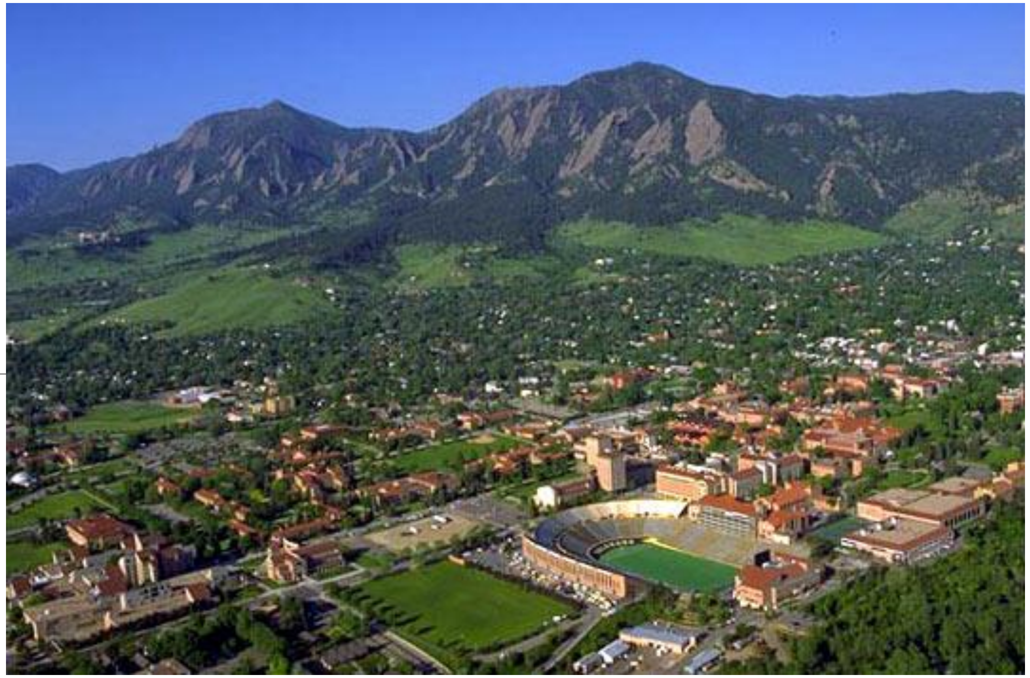
Holography from the Top Down

FERMIONS IN ABJM THEORY

BY OSCAR HENRIKSSON

University of Colorado
Boulder



Why AdS/CFT? Why top-down?

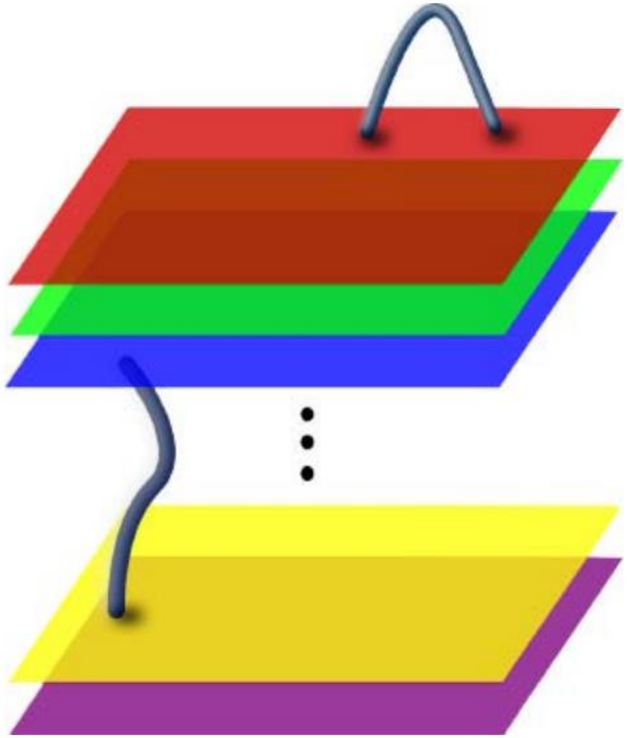
Strong interactions are hard!

…but important in particle physics, condensed matter physics, etc.

Top-down dualities allow us to make concrete statements about specific QFTs

Field theories in question are very special ($\mathcal{N} = 4$ SYM, ABJM, …)

Hope to find general lessons on what aspects of QFTs give interesting behaviors



Outline

ABJM theory and $\mathcal{N} = 8$ gauged supergravity

1st geometry: Regular black holes

2nd geometry: The $SO(3) \times SO(3)$ domain wall

Summary

Two special theories

…THAT ARE RELATED THROUGH ADS/CFT

ABJM theory

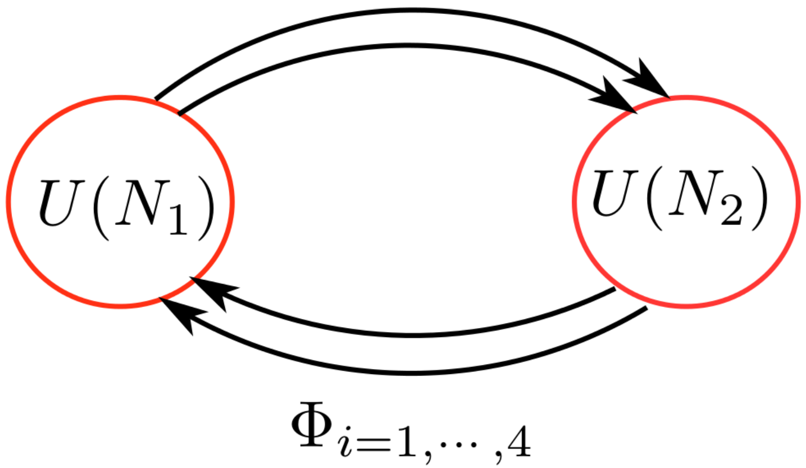
A superconformal 3D Chern-Simons-matter QFT

