Holography, thermalization and heavy-ion collisions III

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1302.0697 [PRL 110 211602 (2013)] with Janik & Witaszczyk

1503.07514 [PRL 115 072501 (2015)] with Spaliński

Overview of Lectures 1 & 2

1202.0981 [PRL 108 191601 (2012)] with Mateos, van der Schee & Tancanelli
1304.5172 [JHEP 1309 026 (2013)] with Mateos, van der Schee & Triana
1305.4919 [PRL 111 181601 (2013)] and **1312.2956** [PRL 112 221602 (2014)]
with Casalderrey-Solana, Mateos & van der Schee

Basic notions

certain states of a class of strongly-coupled QFTs = higher dimensional geometries

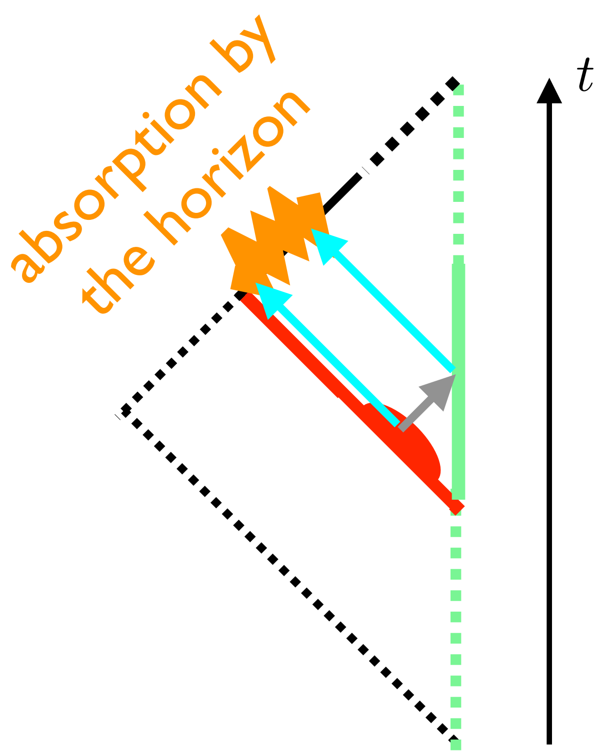
dofs of strongly-coupled QGP = QNMs of dual black branes:

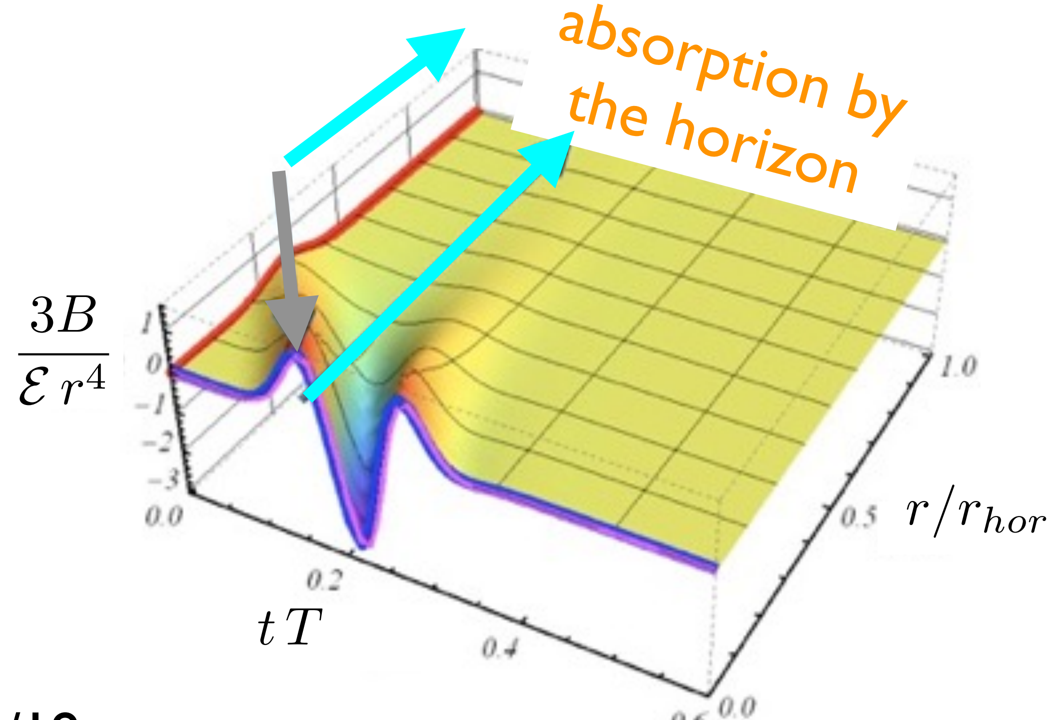
$$\delta\langle T_{\mu\nu}\rangle = \sum_n \int d^3k\, c_n^{\mu\nu} e^{-\omega_n(k)t + i\vec{k}\cdot\vec{x}}$$ with

exponential decay in 1/T

slow decay (hydro)

equilibration in strongly coupled QFTs = dual horizon formation and equilibration:





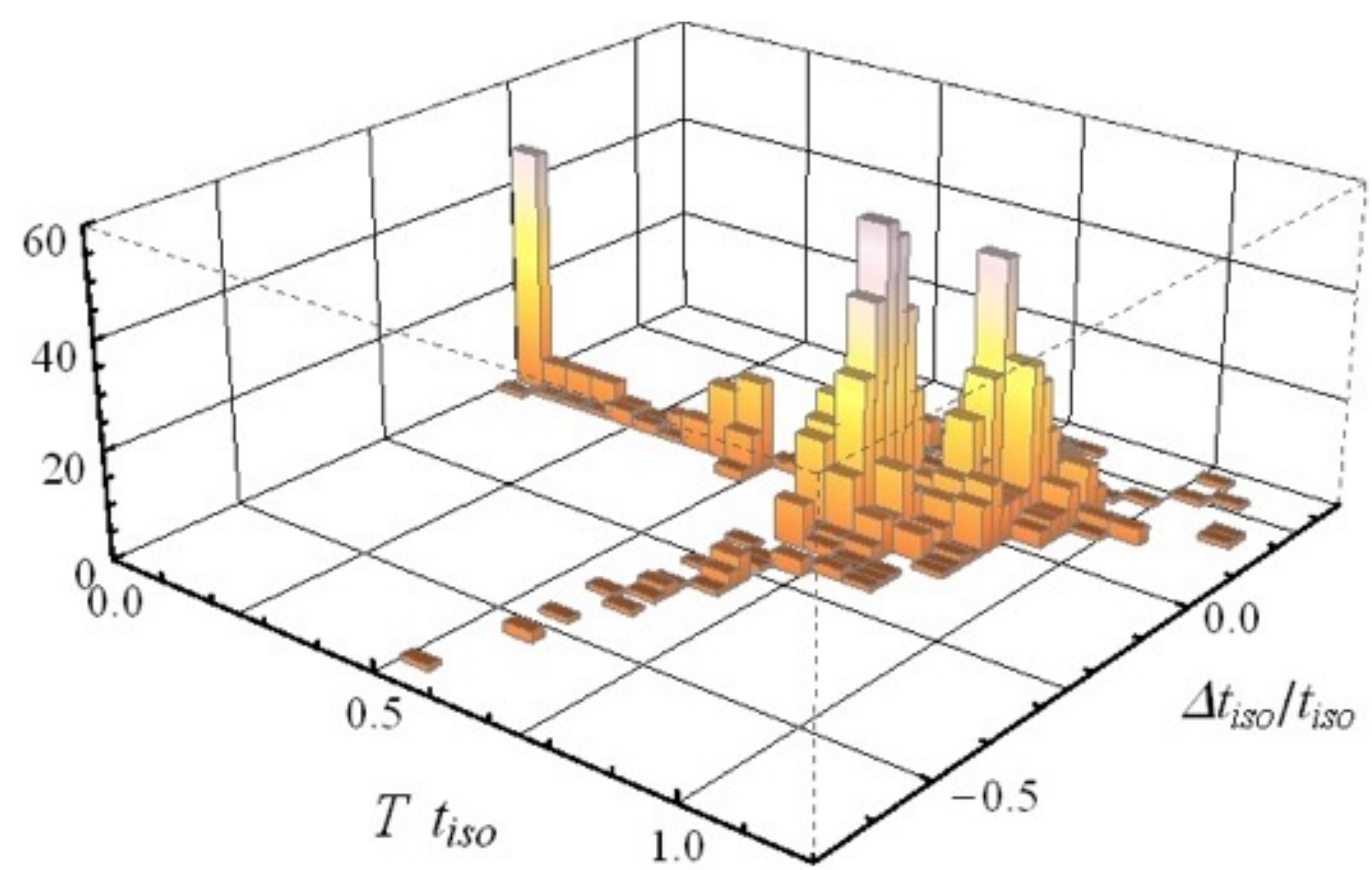
N-eq states and relaxation rates

Real-time dynamics of QFTs requires ∞-many initial conditions, e.g.

$$B = -\frac{b(t)}{2r^4} - \frac{b'(t)}{2r^5} - \frac{7b''(t)}{24r^6} - \frac{b^{(3)}(t)}{8r^7} + \ldots \text{ with } b(t) = \frac{2\pi^2}{3N_c^2}\Delta\mathcal{P}(t)$$

Holography makes it manageable by adding r $\longrightarrow$ $\mathcal{R}_{ab} - \frac{1}{2}\mathcal{R}g_{ab} - \frac{6}{L^2}g_{ab} = 0$

Indications that equilibration in 1/T at strong coupling can be generic:



Confirmed in many other setups*. Is $t_{eq}\,T = O(1)$ becoming new "$\eta/s = 1/4\pi$"?

