AdS/QCD and the Quark Gluon Plasma

Umut Gürsoy

Utrecht University

Zakopane - May, 2016

References

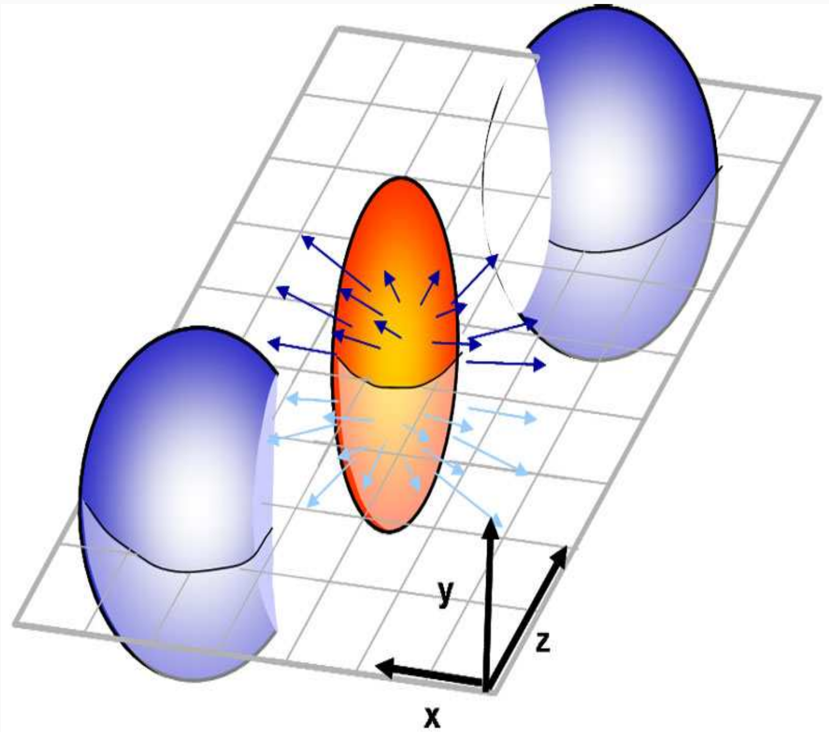
- Based on the papers
  U.G., E. Kiritsis, L. Mazzanti, F. Nitti arXiv:1006.3261
  U.G., E. Kiritsis, F. Nitti, G. Michalogiorgakis arXiv:0906.1890
  U.G., E. Kiritsis, F. Nitti, L. Mazzanti arXiv:0903.2859
  U.G., E. Kiritsis, F.Nitti arXiv:0707.1349
  U.G., E. Kiritsis arXiv:0707.1324

- Reviews on Heavy Ion Collisions and AdS/CFT
  arXiv:1101.0618 — Exhaustive, emphasis on AdS/CFT
  arXiv:0902.3663—Hydrodynamics for HIC
  arXiv:1102.3010—RHIC/LHC results and elliptic flow
  arXiv:1006.546—Non-conformal holographic QCD
  arXiv:1211.6245—Quark-gluon plasma under magnetic fields

Outline

- Lecture I:
  - Heavy ion collisions, observables
  - Relation to QFT correlators
  - Thermalization, hydrodynamics
  - Magnetically induced phenomena
- Lecture II:
  - AdS/CFT basics
  - Top-down and bottom-up models
  - The bulk model: vacuum state, particle spectra
  - The bulk model: finite T, thermodynamics
- Lecture III:
  - The bulk model: flavor sector
  - QGP observables
  - Conclusion and Outlook

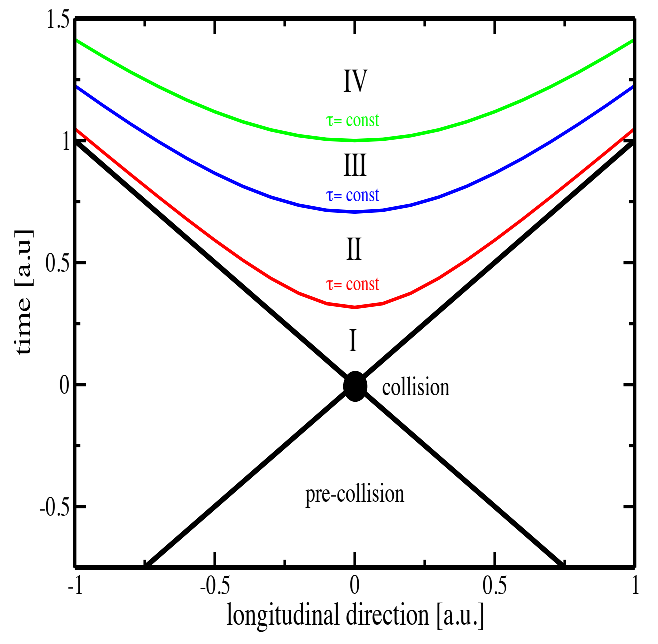
Heavy ion collisions



- RHIC: Au + Au at $\sqrt{s} = 200$ GeV per nucleon; about $T = 200 - 300$ MeV .
- LHC: Pb + Pb at $\sqrt{s} = 2.76$ TeV/n about $T = 300 - 400$ MeV .
- The quark-gluon plasma forms at $\sim$ 0.3-1fm/c and exists for $5 - 10$ fm/c
- Cools down as it expands $\Rightarrow$ and hadronizes around $T = 170$ MeV.

Longitudinal evolution of the QGP

SPS(CERN), RHIC(Brookhaven), LHC(CERN)



Phase I: Pre-equilibrium. Equilibration time $\tau_{QGP} \approx 0.3 - 1$ fm/c.

Phase II: Hydrodynamic phase $\tau \approx 5 - 10$ fm.

Phase III: Hadron gas: after deconfinement at $T \approx 170$ MeV.

Well approximated by the kinetic theory.

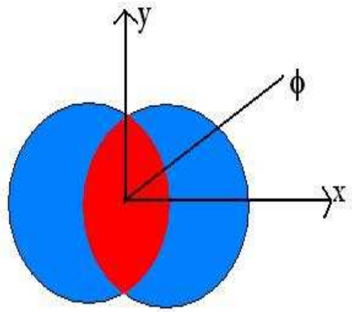
Phase IV: Free streaming of jets towards detectors

Experimental observables

- We want to “look inside" the plasma in phase II
- The time scale of existence 5-10 fm: too short for deep in-elastic scattering, etc.
- Indirect methods to study fluid-like nature of the QGP:
  1. Extract flow parameters from data and compare with hydro simulations
  2. Jet-quenching and energy loss of hard probes

Flow parameters

Azimuthal distribution on the interaction plane:



Definition:

$$p^0 \frac{dN}{dp^3} = v_0(p_\perp, Y) + v_1(p_\perp, Y)\cos(\phi_p) + v_2(p_\perp, Y)\cos(2\phi_p) + \cdots$$

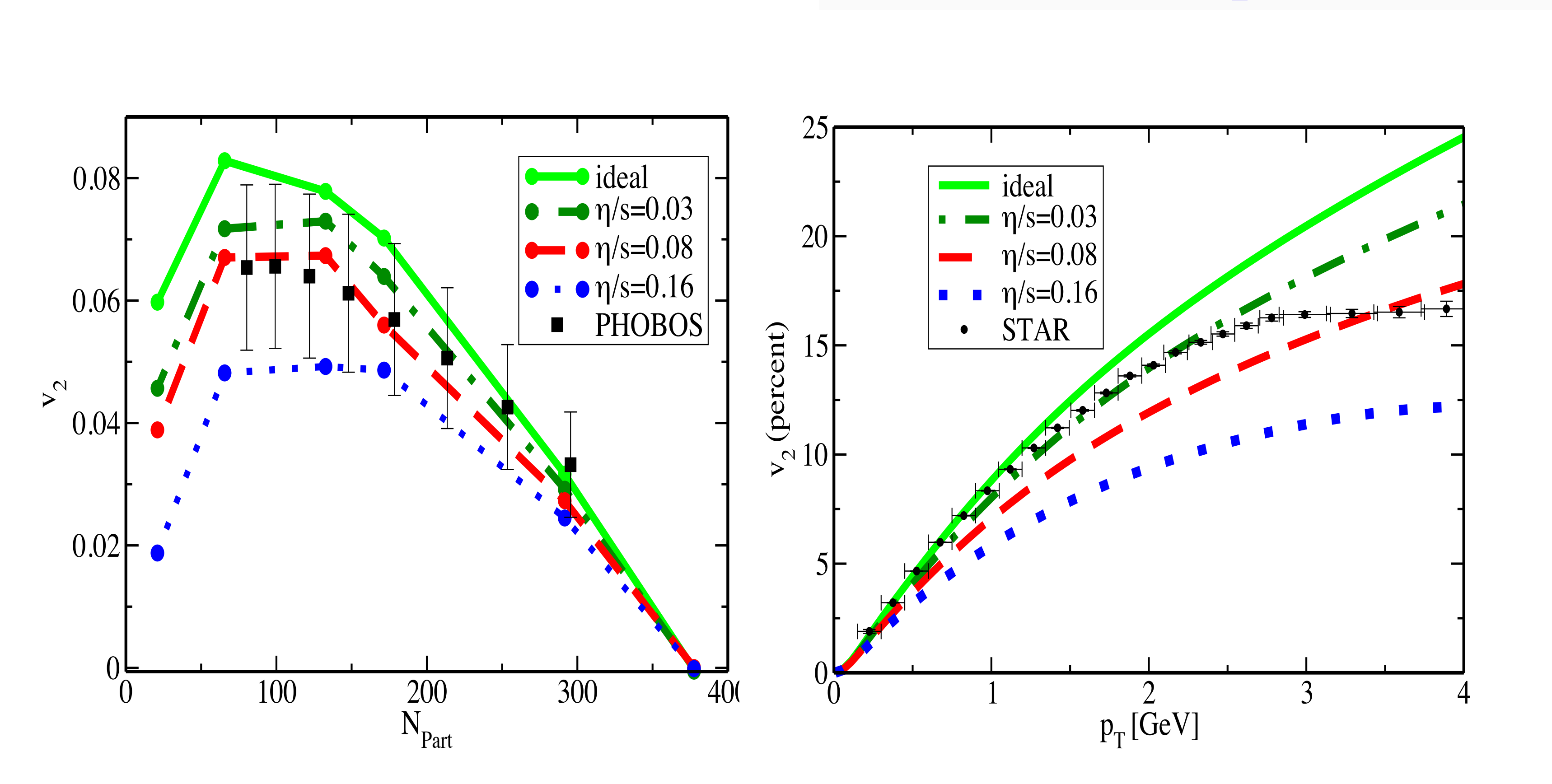
(*) Just $n + n$ collisions and free streaming $\Rightarrow$ purely spherical flow.

(*) Equilibration of QGP at early times $\Rightarrow$ pressure gradients $\Rightarrow$ an-isotropy in space carries over to momentum distribution $\Rightarrow$ elliptic flow, v_2

(*) Off-central collisions: also direct flow, v_1

QGP: viscous fluid vs. ideal fluid

Hydrodynamics provide good account of observed v_2 observables at



Shear viscosity: Response of the fluid to shear deformations.

Larger Shear viscosity η/s: smaller v_2

Simulations vs. RHIC data: $\frac{\eta}{s} \approx 0.03 - 0.15$

The least viscous fluid observed in nature!

$\Rightarrow$ extremely strong interactions.

Hydrodynamics premier

- Effective macroscopic theory for the dynamics of conserved quantities $T^{\mu\nu}$, etc.
- Valid for small k compared to an intrinsic length scale
  e.g. for weak-coupling $\Rightarrow$ kinetic theory of particles
  Hydrodynamic expansion controlled by $k\ell_{mfp}$
- More generally microscopic theory is a QFT.
- If strongly interacting ℓ_{mfp} not well-defined!
- Replaced by some other fundamental scale:
  e.g. confinement scale Λ or temperature T in QCD. (We are interested in the regimes $T \sim \Lambda$)
- Zeroth order in k/T is ideal fluid: just thermodynamics
- Higher orders determined by dissipation coefficients: e.g. η, ζ
  Not computable by hydrodynamics itself: need microscopic description.

Relativistic Hydrodynamics

Consider relativistic, chargeless, one-component fluid characterized by:

- Energy density $\epsilon(t,\vec{x})$,
- Pressure density $p(t,\vec{x})$,
- Four-velocity $u^\mu(t,\vec{x}) = \gamma(v)\binom{1}{\vec{v}}$ with $u^2 = -1$

Ideal fluid: no dissipation (zeroth order in hydro-expansion)

$$T^{\mu\nu}_{(0)} = (\epsilon + p)u^\mu u^\nu + p g^{\mu\nu}$$

Most general Lorentz-covariant expression with
$T^{00} = \epsilon$ and $T^{ij} = p\delta^{ij}$

in local rest frame $u^\mu = (1,\vec{0})$.

Dissipation in relativistic hydrodynamics

- $T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Pi^{\mu\nu}$, what is $\Pi^{\mu\nu}$?
- When no other conserved charge, particle flow is only due energy flow: $u_\mu T^{\mu\nu} = \epsilon u^\mu \Rightarrow u_\mu \Pi^{\mu\nu} = 0$
  Landau reference frame.
- A straightforward exercise: From second law of thermodynamics $\partial_\mu s^\mu = 0$ (with $s^\mu = s u^\mu$)

$$
\begin{aligned}
T^{\mu\nu} &= (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu} \\
&+ P^{\mu\alpha} P^{\nu\beta} \left[\eta \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3} g_{\alpha\beta} \partial \cdot u \right) + \zeta g_{\alpha\beta} \partial \cdot u \right] \\
&+ \mathcal{O}(\partial u)^2; \qquad P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu
\end{aligned}
$$

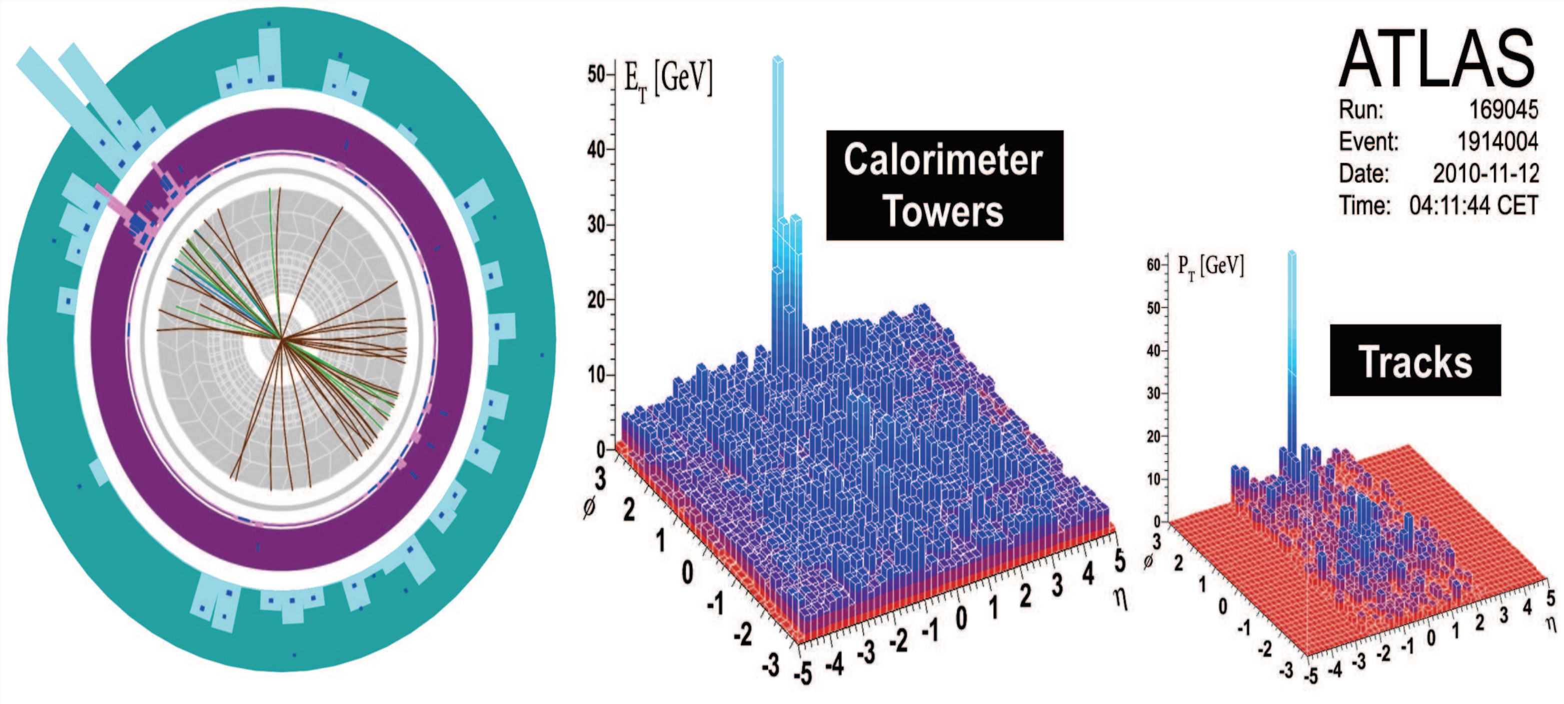
- η: “shear viscosity”; ζ: “bulk viscosity”
- Navier-Stokes and continuity eqs. from $\partial_\mu T^{\mu\nu} = 0$.

Calculation of viscosities in QFT

- Kubo's linear response theory:
  $\mathcal{L} \to \mathcal{L} + \int \mathcal{O}^A \delta\phi_A$,
  then $\langle \mathcal{O}^B \rangle = G_R^{BA} \delta\phi_A$
  where $G_R^{BA}(\omega, \vec{k}) = -i \int d^4x e^{-ik\cdot x} \theta(t) \langle [\mathcal{O}^A(t, \vec{x}), \mathcal{O}^B(0, \vec{0})] \rangle$
- Viscosities: response of $T^{\mu\nu}$ to $g_{\alpha\beta}$.
- $\eta \left(\delta^{il}\delta^{km} + \delta^{im}\delta^{kl} - \frac{2}{3}\delta^{ik}\delta^{lm} \right) + \zeta \delta^{ik}\delta^{lm} = \lim_{\omega\to 0} \frac{i}{\omega} G_R^{ik,lm}(\omega)$
- Read off η from the xy component, and ζ from the $11+22+33$ component.

Hard probes

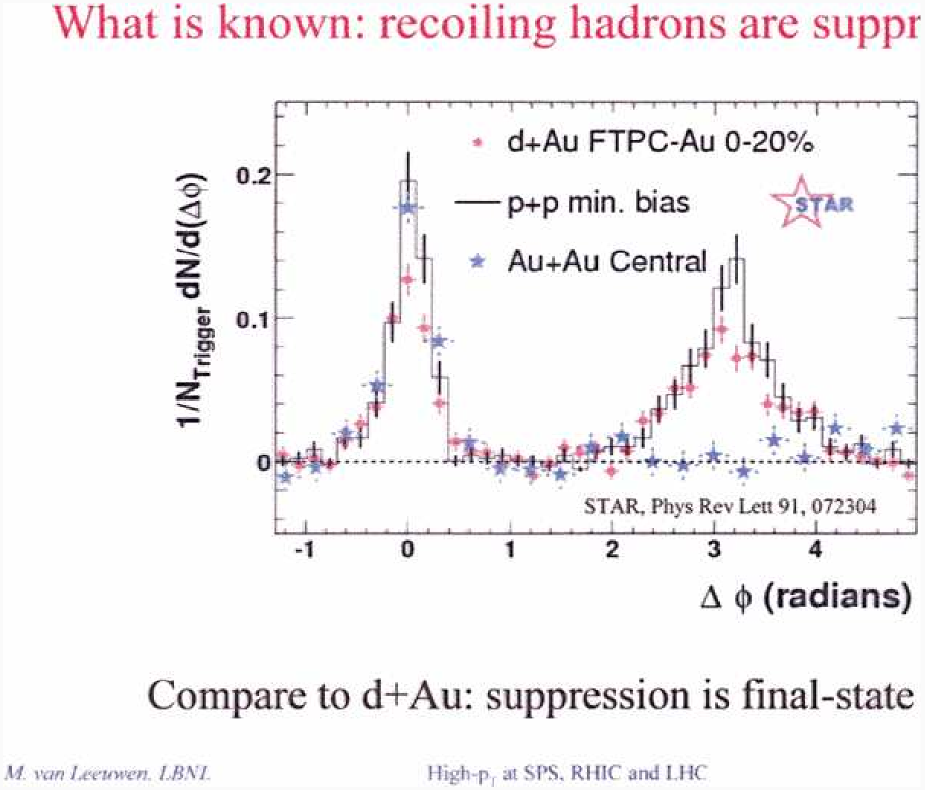
Back-to-back jet production is highly suppressed:

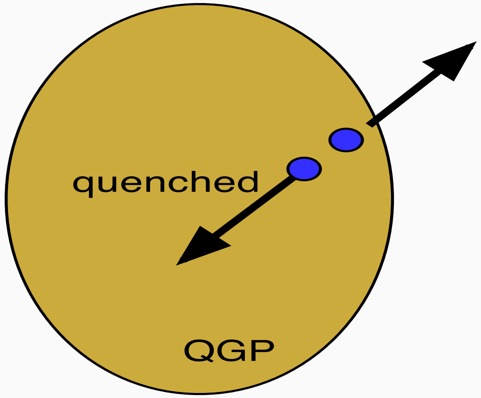


The first direct signals of jet-quenching - November 2010!

