On-Shell Diagrams, Recursion Relations, & Combinatorics

Jacob L. Bourjaily

Cracow School of Theoretical Physics LVI Course, 2016 A Panorama of Holography



The Niels Bohr International Academy

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Organization and Outline

- On-Shell Diagrams: Amalgamations of Scattering Amplitudes
 - Beyond (Mere) Scattering Amplitudes: On-Shell Functions
 - Systematics of Computation and the Auxiliary Grassmannian
 - Building-Up Diagrams with 'BCFW' Bridges
- On-Shell, All-Order Recursion Relations for Scattering Amplitudes
 - Deriving Diagrammatic Recursion Relations for Amplitudes
 - Exempli Gratia: On-Shell Representations of Tree Amplitudes
- 3 Combinatorics, Classification, and Canonical Computation
 - A Combinatorial Classification of On-Shell Functions
 - Building-Up (Representative) Diagrams and Functions with Bridges
 - Asymptotic Symmetries of the S-Matrix: the *Yangian*
- 4 Paths Forward: Beyond the Leading Order of Perturbation Theory
 - On-Shell Representations of Loop-Amplitude Integrands































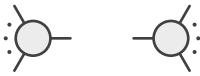


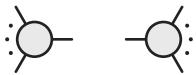


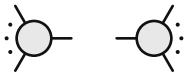


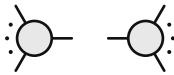


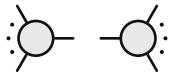


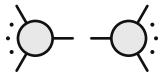


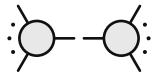


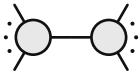


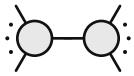


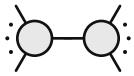


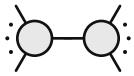


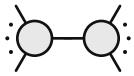


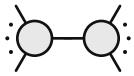


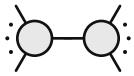




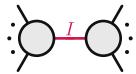






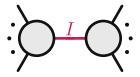


We are interested in the class of functions involving **only** observable quantities



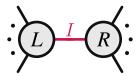
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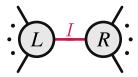
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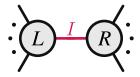
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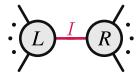
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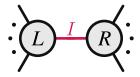
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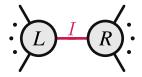
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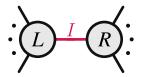
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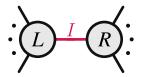
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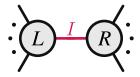
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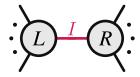
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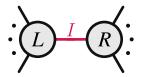
$$\sum_{\text{states } I} \int \frac{d^2 \lambda_I d^2 \widetilde{\lambda}_I}{\text{vol}(GL_1)} \, \mathcal{A}_L(\dots, I) \times \mathcal{A}_R(I, \dots)$$

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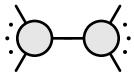


$$\int d^4 \widetilde{\eta}_{I} \int \frac{d^2 \lambda_{I} d^2 \widetilde{\lambda}_{I}}{\operatorname{vol}(GL_1)} \mathcal{A}_{L}(\ldots, I) \times \mathcal{A}_{R}(I, \ldots)$$

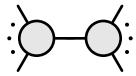
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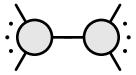


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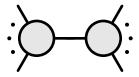
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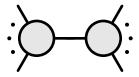
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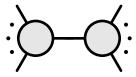
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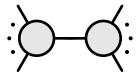
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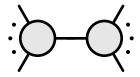
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Counting Constraints:

 n_{δ}

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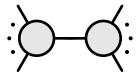


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$$n_{\delta} \equiv 4 \times n_{V}$$

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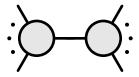


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$$n_{\delta} \equiv 4 \times n_V - 3 \times n_I$$

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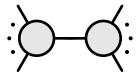


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$$\hat{n}_{\delta} \equiv 4 \times n_V - 3 \times n_I - 4$$

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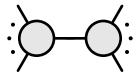


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$$\hat{n}_{\delta} \equiv 4 \times n_V - 3 \times n_I - 4 = \text{number of excess } \delta \text{-functions}$$

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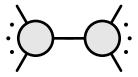
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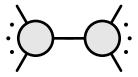
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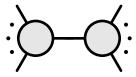
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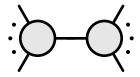


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$$\widehat{n}_{\delta} \equiv 4 \times n_V - 3 \times n_I - 4 = 0$$

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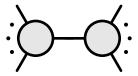
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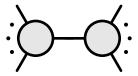
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Counting Constraints:

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 kinematical constraints ordinary (rational) function $< 0 \Rightarrow (-\widehat{n}_{\delta})$ non-trivial integrations

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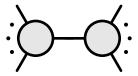
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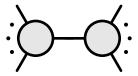
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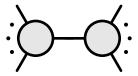
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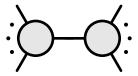
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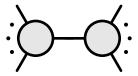
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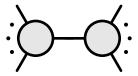
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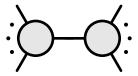
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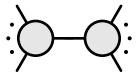
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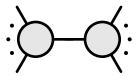
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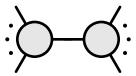




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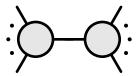




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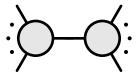




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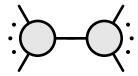






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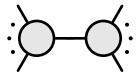




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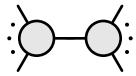




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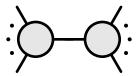




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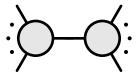




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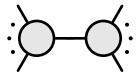




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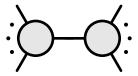




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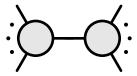




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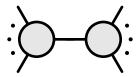


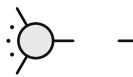


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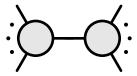


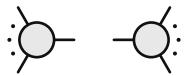


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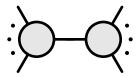


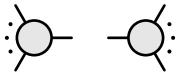
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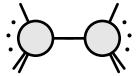


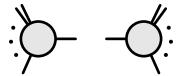
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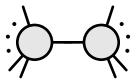
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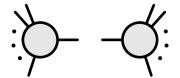




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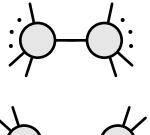
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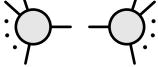




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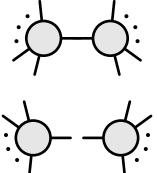


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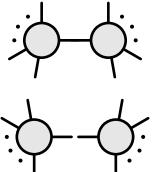
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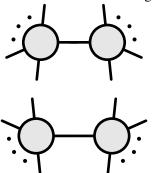
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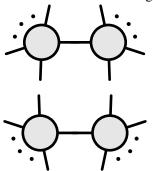
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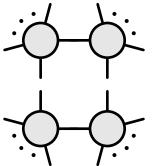
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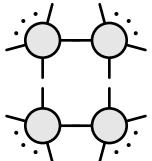
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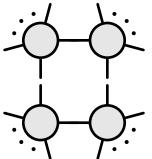
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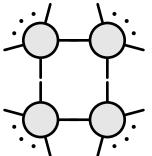
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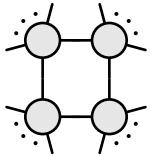
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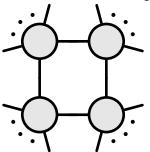
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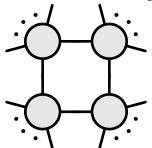
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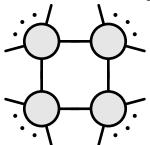
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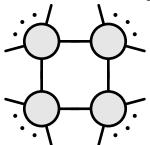
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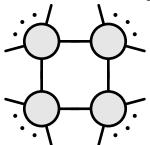
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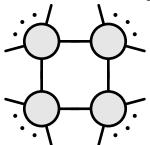
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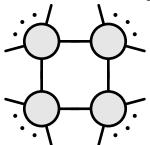
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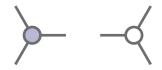
Amalgamating Diagrams from Three-Particle Amplitudes

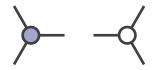
Amalgamating Diagrams from Three-Particle Amplitudes