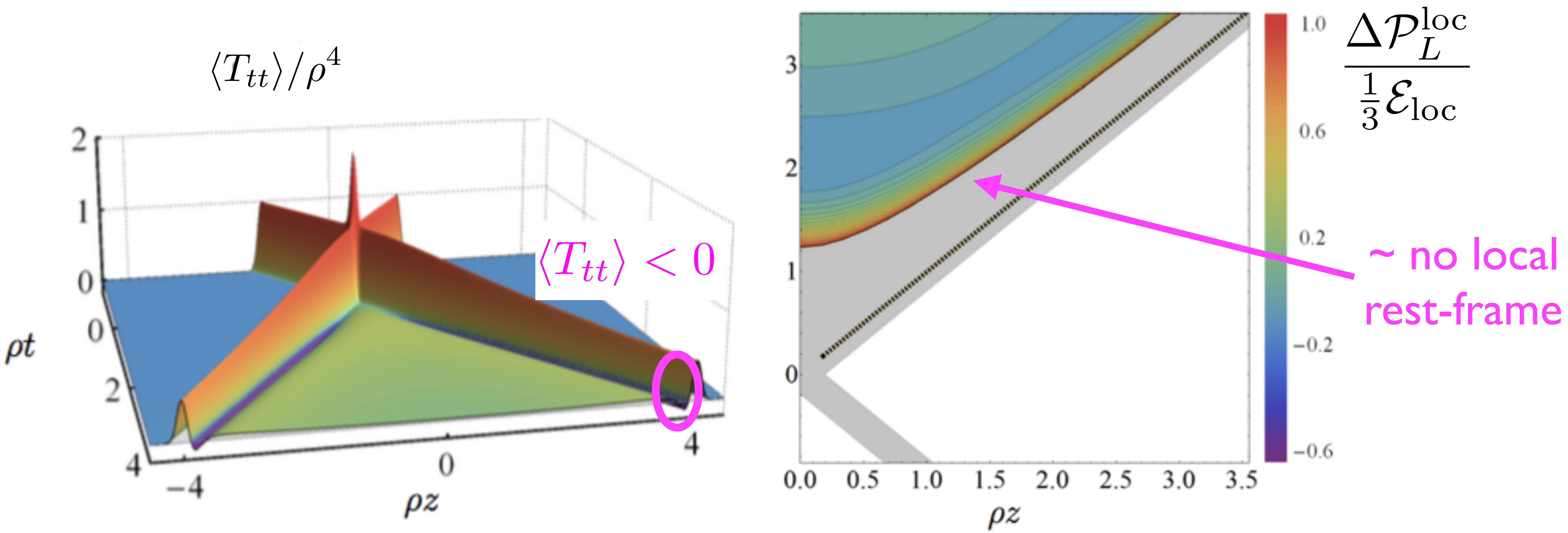
Hydrodynamics & holographic collisions

Hydrodynamics:

$$T^{\mu\nu}_{hydro} = \underbrace{\mathcal{E}u^\mu u^\nu + P(\mathcal{E})\{g^{\mu\nu} + u^\mu u^\nu\}}_{\text{isotropic}} \underbrace{- \eta(\mathcal{E})\,\sigma^{\mu\nu}}_{\text{breaks isotropy}} - \zeta(\mathcal{E})\{g^{\mu\nu} + u^\mu u^\nu\}(\nabla\cdot u) + \ldots$$

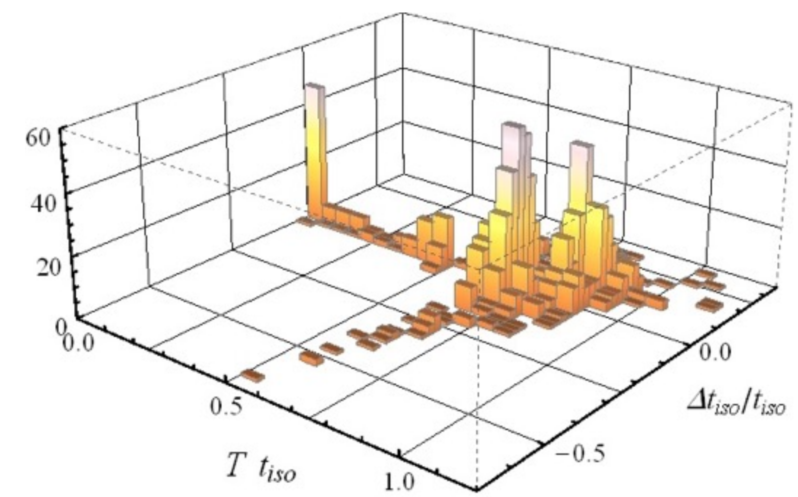
It is clear that $\langle T^{\mu\nu}\rangle$'s will not always be close to satisfying these relations

Rich transient physics before hydro:



Hydrodynamization

$t_{hyd}\,T_{hyd} = \mathcal{O}(1)$ as before:



Fast hydrodynamization:

$\neq$

thermalization or isotropization

$$\frac{1}{3}\mathcal{E}$$

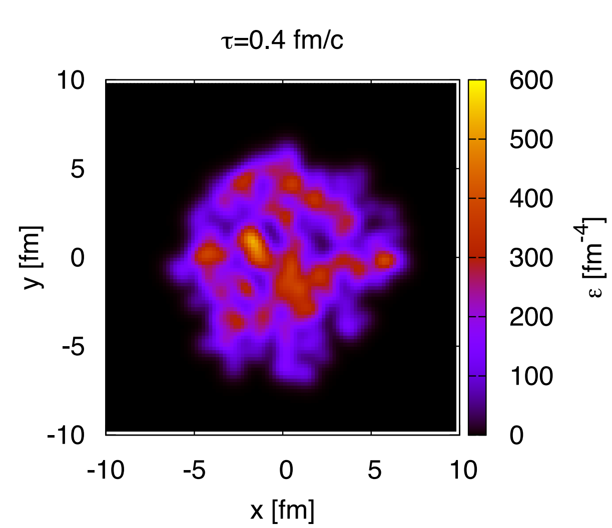
$$/\!/$$

Huge anisotropies at the hydrodynamic threshold: $\mathcal{P}_T - \mathcal{P}_L = 1.35 \times \mathcal{P}_{eq}$

Viscous hydrodynamics constitutive relations work despite:

leading order $\approx$ correction

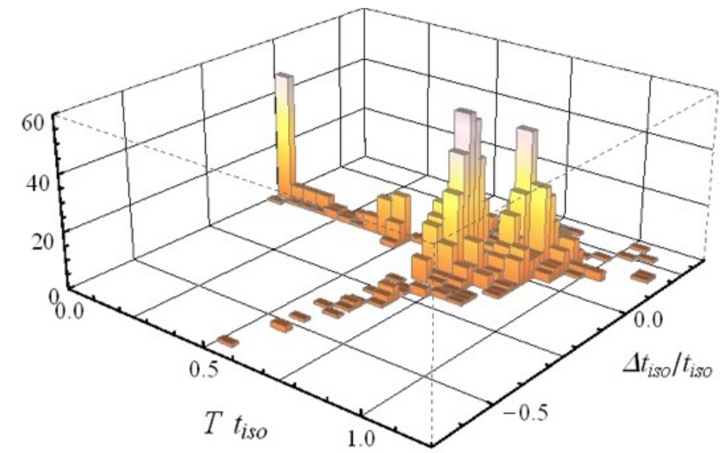
$$T^{\mu\nu}_{hydro} = \boxed{\mathcal{E} u^\mu u^\nu + P_{eq}(\mathcal{E})\left\{g^{\mu\nu} + u^\mu u^\nu\right\}} - \boxed{\eta(\mathcal{E})\,\sigma^{\mu\nu}} + \ldots$$

Great for pheno: 