A clear signal of strongly-coupled plasma.

Quantification of jet quenching Baier et al ’96





Average transverse momentum loss in distance D.

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{D}$$

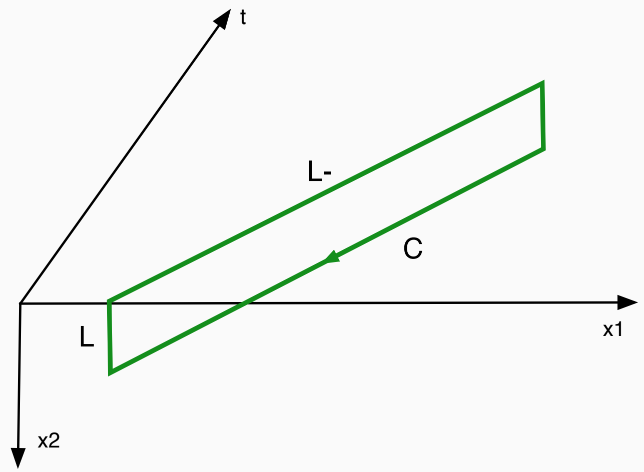
- Can be extracted from the Nuclear Modification factor. Weak-coupling does not explain the data.

Energy loss mechanism I: Langevin diffusion

- Hard probe with velocity v, for times $t \ll \tau_c$ (relaxation time of the probe in plasma)
  Brownian motion is a good approximation:
  $\frac{dp^i}{dt} = -\eta_D^{ij}(\vec{p}^2)p_j + \xi^i(t), \qquad \langle \xi^i(t)\xi^j(t')\rangle = \kappa^{ij}\delta(t-t')$
  η^{ij}: the drag (friction) coefficient; κ^{ij}: Diffusion constants
  Jet quenching parameter: $\hat{q} = 2\frac{\kappa_\perp}{v}$
- Hard probe moving in QGP: $S[X(t)] = S_0 + \int d\tau X_\mu(\tau)\mathcal{F}^\mu(\tau)$
  S_0:free quark action, $\mathcal{F}(\tau)$: drag force—summarizes the d.o.f of the plasma
- Both η^{ij} and κ^{ij} can be obtained from
  $G_R(\omega) = -i\int dt e^{-i\omega t}\theta(t)\langle[F^i(t), F^j(0)]\rangle$
  This is what we will compute using AdS/CFT.

Mechanism II: gluon brehmstahlung

When the main source of energy loss is gluon Brehmstrahlung:
emission of a gluon $\Leftrightarrow$ light-like Wilson loop in Adjoint rep:
Non-perturbative def. of $\hat{q}$:

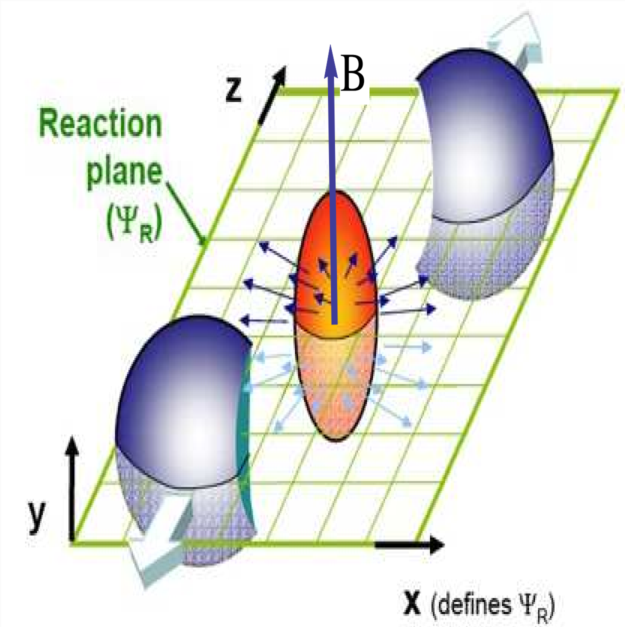


Wiedemann '00

$$\langle W(C) \rangle \approx \exp\left[-\frac{1}{8\sqrt{2}}\hat{q} L^- L^2\right].$$

$\hat{q}$ in this mechanism also computable by AdS/CFT at strong coupling.

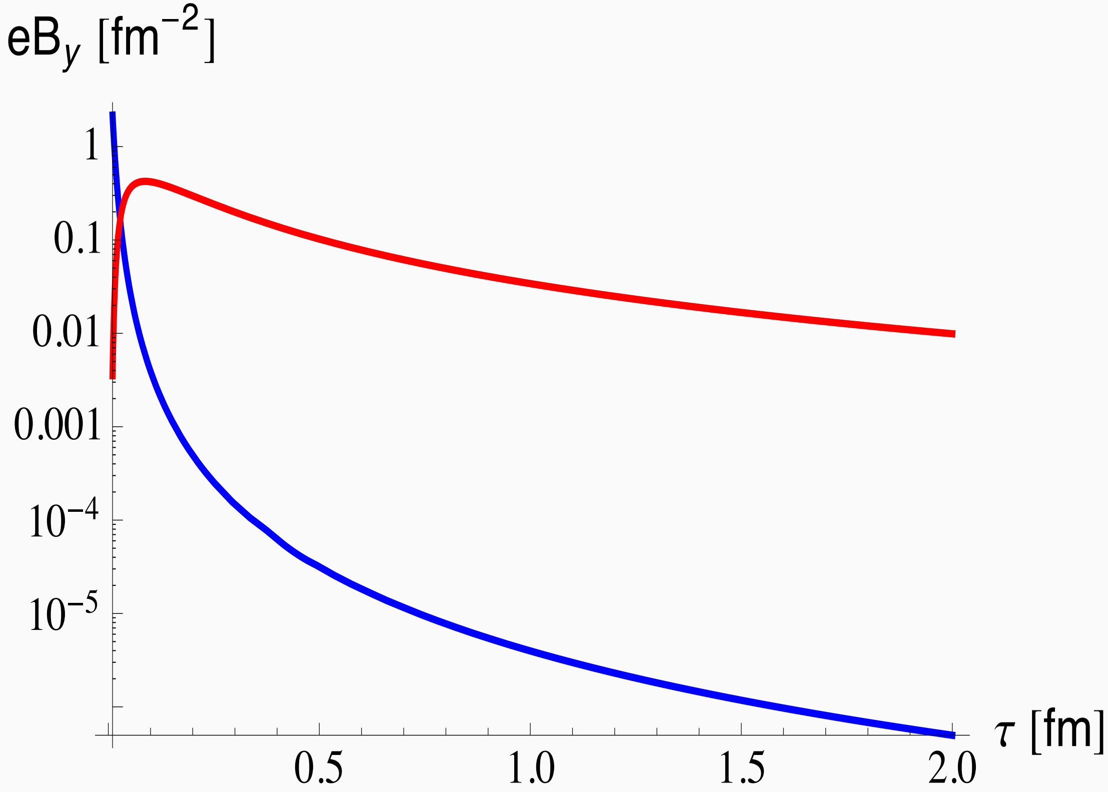
Heavy ion collisions and magnetic fields



- Initial magnitude of B
- Bio-Savart: $B_0 \sim \gamma Z e \frac{b}{R^3} \Rightarrow \sim 10^{18} (10^{19})$ G at RHIC (LHC).
- $B_0 \sim 10^{10} - 10^{13}$G (neutron stars), 10^{15} (magnetars)
- More relevantly $eB \approx 5 - 15 \times m_\pi^2$ RHIC (LHC).

Time profile of B at LHC

Kharzeev, Rajagopal, U.G.'14



- with $\sigma = 0.023\text{fm}^{-1}$ and with $\sigma = 0$:

Magnetically induced currents in QGP

- Quantum mechanical origin
  - Chiral anomaly Kharzeev, McLerran, Warringa ’07
    See lectures of K. Landsteiner
- Classical origin
  - Faraday’s law Kharzeev, Rajagopal, U.G.’14
  - “Hall” effect in expanding plasmas Kharzeev, Rajagopal, U.G.’14
- Both have non-negligible effects on v_2 and also v_1.

Holography: AdS/QCD models

Large-N approximation

QCD with $N_c = 3$ and dynamical quarks is complicated for holography.
Take the large N_c 't Hooft limit:

$$N_c \to \infty, \qquad g^2 \to 0, \qquad \lambda = g^2 N = fixed$$

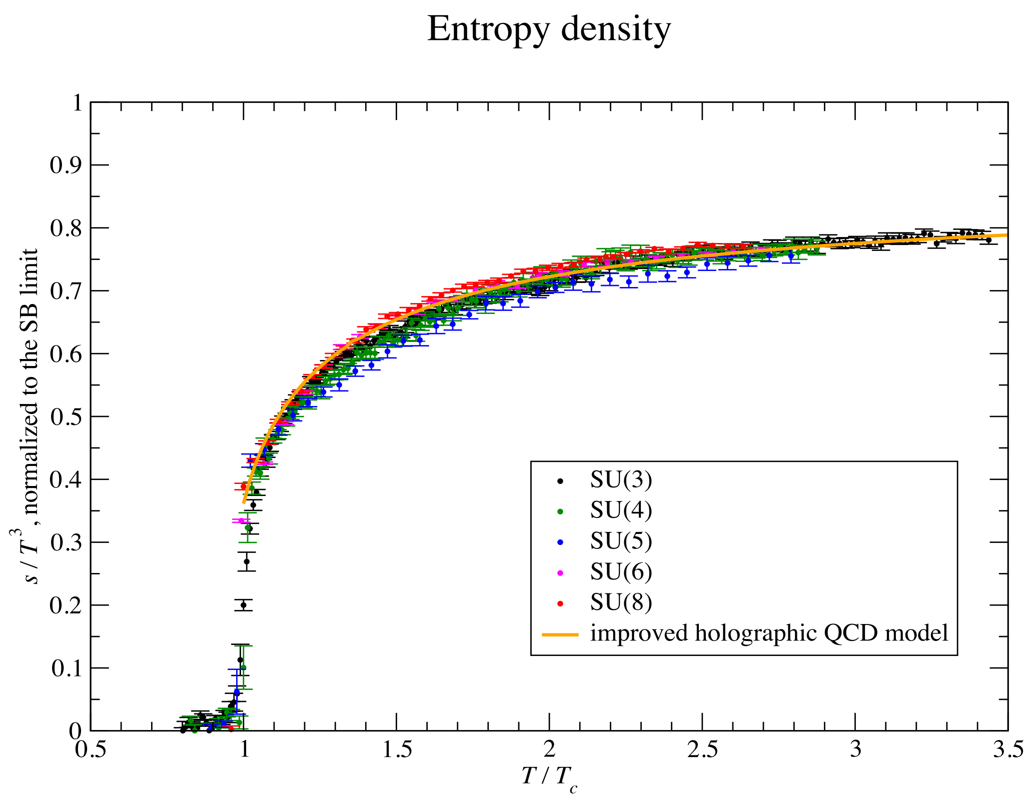
- Only planar Feynman diagrams
- Quarks in loops suppressed by N_f / N_c

$\Rightarrow$ Large N_c pure glue theory with gauge-group $SU(N_c)$
Extrapolation on the lattice: Both at zero T (glueball spectra) and finite T (thermodynamic functions) VERY close to $SU(3)$.

This is what we will assume in the rest of the lectures.

N-dependence of thermodynamic quantities

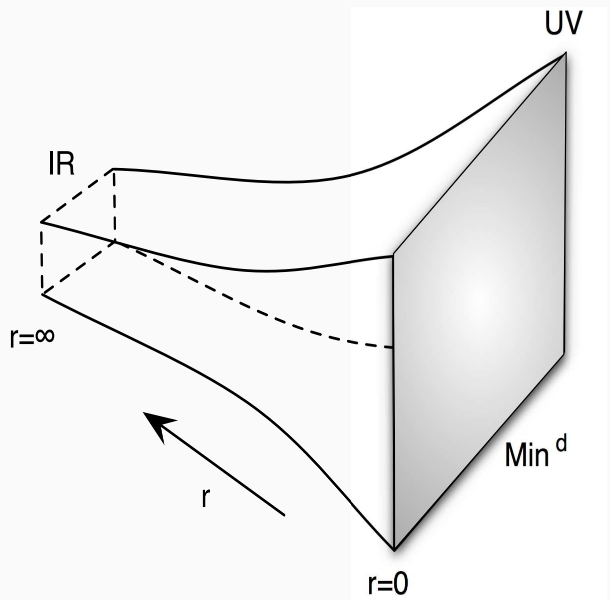
Panero '09



- About 10 % deviation in the hadron spectra
- Thermodynamic observables very close to each other

Holographic duality

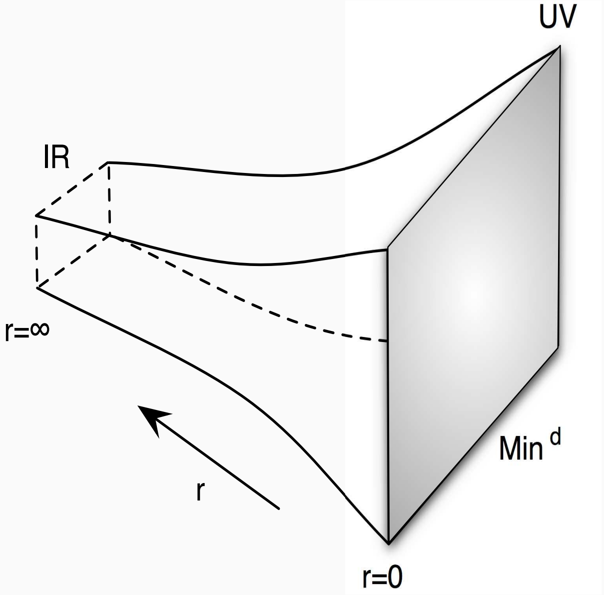
A duality that stems from string theory: String theory in the $d+1$ dim. bulk $\Leftrightarrow$ QFT on the d dim. boundary;



"radial direction" geometrizes the RG energy scale in QFT.
UV $\Leftrightarrow$ boundary; IR $\Leftrightarrow$ deep interior.

- A simple corner of the parameter space: For $g^2 \gg 1$ and $N \gg 1$ stringy physics decouples $\Rightarrow$ classical gravity!
- Compute quantum observables in QFT by solving ODE's in classical GR!
- Works in real time and for finite T

Some details of the duality



Domain-wall type geometries with boundary

Minimal metric:

$$ds^2 = b(r)^2 \left(dr^2 + dx_d^2\right)$$

Rules to compute: Witten; Gubser, Klebanov, Polyakov '98

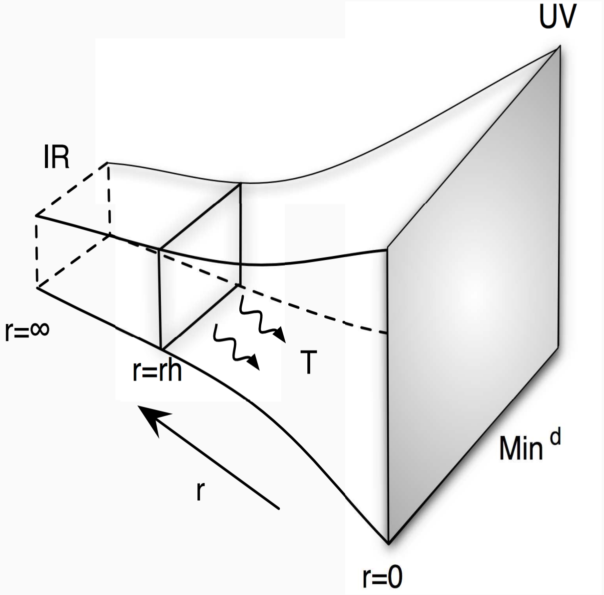
1. A bulk fluctuation $\phi(x,r) \Leftrightarrow \mathcal{O}(x)$ on the boundary.

   Fundamental relation:

   $$\exp(-S_G[\phi(x,r) \to \phi_0(x)]) = \langle \exp(\int \mathcal{O}\phi_0) \rangle$$

   Computes n-point functions $\langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle$ of QFT.

Some details of the duality



Domain-wall type geometries with boundary

Minimal metric:

$$ds^2 = b(r)^2 \left(dr^2 + dx_d^2\right)$$

Rules to compute: Witten; Gubser, Klebanov, Polyakov '98

1. A bulk fluctuation $\phi(x, r) \Leftrightarrow \mathcal{O}(x)$ on the boundary.

   Fundamental relation:

   $$\boxed{\exp(-S_G[\phi(x, r) \to \phi_0(x)]) = \langle \exp(\int \mathcal{O}\phi_0) \rangle}$$

   Computes n-point functions $\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle$ of QFT.

2. Finite temperature in the QFT $\Leftrightarrow$ black-hole in the geometry.

Calculation of correlators

Consider e.g. a scalar operator $\hat{O}$ in QFT $\Leftrightarrow \phi(r,x)$, scalar bulk field with mass m^2

Solution near asymptotically AdS boundary:

$\phi(x,r) = r^{d-\Delta}\phi_0(x) + r^{\Delta}\langle\hat{O}(x)\rangle + \cdots$

One finds $\Delta(d-\Delta) = m^2\ell^2$

- Euclidean signature: Only one correlation function
  $G_E(k_E) = \int d^d x e^{-ik_E\cdot x_E}\langle T_E \hat{O}(x_E)\hat{O}(0)\rangle$
  $\omega_E = 2\pi n T$ the Matsubara frequency.
- Solve the EOM for $\phi(x,r)$ with the boundary conditions:
  1. $\lim_{r\to 0}\phi(x,r) = r^{d-\Delta}\phi_0(x)$,
  2. $\lim_{r\to r_h}\phi(x,r)$ is regular.

  Calculate G_E from $G_E(x) = \frac{\delta^2}{\delta\phi_0(x)\delta\phi_0(0)}\ln Z$ using the fundamental relation.

Real-time correlators

Various different correlation functions: retarded, advanced, symmetric

$G_R(k) = -i \int d^d x e^{-ik\cdot x} \theta(t) \langle [\hat{O}(x), \hat{O}(0)] \rangle$

(analytic in the upper-half ω-plane by causality.)

Analytic continuation:

$$G_R(\omega, \vec{k}) = G_E(\omega_E, \vec{k})\Big|_{\omega_E = -i(\omega + i\epsilon)}$$

Only useful when exact analytic expression for G_E known.

Furthermore, fundamental relation does not analytically continue!

AdS/CFT prescription

Son, Starinets '02; Gubser et al. '08; Iqbal, Liu '09

- Define G_E through Kubo's formula:
  $\mathcal{L} \to \mathcal{L} + \int \mathcal{O}^A \delta\phi_A$ then $\langle \mathcal{O}^B \rangle = G_E^{BA} \delta\phi_A$
- Start from the Euclidean formulation
  $\langle \hat{O}(x) \rangle_{\phi_0} = -\frac{\delta S_G}{\delta\phi_0(x)} = -\lim_{r\to 0} \Pi_E(r,x)$
- Then Kubo's formula give $G_E(k) = -\lim_{r\to 0} \frac{\Pi_E(r,k)}{\phi_E(r,k)}$
- Analytically continue in the bulk:

$$G_R(k) = -\lim_{r\to 0} \frac{\Pi_E(r,k_E)}{\phi_E(r,k_E)}\bigg|_{\omega_E = -i(\omega+i\epsilon)} = \lim_{r\to 0} \frac{\Pi_R(r,k)}{\phi_R(r,k)}$$

- Then the prescription is simply: Solve the EOM for $\phi(x,r)$ with the boundary conditions:
  1. $\lim_{r\to 0} \phi(x,r) = r^{d-\Delta} \phi_0(x)$,
  2. $\lim_{r\to r_h} \phi(x,r) \propto (r_h - r)^{-i\frac{\omega}{4\pi T}}$.
     Infalling boundary conditions

Holographic QCD: top-bottom approach

Top-bottom approach: two descriptions of D-branes.

