Holographic Wilson loops

in $\mathcal{N} = 2^*$ SYM

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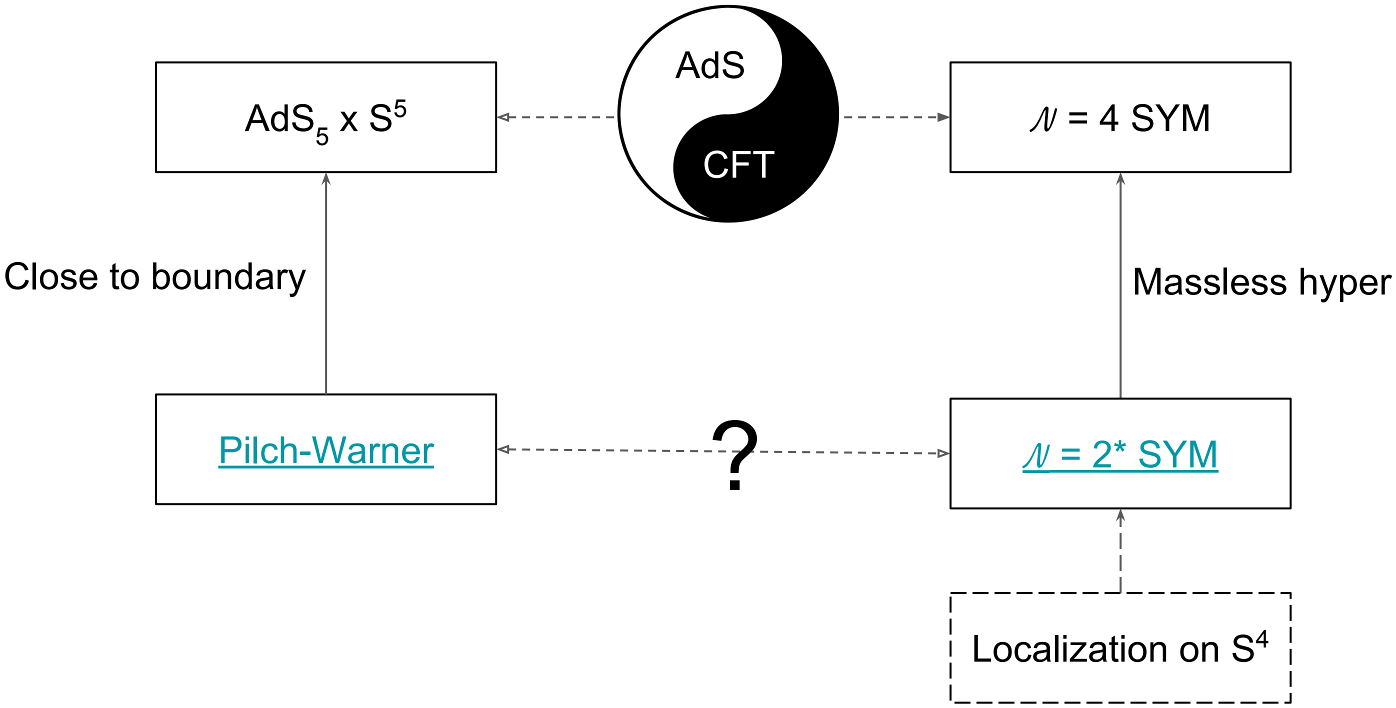
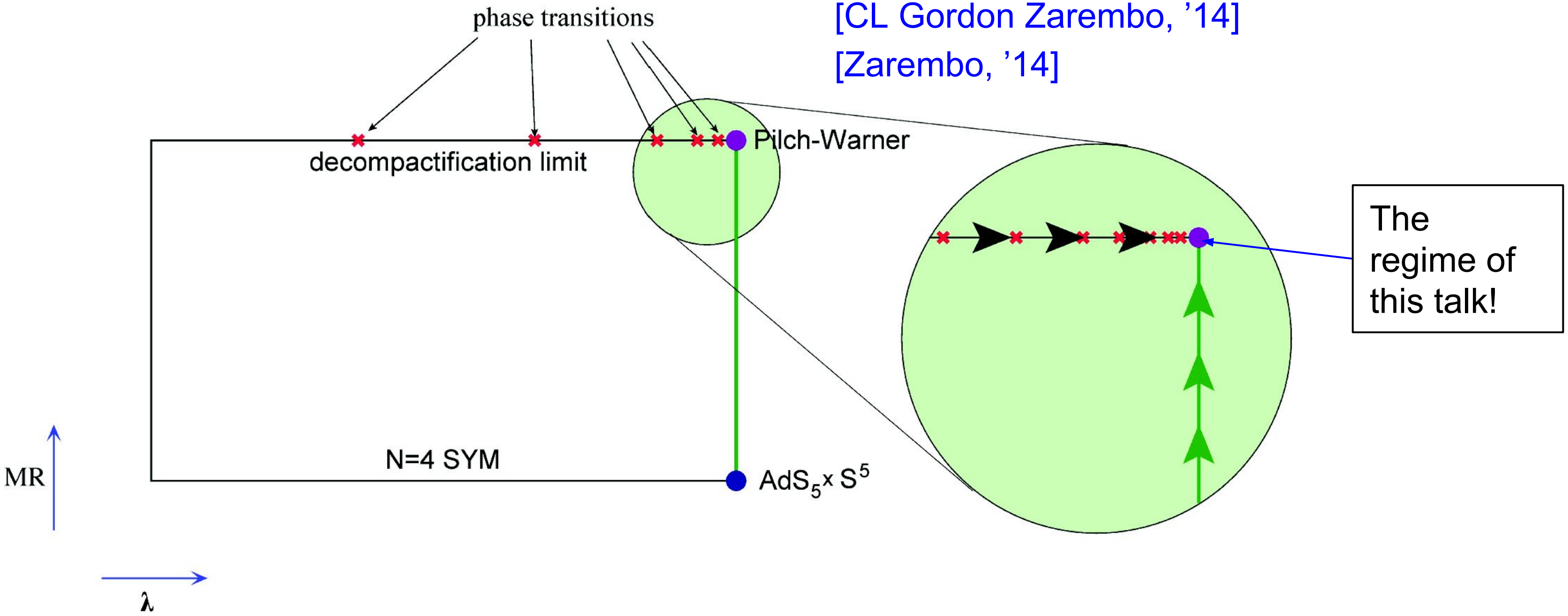
Motivation: (non-conformal) holography

