Yangian Symmetry of N=4 SYM

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Joint work with

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Table of contents

*Shortened due to overlap with Jan Plefka's lecture

$\mathcal{N}=4$ SYM

Maximally supersymmetric gauge theory in 4D

1 gauge field A_μ, 4 spinors Ψ, 4 conjugate spinors $\bar{\Psi}$, 6 scalars ϕ_m

psu(2,2|4) - symmetry algebra of the action

Integrability in N=4 SYM

Spectra of local operators

Wilson loops

Scattering amplitudes

} Increased symmetry algebra: Yangian

Yangian Algebra

Infinitely many levels of generators.

Level 0 - original psu(2,2|4):

$$[J_a, J_b\} = f_{ab}^c J_c$$

Level 1 - first higher level Yangian generators:

$$[J_a, \hat{J}_b\} = f_{ab}^c \hat{J}_c$$

Coproduct (action on multiparticle states)

$$\Delta(J_a) = id \otimes J_a + J_a \otimes id$$

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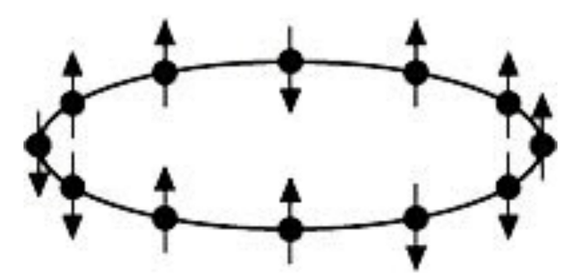
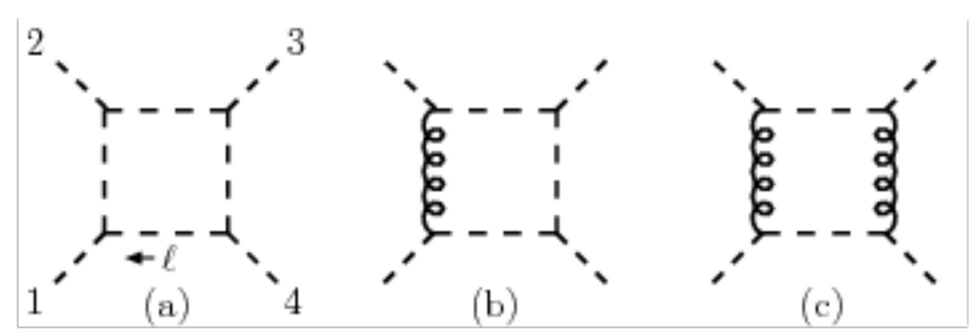
Motivation (for advanced)

We know how Yangian generators act on

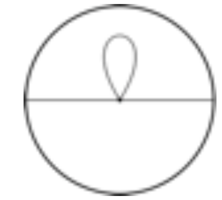
spin chains



scattering amplitudes



Wilson loops



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See Jan Plefka's lecture for details!

(also mentioned in Jacob Bourjaily's talk)

Yangian Action on Action

$$S = \int d^4x Tr(...)$$

Due to the trace, the action is cyclic:

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However:

$$\hat{J}(ABC) \neq \hat{J}(CAB)$$

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Incompatible!

Symmetry of the EoMs

(they are no longer cyclic)

Simplest level-1 generator:

$$\Delta(\hat{P}^{\mu}) = D \wedge P^{\mu} + L^{\mu\nu} \wedge P_{\nu} - \frac{i}{4}\sigma^{\mu\alpha\dot{\alpha}}\bar{Q}^{a}_{\dot{\alpha}} \wedge Q_{a\alpha}$$

Easiest equation of motion:

$$\sigma^{\rho\dot{\alpha}\beta}[D_{\rho}, \Psi_{\beta a}] - ig\epsilon^{\dot{\alpha}\dot{\beta}}\sigma^{m}_{ab}[\Phi_{m}, \bar{\Psi}^{b}_{\dot{\beta}}] = 0$$

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etc...

Symmetry of the EoMs

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Introducing the single field action:

$$\hat{P}^\mu(D^\rho) = \frac{1}{2} g^2 \eta^{\mu\rho} \{\Phi^m, \Phi_m\}$$

$$\hat{P}^\mu(\Psi_{a\alpha}) = \frac{ig}{2} \sigma^\mu_{\alpha\dot{\beta}} \epsilon^{\dot{\beta}\dot{\gamma}} \sigma^m_{ab} \{\bar{\Psi}^b_{\dot{\gamma}}, \Phi_m\}$$

we obtain schematically:

$$\hat{P}(Dirac) = 0$$

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Similarly for all the other equations of motion!

we obtain schematically:

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Off-shell?

