# Holography, thermalization and heavy-ion collisions II

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**1305.4919** [PRL 111 181601 (2013)] and **1312.2956** [PRL 112 221602 (2014)]
with Casalderrey-Solana, Mateos & van der Schee

# Overview of Lecture 1

**1202.0981** [PRL 108 191601 (2012)] with Mateos, van der Schee & Tancanelli
**1304.5172** [JHEP 1309 026 (2013)] with Mateos, van der Schee & Triana

# Basic notions

certain states of a class of strongly-coupled QFTs = higher dimensional geometries

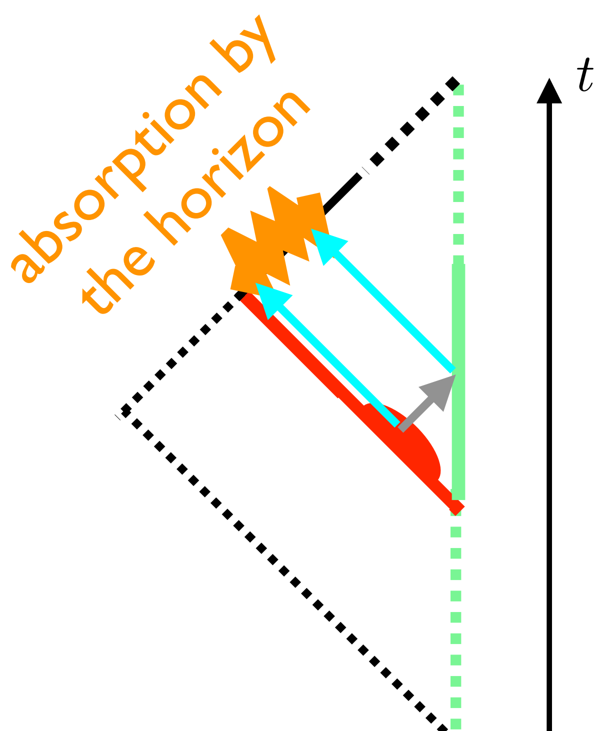
dofs of strongly-coupled QGP = QNMs of dual black branes:

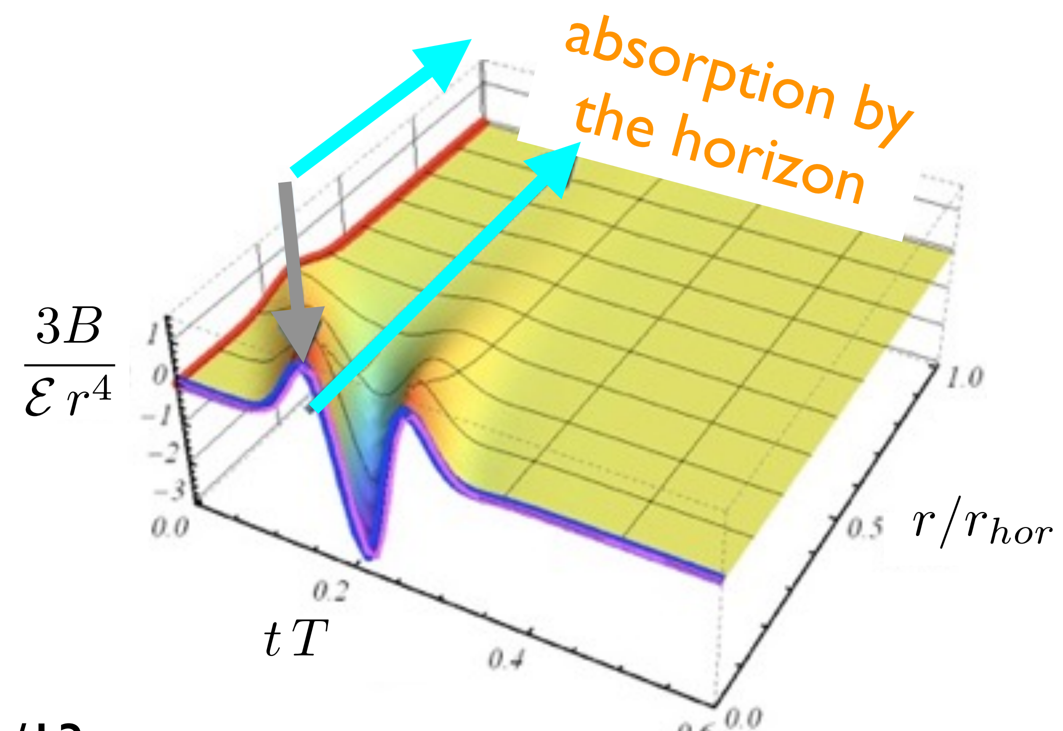
$$\delta\langle T_{\mu\nu}\rangle = \sum_n \int d^3k\, c_n^{\mu\nu} e^{-\omega_n(k)t + i\vec{k}\cdot\vec{x}} \text{ with}$$

**exponential decay in 1/T**

**slow decay (hydro)**

equilibration in strongly coupled QFTs = dual horizon formation and equilibration:



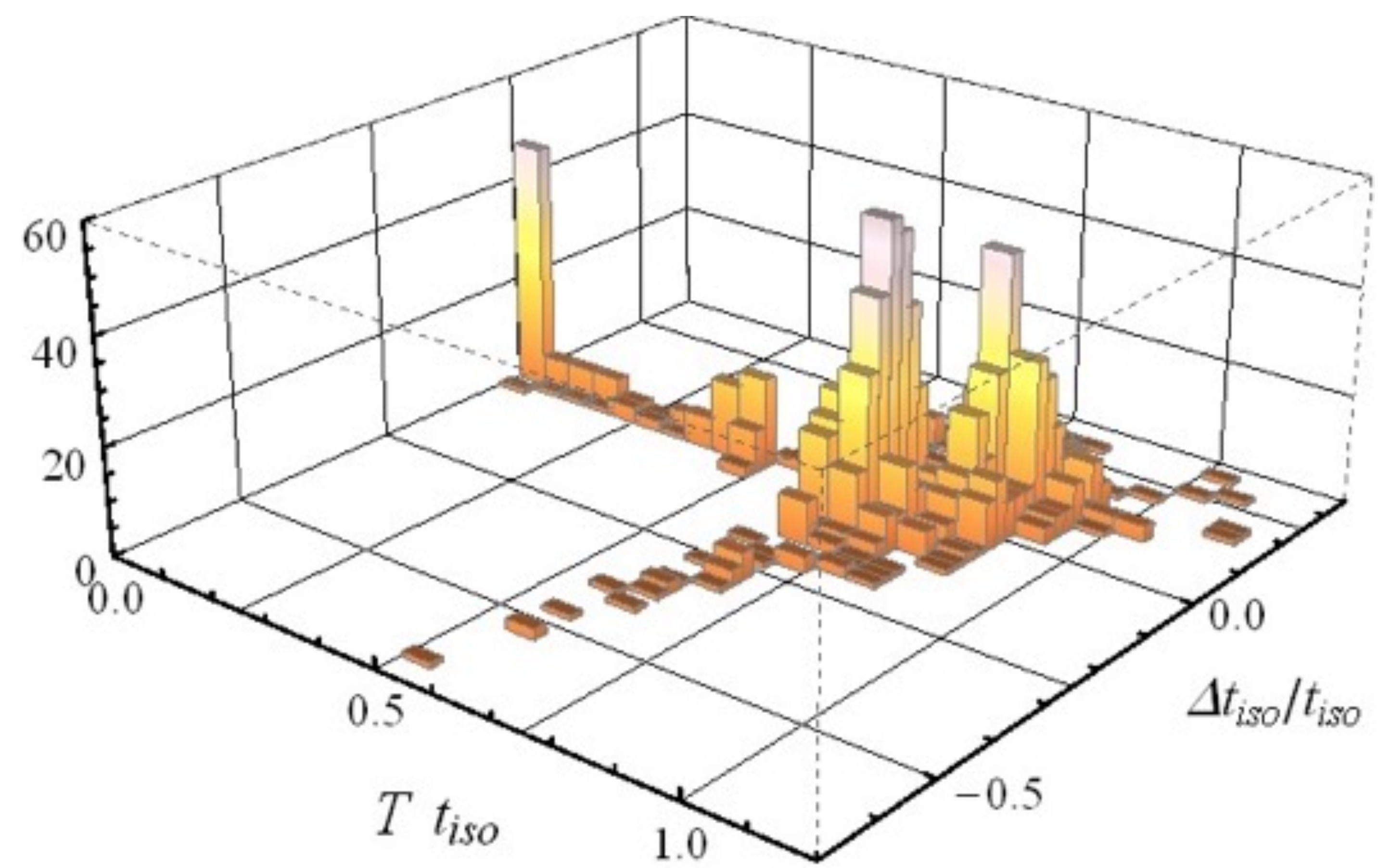
# Lessons: n-eq states and relaxation rates