1. Open strings ⇒ Gauge theory in d dimensions
2. Closed strings ⇒ GR in $d+1$ dimensions

At low energy 1. and 2. decouple and become equivalent!
Examples:

- Witten-Sakai-Sugimoto model: D4 branes on S^1 + D8 flavor branes
  - Infinitely many KK-operators from S^1 and S^4 ⇒ disguised 5D theory
  - Although same universality class as QCD, very different physics (e.g. thermodynamics)
  - To decouple KKs, need $R/\ell_s \ll 1$, world-sheet theory hard.
- D3-D5 and D3-D7 systems
  - Physics associated with running coupling, confinement absent disguised conformal theory.

Holographic QCD: bottom-up approach

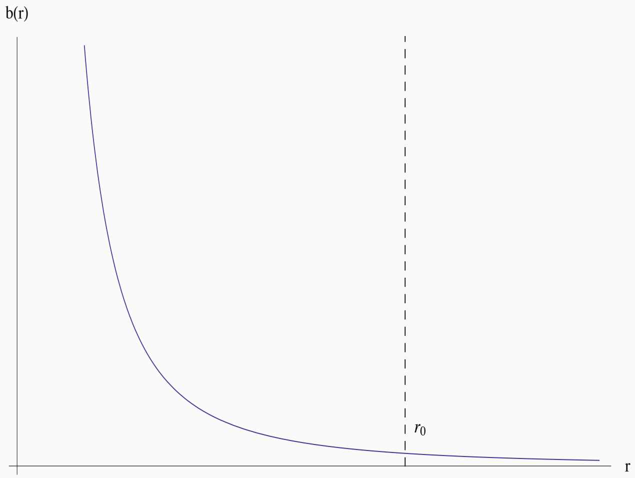
Polchinski, Strassler ’02; Da Rold, Pomarol ’05; Erlich et al ’05

- Construct a consistent GR set-up in the most economic fashion:
  - Dimensions of QFT + 1 (energy scale)
  - Symmetries of QFT in the bulk
  - One bulk field for each relevant + marginal operator
  - Realization of dynamical phenomena (e.g. spontaneous symmetry breaking)
- Declare that this GR theory secretly describes the strong coupling region of the QFT
- Check this by calculations
- For generic GR set-ups $\Rightarrow$ universal lessons

Simplest example: Hard-Wall

Polchinski-Strassler ’02; Erlich et al. ’05; Da Rold, Pomarol ’05

AdS_5 with an IR cut-off



- color confininement
- mass gap
- $\Lambda \sim \frac{1}{r_0}$

- Mesons by adding $D4 - \overline{D}4$ branes in probe approximation
- Fluctuations of the fields on $D4$: meson spectrum
  e.g. $A^L_\mu + A^R_\mu \Leftrightarrow$ vector meson spectrum
- surprisingly successful: certain qualitative features, meson spectra %9 of the lattice

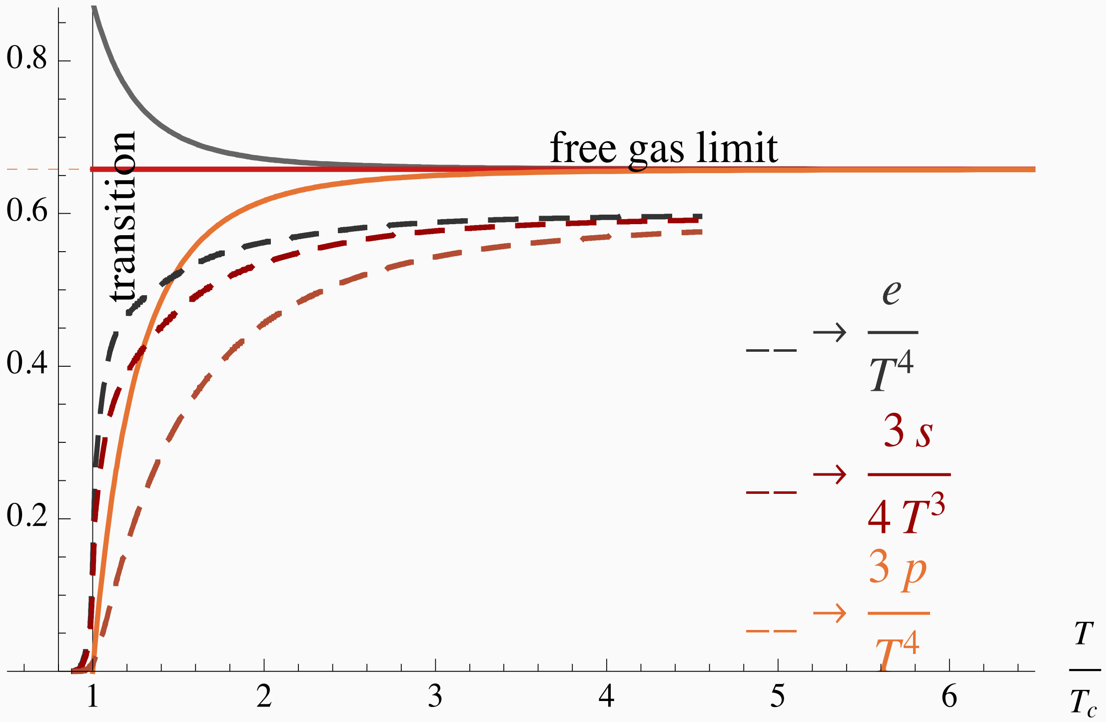
Problems at zero T

- No running gauge coupling, no asymptotic freedom
- Ambiguity with the IR boundary conditions at r_0
- No linear confinement, $m_n^2 \sim n^2, n \gg 1$
- No magnetic screening
- No obvious connection with string theory

Thermodynamics go wrong!

U.G., '08

- Trace anomaly $T^{\mu}_{\mu} = e_{HW} - 3p_{HW} = const$!
- Speed of sound $c_s^2 = 1/3$!
- Entropy $s_{HW} = const$!
- Bulk viscosity $\zeta/s_{HW} = 0$!



Boyd et al. '96

Running coupling

- It is crucial to correct the holographic QCD with running coupling constant:
- Conformal invariance broken by running coupling $\Rightarrow \Lambda_{QCD}$
  - Crucial for non-trivial T-dependence in thermodynamic functions (E, S, F)
  - Non-trivial $\langle \mathrm{Tr} F^2 \rangle$ responsible for deconfinement p.t. (e.g. in pure YM).
- Lattice data on energy and entropy $\Rightarrow$ QGP is almost ($\%80$) free gas of gluons and quarks at $T > 1.5\ T_c$ due to Asymptotic freedom
  (This does not necessarily mean α_s is small above $1.5\ T_c$. e.g. $\mathcal{N} = 4$ sYM.)

The minimal holographic dual at zero T

Lattice QCD data: $SU(N)$ with $N \gg 1$ close to $SU(3)$

- About 10 % deviation in the hadron spectra
- Thermodynamic observables very close to each other
- Consider pure $SU(N)$ at large N, at strong coupling $\Rightarrow$ classical Einstein's GR
- Most economic set-up:
  1. $Mink^4$ + energy scale $\Rightarrow$ 5 dimensions
  2. Only relevant/marginal operators the stress-tensor $T_{\mu\nu}$ and the gluon condensate $\mathrm{Tr}\, F^2$
     $\Rightarrow$ Need metric $g_{\mu\nu} \Leftrightarrow T_{\mu\nu}$ and dilaton $\phi \Leftrightarrow \mathrm{Tr}\, F^2$
  3. Running coupling extremely important for correct thermodynamics $\Rightarrow$ non-conformally invariant background with $e^\phi \propto g^2 N$ a function of r: $\phi = \phi(\Lambda r)$ with $\Lambda \Rightarrow$ dynamically generated QCD scale.

Improved HQCD U.G, Kiritsis; U.G. Kiritsis, Nitti '07

- Gravitational dual in 2∂ effective GR theory:

$$S = M_p^3 N_c^2 \int d^5x \sqrt{g} \left\{ R - \frac{4}{3}(\partial\phi)^2 - V(\phi) \right\}$$

- Look for domain-wall type solutions of the Einstein-dilaton eqs:
$ds^2 = b^2(r)\left(dr^2 - dt^2 + dx_3^2\right), \lambda = \lambda(r) \equiv \exp(\phi(r))$

Dictionary: Geometry vs. QFT:

- Scale factor $b_0(r)$ is the energy scale in the field theory E,
- Dilaton $\lambda(r) \propto \lambda_t(E)$ running 't Hooft coupling,
- Dilaton potential $V(\phi) \Leftrightarrow \beta(\lambda_t)$ the beta-function of the QFT.

Fixing the UV

Two options in the UV for holographic QCD:

1. Asymptotically AdS with power-law corrections:
   $b(r) = \frac{\ell}{r}\left(1 + \mathcal{O}(r^2)\right)$ as $r \to 0$.
   This is a conformal YM theory with mass deformation
   $m^2 = \Delta(4 - \Delta)$
   where Δ is the (renormalized) scale dimension of $\mathrm{Tr}F^2$
   One need to choose very small m^2 slightly relevant Gubser et al, '08.
2. Asymptotically AdS with log-corrections:
   This is an asymptotically free theory with mass deformation
   $m^2 \to 0$
   $\Rightarrow$ scale dimension of $\mathrm{Tr}F^2$ $\Delta \to 4 + \mathcal{O}(\lambda)$ marginal!
   Dilatation Ward identity: $T^\mu_\mu = \frac{\beta(\lambda)}{4\lambda^2}\mathrm{Tr}F^2 \sim \mathrm{Tr}F^2$

We choose the second option U.G, E. Kiritsis '07

UV asymptotics

UV asymptotics of $V(\lambda)$:

$V(\lambda) = v_0 + v_1\lambda + v_2\lambda^2 + \cdots$ as $\lambda \to 0$

- Gaussian f.p. as $\lambda \to 0$ (UV) $\Rightarrow$ AdS, non-zero v_0.
- Log running of $\lambda_t \sim (b_0 \log E)^{-1} \Rightarrow$ non-zero v_1

UV asymptotics of the background:

$$b(r) = \frac{\ell}{r}\left[1 + \frac{4}{9}\frac{1}{\log r\Lambda} - \frac{4}{9}\, b\, \frac{\log(-\log r\Lambda)}{\log^2 r\Lambda} + \ldots\right],$$

$$b_0\lambda(r) = -\frac{1}{\log r\Lambda} + b\, \frac{\log(-\log r\Lambda)}{\log^2 r\Lambda} + \ldots$$

Fixing the IR

For any asymptotically AdS $A(r) \to \frac{\ell}{r} \qquad as \qquad r \to 0,$
Einstein's equations lead to the following IR behaviors:

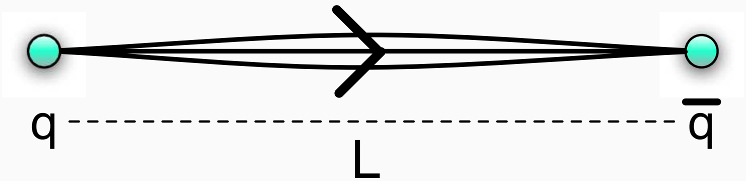
- AdS, $A(r) \to \frac{\ell'}{r}$ with $\ell' \leq \ell$
- Singularity (in the Einstein frame) at a finite point $r = r_0$
- Singularity at infinity $r = \infty$

Phenomenologically preferred asymptotics

- color confininement
- magnetic screening
- linear spectra $m_n^2 \sim n$ for large n

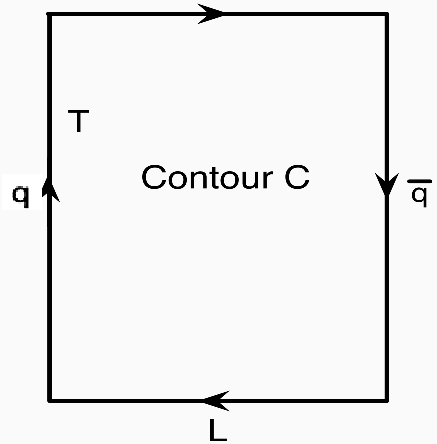
Quark potential and confinement

Linear quark potential from flux tube:



$$V_{q\bar{q}}(L) = \sigma_s L + \cdots$$

K.G. Wilson '74 $\langle W[C] \rangle = \langle \mathrm{Tr} P e^{-\oint_C A_\mu dx^\mu} \rangle = e^{-V_{q\bar{q}}(L)T}$



Gauge-gravity duality:

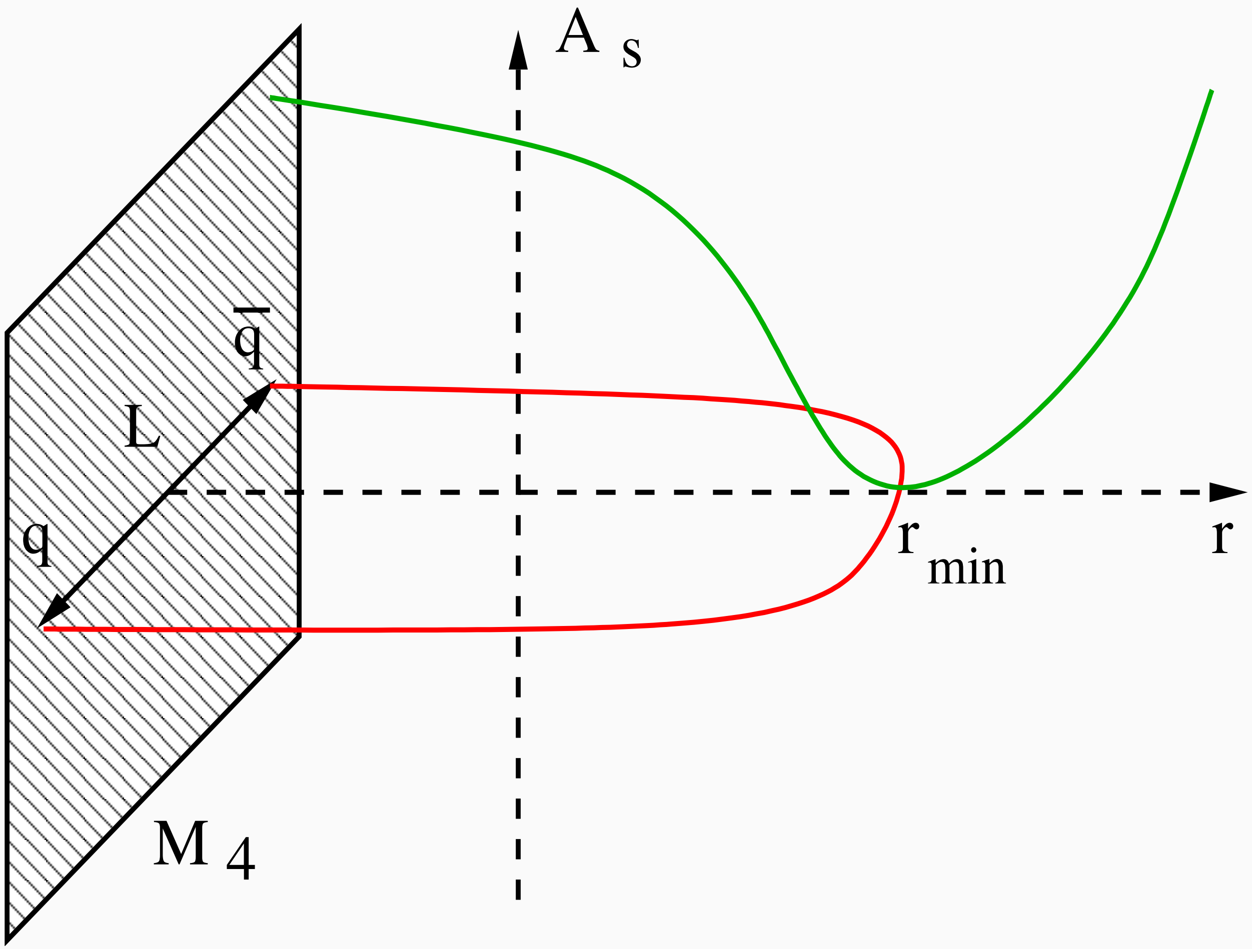
$W[C] \Leftrightarrow$ string world-sheet ending on C

$\langle W[C] \rangle = e^{-S[string;C]}$

J Mal-

dacena '98; S. Rey, J. Yee '98

Color confinement



Linear quark potential $\Leftrightarrow$ $\exists$ minimum of b_s

This constrains large λ asymptotics of the dilaton potential $V(\lambda)$.

Color Confinement - Magnetic Screening

String action: $S_{WS} = \ell_s^{-2} \int \sqrt{\det g_{ab}} + \int \sqrt{\det g_{ab}} R^{(2)} \phi(X)$
in the string frame, $g_{ab} = g^S{}_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$.

- Coupling to dilaton bounded as $L \to \infty$, linear potential, if at least one minimum of $A_s = A_E + \frac{2}{3}\phi$

The quark-anti-quark potential is given by,

$$E_{q\bar{q}} = T_s L = \frac{e^{A_s(r_{min})}}{\ell_s^2} L$$

- String probes the geometry up to r_{min}, parametrically separated from the far interior $r = r_0$, where the dilaton blows up

Confinement criterion

- Space ending at finite r_0
- Space ending at $r = \infty$ with metric vanishing as $e^{-A_\infty r}$ or faster

In terms of a phase space variable $X \equiv -\frac{\phi'}{3A'}$, a diffeo-invariant characterization:

$$X(\phi) \to -\left(C + \frac{D}{\phi}\right), \qquad \phi \to \infty$$

Confinement $\Leftrightarrow C > 1/2$ or $C = 1/2, D > 0$

The phenomenologically preferred backgrounds for the latter case:

$$A(r) \sim -A_\infty r^\alpha \quad \Leftrightarrow \quad C = 1/2, D = \frac{3}{8}\frac{\alpha - 1}{\alpha}$$

Linear confinement in the glueball spectrum for $\alpha = 2$

IR asymptotics

In terms of the potential:

$$V(\phi) \to e^{\frac{4}{3}\phi}\phi^{\frac{\alpha-1}{\alpha}} + \cdots$$

(we will eventually set $\alpha = 2$)
IR asymptotics of the background:

$$b(r) \sim e^{-\left(\frac{r}{L}\right)^{\alpha}}, \qquad \lambda(r) \sim e^{3/2\left(\frac{r}{L}\right)^{\alpha}} \left(\frac{r}{L}\right)^{\frac{3}{4}(\alpha-1)}, \qquad r \to \infty$$

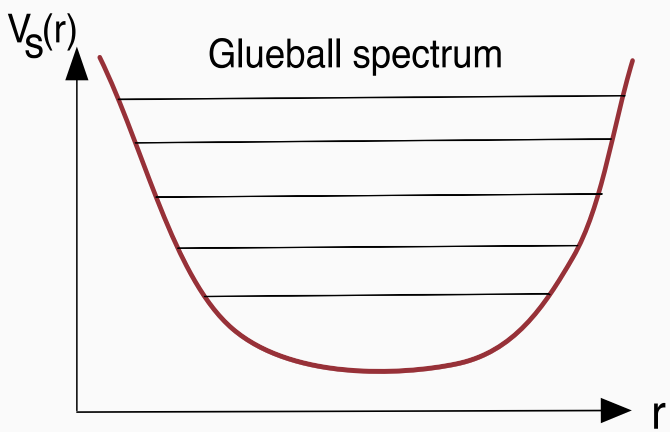
Parameters of the theory

- The dilaton potential:
$$V = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\log\left(1 + V_2\lambda^{\frac{4}{3}} + V_3\lambda^2\right)^{\frac{1}{2}}\right\}$$
- Parameters in the action: V_0, V_2 fixed by scheme independent β-function coefficients (b_0 and b_1), V_1, V_3 fixed by the latent heat L_h and $S(2T_c)$ (lattice)
- The Planck scale M_p also by thermodynamics. matching high T asymptotics of QCD free energy: $M_p = \left(45\pi^2\right)^{-\frac{1}{3}}\ell^{-1}$
- $\Lambda_{QCD}\ell_{AdS}$ the only parameter of the zero T solutions, fixed by $m_{0++} = 1475\ MeV \Rightarrow \Lambda_{QCD} = 292\ MeV$.
- The string length ℓ_s by lattice string σ_s: $\frac{\ell_{AdS}}{\ell_s} \approx 6.5$
  This measures how good the two-derivative approximation is!

The spectra of the theory

Spectrum of 4D glueballs $\Leftrightarrow$ Spectrum of normalizable fluctuations of the bulk fields.