For level-0 symmetry generators we have:

$$0 = JS = \int (JZ_a) \frac{\delta S}{\delta Z_a}$$

Varying again w. r. t. to a field:

Vanishes on-shell

$$J \frac{\delta S}{\delta Z_c} + \frac{\delta (JZ_a)}{\delta Z_c} \frac{\delta S}{\delta Z_a} = 0$$

Off-shell equality regarding behavior of eoms under J.

Off-shell!

Generalize to level-1 generators:

$$\hat{J}^k \frac{\delta S}{\delta Z_c} = -\frac{\delta(\hat{J}^k Z_a)}{\delta Z_c}\frac{\delta S}{\delta Z_a} + f^k{}_{mn}\left((J^m Z_a)\frac{\delta}{\delta Z_a} \wedge \frac{\delta S}{\delta Z_b}\frac{\delta}{\delta Z_c}\right)(J^n Z_b)$$

Vanishes on shell

Holds off-shell!

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$$\hat{J}^k \frac{\delta S}{\delta \Phi_c} = \frac{\delta(\hat{J}^k \Phi_a)}{\delta \Phi_c} \frac{\delta S}{\delta \Phi_a} + f^k{}_{mn} \left((J^m \Phi_a) \frac{\delta}{\delta \Phi_a} \wedge \frac{\delta S}{\delta \Phi_b} \frac{\delta}{\delta \Phi_c} \right) (J^n \Phi_b)$$

Vanishes on shell

Holds off-shell!

Doesn't hold for non-integrable field theories!

Brief recap

- Yangian algebra appears in many observables in N=4 SYM.
- At the first level, the Yangian generators act bilocally (here - in color space).
- Cannot act with them on the action, but they are an on-shell symmetry of equations of motion.
- We put forward an off-shell equality we believe equivalent to the invariance of the action.

Correlation Functions

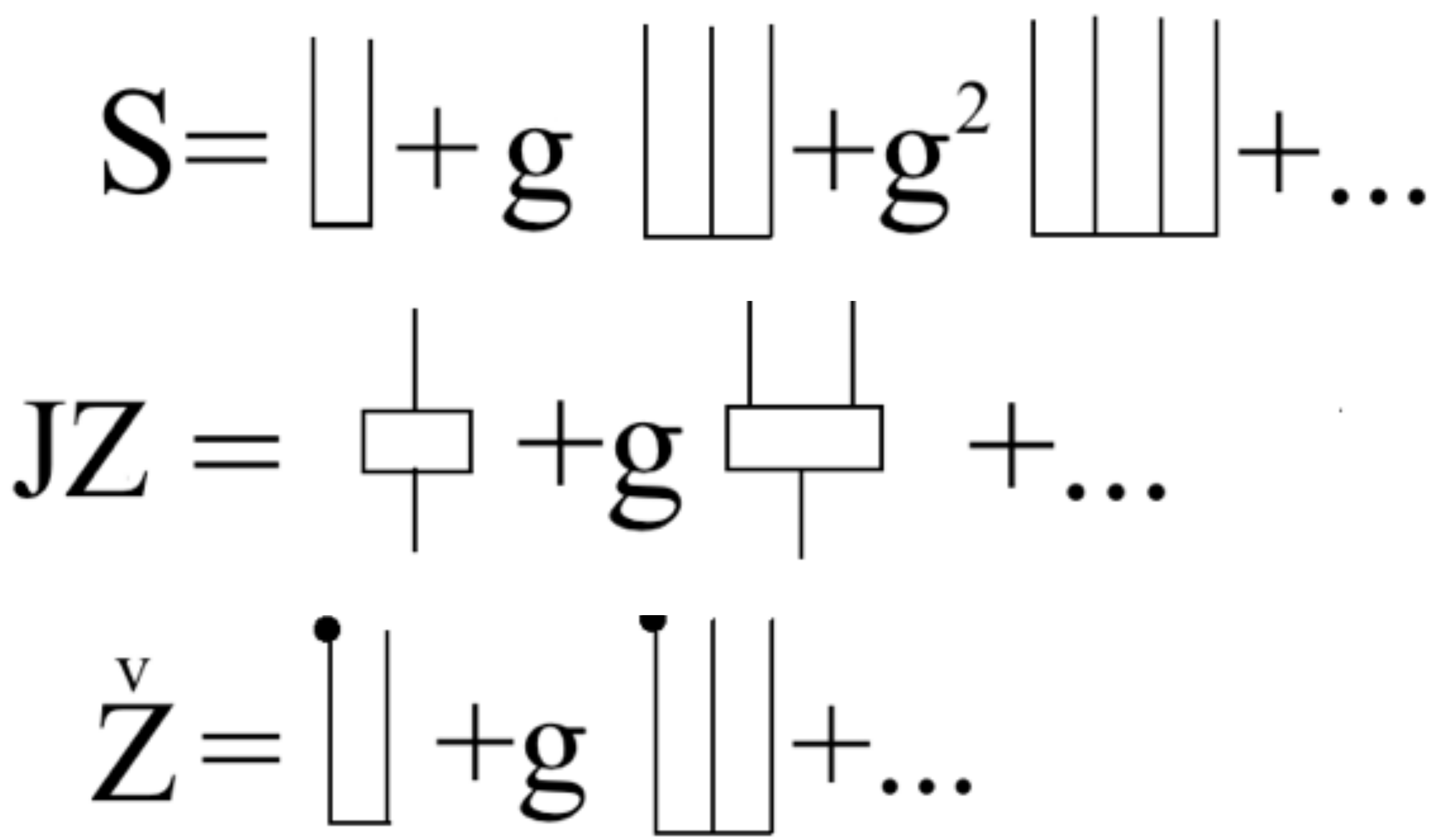
(tree level)

