Holography, thermalization and heavy-ion collisions I

Michał P. Heller
Perimeter Institute for Theoretical Physics, Canada
National Centre for Nuclear Research, Poland

1202.0981 [PRL 108 191601 (2012)] with Mateos, van der Schee & Tancanelli
1304.5172 [JHEP 1309 026 (2013)] with Mateos, van der Schee & Triana

Introduction

New kind of string pheno

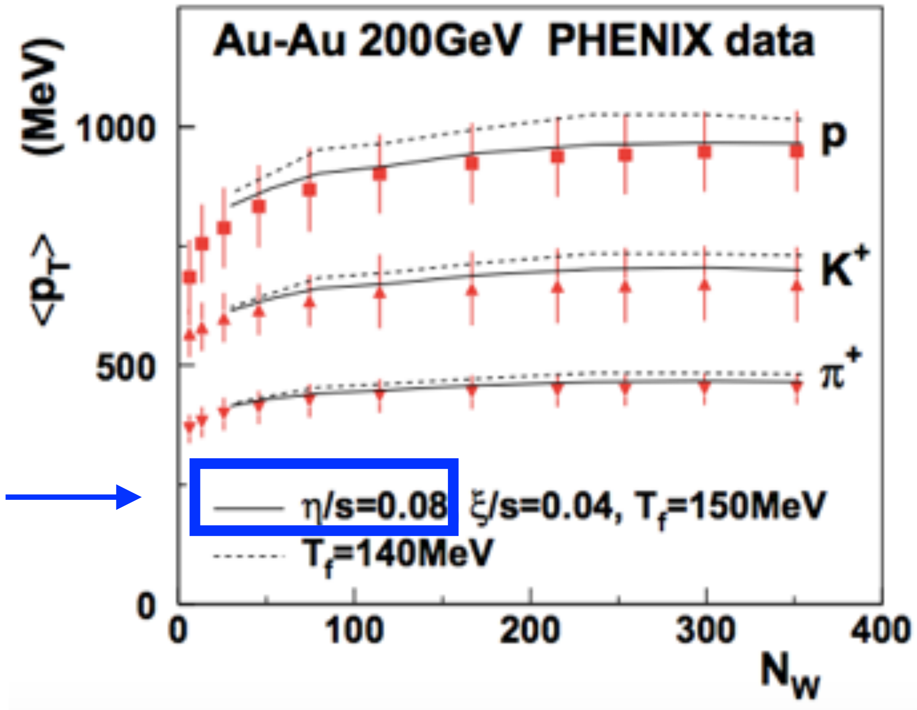
see also lectures by Umut Gursoy and Karl Landsteiner

The natural domain of string pheno is the realm of BSM & early Universe cosmology

1st decade of the 21st century: ↗ holography and 💡 quark-gluon plasma at RHIC

String theory making impact in a brand new way:

“ $\frac{\eta}{s} = \frac{1}{4\pi}$ ” →



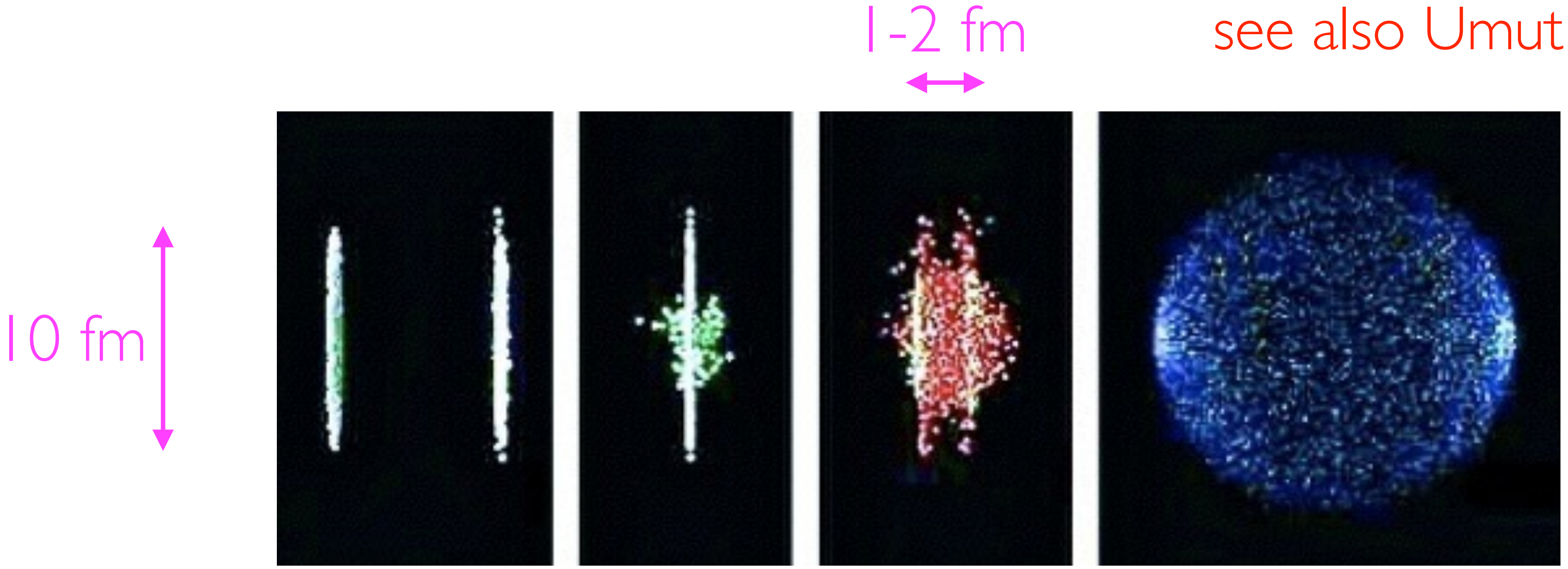
1203.1810

Bożek & Broniowski

Why exciting? Geometrizes certain QFTs. New ab initio tool w/r ↗ weak coupling ↘ lattice

Heavy ion collisions primer (RHIC *2000, LHC *2010)

see also Umut Gursoy's lectures



Successful pheno for soft observables: use hydro, $\langle T^{\mu\nu} \rangle = F[T, u^\alpha]$, as early as **then:**

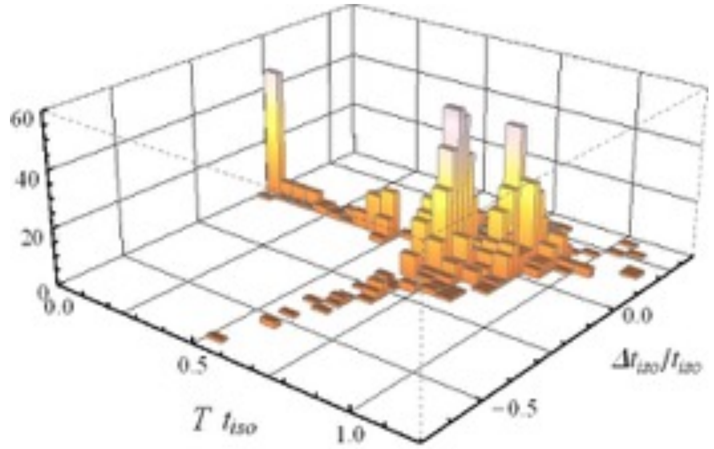
Initial n-eq state is intrinsically anisotropic (expansion axis L vs. transversal plane $\perp$):

$$\left. \frac{\langle T^L{}_L \rangle}{\langle T^\perp{}_\perp \rangle} \right|_{\tau=0} = -1$$

