Tensor networks: part I

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OUTLINE

- **tensor networks**
  - definitions
  - matrix product states
  - multi-scale entanglement renormalization ansatz
- **from 1D to 2D**
  - matrix product states in 2D
  - matrix product states vs projected entangled pair states
- **tensor networks and topological order**
  - ground state degeneracy
  - anyon models: topological S and T matrices
  - edge spectrum

OUTLINE

Piotrek Czarnik
Sunday, 6:05pm

- **tensor networks**
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OUTLINE

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Ania Francuz
Sunday, 5:40pm

- **from 1D to 2D**
  - matrix product states in 2D
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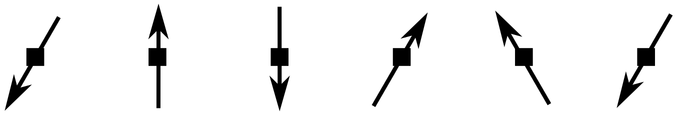
MOTIVATION

- simulating N-body quantum systems



$$\mathcal{H} = \sum_{i=1}^{N} h^{[i,i+1]}$$

- exact diagonalization
  - $\mathcal{H}$ is a $d^N \times d^N$ matrix

$$|\Psi\rangle = \sum_{i_1,\ldots,i_N=1}^{d} \Psi_{i_1\ldots i_N} |i_1 \ldots i_N\rangle$$

  - $\{\Psi_{i_1\ldots i_N}\}$: d^N coefficients
    - current best: T. Häner, D. Steiger, **arXiv: 1704.01127**

      45 spins $\frac{1}{2}$: 8192 cores with 500 terabytes of memory
    - Moore's law: 100 spins by the year 2100

MOTIVATION

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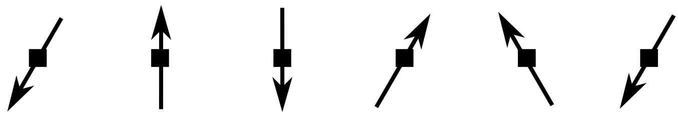

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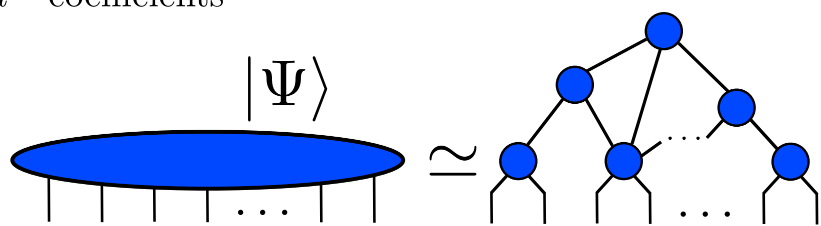
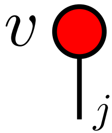
- tensor network

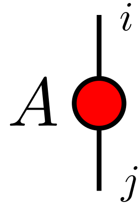


NOTATION

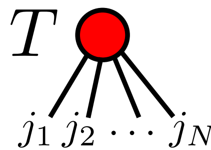
a, number



v_j, vector



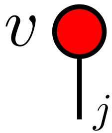
A_{ij}, matrix



$T_{j_1 j_2 \cdots j_N}$, rank-N tensor

NOTATION

a, number



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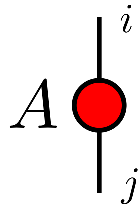

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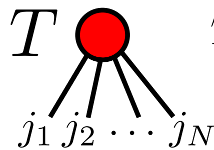

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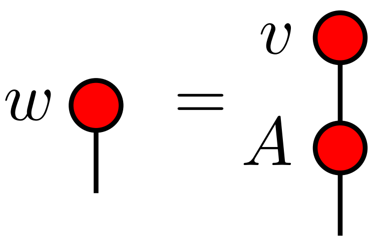
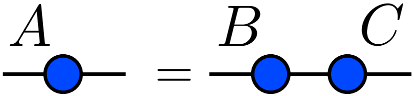
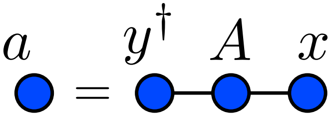


contraction, $w_j = \sum_k v_k A_{kj}$

NOTATION: EXAMPLES

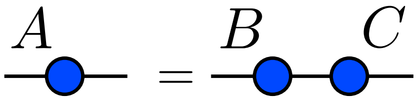


$$A = BC$$



$$a = y^\dagger A x$$

NOTATION: EXAMPLES



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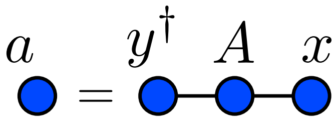

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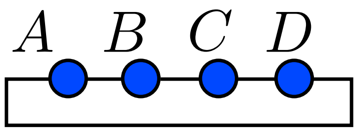

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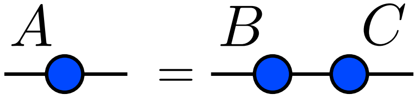

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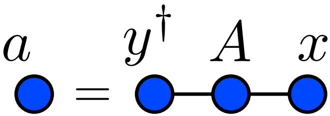

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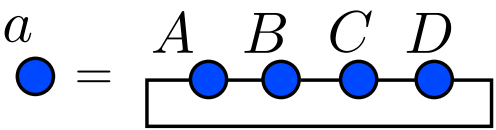
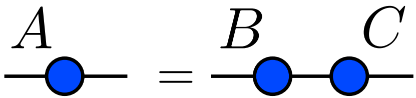


$$a = \mathrm{Tr}(ABCD)$$

NOTATION: EXAMPLES



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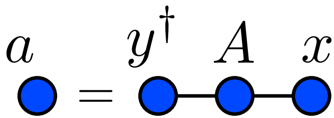
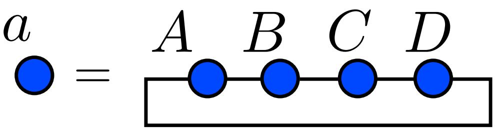
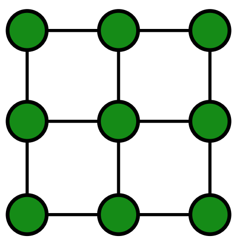


$$a = y^{\dagger} A x$$



$$a = \mathrm{Tr}(ABCD)$$

- why bother?



$$= \sum_{abcdefghijkl} A_{ac}\, B_{bda}\, C_{eb}\, D_{cfh}\, E_{dgif}\, F_{ejg}\, G_{hk}\, H_{ilk}\, I_{jl}$$

MOTIVATION

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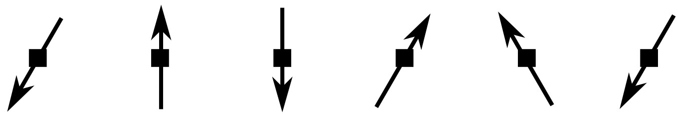

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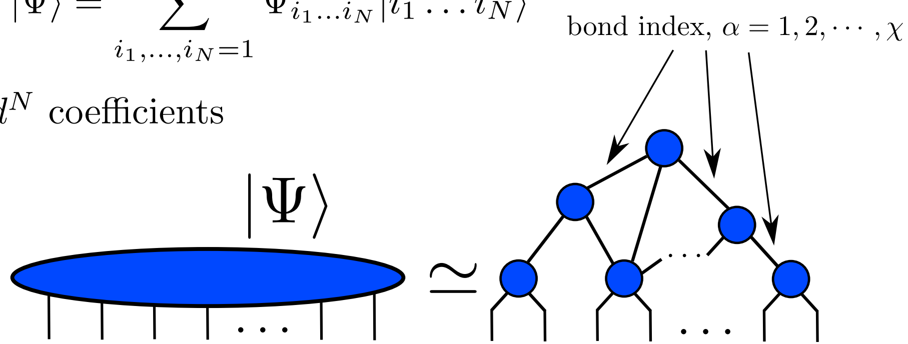
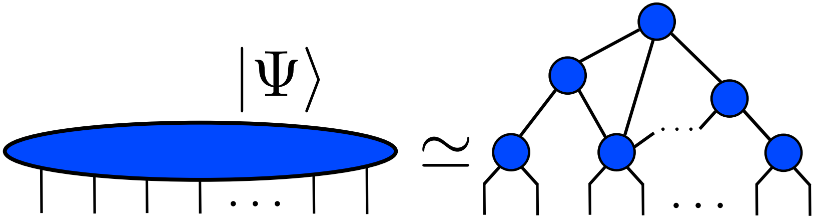
- tensor network



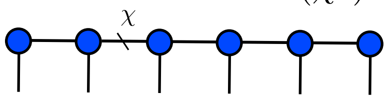
TENSOR NETWORKS



- PROS
  - efficient description of quantum many-body wave-function
  - allow for large (even infinite) systems
  - no sign problem
    - frustrated systems
    - fermions, anyons, ...
- CONS
  - not known

EXAMPLES

- MPS $\mathcal{O}(\chi^3)$



S. R. White, **PRL 69, 2863 (1992)**

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G. Vidal, **PRL 91, 147902 (2003)**

D. Porras, F. Verstraete, J. I. Cirac, **PRL 93, 227205 (2004)**

A. J. Daley, C. Kollath, U. Schollwöck, G. Vidal, **J. Stat. Mech.: Theor. Exp. P04005 (2004)**

G. Vidal, **PRL 93, 040502 (2004)**

G. Vidal, **PRL 98, 070201 (2007)**

R. Orús, G. Vidal, **PRB 78, 155117 (2008)**

... ...