Describes the world-volume of coincident M2-branes

Two vector multiplets with gauge group $U(N) \times U(N)$

Matter supermultiplets in bifundamental representation

Operators of interest (simplified):
$Tr\, X^2$, $Tr\, X\, \lambda$ and $Tr\, \lambda^2$



4D $\mathcal{N} = 8$ gauged supergravity

Large field content:

SUGRA mode	$g_{\mu\nu}$	ψ^i_μ	A^{IJ}_μ	χ_{ijk}	Re ϕ_{ijkl}	Im ϕ_{ijkl}
Dual operator	$T^{\mu\nu}$	$\mathcal{S}^{\mu i}$	$J^{\mu IJ}_R$	Tr $X\lambda$	Tr X^2	Tr λ^2
Conformal dimension	3	5/2	2	3/2	1	2
$SO(8)$ rep	$\mathbf{1}$	$\mathbf{8}_s$	$\mathbf{28}$	$\mathbf{56}_s$	$\mathbf{35}_v$	$\mathbf{35}_c$

$SO(8)$ gauge symmetry

70 scalars parametrize coset space $\frac{E_7}{SU(8)}$

Complicated scalar potential with many critical points – full structure unknown

Fermions in ABJM theory

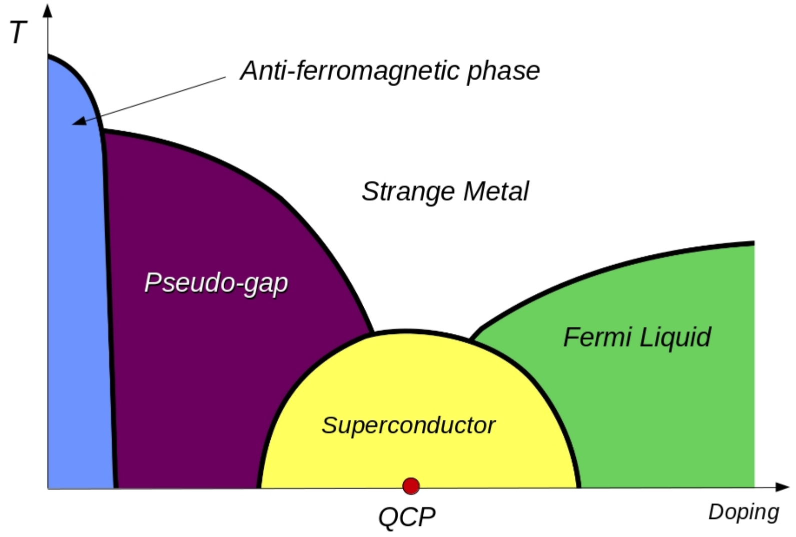
Fermi vs. **non**-Fermi liquids

Fermi liquid theory describes much of observed metallic behavior

But in certain cases, metals do not seem to have (asymptotically) stable quasiparticles
→ “strange metals”

Question:

- Does fermions in ABJM theory at finite density display Fermi or non-Fermi liquid behavior?



The game we play

1) Study truncations of complete $SO(8)$ supergravity Lagrangian
2) Find a solution to classical (bosonic) equations of motion – these describe states of ABJM theory
3) Solve linearized Dirac equations in gravitational background
4) Read of source and response from asymptotic spinor behavior; ratio is the Green's function!
5) Green's functions give information about spectrum: Fermi surfaces, dispersion relations, properties of excitations

1st geometry

REGULAR BLACK HOLES

Regular black holes

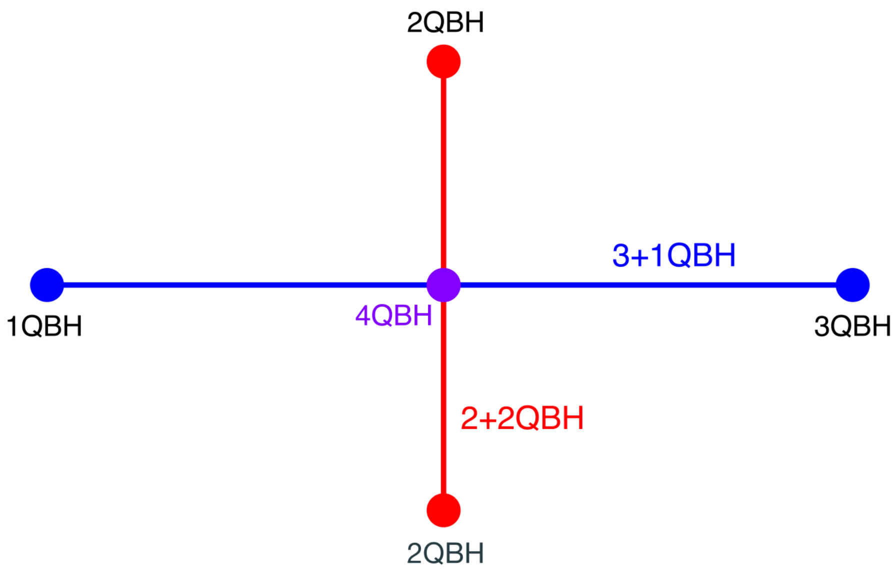
A $U(1)^4$ truncation of the full $SO(8)$ SUGRA:

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\vec{\phi})^2 + \frac{2}{L}(\cosh\phi_1 + \cosh\phi_2 + \cosh\phi_3) - \frac{1}{4}\sum e^{-\lambda_i} F_i^2$$

Admits "generalized Reisner-Nordström" black hole solutions

Dual state has four independent chemical potentials

Study zero-T, finite density states by taking extremal limit of the black holes



Regular black holes

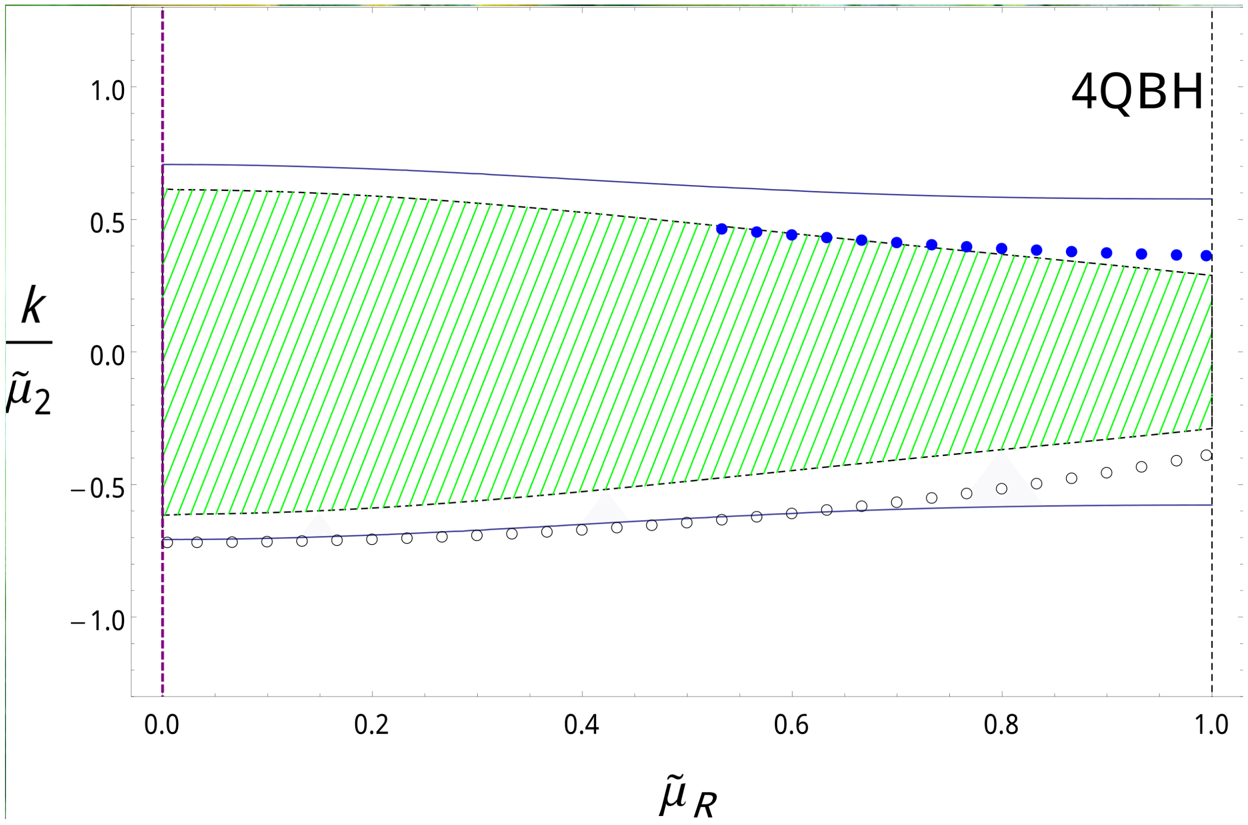
Quadratic fermion Lagrangian (excluding gravitini):