Real-time dynamics of QFTs requires $\infty$-many initial conditions, e.g.

$$B = -\frac{b(t)}{2r^4} - \frac{b'(t)}{2r^5} - \frac{7b''(t)}{24r^6} - \frac{b^{(3)}(t)}{8r^7} + \ldots \text{ with } b(t) = \frac{2\pi^2}{3N_c^2}\Delta\mathcal{P}(t)$$

Holography makes it manageable by adding $r$ $\longrightarrow$ $\mathcal{R}_{ab} - \frac{1}{2}\mathcal{R}g_{ab} - \frac{6}{L^2}g_{ab} = 0$

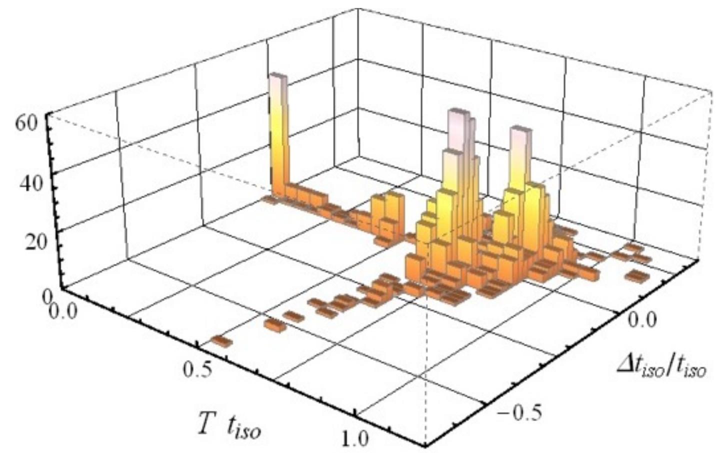
Indications that equilibration in 1/T at strong coupling can be generic:



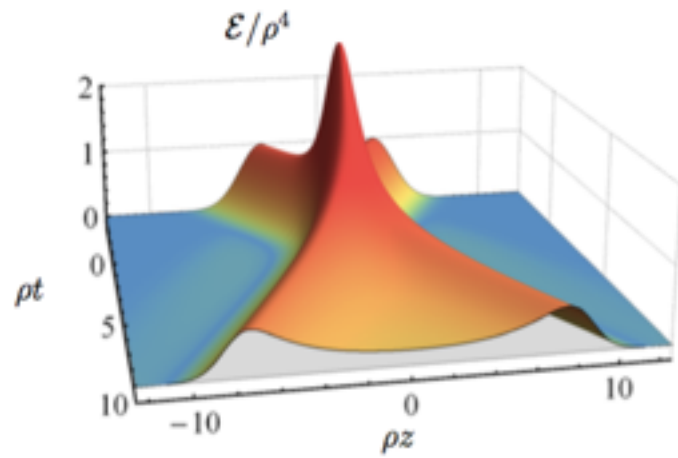

Confirmed in many other setups*. Is $t_{eq}\,T = O(1)$ becoming new "$\eta/s = 1/4\pi$"?

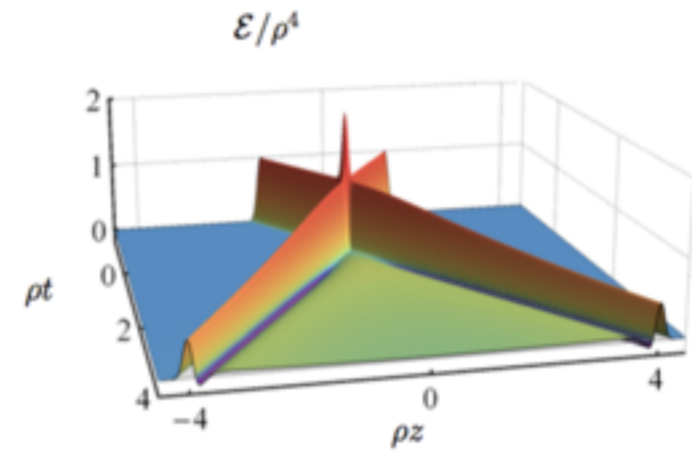
# The plan for today

Lecture I: how long does it take $\langle T^{\mu\nu} \rangle$ to equilibrate in strongly-coupled QFTs?

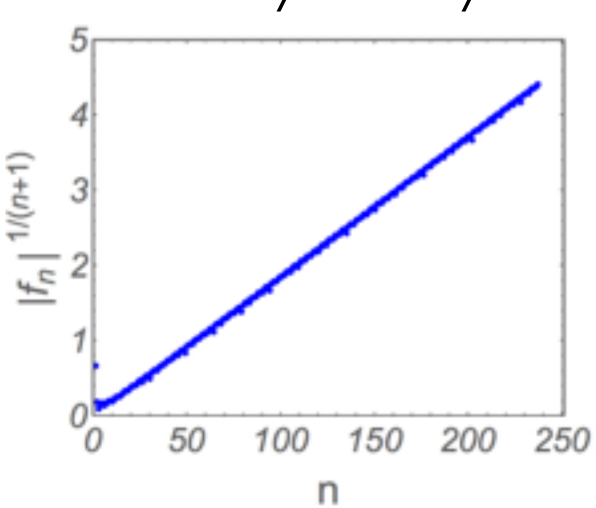


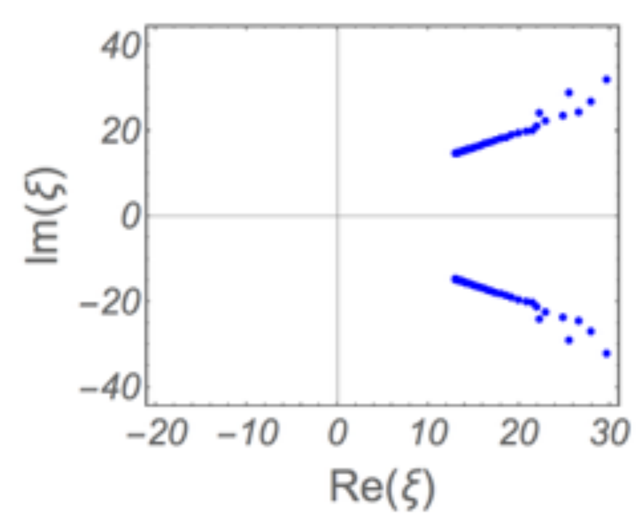

Lecture II: what is $\langle T^{\mu\nu} \rangle (t, \vec{x})$ after a collision of 2 strongly-interacting objects?




Lecture III: what is relativistic hydrodynamics?



# Relativistic hydrodynamics

# Relativistic hydrodynamics

**hydrodynamics is** an EFT of the slow evolution of conserved currents in collective media close to equilibrium

**DOFs**: always local energy density $\mathcal{E}$ and local flow velocity $u^\mu$ $\quad (u_\nu u^\nu = -1)$

**formal EOMs:** conservation eqns $\nabla_\mu \langle T^{\mu\nu} \rangle = 0$ for $\langle T^{\mu\nu} \rangle$ <u>expanded in gradients</u>

$$T^{\mu\nu}_{hydro} = \overbrace{\mathcal{E} u^\mu u^\nu + P(\mathcal{E}) \left\{ g^{\mu\nu} + u^\mu u^\nu \right\}}^{\text{isotropic}} \overbrace{- \eta(\mathcal{E})\, \sigma^{\mu\nu}}^{\text{breaks isotropy}} - \zeta(\mathcal{E}) \left\{ g^{\mu\nu} + u^\mu u^\nu \right\} (\nabla \cdot u) + \ldots$$

**microscopic input:**

EoS $\left(P(\mathcal{E}) = \frac{1}{3} \mathcal{E}\right.$ for CFTs)