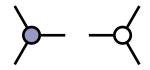


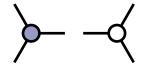


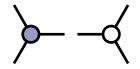


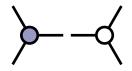


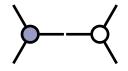


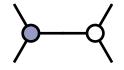


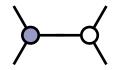




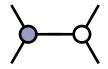


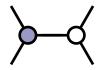






Amalgamating Diagrams from Three-Particle Amplitudes





Amalgamating Diagrams from Three-Particle Amplitudes



Amalgamating Diagrams from Three-Particle Amplitudes

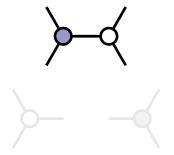


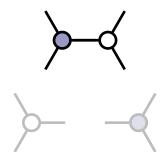


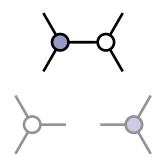


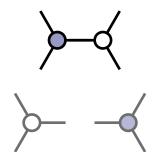


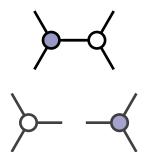


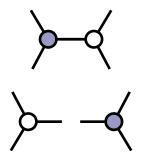


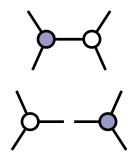


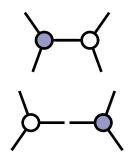


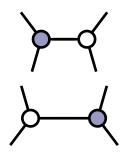


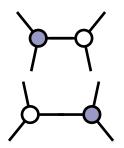


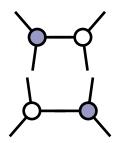


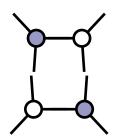


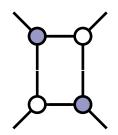


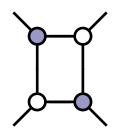


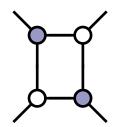


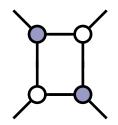


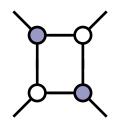


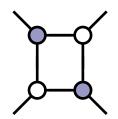


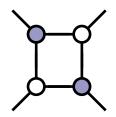


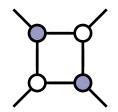


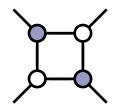


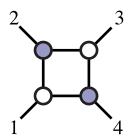


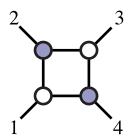


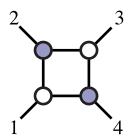


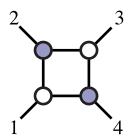


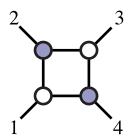


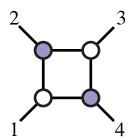


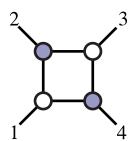


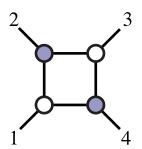


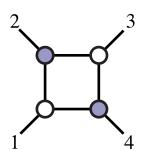


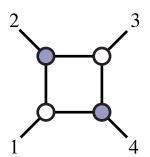


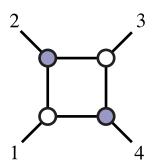


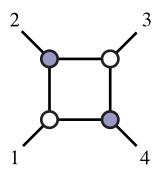


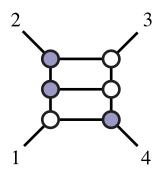


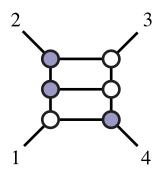


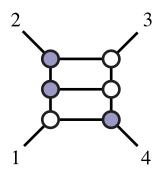


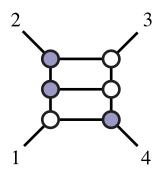


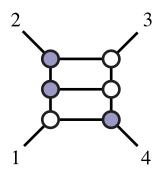


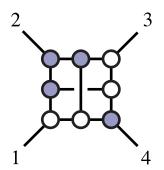


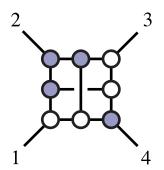


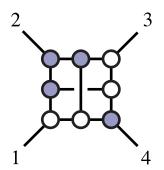


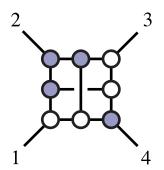


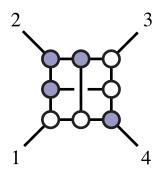


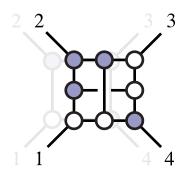


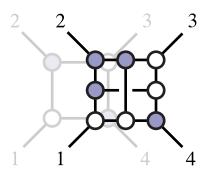


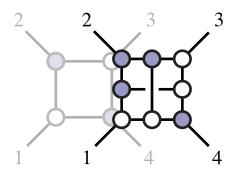


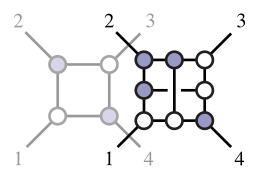


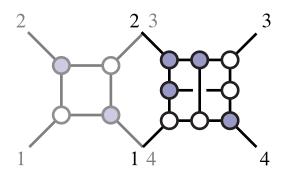


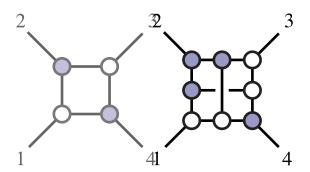


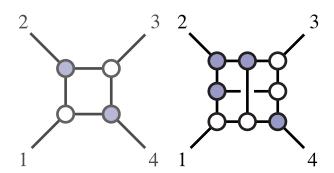


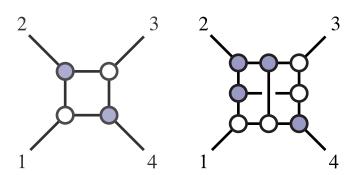


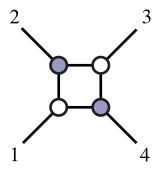


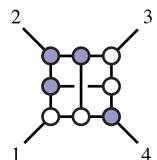


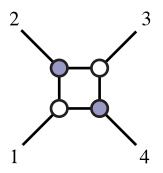


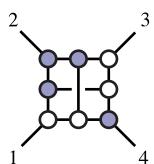


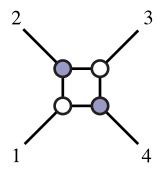


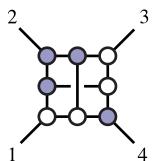


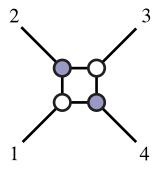


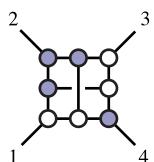


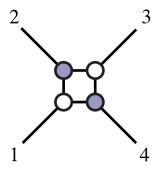


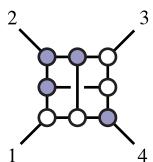


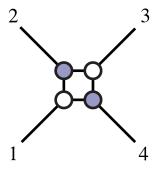


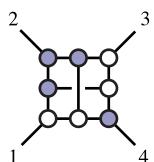


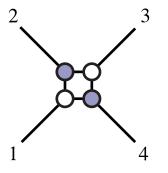


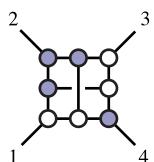


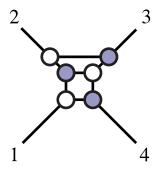


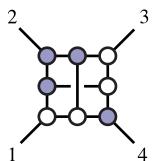


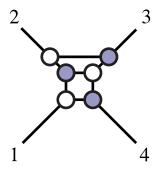


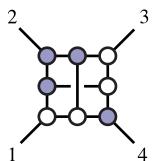


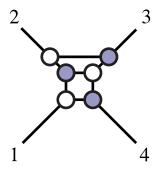


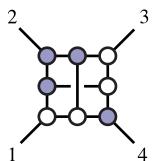


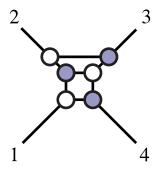


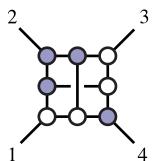


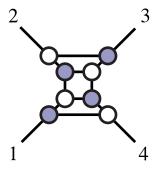


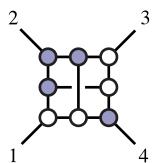


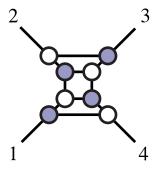


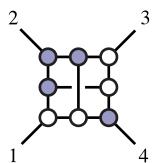


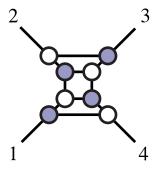


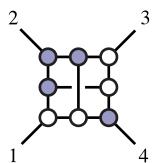


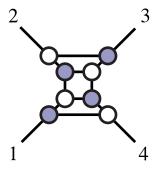


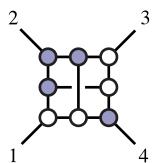


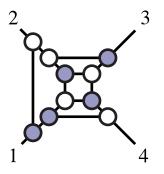


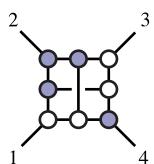


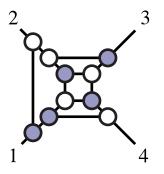


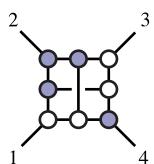


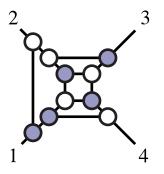


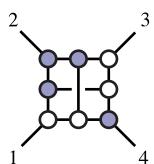


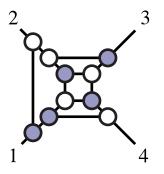


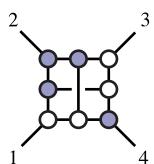


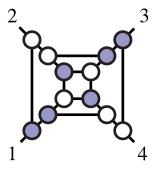


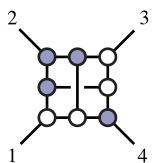


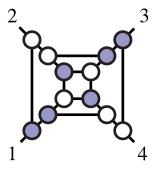


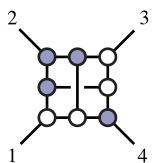


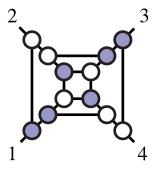


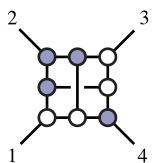


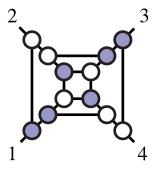


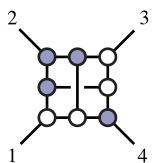


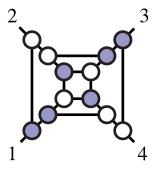


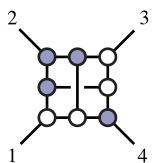


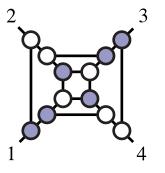


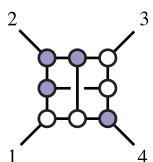


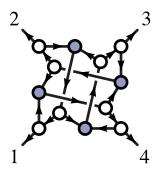


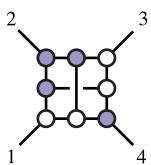


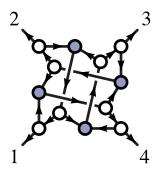


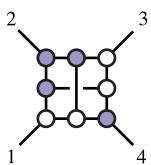


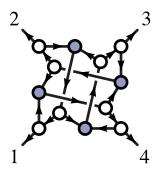


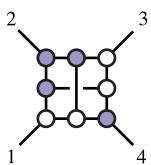


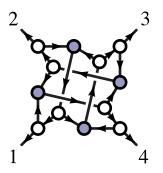


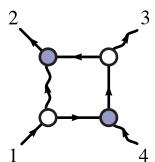


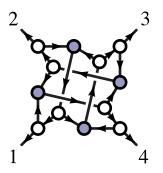


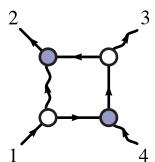


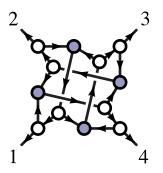


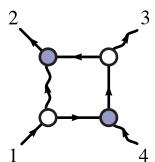


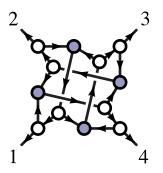


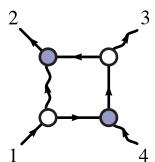


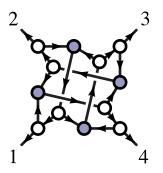


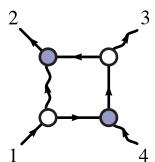












Beyond (Mere) Scattering Amplitudes: On-Shell Functions Systematics of Computation and the Auxiliary Grassmannian Building-Up Diagrams with 'BCFW' Bridges

Amalgamating Diagrams from Three-Particle Amplitudes

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Amalgamating Diagrams from Three-Particle Amplitudes

