AdS/CFT calculations of meson decay rates

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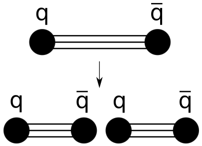
Introduction to Meson Decay: QCD Picture

- Meson decay may be seem as $q\bar{q}$ pair production from a colour field flux tube



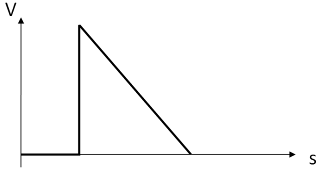
- May use Schwinger formula [1] to calculate decay rate Γ for volume V and energy density ϵ:

$$\begin{aligned}\Gamma &= 2\mathrm{Im}\,\epsilon \\ &= -\frac{2}{V}\,\mathrm{Im}\,\ln\int \mathrm{d}X\,\mathrm{e}^{S}\end{aligned}$$

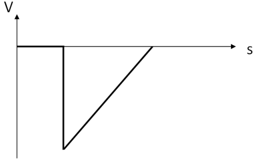
[1] Julian Schwinger. On gauge and vacuum polarizaion. *Physical Review*, 82(5):664, June 1951.

Introduction to Meson Decay: Instanton Method

- After the new $q\bar{q}$ pair is produced it must gain sufficient energy from the field to come on shell. This may be seen as a tunneling process:



- By Wick rotating the time coordinate we flip the potential:



- The calculation may now be done semi-classically.

Point Particle Pair Production

- Consider Minkowski point particle action:

$$S_M = \int \mathrm{d}\tau \, \left(\frac{1}{2} \frac{\dot{X}^2}{e} - \frac{1}{2} e m^2 + A_\mu \dot{X}^\mu \right)$$

- Wick rotate to Euclidean spacetime:

$$\tau \to -i\tau$$
$$X^0 \to -iX^0$$
$$A^0 \to -iA^0$$

- Set periodic boundary condition:

$$X^\mu(\tau + 1) = X^\mu(\tau)$$

- Find Euclidean action:

$$S_E = -\int_0^1 \mathrm{d}\tau \, \left(\frac{\dot{X}^2}{4T} + m^2 T - iA_\mu \dot{X}^\mu \right)$$

Point Particle Pair Production

- Recall $A_\mu = -\frac{1}{2} F_{\mu\nu} X^\nu$
- Find Euler-Lagrange equation for T and eliminate it from action:

$$T = \frac{\sqrt{\dot{X}^2}}{2m} \qquad \Rightarrow \qquad S_E = -\int_0^1 \mathrm{d}\tau \, \left(m\sqrt{\dot{X}^2} + \frac{i}{2} F_{\mu\nu} X^\nu \dot{X}^\mu \right)$$

- Find Euler-Lagrange equation for X_μ:

$$iF_{\nu\mu}\dot{X}^\nu = \frac{2\ddot{X}_\mu}{\sqrt{\dot{X}^2}} - \frac{2\dot{X}_\mu}{\left(\dot{X}^2\right)^{\frac{3}{2}}} \dot{X}^\nu \ddot{X}_\nu + iF_{\mu\nu}\dot{X}^\nu$$

Point Particle Pair Production

- Choose constant field:

$$F_{01} = -F_{10} = -iE$$

- Problem admits the solution:

$$X_\mu = R \begin{pmatrix} \cos(2\pi n\tau) \\ sin(2\pi n\tau) \\ 0 \\ 0 \end{pmatrix}$$

- Subsitute X_μ into action and extremise value of R
- Action evaluates to expected result [2]:

$$S_E = -\frac{\pi m^2}{E} n$$

[2] Gordon W Semenoff and Konstantin Zarembo. Holographic schwinger effect, September 2011. arXiv:1109.2920 [hep-th].

Point Particle in Static Gauge

- The problem may also be done in the static gauge:

$$X_0 = t \qquad X_1 = x \qquad\qquad A_0 = iEx \qquad A_1 = 0$$

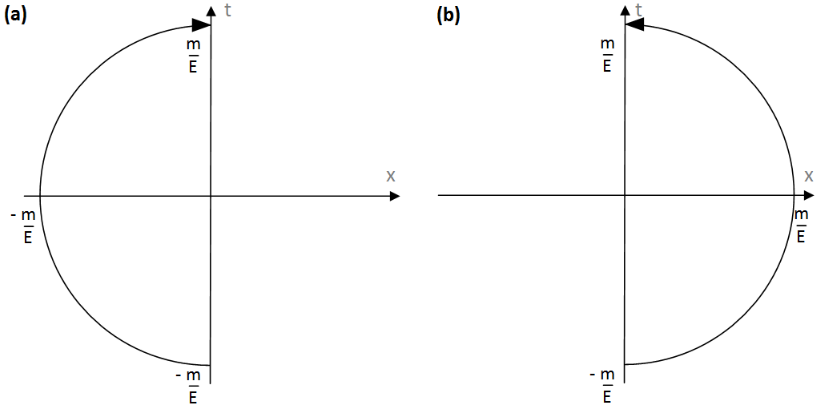
- The Lagrangian reduces to

$$L = -\left(m\sqrt{1+\dot{x}^2} \pm Ex\right)$$

- We obtain the same circular path

$$x = \pm\frac{1}{E}\sqrt{m^2 - E^2 t^2}$$

Point Particle in Static Gauge



Point Particle Static Gauge Action

- We find that

$$
\begin{aligned}
S_{L,Particle} = S_{R,Antiparticle} = &- m \int_{-\frac{m}{E}}^{\frac{m}{E}} \left(\sqrt{\frac{1}{1 - \left(\frac{E}{m}\right)^2 t^2}} \right. \\
&\left. - \sqrt{1 - \left(\frac{E}{m}\right)^2 t^2} \right) \mathrm{d}t \\
= &- \frac{\pi}{2} \frac{m^2}{E}
\end{aligned}
$$

- Therefore the total action is $-\frac{\pi m^2}{E}$, as before.

AdS/CFT Correspondence

- 't Hooft suggested that that for large N_C QCD is equivalent to theory of free strings [3]
- Seems to violate Weinberg-Witten theorem–which forbids the existance of gravitons in QCD [4]
- Maldacena suggested that a QFT in D dimensions corresponds to string theory in $D+1$ dimensions. [5]
- Good correspondence between $\mathcal{N}=4$ super Yang-Mills theory and a string theory in the $AdS_5 \times S^5$

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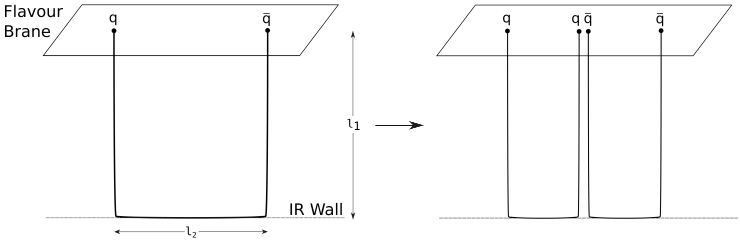
[3] Gerard 't Hooft. A planar diagram theory for strong interactions. *Nuclear Physics B*, 73:461, 1974.

[4] Steven Weinberg & Edward Witten. Limits on massless particles. *Nuclear Physics B*, 96(1-2):59, 1980.

[5] Juan Maldacena. The large-N limit of superconformal field theories and supergravity.*International Journal of Theoretical Physics*, 38(4):1113, 1999.

Meson Decay in the Holographic Picture

- We will eventually want to work in AdS spacetime with a string of the following profile

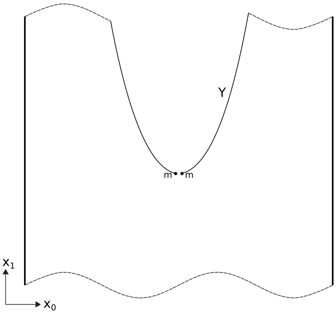


$l_1 \propto$ constituent quark mass $\quad$ $l_2 \propto$ colour flux tube energy

- Will build up to this using simpler examples
- Will work semi-classically using istanton method with a Wick rotated time coordinate

Setting up the Problems

- Consider string with massive endpoints in Eulidean spacetime:



- The action for the string with massive endpoints is:

$$
\begin{aligned}
S_E &= \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \int_0^{\pi} \mathrm{d}\sigma \, \mathcal{L}_{bulk} + \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \, \mathcal{L}_{end} \\
&= -\gamma \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \int_0^{\pi} \mathrm{d}\sigma \sqrt{-\left(\dot{X}\cdot X'\right)^2 + \dot{X}^2 X'^2} \\
&\quad - m \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \left(\sqrt{\dot{X}^2(\tau,\sigma=0)} + \sqrt{\dot{X}^2(\tau,\sigma=\pi)}\right)
\end{aligned}
$$