If $JS = 0$ and J leaves the PI measure invariant, the correlation functions are also invariant (Slavnov-Taylor / Ward identity):

$$J < Z_1(x_1)Z_2(x_2)...Z_n(x_n) >= \sum_{i=1}^{N} < Z_1(x_1)...(JZ_i(x_i))...Z_n(x_n) >= 0$$

Ars Gratia Artis*

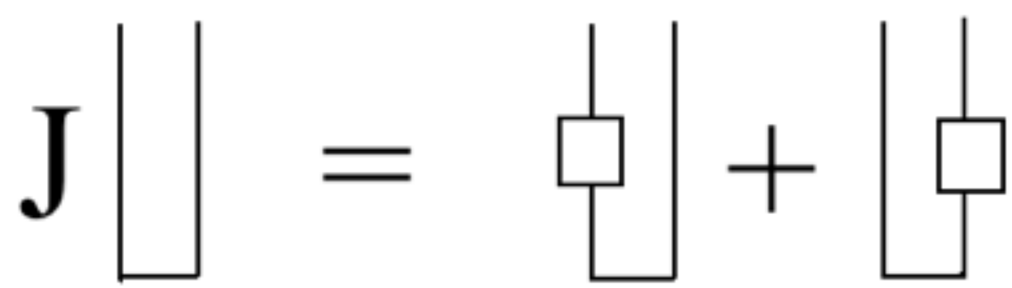
Introduce graphical notation:



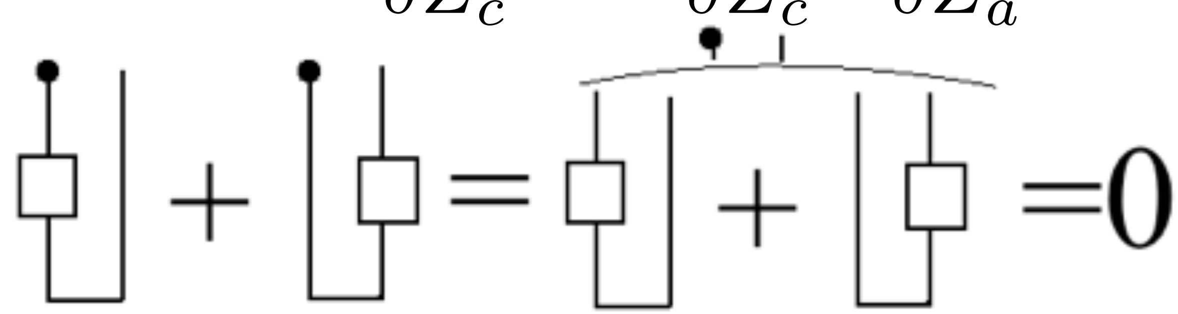
*Art for art's sake

Example:
Propagator Invariance

Level-0 generator J acting on a propagator:



Level-0 magic formula $J\frac{\delta S}{\delta Z_c}+\frac{\delta(JZ_a)}{\delta Z_c}\frac{\delta S}{\delta Z_a}=0$ at 0-th order:



Invariance under level-1

- Draw level-1 magic formula
- Draw action of Yangian generator on the correlation function
- Local action cancels easily - tricky part is the bilocal part of the coproduct
- Level-0 is a symmetry, we can move it around
- Dual coxeter number of psu(2,2|4) vanishes:

$$f^a{}_{bc} f^{bc}{}_d = h \delta^a_b = 0$$

Final recap

- Yangian of psu(2,2|4) - correct symmetry algebra of planar N=4 SYM.
- Invariance of equations of motion under level-1.
- Cannot act on the action, but exists a way to circumvent it.
- Gauge-fixing (BRST) - compatible.
- Implies invariance of the correlation functions.

Final recap

- Yangian of psu(2,2|4) - correct symmetry algebra of planar N=4 SYM.
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- Cannot act on the action, but exists a way to circumvent it.
- Gauge-fixing (BRST) - compatible.
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Holds for ABJM too!

To investigate

- Anomalies
- Algebraic relations for Yangian generators
- Proper Ward identities

Thanks for your attention!