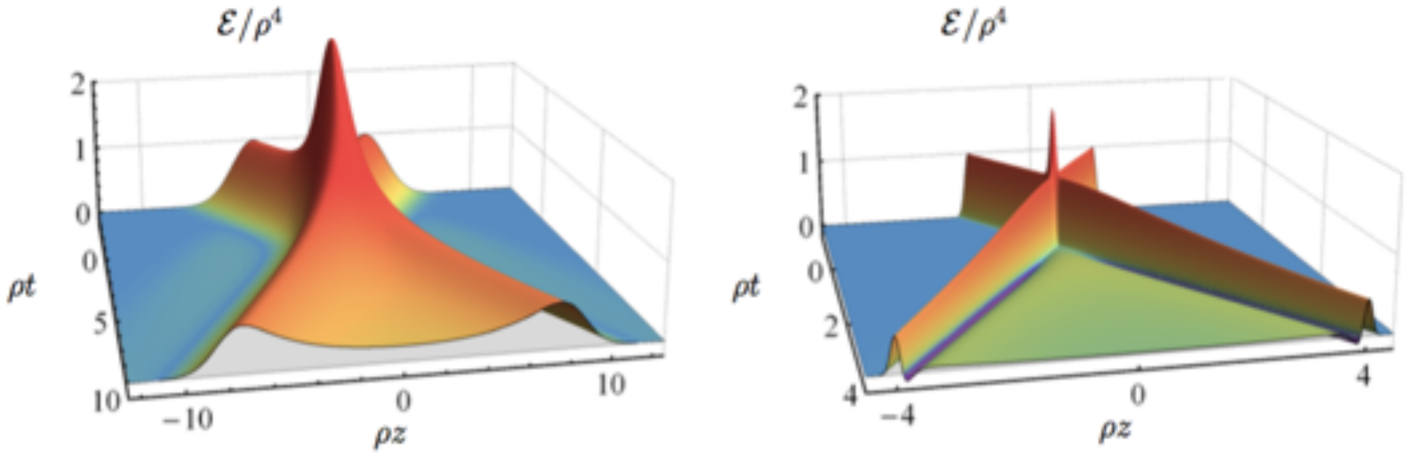
These lectures: ab initio $\langle T^{\mu\nu} \rangle \xrightarrow{\text{time}} F[T, u^\alpha]$ in strongly-coupled QFTs from gravity

Some of the key questions motivating these lectures

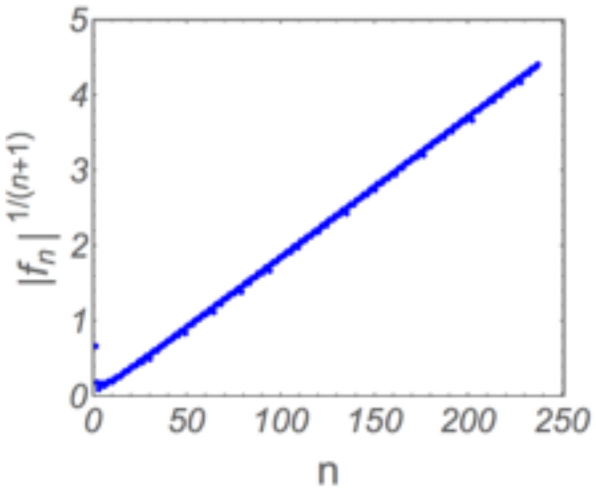
Lecture I: how long does it take $\langle T^{\mu\nu} \rangle$ to equilibrate in strongly-coupled QFTs?

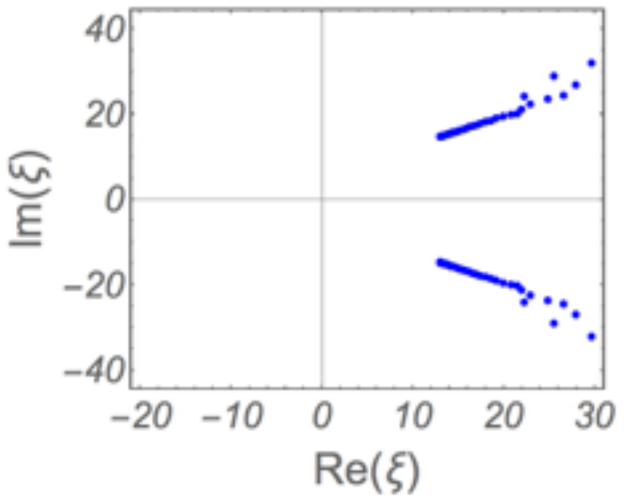


Lecture II: what is $\langle T^{\mu\nu} \rangle(t, \vec{x})$ after a collision of 2 strongly-interacting objects?



Lecture III: what is relativistic hydrodynamics?





AdS gravity (< 2009)

Key notions in holography

Ab initio studies of a large class non-Abelian QFT$_d$'s = understanding geometries$_{d+1}$

Works also for certain non-conformal QFTs, but simplest for (appropriate) CFTs

Geometries$_{4+1}$ (QCD lives in 4D) are governed by the **EOMs** (+ bdry conditions) of:

$$S = \boxed{\frac{1}{2\, l_P^3}} \int d^5x \sqrt{-g} \left(\mathcal{R} + \frac{12}{L^2} + \text{matter} + \boxed{O(\mathcal{R})^2} \right)$$

relying on EOMs: $N_c^2 \sim \dfrac{L^3}{l_P^3} \gg 1$

~ neglecting those: $\lambda = g_{YM}^2 N_c \gg 1$

These lectures: $\mathcal{R}_{ab} - \frac{1}{2}\mathcal{R} g_{ab} - \frac{6}{L^2} g_{ab} = 0$ ~ strongly-coupled $\mathcal{N} = 4$ SYM (CFT)