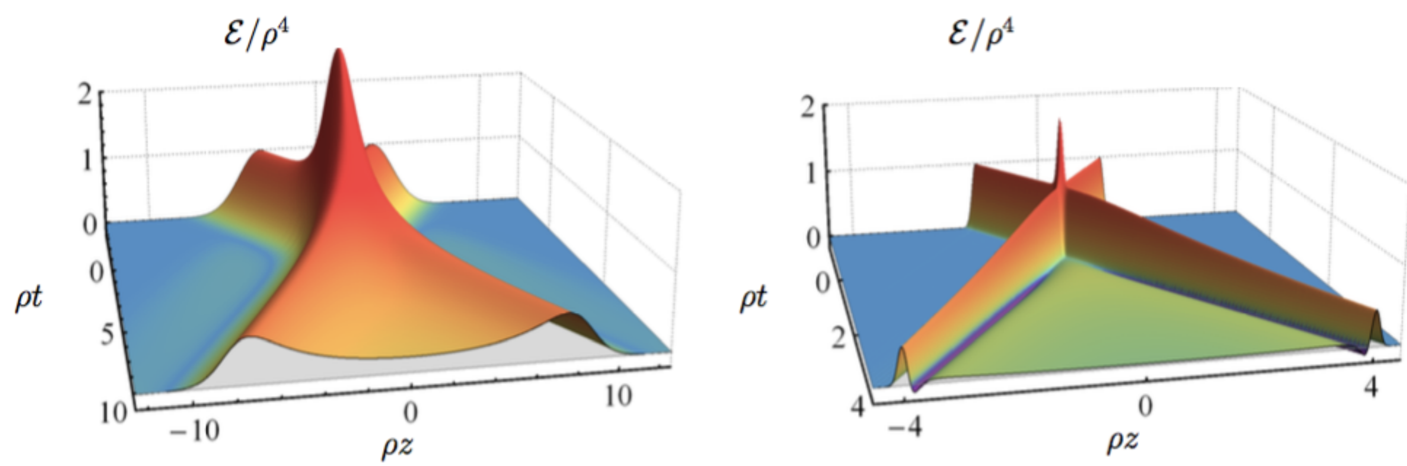
+ pA + pp

The plan for today

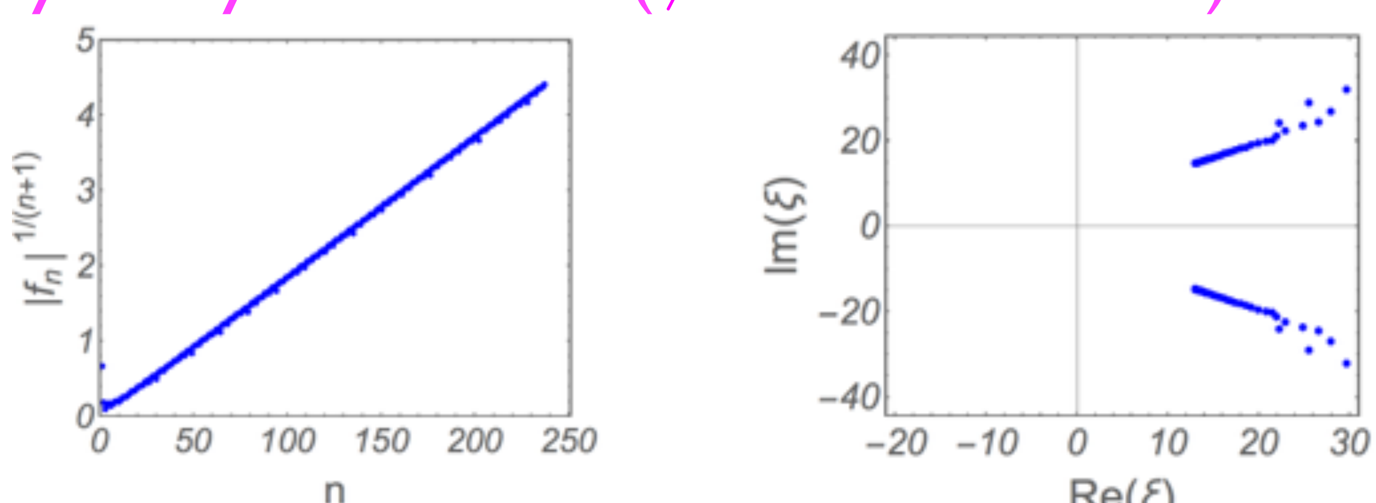
Lecture I: how long does it take $\langle T^{\mu\nu}\rangle$ to equilibrate in strongly-coupled QFTs?



Lecture II: what is $\langle T^{\mu\nu}\rangle(t,\vec{x})$ after a collision of 2 strongly-interacting objects?



Lecture III: why was hydrodynamization ($\neq$ thermalization) at all possible?



Boost-invariant hydrodynamics

1103.3452 [PRL 108 211602 (2012)] with Janik & Witaszczyk

The boost-invariance Bjorken 1982

In order to understand hydrodynamics right at its threshold:

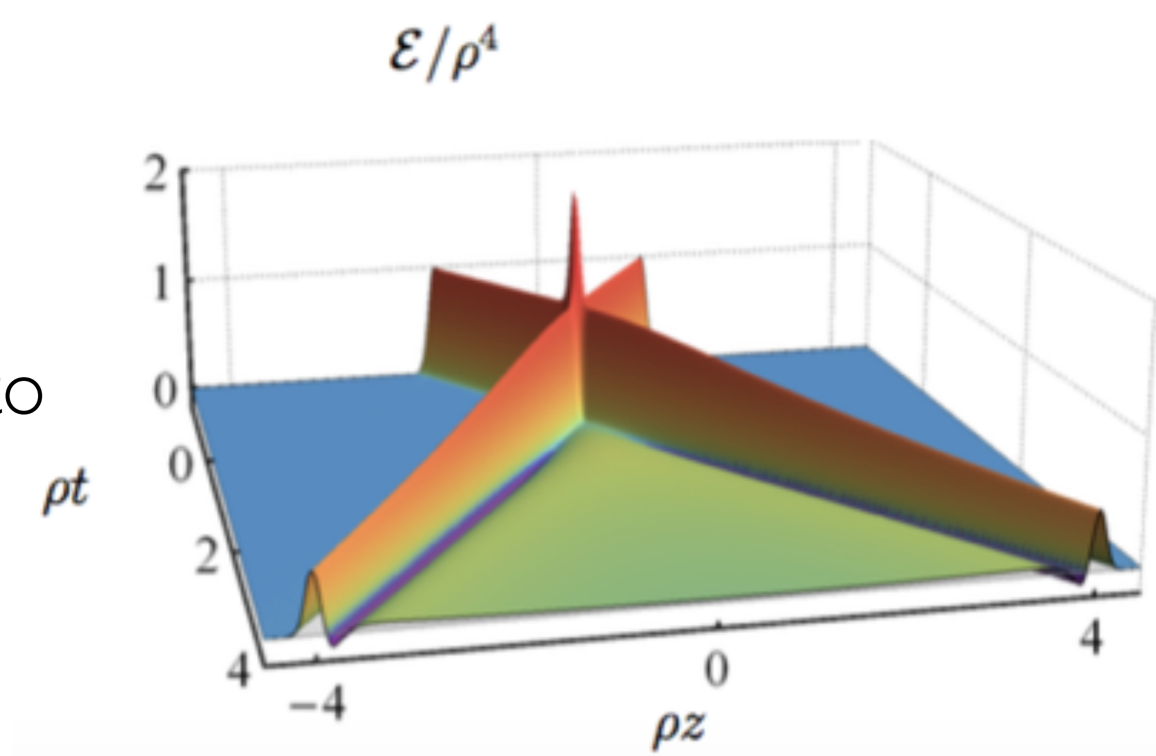
leading order $\approx$ correction

$$T^{\mu\nu}_{hydro} = \boxed{\mathcal{E} u^\mu u^\nu + P_{eq}(\mathcal{E})\left\{g^{\mu\nu} + u^\mu u^\nu\right\}} - \boxed{\eta(\mathcal{E})\,\sigma^{\mu\nu}} + \ldots$$

we have to understand better what is hidden there

Idea: focus on a simple flow in which $\langle T^{\mu\nu}\rangle$ can be expressed as $\langle T^{\tilde{\mu}\tilde{\nu}}\rangle(\tau)$:

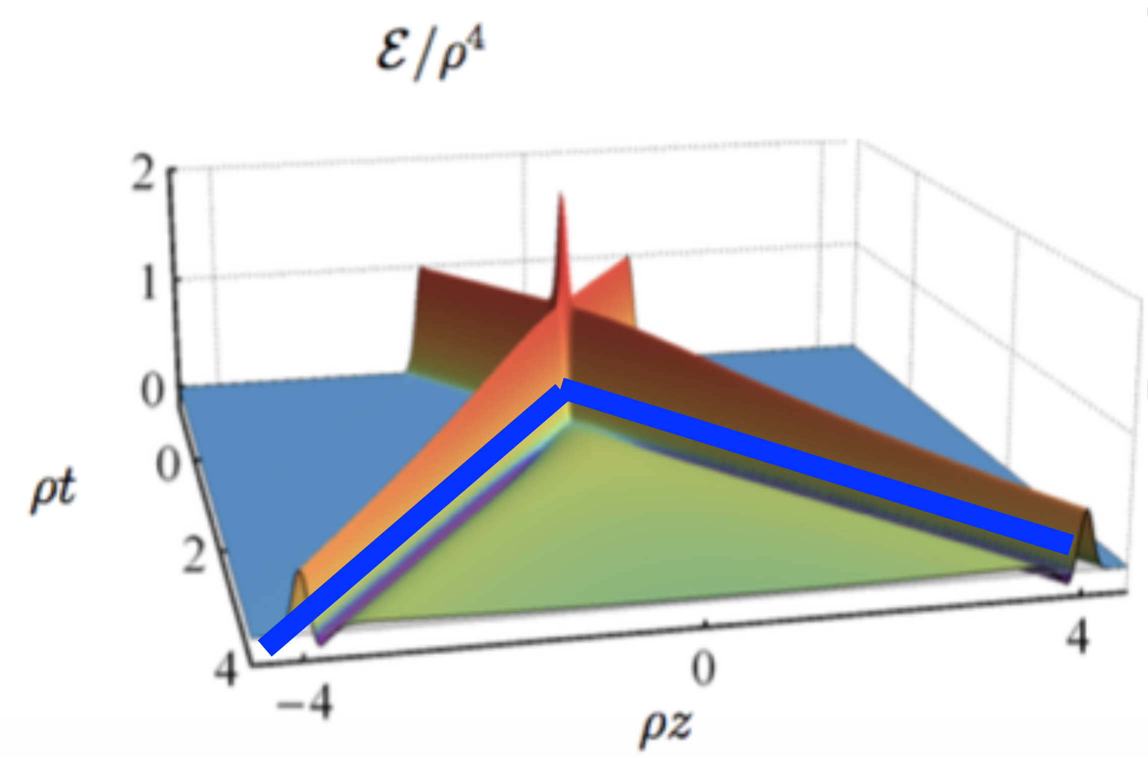
Similar to

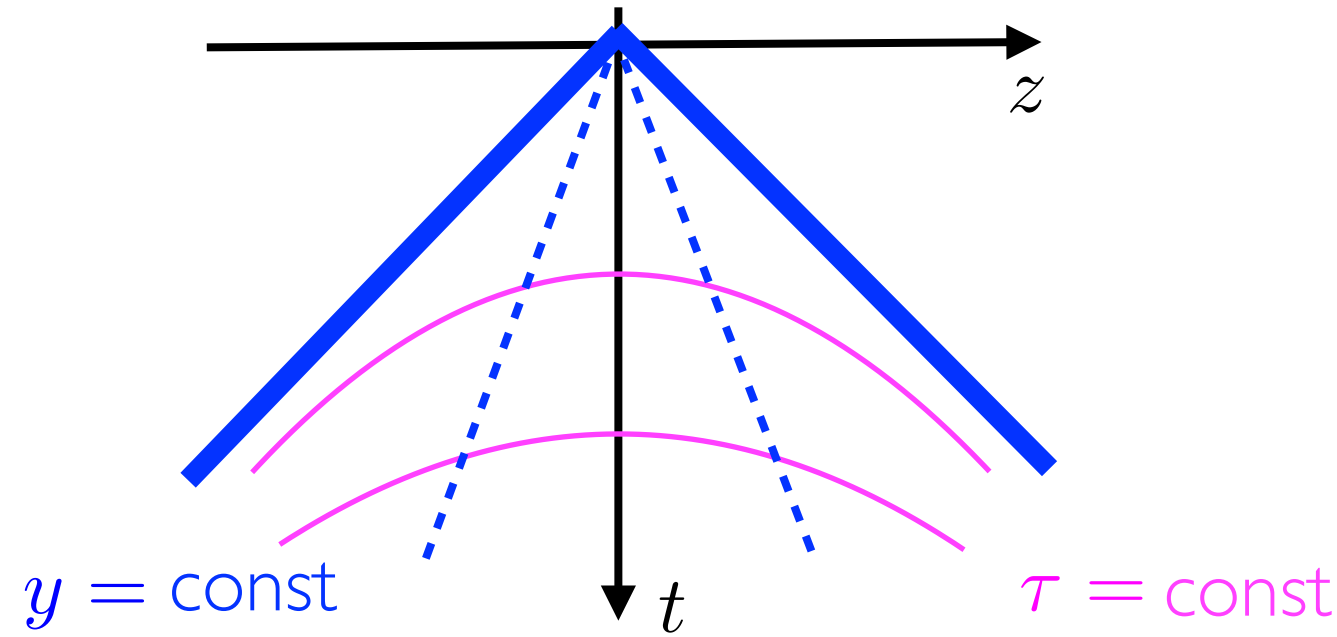


+ additional symmetry under z-boosts:

$$\tau = \sqrt{t^2 - z^2}$$

The boost-invariant flow Bjorken 1982





Boost-invariance: in $(\tau \equiv \sqrt{t^2 - z^2},\ y = \tanh^{-1}\frac{z}{t},\ \mathbf{x}_\perp)$ coords no y-dependence:

Background Minkowski space: $ds^2 = -d\tau^2 + \tau^2 dy^2 + d\mathbf{x}_\perp^2$

CFT: $\langle T_{\mu\nu}\rangle\, dx^\mu dx^\nu = \mathcal{E}(\tau)\, d\tau^2 + \tau^2 \underbrace{(-\mathcal{E} - \tau\,\dot{\mathcal{E}})}_{\mathcal{P}_L}\, dy^2 + \underbrace{(\mathcal{E} + \frac{1}{2}\tau\,\dot{\mathcal{E}})}_{\mathcal{P}_T}\, d\mathbf{x}_\perp^2$

The boost-invariant hydrodynamics

1103.3452 with Janik & Witaszczyk

$$\langle T_{\mu\nu} \rangle \, dx^\mu dx^\nu = \mathcal{E}(\tau)\, d\tau^2 + \tau^2(-\mathcal{E} - \tau\, \dot{\mathcal{E}})\, dy^2 + (\mathcal{E} + \frac{1}{2}\tau\, \dot{\mathcal{E}})\, d\mathbf{x}_\perp^2$$

$$\Big\downarrow \; u^\mu \partial_\mu = \partial_\tau$$

$$T^{\mu\nu}_{hydro} = \mathcal{E} u^\mu u^\nu + P_{eq}(\mathcal{E}) \left\{ g^{\mu\nu} + u^\mu u^\nu \right\} - \eta(\mathcal{E})\, \sigma^{\mu\nu} + \ldots$$

late-time expansion

Qty of interest: $\dfrac{\mathcal{P}_T - \mathcal{P}_L}{\mathcal{P}_{eq}} \sim -\dfrac{\overset{T^3}{\overset{\sim}{\eta(\mathcal{E})}}\, \overset{\frac{1}{\tau}}{\overset{\sim}{\sigma^{\mu\nu}}}}{\underset{\underset{T^4}{\sim}}{\mathcal{E}}} + \ldots \sim \dfrac{1}{\tau\, T} + O\left(\dfrac{1}{(\tau\, T)^2}\right)$