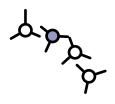


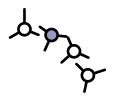


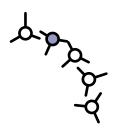


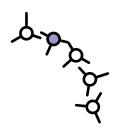


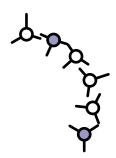


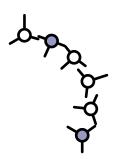


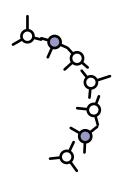


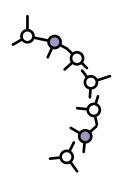


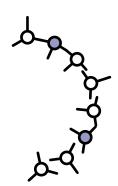


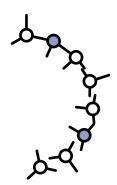


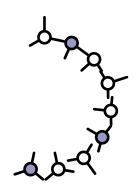


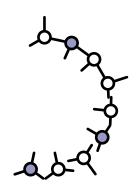


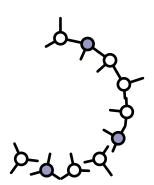


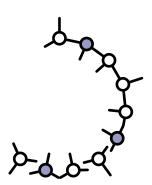


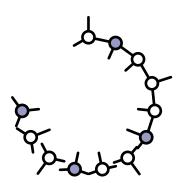


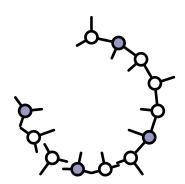


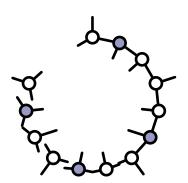


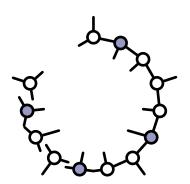


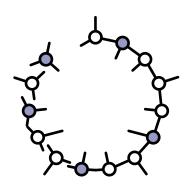


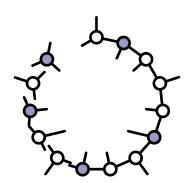


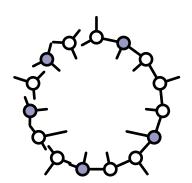


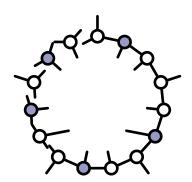


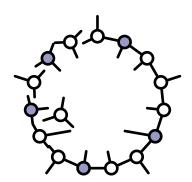


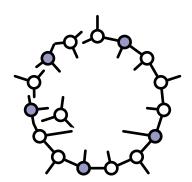


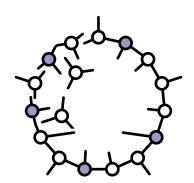


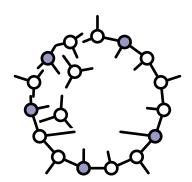


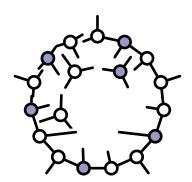


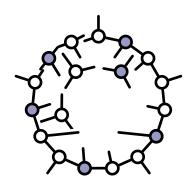


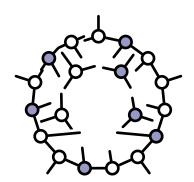


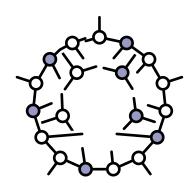


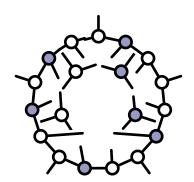


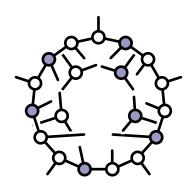


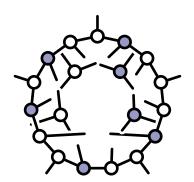


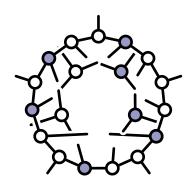


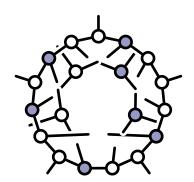


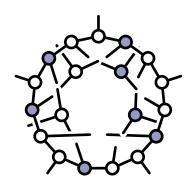


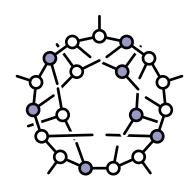


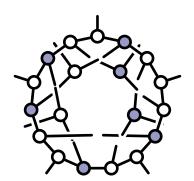


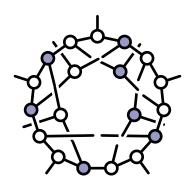


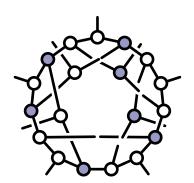


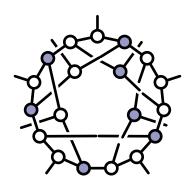


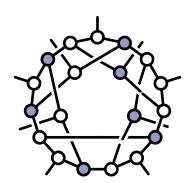


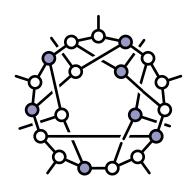


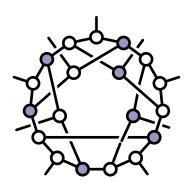


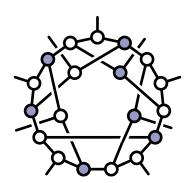


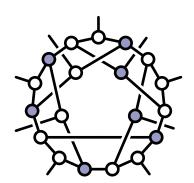


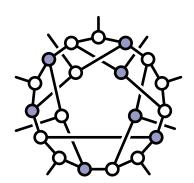


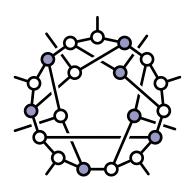


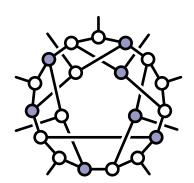


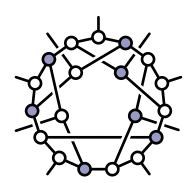


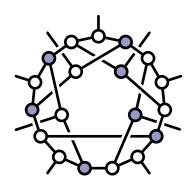


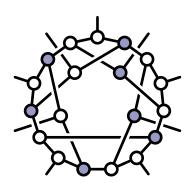


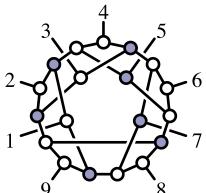


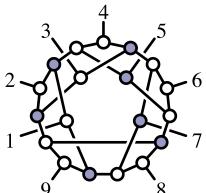


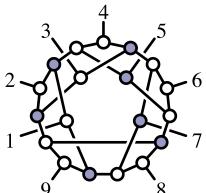


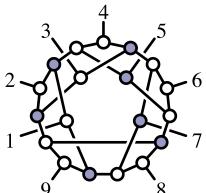


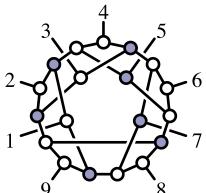


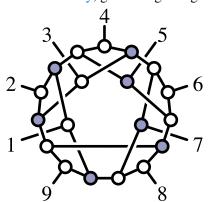




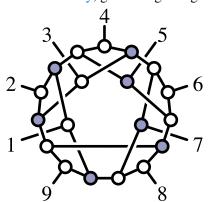




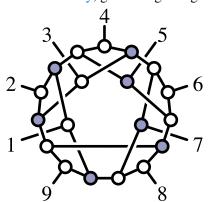




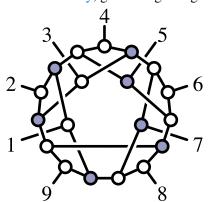
$$=\frac{(\langle 91\rangle\langle 23\rangle\langle 46\rangle-\langle 16\rangle\langle 34\rangle\langle 29\rangle)^2-\delta^{2\times4}\big(\lambda\cdot\widetilde{\eta}\big)\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 67\rangle\langle 78\rangle\langle 81\rangle\langle 14\rangle\langle 42\rangle\langle 29\rangle\langle 96\rangle\langle 63\rangle\langle 39\rangle\langle 91\rangle}_{\equiv}$$



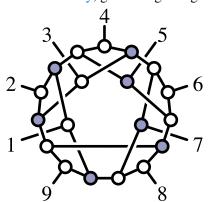
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$$=\frac{(\langle 91\rangle\langle 23\rangle\langle 46\rangle-\langle 16\rangle\langle 34\rangle\langle 29\rangle)^2-\delta^{2\times4}\big(\lambda\cdot\widetilde{\eta}\big)\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 67\rangle\langle 78\rangle\langle 81\rangle\langle 14\rangle\langle 42\rangle\langle 29\rangle\langle 96\rangle\langle 63\rangle\langle 39\rangle\langle 91\rangle}_{\equiv}$$

Beyond (Mere) Scattering Amplitudes: On-Shell Functions Systematics of Computation and the Auxiliary Grassmannian Building-Up Diagrams with 'BCFW' Bridges

Grassmannian Representations of Three-Point Amplitudes

In order to **linearize** momentum conservation at each three-particle vertex

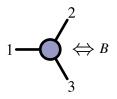
Beyond (Mere) Scattering Amplitudes: On-Shell Functions Systematics of Computation and the Auxiliary Grassmannian Building-Up Diagrams with 'BCFW' Bridges

Grassmannian Representations of Three-Point Amplitudes

In order to **linearize** momentum conservation at each three-particle vertex, (and to specify *which* of the solutions to three-particle kinematics to use)



$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4}(\lambda\cdot\widetilde{\eta})}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} \delta^{2\times2}(\lambda\cdot\widetilde{\lambda})$$



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$$1 \longrightarrow \left(\begin{matrix} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4}(\lambda\cdot\widetilde{\eta})}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} \delta^{2\times2}(\lambda\cdot\widetilde{\lambda})$$

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$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d^{2\times3} B}{\mathrm{vol}(GL_{2})} \frac{\delta^{2\times4} \left(B \cdot \widetilde{\eta}\right)}{\langle 12 \rangle (23) \langle 31 \rangle} \delta^{2\times2} \left(B \cdot \widetilde{\lambda}\right) \delta^{1\times2} \left(\lambda \cdot B^{\perp}\right)$$