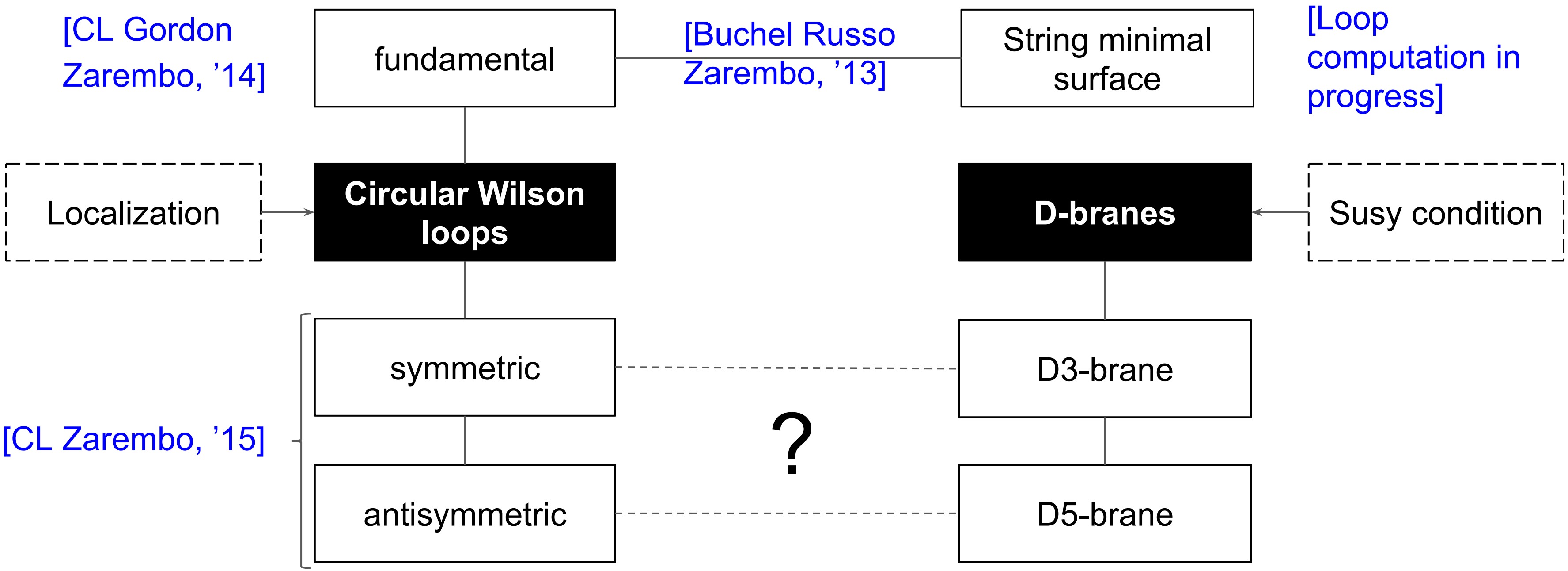


Motivation: phase transitions

[Russo Zarembo, '13]
[CL Gordon Zarembo, '14]
[Zarembo, '14]



Observables



Part I: Wilson loops in $\mathcal{N} = 2^*$ SYM

Partition function of $\mathcal{N}$ = 2* SYM on S^4

Localization reduces it to an effective **matrix model** [Pestun, '07]:

$$Z = \int d^{N-1}a \prod_{i<j} \frac{(a_i - a_j)^2 H^2(a_i - a_j)}{H(a_i - a_j - MR)H(a_i - a_j + MR)} \; |\mathcal{Z}_{inst}(a)|^2 \; e^{-\frac{8\pi^2 N}{\lambda} \sum_j^N a_j^2}$$

$$H(x) \equiv \prod_{n=1}^{\infty} \left(1 + \frac{x^2}{n^2}\right)^n e^{-\frac{x^2}{n}}$$

VEV of the scalar of the vector multiplet = diag $(a_1, \ldots, a_n)$

M = mass of the hypermultiplet, R = radius of S^4