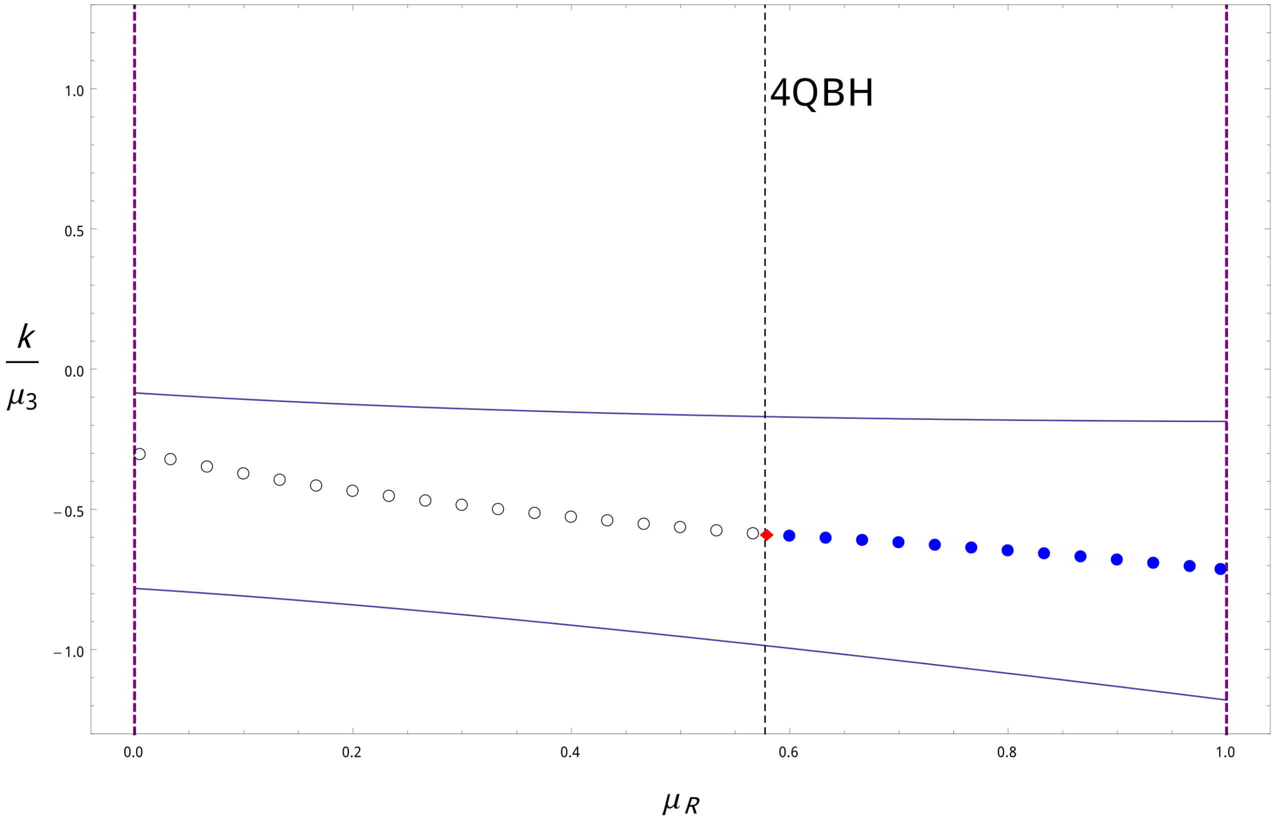
$$
\begin{aligned}
e^{-1}\mathcal{L} =& \frac{i}{6}\bar{\chi}^{ijk}\gamma^{\mu}\nabla_{\mu}\chi_{ijk} + \frac{i}{8L}\bar{\chi}^{ijk}\gamma^{\mu}A_{\mu}{}^{l}{}_{i}\chi_{ljk} + \frac{1}{72L}\epsilon^{ijklmnpq}A^{2}{}_{rlmn}\bar{\chi}_{ijk}\chi_{pq}{}^{r} \\
& - \frac{1}{576}F_{\mu\nu ij}S^{ijkl}(u^{-1})_{klmn}\epsilon^{mnpqrstu}\bar{\chi}_{pqr}\sigma^{\mu\nu}\chi_{stu}\,.
\end{aligned}
$$

Think of χ_{ijk} as a 56-component vector →

$$
e^{-1}\mathcal{L} = \frac{1}{2}\bar{\vec{\chi}}(i\gamma^{\mu}\nabla_{\mu}\mathbf{1} + \mathbf{Q} + \mathbf{M} + \mathbf{P})\vec{\chi}
$$

Diagonalize and solve resulting uncoupled Dirac equations!





Regular black holes: Results

We find Fermi surfaces with exclusively **non-Fermi liquid behavior**!

Superconductors & gapped fermions

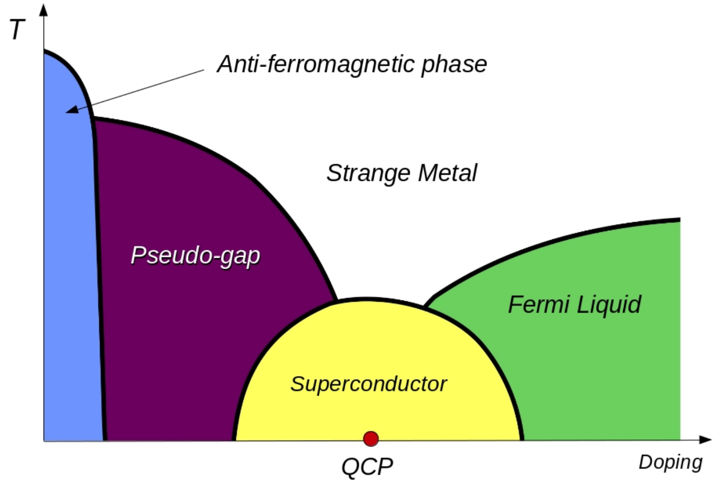
Superconductivity is a generic low-temperature phase

A $U(1)$ Higgs mechanism makes the photon effectively massive and gaps fermions.

Can study superconductivity through holography by turning on a charged scalar in the gravity theory

Questions:

- Does ABJM theory at finite density have a superconducting phase?
- Will the fermions display a gap?



2^{nd} geometry

THE $SO(3) \times SO(3)$ DOMAIN WALL

The $SO(3) \times SO(3)$ truncation

Truncate full supergravity to an $SO(3) \times SO(3)$ invariant sector:

$$e^{-1}\mathcal{L} = \frac{1}{2}R - (\partial\lambda)^2 - \frac{\sinh^2(2\lambda)}{4}(\partial\alpha - gA)^2 + \mathcal{P} - \frac{1}{4}F^2$$

Scalar potential $\mathcal{P}$ has two critical points:

- $\lambda = 0$ with $SO(8)$ symmetry and SUSY
- $\lambda \neq 0$ with $SO(3) \times SO(3)$ symmetry and **no** SUSY

