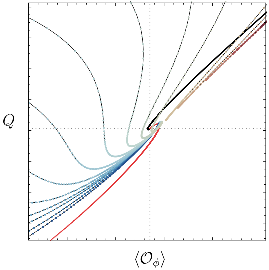
Numerical Investigation of Hairy Black Hole and Soliton Solutions in $\mathrm{AdS}_5 \times S^5$

Julija Markevičiūtė (Cambridge, DAMTP)

Cracow School of Theoretical Physics, Zakopane, 30/05/16



With Jorge E. Santos (Cambridge, DAMTP)

arXiv:1602.03893

Outline and motivation

- We numerically constructed novel AdS_5 hairy black holes.

Outline and motivation

- We numerically constructed novel AdS_5 hairy black holes.
- These solutions are relevant to the AdS/CFT correspondence.

Outline and motivation

- We numerically constructed novel AdS_5 hairy black holes.
- These solutions are relevant to the AdS/CFT correspondence.
  - The best known realization is the duality of strongly coupled, large N $\mathcal{N} = 4$ SYM and type IIB SUGRA on $AdS_5 \times S^5$ [Maldacena et al. 1997].

Outline and motivation

- We numerically constructed novel AdS_5 hairy black holes.
- These solutions are relevant to the AdS/CFT correspondence.
  - The best known realization is the duality of strongly coupled, large N $\mathcal{N} = 4$ SYM and type IIB SUGRA on $\mathrm{AdS}_5 \times \mathrm{S}^5$ [Maldacena et al. 1997].
- Black holes correspond to thermodynamic saddle points.

Outline and motivation

- We numerically constructed novel AdS_5 hairy black holes.
- These solutions are relevant to the AdS/CFT correspondence.
  - The best known realization is the duality of strongly coupled, large N $\mathcal{N} = 4$ SYM and type IIB SUGRA on $\mathrm{AdS}_5 \times \mathrm{S}^5$ [Maldacena et al. 1997].
- Black holes correspond to thermodynamic saddle points.
  - New phases of the dual gauge theory.

Outline and motivation

- We numerically constructed novel AdS_5 hairy black holes.
- These solutions are relevant to the AdS/CFT correspondence.
  - The best known realization is the duality of strongly coupled, large N $\mathcal{N}=4$ SYM and type IIB SUGRA on $\text{AdS}_5\times\text{S}^5$ [Maldacena et al. 1997].
- Black holes correspond to thermodynamic saddle points.
  - New phases of the dual gauge theory.
- We focus on d=5 $\mathcal{N}=8$ gauged supergravity which is a consistent truncation of IIB supergravity on $\text{AdS}_5\times\text{S}^5$.

Outline and motivation

- We numerically constructed novel AdS_5 hairy black holes.
- These solutions are relevant to the AdS/CFT correspondence.
  - The best known realization is the duality of strongly coupled, large N $\mathcal{N}=4$ SYM and type IIB SUGRA on $\mathrm{AdS}_5\times\mathrm{S}^5$ [Maldacena et al. 1997].
- Black holes correspond to thermodynamic saddle points.
  - New phases of the dual gauge theory.
- We focus on d=5 $\mathcal{N}=8$ gauged supergravity which is a consistent truncation of IIB supergravity on $\mathrm{AdS}_5\times\mathrm{S}^5$.
  - Not an easy task! $\longrightarrow$ Work with a further truncation.