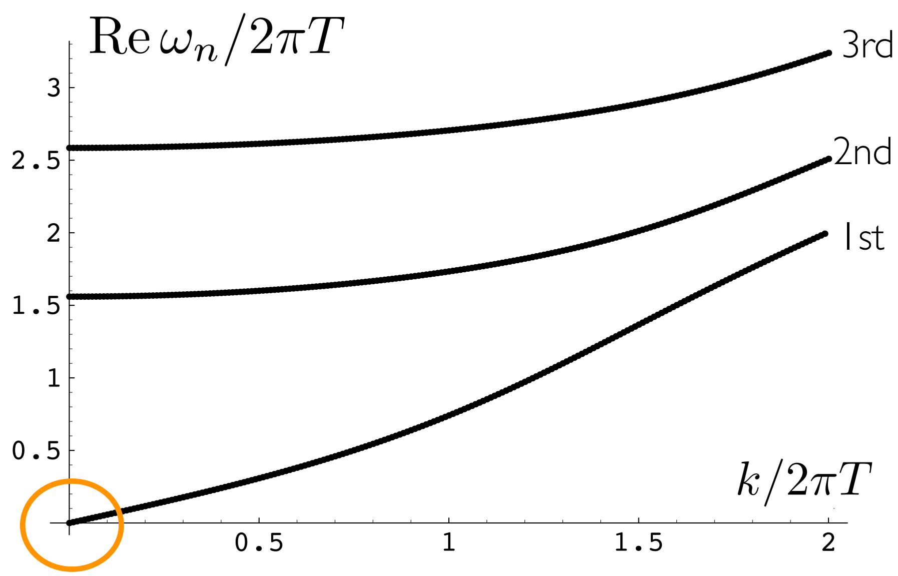
shear viscosity

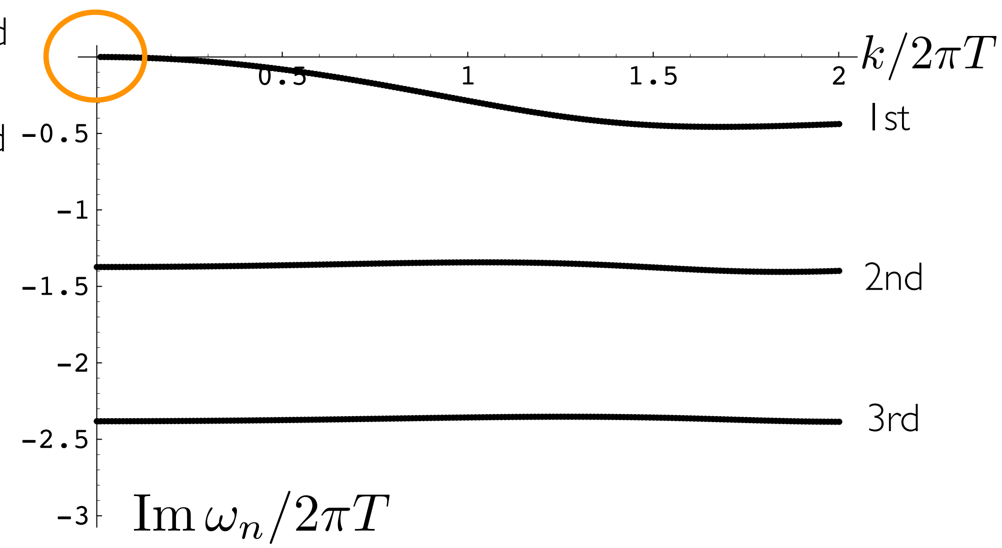
bulk viscosity (vanishes for CFTs)

It is clear that $\langle T^{\mu\nu} \rangle$'s will not always be close to satisfying these relations

# Quasinormal modes and hydrodynamics

QNMs:




$$\nabla_\mu\Big(\mathcal{E}\,u^\mu u^\nu + P(\mathcal{E})\,\{g^{\mu\nu}+u^\mu u^\nu\} - \eta(\mathcal{E})\,\sigma^{\mu\nu} + \ldots\Big) = 0 \text{ for}$$

$$u^\mu\,\partial_\mu = \partial_t + \delta u^\mu\, e^{-i\omega(k)t + i\vec{k}\cdot\vec{x}}\,\partial_\mu$$

$$\mathcal{E} = \mathcal{E}_{th} + \delta\mathcal{E}\, e^{-i\omega(k)t + i\vec{k}\cdot\vec{x}}$$

$$\longrightarrow \quad \omega_1(k) = \pm\frac{1}{\sqrt{3}}k - i\,\frac{2}{3T}\,\frac{\eta}{s}\,k^2 + \ldots$$

$\boxed{\text{but not } \omega_{n\geq 2}}$

# Testing the applicability of hydrodynamics

How to see if $\langle T^{\mu\nu} \rangle$ has reached its hydrodynamic form?

Previously, we secretly defined: $T^{\mu\nu}_{hydro}\, u_\nu = -\mathcal{E}\, u^\mu$ with $u_\nu u^\nu = -1 \longrightarrow$ algorithm:

**step I:** for a generic $\langle T^{\mu\nu} \rangle$, use to find would-be hydrodynamic $\mathcal{E}$ and $u^\mu$

**step II:** evaluate truncated $T^{\mu\nu}_{hydro} = \mathcal{E} u^\mu u^\nu + P(\mathcal{E})\, \{g^{\mu\nu} + u^\mu u^\nu\} - \eta(\mathcal{E})\, \sigma^{\mu\nu} + \ldots$

**step III:** compare $\langle T^{\mu\nu} \rangle$ with $T^{\mu\nu}_{hydro}$. Hydro works* ever since they differ by a few %

Let us see how it works with the homogeneous isotropization:

$$\langle T^{\mu\nu} \rangle = \mathrm{diag} \left\{ \mathcal{E},\, \frac{\mathcal{E}}{3} - \frac{2}{3} \Delta \mathcal{P}(t),\, \frac{\mathcal{E}}{3} + \frac{1}{3} \Delta \mathcal{P}(t),\, \frac{\mathcal{E}}{3} + \frac{1}{3} \Delta \mathcal{P}(t) \right\}^{\mu\nu}$$

We get $u^\mu \partial_\mu = \partial_t$ and $\mathcal{E}$ = const $\longrightarrow$ $T^{\mu\nu}_{hydro} = \mathrm{diag} \left\{ \mathcal{E},\, \frac{1}{3}\mathcal{E},\, \frac{1}{3}\mathcal{E},\, \frac{1}{3}\mathcal{E} \right\}^{\mu\nu}$

# Holographic models of heavy-ion collisions

**1305.4919** [PRL 111 181601 (2013)] and **1312.2956** [PRL 112 221602 (2014)]
with Casalderrey-Solana, Mateos & van der Schee

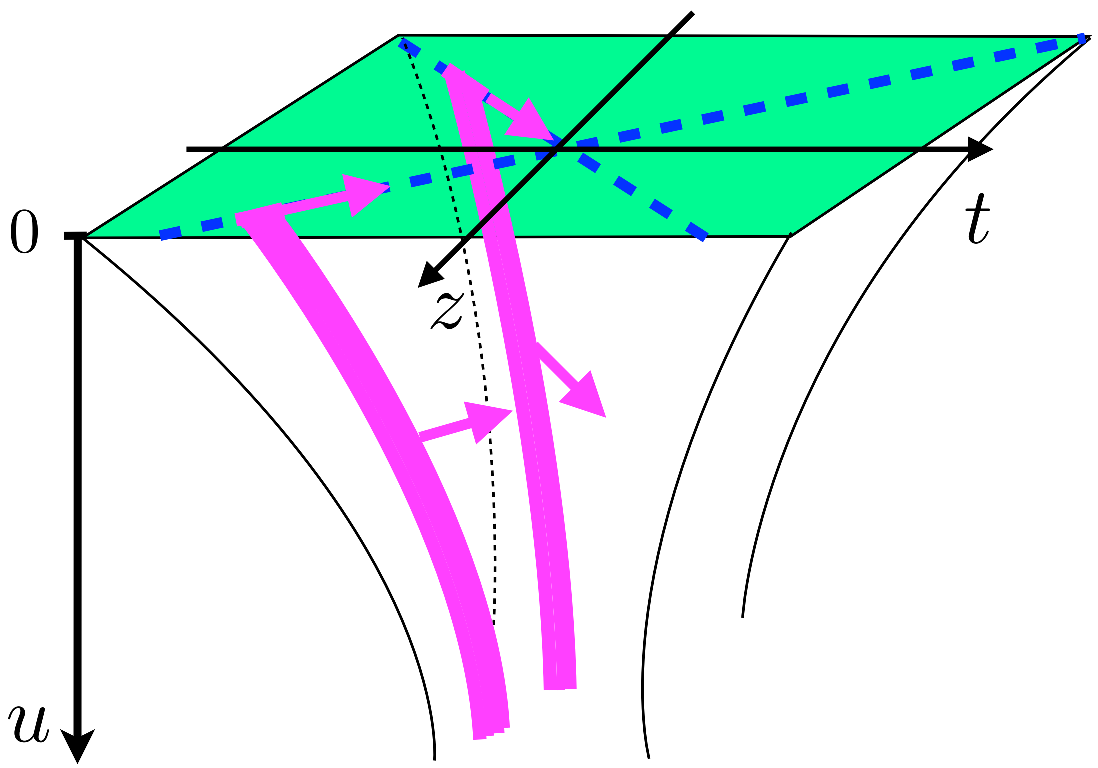
# How to model HIC at strong coupling?