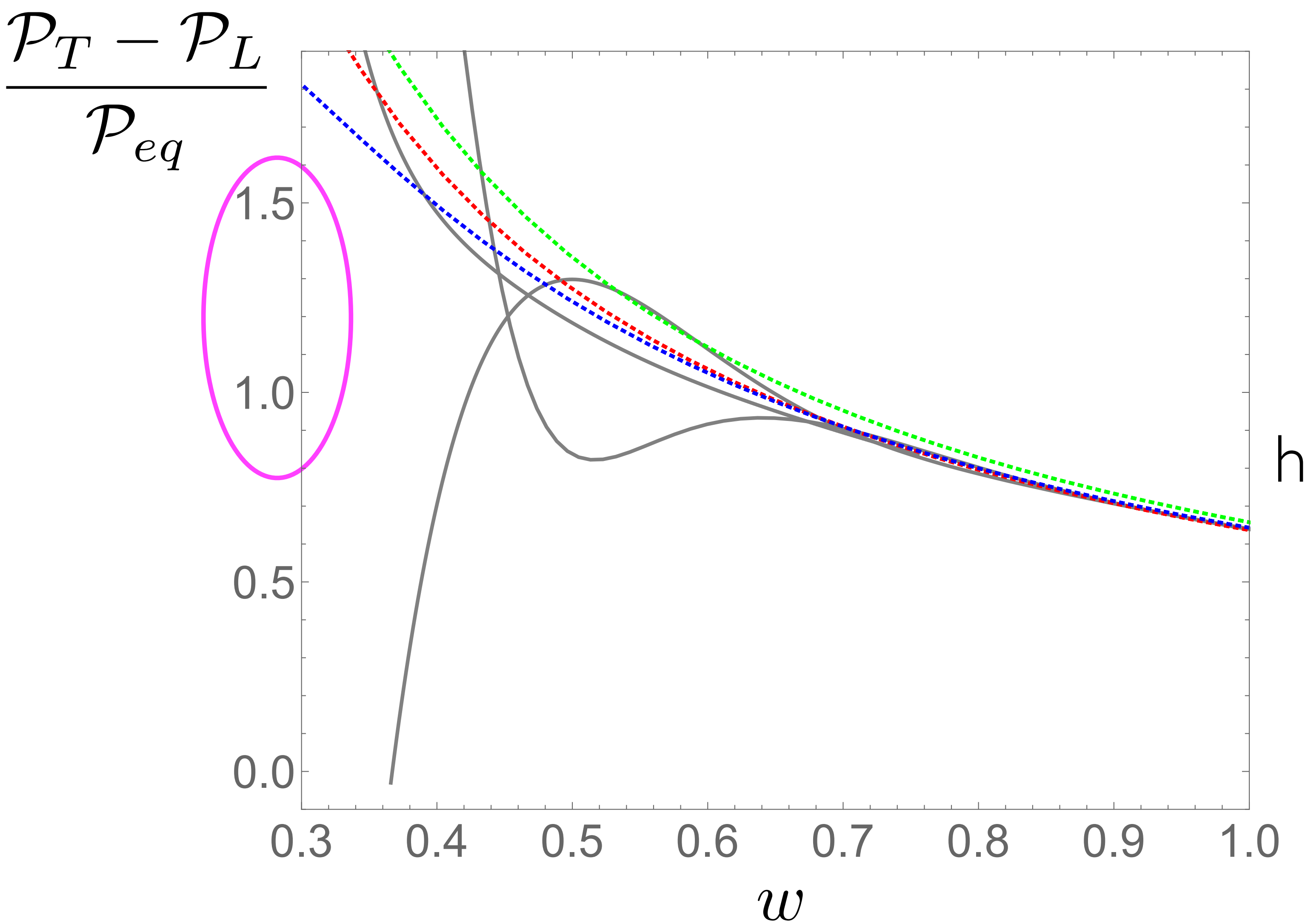
$\equiv \dfrac{1}{w}$

Conclusion: gradient expansion in the boost-invariant flow is an expansion in $\dfrac{1}{w}$

Hydrodynamization in the boost-invariant flow

1103.3452 with Janik & Witaszczyk

Ab initio calculation in N=4 SYM at strong coupling:



Hydrodynamics works despite huge anisotropy captured by $-\eta\,\sigma^{\mu\nu}$

$$\frac{\mathcal{P}_T - \mathcal{P}_L}{\mathcal{P}_{eq}} = \frac{0.64}{w} + \frac{0.02}{w^2} + \frac{0.01}{w^3} + \dots$$

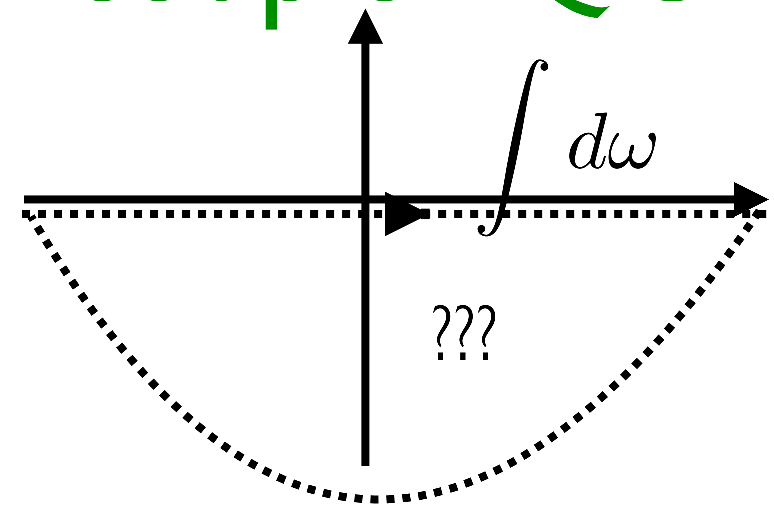
Why can hydrodynamization occur?

M. P. Heller, R. A. Janik and P. Witaszczyk,

Phys. Rev. Lett. 110, 211602 (2013), **1302.0697**

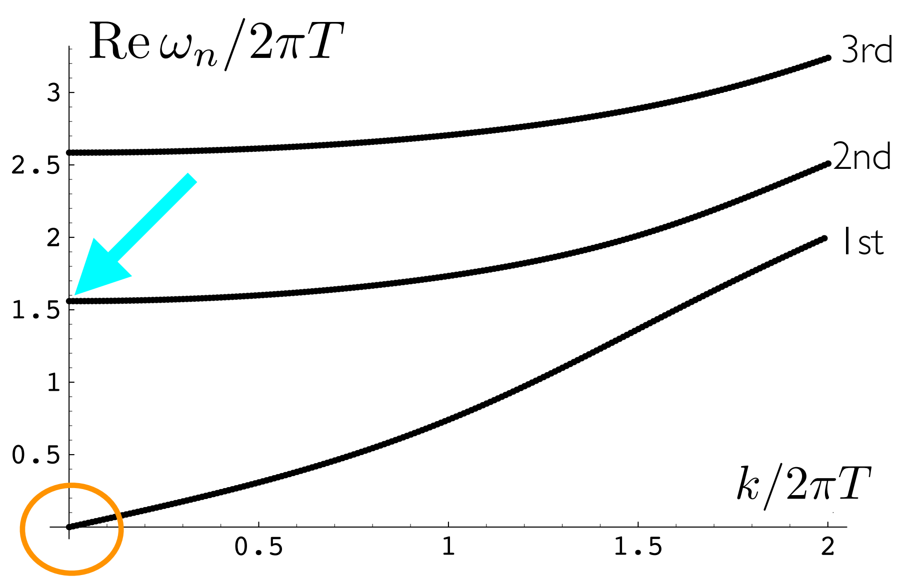
Quasinormal modes: dofs of strongly-coupled QGP

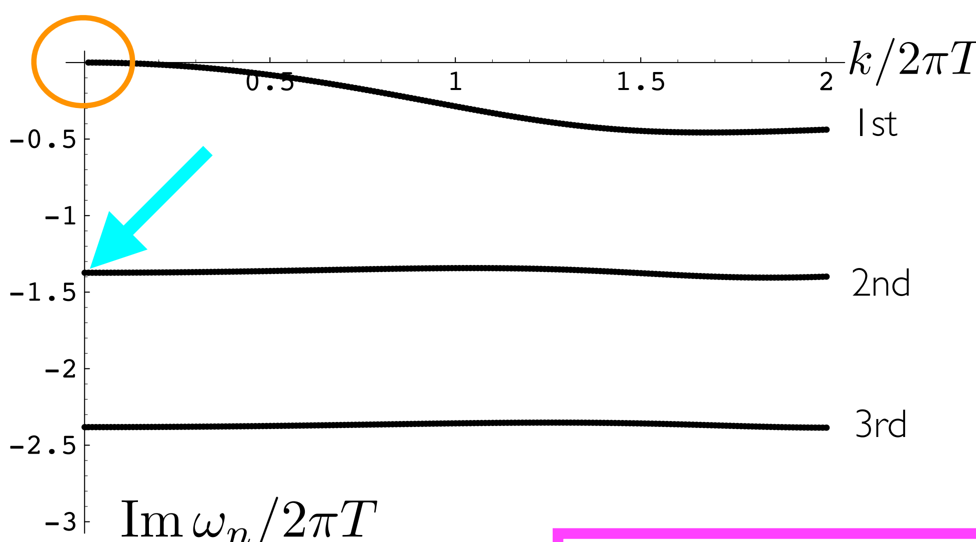
$$\delta\langle T_{\mu\nu}\rangle = \left\{ \int d^3k \int d\omega \, e^{-i\omega t + i\vec{k}\cdot\vec{x}} G_R(\omega, k) \cdot \delta g \right\}_{\mu\nu} :$$



Singularities in the lower-half ω-plane are single poles (QNMs) for each value of k

see hep-th/0506184 by Kovtun & Starinets





Thus $\delta\langle T_{\mu\nu}\rangle = \sum_n \int d^3k \, c_n^{\mu\nu} e^{-\omega_n(k)t + i\vec{k}\cdot\vec{x}}$ with