Setup

- The consistent truncation which keeps only one scalar field was first given in [Bhattacharya et al., 2010]. The action reads:

$$S = \frac{1}{16\pi G_5}\int \mathrm{d}^5x\sqrt{g}\left\{R[g] + 12 - \frac{3}{4}F_{\mu\nu}F^{\mu\nu} \right.$$
$$\left. -\frac{3}{8}\left[(D_\mu\phi)(D^\mu\phi)^\dagger - \frac{\nabla_\mu\lambda\,\nabla^\mu\lambda}{4(4+\lambda)} - 4\lambda\right]\right\}$$

Setup

- The consistent truncation which keeps only one scalar field was first given in [Bhattacharya et al., 2010]. The action reads:

$$S = \frac{1}{16\pi G_5} \int \mathrm{d}^5 x \sqrt{g} \left\{ R[g] + 12 - \frac{3}{4} F_{\mu\nu} F^{\mu\nu} \right. \\ \left. - \frac{3}{8} \left[(D_\mu \phi)(D^\mu \phi)^\dagger - \frac{\nabla_\mu \lambda \, \nabla^\mu \lambda}{4(4+\lambda)} - 4\lambda \right] \right\}$$

$$F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]},\ D_\mu \phi = \nabla_\mu \phi - i\, e A_\mu \phi,\ e = 2,\ m_\phi^2 = -4,\ \lambda = \phi\phi^\dagger.$$

Setup

- The consistent truncation which keeps only one scalar field was first given in [Bhattacharya et al., 2010]. The action reads:

$$S = \frac{1}{16\pi G_5}\int \mathrm{d}^5x\sqrt{g}\left\{R[g] + 12 - \frac{3}{4}F_{\mu\nu}F^{\mu\nu} - \frac{3}{8}\left[(D_\mu\phi)(D^\mu\phi)^\dagger - \frac{\nabla_\mu\lambda\,\nabla^\mu\lambda}{4(4+\lambda)} - 4\lambda\right]\right\}$$

$$F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]},\ D_\mu\phi = \nabla_\mu\phi - i\,eA_\mu\phi,\ e = 2,\ m_\phi^2 = -4,\ \lambda = \phi\phi^\dagger.$$

- If the e is sufficiently large, black holes in suffer from superradiant instability.

Setup

- The consistent truncation which keeps only one scalar field was first given in [Bhattacharya et al., 2010]. The action reads:

$$S = \frac{1}{16\pi G_5}\int \mathrm{d}^5x\sqrt{g}\left\{R[g] + 12 - \frac{3}{4}F_{\mu\nu}F^{\mu\nu}\right.$$
$$\left. -\frac{3}{8}\left[(D_\mu\phi)(D^\mu\phi)^\dagger - \frac{\nabla_\mu\lambda\,\nabla^\mu\lambda}{4(4+\lambda)} - 4\lambda\right]\right\}$$

$$F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]},\ D_\mu\phi = \nabla_\mu\phi - i\,eA_\mu\phi,\ e = 2,\ m_\phi^2 = -4,\ \lambda = \phi\phi^\dagger.$$

- If the e is sufficiently large, black holes in suffer from superradiant instability.
  - This turns out to be a marginal case.

Setup cont.

- We look for static, spherically symmetric and asymptotically AdS_5 solutions. Our metric *ansatz*:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + g(r)\mathrm{d}r^2 + \Sigma(r)^2\mathrm{d}\Omega_3^2,$$
$$A_\mu \mathrm{d}x^\mu = A(r)\mathrm{d}t, \quad \phi = \phi^* = \phi(r).$$

Setup cont.

- We look for static, spherically symmetric and asymptotically AdS_5 solutions. Our metric *ansatz*:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + g(r)\mathrm{d}r^2 + \Sigma(r)^2\mathrm{d}\Omega_3^2,$$
$$A_\mu \mathrm{d}x^\mu = A(r)\mathrm{d}t, \quad \phi = \phi^* = \phi(r).$$

- A well known solution is the Reissner-Nordström AdS_5 black hole

Setup cont.