$$1 - \bigcirc \left(\Rightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 - \bigcirc \left(\begin{cases} 2 \\ 3 \end{cases} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4}(\lambda\cdot\widetilde{\eta})}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} \delta^{2\times2}(\lambda\cdot\widetilde{\lambda}) \equiv \int \frac{d^{2\times3}B}{\operatorname{vol}(GL_{2})} \frac{\delta^{2\times4}(B\cdot\widetilde{\eta})}{(12)(23)(31)} \delta^{2\times2}(B\cdot\widetilde{\lambda}) \delta^{1\times2}(\lambda\cdot B^{\perp})$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4} \left(\widetilde{\lambda}^{\perp} \cdot \widetilde{\eta}\right)}{[12][23][31]} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d^{2\times3} B}{\mathrm{vol}(GL_{2})} \frac{\delta^{2\times4} \big(B \cdot \widetilde{\eta}\big)}{(12)(23)(31)} \delta^{2\times2} \big(B \cdot \widetilde{\lambda}\big) \delta^{1\times2} \big(\lambda \cdot B^{\perp}\big)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4} \left(\widetilde{\lambda}^{\perp} \cdot \widetilde{\eta}\right)}{[12][23][31]} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d^{2\times3} B}{\mathrm{vol}(GL_{2})} \frac{\delta^{2\times4} \big(B \cdot \widetilde{\eta}\big)}{(12)(23)(31)} \delta^{2\times2} \big(B \cdot \widetilde{\lambda}\big) \delta^{1\times2} \big(\lambda \cdot B^{\perp}\big)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4} \left(\widetilde{\lambda}^{\perp} \cdot \widetilde{\eta}\right)}{[12][23][31]} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d^{2\times3}B}{\mathrm{vol}(GL_{2})} \frac{\delta^{2\times4} \big(B \cdot \widetilde{\eta}\big)}{(12)(23)(31)} \, \delta^{2\times2} \big(B \cdot \widetilde{\lambda}\big) \, \, \delta^{1\times2} \big(\lambda \cdot B^{\perp}\big)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{[12][23][31]}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \frac{\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right)}{\left(1\right)\left(2\right)\left(3\right)} \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d^{2\times3}B}{\mathrm{vol}(GL_{2})} \frac{\delta^{2\times4} \big(B \cdot \widetilde{\eta}\big)}{(12)(23)(31)} \, \delta^{2\times2} \big(B \cdot \widetilde{\lambda}\big) \, \, \delta^{1\times2} \big(\lambda \cdot B^{\perp}\big)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{[12][23][31]}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \frac{\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right)}{\left(1\right)\left(2\right)\left(3\right)} \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d^{2\times3}B}{\mathrm{vol}(GL_{2})} \frac{\delta^{2\times4} \big(B \cdot \widetilde{\eta}\big)}{(12)(23)(31)} \, \delta^{2\times2} \big(B \cdot \widetilde{\lambda}\big) \, \, \delta^{1\times2} \big(\lambda \cdot B^{\perp}\big)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{[12][23][31]}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \frac{\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right)}{\left(1\right)\left(2\right)\left(3\right)} \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d^{2\times3}B}{\mathrm{vol}(GL_{2})} \frac{\delta^{2\times4} \big(B \cdot \widetilde{\eta}\big)}{(12)(23)(31)} \, \delta^{2\times2} \big(B \cdot \widetilde{\lambda}\big) \, \, \delta^{1\times2} \big(\lambda \cdot B^{\perp}\big)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{[12][23][31]}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \frac{\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right)}{\left(1\right)\left(2\right)\left(3\right)} \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \ \frac{\delta^{2\times4} \big(\lambda \cdot \widetilde{\eta}\big)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \, \delta^{2\times2} \big(\lambda \cdot \widetilde{\lambda}\big) \ \equiv \int \frac{d^{2\times3} B}{\mathrm{vol}(GL_2)} \, \frac{\delta^{2\times4} \big(B \cdot \widetilde{\eta}\big)}{(12)(23)(31)} \, \delta^{2\times2} \big(B \cdot \widetilde{\lambda}\big) \, \, \delta^{1\times2} \big(\lambda \cdot B^\perp\big)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{[12][23][31]}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \frac{\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right)}{\left(1\right)\left(2\right)\left(3\right)} \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \ \frac{\delta^{2\times4} \big(\lambda \cdot \widetilde{\eta}\big)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \, \delta^{2\times2} \big(\lambda \cdot \widetilde{\lambda}\big) \ \equiv \int \frac{d^{2\times3} B}{\mathrm{vol}(GL_2)} \, \frac{\delta^{2\times4} \big(B \cdot \widetilde{\eta}\big)}{(12)(23)(31)} \, \delta^{2\times2} \big(B \cdot \widetilde{\lambda}\big) \, \, \delta^{1\times2} \big(\lambda \cdot B^\perp\big)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{[12][23][31]}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \frac{\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right)}{\left(1\right)\left(2\right)\left(3\right)} \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{ccc} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} 1 & 0 & b_3^1 \\ 0 & 1 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{ccc} 2 \\ \Leftrightarrow W \equiv \begin{pmatrix} 1 & w_2^1 & w_3^1 \end{pmatrix} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4}\left(\lambda\cdot\widetilde{\eta}\right)}{\langle12\rangle\langle23\rangle\langle31\rangle}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{d\,b_{3}^{1}}{b_{3}^{1}} \wedge \frac{d\,b_{3}^{2}}{b_{3}^{2}}\,\delta^{2\times4}\!\!\left(\boldsymbol{B}\cdot\widetilde{\eta}\right) \,\,\delta^{2\times2}\!\left(\boldsymbol{B}\cdot\widetilde{\lambda}\right)\,\delta^{1\times2}\!\left(\lambda\cdot\boldsymbol{B}^{\perp}\right)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta})}{[12][23][31]}\delta^{2\times2}(\lambda\cdot\widetilde{\lambda}) \equiv \int \frac{dw_{1}^{1}}{w_{2}^{1}} \wedge \frac{dw_{3}^{1}}{w_{3}^{1}}\delta^{1\times4}(W\cdot\widetilde{\eta}) \delta^{1\times2}(W\cdot\widetilde{\lambda})\delta^{2\times2}(\lambda\cdot W^{\perp})$$

$$1 \longrightarrow \begin{pmatrix} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & 1 & 0 \\ b_1^2 & 0 & 1 \end{pmatrix} \qquad 1 \longrightarrow \begin{pmatrix} 2 \\ \Leftrightarrow W \equiv \begin{pmatrix} w_1^1 & 1 & w_3^1 \end{pmatrix}$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d \, b_{1}^{1}}{b_{1}^{1}} \wedge \frac{d \, b_{1}^{2}}{b_{1}^{2}} \, \delta^{2\times4} \! \left(\boldsymbol{B} \cdot \widetilde{\boldsymbol{\eta}}\right) \, \, \delta^{2\times2} \! \left(\boldsymbol{B} \cdot \widetilde{\lambda}\right) \, \delta^{1\times2} \! \left(\lambda \cdot \boldsymbol{B}^{\perp}\right)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4} \left(\widetilde{\lambda}^{\perp} \cdot \widetilde{\eta}\right)}{[12][23][31]} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d w_{3}^{1}}{w_{3}^{1}} \wedge \frac{d w_{1}^{1}}{w_{1}^{1}} \, \delta^{1\times4} \left(W \cdot \widetilde{\eta}\right) \, \, \delta^{1\times2} \left(W \cdot \widetilde{\lambda}\right) \delta^{2\times2} \left(\lambda \cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} 0 & b_2^1 & 1 \\ 1 & b_2^2 & 0 \end{array} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv \left(w_1^1 w_2^1 & 1 \right) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d \, b_{2}^{1}}{b_{2}^{1}} \wedge \frac{d \, b_{2}^{2}}{b_{2}^{2}} \, \delta^{2\times4} \! \left(\boldsymbol{B} \cdot \widetilde{\boldsymbol{\eta}}\right) \, \, \, \delta^{2\times2} \! \left(\boldsymbol{B} \cdot \widetilde{\boldsymbol{\lambda}}\right) \, \, \delta^{1\times2} \! \left(\lambda \cdot \boldsymbol{B}^{\perp}\right)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{[12][23][31]}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{dw_{1}^{1}}{w_{1}^{1}} \wedge \frac{dw_{2}^{1}}{w_{2}^{1}}\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right) \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \ \frac{\delta^{2\times4} \big(\lambda \cdot \widetilde{\eta}\big)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \, \delta^{2\times2} \big(\lambda \cdot \widetilde{\lambda}\big) \ \equiv \int \!\! \frac{d^{2\times3} B}{\mathrm{vol}(GL_2)} \frac{\delta^{2\times4} \! \big(B \cdot \widetilde{\eta}\big)}{(12)(23)(31)} \, \delta^{2\times2} \big(B \cdot \widetilde{\lambda}\big) \, \, \delta^{1\times2} \big(\lambda \cdot B^\perp\big)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{[12][23][31]}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \frac{\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right)}{\left(1\right)\left(2\right)\left(3\right)} \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d^{2\times3}B}{\mathrm{vol}(GL_{2})} \frac{\delta^{2\times4} \big(B \cdot \widetilde{\eta}\big)}{(12)(23)(31)} \, \delta^{2\times2} \big(B \cdot \widetilde{\lambda}\big) \, \, \delta^{1\times2} \big(\lambda \cdot B^{\perp}\big)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{[12][23][31]}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \frac{\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right)}{\left(1\right)\left(2\right)\left(3\right)} \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d^{2\times3}B}{\mathrm{vol}(GL_{2})} \frac{\delta^{2\times4} \big(B \cdot \widetilde{\eta}\big)}{(12)(23)(31)} \, \delta^{2\times2} \big(B \cdot \widetilde{\lambda}\big) \, \, \delta^{1\times2} \big(\lambda \cdot B^{\perp}\big)$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{[12][23][31]}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \frac{\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right)}{\left(1\right)\left(2\right)\left(3\right)} \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4} \left(\lambda \cdot \widetilde{\eta}\right)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} \delta^{2\times2} \left(\lambda \cdot \widetilde{\lambda}\right) \equiv \int \frac{d^{2\times3}B}{\operatorname{vol}(GL_{2})} \frac{\delta^{2\times4} \left(B \cdot \widetilde{\eta}\right)}{(12)(23)(31)} \delta^{2\times2} \left(B \cdot \widetilde{\lambda}\right) \underbrace{\delta^{1\times2} \left(\lambda \cdot B^{\perp}\right)}_{} \underbrace{\delta^{1\times2} \left(\lambda \cdot B^{\perp}\right)}_{$$

$$\mathcal{A}_{3}^{(1)} = \ \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{\left[12\right]\left[23\right]\left[31\right]} \delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \ \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \ \frac{\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right)}{\left(1\right)\left(2\right)\left(3\right)} \ \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4}(\lambda\cdot\widetilde{\eta})}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} \delta^{2\times2}(\lambda\cdot\widetilde{\lambda}) \equiv \int \frac{d^{2\times3}B}{\operatorname{vol}(GL_{2})} \frac{\delta^{2\times4}(B\cdot\widetilde{\eta})}{(12)(23)(31)} \delta^{2\times2}(B\cdot\widetilde{\lambda}) \underbrace{\delta^{1\times2}(\lambda\cdot B^{\perp})}_{B\mapsto B^{*}}$$

$$\mathcal{A}_{3}^{(1)} = \frac{\delta^{1\times4}\left(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\right)}{[12][23][31]}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \frac{\delta^{1\times4}\left(W\cdot\widetilde{\eta}\right)}{\left(1\right)\left(2\right)\left(3\right)} \delta^{1\times2}\left(W\cdot\widetilde{\lambda}\right)\delta^{2\times2}\left(\lambda\cdot W^{\perp}\right)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4}(\lambda\cdot\widetilde{\eta})}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} \delta^{2\times2}(\lambda\cdot\widetilde{\lambda}) \equiv \int \frac{d^{2\times3}B}{\operatorname{vol}(GL_{2})} \frac{\delta^{2\times4}(B\cdot\widetilde{\eta})}{(12)(23)(31)} \delta^{2\times2}(B\cdot\widetilde{\lambda}) \underbrace{\delta^{1\times2}(\lambda\cdot B^{\perp})}_{B\mapsto B^{*}=\lambda}$$

$$\mathcal{A}_{3}^{(1)} = \ \frac{\delta^{1\times4}\big(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\big)}{[12][23][31]} \delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big) \ \equiv \int \frac{d^{1\times3}W}{\operatorname{vol}(GL_{1})} \ \frac{\delta^{1\times4}\big(W\cdot\widetilde{\eta}\big)}{(1)\ (2)\ (3)} \ \delta^{1\times2}\big(W\cdot\widetilde{\lambda}\big)\delta^{2\times2}\big(\lambda\cdot W^{\perp}\big)$$

$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

$$\mathcal{A}_{3}^{(2)} = \frac{\delta^{2\times4}(\lambda\cdot\widetilde{\eta})}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} \delta^{2\times2}(\lambda\cdot\widetilde{\lambda}) \equiv \int \frac{d^{2\times3}B}{\operatorname{vol}(GL_{2})} \frac{\delta^{2\times4}(B\cdot\widetilde{\eta})}{(12)(23)(31)} \delta^{2\times2}(B\cdot\widetilde{\lambda}) \underbrace{\delta^{1\times2}(\lambda\cdot B^{\perp})}_{B\mapsto B^{*}=\lambda}$$