Solved it using the **saddle-point method** for large N [CL Gordon Zarembo, '14].

Large N and strong coupling (λ)

Saddle point equation in the continuous approximation:

$$PV \int_{-\mu}^{\mu} dy\, \rho(y) \left(\frac{1}{x-y} + G(x-y, MR) \right) = \frac{8\pi^2}{\lambda}\, x$$

At leading order in strong coupling (valid for the bulk):

$$PV \int_{-\mu}^{\mu} dy\, \rho(y) \frac{1+(MR)^2}{x-y} = \frac{8\pi^2}{\lambda}\, x$$

Because [Buchel Russo Zarembo, '13]:

$$G(x, M) \equiv \frac{1}{2} K(x+M) + \frac{1}{2} K(x-M) - K(x) \approx \frac{M^2}{x}, \quad K(x) \equiv -\frac{H'(x)}{H(x)}$$

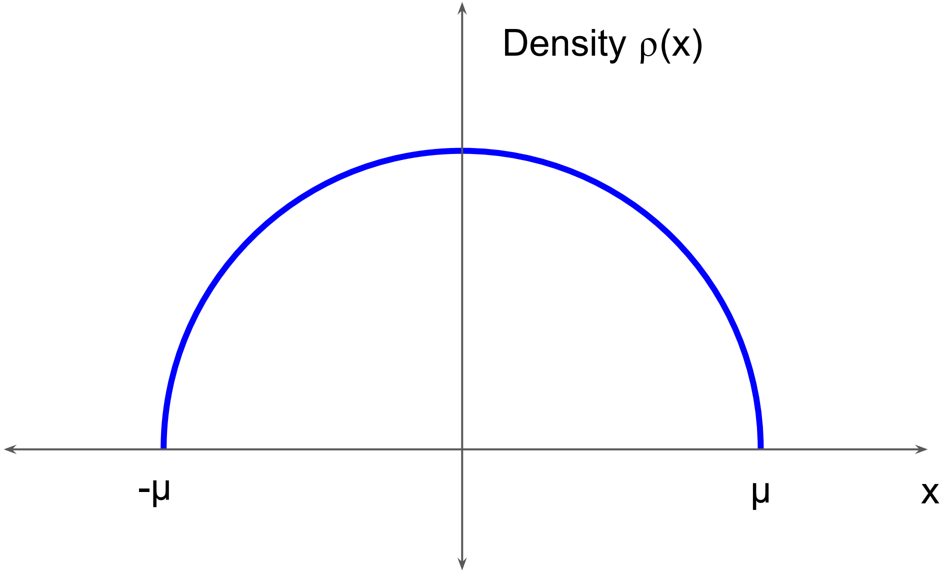
Solution: Wigner semi-circle

$$PV \int_{-\mu}^{\mu} dy\, \rho(y) \frac{1+(MR)^2}{x-y} = \frac{8\pi^2}{\lambda}\, x$$