- Spin 2: $h_{\mu\nu}^{TT}$; Spin 0: mixture of h_μ^μ and $\delta\Phi$;
- For a particle in 4D with wave-funciton $\psi(x)$ the corresponding bulk fluctuation is $\phi(x,r) = \psi(x)\zeta(r)$ For $\zeta(r)$ square integrable on r, the fluctuation eq. is a Schrödinger equation: $\mathcal{H}\zeta \equiv -\ddot{\zeta} + V_s(r)\zeta = m^2\zeta$ where $V_s = V_s[b(r), \lambda(r)]$
- Both mass gap and discrete spectra m^2 follows if V_s has a well-shape $\Leftrightarrow$ linear quark potential. Happens only for $\alpha \geq 1$ $\Rightarrow$ a good type, repulsive singularity.



Comparison with one lattice study Meyer, '02

J^{PC}	Lattice (MeV)	Our model (MeV)	Mismatch
0^{++}	1475 (4%)	1475	0
2^{++}	2150 (5%)	2055	4%
0^{++*}	2755 (4%)	2753	0
2^{++*}	2880 (5%)	2991	4%
0^{++**}	3370 (4%)	3561	5%
0^{++***}	3990 (5%)	4253	6%

$$0^{++} : TrF^2; \qquad 2^{++} : TrF_{\mu\rho}F^{\rho}_{\nu}.$$

Summary of zero T results

- Requirement of a marginal deformation $\mathrm{Tr} F^2$ fixes the UV asymptotics as
  $V(\lambda) = v_0 + v_1 \lambda + \cdots , \qquad \lambda \to 0$
- Requirement of linear color confinement fixes the IR asymptotics as
  $V(\lambda) \propto \lambda^{\frac{4}{3}} \log^{\frac{1}{2}} \lambda + \cdots , \qquad \lambda \to \infty$
- Then 1) mass gap 2) first order T_c is automatic
- Spectrum of glueballs can be computed with no IR ambiguity

Thermodynamics in QFT

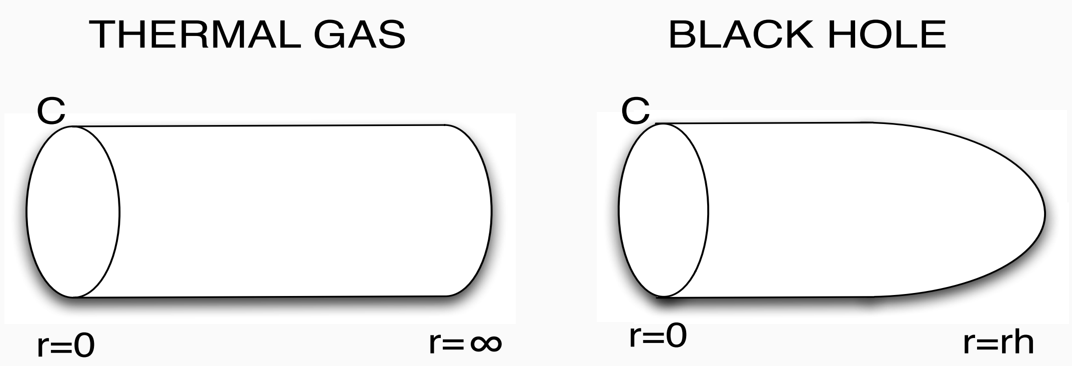
- Finite T field theory $\mathrm{Tr} e^{-H/T} = \oint \mathcal{D}\Phi e^{-S[\Phi]}$
  with Euclidean and compact time: $t \to -it$ and $t \sim t + 1/T$.
- What happens if we heat up QCD?

Thermodynamics in QFT

- Finite T field theory $\mathrm{Tr} e^{-H/T} = \oint \mathcal{D}\Phi e^{-S[\Phi]}$
  with Euclidean and compact time: $t \to -it$ and $t \sim t + 1/T$.
- What happens if we heat up QCD?
  Life becomes COLORFUL!
- A fact from lattice studies: Color degrees of freedom deconfines at $T_c \approx 260$ MeV ($N = 3$) in a first-order transition.
  Order parameter is the expectation value of Polyakov loop:
  $W_T[C] = \mathrm{Tr} P e^{-\oint_C A_0 dt}$.
- Physically: $\langle W_T \rangle \propto e^{-F_q}$, $F_q =$ cost of adding a quark.
  - In the confined phase $\langle W_T \rangle = 0$
  - In the confined phase $\langle W_T \rangle \neq 0$
- Mathematically: $W_T[C]$ is the holonomy of $SU(N)$ on $S^1 \times \mathbb{R}^3$:
  Under the center $W_T[C] \to h W_T[C]$; $h \in \Sigma[SU(N)] = \mathbb{Z}_N$
- Deconfinement phase transition is spontaneous breakdown of the center of $SU(N)$.

Dual gravitational theory UG et al. '08

- Look for solutions to Einstein-dilaton gravity at finite T:
  As $t \sim t + 1/T$ less symmetry $SO(1,3) \to SO(3)$
- Two sol'ns with AdS asymptotics $ds^2 = e^{A(r)}\left(dt^2 f(r) + dx_3^2 + \frac{dr^2}{f(r)}\right)$:
  * Thermal gas of gravitions $f(r) = 1,\ b(r) = b_0(r),\ \lambda(r) = \lambda_0(r)$.
  * Black-holes Horizon at $r = r_h$ where $f = 0$.
- Compare the gravitational energies: $S[BH] - S[TG]$
- The analog of deconfinement: Hawking-Page transition
  At $T = T_c$ gas of gravitons $\to$ nucleation of black holes!
- Gauge-gravity duality: $W_T[C] \Leftrightarrow$ a string that wraps the time circle.

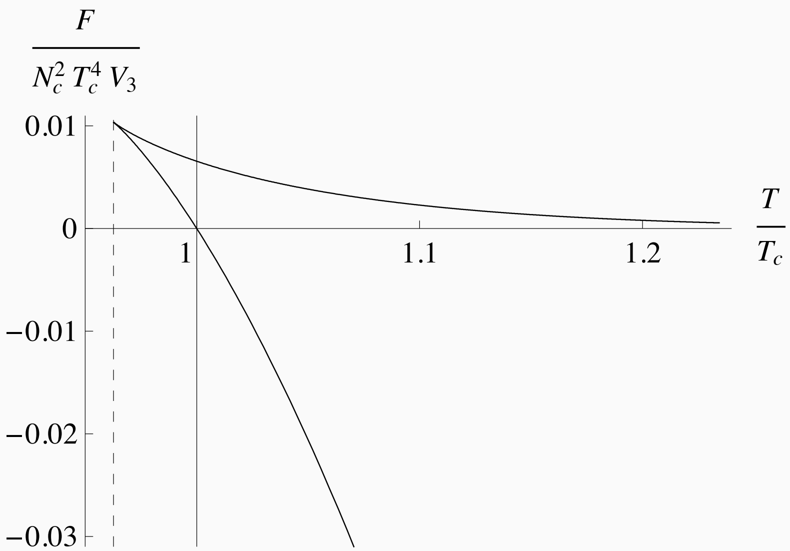


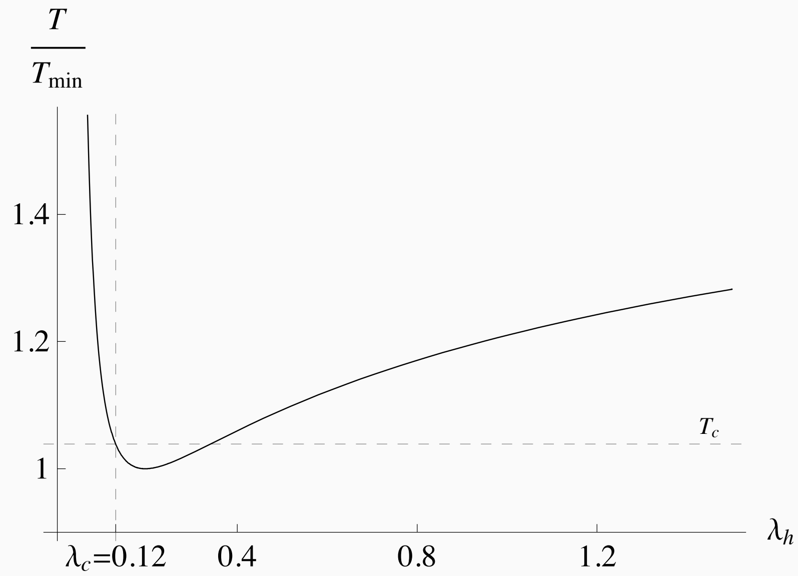
Thermodynamics: results

- Fix the dilaton potential:
$$V = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\log\left(1 + V_2\lambda^{\frac{4}{3}} + V_3\lambda^2\right)^{\frac{1}{2}}\right\}$$
- Two sol'ns with AdS asymptotics $ds^2 = e^{A(r)}\left(dt^2 f(r) + dx_3^2 + \frac{dr^2}{f(r)}\right)$:
  - Thermal Gas $\Leftrightarrow$ thermal gas of glueballs.
  - Black-hole $\Leftrightarrow$ quark-gluon plasma.
  - Hawking-Page transition $\Leftrightarrow$ deconfienment transition at T_c.
- Free energy from $S_{BH} - S_{TG}$.
- Parameter fixing: V_0, V_2 fixed by scheme independent β-function coefficients (b_0 and b_1), V_1, V_3 fixed by the latent heat L_h and $S(2T_c)$ (lattice).
- Deconfiniment transition at $T_c = 247\ MeV$ (lattice: $T_c = 260$ MeV.) Comparison to Boyd et al. '96

iHQCD Thermodynamics continued

The free energy:

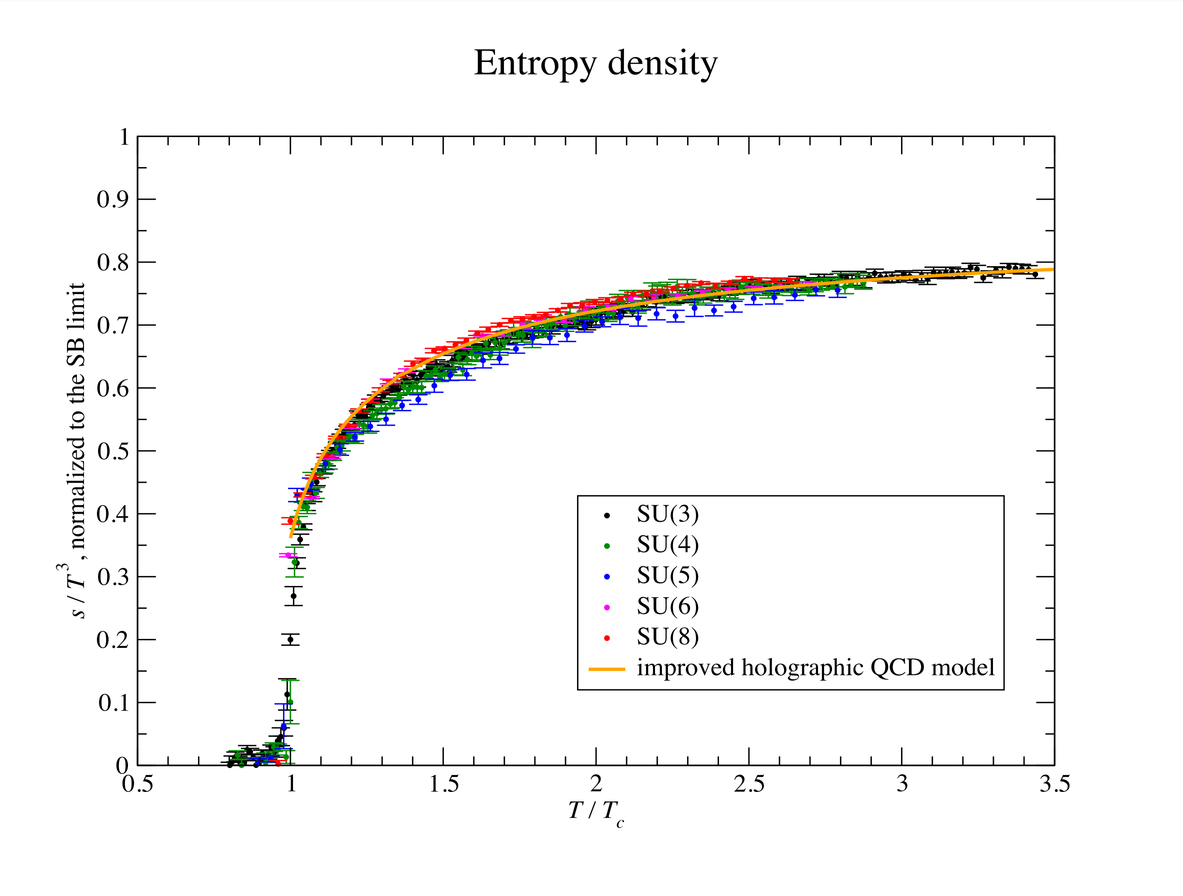




- Big and Small black-hole solutions, like $\mathcal{N} = 4$ on R^3
- Existence of $T_{min} \Leftrightarrow$ phase transition at $T_c > T_{min}$

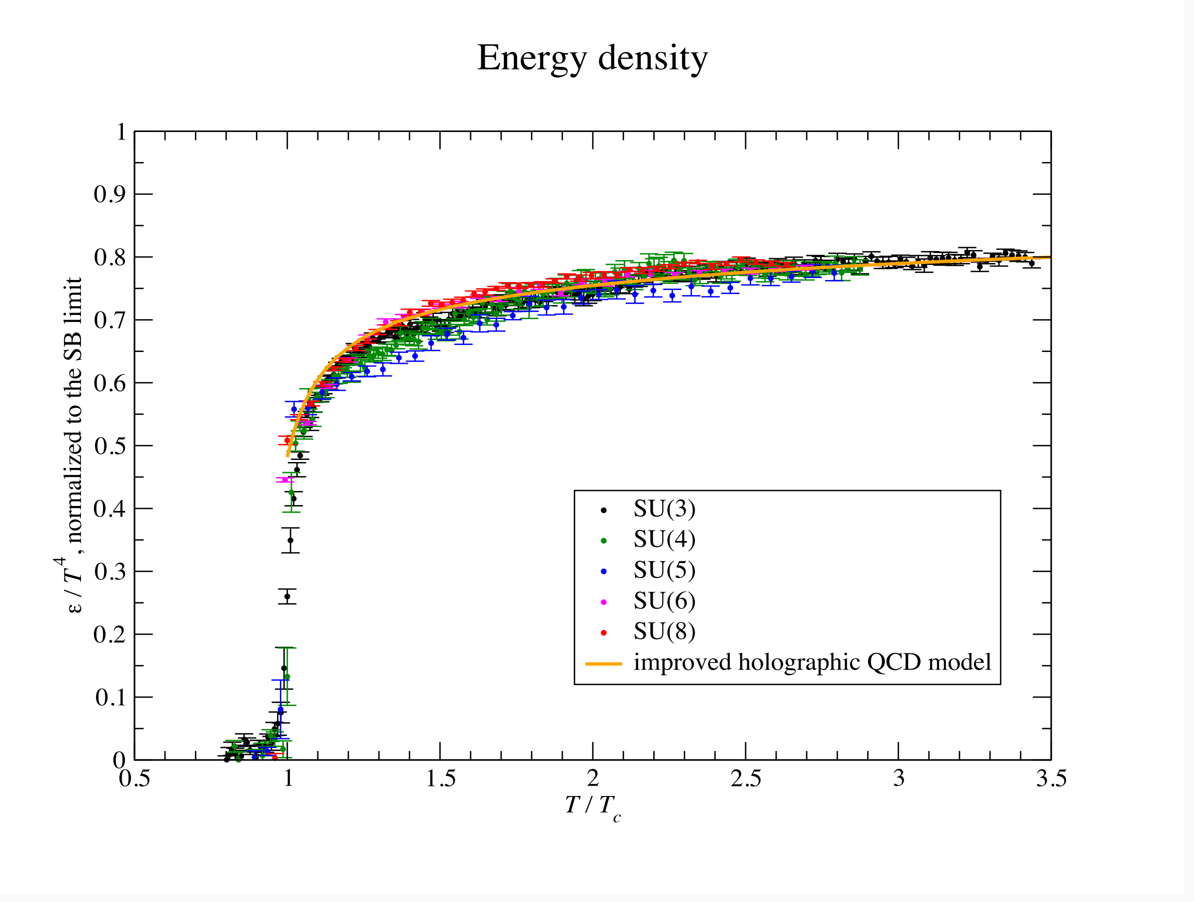
Survey of thermodynamical quantities I

Comparison to Panero '09



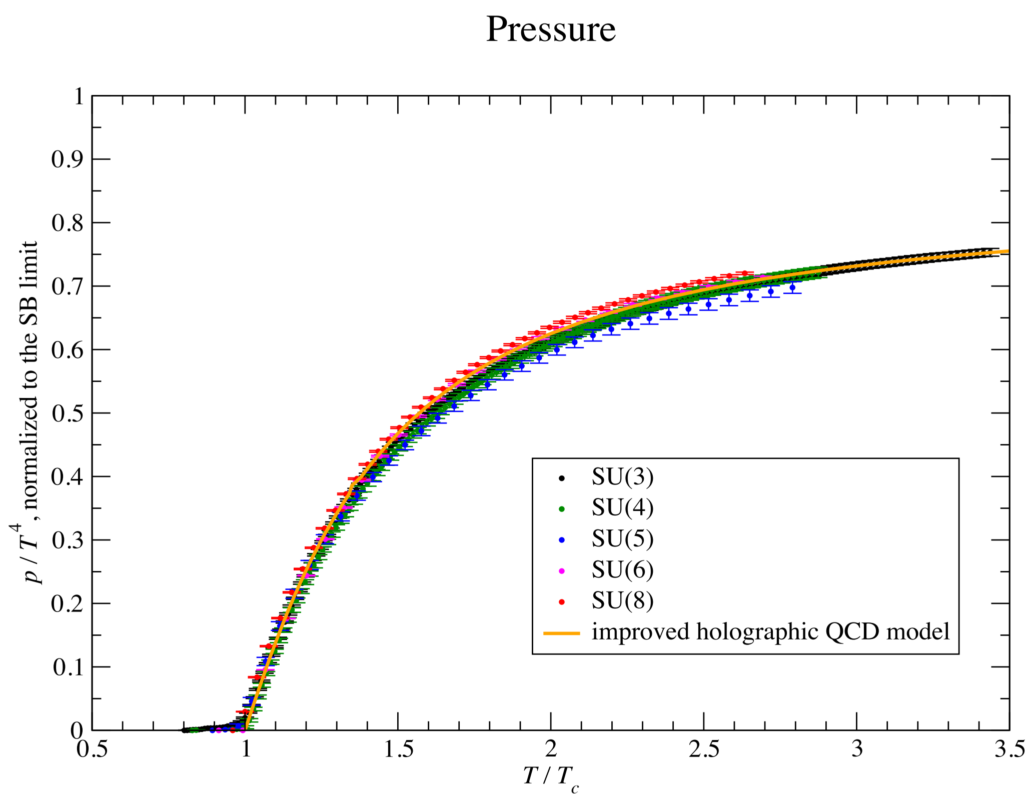
Survey of thermodynamical quantities II

Comparison to Panero '09



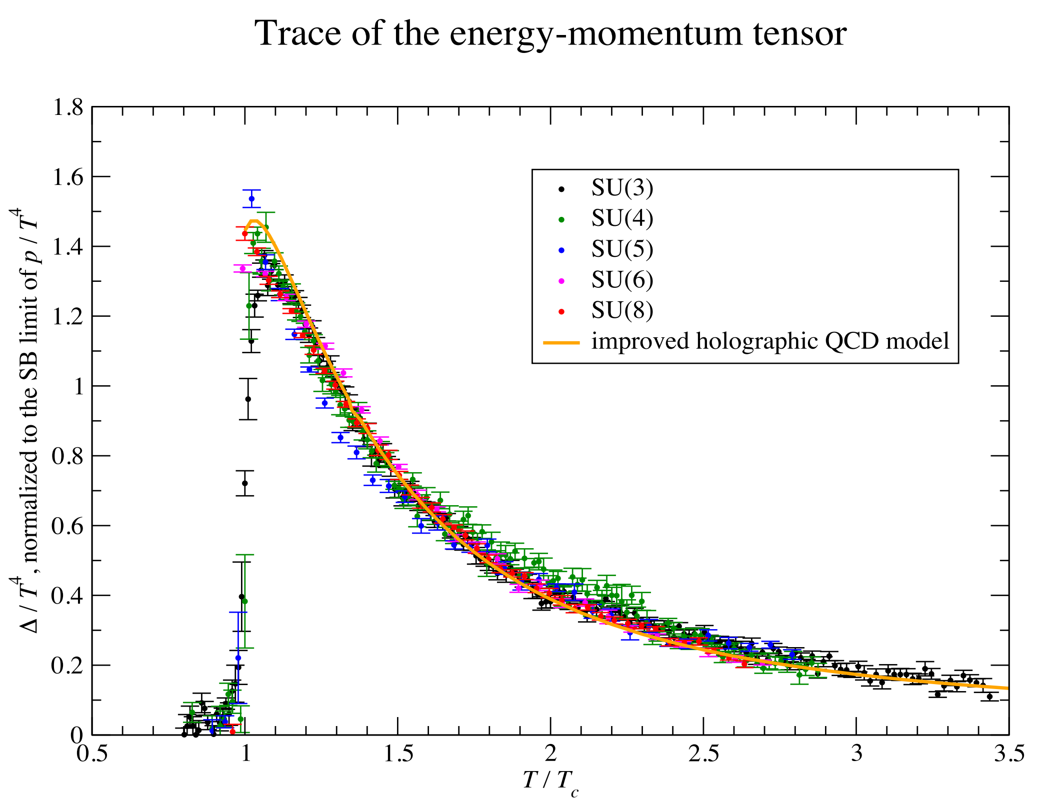
Survey of thermodynamical quantities III