- We look for static, spherically symmetric and asymptotically AdS_5 solutions. Our metric *ansatz*:

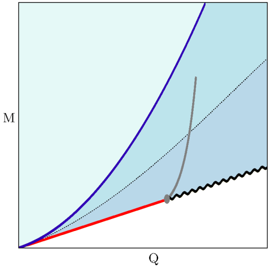
$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + g(r)\mathrm{d}r^2 + \Sigma(r)^2\mathrm{d}\Omega_3^2,$$
$$A_\mu \mathrm{d}x^\mu = A(r)\mathrm{d}t, \quad \phi = \phi^* = \phi(r).$$

- A well known solution is the Reissner-Nordström AdS_5 black hole (in gauge $\Sigma(r) = r$)

$$f(r) = \frac{\mu^2 R^4}{r^4} - \frac{(R^2+\mu^2+1)R^2}{r^2} + r^2 + 1,$$
$$g(r) = \frac{1}{f(r)}, \quad A(r) = \mu\left(1 - \frac{R^2}{r^2}\right), \quad \phi(r) = 0.$$

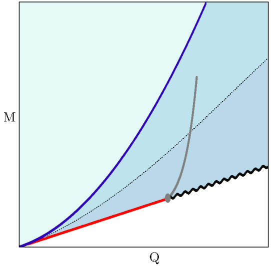
Previous results

- In Bhattacharya et al., small hairy black holes were constructed in a matched asymptotic expansion.



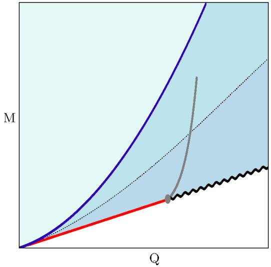
Previous results

- In Bhattacharya et al., small hairy black holes were constructed in a matched asymptotic expansion.
  - Do large black holes extend down to the BPS bound $M = 3Q$?

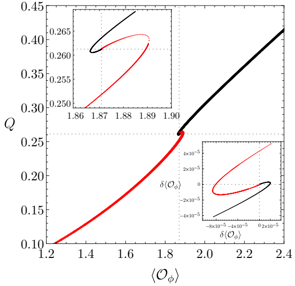


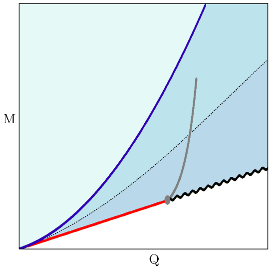
Previous results

- In Bhattacharya et al., small hairy black holes were constructed in a matched asymptotic expansion.
  - Do large black holes extend down to the BPS bound $M = 3Q$?
- For small charges, the solutions join to the smooth supersymmetric solitons.



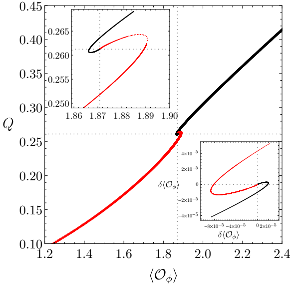
Previous results

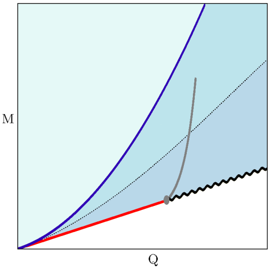




- Two branches of supersymmetric solitons connect at the special solution Q_c.

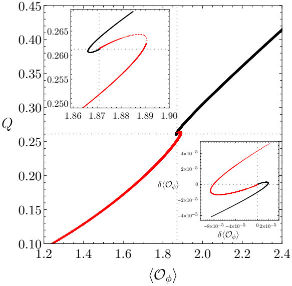
Previous results

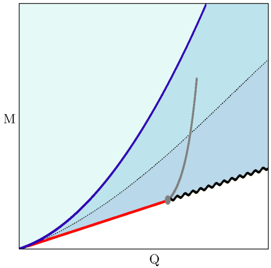




- Two branches of supersymmetric solitons connect at the special solution Q_c.
- The $Q - \langle \mathcal{O}_\phi \rangle$ space reveals an intricate spiralling behaviour (the vev of the dual operator $\phi \sim \langle \mathcal{O}_\phi \rangle / r^2$).