Solution:

$$\rho(x) = \frac{2}{\pi\mu^2}\sqrt{\mu^2 - x^2}$$

$$\mu = \frac{\sqrt{(1+(MR)^2)\lambda}}{2\pi}$$



At the decompactification limit *M R* >> 1:

$\mu = MR\,\frac{\sqrt{\lambda}}{2\pi}$, i.e. rescaled $\mathcal{N}$ = 4 SYM matrix model solution!

Wilson loops insertions

Definition:

$$W_{\mathcal{R}} =< \mathrm{tr}_{\mathcal{R}} U >, \quad U = P \exp \left[\oint_C ds \, (i \, \dot{x}^\mu A_\mu + |\dot{x}| \Phi) \right] \in SU(N)$$

k-symmetric (+) and k-antisymmetric (-) representations, rescaled by L = 2 π R:

$$W_k^{\pm} = L \left\langle \int_{C-i\pi}^{C+i\pi} \frac{d\nu}{2\pi i} \; \mathrm{e}^{\,kL\nu} \prod_j \left[1 \mp \mathrm{e}^{\,L(a_j - \nu)} \right]^{\mp 1} \right\rangle$$

- Subleading in N, so at large N, the density ρ(x) applies here.
- Large k with k/N fixed, in order to use the saddle-point method again.

Solution at strong coupling

Saddle point equation:

$$\frac{k}{N} = \int_{-\mu}^{\mu} \frac{dx\,\rho(x)}{\mathrm{e}^{L(\nu_* - x)} \mp 1}$$

The solutions are the same as in $\mathcal{N} = 4$ SYM but rescaled by MR:

$$\ln W_k^- = NMR\frac{\sqrt{2\lambda}}{3\pi}\sin^3\theta, \quad \theta - \frac{1}{2}\sin 2\theta = \pi\frac{k}{N}, \quad \cos\theta = \frac{\nu_*}{\mu}$$

$$\ln W_k^+ = 2N(MR)^2 f\left(\frac{\kappa}{MR}\right), \quad f(x) = x\sqrt{1+x^2} + \operatorname{arcsinh} x, \quad \kappa \equiv \frac{\sqrt{\lambda}k}{4N}$$

[CL Zarembo, ’15] [Yamaguchi, ’06] [Hartnoll Kumar, ’06] [Drukker Fiol, ’05]

Part II: D-branes in Pilch-Warner

Pilch-Warner background

$$\mathrm{ds}^2 = V_x\,\mathrm{dx}^\mu \mathrm{dx}_\mu - V_r\,\mathrm{dr}^2 - \left(V_\theta\,\mathrm{d}\theta^2 + V_1\,\sigma 1^2 + V_2\,\{\sigma 2^2 + \sigma 3^2\} + V_\phi\,\mathrm{d}\phi^2\right)$$

All V depend on r and θ, see here.

All fluxes are turned on!

Limit to AdS_5 x S^5: when c(r) → 1

$$\mathrm{ds}^2 = e^{2r}\mathrm{dx}^\mu \mathrm{dx}_\mu - \mathrm{dr}^2 - \left(\mathrm{d}\theta^2 + \cos(\theta)\{\sigma 1^2 + \sigma 2^2 + \sigma 3^2\} + \sin(\theta)\mathrm{d}\phi^2\right)$$

[Pilch Warner, '00]