String Equations of Motion

- Find variation of action:

$$0 = \delta S = -\int_{\tau_1}^{\tau_2} \mathrm{d}\tau \int_0^{\pi} \mathrm{d}\sigma \left(\frac{\partial}{\partial \tau} \left(\frac{\partial \mathcal{L}_{bulk}}{\partial \dot{X}_\mu} \right) + \frac{\partial}{\partial \sigma} \left(\frac{\partial \mathcal{L}_{bulk}}{\partial X'_\mu} \right) \right) \delta X(\tau, \sigma)$$
$$- \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \left(\frac{\partial}{\partial \tau} \left(\frac{\partial \mathcal{L}_{end,\, \sigma=0}}{\partial \dot{X}_\mu} \right) + \frac{\partial \mathcal{L}_{bulk}}{\partial {X'}_\mu} \right) \Bigg|_{\sigma=0} \delta X_\mu(\tau, \sigma = 0)$$
$$- \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \left(\frac{\partial}{\partial \tau} \left(\frac{\partial \mathcal{L}_{end,\, \sigma=\pi}}{\partial \dot{X}_\mu} \right) - \frac{\partial \mathcal{L}_{bulk}}{\partial {X'}_\mu} \right) \Bigg|_{\sigma=\pi} \delta X_\mu(\tau, \sigma = \pi)$$

String Equations of Motion

- Find bulk equation of motion ($0 < \sigma < \pi$):

$$\frac{\partial}{\partial \tau}\left(\frac{\partial \mathcal{L}_{bulk}}{\partial \dot{X}_\mu}\right) + \frac{\partial}{\partial \sigma}\left(\frac{\partial \mathcal{L}_{bulk}}{\partial X'_\mu}\right) = 0$$

- Find boundary equations of motions:

$$\left(\frac{\partial}{\partial \tau}\left(\frac{\partial \mathcal{L}_{end,\,\sigma=0}}{\partial \dot{X}_\mu}\right) + \frac{\partial \mathcal{L}_{bulk}}{\partial {X'}_\mu}\right)\Bigg|_{\sigma=0} \delta X_\mu(\tau, \sigma = 0) = 0$$

$$\left(\frac{\partial}{\partial \tau}\left(\frac{\partial \mathcal{L}_{end,\,\sigma=\pi}}{\partial \dot{X}_\mu}\right) - \frac{\partial \mathcal{L}_{bulk}}{\partial {X'}_\mu}\right)\Bigg|_{\sigma=\pi} \delta X_\mu(\tau, \sigma = \pi) = 0$$

Straight Endpoint String Solution

- Bardeen et al suggest a more general analytic Lorentzian solution for a string with two massive free moving endpoints: [6]

$$
\begin{aligned}
X_0 &= \tau \\
X_1 &= \pm\left(\frac{2\sigma}{\pi} - 1\right)\left(\sqrt{(\tau-\tau_0)^2 + k^2} + x_0\right)
\end{aligned}
$$

- Wick rotating, we therefore find:

$$
\begin{aligned}
X_{L0} = X_{R0} &= \tau \\
X_{L1} = x_L &= -\frac{\sigma}{\pi}\left(\sqrt{-\tau^2 + k^2} - x_0\right) \\
X_{R1} = x_R &= \left(1 - \frac{\sigma}{\pi}\right)\left(\sqrt{-\tau^2 + k^2} + x_0\right) + 2x_0\frac{\sigma}{\pi}
\end{aligned}
$$

[6] W.A. Bardeen, Itzhak Bars, Andrew J. Hanson, and R.D. Peccei. Study of the longitudinal kink modes of the string. *Physical Review D: Particles and Fields*, 13(8):2364-2382, 1976.

Applying Boundary Conditions

- We consider the Neumann boundary condition at $\sigma = \pi$ for the left hand string. We find

$$\frac{\partial}{\partial \tau}\left(\frac{\partial \mathcal{L}_{end,\sigma=\pi}}{\partial \dot{X}_\mu}\right) - \frac{\partial \mathcal{L}_{bulk}}{\partial X'_\mu}\bigg|_{\sigma=\pi} = 0$$

$$-m\frac{\partial}{\partial \tau}\left(\frac{\dot{X}^\mu}{\sqrt{\dot{X}^2}}\right) + \gamma\frac{-(\dot{X}\cdot X')\dot{X}^\mu + X'^\mu \dot{X}^2}{\sqrt{-(\dot{X}\cdot X')^2 + \dot{X}^2 X'^2}}\bigg|_{\sigma=\pi} = 0$$