The crudest projectile in strongly-coupled CFTs:

hep-th/0512162 by Janik & Peschanski
1011.3562 by Chesler & Yaffe

$$\langle T^{tt}\rangle = \langle T^{zz}\rangle = \mp\langle T^{tz}\rangle = \frac{N_c^2}{2\pi^2} h(t \pm z) \text{ with any } h \geq 0$$

It ignores transversal structure. Holographic collision:




$$ds^2 = \frac{L^2}{u^2}\Big\{du^2 + \eta_{\mu\nu}dx^\mu dx^\nu + \boxed{+u^2 h(z_+)\, dz_+^2 + u^2 h(z_-)\, dz_-^2 \Big\}}$$

valid solution before the collision ($t = 0$)

later: $\mathcal{R}_{ab} - \frac{1}{2}\mathcal{R} g_{ab} - \frac{6}{L^2} g_{ab} = 0$

We take $h(t \pm z) = \rho^4 \exp\left[-(t \pm z)^2/2\sigma^2\right]$ where $\rho\,\sigma \sim \gamma^{-1/2}$ in HIC

Large $\rho\,\sigma$

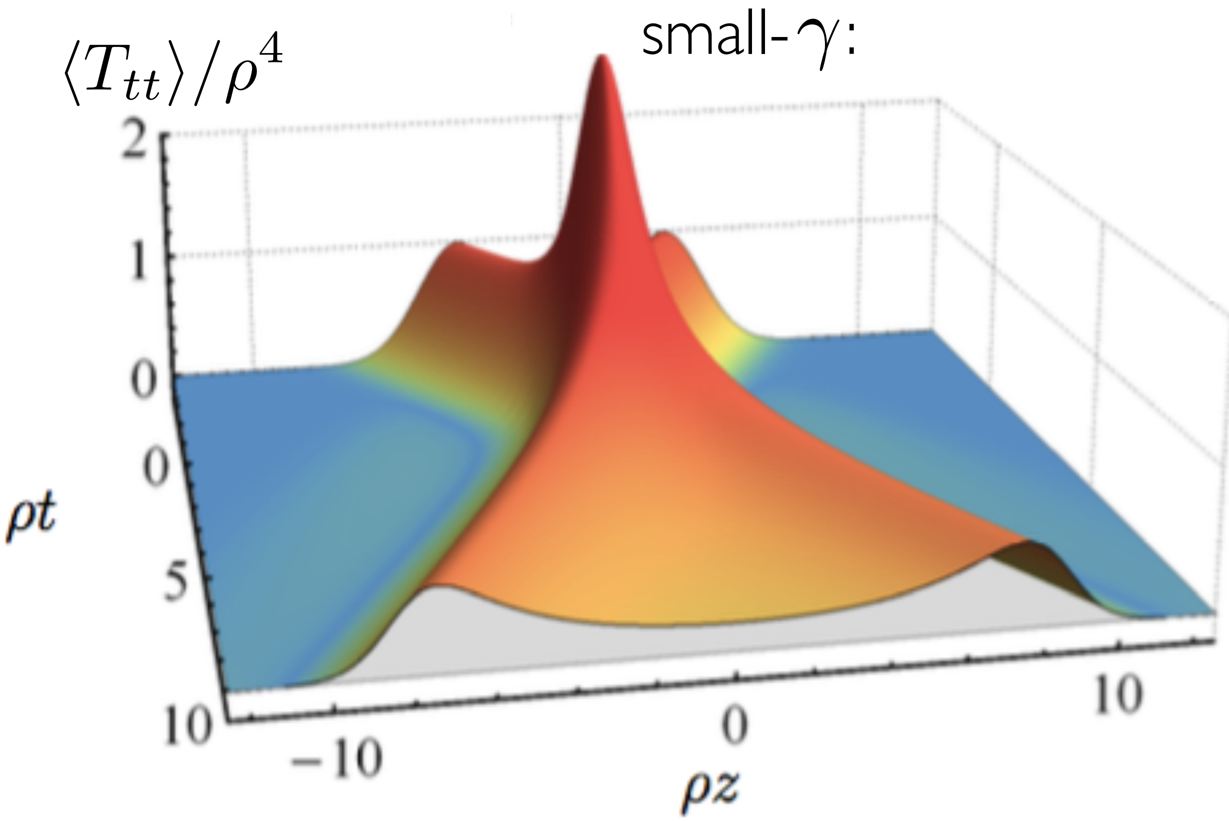
collisions at "low energies"