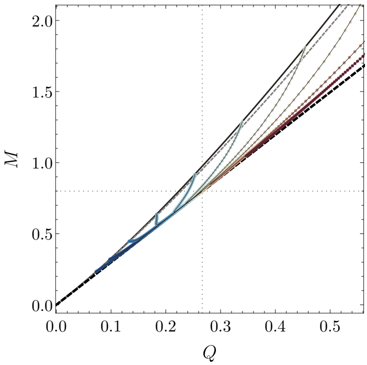
Previous results



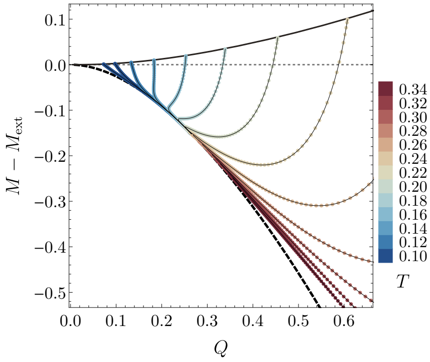


- Two branches of supersymmetric solitons connect at the special solution Q_c.
- The $Q - \langle\mathcal{O}_\phi\rangle$ space reveals an intricate spiralling behaviour (the vev of the dual operator $\phi \sim \langle\mathcal{O}_\phi\rangle/r^2$).
  - The susy solutions hint at the behavior of the whole phase space.

Results: the Phase Space

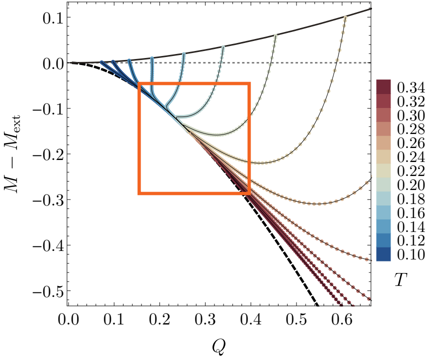


Results: the Phase Space



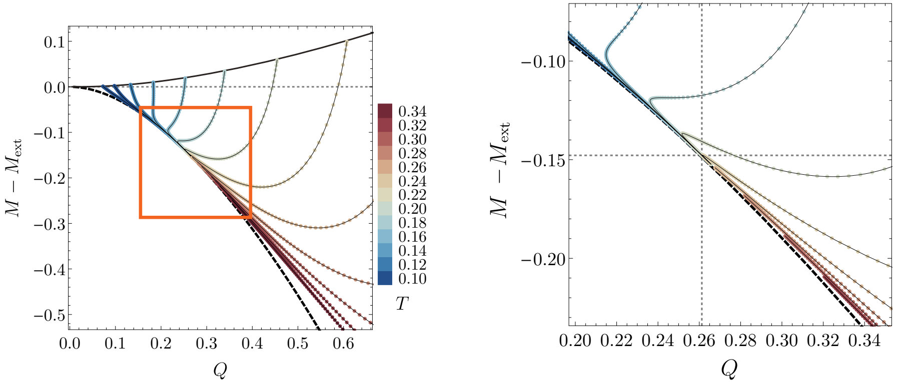
- We find that the hairy black holes exist between the curve of the onset of superradiant instability and the BPS bound for all charges.

Results: the Phase Space



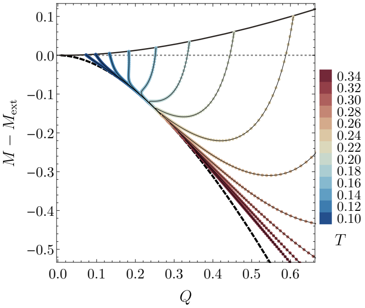
- We find that the hairy black holes exist between the curve of the onset of superradiant instability and the BPS bound for all charges.