Explicit functions

$$V_x = \frac{M^2 c^{1/8} A^{1/4} X_1^{1/8} X_2^{1/8}}{(c^2-1)^{1/2}}, \quad V_r = \frac{c^{1/8} X_1^{1/8} X_2^{1/8}}{A^{1/12}}, \quad V_\theta = \frac{X_1^{1/8} X_2^{1/8}}{c^{3/8} A^{1/4}},$$

$$V_1 = \frac{A^{1/4} X_1^{1/8}}{c^{3/8} X_2^{3/8}}, \quad V_2 = \frac{c^{1/8} A^{1/4} X_2^{1/8}}{X_1^{3/8}}, \quad V_\phi = \frac{c^{1/8} X_1^{1/8}}{A^{1/4} X_2^{3/8}}$$

$$X_1 = \cos^2\theta + cA\sin^2\theta, \quad X_2 = c\cos^2\theta + A\sin^2\theta$$

$$A = c + (c^2-1)\frac{1}{2}\ln\left(\frac{c-1}{c+1}\right)$$

Go to D3-brane

We will work in c coordinate!
It relates with the Poincaré coordinate z for c close to 1:
$c = 1 + z^2 M^2 / 2$

$$\frac{dc}{dr} = A^{2/3}(1-c^2)$$

D-brane action (in string frame)

Dirac-Born-Infeld + Wess-Zumino + String charge (k) term

$$
\begin{aligned}
S \;=\; & T_{D_p} \int_{\mathcal{M}} d\sigma^{p+1} e^{-\Phi} \sqrt{\det_{ij}\left(g_{ij} + B_{ij} + \frac{1}{T_{F_1}} F_{ij}\right)} \\
& - T_{D_p} \int_{\mathcal{M}} e^{F} \wedge C \\
& - ik \int_{\Sigma} d\sigma^2 \, \frac{1}{2} \epsilon^{ab} F_{ab}
\end{aligned}
$$

Solving the equations of motion is in general complicated!

SUSY condition

A supersymmetric D-brane configuration must satisfy:

$$\boxed{\Gamma\epsilon = \epsilon}$$

ϵ is the Killing spinor [Pilch Warner, ’03]

Γ is the kappa-symmetry projector, defined as (we follow [Skenderis Taylor, ’02]):

$$d^{p+1}\xi\,\Gamma = -e^{-\Phi}L_{\rm DBI}^{-1}e^{\mathcal{F}}\wedge X|_{\rm Vol}$$

$$L_{\rm DBI} = e^{-\Phi}\sqrt{-\det(g+\mathcal{F})} \quad ; \quad X = \bigoplus_n \gamma_{(2n)}K^n I$$

$$\gamma_{(n)} = \frac{1}{n!}\partial_{i_1}X^{\mu_1}\ldots\partial_{i_n}X^{\mu_n}\gamma_{\mu_1\ldots\mu_n}d\xi^{i_n}\wedge\ldots\wedge d\xi^{i_1}, \quad K\epsilon = \epsilon^*, \quad I\epsilon = -i\epsilon$$

D3-brane configuration

Embedding (induced metric in string frame, with $\theta = \pi / 2$ and $\phi = 0$):

$$ds^2 = \frac{A(c)M^2}{c^2-1}\left(dx^2 - \rho(c)^2 d\Omega_2^2\right) - \left(\frac{1}{A(c)\left(c^2-1\right)^2} + \frac{A(c)M^2\rho'(c)^2}{c^2-1}\right)dc^2$$

Gauge field is nontrivial in (x, c) component. *A*(c) defined here.

Note that ρ here represents the radial coordinate of $\mathrm{dx}^\mu \mathrm{dx}_\mu$.

The goal is to find ρ(c) and the gauge field *F*, then, compute the D3-brane action in Euclidean signature, which is related to the dual Wilson loop at strong coupling:

$$\boxed{\ln W_k^+ = -S_{D3}}$$

D3-brane solution

Constraints from SUSY condition [CL Dekel Zarembo, '15]:

$$\rho'(c) = \frac{c\rho(c)^2}{\kappa}(c^2-1)^{-3/2}, \quad \kappa = \frac{\sqrt{\lambda}\,k}{4\,N} \quad \Rightarrow \quad \rho(c) = \kappa\,\sqrt{c^2-1}$$

$$F_{xc}(c) = -\frac{\sqrt{\lambda}M}{2\pi}(c^2-1)^{-3/2}$$

$$\frac{1}{2}\left(1-\Gamma_{15}K\right)\epsilon = 0$$

Reduces to the AdS solution when:
$c = 1 + z^2/2$

Action at the solution: DBI + WZ = 0, we are left only with

$$S = MR\sqrt{\lambda}\,k\int_{1+\epsilon^2/2}^{\infty} dc\,(c^2-1)^{-3/2} = MR\sqrt{\lambda}\,k\left(\frac{1}{\epsilon}-1\right)$$