$$-m\frac{\partial}{\partial \tau}\left(\frac{\dot{X}^\mu}{\sqrt{1+\dot{x}^2}}\right) + \gamma\frac{-\dot{x}x'\dot{X}^\mu + (1+\dot{x}^2)X'^\mu}{\sqrt{-\dot{x}^2 x'^2 + (1+\dot{x}^2)x'^2}}\bigg|_{\sigma=\pi} = 0$$

$$-m\frac{\partial}{\partial \tau}\left(\frac{\dot{X}^\mu}{\sqrt{1+\dot{x}^2}}\right) + \gamma\left(-\dot{x}\dot{X}^\mu + \left(\frac{1+\dot{x}^2}{x'}\right)X'^\mu\right)\bigg|_{\sigma=\pi} = 0.$$

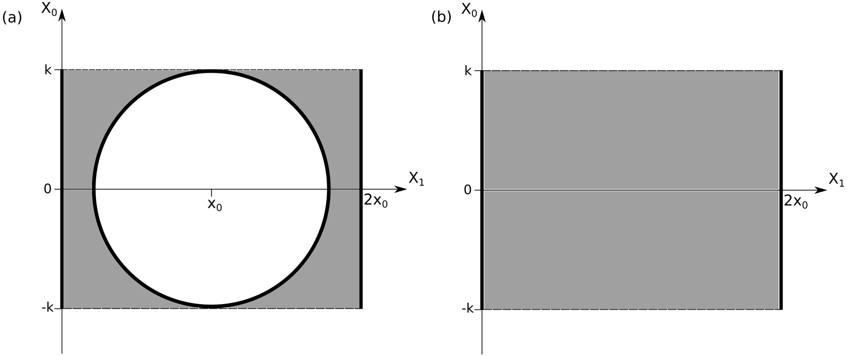
Applying Boundary Conditions

- For $\mu = 1$ this gives

$$
\begin{aligned}
-m\frac{\partial}{\partial\tau}\left(\frac{\dot{x}^{\mu}}{\sqrt{1+\dot{x}^2}}\right)+\gamma =& 0\\
-m\frac{\partial}{\partial\tau}\left(\frac{\tau}{\sqrt{-\tau^2+k^2}}\frac{\sqrt{-\tau^2+k^2}}{k}\right)+\gamma =& 0\\
-\frac{m}{k}+\gamma =& 0
\end{aligned}
$$

- This fixes $k = \frac{m}{\gamma}$. We get the same value of k from setting $\mu = 0$.

String Solution



Evaluating Action

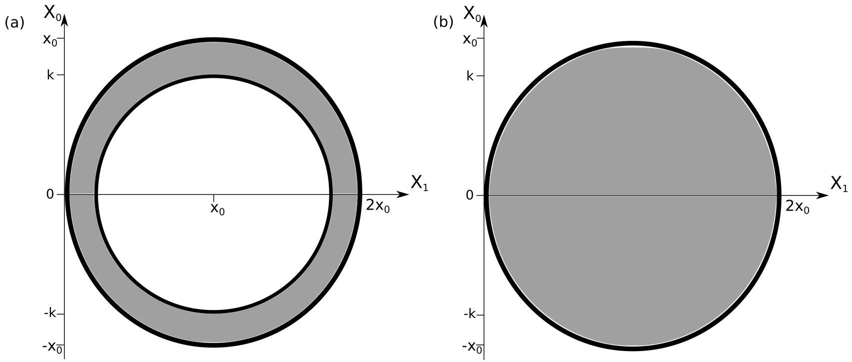
- Subtracting the background action:

$$
\begin{aligned}
S =& S_{sol} - S_{back} \\
=& -\int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(m\sqrt{1+\dot{x}_L^2(\tau,\sigma=\pi)} + \gamma x_L(\tau,\sigma=\pi) \right) \\
& -\int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(m\sqrt{1+\dot{x}_R^2(\tau,\sigma=0)} - \gamma x_R(\tau,\sigma=0) \right) = -\frac{\pi m^2}{\gamma}
\end{aligned}
$$