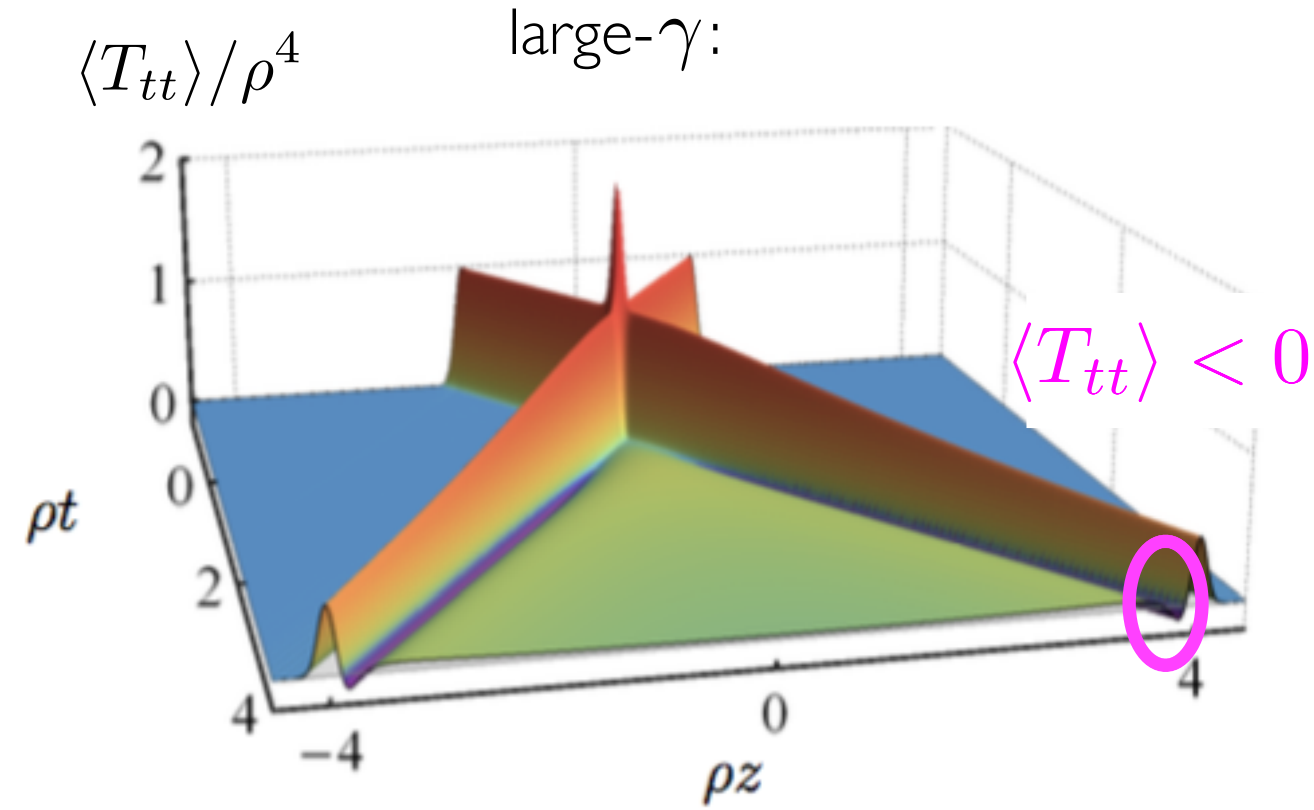
Small $\rho\,\sigma$

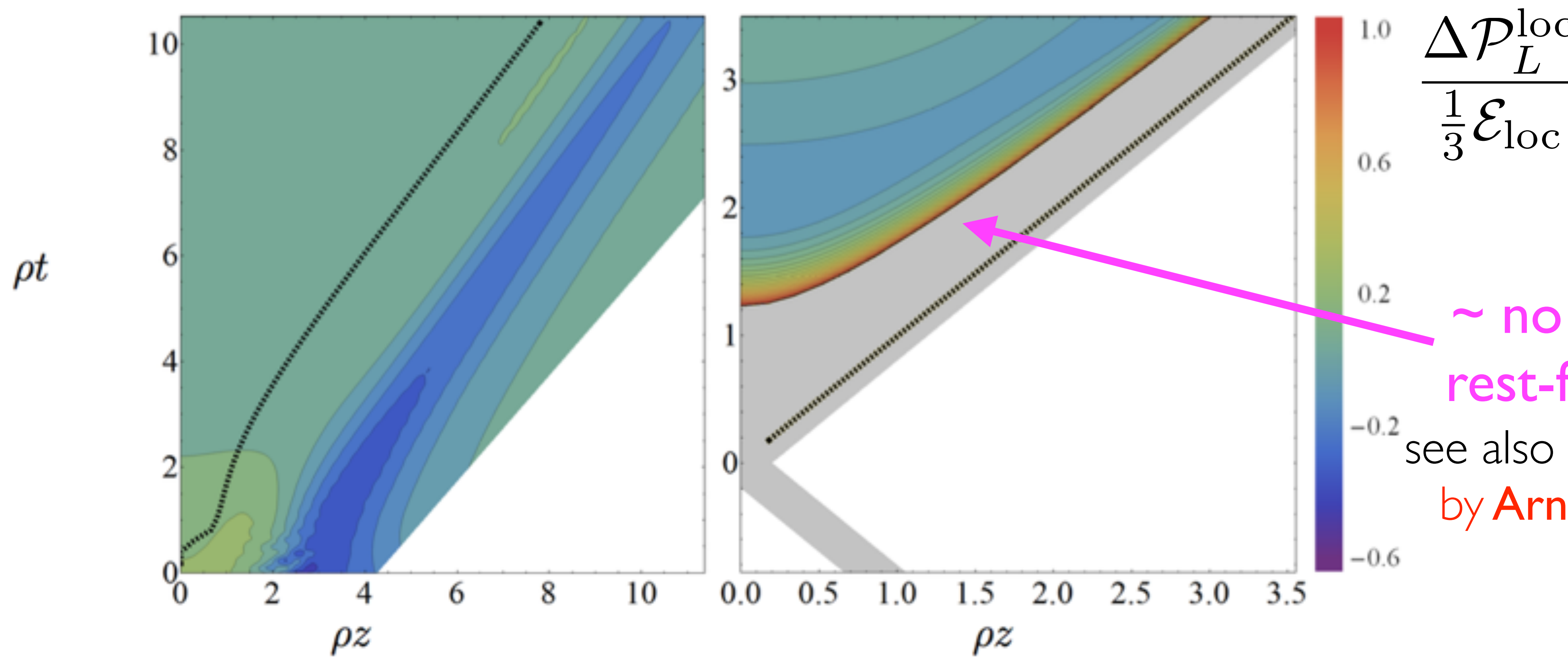
collisions at "high energies"

Rich transient physics as a function of $\gamma$

# Collision of symmetric projectiles




~ no local rest-frame

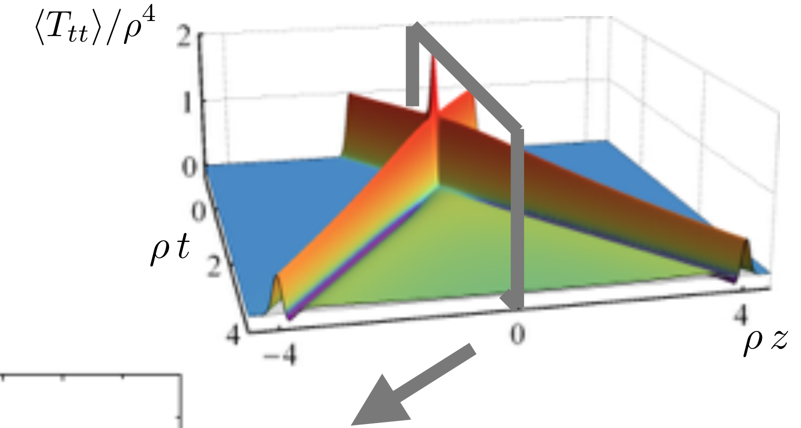
see also 1408.2518 by Arnold et al.

# Hydrodynamization

**1305.4919** [PRL 111 181601 (2013)] and **1312.2956** [PRL 112 221602 (2014)]
with Casalderrey-Solana, Mateos & van der Schee

see 0906.4426 and 1011.3562 by Chesler & Yaffe for the first observation of hydrodynamization,
as well as 1103.3452 with Janik & Witaszczyk

# Hydrodynamization $\neq$ thermalization

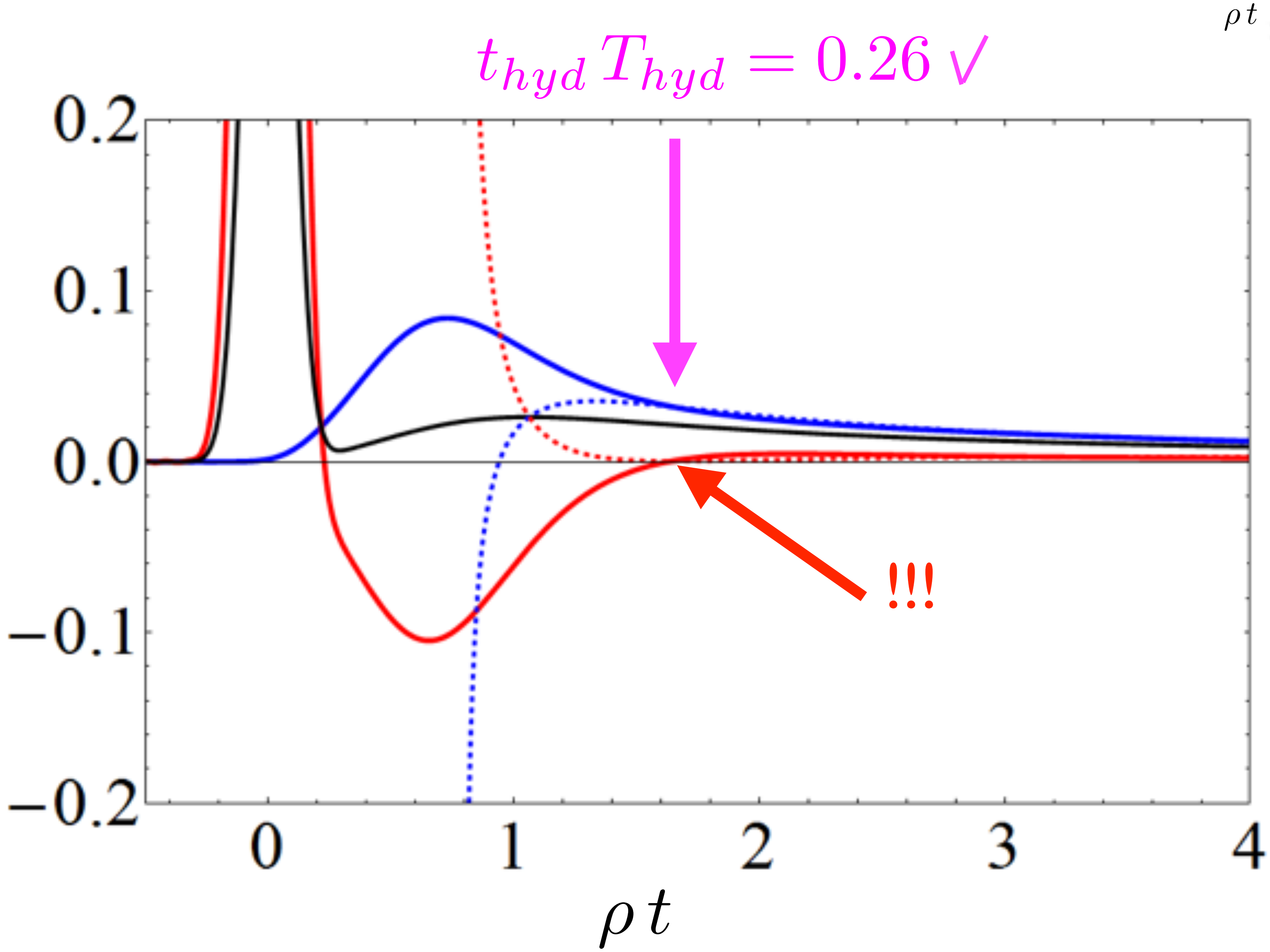



$\langle T_{tt}\rangle = \mathcal{E}$

$\langle T_{zz}\rangle = \mathcal{P}_L$

$\langle T_{\perp\perp}\rangle = \mathcal{P}_\perp$

dotted = viscous hydro




$\frac{1}{3}\mathcal{E}$

Huge anisotropies at the hydrodynamic threshold: $\mathcal{P}_T - \mathcal{P}_L = 1.35 \times \mathcal{P}_{eq}$

Viscous hydrodynamics constitutive relations work despite:

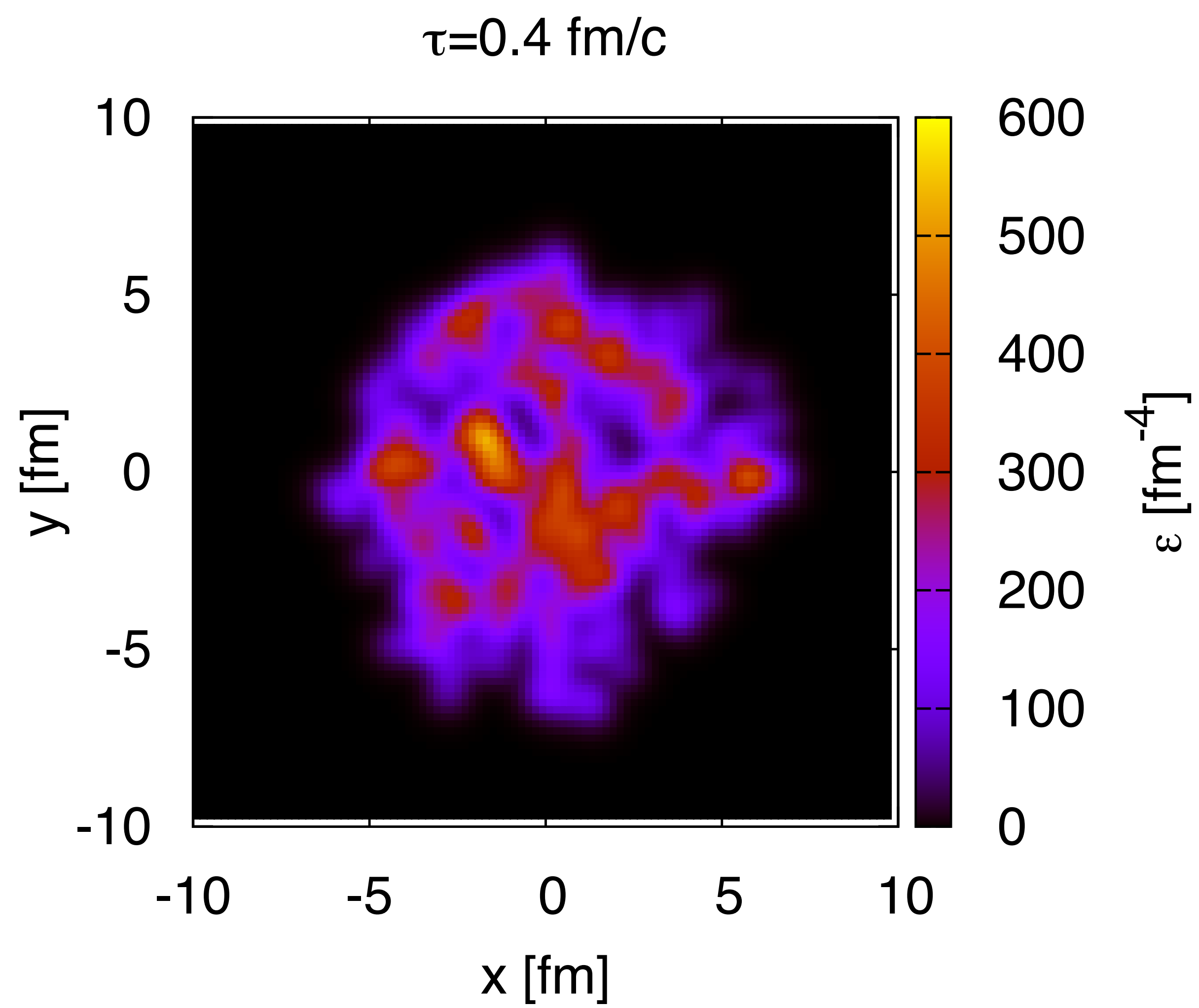
leading order $\approx$ correction

$$T^{\mu\nu}_{hydro} = \boxed{\mathcal{E} u^\mu u^\nu + P_{eq}(\mathcal{E})\,\{g^{\mu\nu} + u^\mu u^\nu\}} - \boxed{\eta(\mathcal{E})\,\sigma^{\mu\nu}} + \ldots$$

# Hydrodynamization: why interesting?

Nuclear physicists necessarily initiate hydro evolution in HICs very early on:

Initial conditions for hydro can be then very extreme: ($\mathcal{E}$ in the transversal plane) →




1009.3244 by Schenke, Jeon & Gale

We just showed that despite large gradients viscous hydro can nevertheless be OK

This is a valuable and unanticipated pheno insight especially relevant for pA and pp

→ Lecture III (Tue 10:00-10:55): why hydrodynamization does make sense.