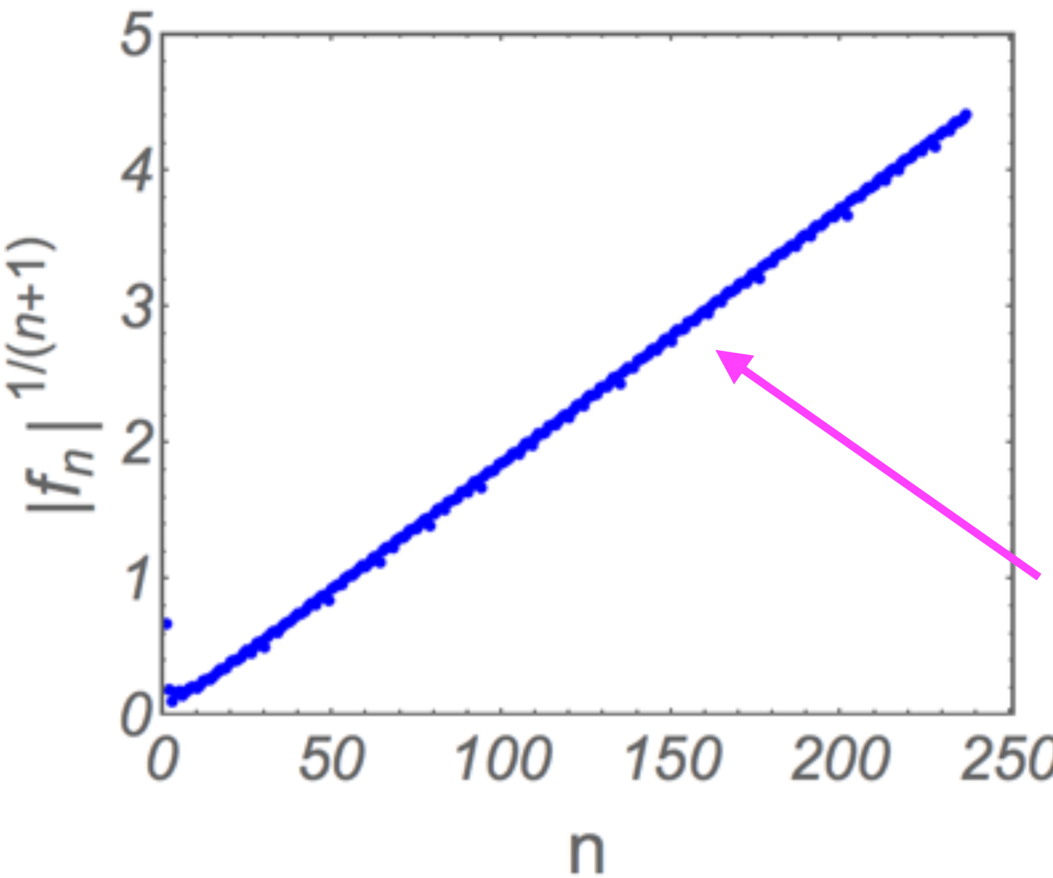
$\boxed{\text{exponential decay in 1/T}}$

slow decay (hydro)

Hydrodynamic gradient expansion is divergent

In 1302.0697 we computed $f(w) = \frac{2}{3} + \frac{1}{18}\frac{\mathcal{P}_T - \mathcal{P}_L}{\mathcal{P}_{eq}}$ up to $O(w^{-240})$:

$$f(w) = \sum_{n=0}^{\infty} f_n w^{-n} =$$
$$= \frac{2}{3} + \frac{1}{9\pi} w^{-1} + \ldots$$



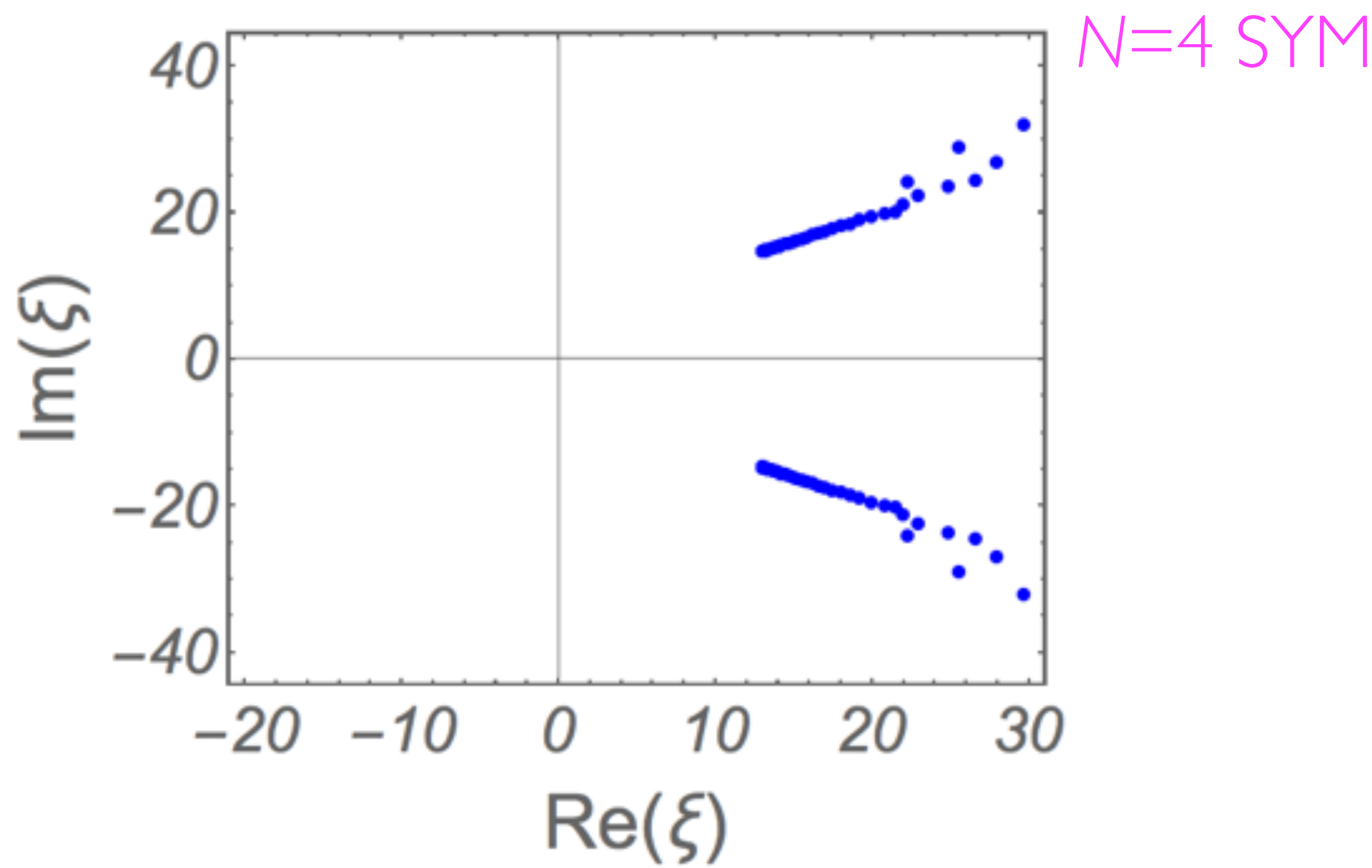
N=4 SYM

$$(n!)^{1/(n+1)}\Big|_{n\to\infty} \approx \frac{1}{e}\cdot n$$

Hydrodynamics and QNMs

1302.0697

Analytic continuation of $f_B(\xi) \approx \sum_{n=0}^{240} \frac{1}{n!} f_n \, \xi^n$ revealed the following singularities:



Branch cut singularities start at $\frac{3}{2} \, i \, \omega_{QNM_1}$!

Resumming gradient expansion in MIS theory

1503.07514 [PRL 115 072501 (2015)] with Spaliński

see also 1509.05046 by Basar & Dunne, as well as 1511.06358 by Aniceto & Spaliński