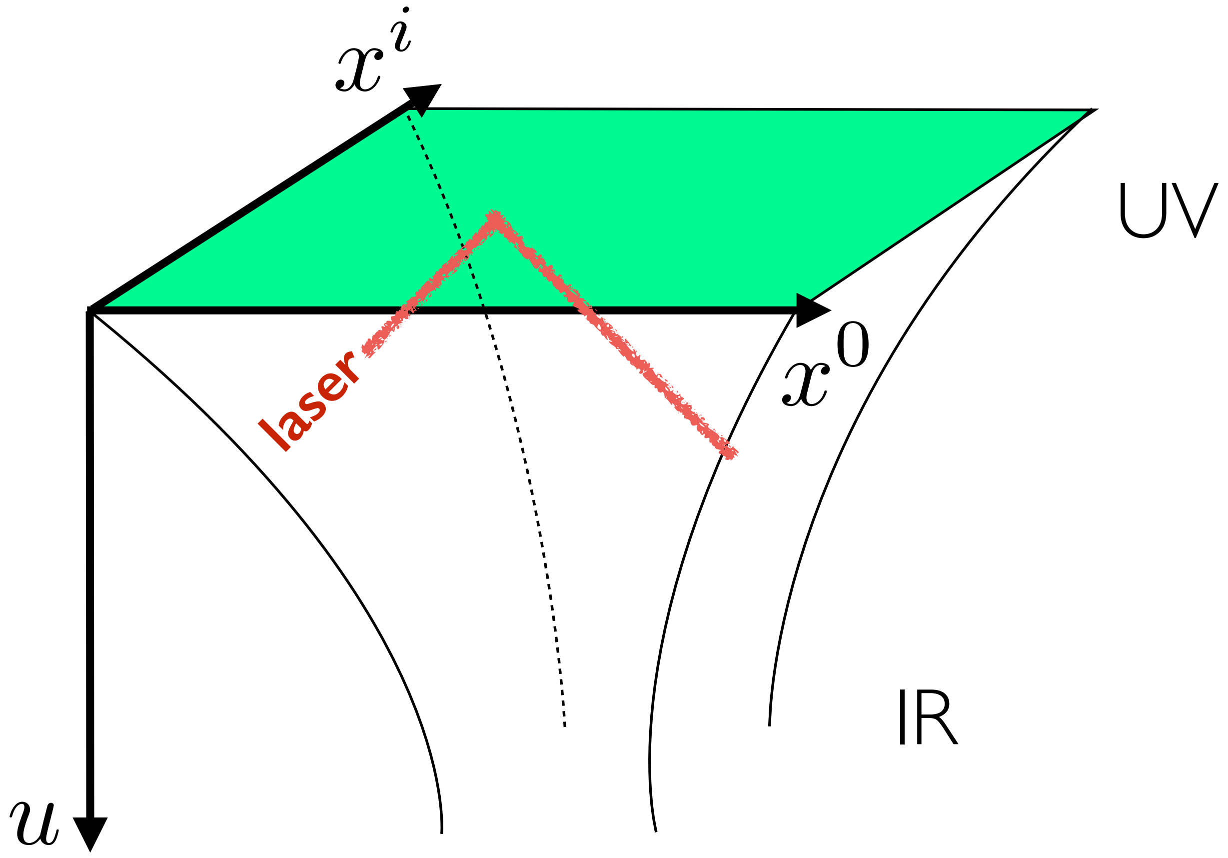
see also Jan Plefka's lectures

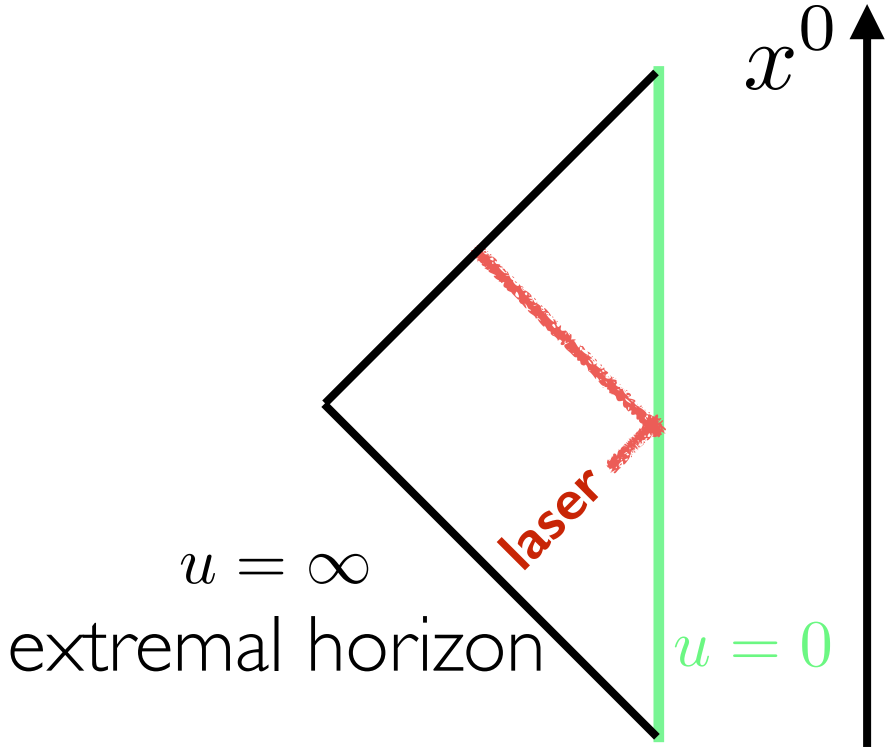
Properties of Anti-de Sitter (AdS) spacetime

CFT$_4$ vacuum = AdS$_5$: $SO(2,4)$ are its isometries

We want $\mathcal{N}=4$ SYM to live in Minkowski space $\eta_{\mu\nu}$ $\longrightarrow$ Poincaré patch:

$$ds^2 = g_{ab}\, dx^a dx^b = \frac{L^2}{u^2}\left\{du^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right\}$$





EOMs require boundary conditions at $u=0$: for g_{ab}, $\eta_{\mu\nu}$ plays this role here

Einstein's equations in AdS and dual $\langle T^{\mu\nu} \rangle$

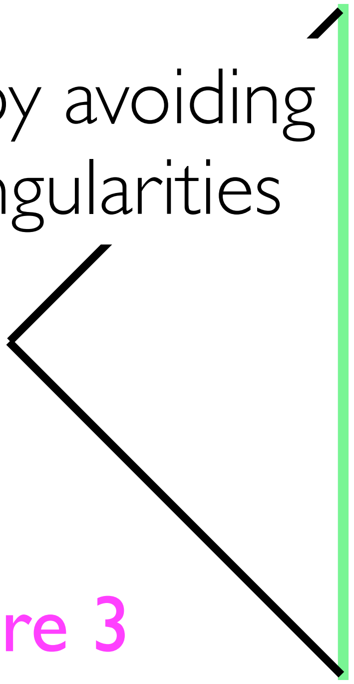
Of course, we are interested in excited states: $ds^2 = \frac{L^2}{u^2}\left\{du^2 + g_{\mu\nu}(u,x)dx^\mu dx^\nu\right\}$

Solving $\mathcal{R}_{ab} - \frac{1}{2}\mathcal{R}g_{ab} - \frac{6}{L^2}g_{ab} = 0$ for $g_{\mu\nu}(u,x)$ around $u = 0$ gives*:

$$g_{\mu\nu}(u,x) = \eta_{\mu\nu} + t_{\mu\nu}\, u^4 + \ldots \quad \text{with} \;\; \eta^{\mu\nu}t_{\mu\nu} = 0 \;\; \& \;\; \partial^\mu t_{\mu\nu} = 0$$

Indeed, one can show that $t_{\mu\nu} = \mathcal{C} \times \langle T_{\mu\nu} \rangle$ with, for $\mathcal{N} = 4$ SYM, $\mathcal{C} = \frac{2\pi^2}{N_c^2}$

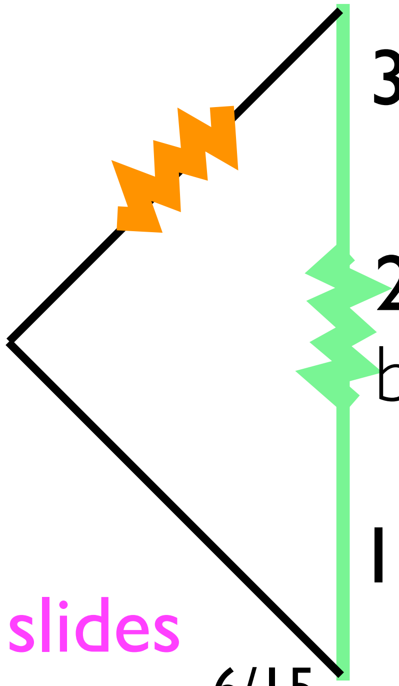
Points of departure:



2) fix it by avoiding naked singularities

1) assume $\langle T_{\mu\nu} \rangle$

Lecture 3

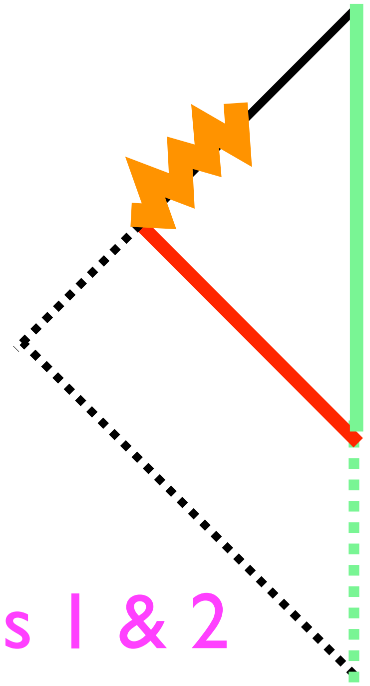


3) get $\langle T_{\mu\nu} \rangle$

2) perturb bdry metric

1) static

next 2 slides



3) get $\langle T_{\mu\nu} \rangle$

2) solve EOMs

1) set $g_{ab}\Big|_{t=0}$

Lectures 1 & 2

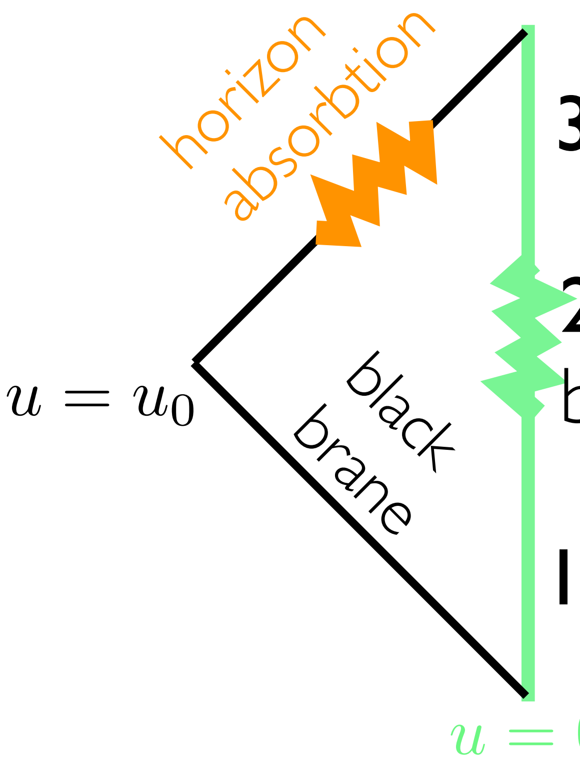
Strongly-coupled QGP = black brane

Equilibrium strongly-coupled QGP: $\langle T_{\mu\nu}\rangle = \mathrm{diag}(\mathcal{E}, P, P, P)_{\mu\nu}$ and $\mathcal{E} = O(N_c^2)$:

$$ds^2 = \frac{L^2}{u^2}\left\{du^2 - \frac{(1-u^4/u_0^4)^2}{1+u^4/u_0^4}dt^2 + (1+u^4/u_0^4)d\vec{x}^2\right\}$$

$$S = \frac{A_{hor}}{4G_N} \xrightarrow{\qquad\qquad} T\,dS = d\mathcal{E}$$

Simplest n-eq states: linear response theory at finite temperature:



3) get $\delta\langle T_{\mu\nu}\rangle = \left\{\int d^3k \int d\omega\, e^{-i\omega t + i\vec{k}\cdot\vec{x}} G_R(\omega, k)\cdot \delta g\right\}_{\mu\nu}$

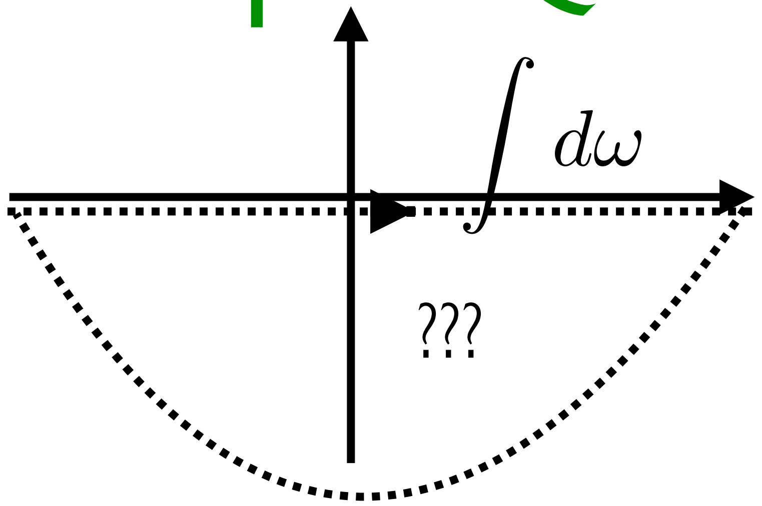
2) perturb bdry metric a bit

1) static plasma

Holographic thermalization = horizon formation* and subsequent equilibration

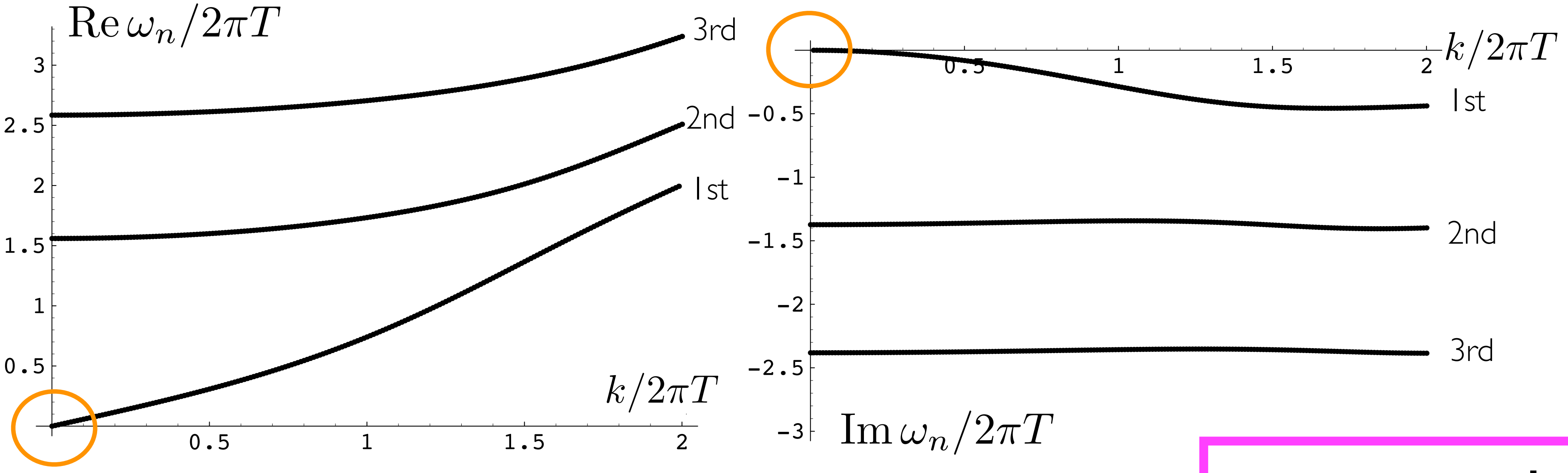
Quasinormal modes: dofs of strongly-coupled QGP

$$\delta\langle T_{\mu\nu}\rangle = \left\{ \int d^3k \int d\omega\, e^{-i\omega t + i\vec{k}\cdot\vec{x}} G_R(\omega, k)\cdot \delta g \right\}_{\mu\nu} :$$