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$$1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix} \right) \qquad 1 \longrightarrow \left(\begin{array}{c} 2 \\ \Leftrightarrow W \equiv (w_1^1 & w_2^1 & w_3^1) \\ 3 \end{array} \right)$$

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Grassmannian Representations of Three-Point Amplitudes

In order to **linearize** momentum conservation at each three-particle vertex, (and to specify *which* of the solutions to three-particle kinematics to use) we introduce **auxiliary** $B \in G(2,3)$ and $W \in G(1,3)$ for each vertex:

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$$C \in G(k,n)$$

$$k \equiv 2n_B + n_W - n_I$$

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$$1$$

$$\frac{1}{(1 + w_{2} + w_{I})}$$

$$4$$

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Indiving us to represent all on-shell functions in the form:
$$f \equiv \int \Omega_C \ \delta^{k \times 4}(C \cdot \widetilde{\eta}) \delta^{k \times 2}(C \cdot \widetilde{\lambda}) \delta^{2 \times (n-k)}(\lambda \cdot C^{\perp})$$

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$$k \equiv 2n_{B} + n_{W} - n_{I}$$

$$1 \qquad \qquad 4$$

$$\frac{1 \quad 2 \quad \mathbf{I}}{(1 \quad w_{2} \quad w_{I})} \qquad \frac{\mathbf{I'} \quad 3 \quad 4}{(1 \quad 0 \quad b^{1})}$$

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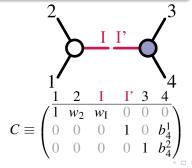
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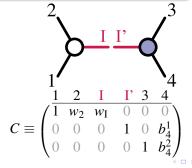
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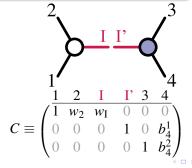
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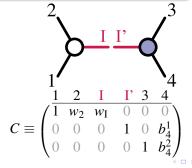
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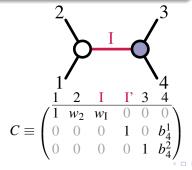
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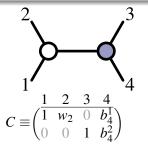
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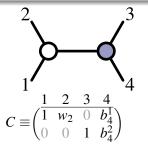
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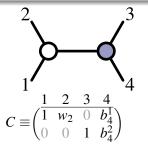
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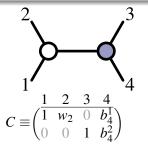
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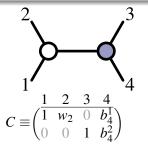
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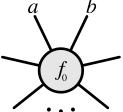
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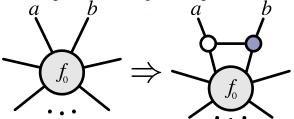


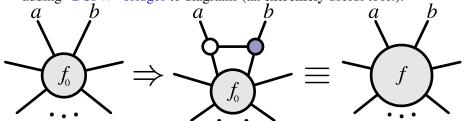
Beyond (Mere) Scattering Amplitudes: On-Shell Functions Systematics of Computation and the Auxiliary Grassmannian Building-Up Diagrams with 'BCFW' Bridges

Building-Up On-Shell Diagrams with "BCFW" Bridges

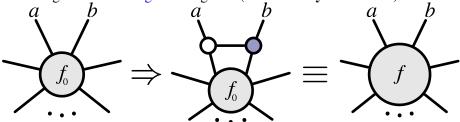
Very complex on-shell diagrams can be constructed by successively adding "BCFW" bridges to diagrams



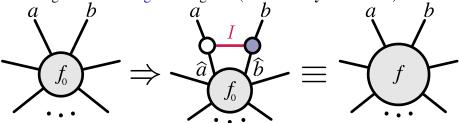




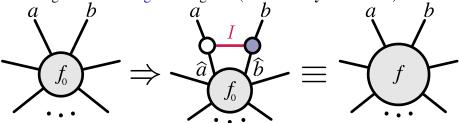
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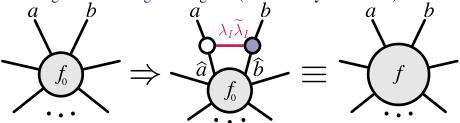


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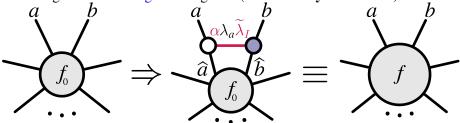
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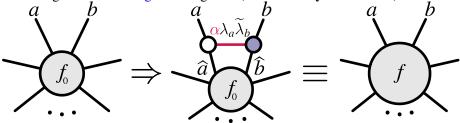
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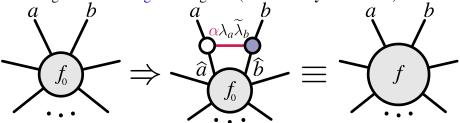
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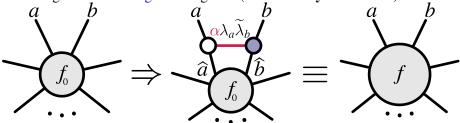
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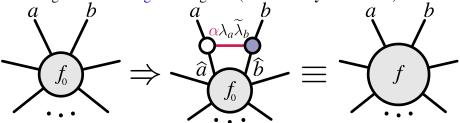
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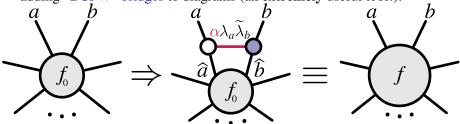
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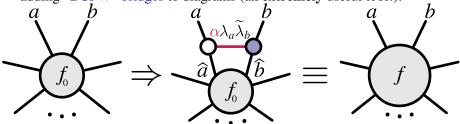
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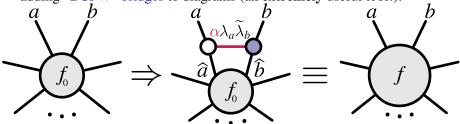
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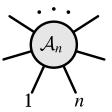


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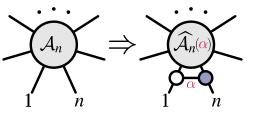
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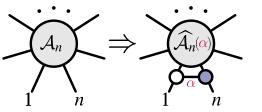
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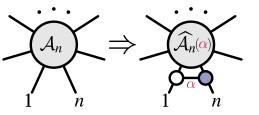


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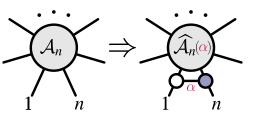
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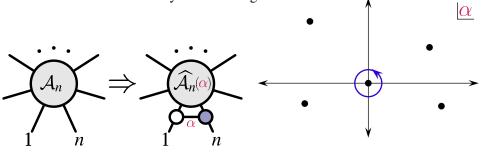
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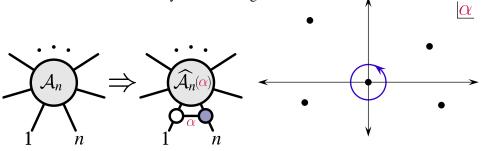
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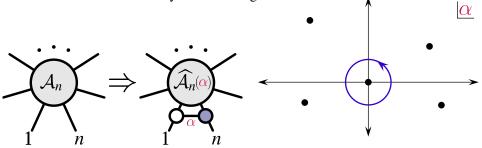
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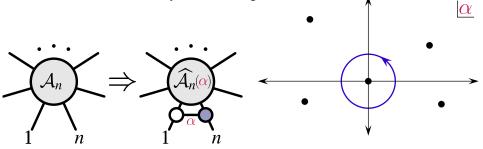
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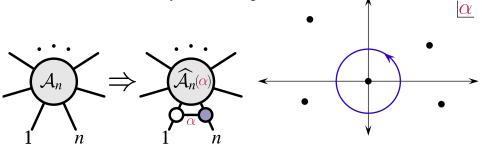
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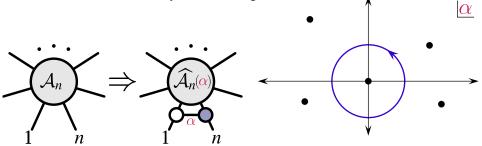
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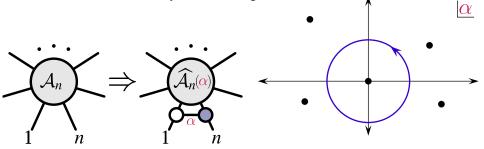
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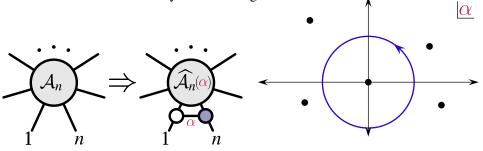
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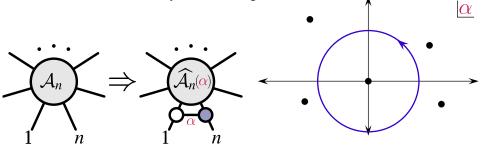
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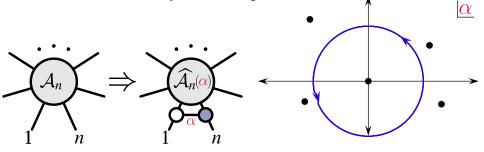
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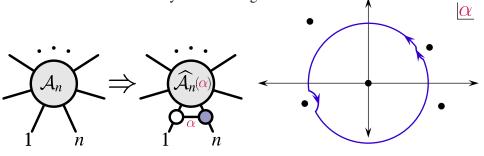
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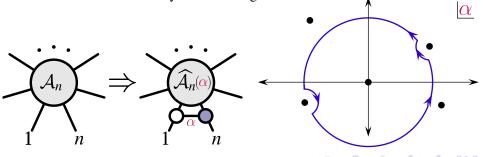
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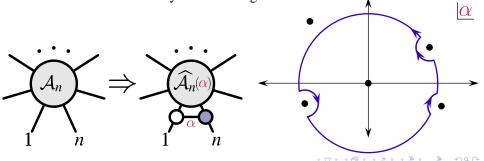
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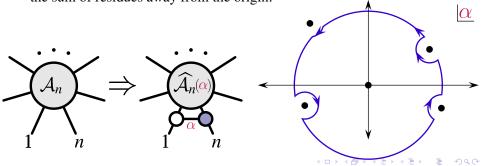
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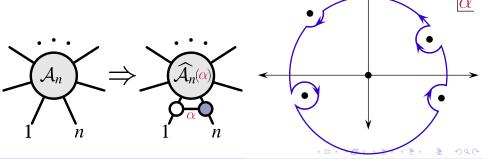
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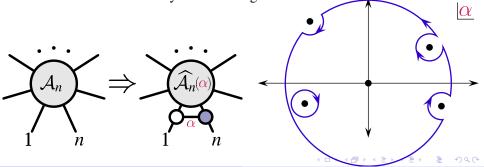
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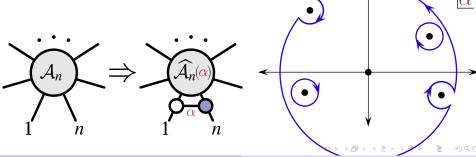
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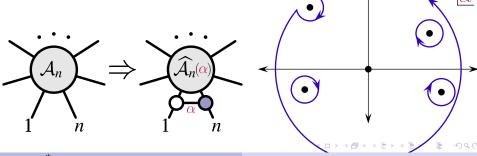