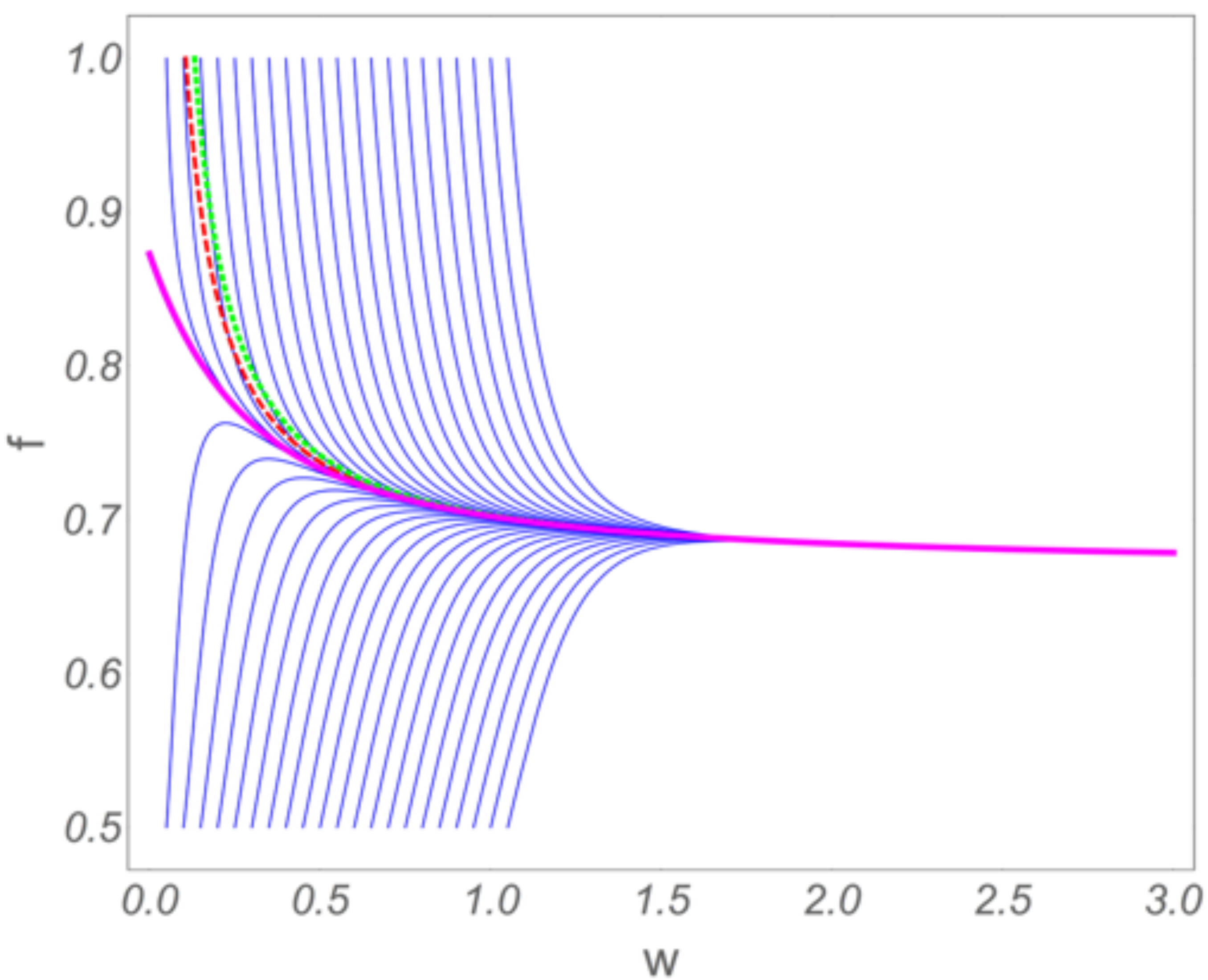
The boost-invariant MIS theory

1503.07514

$$(\tau_\Pi \mathcal{D} + 1)\,\Pi_{\mu\nu} = -\eta\sigma_{\mu\nu} + \ldots$$

$C_\eta = \frac{\eta}{s}$ and $C_{\tau_\Pi} = \tau_\Pi T$

$$f' = -\frac{1}{C_{\tau_\Pi}} + \frac{2}{3\,C_{\tau_\Pi}\,f} + \frac{16}{3\,w} - \frac{16}{9\,w\,f} + \frac{4C_\eta}{9\,C_{\tau_\Pi}\,w\,f} - \frac{4\,f}{w} + \ldots$$



attractor

different solutions

$f(w) = f_0 + f_1 w^{-1}$

$f(w) = f_0 + f_1 w^{-1} + f_2 w^{-2}$

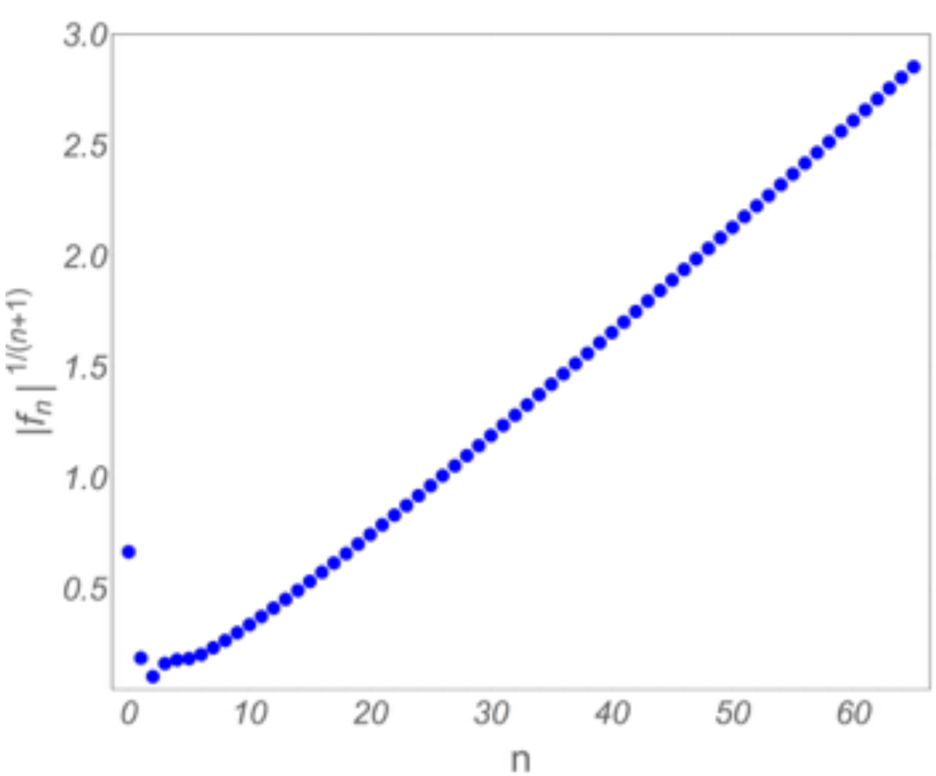
Gradient expansion

1503.07514

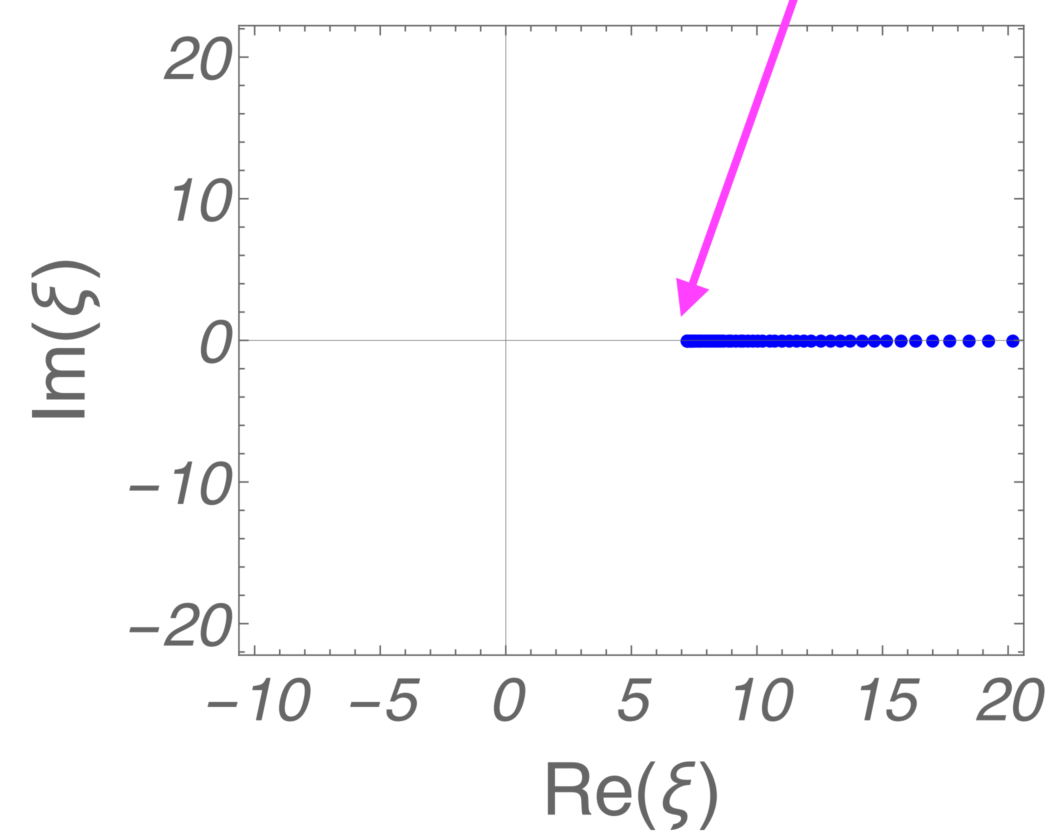
$$f' = -\frac{1}{C_{\tau\Pi}} + \frac{2}{3\,C_{\tau\Pi}\,f} + \frac{16}{3\,w} - \frac{16}{9\,w\,f} + \frac{4C_\eta}{9\,C_{\tau\Pi}\,w\,f} - \frac{4\,f}{w} + \ldots$$