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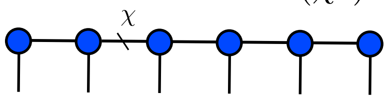
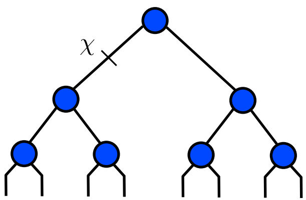
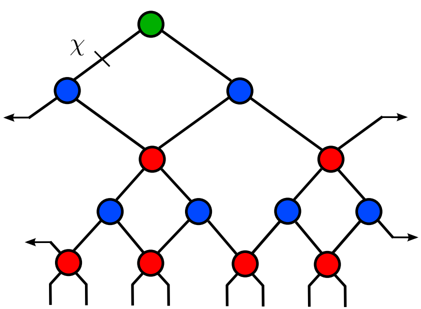


- TTN $\mathcal{O}(\chi^4)$



- 1D MERA $\mathcal{O}(\chi^9)$



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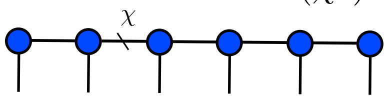

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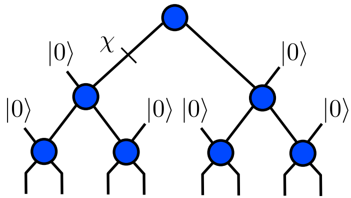

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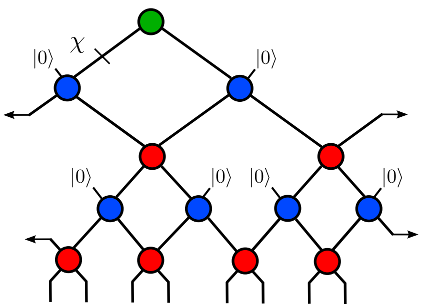

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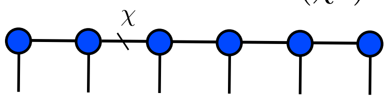

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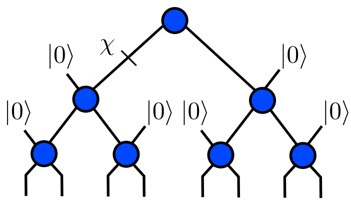

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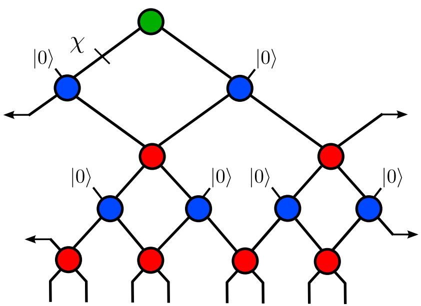
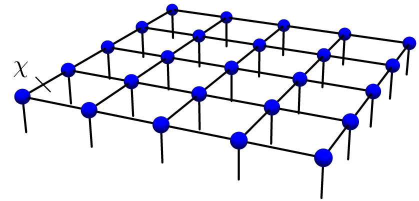


- PEPS $\mathcal{O}(\chi^{10\ldots12})$



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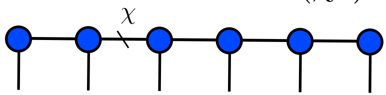

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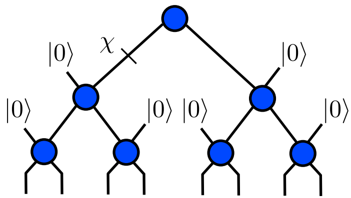

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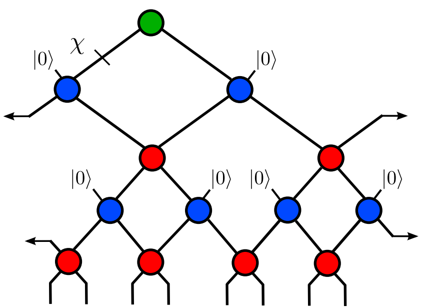

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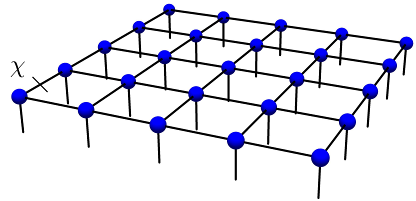
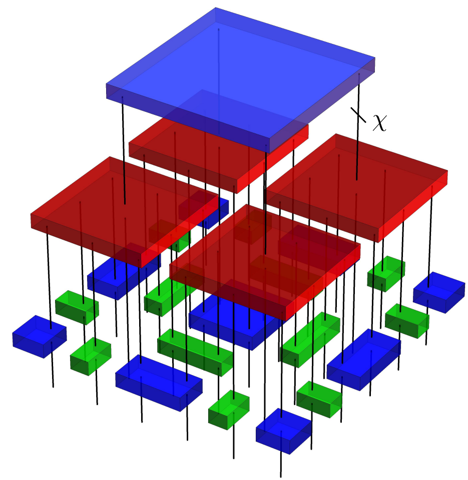


- 2D MERA $\mathcal{O}(\chi^{16})$



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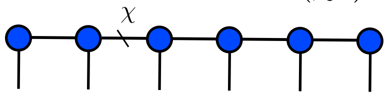

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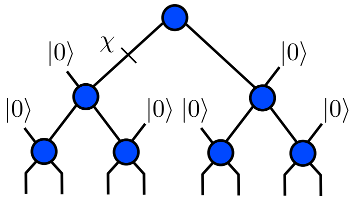

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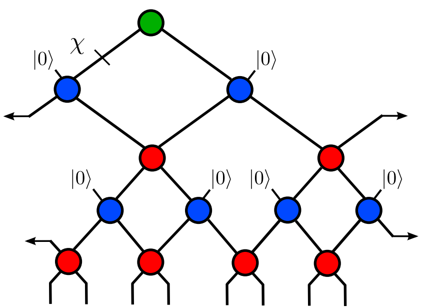

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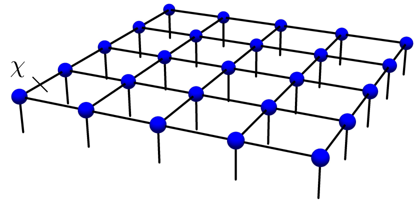

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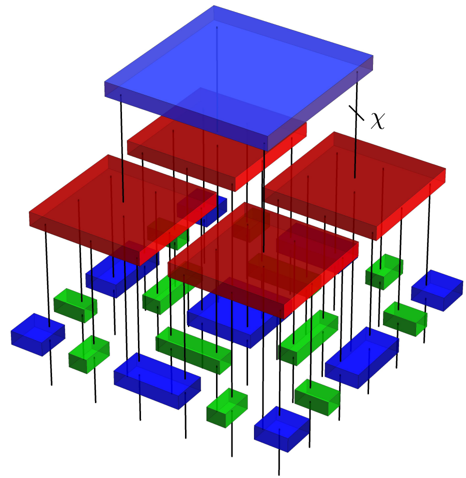

TENSOR NETWORKS

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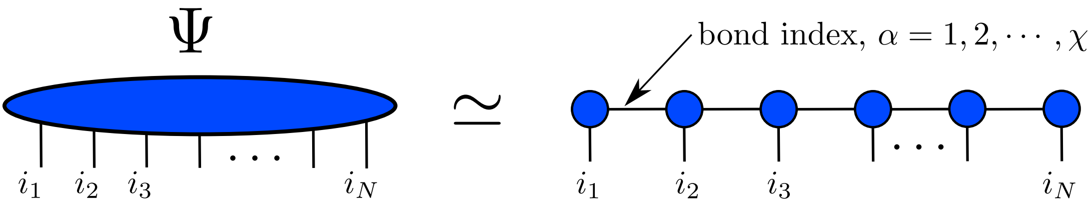
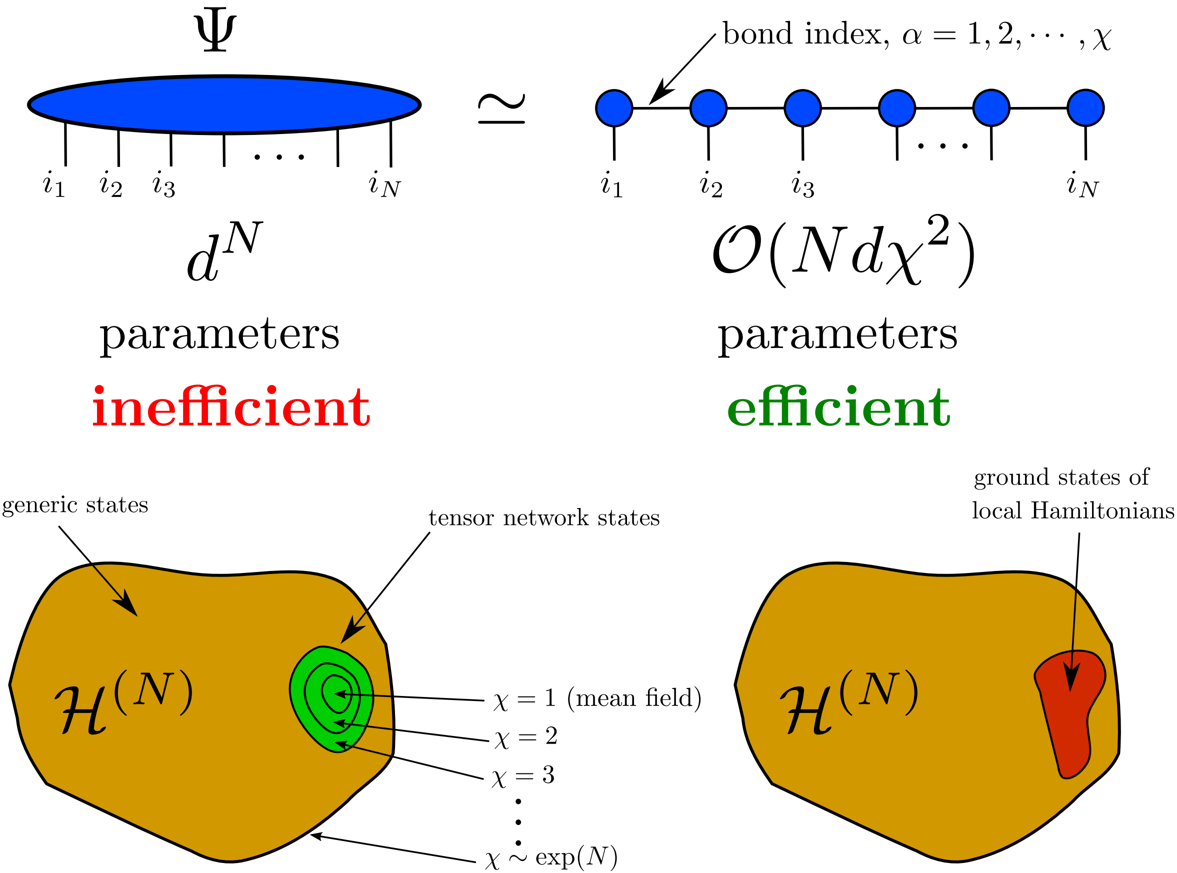