Critical points are *perturbatively stable*

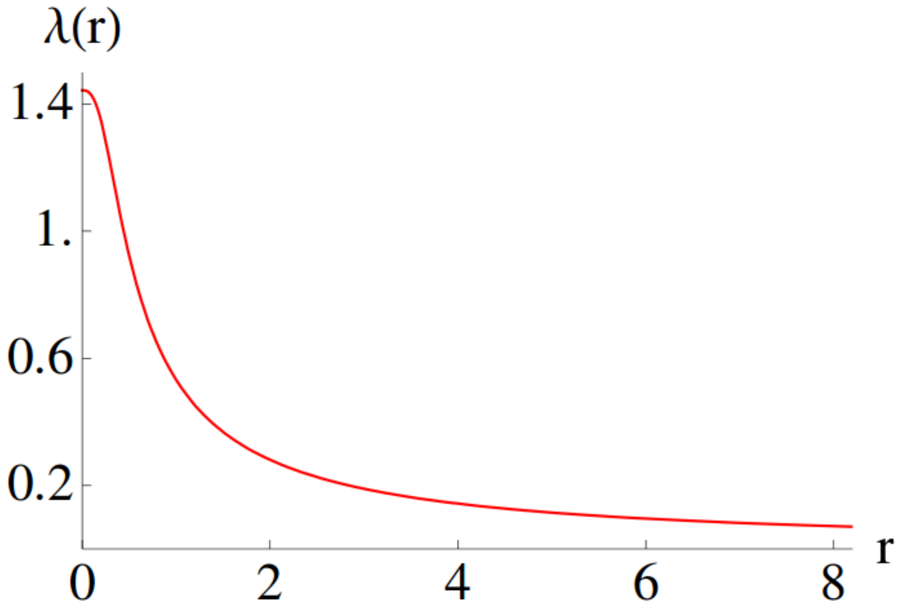
The $SO(3) \times SO(3)$ domain wall

Bobev et al. constructed domain wall between the two critical points

A zero temperature, zero entropy geometry

Boundary $U(1)$ is **explicitly** broken
→ a deformation of ABJM theory
→ not quite a true superconductor

Background leads to **coupled** Dirac equations

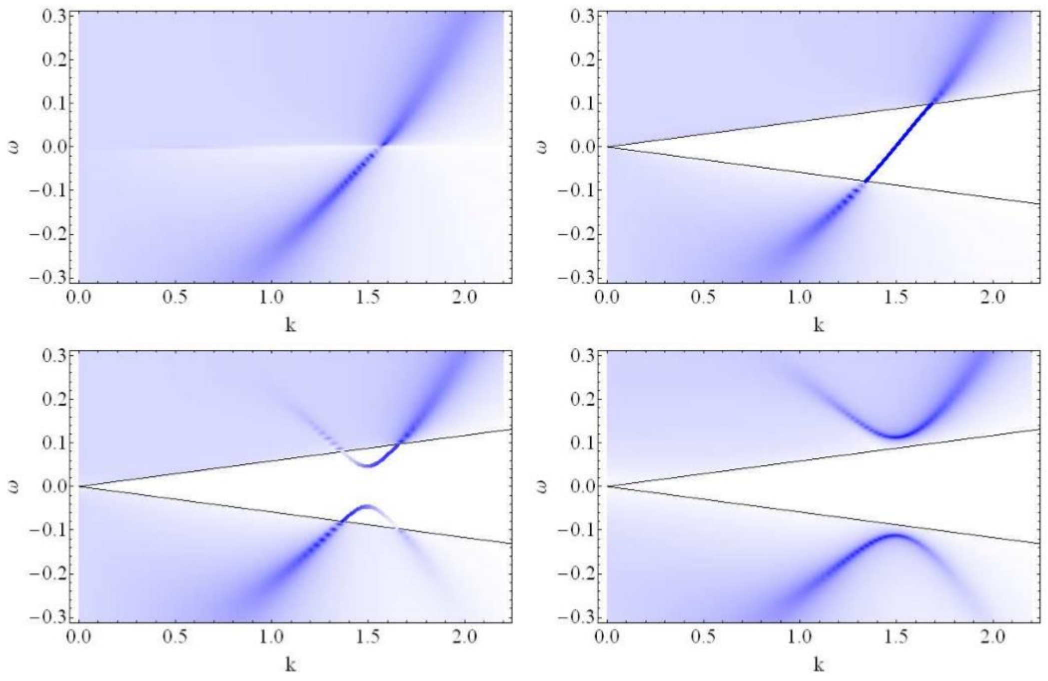


$$\mathcal{G} = \begin{pmatrix} \mathcal{G}_{\mathcal{O}_1\mathcal{O}_1} & \mathcal{G}_{\mathcal{O}_1\mathcal{O}_2} & \mathcal{G}_{\mathcal{O}_1\mathcal{O}_3} & \mathcal{G}_{\mathcal{O}_1\mathcal{O}_4} \\ \cdots & & & \\ \cdots & & & \\ \cdots & & & \mathcal{G}_{\mathcal{O}_4\mathcal{O}_4} \end{pmatrix}$$

Comparison: Faulkner et al. (0911.3402)