Results: the Phase Space



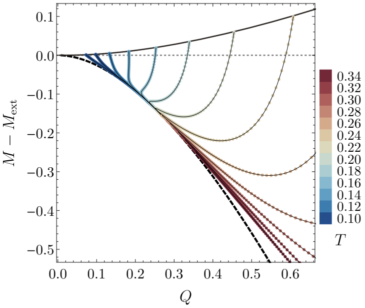
- We find that the hairy black holes exist between the curve of the onset of superradiant instability and the BPS bound for all charges.

Results: the Phase Space cont.



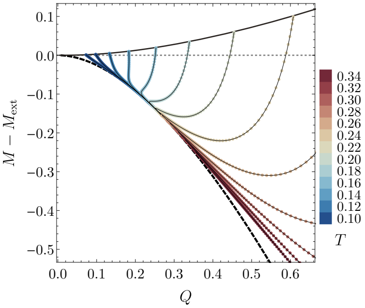
- For $T \to 0$, we recover the whole smooth soliton: the central black hole shrinks down.

Results: the Phase Space cont.



- For $T \to 0$, we recover the whole smooth soliton: the central black hole shrinks down.
- Planar black holes approach the planar singular soliton.

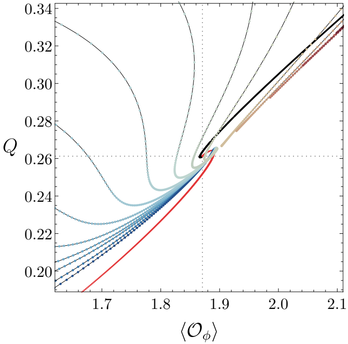
Results: the Phase Space cont.



- For $T \to 0$, we recover the whole smooth soliton: the central black hole shrinks down.
- Planar black holes approach the planar singular soliton.
- The isotherms reduce to the singular soliton for all non-zero finite temperatures.

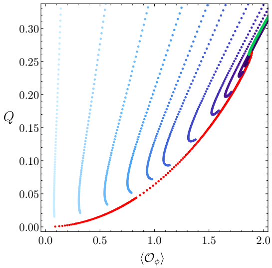
Results: The spirals

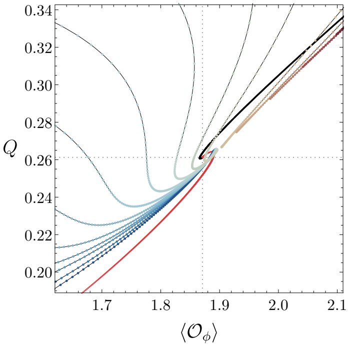
- When isotherms approach the singular branch, in the $Q - \langle \mathcal{O}_\phi \rangle$ space, we see the characteristic spirals..



Results: The spirals

- When isotherms approach the singular branch, in the $Q - \langle \mathcal{O}_\phi \rangle$ space, we see the characteristic spirals..
- .. and also when $T \to 0$. This picture gives us the whole phase space.



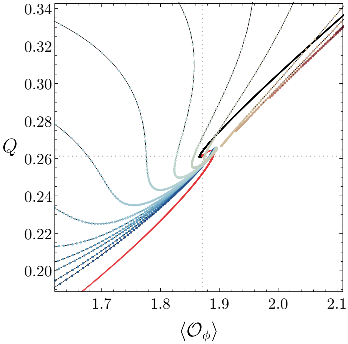


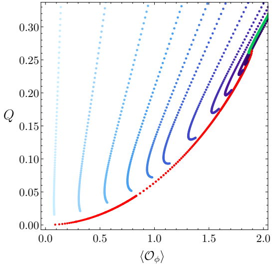
Results: The spirals

- When isotherms approach the singular branch, in the $Q - \langle \mathcal{O}_\phi \rangle$ space, we see the characteristic spirals..

- .. and also when $T \to 0$. This picture gives us the whole phase space.

- Black hole non-uniqueness!