d^N	$\mathcal{O}(Nd\chi^2)$
parameters	parameters
inefficient	**efficient**

TENSOR NETWORKS

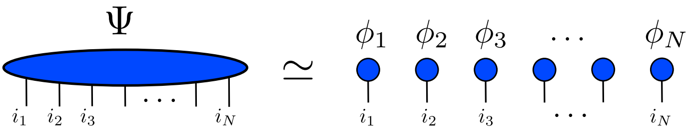


WHY BOND INDICES?

$$|\Psi\rangle = \sum_{i_1,\ldots,i_N=1}^{d} \Psi_{i_1\ldots i_N} |i_1 \ldots i_N\rangle$$

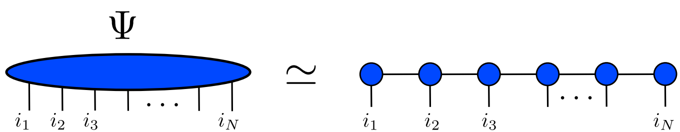
- product (unentangled) state

$$|\Psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_N\rangle$$



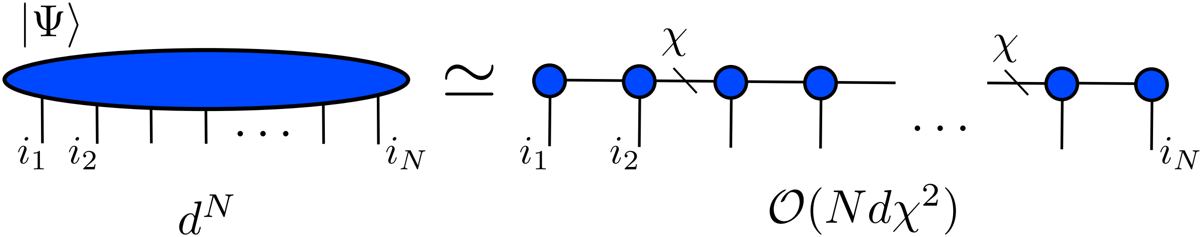
- entangled state

$$|\Psi\rangle \neq |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_N\rangle$$



MPS: efficiency

- number of parameters needed to specify wave-function



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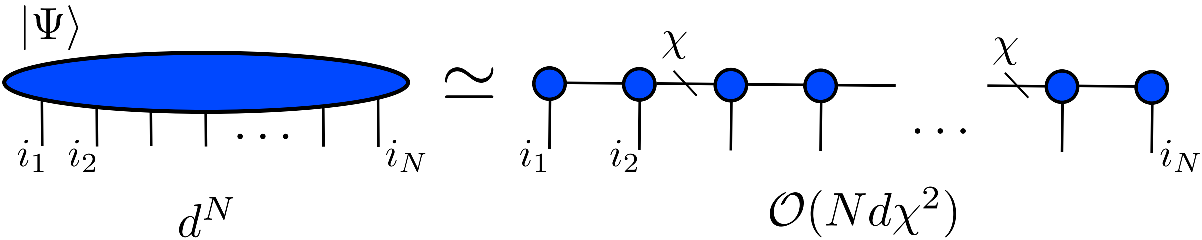
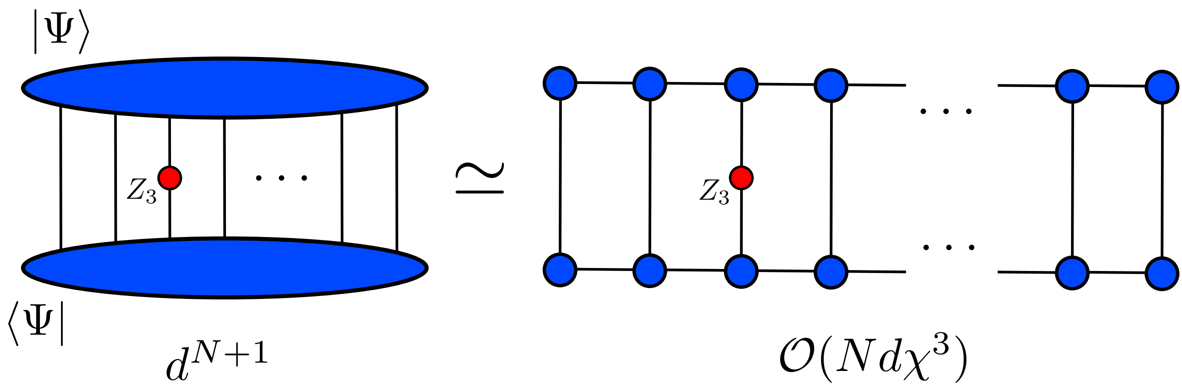


- computational cost

calculate $\langle\Psi|Z_3|\Psi\rangle$



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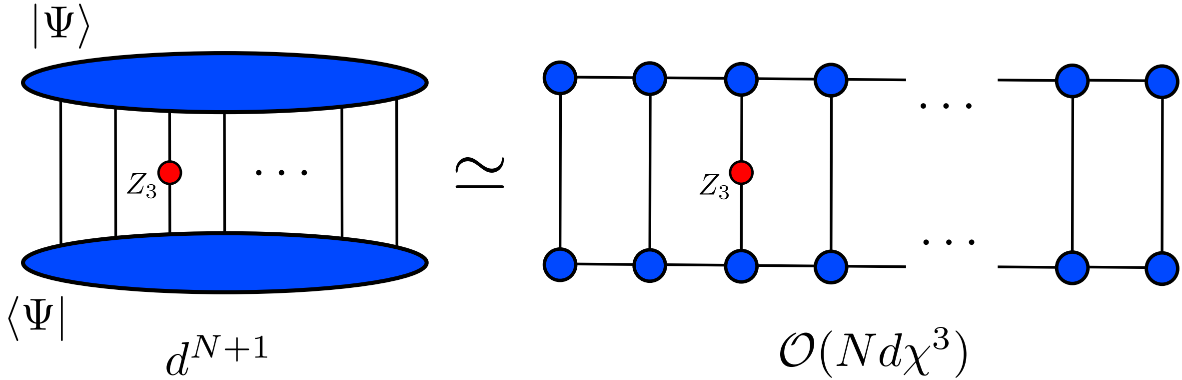

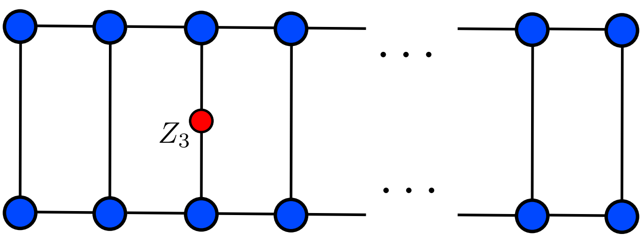

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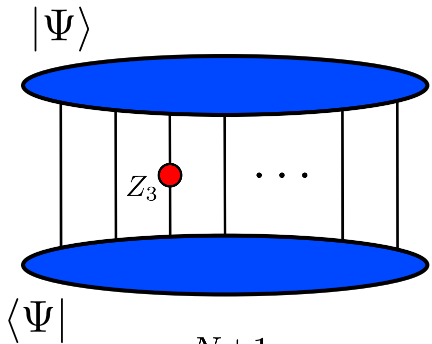

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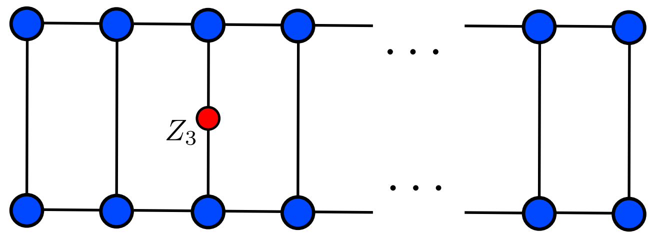
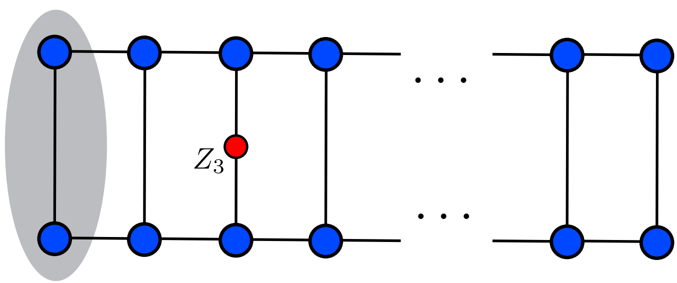


d^{N+1}

$\mathcal{O}(Nd\chi^3)$



cost:

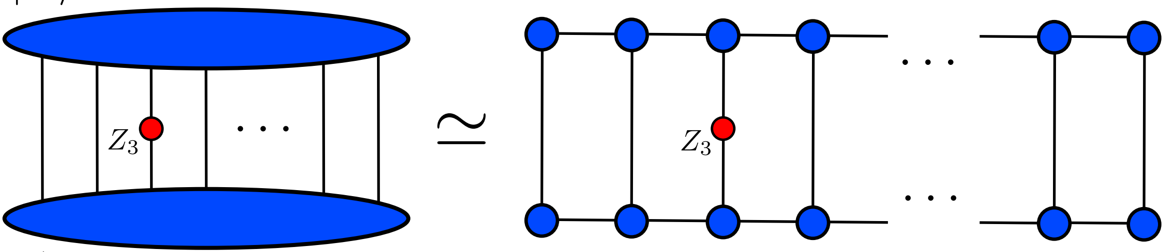
$d\chi^2$

MPS: EFFICIENCY

- computational cost

calculate $\langle\Psi|Z_3|\Psi\rangle$

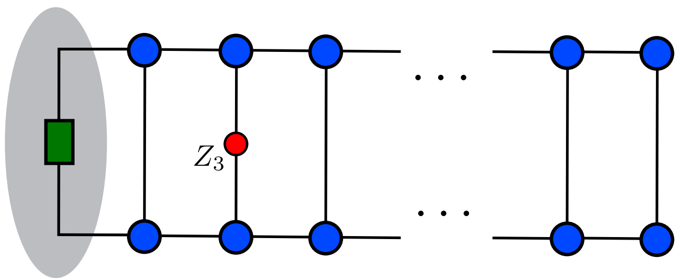
$|\Psi\rangle$



$\langle\Psi|$

d^{N+1}

$\mathcal{O}(Nd\chi^3)$



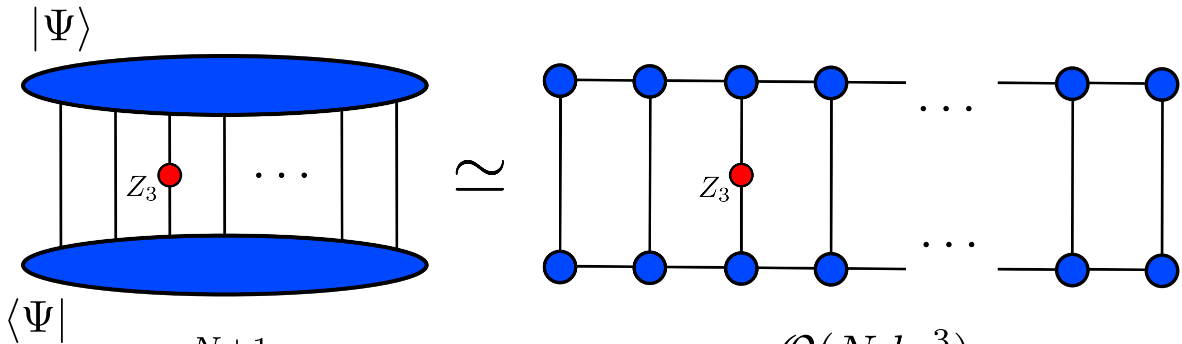
cost:

$d\chi^2$

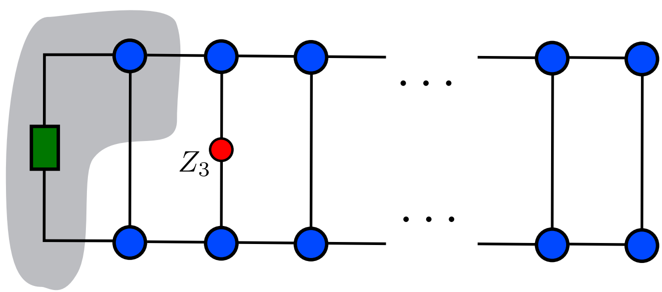
MPS: EFFICIENCY

- computational cost

calculate $\langle\Psi|Z_3|\Psi\rangle$



d^{N+1} $\mathcal{O}(Nd\chi^3)$

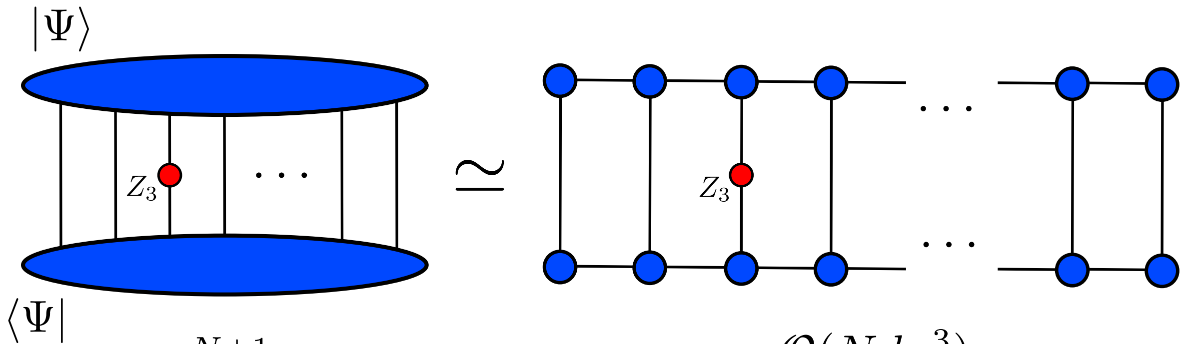


cost:
$d\chi^3$

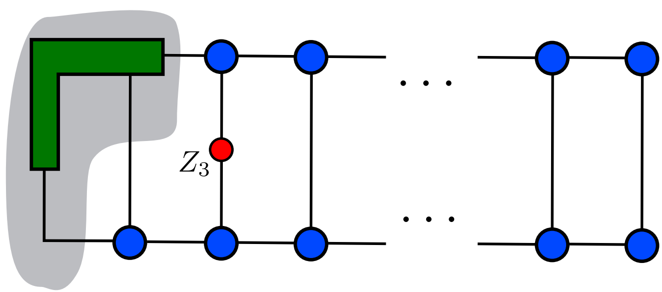
MPS: EFFICIENCY

- computational cost

calculate $\langle\Psi|Z_3|\Psi\rangle$



d^{N+1} $\mathcal{O}(Nd\chi^3)$

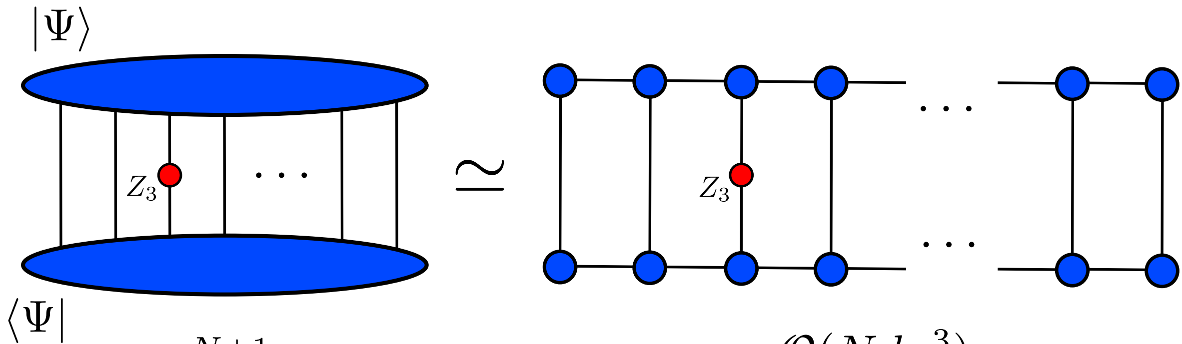


cost: $d\chi^3$

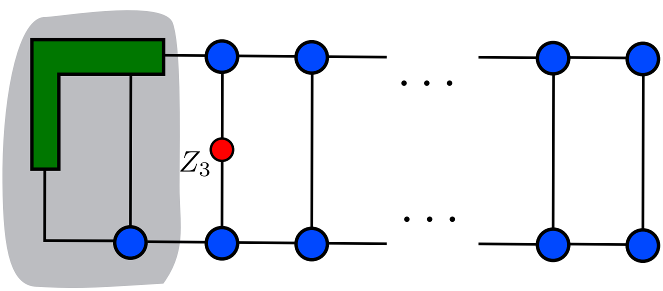
MPS: EFFICIENCY

- computational cost

calculate $\langle\Psi|Z_3|\Psi\rangle$



d^{N+1} $\mathcal{O}(Nd\chi^3)$



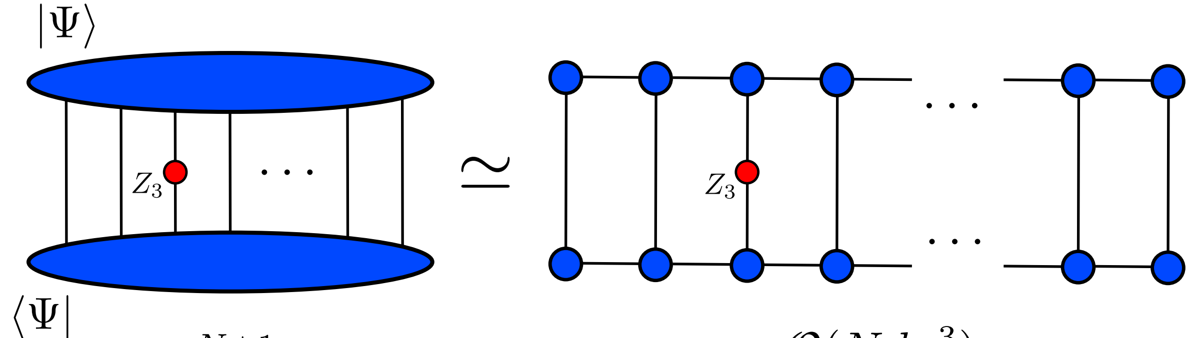
cost:

$d\chi^3$

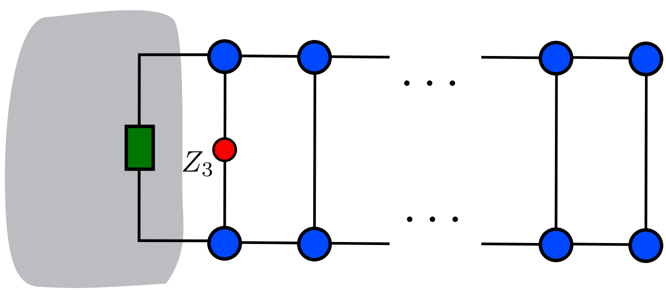
MPS: EFFICIENCY

- computational cost

calculate $\langle\Psi|Z_3|\Psi\rangle$



d^{N+1} $\mathcal{O}(Nd\chi^3)$

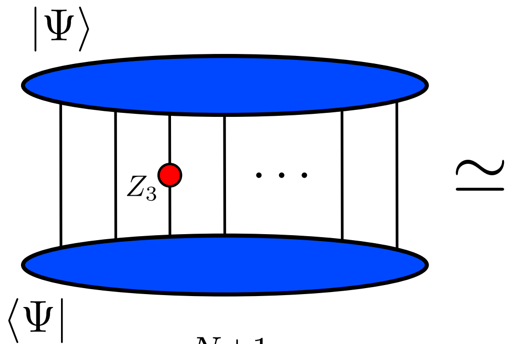


cost:
$d\chi^3$

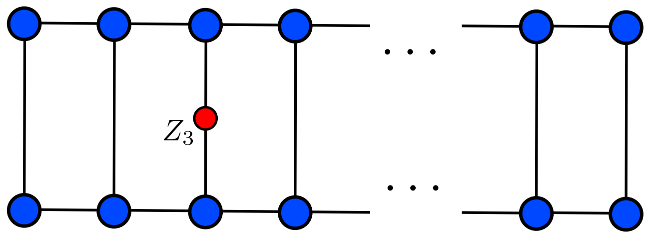
MPS: EFFICIENCY

- computational cost

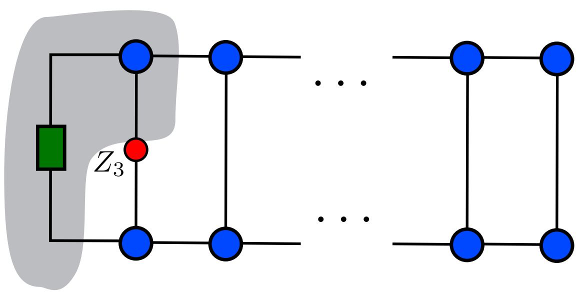
calculate $\langle\Psi|Z_3|\Psi\rangle$



d^{N+1}



$\mathcal{O}(Nd\chi^3)$

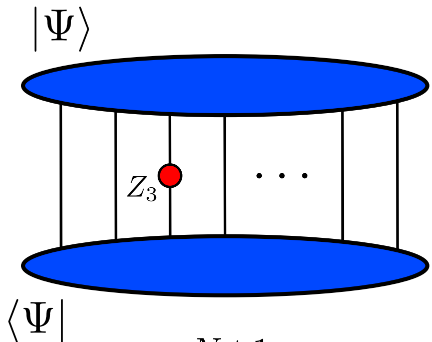


cost:
$d\chi^3$

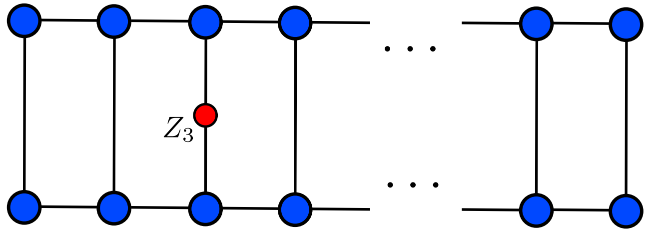
MPS: EFFICIENCY

- computational cost

calculate $\langle \Psi | Z_3 | \Psi \rangle$

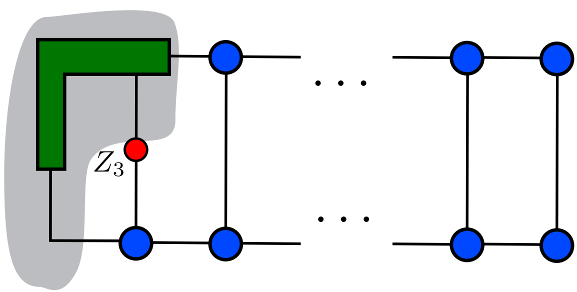


$\simeq$



d^{N+1}

$\mathcal{O}(N d \chi^3)$



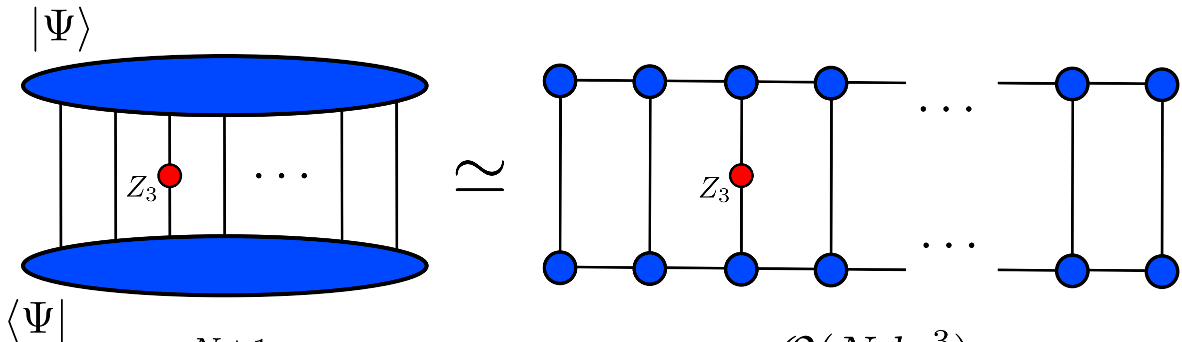
cost:

$d\chi^3$

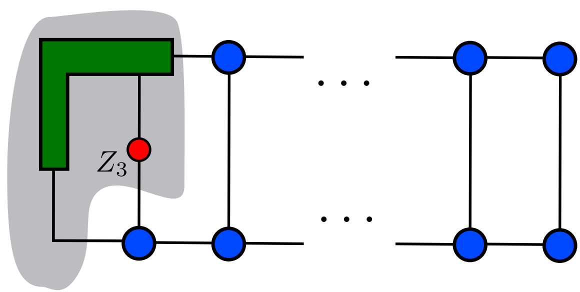
MPS: EFFICIENCY

- computational cost

calculate $\langle \Psi | Z_3 | \Psi \rangle$



d^{N+1} $\qquad$ $\mathcal{O}(N d \chi^3)$



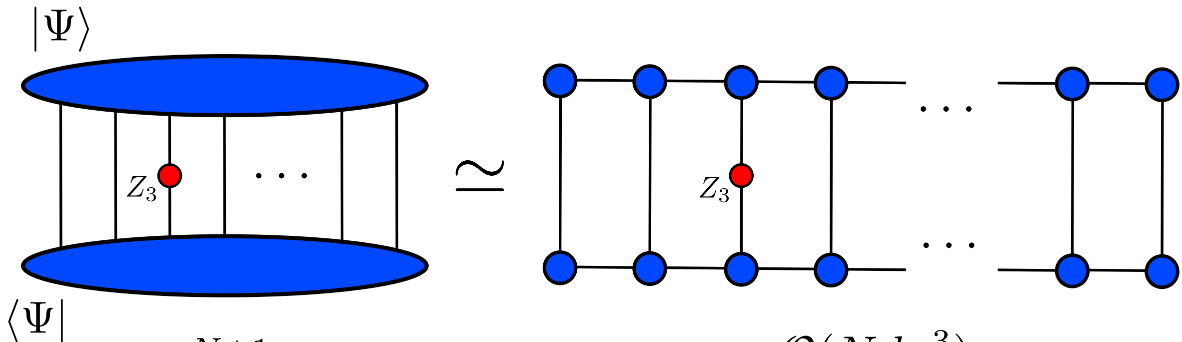
cost:

$d^2 \chi^2$

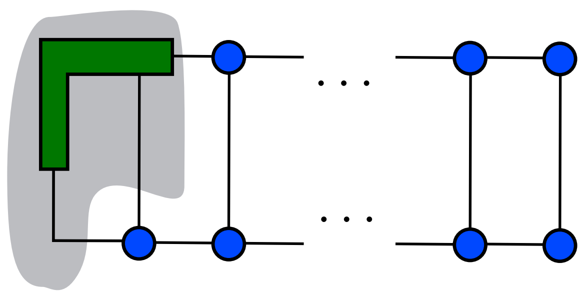
MPS: EFFICIENCY

- computational cost

calculate $\langle\Psi|Z_3|\Psi\rangle$



d^{N+1} $\mathcal{O}(Nd\chi^3)$

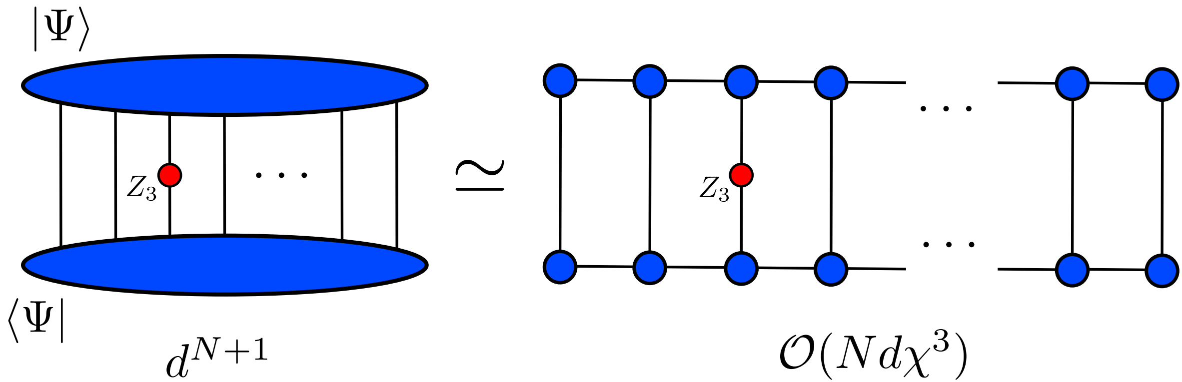


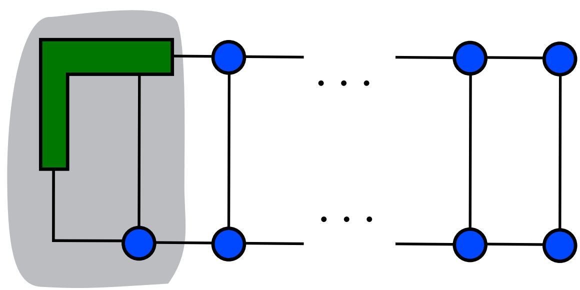
cost:
$d^2\chi^2$

MPS: EFFICIENCY

- computational cost

calculate $\langle\Psi|Z_3|\Psi\rangle$



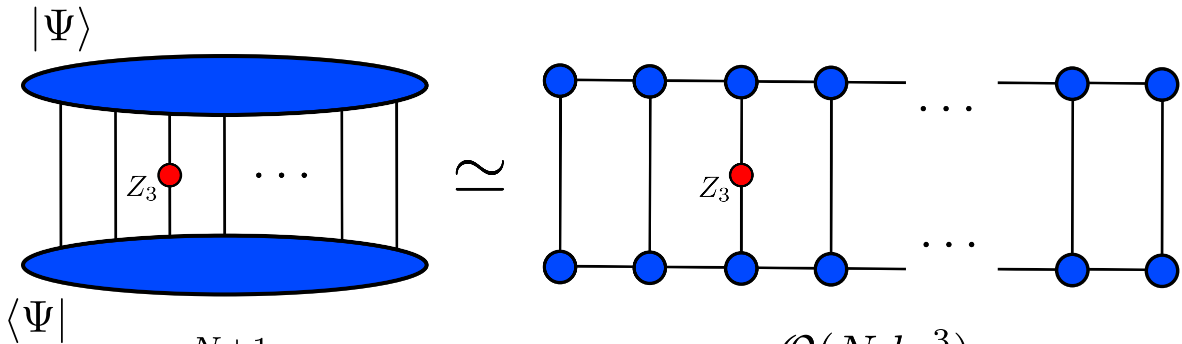


cost:
$d\chi^3$

MPS: EFFICIENCY

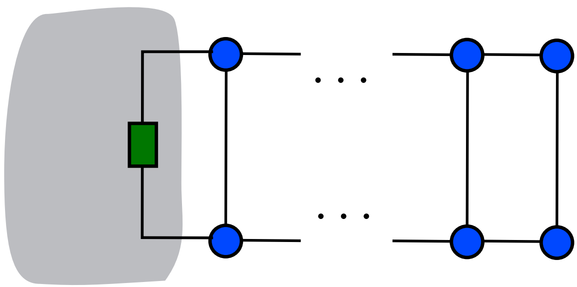
- computational cost

calculate $\langle\Psi|Z_3|\Psi\rangle$



d^{N+1}

$\mathcal{O}(Nd\chi^3)$



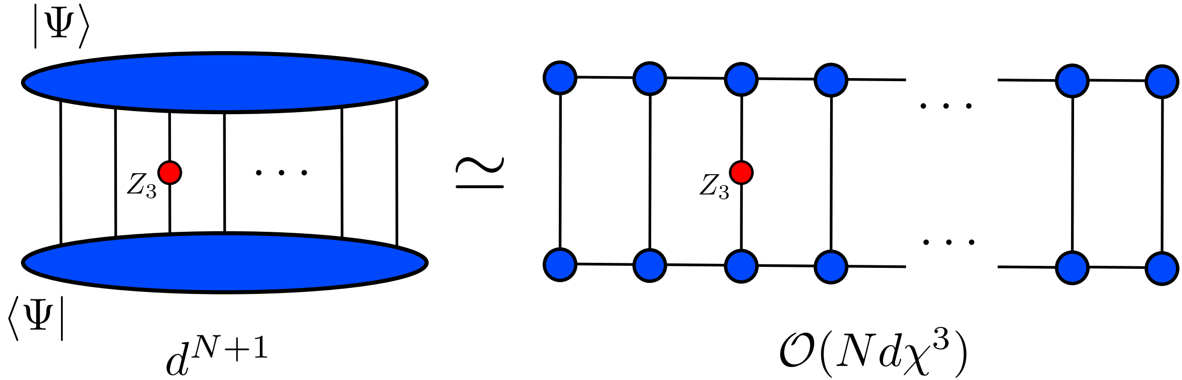
cost:

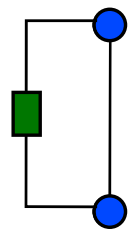
$d\chi^3$

MPS: EFFICIENCY

- computational cost

calculate $\langle\Psi|Z_3|\Psi\rangle$

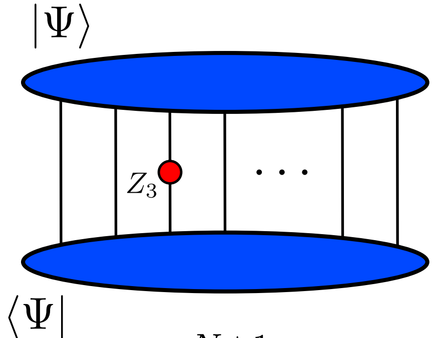




MPS: EFFICIENCY

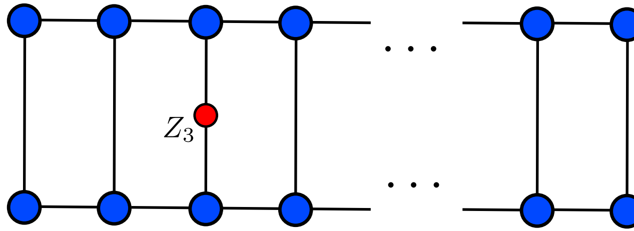
- computational cost

calculate $\langle \Psi | Z_3 | \Psi \rangle$

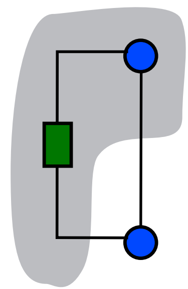


d^{N+1}

$\simeq$



$\mathcal{O}(Nd\chi^3)$



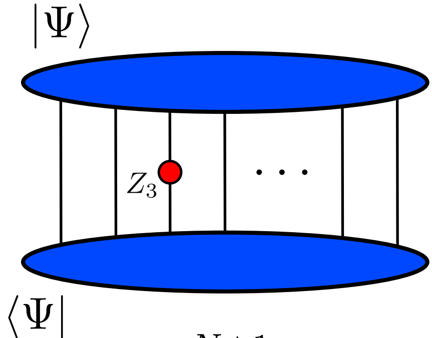
cost:

$d\chi^3$

MPS: EFFICIENCY

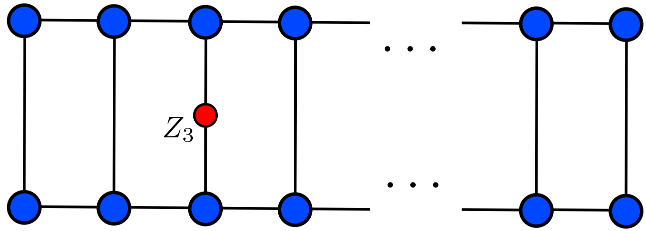
- computational cost

calculate $\langle \Psi | Z_3 | \Psi \rangle$

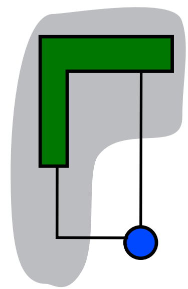


d^{N+1}

$\simeq$



$\mathcal{O}(N d \chi^3)$

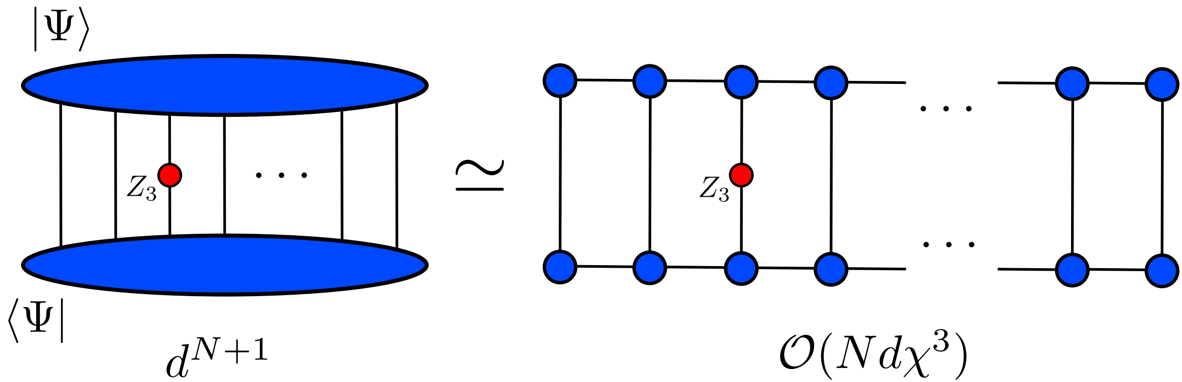


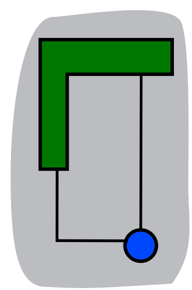
cost:
$d\chi^3$

MPS: efficiency

- computational cost

calculate $\langle\Psi|Z_3|\Psi\rangle$





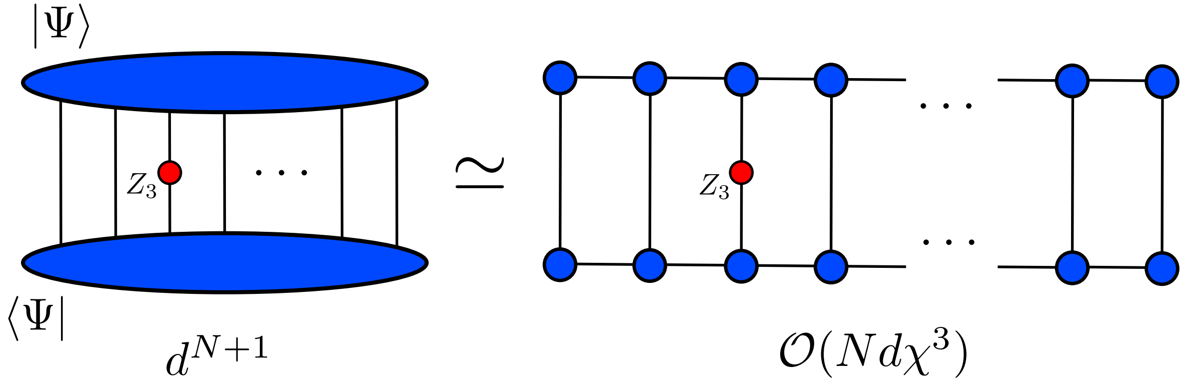
cost:

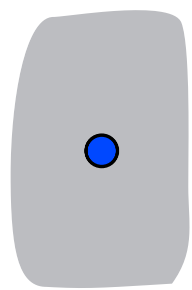
$d\chi^3$

MPS: EFFICIENCY

- computational cost

calculate $\langle \Psi | Z_3 | \Psi \rangle$





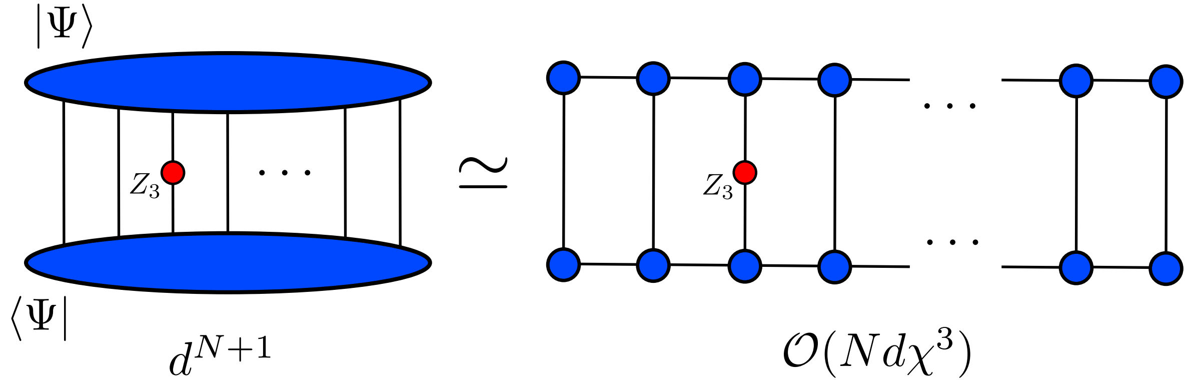
cost:

$d\chi^3$

MPS: EFFICIENCY

- computational cost

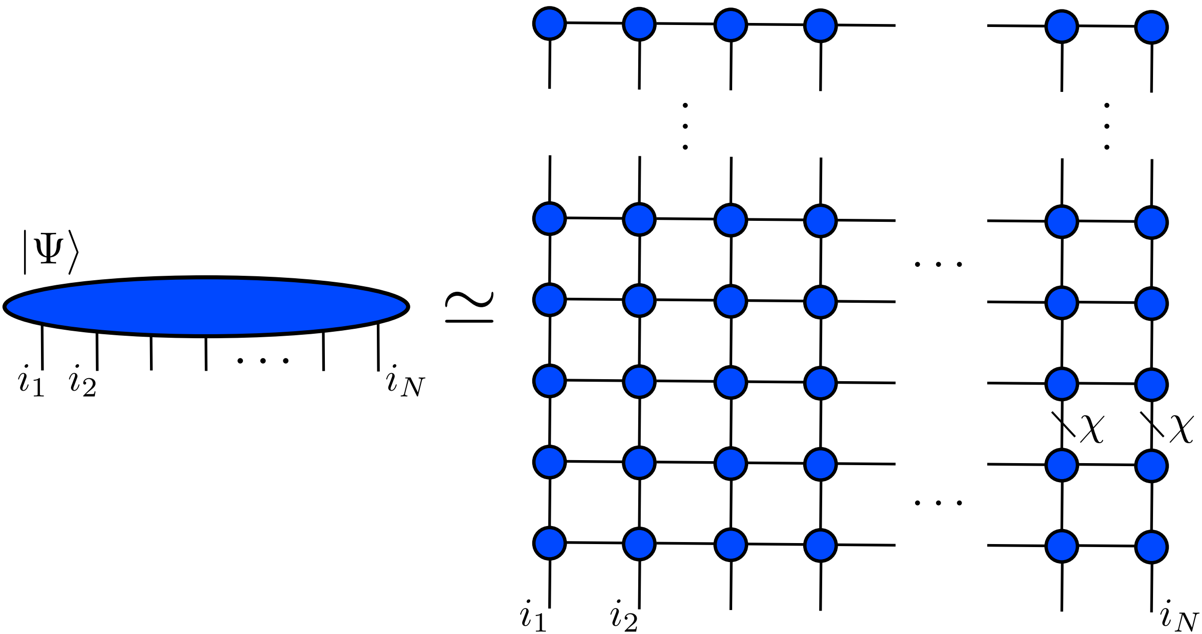
calculate $\langle\Psi|Z_3|\Psi\rangle$



overall cost: $\mathcal{O}(Nd\chi^3)$

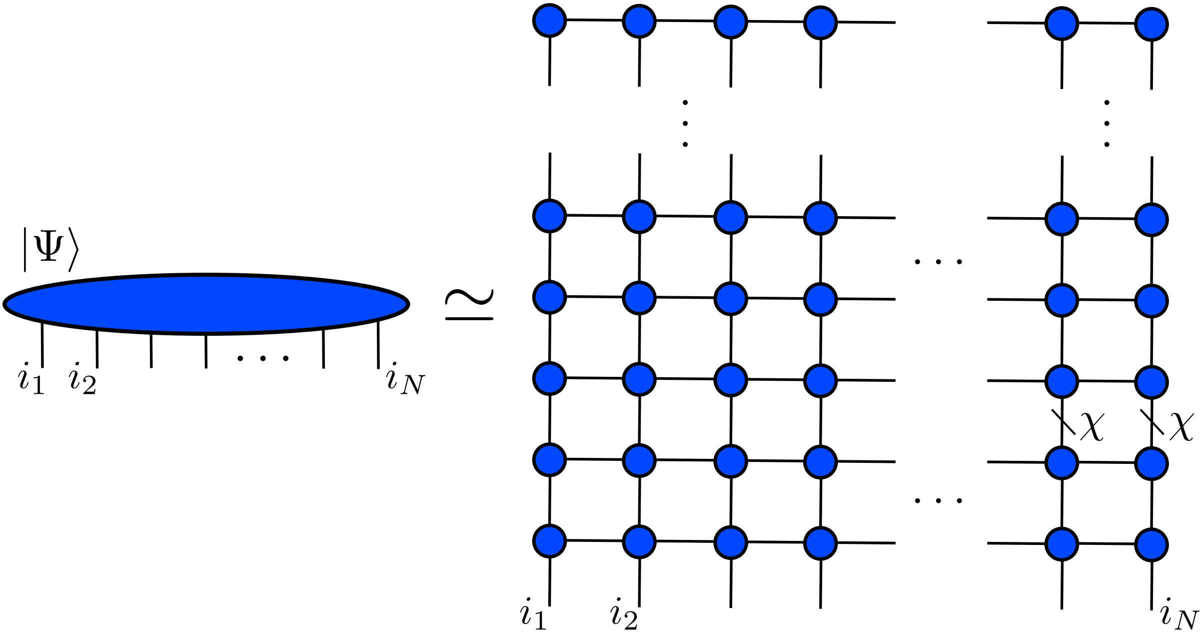
MPS: EFFICIENCY

not every tensor network gives an efficient representation



MPS: EFFICIENCY

not every tensor network gives an efficient representation

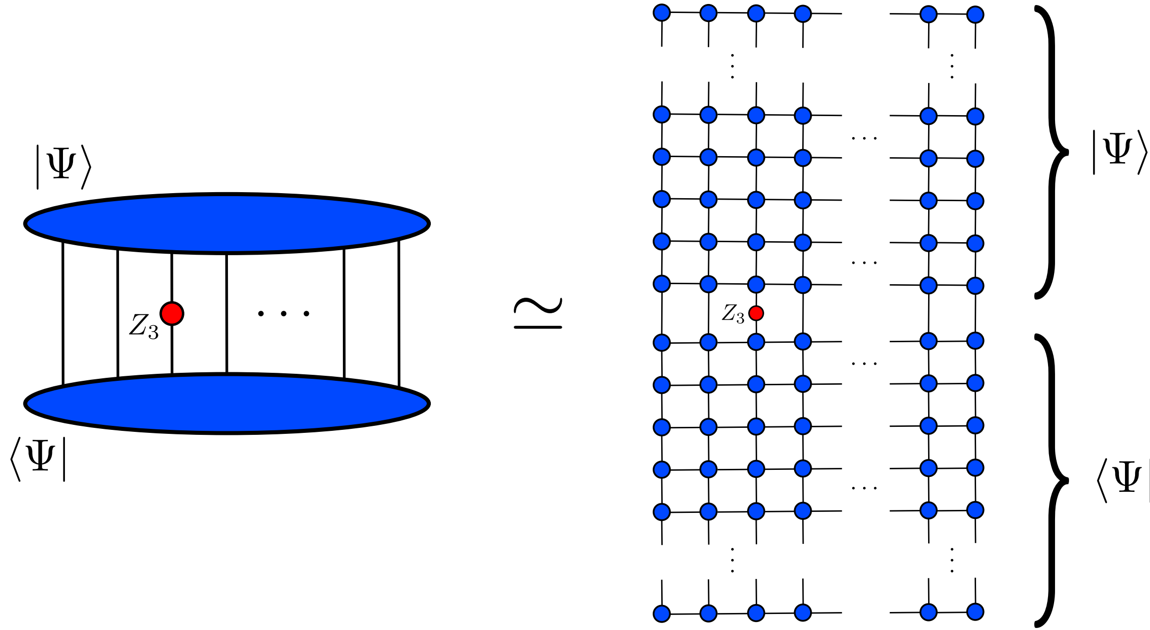


- number of parameters needed to specify wave-function

d^N $\mathcal{O}(N^2\chi^4)$

MPS: EFFICIENCY

not every tensor network gives an efficient representation

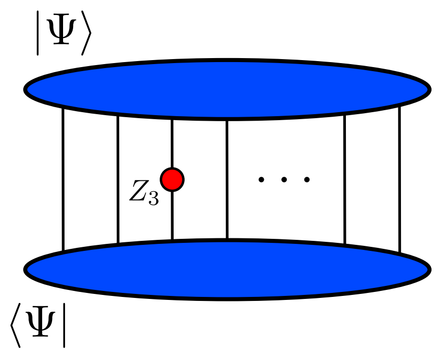


- computational cost

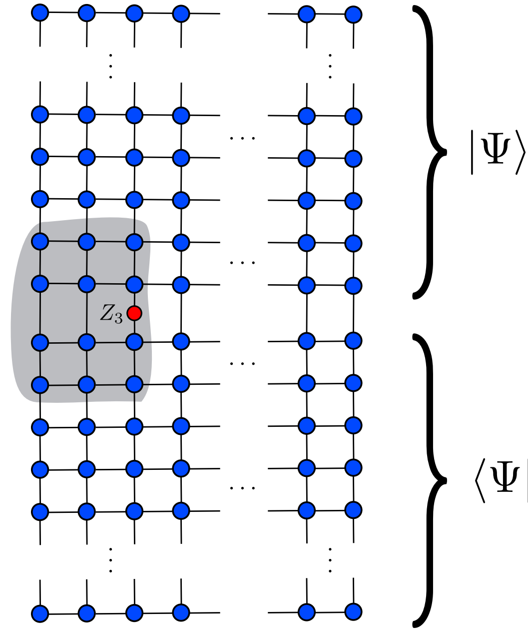
d^{N+1} $\qquad\qquad$ $\exp(N)$

MPS: EFFICIENCY

not every tensor network gives an efficient representation



$$\simeq$$

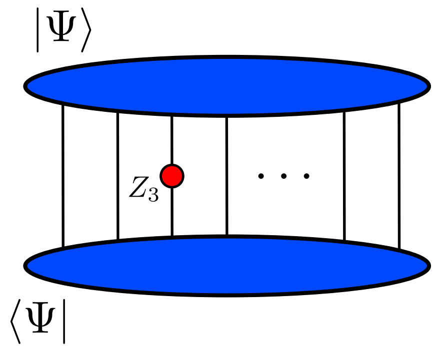


- computational cost

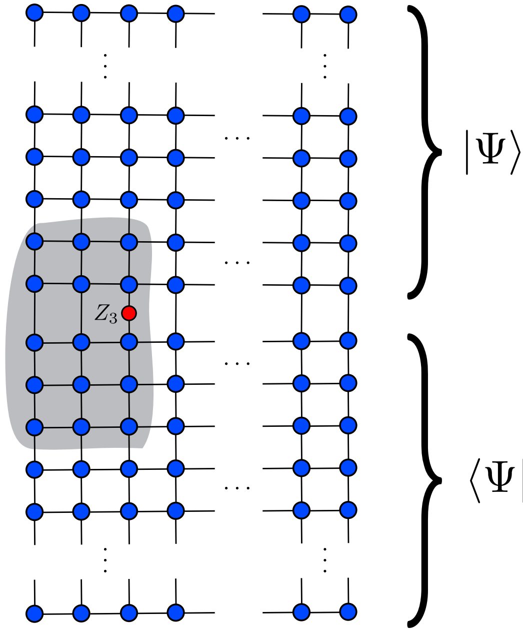
$$d^{N+1} \qquad\qquad \exp(N)$$

MPS: efficiency

not every tensor network gives an efficient representation



$\simeq$



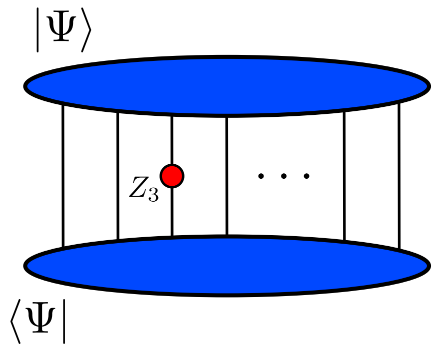
- computational cost

d^{N+1}

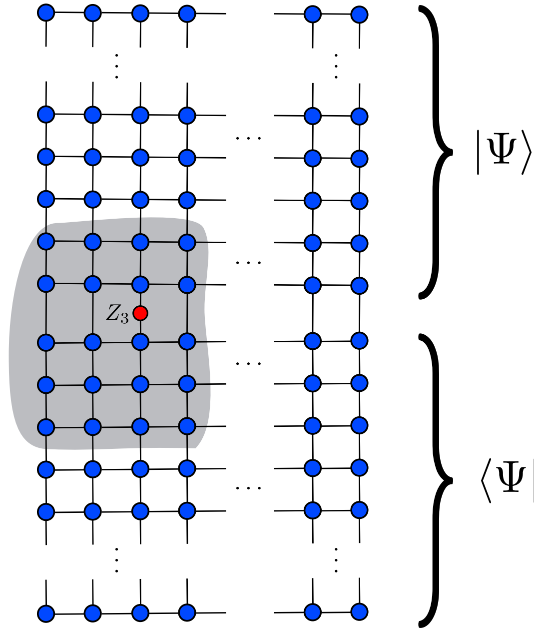
$\exp(N)$

MPS: EFFICIENCY

not every tensor network gives an efficient representation



$\simeq$

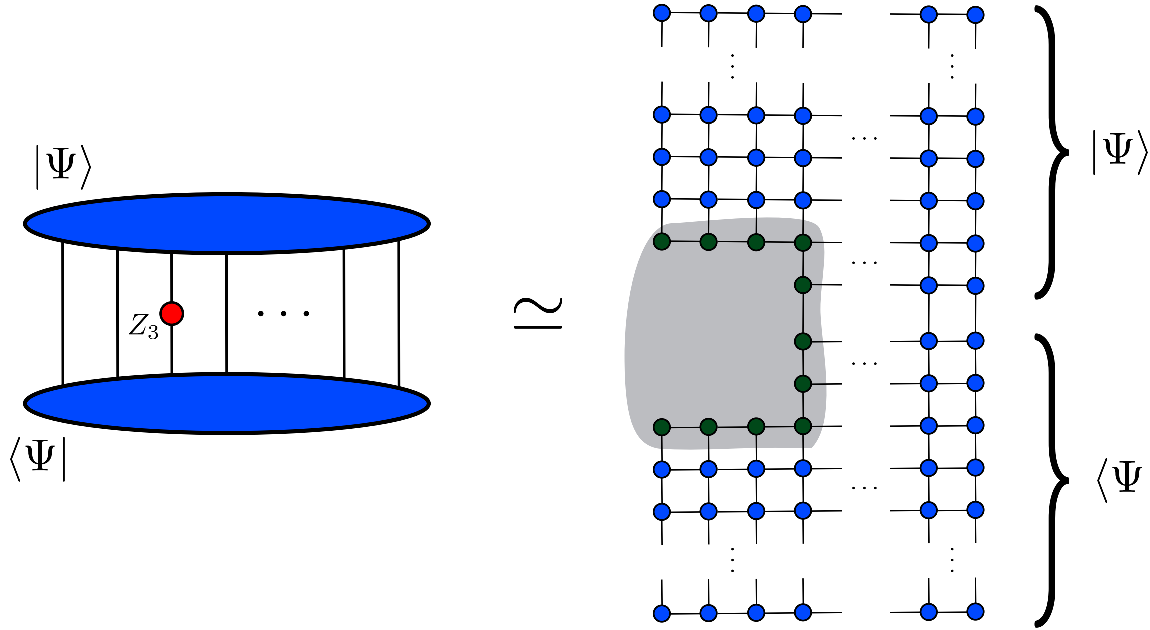


- computational cost

d^{N+1} $\qquad$ $\exp(N)$

MPS: EFFICIENCY

not every tensor network gives an efficient representation



- computational cost

d^{N+1} poly(N)