A bottom-up study of fermions in a superconducting geometry

Studied special scalar-fermion coupling involving Γ_5 that **guarantees** a gap



The $SO(3) \times SO(3)$ domain wall: Results

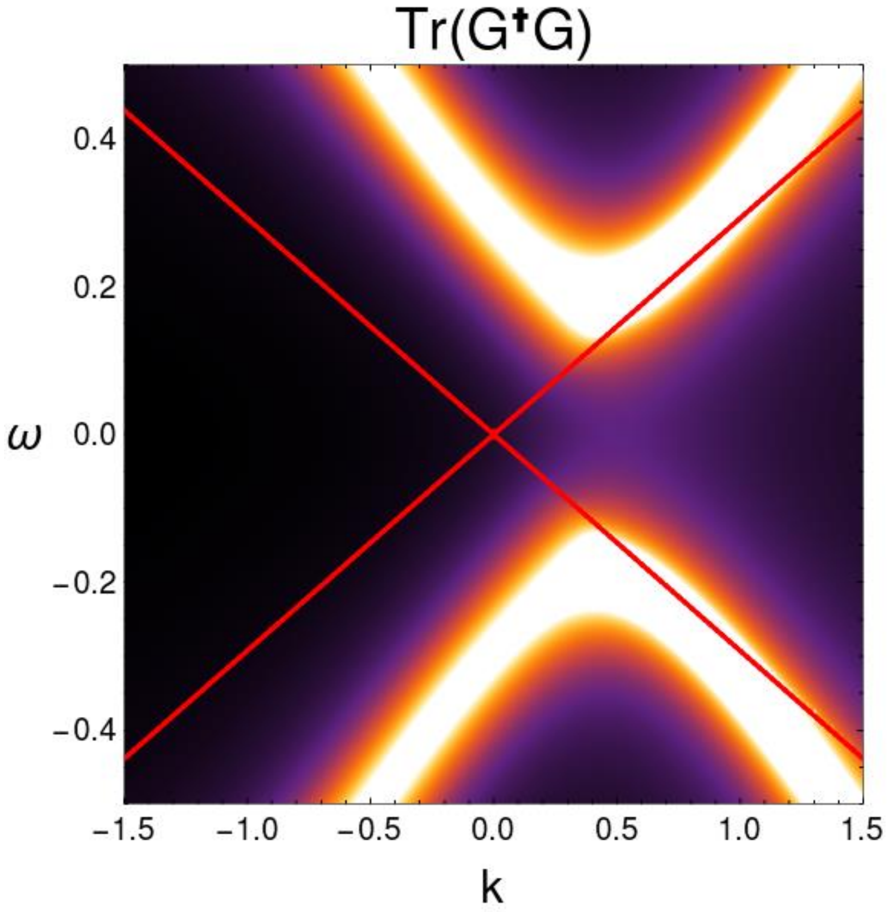
We find similar Γ_5 coupling

Fermions are gapped!

In stable region, Green's functions display poles

For timelike 4-momenta, unstable region; finite non-zero spectral weight and no stable modes

→ sector that mediates fermion decay?