Singularities in the lower-half ω-plane are single poles (QNMs) for each value of k

see hep-th/0506184 by Kovtun & Starinets



Thus $\delta\langle T_{\mu\nu}\rangle = \sum_n \int d^3k\, c_n^{\mu\nu} e^{-\omega_n(k)t + i\vec{k}\cdot\vec{x}}$ with

exponential decay in 1/T

slow decay (hydro)

These lectures: ~ nonlinear interactions between QNMs studied using AdS gravity

Going non-equilibrium (2009 ++): homogeneous isotropization

1202.0981 [PRL 108 191601 (2012)] with Mateos, van der Schee & Tancanelli
1304.5172 [JHEP 1309 026 (2013)] with Mateos, van der Schee & Triana

Homogeneous isotropization

see also **0812.2053** with **Chesler & Yaffe**

What is the simplest n-eq setup available? It is:

$$\langle T_{\mu\nu}\rangle = \text{diag}\left\{\mathcal{E},\, \frac{\mathcal{E}}{3} - \frac{2}{3}\Delta\mathcal{P}(t),\, \frac{\mathcal{E}}{3} + \frac{1}{3}\Delta\mathcal{P}(t),\, \frac{\mathcal{E}}{3} + \frac{1}{3}\Delta\mathcal{P}(t)\right\}_{\mu\nu}$$

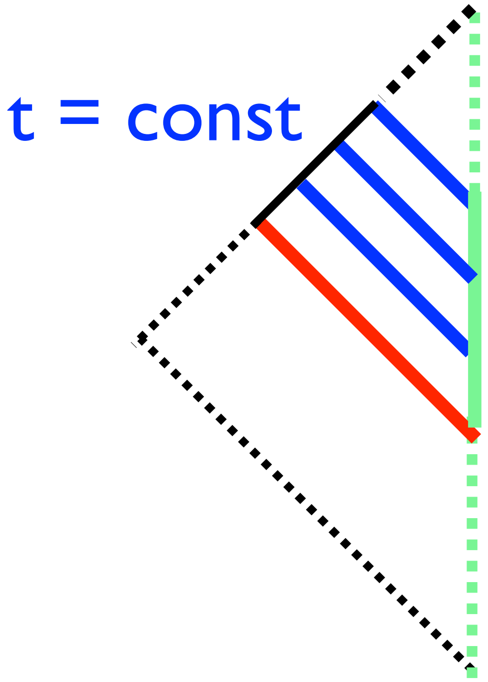
$\mathcal{E} =$ const $= \frac{3}{8} N_c^2 \pi^2 T^4$, no $\vec{x}$ -dependence → no hydro, but sensitive to nonlinearities

functions of t and r

Dual metric ansatz: $$ds^2/L^2 = -\frac{2\,dt\,dr}{r^2} - A\,dt^2 + \Sigma^2 e^{-2B} dx_L^2 + \Sigma^2 e^{B} d\mathbf{x}_P^2$$

$\sim \mathcal{E}$

$\sim \Delta\mathcal{P}(t)$



t = const

3) get $\Delta\mathcal{P}(t)$

2) solve EOMs

1) set $g_{ab}\Big|_{t=0}$

numerical GR →

see **1309.1439** by **Chesler & Yaffe** & **1407.1849** by **van der Schee**

3 dynamical eqs. to ~get $\partial_t A\,, \partial_t \Sigma$ & $\partial_t B$

2 constraints (1 on initial data)

Initial conditions

Let us solve all Einstein's eqs. near the boundary, i.e. for $r = 0$ and we look at B :

$$B = -\frac{b(t)}{2r^4} - \frac{b'(t)}{2r^5} - \frac{7b''(t)}{24r^6} - \frac{b^{(3)}(t)}{8r^7} + \ldots \text{ with } b(t) = \frac{2\pi^2}{3N_c^2}\Delta\mathcal{P}(t)$$

Solving $\langle T_{\mu\nu}\rangle(t,\vec{x})$ for $t > 0$ for requires knowing $(\partial_t)^n \langle T_{\mu\nu}\rangle\Big|_{t=0}$ for all $n \geq 0$

~ you need to specify occupation numbers $c_{\mu\nu}$ for all QNMs

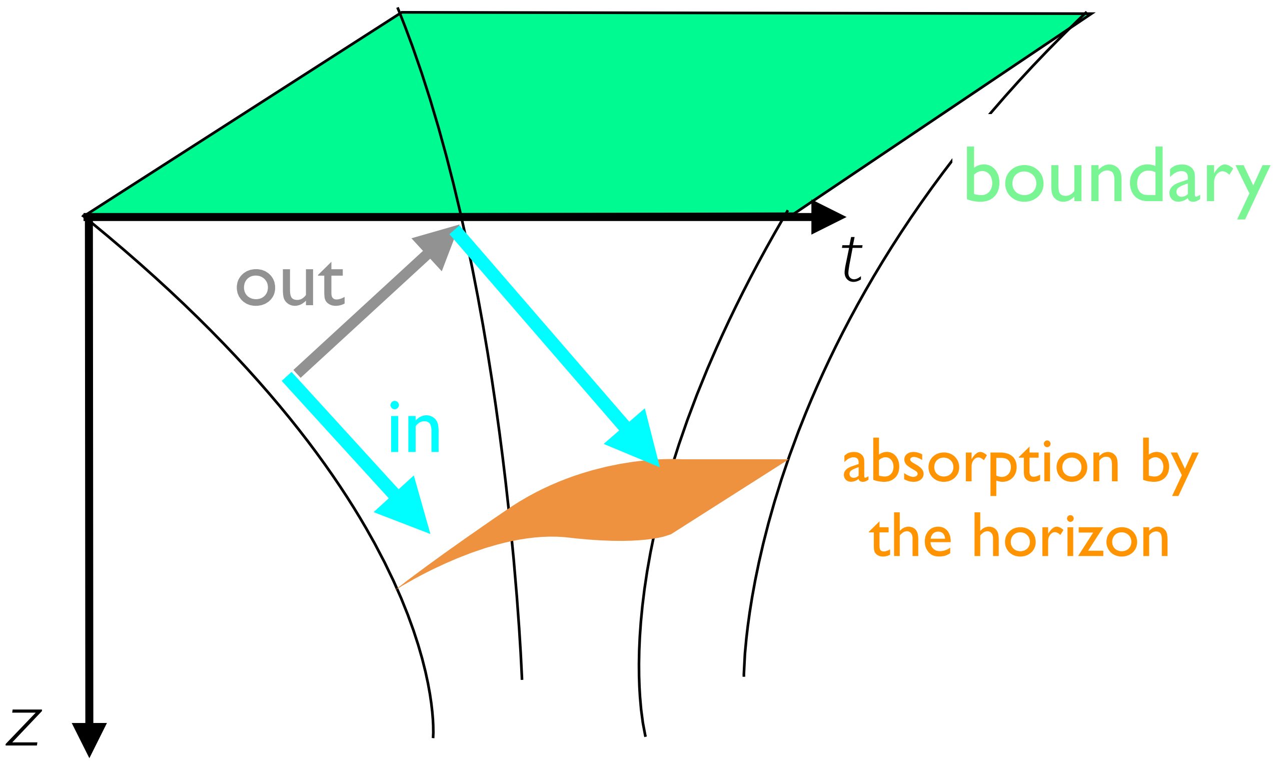
see also **0906.4423** with **Beuf, Janik & Peschanski**