Comparison to Panero '09



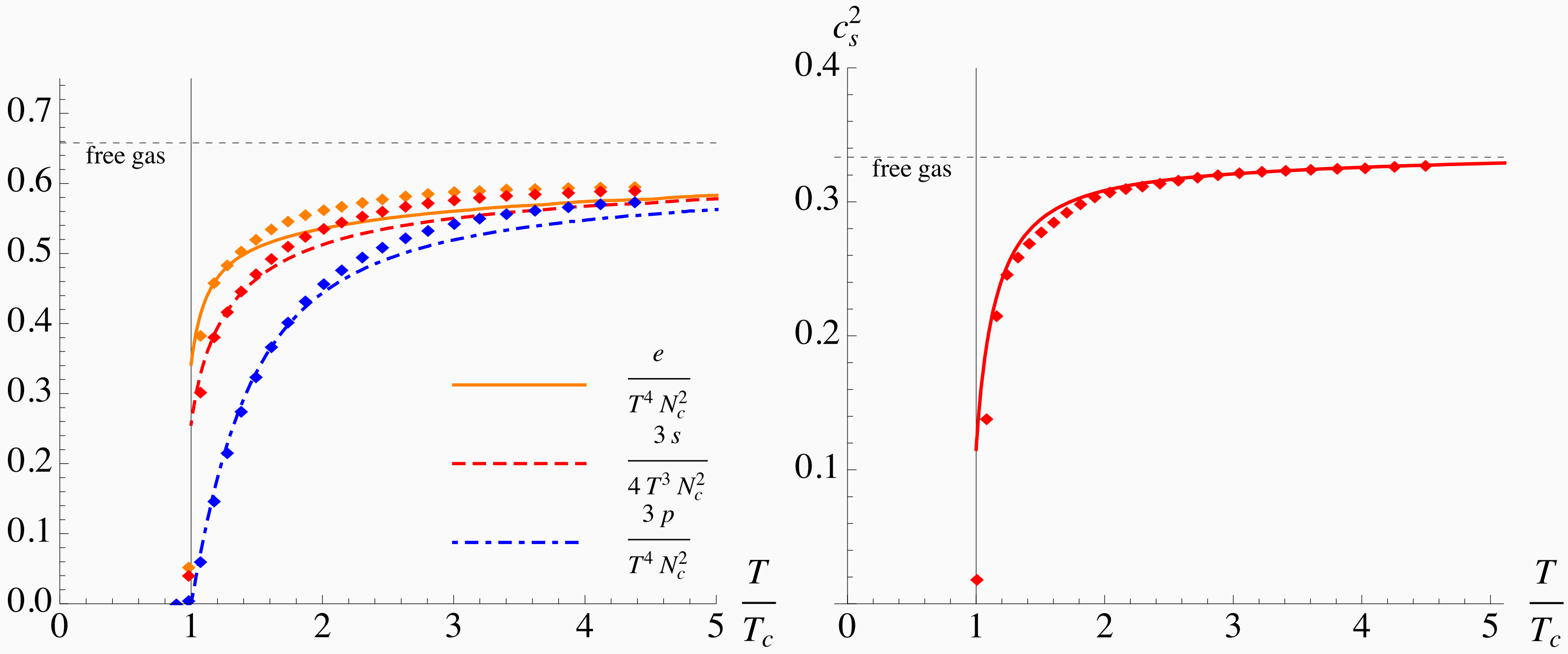
Survey of thermodynamical quantities IV

Comparison to Panero '09



Survey of thermodynamic quantities V

Comparison to Boyd et al. ’96 Thermodynamic functions and the speed of sound:



Summary of ihQCD results

- A single parameter α rules all IR physics $A(r) \to -A_\infty r^\alpha$ as $r \to \infty, V(\phi) \to V_\infty e^{\frac{4}{3}\phi} \phi^{\frac{\alpha-1}{\alpha}}$
- For $\alpha \geq 1$:
  - Linear confinement
  - Repulsive singularity, mass gap, discrete glueball spectrum ($m^2 \sim n$ for $\alpha = 2$)
  - Three phases at $T > T_{min}$: Big BH (QGP) , small BH and TG (confined phase)
  - There exists a first order conf-deconf transition: Big BH always dominates above a certain $T_c > T_{min}$
  - Can fix the potential s.t. one reproduces the observed thermodynamic functions to very good accuracy

AdS/QCD with flavors

Flavor physics in ihQCD

Casero, Paredes, Kiritsis '06, U.G, Kiritsis, Nitti '07

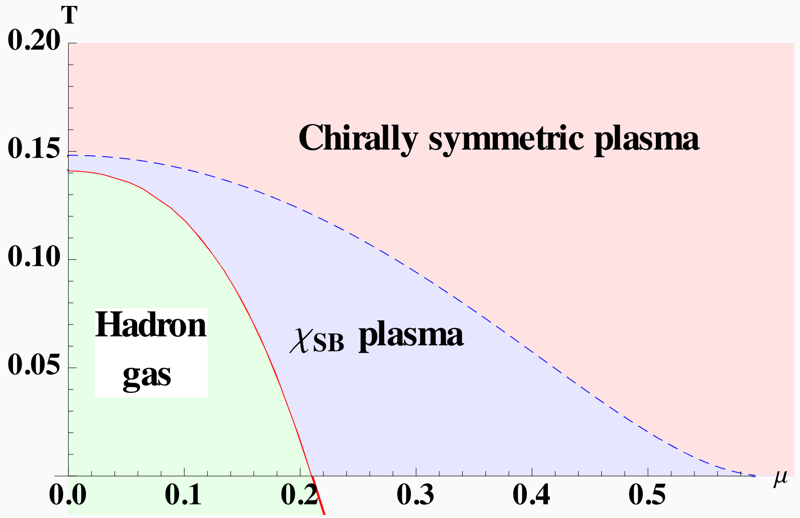
- ihQCD also suitable for large N_c, finite N_f
- Flavor physics introduced through space-filling $D4 - \overline{D4}$ branes
- Chiral symmetry $SU(N_f)_L \times SU(N_f)_R$ from $A^a_{L,\mu}$, $A^a_{R,\mu}$ gauge fields
- Quark mass operator $\bar{q}^a q^b$ couples to complex scalar T_{ab}, "open string tachyon"
- Non-trivial profile $T_{ab}(r) \Rightarrow \langle \bar{q}^a q^b \rangle \neq 0$: χSB
  $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_{L+R}$
- Scalar and vector mesons from fluctuations of T_{ab} and $A^{a,\mu}_{L,R}$.
- However in QCD $N_f \sim N_c$
- Perhaps Veneziano limit:
  $N_c \to \infty, \quad N_f \to \infty, \quad x = N_f/N_c = fixed$ more realistic.
- One has to back react flavor branes on the background!

V-QCD Jarvinen, Kiritsis '12

- Total action $S = S_g + S_f$ with
  $S_g = M_p^3 N_c^2 \int d^5x \sqrt{-g} \left(R - \frac{4}{3}(\partial\phi)^2 + V_g(\phi) \right)$ and
  $S_f =$
  $-\frac{1}{2} M_p^3 N_c \mathbb{T}\mathrm{r} \int d^5x \left(V_f(\lambda, T^\dagger T) \sqrt{-\det \mathbf{A}_L} + V_f((\lambda, TT^\dagger) \sqrt{-\det \mathbf{A}_R} \right)$
  $\mathbf{A}_{L\mu\nu} =$
  $g_{\mu\nu} + w(\lambda, T) F^L_{\mu\nu} + \frac{\kappa(\lambda,T)}{2} \left[(D_\mu T)^\dagger (D_\nu T) + (D_\nu T)^\dagger (D_\mu T) \right]$
  $\mathbf{A}_{R\mu\nu} =$
  $g_{\mu\nu} + w(\lambda, T) F^R_{\mu\nu} + \frac{\kappa(\lambda,T)}{2} \left[(D_\mu T)(D_\nu T)^\dagger + (D_\nu T)(D_\mu T)^\dagger \right]$
  $D_\mu T = \partial_\mu T + iT A^L_\mu - i A^R_\mu T.$
- Inspired by the non-Abelian DBI action Sen '04 choose tachyon potential as $V_f(\lambda, TT^\dagger) = V_{f0}(\lambda) e^{-a(\lambda)TT^\dagger}$
- Asymptotics of $V_{f0}(\lambda), a(\lambda), \kappa(\lambda), w(\lambda)$ fixed by physical requirements: linear meson spectra, χSB, anomalies etc.
- More freedom in the flavor sector, less predictability.

V-QCD thermodynamics

Alho et al '14 A typical phase diagram for $m_q = 0$:



with baryon chemical potential $\mu = \frac{1}{2}\left(A_L + A_R\right)\Big|_{r=0}$

QGP observables from AdS/QCD

Hydrodynamics at first order

- Relativistic fluid with 4-velocity u^μ, energy density ϵ and pressure p.
- Navier-Stokes & continuity equations from the energy-momentum tensor:

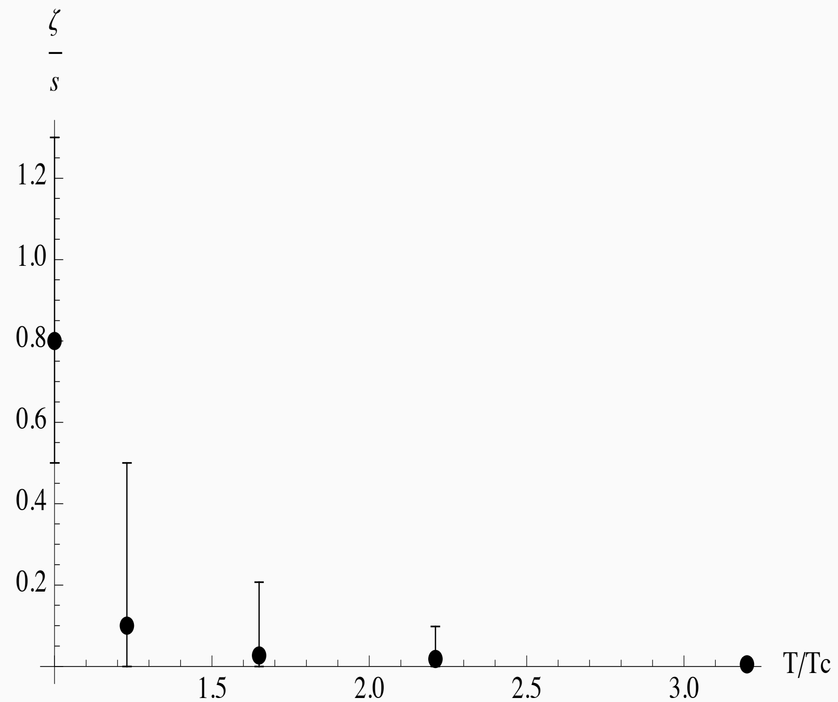
$$\begin{aligned} T_{\mu\nu} &= (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} \\ &+ P^{\mu\alpha}P^{\nu\beta}\left[\eta\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3}g_{\alpha\beta}\partial\cdot u\right) + \zeta g_{\alpha\beta}\partial\cdot u\right] \\ &+ \mathcal{O}(\partial u)^2 \end{aligned}$$

The characteristic parameters of the fluid at $\mathcal{O}(\partial u)$

- Shear viscosity η: For all 2∂ theories $\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$
  Buchel and Liu '03
- Bulk viscosty ζ: What is already known from field theory and lattice ?

A lattice computation

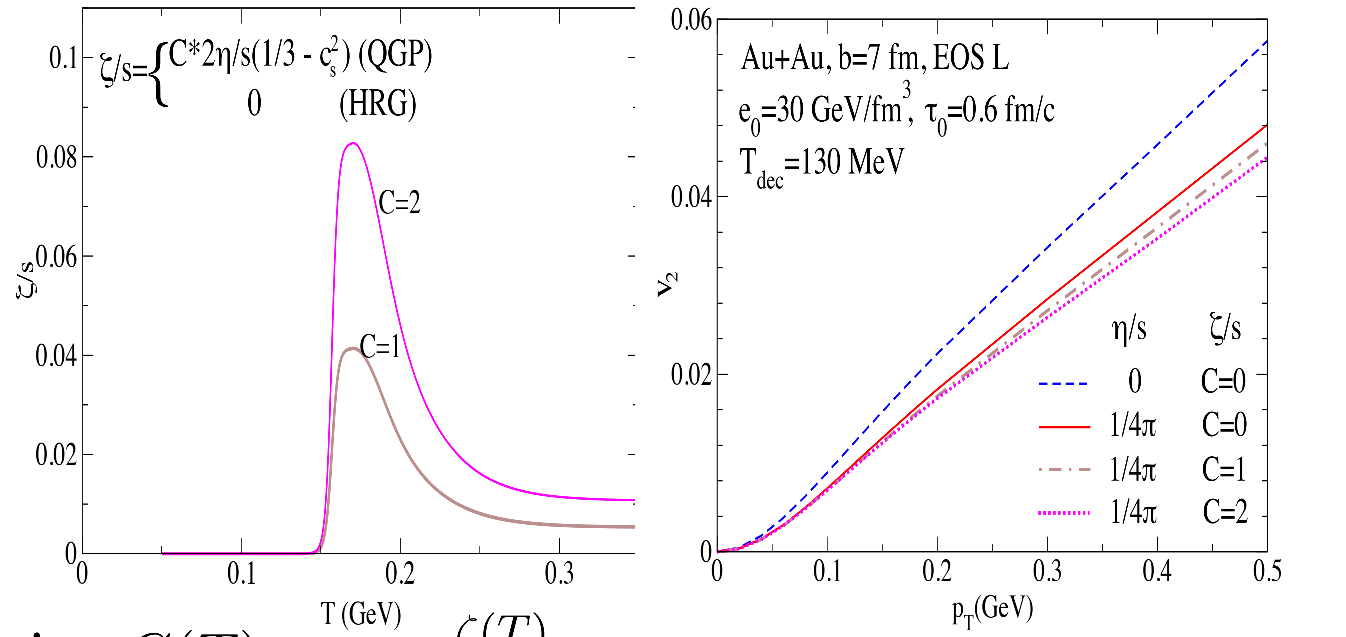
Meyer '08



- Huge statistical errors
- Huge systematic errors due to analytic continuation, assumption on the spectrum.

Effects of bulk viscosity on observables

Hydrodynamic simulations: H.Song, U.Heinz '09



The function $C(T) = \frac{\zeta(T)}{2\eta(1/3 - c_s^2(T))}$

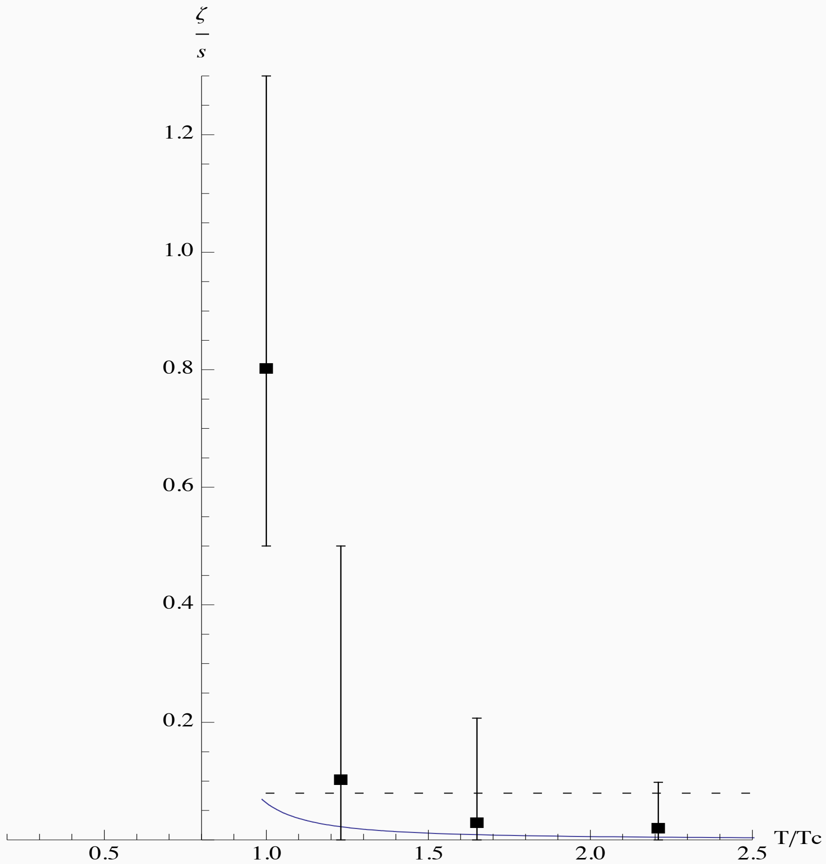
Buchel '07 A dynamical bound on $\zeta \Rightarrow C(T) > 1$.

However direct lattice Meyer '08 $\Rightarrow \zeta/s \sim 0.8$ near T_c!

Questions to be answered by holography

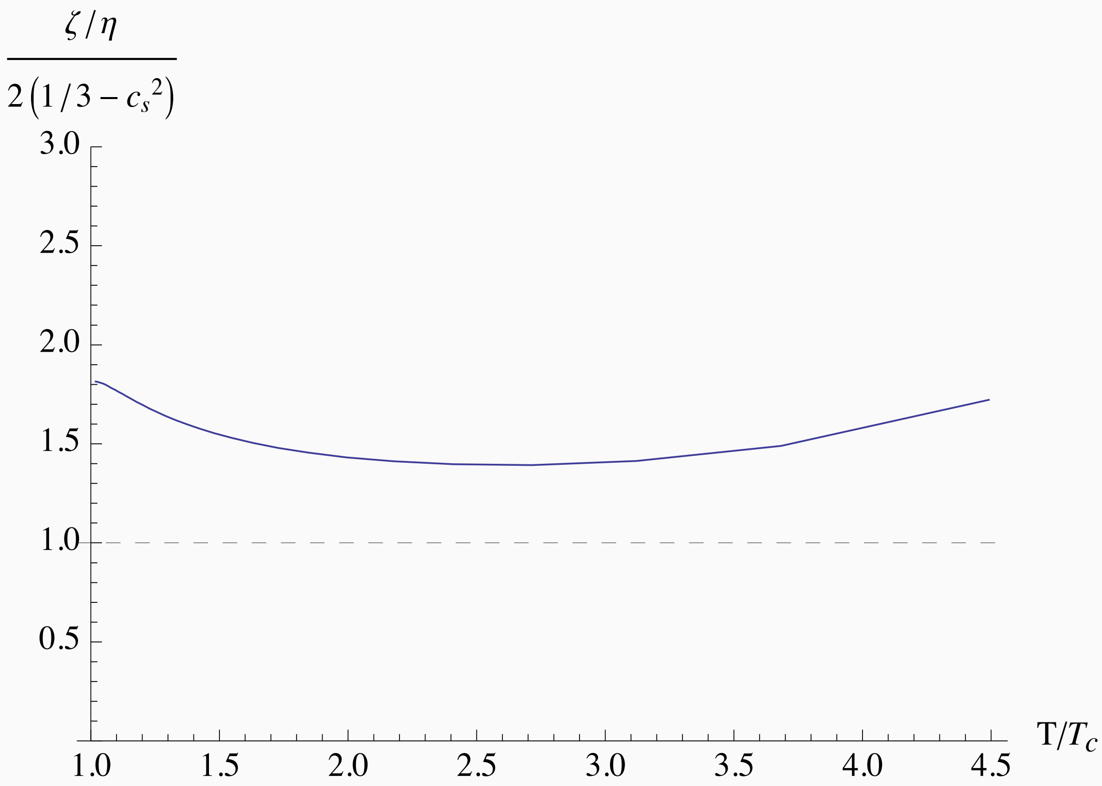
- How significant is ζ near T_c?
- Does ζ/s increase as $T \to T_c$ as the field theory and lattice indicates?
- If so, what is the holographic reason for the rise near T_c?
- More generally: Do we see similar profile for $\zeta/s(T)$ in holographic models?