D3-brane vs WL in symmetric representation

Drop the perimeter divergence, the renormalized D3-brane action is:

$$\boxed{S_{D3} = -MR\sqrt{\lambda}\,k}$$

Wilson loop in k-symmetric representation

$$\ln W_k^+ = 2N(MR)^2 f\left(\frac{\kappa}{MR}\right), \quad f(x) = x\sqrt{1+x^2} + \operatorname{arcsinh} x, \quad \kappa \equiv \frac{\sqrt{\lambda}\,k}{4N}$$

κ / MR <<1 : $\boxed{\ln W_k^+ = MR\sqrt{\lambda}\,k}$ i.e. k-wrapped fundamental rep.!

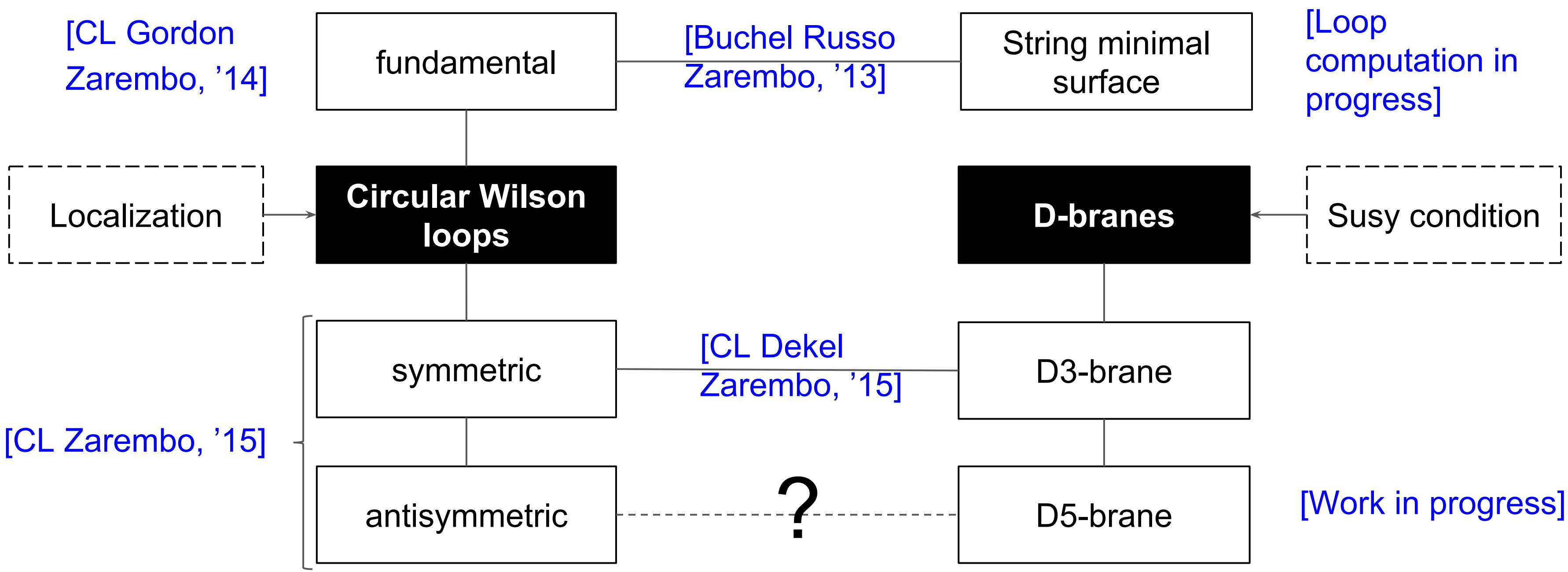
Conclusion

- The results match:

$$\ln W_k^+ = -S_{D3}$$

- However, the field theory calculation applies to a more general scaling limit, that we do not know how to analyze holographically!

Observables



Outlook

- Solve for D5-brane
- D5-brane fluctuations to probe the large N phase transitions
- Develop a systematic algorithm to solve supersymmetric D-branes using the susy condition

Thank you for your attention!