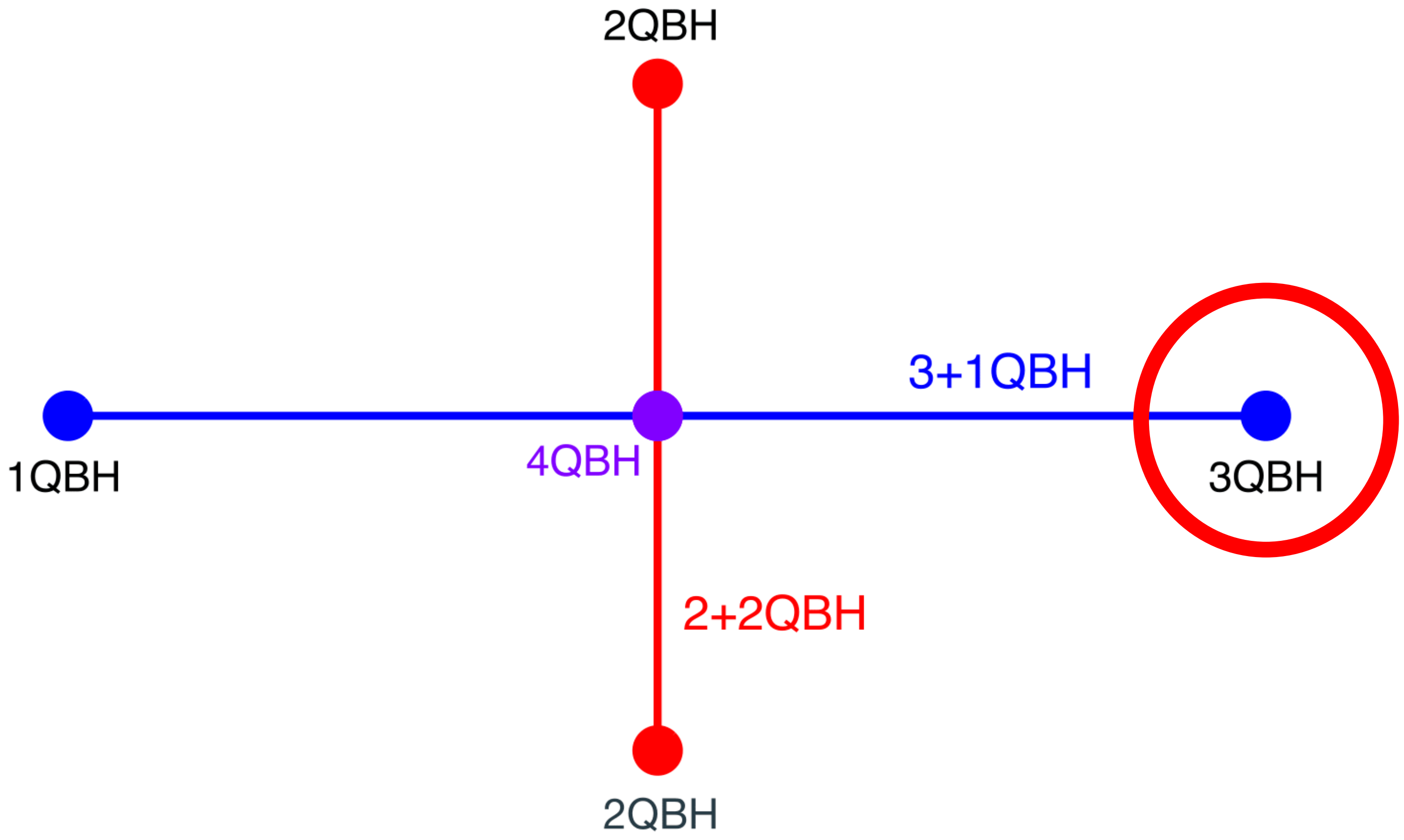
Summary

- Studied fermions at finite density in ABJM/SUGRA duality
- Found Fermi surfaces with exclusively **non-Fermi liquid** excitations in extremal black hole geometries
- The $SO(3) \times SO(3)$ domain wall is a good candidate ground state for *deformed* ABJM theory
- In domain wall, fermions become **gapped** through Γ_5-couplings

Thank you for listening!

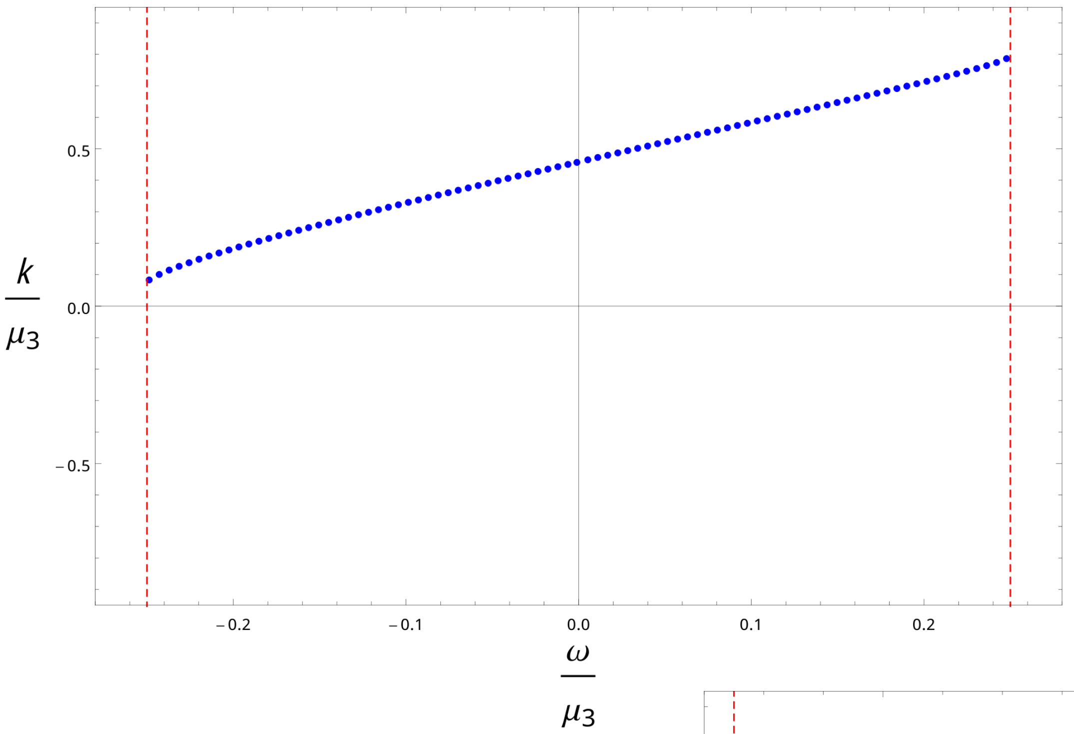
Extra: A special black hole!

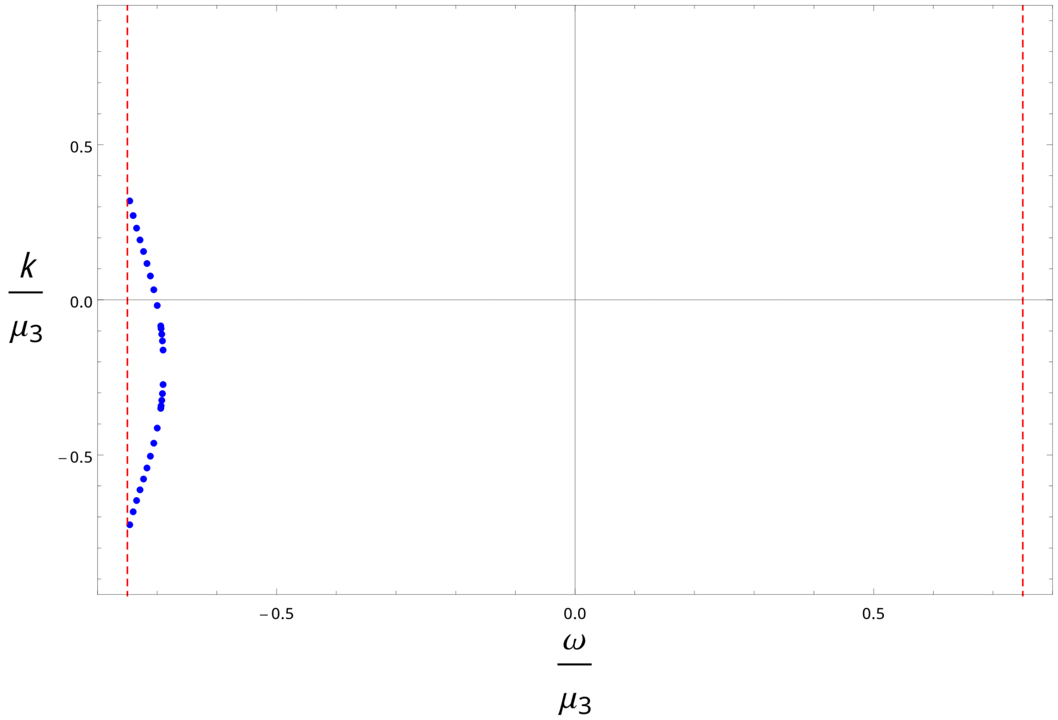
- The "three-charge" black hole is a special limit of the regular black holes
- In this geometry, T → 0 gives S → 0



Extra: A special black hole!

