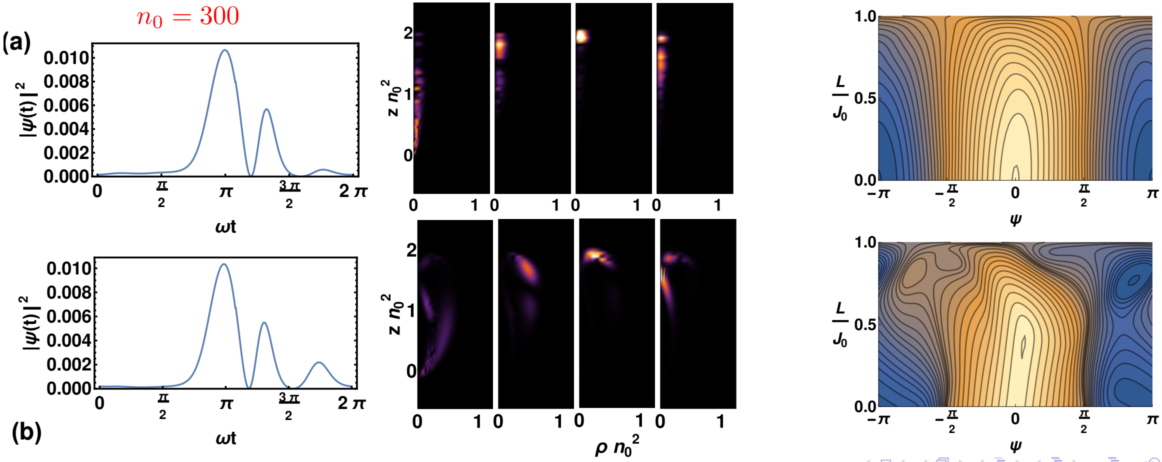




Anderson localization in the time domain 3D

$$H = \frac{\mathbf{p}^2}{2} - \frac{1}{r} - F_s z + F z f(t),$$

$n_0 = 300$

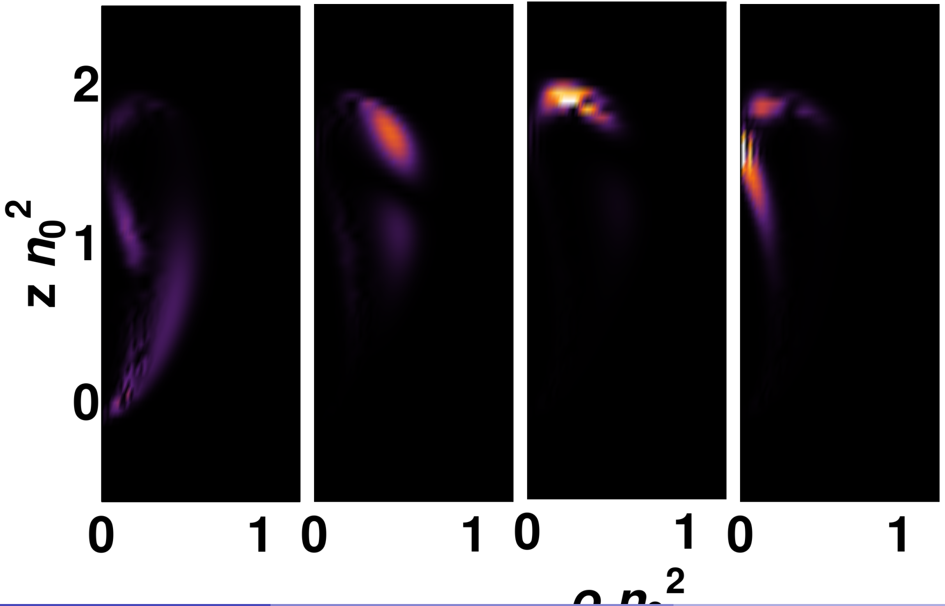


Conclusions

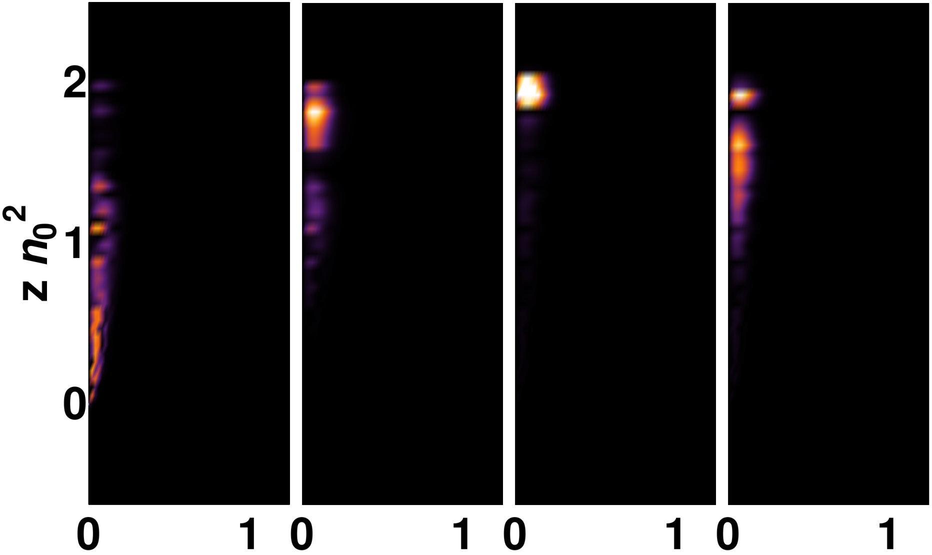
- We have considered perturbation of a system that is regular in space but disordered in time.
- It turns out that temporal disorder induces Anderson Localization in the time domain.
- Anderson localization in time may be realized in different systems (hydrogen atom, BEC, possibly solid state systems).

Sacha, K. 2015, Scientific Reports, 5, 10787
Sacha, K., & Delande, D. 2016, PRA, 94, 023633
Giergiel, K., Sacha, K., 2017, PRA, 95

Thank you!



Thank you for your question!



Thank you for your question!