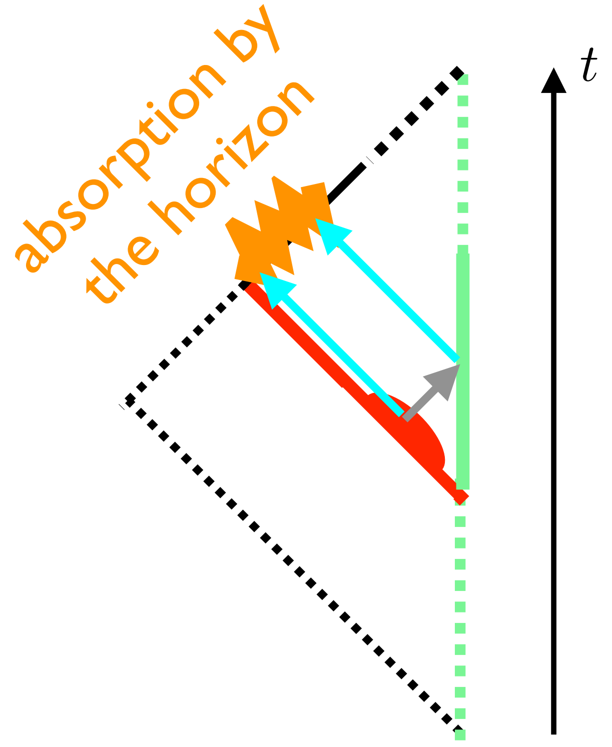
Not all $B\Big|_{t=0}$ will do: some lead to naked singularities. Nontrivial conditions on $\langle T_{\mu\nu}\rangle$

see also **0806.2141** by **Janik & Witaszczyk**

Holographic thermalization

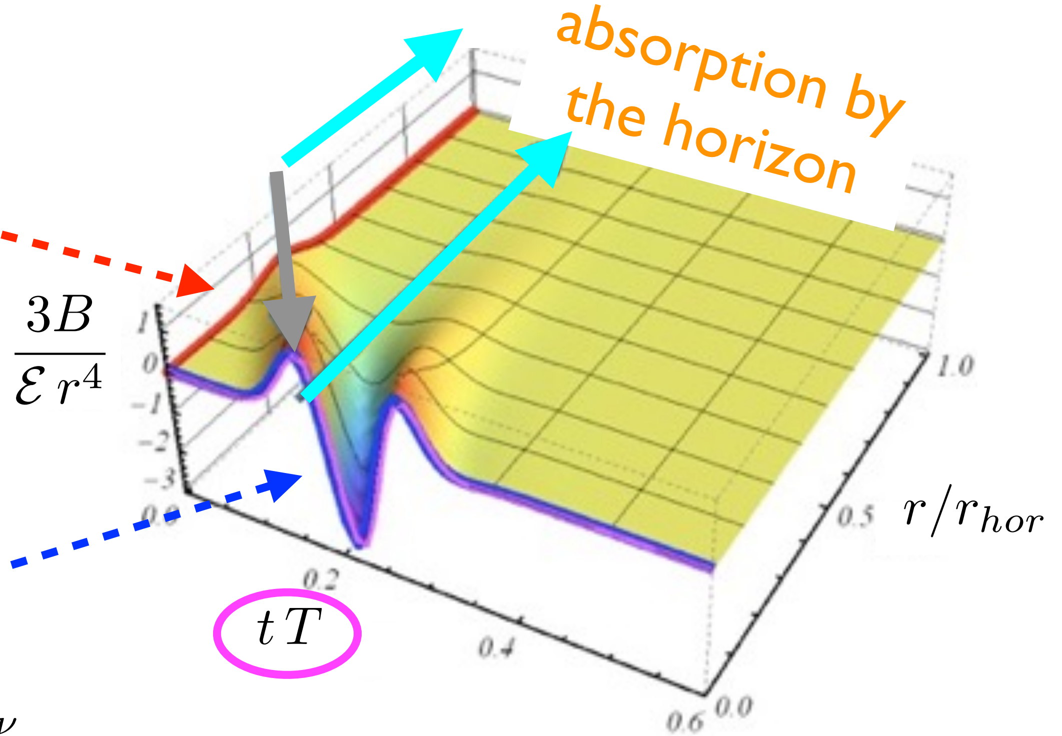
Theory:





Numerical experiment:

initial profile

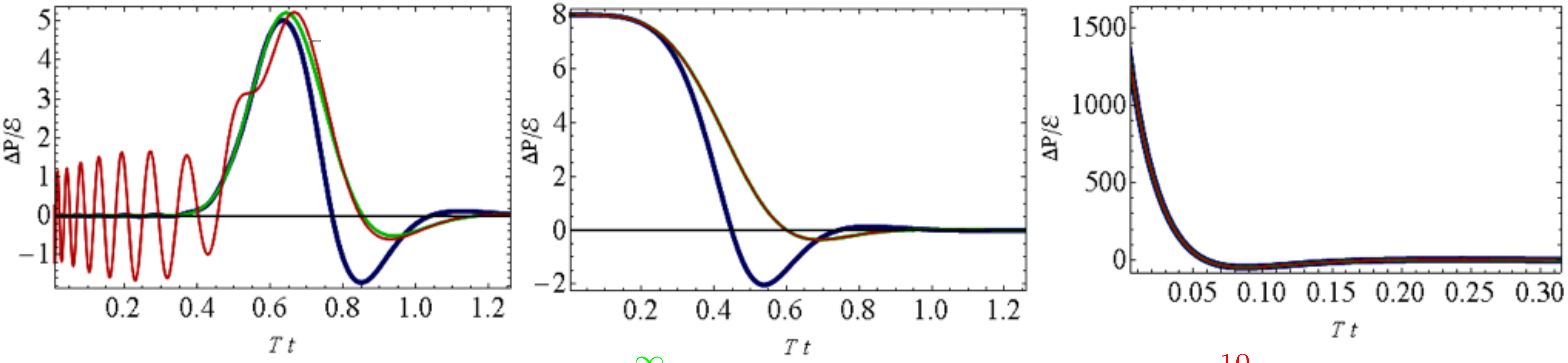


$$\langle T_{\mu\nu}\rangle = \mathrm{diag}\left\{\mathcal{E},\ \frac{\mathcal{E}}{3}-\frac{2}{3}\Delta\mathcal{P}(t),\ \frac{\mathcal{E}}{3}+\frac{1}{3}\Delta\mathcal{P}(t),\ \frac{\mathcal{E}}{3}+\frac{1}{3}\boxed{\Delta\mathcal{P}(t)}\right\}_{\mu\nu}$$

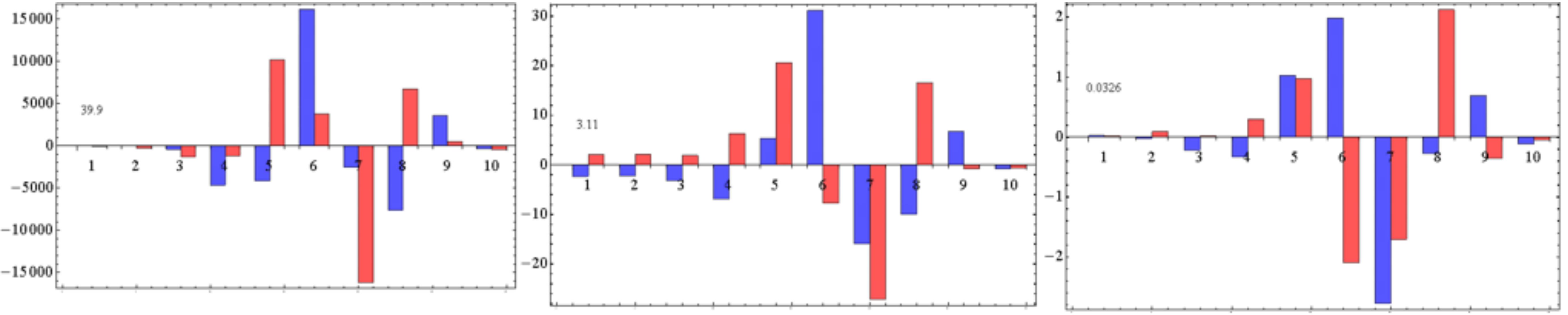
We watch genuinely n-eq states relax the way they want to relax (bdry: $\eta_{\mu\nu}$)!