- Has “stable region”: interval of frequencies where fermionic excitations are stable
- Both gapped and ungapped fermions





Extra: Group theory and fermion mixing

Under $SO(8) \to SO(3) \times SO(3)$, the fermions decompose as

$56 \to 2(1,1) \oplus 3(3,1) \oplus 3(1,3) \oplus 4(3,3)$

Fermions in the four $(3,3)$ cannot mix with the gravitini

…but they can and do mix with each other!

→ **Coupled** Dirac equations

Must consider **matrix** of Green's function:

$$\mathcal{G} = \begin{pmatrix} \mathcal{G}_{\mathcal{O}_1\mathcal{O}_1} & \mathcal{G}_{\mathcal{O}_1\mathcal{O}_2} & \mathcal{G}_{\mathcal{O}_1\mathcal{O}_3} & \mathcal{G}_{\mathcal{O}_1\mathcal{O}_4} \\ \cdots & & & \\ \cdots & & & \\ \cdots & & & \mathcal{G}_{\mathcal{O}_4\mathcal{O}_4} \end{pmatrix}$$

Extra: 4D $\mathcal{N} = 8$ SUGRA Lagrangian

$$
\begin{aligned}
\mathscr{L} = & -\tfrac{1}{2}eR(e,\omega) - \tfrac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(\bar\psi^i_\mu\gamma_\nu D_\rho\psi_{\sigma i} - \bar\psi^i_\mu\overleftarrow{D}_\rho\gamma_\nu\psi_{\sigma i}) \\
& -\tfrac{1}{12}e(\bar\chi^{ijk}\gamma^\mu D_\mu\chi_{ijk} - \bar\chi^{ijk}\overleftarrow{D}_\mu\gamma^\mu\chi_{ijk}) - \tfrac{1}{96}e\mathcal{A}_\mu{}^{ijkl}\mathcal{A}^\mu{}_{ijkl} \\
& -\tfrac{1}{8}e[F^+_{\mu\nu IJ}(2S^{IJ,KL} - \delta^{IJ}_{KL})F^{+\mu\nu}{}_{KL} + \text{h.c.}] \\
& -\tfrac{1}{2}e[F^+_{\mu\nu IJ}S^{IJ,KL}O^{+\mu\nu KL} + \text{h.c.}] \\
& -\tfrac{1}{4}e[O^+_{\mu\nu}{}^{IJ}(S^{IJ,KL} + u^{ij}{}_{IJ}v_{ijKL})O^{+\mu\nu KL} + \text{h.c.}] \\
& -\tfrac{1}{24}e[\bar\chi_{ijk}\gamma^\nu\gamma^\mu\psi_{\nu l}(\hat{\mathcal{A}}_\mu{}^{ijkl} + \mathcal{A}_\mu{}^{ijkl}) + \text{h.c.}] \\
& -\tfrac{1}{2}e\bar\psi^{[i}_\mu\psi^{j]}_\nu\bar\psi^\mu_i\psi^\nu_j \\
& +\tfrac{1}{4}\sqrt{2}e[\bar\psi^i_\lambda\sigma^{\mu\nu}\gamma^\lambda\chi_{ijk}\bar\psi^j_\mu\psi^k_\nu + \text{h.c.}] \\
& +e[\tfrac{1}{144}\eta\varepsilon_{ijklmnpq}\bar\chi^{ijk}\sigma^{\mu\nu}\chi^{lmn}\bar\psi^p_\mu\psi^q_\nu \\
& +\tfrac{1}{8}\bar\psi^i_\lambda\sigma^{\mu\nu}\gamma^\lambda\chi_{ikl}\bar\psi_{\mu j}\gamma_\nu\chi^{jkl} + \text{h.c.}] \\
& +\tfrac{1}{864}\sqrt{2}\eta e[\varepsilon^{ijklmnpq}\bar\chi_{ijk}\sigma^{\mu\nu}\chi_{lmn}\bar\psi^r_\mu\gamma_\nu\chi_{pqr} + \text{h.c.}] \\
& +\tfrac{1}{32}e\bar\chi^{ikl}\gamma^\mu\chi_{jkl}\bar\chi^{jmn}\gamma_\mu\chi_{imn} - \tfrac{1}{96}e(\bar\chi^{ijk}\gamma^\mu\chi_{ijk})^2 .
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}_g + \mathscr{L}_{g^2} = & \{\sqrt{2}geA_{1ij}\bar\psi^i_\mu\sigma^{\mu\nu}\psi^j_\nu + \tfrac{1}{6}geA_{2i}{}^{jkl}\bar\psi^i_\mu\gamma^\mu\chi_{jkl} \\
& +\tfrac{1}{144}\sqrt{2}g\eta e\varepsilon^{ijkpqrlm}A^n_{2pqr}\bar\chi_{ijk}\chi_{lmn} + \text{h.c.}\} \\
& +g^2e\{\tfrac{3}{4}|A^{ij}_1|^2 - \tfrac{1}{24}|A^i_{2jkl}|^2\} .
\end{aligned}
$$