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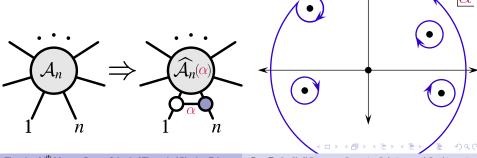


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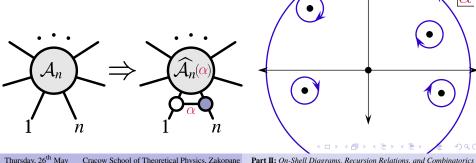


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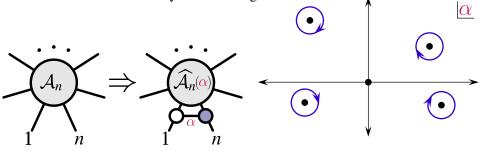
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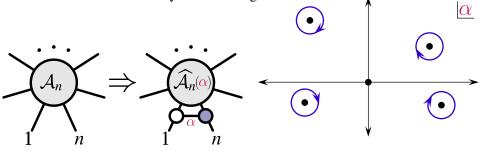
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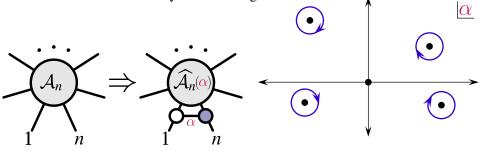
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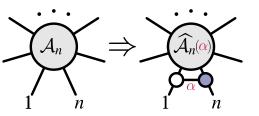
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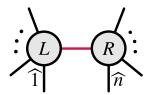
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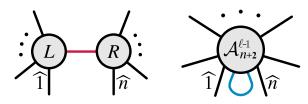
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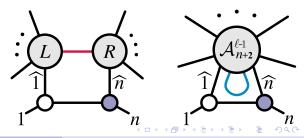
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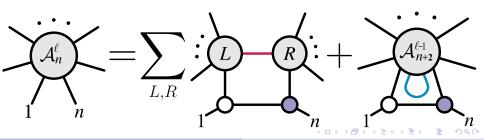
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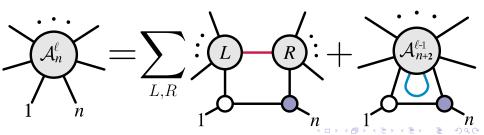


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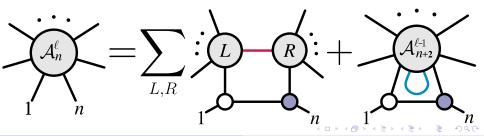


Forward-limits and loop-momenta:

$$=\sum_{L,R} \frac{1}{1} \frac{1}{n} = \sum_{n=1}^{\infty} \frac{1}{n} \frac{A_{n+2}^{\ell-1}}{n}$$

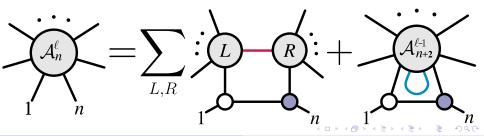
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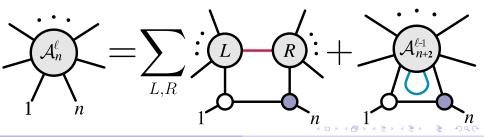
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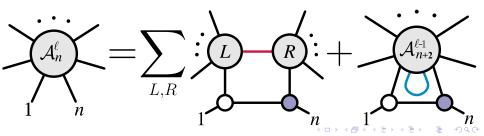
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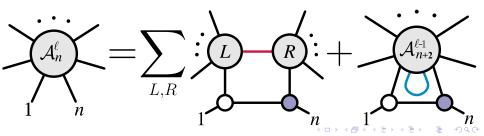
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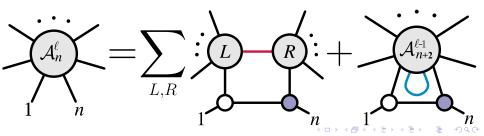
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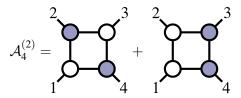
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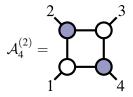
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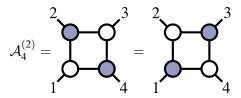


The BCFW recursion relations realize an incredible fantasy: they **directly** produces the **Parke-Taylor** formula for all amplitudes with k=2, $\mathcal{A}_n^{(2)}$!

$$A_4^{(2)} =$$



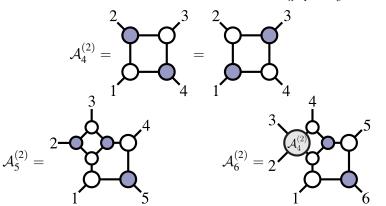


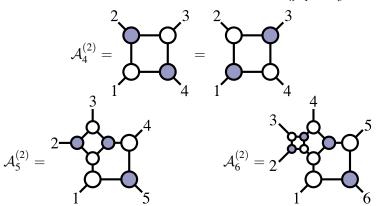


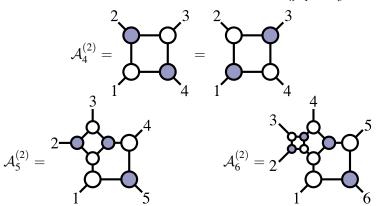
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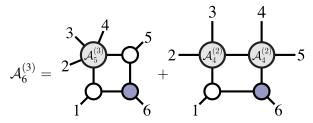


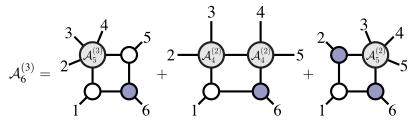


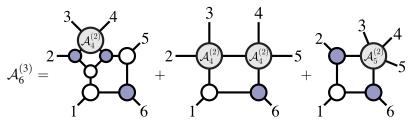


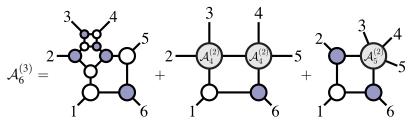
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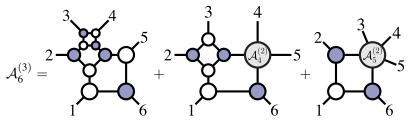
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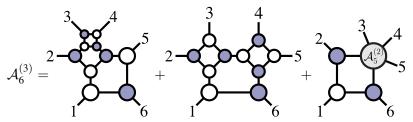


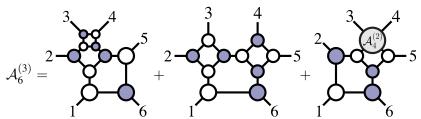


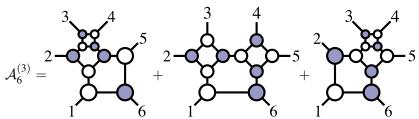




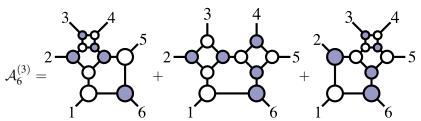




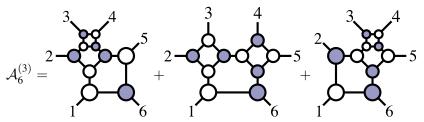




The BCFW recursion relations realize an incredible fantasy: they **directly** produces the **Parke-Taylor** formula for all amplitudes with k=2, $\mathcal{A}_n^{(2)}$! And it generates **very concise** formulae for all other amplitudes—e.g. $\mathcal{A}_6^{(3)}$:



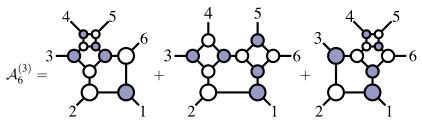
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Observations regarding recursed representations of scattering amplitudes:

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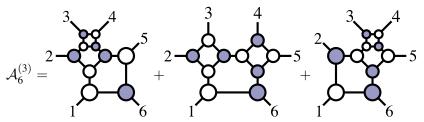
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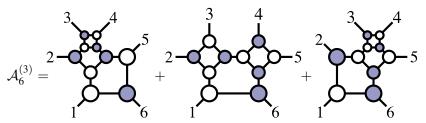
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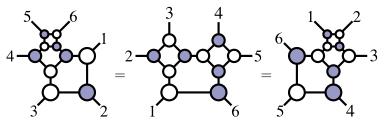
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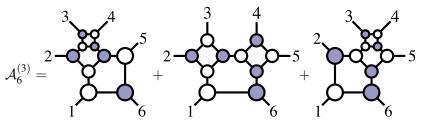
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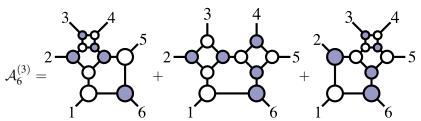
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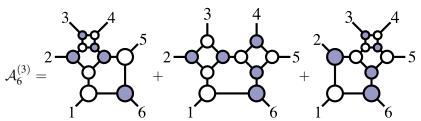


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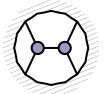
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A Combinatorial Classification of On-Shell Functions Building-Up (Representative) Diagrams and Functions with Bridges Asymptotic Symmetries of the S-Matrix: the *Yangian*

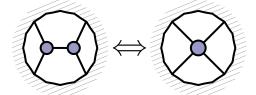
Combinatorial Characterization of On-Shell Diagrams

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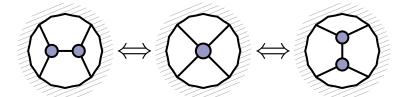
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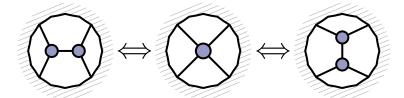
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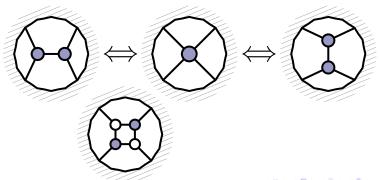
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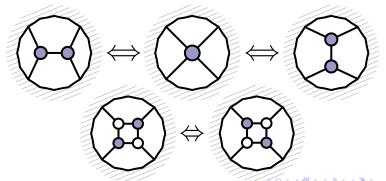
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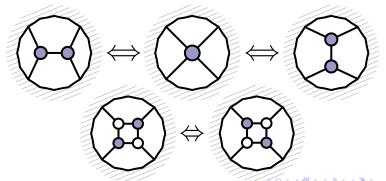
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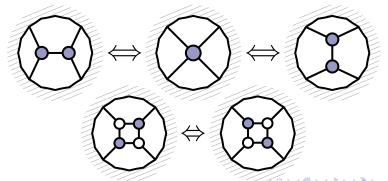
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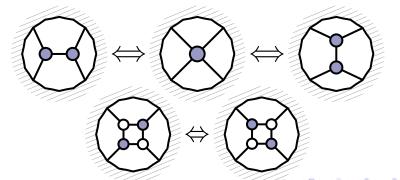