# Summary of Lecture 11

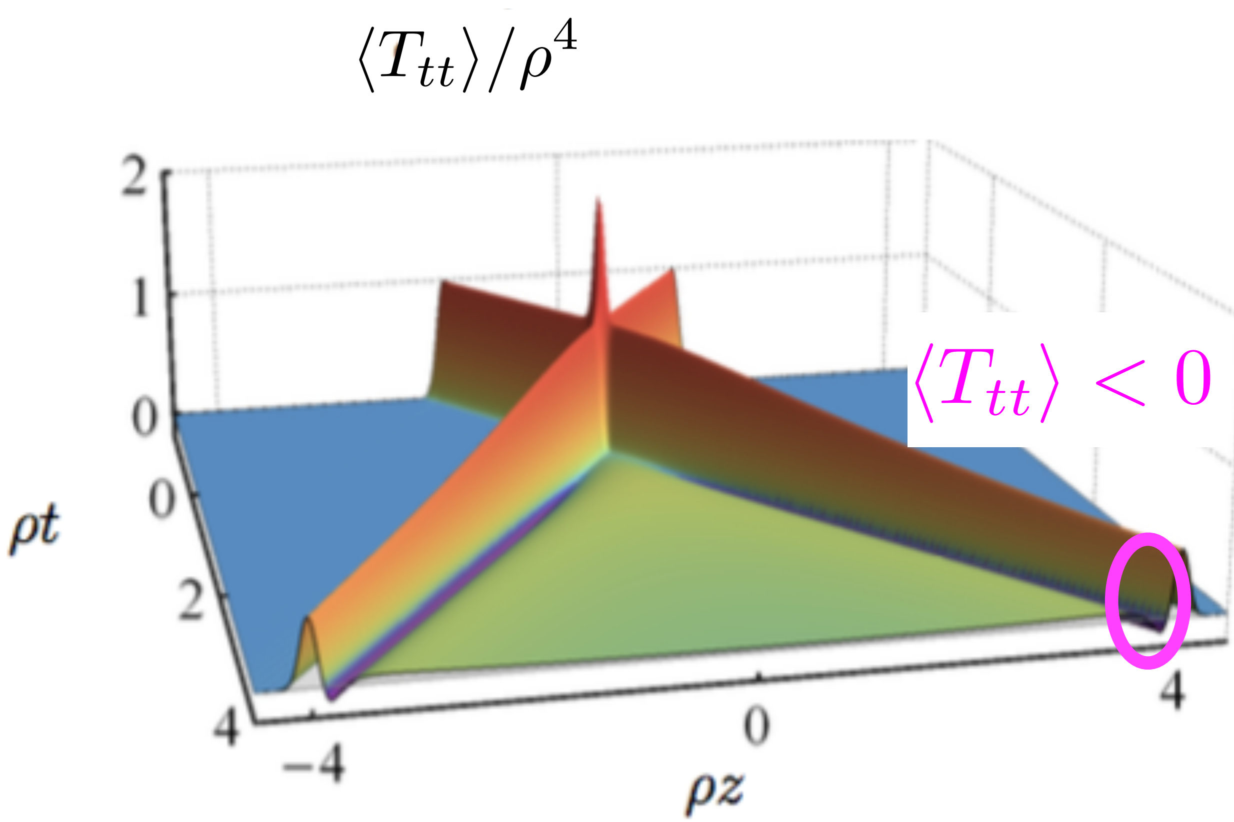
# Notions

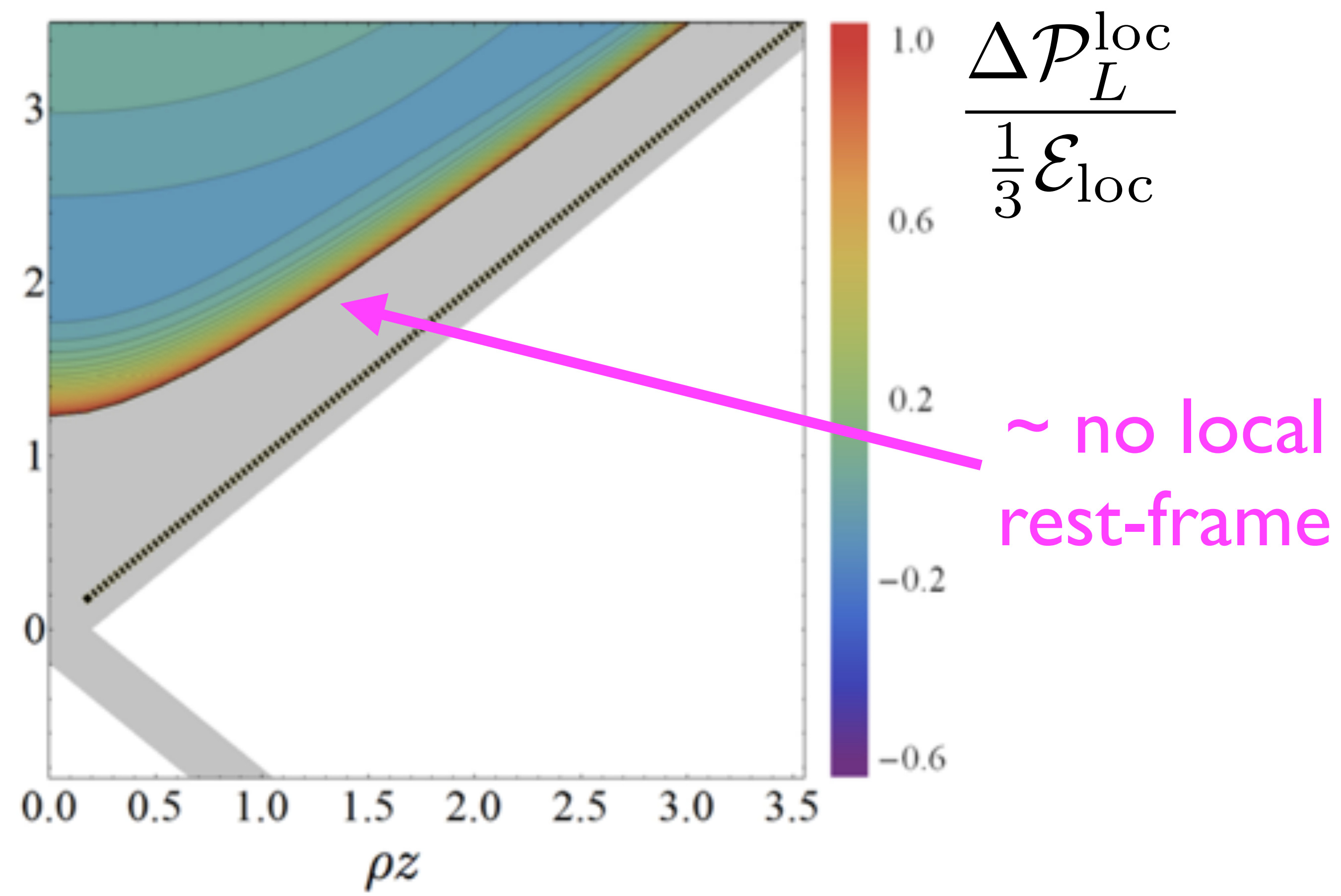
Hydrodynamics:

$$T^{\mu\nu}_{hydro} = \overbrace{\mathcal{E} u^\mu u^\nu + P(\mathcal{E})\left\{g^{\mu\nu} + u^\mu u^\nu\right\}}^{\text{isotropic}} \overbrace{- \eta(\mathcal{E})\,\sigma^{\mu\nu}}^{\text{breaks isotropy}} - \zeta(\mathcal{E})\left\{g^{\mu\nu} + u^\mu u^\nu\right\}(\nabla \cdot u) + \ldots$$

It is clear that $\langle T^{\mu\nu} \rangle$'s will not always be close to satisfying these relations

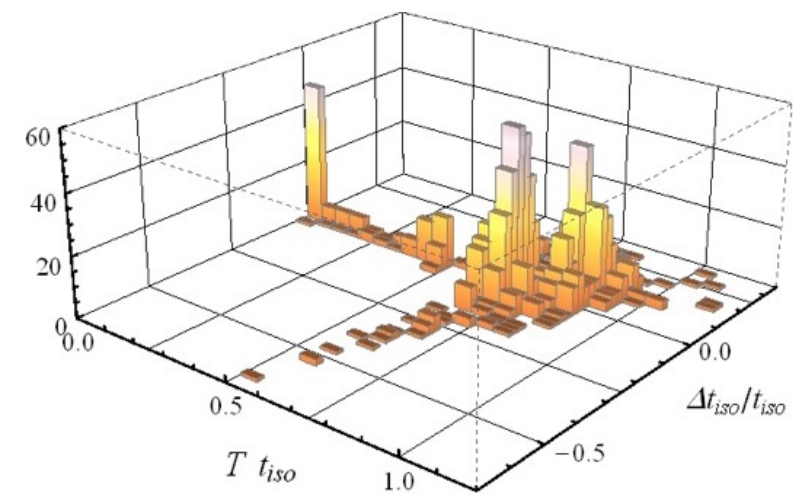
Rich transient physics before hydro:



# Lessons

$t_{hyd}\, T_{hyd} = \mathcal{O}(1)$ as before:




Fast hydrodynamization:

$\neq$

thermalization or isotropization

$\frac{1}{3}\mathcal{E}$

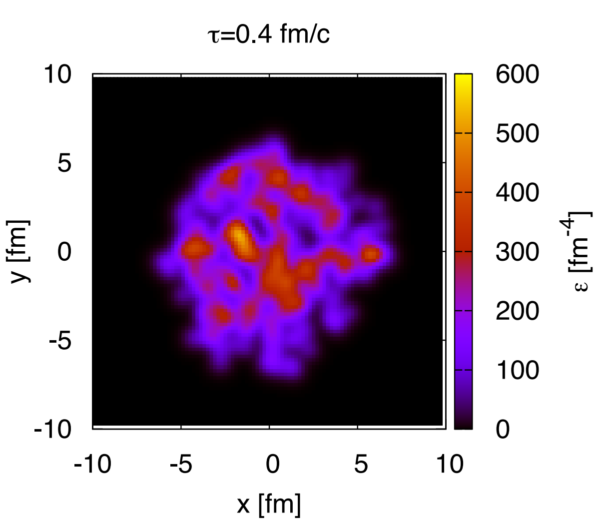
Huge anisotropies at the hydrodynamic threshold: $\mathcal{P}_T - \mathcal{P}_L = 1.35 \times \mathcal{P}_{eq}$

Viscous hydrodynamics constitutive relations work despite:

leading order $\approx$ correction

$$T^{\mu\nu}_{hydro} = \boxed{\mathcal{E} u^\mu u^\nu + P_{eq}(\mathcal{E})\left\{g^{\mu\nu} + u^\mu u^\nu\right\}} - \boxed{\eta(\mathcal{E})\, \sigma^{\mu\nu}} + \ldots$$

Great for pheno:



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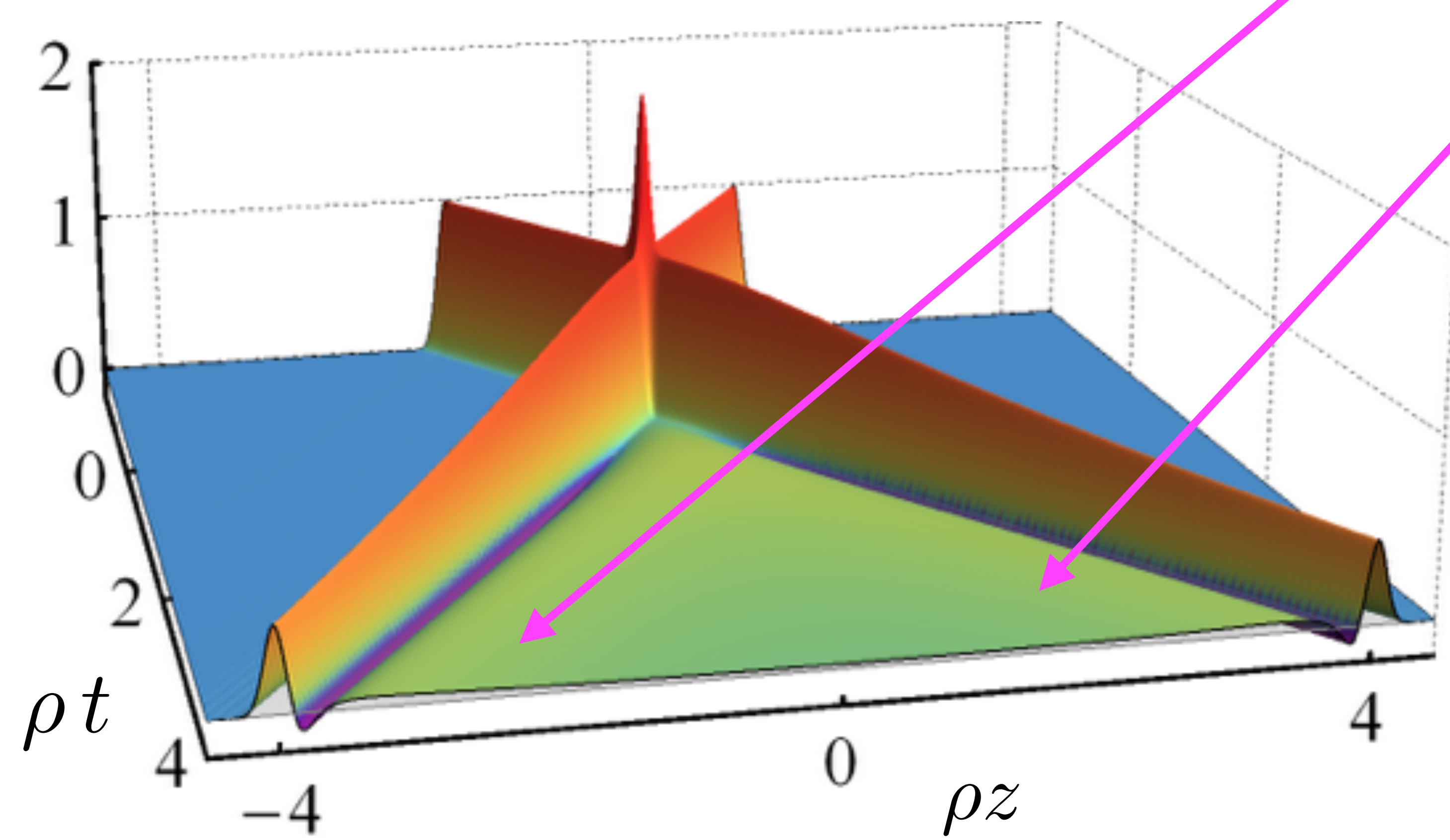
# Advertisement

$$\langle T_{tt}(t, z_1)\, T_{tt}(t, z_2)\rangle = ???$$




Preliminary steps towards this goal:

Philipp Kleinert "Thermalization of Wightman 2-Point Functions in AdS/CFT"

# TODAY 17:00 - 17:20

# Extra

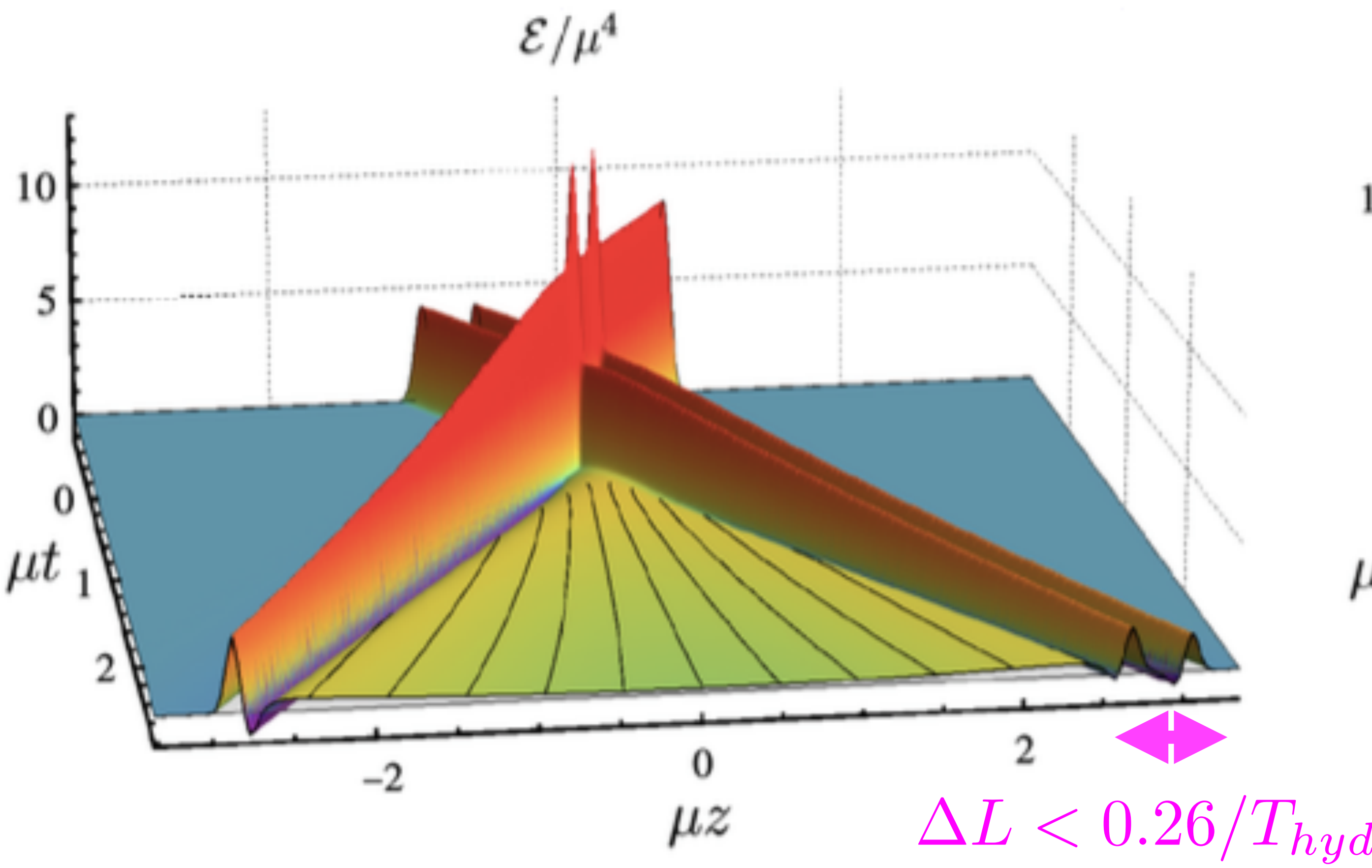
**1312.2956** [PRL 112 221602 (2014)]

with Casalderrey-Solana, Mateos & van der Schee

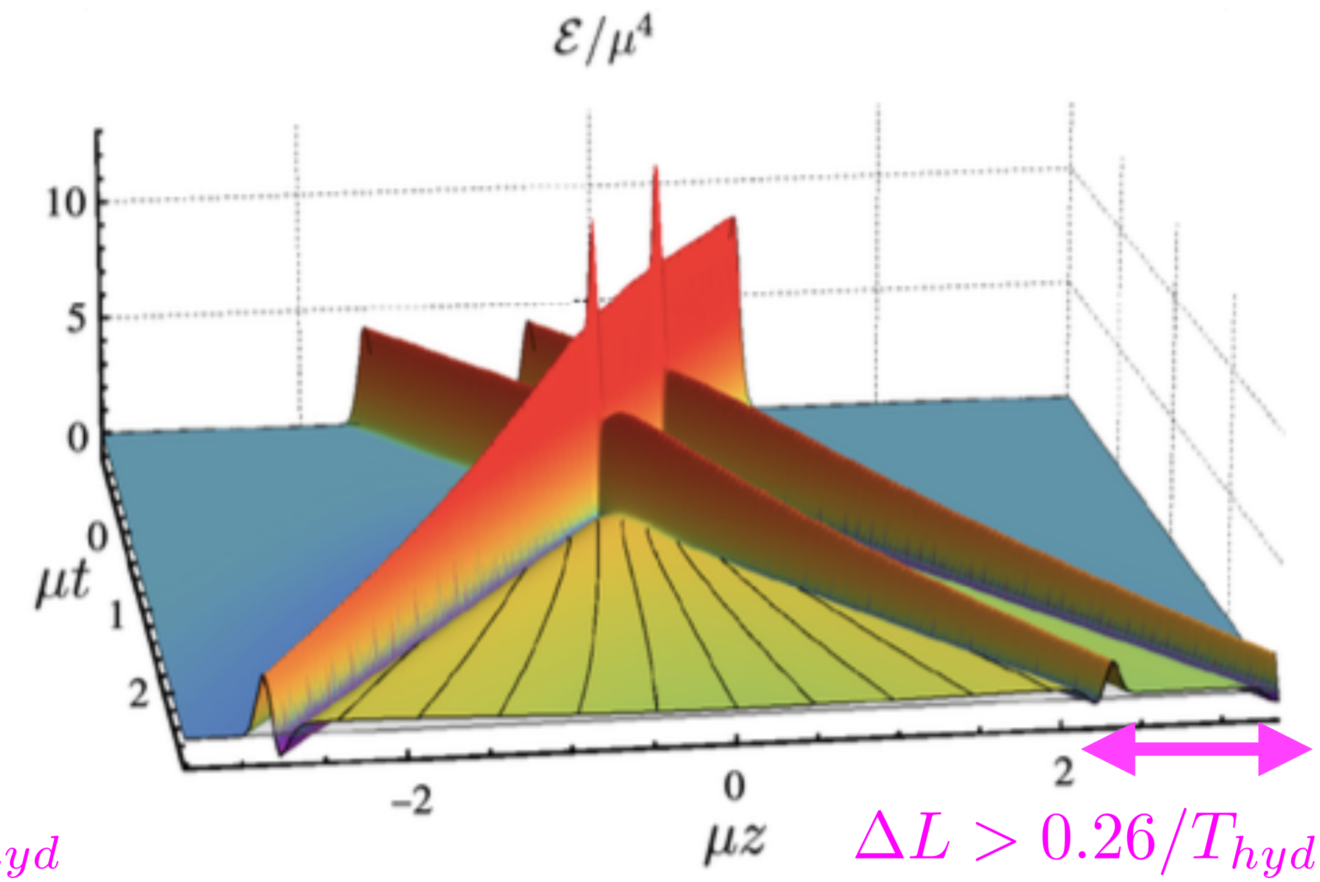
# Collision of asymmetric projectiles

Idea: modelling granular structure of colliding nuclei in the longitudinal direction

centre of mass frame:




behaves coherently (“high energy collisions”)




2 independent collisions

extra