Results I: Comparison to Meyer '08



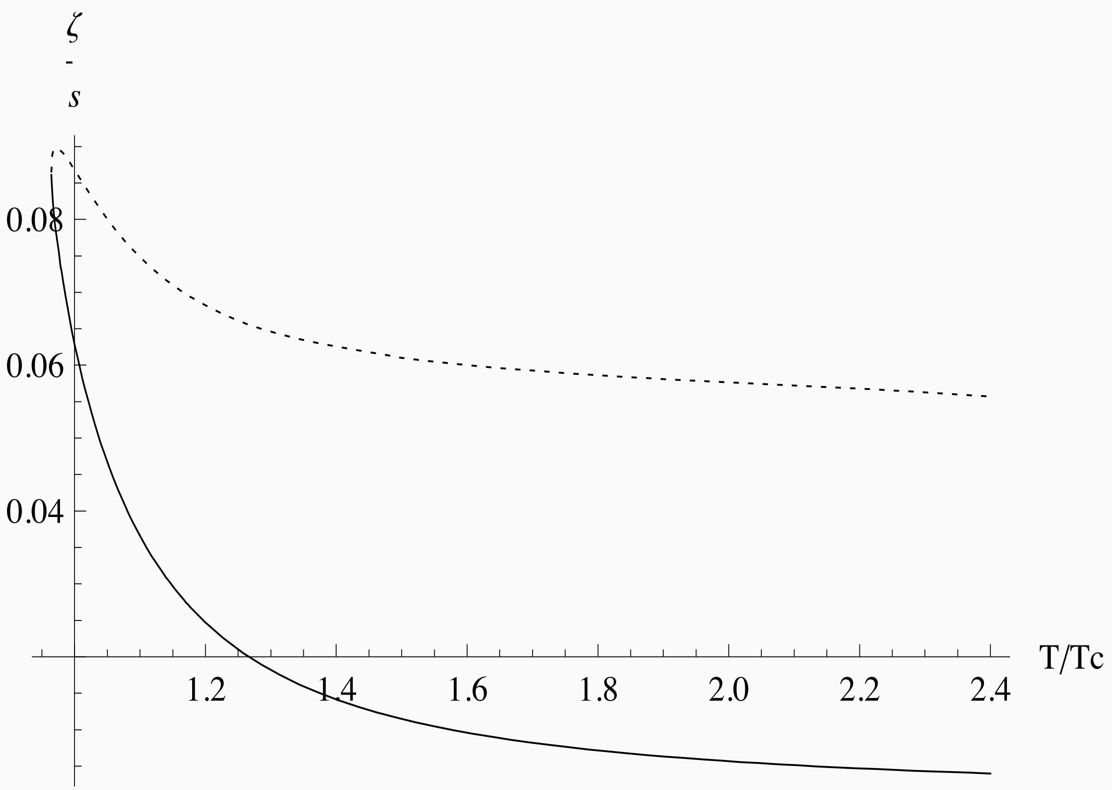
- Near UV, vanishes as expected: ideal gluon gas at high T
- Near T_c Peak, much smaller than lattice expectations!
- Agreement with another holographic model Gubser et al. 08

Results II: The function $C(T)$



- Buchel's bound is satisfied: $C(T) > 1$.
- Indeed $C \approx 1.5$ as the hydro simulations assumes
- Effects of ζ on the elliptic flow are small:
  At $p_T = 0.5$ GeV: $\eta/s = 0.08 \Rightarrow 16\%$ suppression,
  $\zeta/s(T_c) = 0.07 \Rightarrow 8\%$ suppression in v_2.

Holographic reason for the peak



- Holographic explanation of the rise: due to small BH branch!
- Existence of T_{min} $\Rightarrow$ rise near T_c.
- Color confinement in zero-T theory $\Leftrightarrow$ peak near T_c at finite T!

U.G, Kiritsis, Nitti, Mazzanti '08

Energy loss of a heavy quark

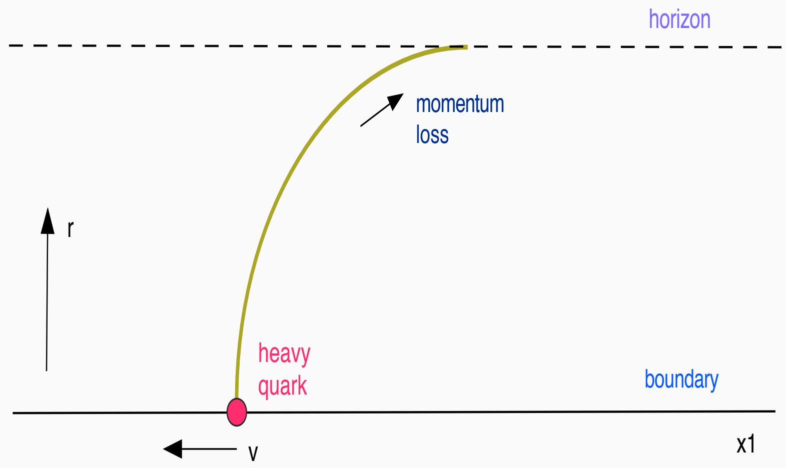
- Highly energetic partons produced in head-on nuclei collisions are very important probes
- In weakly coupled QGP: main source of energy loss is collisions with thermal gluons and quarks.

  D. Teaney '03
- What happens in a strongly coupled plasma?
  Combination of two distinct mechanisms:
  1. Energy loss by Langevin diffusion process
  2. Energy loss by gluon Brehmstahlung

Dual picture

Herzog et al; Gubser '06

Holography: Represent the (infinitely) heavy quark with a trailing string moving with constant v:



Drag force on a heavy quark in a hot wind:

$$F = \frac{dp}{dt} = \frac{1}{v}\frac{dE}{dt} = -\mu p + \zeta(t)$$

Ignore stochastic force $\zeta(t)$ in this talk $\Leftrightarrow$ fluctuations of the trailing string $\Rightarrow$ diffusion constant.

Standard calculation:

- Pick up the static gauge: $\sigma^0 = t, \sigma^1 = r$.
- String ansatz $x^1 = vt + \delta(r)$
- Minimize the area (in the string frame!)
- Compute the WS momentum flowing into the BH horizon

$F = \frac{1}{v}\frac{dE}{dt} = -\frac{1}{2\pi\ell_s^2}\ v\ e^{2A(r_s)}\lambda(r_s)^{\frac{4}{3}}$, r_s defined by $f(r_s) = v^2$.

Relativistic limit, $v \to 1$: $F = -\frac{\ell^2}{\ell_s^2}\sqrt{\frac{45\ Ts(T)}{4N_c^2}}\frac{v}{\sqrt{1-v^2}\left(-\frac{\beta_0}{4}\log[1-v^2]\right)^{\frac{4}{3}}} + \cdots$

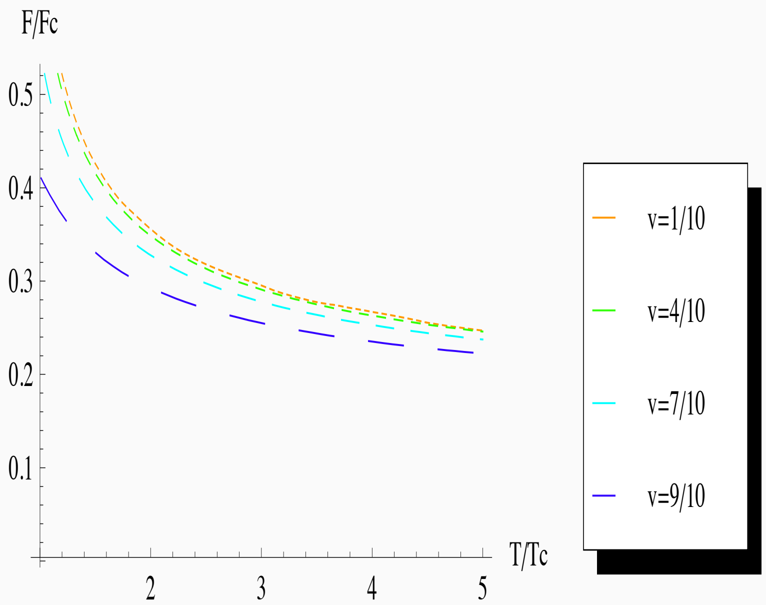
Non-relativistic limit $v \to 0$: $F = -\frac{\ell^2}{\ell_s^2}\left(\frac{45\pi\ s(T)}{N_c^2}\right)^{\frac{2}{3}}\frac{\lambda(r_h)^{\frac{4}{3}}}{2\pi}v + \cdots$

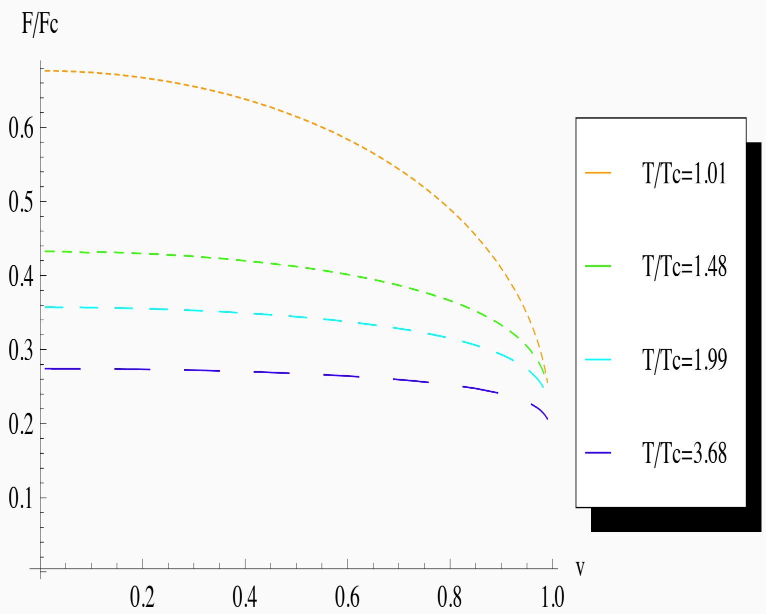
Comparison to conformal case

The AdS result: $F_{conf} = \frac{\pi}{2}\sqrt{\lambda}T^2 \frac{v}{\sqrt{1-v^2}}$

Fix ℓ_s in our model by the lattice string tension

Fix $\lambda = 5.5$ in $\mathcal{N} = 4$ SYM:





We clearly see the effects of asymptotic freedom!

Comparison schemes

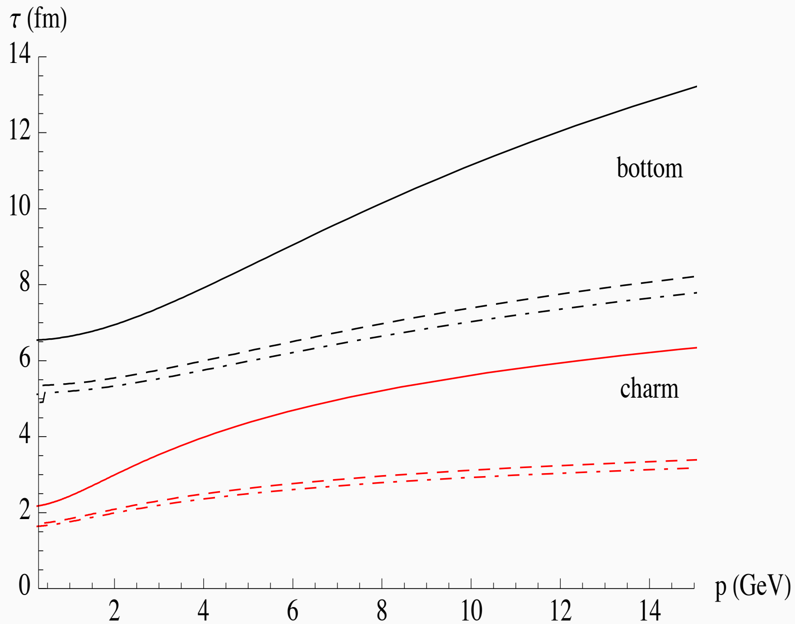
- An important detail:: How to compare to QCD?
- Direct scheme: $T_{QGP} = T_{our}$

In the range $1.5T_c < T < 3T_c$ $E_{QGP} \propto E_{GP} \propto T^4$

- Alternative schemes: $E_{QGP} = E_{our}$ or $s_{QGP} = s_{our}$
- We try all possible schemes.

Predictions for experiments

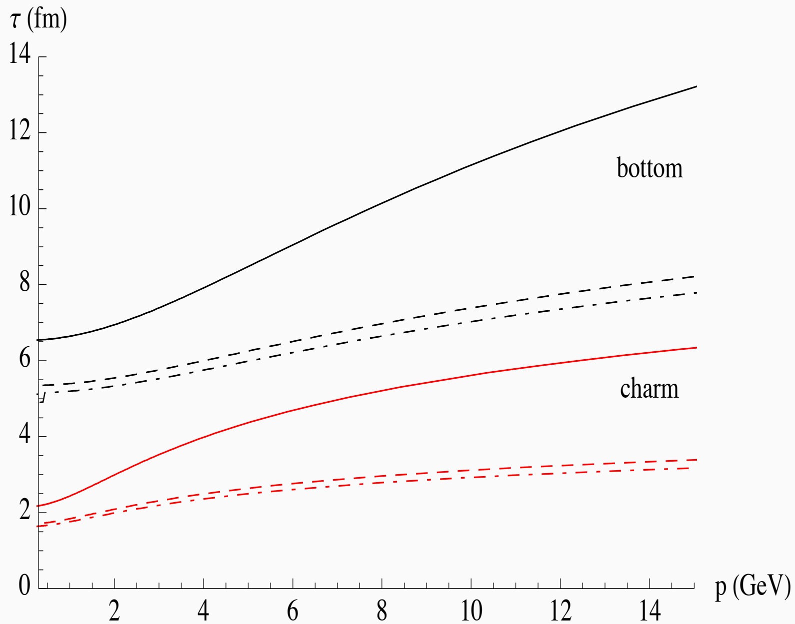
Equilibration times for charm and bottom:



Solid: direct, dashed: energy, dot-dashed entropy schemes.

Predictions for experiments

Equilibration times for charm and bottom:



Solid: direct, dashed: energy, dot-dashed entropy schemes.

Some experimental studies + models PHENIX col. '06, van Hees et al '05:

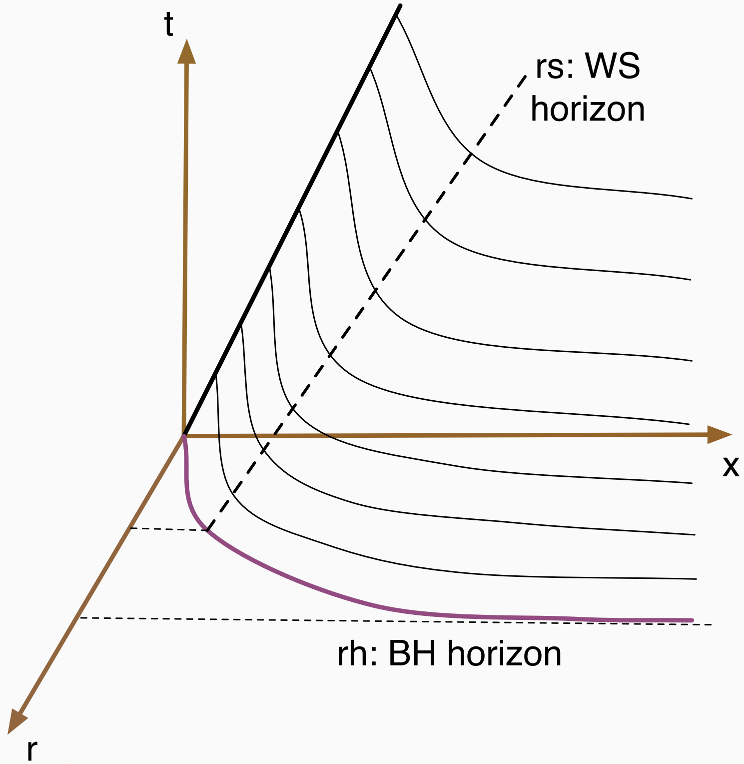
For $p = 10$ GeV, $\tau_e \approx 4.5\ fm$ (charm)

We have $3 < \tau_e < 5.5\ fm$

Diffusion constants

- In Fourier space
  $\kappa = \lim_{\omega\to 0} G_{sym}(\omega) = \lim_{\omega\to 0} \coth(\frac{\omega}{4T_s}) Im\, G_R(\omega)$
  where T_s is the world-sheet temperature.
- G_R extracted from fluctuations on the trailing string solution:
  $X^1 = vt + \zeta(r) + \delta X^1, \qquad X^T = \delta X^T.$
- There is a "horizon" on the world-sheet:
  $ds^2 = b^2 \left[-(f(r) - v^2) d\tau^2 + \frac{dr^2}{f - v^2 b^4(r_s)/b^4(r)} \right]$
  WS horizon at $f(r_s) = v^2$.
- $\kappa_\perp = \frac{2}{\pi \ell_s^2} b^2(r_s) T_s, \qquad \kappa_\| = \frac{32\pi}{\ell_s^2} \frac{b^2(r_s)}{f'(r_s)^2} T_s^3$

Physical picture



- A black-hole horizon on the WS at r_s:
  Fluctuations on the string fall into the horizon $\Rightarrow$ energy loss
- However, there is Hawking radiation at r_s towards the boundary
  $\Rightarrow$ momentum broadening.

Two universal results

- In the conformal case $\mathcal{N}=4$ sYM one finds,

$$\begin{aligned} \kappa_{\perp \mathcal{N}=4} &= \pi\sqrt{\lambda_{\mathcal{N}=4}}\gamma^{1/2}T^3 \\ \kappa_{\parallel \mathcal{N}=4} &= \pi\sqrt{\lambda_{\mathcal{N}=4}}\gamma^{5/2}T^3 \ . \end{aligned}$$

$\Rightarrow \frac{\kappa_\parallel}{\kappa_\perp} = \gamma^2 > 1$
This is just do to Lorentz contraction.

- In the non-conformal case, one further finds: $\Rightarrow \frac{\kappa_\parallel}{\gamma^2\kappa_\perp} \geq 1.$
- The effective temperature for the Langevin diffusion process is T_s instead of T!
In the conformal case: $T_{s,conf} = T/\sqrt{\gamma}$
In the non-conformal case: $T_s \leq T_{s,conf}$
equality only in the limit $T \gg 1$ or $v \to 1$.

Numerical results for jet quenching

- From the nuclear modification factors R_{AA} at RHIC and comparison with hydro simulations: $\hat{q}_\perp \sim 5 - 15\ GeV^2/fm$.
- If Langevin dynamics satisfied: $\hat{q}_\perp = \frac{2\kappa_\perp}{v}$.
- In the extreme relativistic limit $v \approx 1$, one derives:

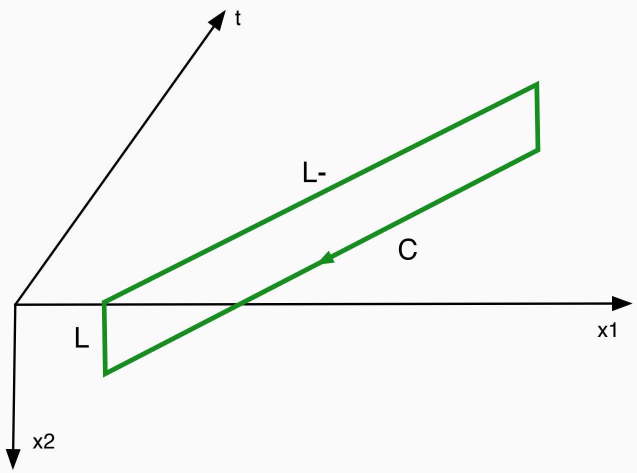
$$\kappa_\perp \approx \frac{(45\pi^2)^{\frac{3}{4}}}{\sqrt{2}\pi^2} \frac{\ell^2}{\ell_s^2} \frac{(sT)^{\frac{3}{4}}}{(1-v^2)^{\frac{1}{4}}} \left(-\frac{b_0}{4}\log(1-v^2)\right)^{-\frac{4}{3}}$$

- $\boxed{\hat{q}_\perp = 5.2 \text{ (direct)}, 12 \text{ (energy)}, 13.13 \text{ (entropy)}\ GeV^2/fm,}$

for a charm quark traveling at $p = 10GeV$ at $T = 250$ MeV.