- This makes sense looking at the problem geometrically, as the action for system is effectively

$$
\begin{aligned}
S =& -m(\text{Circumeference of circle}) + \gamma(\text{Area of circle}) \\
=& -m(2\pi k) + \gamma(\pi k^2) \\
=& -2\frac{\pi m^2}{\gamma} + \frac{\pi m^2}{\gamma} = -\frac{\pi m^2}{\gamma}.
\end{aligned}
$$

Concentric Circle Solution



Concentric Circle Solution

- We find the solution

$$
\begin{aligned}
X_{L0} &= X_{R0} = \tau \\
X_{L1} &= x_L = -\frac{\sigma}{\pi}\left(\sqrt{-\tau^2+k^2}-x_0\right)+\left(1-\frac{\sigma}{\pi}\right)\left(\sqrt{-\tau^2+{x_0}^2}\right) \\
X_{R1} &= x_R = \left(1-\frac{\sigma}{\pi}\right)\left(\sqrt{-\tau^2+k^2}+x_0\right)+\frac{\sigma}{\pi}\left(\sqrt{-\tau^2+{x_0}^2}+x_0\right)
\end{aligned}
$$

- As before, we find

$$S = S_{sol} - S_{back} = -\frac{\pi m^2}{\gamma}$$

Motivating Further Work

- Appropriate metric[7] is:

$$\mathrm{d}s^2 = Y_\mu Y^\mu = \frac{U^{\frac{3}{2}}}{{R_0}^{\frac{3}{2}}}\left(\mathrm{d}X^\mu \mathrm{d}X^\nu \eta_{\mu\nu} + f(U)\mathrm{d}\psi^2\right) + K(U)\left(\mathrm{d}\rho^2 + \rho^2 \mathrm{d}{\Omega_4}^2\right)$$

$$= \frac{U^{\frac{3}{2}}}{{R_0}^{\frac{3}{2}}}\left(\mathrm{d}X^\mu \mathrm{d}X^\nu \eta_{\mu\nu} + f(U)\mathrm{d}\psi^2\right) + K(U)\left(\mathrm{d}\lambda^2 + \lambda^2 \mathrm{d}{\Omega_2}^2 + \mathrm{d}r^2 + r^2 \mathrm{d}\phi^2\right),$$

where

$$f(U) = 1 - \frac{{U_\Lambda}^3}{U^3}$$

$$U(\rho) = \left(\rho^{\frac{3}{2}} + \frac{{U_\Lambda}^3}{4\rho^{\frac{3}{2}}}\right)^{\frac{2}{3}}$$

$$K(U) = {R_0}^{\frac{3}{2}} U^{\frac{1}{2}} \rho^{-2},$$

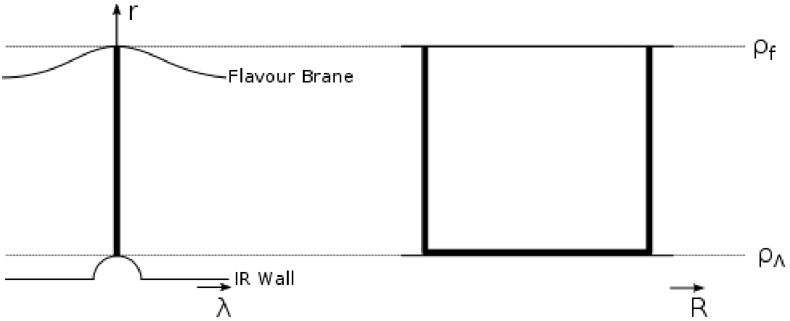
and, after Wick rotating the time coordinate,

$$\mathrm{d}X^\mu \mathrm{d}X^\nu \eta_{\mu\nu} = \left(\mathrm{d}X^0\right)^2 + \mathrm{d}R^2 + R^2 \mathrm{d}\theta^2 + \left(\mathrm{d}X^3\right)^2.$$

[7] Martin Kruczenski, Leopoldo A. Pando Zayas, Jacob Sonnenschein, and Varman Diana. Regge trajectories for mesons in the holographic dual of large-N_c QCD. *Journal of High Energy Physics*, 2005(06):046, June 2005. arXiv:hep-th/0410035.

Motivating Further Work

- Use string profile[8]:



[8]Kasper Peeters, Jacob Sonnenschein, and Marija Zamaklar. Holographic decays of large spin mesons. JHEP, 0602:009, 2005. arXiv:hep-th/0511044.

Summary

- We have shown that we can use holographic and instanton methods for the case of point particle production from an electric field.
- We hope to move forward with using these methods to evaluate the action for meson decay rate.