Results: Thermodynamics

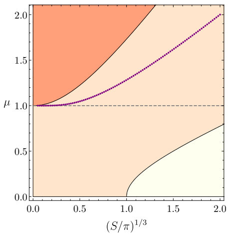
- These intricate shapes result in interesting thermodynamic properties.
- We analysed three ensembles - micro canonical, canonical and grand-canonical.

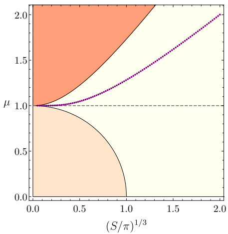
Results: Thermodynamics

- These intricate shapes result in interesting thermodynamic properties.
- We analysed three ensembles - micro canonical, canonical and grand-canonical.
- Competition between the pure AdS, hairy black holes and RNAdS.
  - We find that the hairy solutions maximise entropy.

Results: Thermodynamics

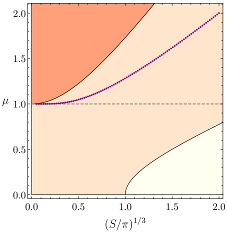
- These intricate shapes result in interesting thermodynamic properties.
- We analysed three ensembles - micro canonical, canonical and grand-canonical.
- Competition between the pure AdS, hairy black holes and RNAdS.
  - We find that the hairy solutions maximise entropy.

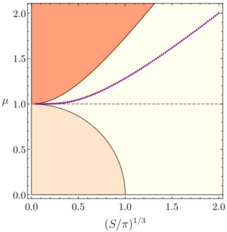




Results: Thermodynamics

- These intricate shapes result in interesting thermodynamic properties.
- We analysed three ensembles - micro canonical, canonical and grand-canonical.
- Competition between the pure AdS, hairy black holes and RNAdS.
  - We find that the hairy solutions maximise entropy.
  - Hairy black holes never dominate the other two ensembles.





Summary and outlook

- With the advance of numerical methods we can analyse higher dimensional systems at full non-linearity.

Summary and outlook

- With the advance of numerical methods we can analyse higher dimensional systems at full non-linearity.
- Our novel HBH solutions extend all the way to the BPS limit and therefore are interesting from the field theory perspective.

Summary and outlook

- With the advance of numerical methods we can analyse higher dimensional systems at full non-linearity.
- Our novel HBH solutions extend all the way to the BPS limit and therefore are interesting from the field theory perspective.
- This setup can be used to analyse other consistent truncations, or other theories, e.g. on $\mathrm{AdS}_4\times\mathrm{S}^7$.

Summary and outlook

- With the advance of numerical methods we can analyse higher dimensional systems at full non-linearity.
- Our novel HBH solutions extend all the way to the BPS limit and therefore are interesting from the field theory perspective.
- This setup can be used to analyse other consistent truncations, or other theories, e.g. on $AdS_4 \times S^7$.
- We extended this work by adding rotation to the system starting from equally-rotating *Myers-Perry* AdS_5 *ansatz*.

Summary and outlook

- With the advance of numerical methods we can analyse higher dimensional systems at full non-linearity.
- Our novel HBH solutions extend all the way to the BPS limit and therefore are interesting from the field theory perspective.
- This setup can be used to analyse other consistent truncations, or other theories, e.g. on $AdS_4 \times S^7$.
- We extended this work by adding rotation to the system starting from equally-rotating *Myers-Perry* AdS_5 *ansatz*.
  - For this system it is known that there exist a one parameter family of supersymmetric black holes, the *Gutowski-Reall* black holes.

Summary and outlook

- With the advance of numerical methods we can analyse higher dimensional systems at full non-linearity.
- Our novel HBH solutions extend all the way to the BPS limit and therefore are interesting from the field theory perspective.
- This setup can be used to analyse other consistent truncations, or other theories, e.g. on $AdS_4 \times S^7$.
- We extended this work by adding rotation to the system starting from equally-rotating *Myers-Perry* AdS_5 *ansatz*.
  - For this system it is known that there exist a one parameter family of supersymmetric black holes, the *Gutowski-Reall* black holes.

Thank you!