Jet quenching, non-perturbative

Non-perturbative def. of $\hat{q}$:



Wiedemann '00

$$\langle W(C)\rangle \approx \exp\left[-\frac{1}{8\sqrt{2}}\hat{q}L^- L^2\right].$$

Holographic computation Liu, Rajagopal, Wiedemann '06: $\langle W(C)\rangle = e^{iS}$

Pick up gauge: $x^- \equiv x_1 - t = \tau, x_2 = \sigma$, Compute minimal area:

- $\hat{q} = \frac{\sqrt{2}}{\pi \ell_s^2} \frac{1}{\int_0^{r_h} \frac{dr}{e^{2A_s}\sqrt{f(1-f)}}}$

Results

T_{QGP}, MeV	$\hat{q}\ (GeV^2/fm)$ (direct)	$\hat{q}\ (GeV^2/fm)$ (energy)	$\hat{q}\ (GeV^2/fm)$ (entropy)
220	-	0.89	1.01
250	0.53	1.21	1.32
280	0.79	1.64	1.73
310	1.07	2.14	2.21
340	1.39	2.73	2.77
370	1.76	3.37	3.42
400	2.18	4.20	4.15

Close to AdS somewhat smaller than pQCD + fit to data Eskola et al '05

$\hat{q}_{expect} \sim 5 - 12\ GeV^2/fm$

QGP in magnetic fields

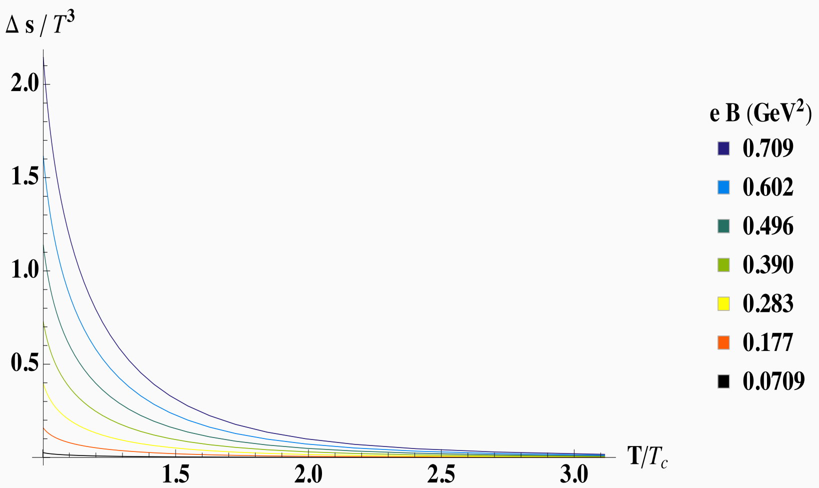
- Dependence on B at strong coupling: Landau levels, thermodynamic functions, transport coefficients, equilibration times
- Introduce B on diagonal through $U(1)_{L+R} \in U(N_f)_L \times U(N_f)_R$ gauge field on flavor branes
- B breaks $SO(3) \to U(1)$: $ds^2 = e^{2A(r)} \left(-e^{g(r)}dt^2 + dx_1^2 + dx_2^2 + e^{2W(r)}dx_3^2 + e^{-g(r)}dr^2\right), \qquad \lambda = \lambda(r)$
- Solve the coupled system of Einstein, Maxwell and dilaton and tachyon equations

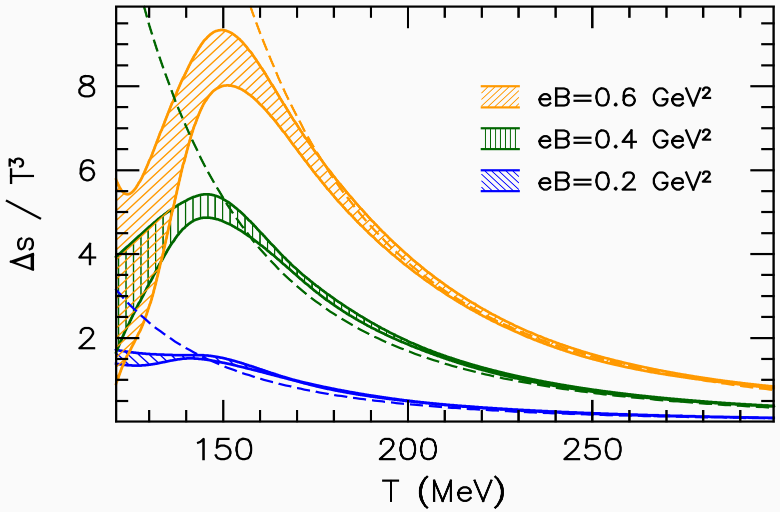
Thermodynamic functions

- Determine the entropy $S(B,T)$ from horizon area

  T. Drwenski, U.G., I. Iatrakis '15

  Compare with lattice Bali et al, '14

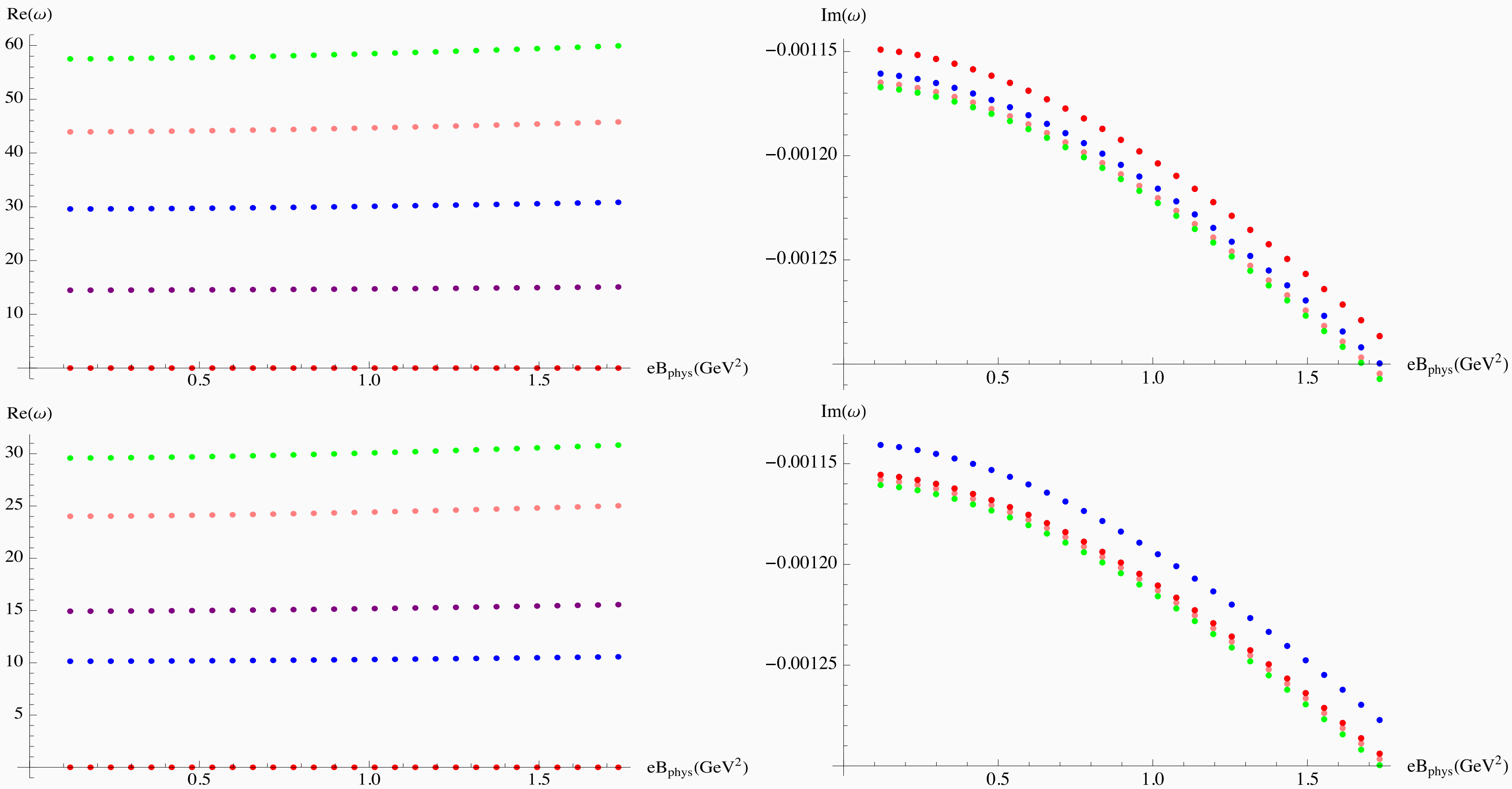




- Backreacted ihQCD with $N_f/N_c = 1/10$.

Equilibration times

- Equilibration profile of linear fluctuations ⇔ quasi-normal mode spectra of black-holes See M. Heller's lectures
- Shear and scalar modes - dependence on B T. Demircik, U.G '16



Summary

- A realistic holographic model for QCD at strong coupling
- Baryon and meson spectra, thermodynamics, transport coefficients, equilibration of linear fluctuations, thermalization.
- Universal results:
  - Linear confinement in the vacuum $\Rightarrow$ first order transition at T_c, mass gap, discrete spectrum
  - ζ/s increase monotonically as $T \to T_c$ from above
  - $\kappa_{\parallel} \geq \kappa_{\perp}$
  - All of this physics tied to linear confinement at zero T.
- Also very good quantitative agreement
- Not everything works: qualities sensitive to UV physics, e.g η/S at weak coupling

Outlook

- Full phase diagram in $\mu_c - T - B$.
- Higher order hydrodynamic coefficients
- Dependence of η/S and ζ/S on B
- Thermalization with full back reaction, with μ and B
- Meson sector. How to fix extra functions in the model?
- Anomalous transport K. Landsteiner's lectures: Renormalization of CME and CVE transport coefficients at strong coupling, Calculation of μ_5 in time-dependent setting, etc.
- Dynamical instabilities in the QGP R. Janik, J. Jankowski, H. Soltapanahi 1603.05950; U.G, A. Jansen, W. van der Schee 1603.07724

THANK YOU!

Witten's Model '98

- YM_5 on D4 Branes
- Antiperiodic boundary conditions on for the fermions on S^1 $m_\psi \sim \frac{1}{R}, \qquad m_\phi \sim \frac{\lambda_4}{R}$
- UV cut-off in the 4D theory at $E = 1/R$
- Pure YM_4 in the IR

Issues with Witten's Model

Issues with Witten's Model

- Dual geometry involve $S^1 \times S^4$ cycles
- Infinite number of bulk fields from KK modes!

Issues with Witten's Model

- Dual geometry involve $S^1 \times S^4$ cycles
- Infinite number of bulk fields from KK modes!
- In order to decouple: R_1 and R_4 should be small
- $\Rightarrow$ Highly curved regime!

Issues with Witten's Model

- Dual geometry involve $S^1 \times S^4$ cycles
- Infinite number of bulk fields from KK modes!
- In order to decouple: R_1 and R_4 should be small
- $\Rightarrow$ Highly curved regime!

In the super-gravity approximation: Dual field theory with infinite number of operators
$\Rightarrow$ a disguised 5D field theory

Issues with Witten's Model

- Dual geometry involve $S^1 \times S^4$ cycles
- Infinite number of bulk fields from KK modes!
- In order to decouple: R_1 and R_4 should be small
- $\Rightarrow$ Highly curved regime!

In the super-gravity approximation: Dual field theory with infinite number of operators
$\Rightarrow$ a disguised 5D field theory
Although in the same universality class as QCD, very different physics.

Need the full world-sheet theory $\Rightarrow$ Very hard open problem...

Thermodynamics of Hard-Wall

Herzog ’06; U.G., ’08

- Two competing solutions: (i) Thermal gas in AdS cavity, (ii) the AdS Black-hole

Thermodynamics of Hard-Wall

Herzog '06; U.G., '08

- Two competing solutions: (i) Thermal gas in AdS cavity, (ii) the AdS Black-hole
- Free energy from $S_{BH} - S_{TG}$:

$$f_{HW} = -p_{HW} = (M_p \ell)^3 \Lambda^4 \left[2 - \pi^4 \left(\frac{T}{\Lambda} \right)^4 \right]$$

Thermodynamics of Hard-Wall

Herzog '06; U.G., '08

- Two competing solutions: (i) Thermal gas in AdS cavity, (ii) the AdS Black-hole
- Free energy from $S_{BH} - S_{TG}$:

$$f_{HW} = -p_{HW} = (M_p \ell)^3 \Lambda^4 \left[2 - \pi^4 \left(\frac{T}{\Lambda} \right)^4 \right]$$

- How to fix the Planck Mass? U.G. et al. '08

Thermodynamics of Hard-Wall

Herzog '06; U.G., '08

- Two competing solutions: (i) Thermal gas in AdS cavity, (ii) the AdS Black-hole
- Free energy from $S_{BH} - S_{TG}$:

$$f_{HW} = -p_{HW} = (M_p \ell)^3 \Lambda^4 \left[2 - \pi^4 \left(\frac{T}{\Lambda} \right)^4 \right]$$

- How to fix the Planck Mass? U.G. et al. '08
- At high T, QCD $\Rightarrow$ free gas of gluons $f/T^4 \to -\pi^2/45$
- Comparison to f_{HW} fixes universally $M_p \ell = (45\pi^2)^{-\frac{1}{3}}$.

Thermodynamics of Hard-Wall

Herzog '06; U.G., '08

- Two competing solutions: (i) Thermal gas in AdS cavity, (ii) the AdS Black-hole
- Free energy from $S_{BH} - S_{TG}$:

$$f_{HW} = -p_{HW} = (M_p \ell)^3 \Lambda^4 \left[2 - \pi^4 \left(\frac{T}{\Lambda} \right)^4 \right]$$

- How to fix the Planck Mass? U.G. et al. '08
- At high T, QCD $\Rightarrow$ free gas of gluons $f/T^4 \to -\pi^2/45$
- Comparison to f_{HW} fixes universally $M_p \ell = (45\pi^2)^{-\frac{1}{3}}$.
- Fix Λ by lowest ρ meson $\Rightarrow \Lambda = 323$ Mev.

Thermodynamics of Hard-Wall

Herzog '06; U.G., '08

- Two competing solutions: (i) Thermal gas in AdS cavity, (ii) the AdS Black-hole
- Free energy from $S_{BH} - S_{TG}$:

$$f_{HW} = -p_{HW} = (M_p \ell)^3 \Lambda^4 \left[2 - \pi^4 \left(\frac{T}{\Lambda} \right)^4 \right]$$

- How to fix the Planck Mass? U.G. et al. '08
- At high T, QCD $\Rightarrow$ free gas of gluons $f/T^4 \to -\pi^2/45$
- Comparison to f_{HW} fixes universally $M_p \ell = (45\pi^2)^{-\frac{1}{3}}$.
- Fix Λ by lowest ρ meson $\Rightarrow \Lambda = 323$ Mev.
- Deconfinement transition at $T_c = 2^{1/4}\Lambda/\pi$, for $\Lambda = 323$ MeV, $T_c = 122$ MeV.
- Latent heat $L_h = 4\pi^2/45T_c^4 = (0.97T_c)^4$. Compare to Lucini et al. '05 $L_h = (0.77T_c)^4$

Local approximation

Local approximation

- Suppose correlations vanish for $t - t' \gg \tau_c$:

Local approximation

- Suppose correlations vanish for $t - t' \gg \tau_c$:
  $A^{ij}(t - t') \approx \kappa^{ij}\delta(t - t'), \qquad C^{ij}(t) = \frac{d}{dt}\gamma^{ij}(t)$

Local approximation

- Suppose correlations vanish for $t - t' \gg \tau_c$:
  $A^{ij}(t-t') \approx \kappa^{ij}\delta(t-t'), \qquad C^{ij}(t) = \frac{d}{dt}\gamma^{ij}(t)$
- EOM for the hard probe becomes,
  $\frac{\delta S_0}{\delta X_i(t)} + \eta^{ij}\dot{X}_j(t) = \xi^i(t), \qquad \langle \xi^i(t)\xi^j(t')\rangle = \kappa^{ij}\delta(t-t')$
  with $\kappa^{ij} = \lim_{\omega\to 0} A^{ij}(\omega)$,
  $\eta^{ij} \equiv \int_0^\infty d\tau\, \gamma^{ij}(\tau) = -\lim_{\omega\to 0}\frac{\mathrm{Im}\, C^{ij}(\omega)}{\omega}$

Local approximation

- Suppose correlations vanish for $t - t' \gg \tau_c$:
  $A^{ij}(t-t') \approx \kappa^{ij}\delta(t-t')$, $\qquad C^{ij}(t) = \frac{d}{dt}\gamma^{ij}(t)$
- EOM for the hard probe becomes,
  $\frac{\delta S_0}{\delta X_i(t)} + \eta^{ij}\dot{X}_j(t) = \xi^i(t)$, $\qquad \langle \xi^i(t)\xi^j(t')\rangle = \kappa^{ij}\delta(t-t')$
  with $\kappa^{ij} = \lim_{\omega\to 0} A^{ij}(\omega)$,
  $\eta^{ij} \equiv \int_0^\infty d\tau\, \gamma^{ij}(\tau) = -\lim_{\omega\to 0} \frac{\mathrm{Im}\, C^{ij}(\omega)}{\omega}$
- For QGP at equilibrium at temperature T:
  $A^{ij}(\omega) = -\coth(\frac{\omega}{2T}) Im G_R^{ij}(\omega)$, $\qquad C^{ij}(\omega) = Im G_R^{ij}(\omega)$

Local approximation

- Suppose correlations vanish for $t - t' \gg \tau_c$:
  $A^{ij}(t-t') \approx \kappa^{ij}\delta(t-t'), \qquad C^{ij}(t) = \frac{d}{dt}\gamma^{ij}(t)$
- EOM for the hard probe becomes,
  $\frac{\delta S_0}{\delta X_i(t)} + \eta^{ij}\dot{X}_j(t) = \xi^i(t), \qquad \langle \xi^i(t)\xi^j(t')\rangle = \kappa^{ij}\delta(t-t')$
  with $\kappa^{ij} = \lim_{\omega\to 0} A^{ij}(\omega)$,
  $\eta^{ij} \equiv \int_0^\infty d\tau\, \gamma^{ij}(\tau) = -\lim_{\omega\to 0} \frac{\mathrm{Im}\, C^{ij}(\omega)}{\omega}$
- For QGP at equilibrium at temperature T:
  $A^{ij}(\omega) = -\coth(\frac{\omega}{2T}) Im G_R^{ij}(\omega), \qquad C^{ij}(\omega) = Im G_R^{ij}(\omega)$
- Thus, it is sufficient to calculate
  $G_R(\omega) = -i\int dt e^{-i\omega t}\theta(t)\langle [F^i(t), F^j(0)]\rangle$
  This is what we will compute using AdS/CFT.

Glueballs

Spectrum of 4D glueballs $\Leftrightarrow$ Spectrum of normalizable fluctuations of the bulk fields.

Spin 2: $h^{TT}_{\mu\nu}$; Spin 0: mixture of h^{μ}_{μ} and $\delta\Phi$;

Glueballs

Spectrum of 4D glueballs $\Leftrightarrow$ Spectrum of normalizable fluctuations of the bulk fields.

Spin 2: $h^{TT}_{\mu\nu}$; Spin 0: mixture of h^{μ}_{μ} and $\delta\Phi$;

Quadratic action for fluctuations:

$$S \sim \frac{1}{2} \int d^4x dr e^{2B(r)} \left[\dot{\zeta}^2 + (\partial_\mu \zeta)^2\right]$$

$$\ddot{\zeta} + 3\dot{B}\dot{\zeta} + m^2\zeta = 0, \quad \partial^\mu \partial_\mu \zeta = -m^2 \zeta$$

Glueballs

Spectrum of 4D glueballs $\Leftrightarrow$ Spectrum of normalizable fluctuations of the bulk fields.
Spin 2: $h_{\mu\nu}^{TT}$; Spin 0: mixture of h_μ^μ and $\delta\Phi$;
Quadratic action for fluctuations:

$$S \sim \frac{1}{2}\int d^4x dr e^{2B(r)} \left[\dot{\zeta}^2 + (\partial_\mu \zeta)^2\right]$$

$$\ddot{\zeta} + 3\dot{B}\dot{\zeta} + m^2\zeta = 0, \quad \partial^\mu\partial_\mu\zeta = -m^2\zeta$$

- Scalar : $B(r) = 3/2A(r) + \log(\dot{\Phi}/\dot{A})$
- Tensor : $B(r) = 3/2A(r)$

Reduction to a Schrödinger problem

Define:$\zeta(r) = e^{-B(r)}\Psi(r)$ Schrödinger equation:

$$\mathcal{H}\Psi \equiv -\ddot{\Psi} + V(r)\Psi = m^2\Psi \qquad V_s(r) = \dot{B}^2 + \ddot{B}$$

Reduction to a Schrödinger problem

Define:$\zeta(r) = e^{-B(r)}\Psi(r)$ Schrödinger equation:

$$\mathcal{H}\Psi \equiv -\ddot{\Psi} + V(r)\Psi = m^2\Psi \qquad V_s(r) = \dot{B}^2 + \ddot{B}$$

- The normalizability condition: $\int dr \, |\Psi|^2 < \infty$
- Normalizability in the UV, picks normalizable UV asymptotics for ζ
- Normalizability in the IR, restricts discrete m^2, for confining V_s.