Sample processes vs. linear response theory

1202.0981, 1304.5172



above: $\Delta\mathcal{P}/\mathcal{E}$ full $\quad \Delta\mathcal{P}/\mathcal{E} = \sum_{n=1}^{\infty} c_n\, e^{-i\,\omega_n(k)t} + \mathrm{cc} \quad \Delta\mathcal{P}/\mathcal{E} = \sum_{n=1}^{10} c_n\, e^{-i\,\omega_n(k)t} + \mathrm{cc}$



above: the corresponding Re and Im of c_n's

Surprising linearity despite seemingly large deviations from equilibrium

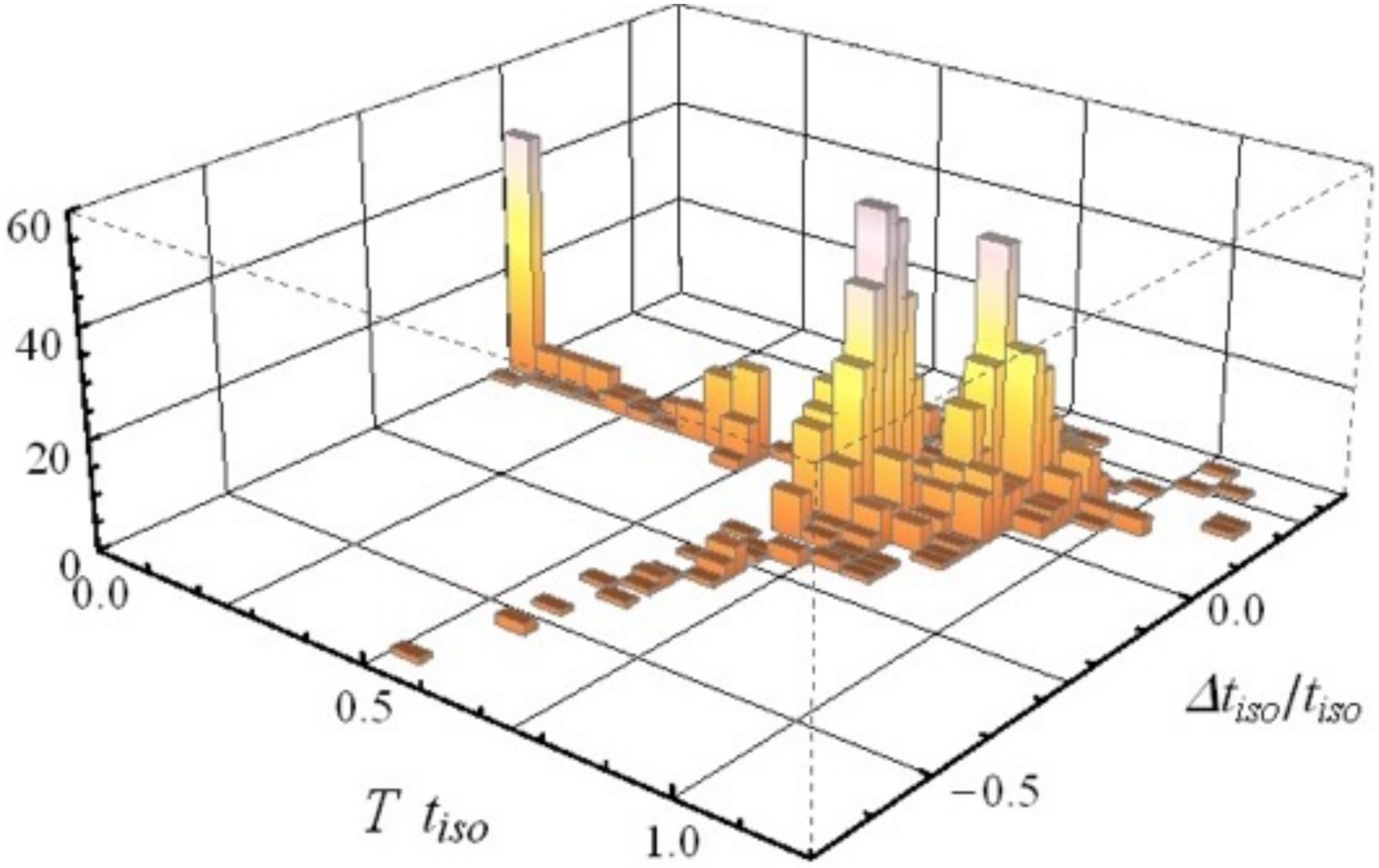
Genericity of 1/T relaxation time at strong coupling

Homogeneous isotropization: $\langle T_{\mu\nu}\rangle = \mathrm{diag}\left(\mathcal{E}, \frac{1}{3}\mathcal{E} - \frac{2}{3}\Delta\mathcal{P}(t), \frac{1}{3}\mathcal{E} + \frac{1}{3}\Delta\mathcal{P}(t), \frac{1}{3}\mathcal{E} + \frac{1}{3}\Delta\mathcal{P}(t)\right)_{\mu\nu}$

1000 different excited states:

all equilibrate within 1.2 / T

pheno: 1 fm × 400 MeV = $O(1)$



surprising linearity

By now confirmed in many other setups (see also Lecture 2)

Summary of Lecture 1

Notions

certain states of a class of strongly-coupled QFTs = higher dimensional geometries

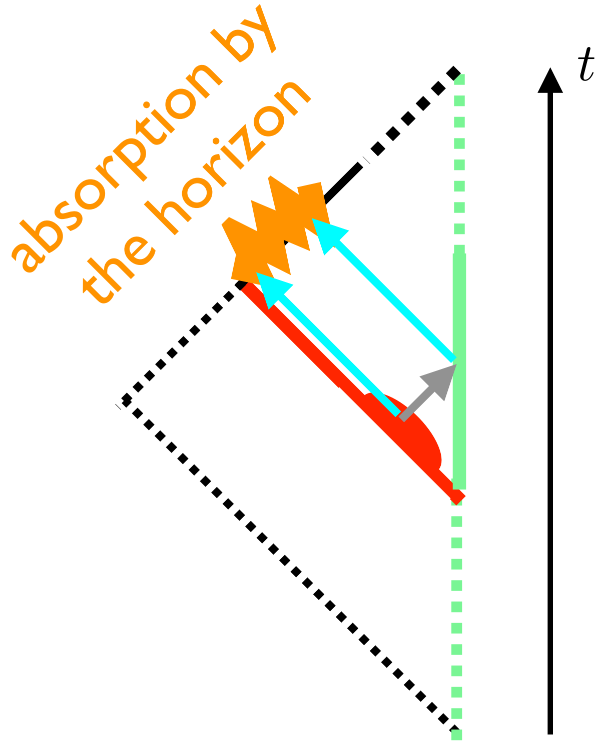
dofs of strongly-coupled QGP = QNMs of dual black branes:

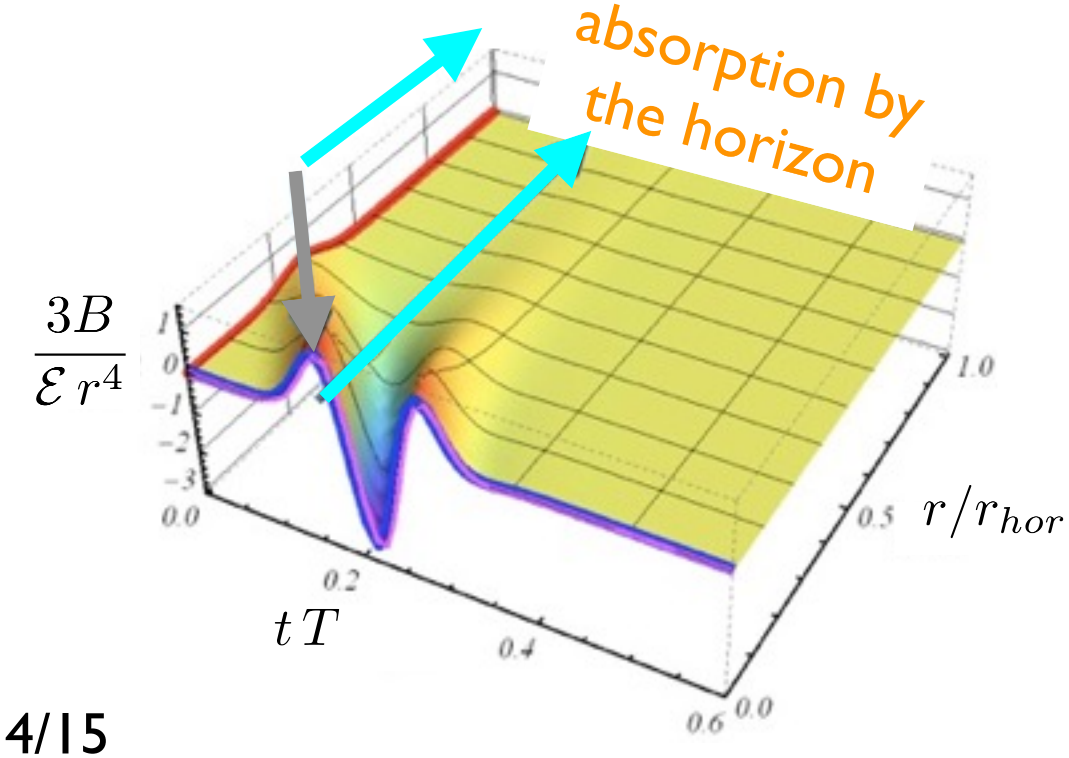
$$\delta\langle T_{\mu\nu}\rangle = \sum_n \int d^3k\, c_n^{\mu\nu} e^{-\omega_n(k)t+i\vec{k}\cdot\vec{x}}$$ with

exponential decay in 1/T

slow decay (hydro)

equilibration in strongly coupled QFTs = dual horizon formation and equilibration:





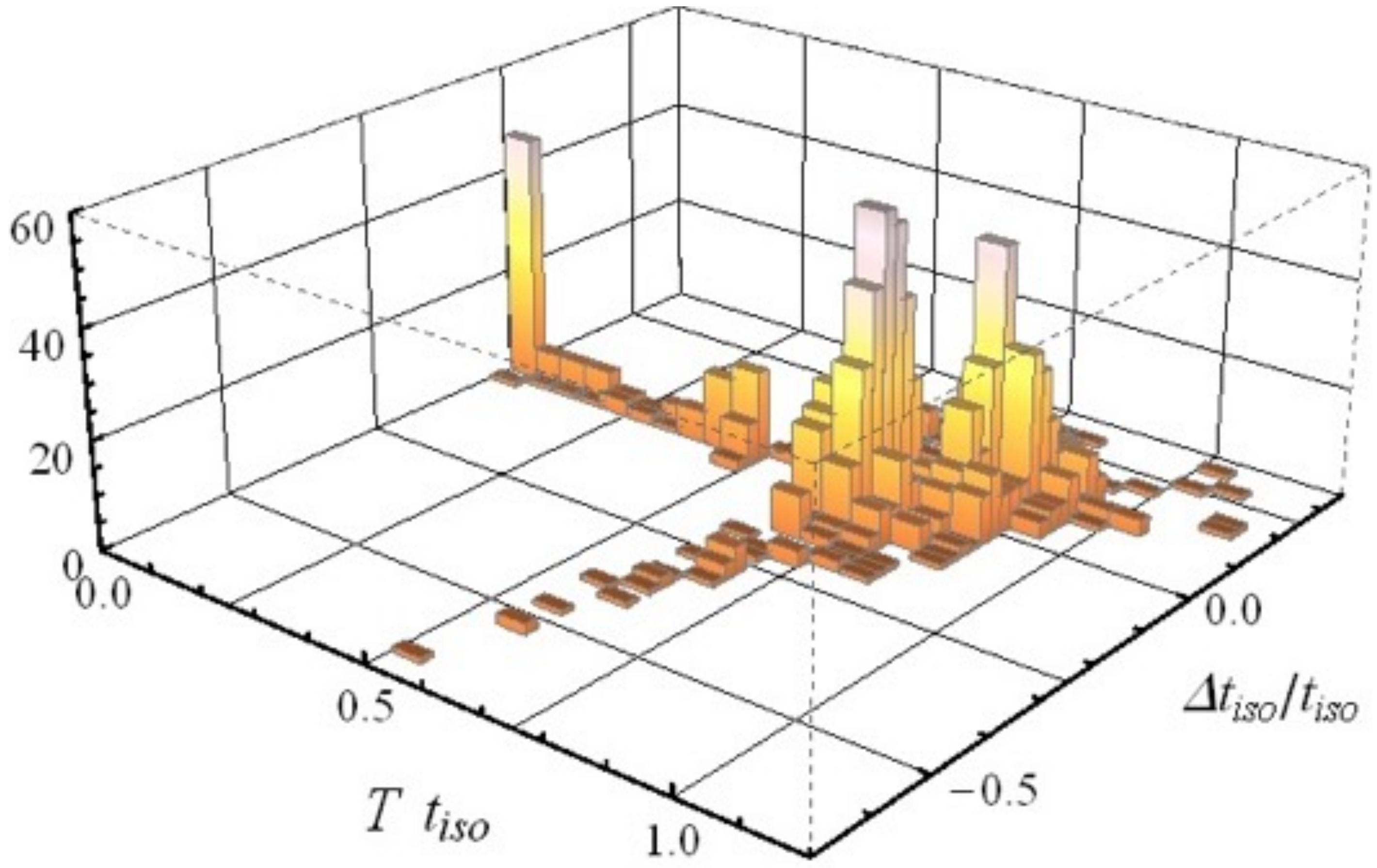
Lessons

Real-time dynamics of QFTs requires ∞-many initial conditions:

$$B = -\frac{b(t)}{2r^4} - \frac{b'(t)}{2r^5} - \frac{7b''(t)}{24r^6} - \frac{b^{(3)}(t)}{8r^7} + \dots \text{ with } b(t) = \frac{2\pi^2}{3N_c^2}\Delta\mathcal{P}(t)$$

Holography makes it manageable by adding r $\longrightarrow$ $\mathcal{R}_{ab} - \frac{1}{2}\mathcal{R}g_{ab} - \frac{6}{L^2}g_{ab} = 0$

Indications that equilibration in 1/T at strong coupling can be generic:



Confirmed in many other setups*. Is $t_{eq}\,T = O(1)$ becoming new "$\eta/s = 1/4\pi$"?