$$f = \sum_{n=0}^{\infty} \frac{f_n}{w^n} + \underbrace{\delta f}_{\exp\left(-\frac{3}{2C_{\tau\Pi}}w\right) \times \ldots} + \mathcal{O}(\delta f^2)$$

$$\xi_{sing} = \frac{3}{2C_{\tau\Pi}}$$



$$f_B(\xi) \approx \sum_{n=0}^{200} \frac{1}{n!} f_n\,\xi^n$$

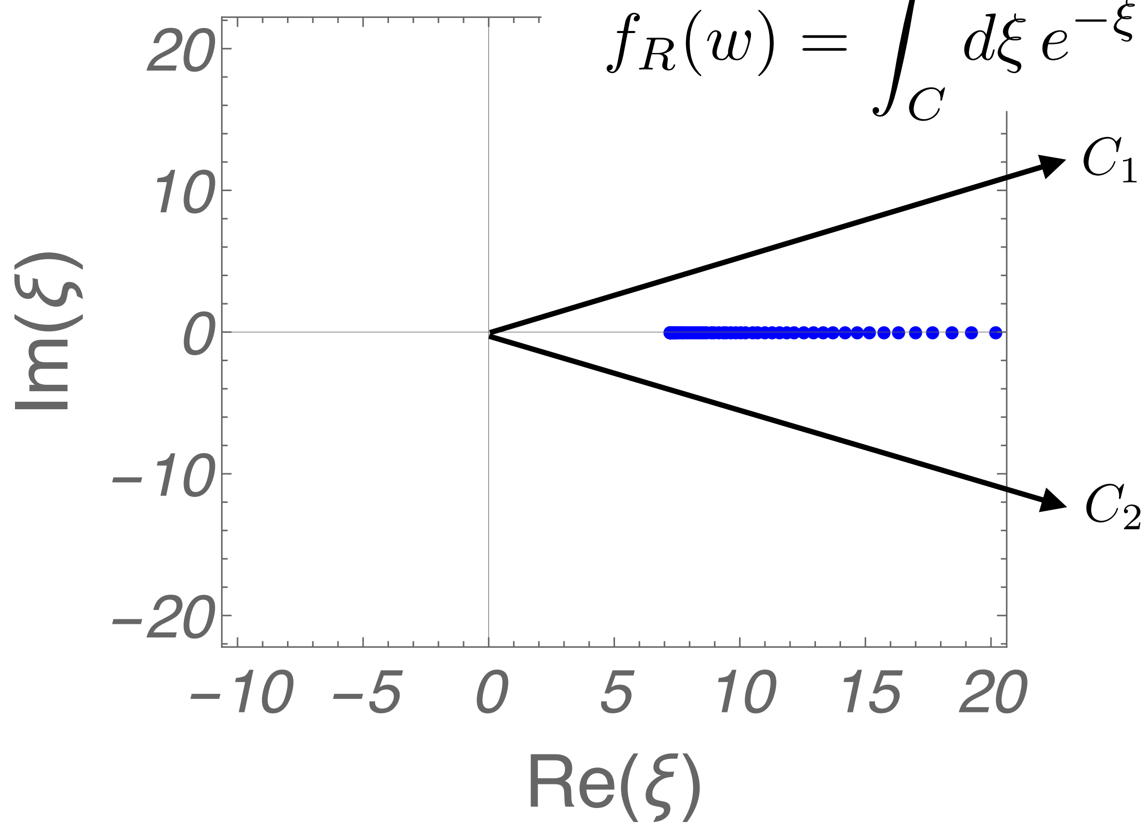


Transseries

1503.07514

Hydrodynamic gradient expansion is intrinsically ambiguous:

$$f_R(w) = \int_C d\xi\, e^{-\xi} f_B(\xi/w) = w \int_C d\xi\, e^{-w\xi} f_B(\xi)$$



The ambiguity goes away upon including the quasinormal mode ($f_n = a_{0,n}$)

$$f = \sum_{m=0}^{\infty} (c_{amb} + r)^m \left\{ w^{\frac{C_\eta}{C_{\tau_\Pi}}} \exp\left(-\frac{3}{2C_{\tau_\Pi}} w \right) \right\}^m \sum_{n=0}^{\infty} a_{m,n} w^{-n}$$

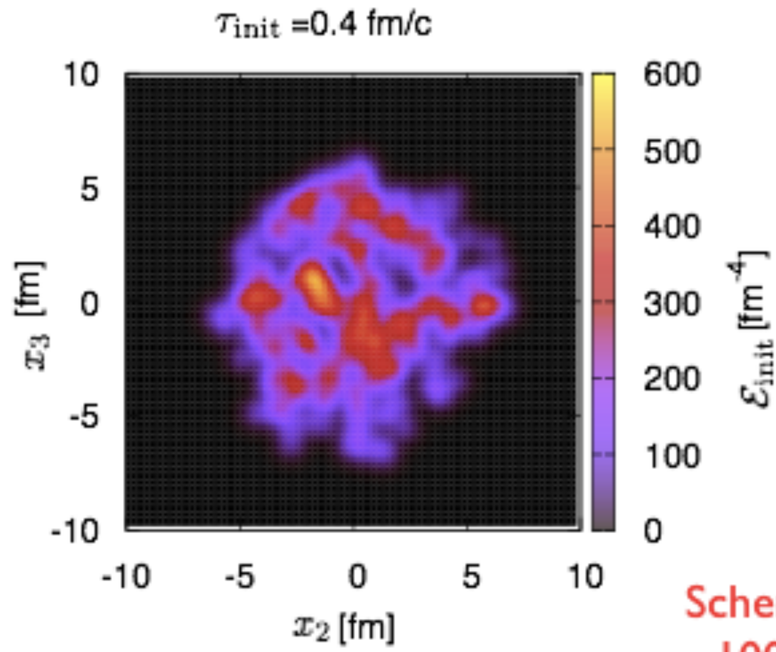
initial condition

Homework: write an ansatz for a transseries for f in N=4 SYM

Summary of Lecture III

1302.0697 [PRL 110 211602 (2013)] with Janik & Witaszczyk
1503.07514 [PRL 115 072501 (2015)] with Spaliński

pheno:
hydrodynamization



Schenke et al.
1009.3244

hydrodynamic gradient expansion diverges → precision calculations in NumHol

towards genericity

new connections:
transseries & resurgence
(very active area in QM & QFT)

Take-home messages

1202.0981 [PRL 108 191601 (2012)] with Mateos, van der Schee & Tancanelli
1304.5172 [JHEP 1309 026 (2013)] with Mateos, van der Schee & Triana
1305.4919 [PRL 111 181601 (2013)] and **1312.2956** [PRL 112 221602 (2014)]
with Casalderrey-Solana, Mateos & van der Schee
1103.3452 [PRL 108 211602 (2012)] with Janik & Witaszczyk
1302.0697 [PRL 110 211602 (2013)] with Janik & Witaszczyk
1503.07514 [PRL 115 072501 (2015)] with Spaliński

Take-home messages:

Key idea: novel ab initio methods allow to test our understanding of collective phenomena in QFTs

Lesson I: hydrodynamization at strong coupling takes $t_{hyd} = O(1/T_{hyd})$

Lesson II: a priori, hydrodynamization $\neq$ thermalization / isotropization

Lesson III: hydrodynamic gradient expansion diverges; as a result gradients need not to be parametrically small in order for the viscous hydrodynamics to work

All three lessons directly lead to valuable insights for the HIC pheno

Open problems

Open problems

Problem I: how does the breaking of the conformal symmetry affect lessons I-III ?

$\longrightarrow$ **1503.07114** with Buchel & Myers, **1503.07149** by Janik et al. & **1604.06439** by Attems et al.

Problem II: how about higher curvature corrections to $\mathcal{R}_{ab} - \frac{1}{2}\mathcal{R}g_{ab} - \frac{6}{L^2}g_{ab} = 0$?

$\longrightarrow$ **1605.02173** by Grozdanov, Kaplis & Starinets

Problem III: are ~AdS-instability-like processes relevant for any realistic equilibration ?

$\longrightarrow$ **1311.7560** by Craps et al.

Problem IV: understanding large-order hydro gradient expansion for a general flow ?

$\longrightarrow$ **1507.02461** by Grozdanov & Kaplis

Problem V: observational signatures of "QNMs" in experiments (HIC / cold atoms) ?

$\longrightarrow$ **1508.01199** by Brewer & Romatschke

Problem VI: new insights in weakly-coupled QFTs from holographic lessons ?

$\longrightarrow$ **1506.06647** by Kurkela & Zhu, as well as **1512.05347** by Keegan et al.

Problem VII: nonlocal correlation functions beyond the AdS-Vaidya paradigm?

$\longrightarrow$ **1211.0343** by Chesler & Teaney

Problem VIII: relax symm. assumptions / better projectiles for holographic collisions?

$\longrightarrow$ **1501.04644** by Chesler & Yaffe