Mass gap

$$\mathcal{H} = (\partial_r + \partial_r B)(-\partial_r + \partial_r B) = \mathcal{P}^\dagger \mathcal{P} \geq 0 :$$

- Spectrum is non-negative
- Can prove that no normalizable zero-modes
- If $V(r) \to \infty$ as $r \to +\infty$: Mass Gap

Mass gap

$$\mathcal{H} = (\partial_r + \partial_r B)(-\partial_r + \partial_r B) = \mathcal{P}^\dagger \mathcal{P} \geq 0 :$$

- Spectrum is non-negative
- Can prove that no normalizable zero-modes
- If $V(r) \to \infty$ as $r \to +\infty$: Mass Gap
- This precisely coincides with the condition from color confinement
- For $A(r) \to -Cr^\alpha, \qquad \Leftrightarrow \qquad V(r) \sim r^{\alpha-1}$
  Confinement AND mass gap for $\alpha \geq 1$.

Field theory computation

Kharzeev, Tuchin,Karsch ’07

Field theory computation

Kharzeev, Tuchin,Karsch '07

- Kubo's linear response theory:

$$\zeta = -\frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega} Im G_R(\omega,0) = \frac{\pi}{9}\lim_{\omega\to 0}\frac{1}{\omega}\rho(\omega)$$

where $G_R(\omega,\vec{p}) = -i\int d^3x dt e^{i\omega t - i\vec{p}\dot{\vec{x}}}\theta(t)\sum_{i,j=1}^{3}\langle[T_{ii}(t,\vec{x}),T_{jj}(0,0)]\rangle.$

Field theory computation

Kharzeev, Tuchin,Karsch ’07

- Kubo’s linear response theory:

$$\zeta = -\frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega} Im G_R(\omega,0) = \frac{\pi}{9}\lim_{\omega\to 0}\frac{1}{\omega}\rho(\omega)$$

where $G_R(\omega,\vec{p}) = -i\int d^3x dt e^{i\omega t - i\vec{p}\vec{x}}\theta(t)\sum_{i,j=1}^{3}\langle[T_{ii}(t,\vec{x}), T_{jj}(0,0)]\rangle.$

- Low-energy theorems (SVZ) at finite T:
$\lim_{\omega\to 0} G^E(\omega,0) = T\frac{d}{dT}\langle T^\mu_\mu\rangle$

- LHS by analytic continuation $G^E(\omega,0) = 2\int_0^\infty d\omega\frac{\rho(\omega)}{\omega}$.
RHS by thermodynamics $T^\mu_\mu = e(T) - 3p(T)$.

Field theory computation

Kharzeev, Tuchin,Karsch '07

- Kubo's linear response theory:

$$\zeta = -\frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega} Im G_R(\omega,0) = \frac{\pi}{9}\lim_{\omega\to 0}\frac{1}{\omega}\rho(\omega)$$

where $G_R(\omega,\vec{p}) = -i\int d^3x dt e^{i\omega t - i\vec{p}\dot{\vec{x}}}\theta(t)\sum_{i,j=1}^{3}\langle[T_{ii}(t,\vec{x}),T_{jj}(0,0)]\rangle.$

- Low-energy theorems (SVZ) at finite T:
  $\lim_{\omega\to 0} G^E(\omega,0) = T\frac{d}{dT}\langle T^\mu_\mu\rangle$
- LHS by analytic continuation $G^E(\omega,0) = 2\int_0^\infty d\omega \frac{\rho(\omega)}{\omega}$.
  RHS by thermodynamics $T^\mu_\mu = e(T) - 3p(T)$.
- One needs to make an ansatz for density to extract ζ:
  $\rho(\omega) = \frac{9\zeta}{\pi}\frac{\omega_0(T)\omega}{\omega_0^2(T)+\omega^2}$.

Field theory computation

Kharzeev, Tuchin,Karsch '07

- Kubo's linear response theory:

$$\zeta = -\frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega} Im G_R(\omega,0) = \frac{\pi}{9}\lim_{\omega\to 0}\frac{1}{\omega}\rho(\omega)$$

where $G_R(\omega,\vec{p}) = -i\int d^3x dt e^{i\omega t - i\vec{p}\dot{\vec{x}}}\theta(t)\sum_{i,j=1}^{3}\langle[T_{ii}(t,\vec{x}),T_{jj}(0,0)]\rangle$.

- Low-energy theorems (SVZ) at finite T:
  $\lim_{\omega\to 0} G^E(\omega,0) = T\frac{d}{dT}\langle T^\mu_\mu\rangle$
- LHS by analytic continuation $G^E(\omega,0) = 2\int_0^\infty d\omega\frac{\rho(\omega)}{\omega}$.
  RHS by thermodynamics $T^\mu_\mu = e(T) - 3p(T)$.
- One needs to make an ansatz for density to extract ζ:
  $\rho(\omega) = \frac{9\zeta}{\pi}\frac{\omega_0(T)\omega}{\omega_0^2(T)+\omega^2}$.

One finds that ζ/s increases as $T\to T_c$ from above, but the over-all size of ζ is ambigious... need to know $\rho(\omega)$ precisely.

Holographic computation

- Kubo's linear response theory:
  $\zeta = -\frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} Im G_R(w, 0)$
- More complicated than shear because h_{ii} mix with dilaton fluctuations $\delta\phi$.
- Derive the fluctuation equations for h_{ii}, pick up the gauge $\delta\phi = 0$,
- Fluctuations decouple in the smart gauge! Gubser et al '08: Define $X = \phi'/3A'$
- $h''_{ii} + \left(3A' + 2\frac{X'}{X} + \frac{f'}{f}\right) h'_{ii} + \left(\frac{\omega^2}{f^2} - \frac{f'X'}{fX}\right) h_{ii} = 0$
- Boundary conditions:
  - $h_{ii}(\phi = -\infty) = 1$ and,
  - In-falling wave at horizon $h_{ii} \to c_b(r_h - r)^{-\frac{i\omega}{4\pi T}}$
- Read off $c_b(\omega, T)$

Spectral densities

Spectral densities

The fluctuation equations for shear and bulk fluctuations can also be solved for finite ω. to obtain $\rho(\omega) = -\frac{1}{\pi} Im\, G_R(\omega)$.

- One has $\rho(\omega) = -\frac{1}{\pi} Im\, G_R(\omega, \vec{p})$,

Spectral densities

The fluctuation equations for shear and bulk fluctuations can also be solved for finite ω. to obtain $\rho(\omega) = -\frac{1}{\pi} Im\, G_R(\omega)$.

- One has $\rho(\omega) = -\frac{1}{\pi} Im\, G_R(\omega, \vec{p})$,
- Numerical computation for arbitrary ω is ongoing work.

Spectral densities

The fluctuation equations for shear and bulk fluctuations can also be solved for finite ω. to obtain $\rho(\omega) = -\frac{1}{\pi} Im\, G_R(\omega)$.

- One has $\rho(\omega) = -\frac{1}{\pi} Im\, G_R(\omega, \vec{p})$,
- Numerical computation for arbitrary ω is ongoing work.
- One can obtain exact results by WKB approximation:
  Transform the fluctuation equations into Schrodinger form and apply WKB for $\omega/T \gg 1$: U.G., Kiritsis, to appear

$$\rho_s(\omega) = \frac{N_c^2}{360\pi^2}\omega^4, \qquad \rho_b(\omega) = \frac{N_c^2}{540\pi^2}\omega^4 b_0^2 \lambda_t^2.$$

Spectral densities

The fluctuation equations for shear and bulk fluctuations can also be solved for finite ω. to obtain $\rho(\omega) = -\frac{1}{\pi} Im\, G_R(\omega)$.

- One has $\rho(\omega) = -\frac{1}{\pi} Im\, G_R(\omega, \vec{p})$,
- Numerical computation for arbitrary ω is ongoing work.
- One can obtain exact results by WKB approximation: Transform the fluctuation equations into Schrodinger form and apply WKB for $\omega/T \gg 1$: U.G., Kiritsis, to appear

$$\rho_s(\omega) = \frac{N_c^2}{360\pi^2}\omega^4, \qquad \rho_b(\omega) = \frac{N_c^2}{540\pi^2}\omega^4 b_0^2 \lambda_t^2.$$

- Contrast them with the perturbative Yang-Mills computations:

H. Meyer '07

$$\rho_s(\omega) = \frac{N_c^2}{160\pi^2}\omega^4, \qquad \rho_b(\omega) = \frac{N_c^2}{64\pi^2}\omega^4 b_0^2 \lambda_t^2.$$

Spectral densities

The fluctuation equations for shear and bulk fluctuations can also be solved for finite ω. to obtain $\rho(\omega) = -\frac{1}{\pi} Im\, G_R(\omega)$.

- One has $\rho(\omega) = -\frac{1}{\pi} Im\, G_R(\omega, \vec{p})$,
- Numerical computation for arbitrary ω is ongoing work.
- One can obtain exact results by WKB approximation: Transform the fluctuation equations into Schrodinger form and apply WKB for $\omega/T \gg 1$: U.G., Kiritsis, to appear

$$\rho_s(\omega) = \frac{N_c^2}{360\pi^2}\omega^4, \qquad \rho_b(\omega) = \frac{N_c^2}{540\pi^2}\omega^4 b_0^2 \lambda_t^2.$$

- Contrast them with the perturbative Yang-Mills computations:

H. Meyer '07

$$\rho_s(\omega) = \frac{N_c^2}{160\pi^2}\omega^4, \qquad \rho_b(\omega) = \frac{N_c^2}{64\pi^2}\omega^4 b_0^2 \lambda_t^2.$$

- Dependence on ω, N_c and λ_t match, as it should.
- Overall coefficients do not —we expect α' corrections in UV.

Langevin diffusion process

Langevin diffusion process

- Hard probe moving in QGP: $S[X(t)] = S_0 + \int d\tau X_\mu(\tau)\mathcal{F}^\mu(\tau)$
  S_0:free quark action, $\mathcal{F}(\tau)$: drag force—summarizes the d.o.f of the plasma
- EOM of the hard probe:

Langevin diffusion process

- Hard probe moving in QGP: $S[X(t)] = S_0 + \int d\tau X_\mu(\tau)\mathcal{F}^\mu(\tau)$
  S_0:free quark action, $\mathcal{F}(\tau)$: drag force—summarizes the d.o.f of the plasma
- EOM of the hard probe:
  $\frac{\delta S_0}{\delta X_i(t)} = \int_{-\infty}^{+\infty} d\tau\ \theta(\tau)C^{ij}(\tau)X_j(t-\tau) + \xi^i(t), \qquad i = 1,2,3$
  with $\langle \xi^i(t)\xi^j(t')\rangle = A^{ij}(t-t')$

Langevin diffusion process

- Hard probe moving in QGP: $S[X(t)] = S_0 + \int d\tau X_\mu(\tau)\mathcal{F}^\mu(\tau)$
  S_0:free quark action, $\mathcal{F}(\tau)$: drag force—summarizes the d.o.f of the plasma
- EOM of the hard probe:
  $\frac{\delta S_0}{\delta X_i(t)} = \int_{-\infty}^{+\infty} d\tau \, \theta(\tau) C^{ij}(\tau) X_j(t-\tau) + \xi^i(t), \qquad i = 1, 2, 3$
  with $\langle \xi^i(t)\xi^j(t')\rangle = A^{ij}(t-t')$
- The entire information is stored in:
  $C^{ij}(t) \equiv -i\langle \big[\mathcal{F}^i(t), \mathcal{F}^j(0)\big]\rangle,$
  $A^{ij}(t) \equiv -\frac{i}{2}\langle \big\{\mathcal{F}^i(t), \mathcal{F}^j(0)\big\}\rangle.$

Local approximation

Local approximation

- Suppose correlations vanish for $t - t' \gg \tau_c$:

Local approximation

- Suppose correlations vanish for $t - t' \gg \tau_c$:
  $A^{ij}(t - t') \approx \kappa^{ij} \delta(t - t'), \qquad C^{ij}(t) = \frac{d}{dt}\gamma^{ij}(t)$

Local approximation

- Suppose correlations vanish for $t - t' \gg \tau_c$:
  $A^{ij}(t-t') \approx \kappa^{ij}\delta(t-t'), \qquad C^{ij}(t) = \frac{d}{dt}\gamma^{ij}(t)$
- EOM for the hard probe becomes,

$$\boxed{\frac{\delta S_0}{\delta X_i(t)} + \eta^{ij}\dot{X}_j(t) = \xi^i(t), \qquad \langle \xi^i(t)\xi^j(t')\rangle = \kappa^{ij}\delta(t-t')}$$

with

$$\kappa^{ij} = \lim_{\omega\to 0} A^{ij}(\omega),$$

$$\eta^{ij} \equiv \int_0^\infty d\tau\, \gamma^{ij}(\tau) = -\lim_{\omega\to 0}\frac{\mathrm{Im}\, C^{ij}(\omega)}{\omega}$$

Local approximation

- Suppose correlations vanish for $t - t' \gg \tau_c$:
  $A^{ij}(t-t') \approx \kappa^{ij}\delta(t-t'), \qquad C^{ij}(t) = \frac{d}{dt}\gamma^{ij}(t)$
- EOM for the hard probe becomes,

$$\boxed{\frac{\delta S_0}{\delta X_i(t)} + \eta^{ij}\dot{X}_j(t) = \xi^i(t), \qquad \langle \xi^i(t)\xi^j(t')\rangle = \kappa^{ij}\delta(t-t')}$$ with

$\kappa^{ij} = \lim_{\omega\to 0} A^{ij}(\omega),$

$\eta^{ij} \equiv \int_0^\infty d\tau\, \gamma^{ij}(\tau) = -\lim_{\omega\to 0} \frac{\mathrm{Im}\, C^{ij}(\omega)}{\omega}$

- For QGP at equilibrium at temperature T:
  $A^{ij}(\omega) = -\coth(\frac{\omega}{2T}) ImG_R^{ij}(\omega), \qquad C^{ij}(\omega) = ImG_R^{ij}(\omega)$

Local approximation

- Suppose correlations vanish for $t - t' \gg \tau_c$:
  $A^{ij}(t-t') \approx \kappa^{ij}\delta(t-t'), \qquad C^{ij}(t) = \frac{d}{dt}\gamma^{ij}(t)$
- EOM for the hard probe becomes,
  $$\boxed{\frac{\delta S_0}{\delta X_i(t)} + \eta^{ij}\dot{X}_j(t) = \xi^i(t), \qquad \langle \xi^i(t)\xi^j(t')\rangle = \kappa^{ij}\delta(t-t')}$$ with
  $\kappa^{ij} = \lim_{\omega\to 0} A^{ij}(\omega)$,
  $\eta^{ij} \equiv \int_0^\infty d\tau\, \gamma^{ij}(\tau) = -\lim_{\omega\to 0} \frac{\mathrm{Im}\, C^{ij}(\omega)}{\omega}$
- For QGP at equilibrium at temperature T:
  $A^{ij}(\omega) = -\coth(\frac{\omega}{2T}) ImG_R^{ij}(\omega), \qquad C^{ij}(\omega) = ImG_R^{ij}(\omega)$
- Thus, it is sufficient to calculate
  $G_R(\omega) = -i\int dte^{-i\omega t}\theta(t)\langle[F^i(t), F^j(0)]\rangle$

Momentum broadening

Momentum broadening

For $\Delta t \ll \tau_c$, short-time solution to the EOM for the hard-probe:
(in momentum space, around liner trajectory: $\vec{p} \simeq p_0 \vec{v}/v + \delta \vec{p}$.

Momentum broadening

For $\Delta t \ll \tau_c$, short-time solution to the EOM for the hard-probe:
(in momentum space, around liner trajectory: $\vec{p} \simeq p_0 \vec{v}/v + \delta\vec{p}$.

$\frac{d\delta p^{\perp}}{dt} = -\eta^{\perp}\delta p^{\perp} + \xi^{\perp}$

Momentum broadening

For $\Delta t \ll \tau_c$, short-time solution to the EOM for the hard-probe:
(in momentum space, around liner trajectory: $\vec{p} \simeq p_0 \vec{v}/v + \delta\vec{p}$.

$\frac{d\delta p^\perp}{dt} = -\eta^\perp \delta p^\perp + \xi^\perp$

Solution with initial conditions $\delta\vec{p}(t=0) = 0$:

$p^\perp(t) = \int_0^t dt'\, e^{\eta^\perp(t'-t)} \xi^\perp(t'),$

Momentum broadening

For $\Delta t \ll \tau_c$, short-time solution to the EOM for the hard-probe:
(in momentum space, around liner trajectory: $\vec{p} \simeq p_0 \vec{v}/v + \delta\vec{p}$.

$\frac{d\delta p^\perp}{dt} = -\eta^\perp \delta p^\perp + \xi^\perp$

Solution with initial conditions $\delta\vec{p}(t=0) = 0$:

$p^\perp(t) = \int_0^t dt' \, e^{\eta^\perp (t'-t)} \xi^\perp(t')$,

Compute the noise-average of fluctuations:

$\langle (p^\perp)^2 \rangle = \int_0^t dt' \, \int_0^t dt'' \, e^{\eta^\perp (t'+t''-2t)} \langle \xi^\perp(t') \xi^\perp(t'') \rangle$

Momentum broadening

For $\Delta t \ll \tau_c$, short-time solution to the EOM for the hard-probe:

(in momentum space, around liner trajectory: $\vec{p} \simeq p_0 \vec{v}/v + \delta\vec{p}$.

$\frac{d\delta p^\perp}{dt} = -\eta^\perp \delta p^\perp + \xi^\perp$

Solution with initial conditions $\delta\vec{p}(t=0) = 0$:

$p^\perp(t) = \int_0^t dt'\, e^{\eta^\perp (t'-t)} \xi^\perp(t')$,

Compute the noise-average of fluctuations:

$\langle (p^\perp)^2 \rangle = \int_0^t dt' \ \int_0^t dt''\, e^{\eta^\perp (t'+t''-2t)} \langle \xi^\perp(t') \xi^\perp(t'') \rangle$

Use $\langle \xi(t')\xi(t'') \rangle = \kappa \delta(t' - t'')$, for $t\eta^\perp \ll 1$:

$\langle (p^\perp)^2 \rangle = 2\kappa^\perp t$

Momentum broadening

For $\Delta t \ll \tau_c$, short-time solution to the EOM for the hard-probe:
(in momentum space, around liner trajectory: $\vec{p} \simeq p_0 \vec{v}/v + \delta\vec{p}$.

$\frac{d\delta p^\perp}{dt} = -\eta^\perp \delta p^\perp + \xi^\perp$

Solution with initial conditions $\delta\vec{p}(t=0) = 0$:

$p^\perp(t) = \int_0^t dt'\, e^{\eta^\perp (t'-t)} \xi^\perp(t')$,

Compute the noise-average of fluctuations:

$\langle (p^\perp)^2 \rangle = \int_0^t dt' \, \int_0^t dt''\, e^{\eta^\perp (t'+t''-2t)} \langle \xi^\perp(t') \xi^\perp(t'') \rangle$

Use $\langle \xi(t')\xi(t'') \rangle = \kappa \delta(t'-t'')$, for $t\eta^\perp \ll 1$:

$\langle (p^\perp)^2 \rangle = 2\kappa^\perp t$

thus jet-quenching parameter:

$\hat{q}^\perp = \frac{\langle (p^\perp)^2 \rangle}{vt} = 2\frac{\kappa^\perp}{v}$.

How to calculate in the bulk dual?

Recall $S[X(t)] = S_0 + \int d\tau X_\mu(\tau)\mathcal{F}^\mu(\tau)$

To calculate $\langle\{\mathcal{F}^\perp(t), \mathcal{F}^\perp(0)\}\rangle = \mathcal{O}(0) + \langle\{\xi^\perp(t), \xi^\perp(0)\}\rangle$

We need to calculate the fluctuations $\delta X^\perp(t)$.