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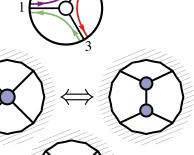
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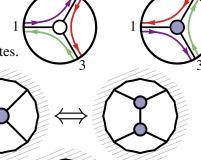


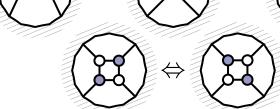


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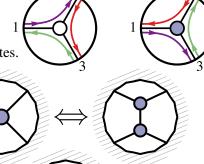


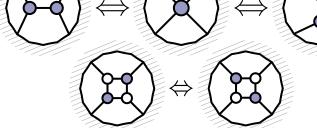


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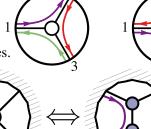


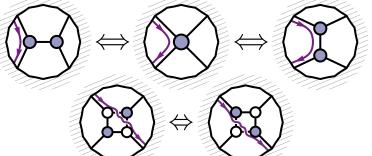


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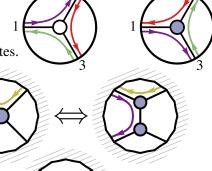


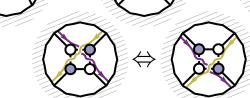


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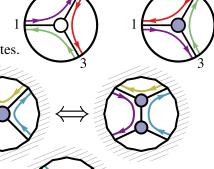




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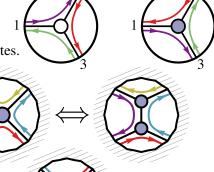


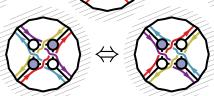


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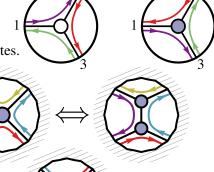


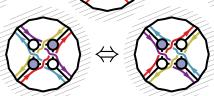


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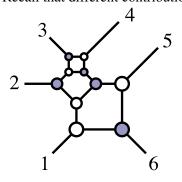
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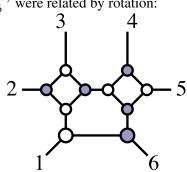
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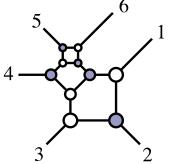
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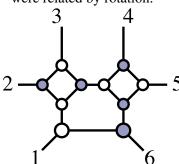




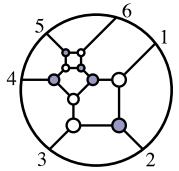
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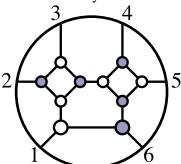
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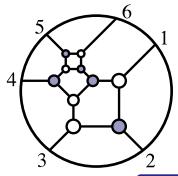


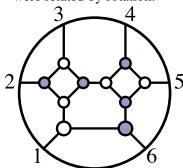
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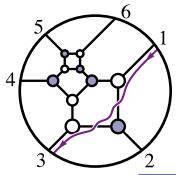
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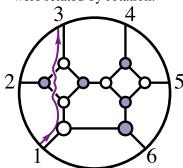




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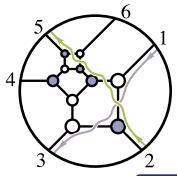
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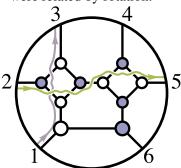




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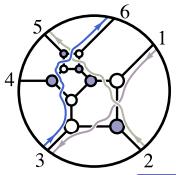
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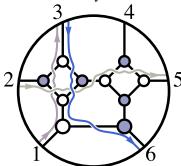




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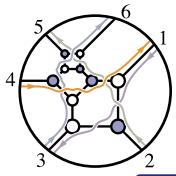
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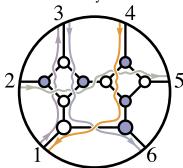




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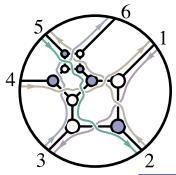
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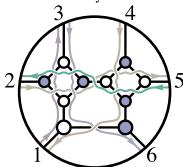




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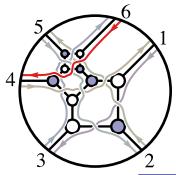
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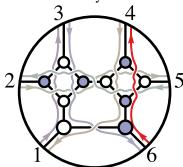




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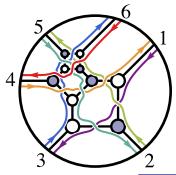
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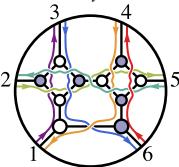




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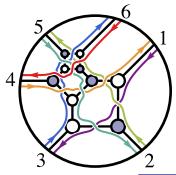
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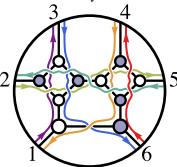




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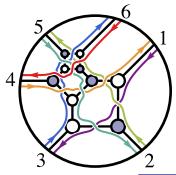
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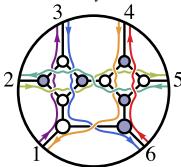






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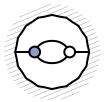


Notice that the merge and square moves leave the number of 'faces' of an on-shell diagram invariant.

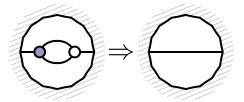
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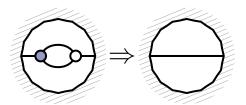
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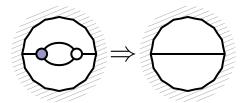
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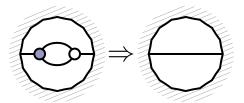
Bubble-deletion does not, however, relate 'identical' on-shell diagrams:

• it leaves behind an overall factor of $d\alpha/\alpha$ in the on-shell function



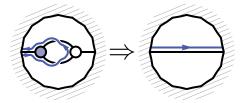
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- and it alters the corresponding left-right path permutation



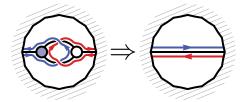
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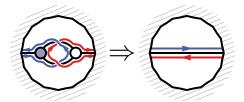


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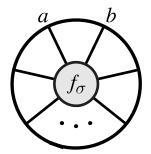
Such factors of $d\alpha/\alpha$ arising from bubble deletion encode loop integrands!

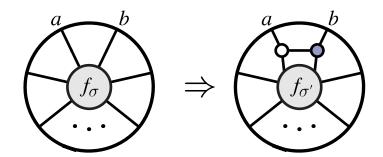


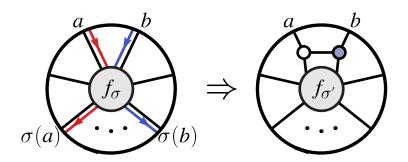
A Combinatorial Classification of On-Shell Functions Building-Up (Representative) Diagrams and Functions with Bridges Asymptotic Symmetries of the S-Matrix: the *Yangian*

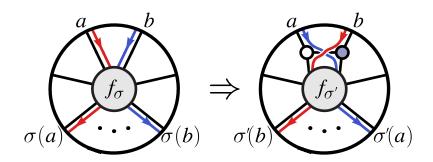
Canonical Coordinates for Computing On-Shell Functions

Recall that attaching 'BCFW bridges' can lead to very rich on-shell diagrams.

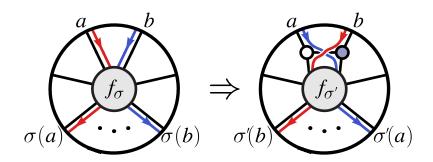




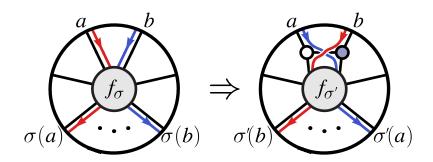




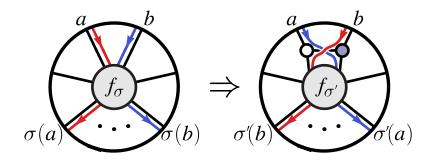
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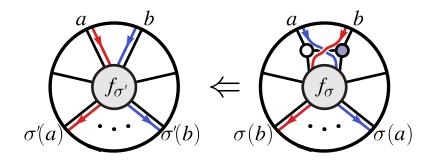
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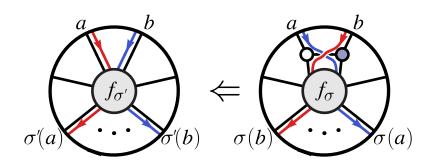
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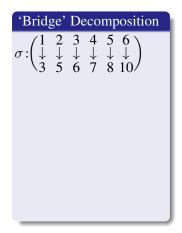
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Recall that attaching 'BCFW bridges' can lead to very rich on-shell diagrams. Read the other way, we can 'peel-off' bridges and thereby decompose a permutation into transpositions according to $\sigma = (ab) \circ \sigma'$



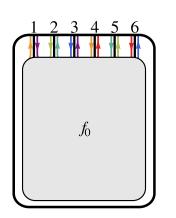
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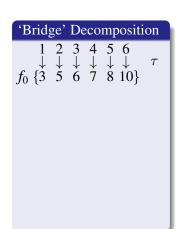


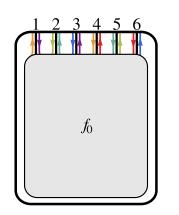
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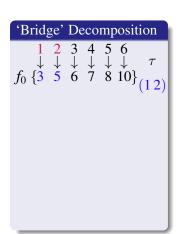






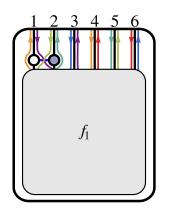






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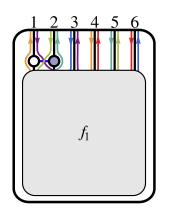
$$f_0 = \frac{d\alpha_1}{\alpha_1} f_1$$



'Bridge' Decomposition 1 2 3 4 5 6 $\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \tau$ $f_0 \{3 5 6 7 8 10\}$ $f_1 \{5 3 6 7 8 10\}$ (12)

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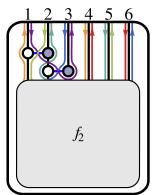
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'Bridge' Decomposition

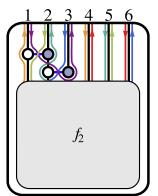
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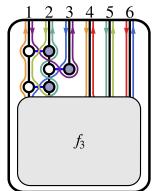
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'Bridg	ge' D	ecomp	position
$f_0 \begin{cases} 3 \\ f_1 \end{cases} \begin{cases} 5 \\ f_2 \end{cases} \begin{cases} 5 \end{cases}$	2 3 ↓ ↓ ↓ 5 6 3 6 6 3	4 5 ↓ ↓ ↓ 7 8 7 8 7 8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10\}(12) \\ 10\}(23) \\ 10\}(12) \end{array} $

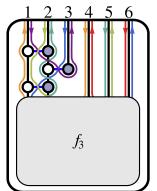
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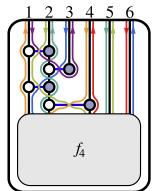
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'Brid	ge' De	ecom	positi	on
$f_0 \begin{cases} 3 \\ 5 \\ f_1 \end{cases} \begin{cases} 5 \\ f_2 \end{cases} \begin{cases} 5 \\ f_3 \end{cases} \begin{cases} 6 \end{cases}$	2 3 ↓ ↓ ↓ 5 6 3 6 6 3 5 3	4 5 7 8 7 8 7 8 7 8	5 6 ↓ 3 10} 3 10} 3 10} 3 10}	τ 12) 23) 12) 24)

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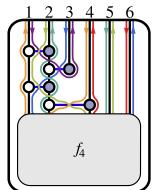
$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} f_4$$



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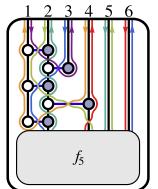
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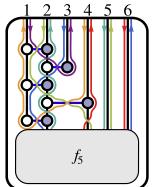
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'Bridge' Decomposition

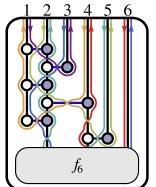
$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} f_5$$



'Bridg	e' I	Deco	mpc	sition
	2	2 4		c

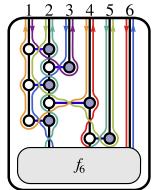
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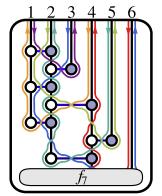
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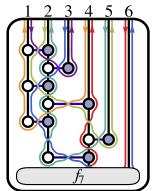
'Bridge' Decomposition							
1	2 ↓ 5 3 6 5 7 6	3 6 6 3 3 3	4 → 7 7 7 7 5 5	5 ↓ 8 8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10\} \\ 10\} \\ 23) \\ 10\} \\ 12) \\ 10\} \\ 24) \\ 10\} \\ 10\} \\ 45) \\ 10\} \\ 24) $		

$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} f_7$$



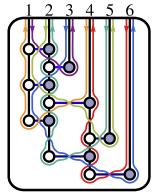
'Brid	ge'	De	eco	mp	osition
f_{0} {3} f_{1} {5} f_{2} {5} f_{3} {6} f_{4} {6} f_{5} {7	2 5 3 6 5 7 6	3	4 → 7 7 7 7 5 5	5 ↓ 8 8	$\begin{array}{c} 6 \\ \downarrow \tau \\ 10\} (12) \\ 10\} (23) \\ 10\} (12) \\ 10\} (24) \\ 10\} (12) \\ 10\} (45) \\ 10\} (24) \\ \end{array}$
$f_6 \{ 7 \\ f_7 \{ 7 \} $	8	3	8	5	10 $\{(24)$

$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} f_7$$



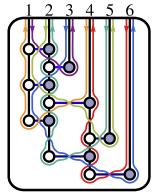
	ion
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	au (12) (23)

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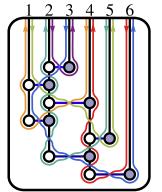
'Bridg	ge'	De	ecoi	mţ	osition
1	2 ↓	3	4 ↓		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	5	↓ 6	₇	8	$10\}_{(1,2)}$
f_1 {5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2 {5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $
f_3 {6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$
f_6 {7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7 {7	8	3	6	5	10}(12) 10}(24) 10}(12) 10}(45) 10}(24) 10}(46)
f_8 {7	8	3	10	5	$\{6\}^{(40)}$

$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$



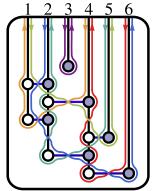
'Brid	ge'	De	ecoi	mţ	osition
1	2	3	4		6 τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	↓ 6	↓ 7	8	
f_1 {5	3	6	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $
f_2 {5	6	3	7	8	$10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10\}_{(45)}^{(12)}$
f_6 {7	6	3	8	5	$10\}_{(24)}^{(73)}$
f_7 {7	8	3	6	5	10}(12) 10}(24) 10}(12) 10}(45) 10}(24) 10}(46)
f_8 {7	8	3	10	5	$\left\{ 6\right\} ^{(40)}$

$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$



'Brid	lge'	De	ecoi	mp	osi	tion
	2 ↓					au
f_1 {5	3	6	7	8	10	\{(23)\{(12)\}(24)\}((45)\}((46))
$f_2 \{ 5 \}$	6	3	7	8	10	$\binom{(23)}{(12)}$
$f_2 \{5 \\ f_3 \{6 \}$	5	3	7	8	10	$\binom{(12)}{(24)}$
$f_4 \{ 6 \}$	7		5	8	10	$\binom{(24)}{(12)}$
f_5 {7	6	3	5	8	10	$\binom{12}{45}$
f_6 {7	6	3	8	5	10	$\{\frac{(43)}{(24)}\}$
f_7 {7	8	3	6	5	10	$\binom{(44)}{(46)}$
f_8 {7	8	3	10	5	6	}(+0)

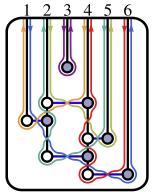
$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$



'Brid	ge'	De	eco	mŗ	osi	tion
1	2	3	4 ↓	5	6	au
*	*	*	*	*	*	·
£ (5	6	2	7	0	10	1
f_2 {5	6	2	7	0	10	(12)
f_3 {6	3	3	/	8	10	$\frac{1}{(2.4)}$
f_4 {6	7	3	5	8	10	$\{(1,2)^{(1,2)}\}$
f_5 {7	6	3	5	8	10	$\binom{1}{(15)}$
f_6 {7	6	3	8	5	10	\{(12) \{(24) \{(12) \{(45) \{(24) \{(46)}
f_7 {7	8	3	6	5	10	$\binom{(24)}{(46)}$
f_8 {7	8	3	10	5	6	\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \

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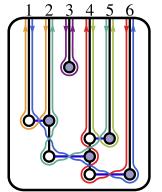
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'Bridge' Decomposition 1 2 3 4 5 6 f_3 {6 5 3 7 8 10}(24) f_4 {6 7 3 5 8 10}(12) f_5 {7 6 3 5 8 10}(45) f_6 {7 6 3 8 5 10}(24) f_7 {7 8 3 6 5 10}(24) f_8 {7 8 3 10 5 6}

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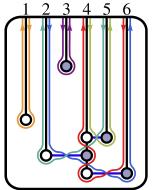
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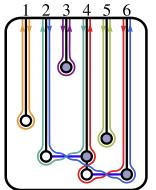


'Bridge' Decomposition 1 2 3 4 5 6

$$f_5 \{7 \ 6 \ 3 \ 5 \ 8 \ 10\}_{(45)}$$
 $f_6 \{7 \ 6 \ 3 \ 8 \ 5 \ 10\}_{(24)}$
 $f_7 \{7 \ 8 \ 3 \ 6 \ 5 \ 10\}_{(46)}$
 $f_8 \{7 \ 8 \ 3 \ 10 \ 5 \ 6\}_{(46)}$

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a \ b)$ such that $\sigma(a) < \sigma(b)$:

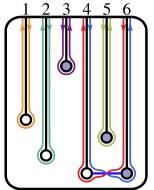
$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$



1 2 3 4 5 6 $f_6 \{7 \ 6 \ 3 \ 8 \ 5 \ 10\}_{(2 \ 4)}$ $f_7 \{7 \ 8 \ 3 \ 6 \ 5 \ 10\}_{(4 \ 6)}$

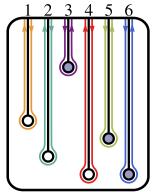
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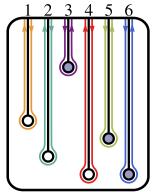
'Bridge' Decomposition 1 2 3 4 5 6 $f_7 \{7 \ 8 \ 3 \ 6 \ 5 \ 10\}$ $f_8 \{7 \ 8 \ 3 \ 10 \ 5 \ 6 \}$ (46)

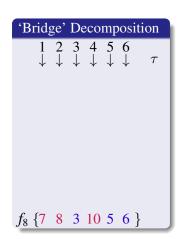
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$$f_8 = \prod_{a=\sigma(a)+n} \left(\delta^4(\widetilde{\eta}_a) \delta^2(\widetilde{\lambda}_a) \right) \prod_{b=\sigma(b)} \left(\delta^2(\lambda_b) \right)$$

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$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{0} & \frac{3}{0} & \frac{4}{0} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

'Bridge' Decomposition 1 2 3 4 5 6

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$$f_8 = \delta^{3\times4} \big(C \cdot \widetilde{\eta}\big) \delta^{3\times2} \big(C \cdot \widetilde{\lambda}\big) \delta^{2\times3} \big(\lambda \cdot C^\perp\big)$$

$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{0} & \frac{3}{0} & \frac{4}{0} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

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$$f_7 = \frac{d\alpha_8}{\alpha_8} \delta^{3\times 4} (C \cdot \widetilde{\eta}) \delta^{3\times 2} (C \cdot \widetilde{\lambda}) \delta^{2\times 3} (\lambda \cdot C^{\perp})$$

$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{0} & \frac{3}{0} & \frac{4}{0} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{pmatrix}$$

$$(46): c_6 \mapsto c_6 + \alpha_8 c_4$$

'Bridge' Decomposition 1 2 3 4 5 6

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$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$

$$f_6 = \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} \delta^{3\times4} (C \cdot \widetilde{\eta}) \delta^{3\times2} (C \cdot \widetilde{\lambda}) \delta^{2\times3} (\lambda \cdot C^{\perp})$$

$$f_6 \{7 \ 6 \ 3 \ 8 \ 5 \ 10\}_{(24)}$$

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There are many ways to decompose a permutation into transpositions—e.g., always choose the first transposition $\tau \equiv (a b)$ such that $\sigma(a) < \sigma(b)$:

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$$f_5 = \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} \delta^{3\times 4} (C \cdot \widetilde{\eta}) \delta^{3\times 2} (C \cdot \widetilde{\lambda}) \delta^{2\times 3} (\lambda \cdot C^{\perp})$$

$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{0} & \frac{3}{0} & \frac{4}{0} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & 0 & \alpha_7 & \alpha_6 \alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix} \qquad f_5 \left\{ 7 & 6 & 3 & 5 & 8 & 10 \right\} (45) \\ (45): c_5 \mapsto c_5 + \alpha_6 c_4 \qquad f_8 \left\{ 7 & 8 & 3 & 6 & 5 & 10 \right\} (46) \\ \frac{1}{6} \left\{ 7 & 8 & 3 & 6 & 5 & 10 \right\} (46)$$

$$f_5$$
 {7 6 3 5 8 10}(45)
 f_6 {7 6 3 8 5 10}(24)
 f_7 {7 8 3 6 5 10}(46)
 f_8 {7 8 3 10 5 6}

There are many ways to decompose a permutation into transpositions—e.g., always choose the first transposition $\tau \equiv (a b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$

$$f_4 = \frac{d\alpha_5}{\alpha_5} \cdots \frac{d\alpha_8}{\alpha_8} \delta^{3\times 4} (C \cdot \widetilde{\eta}) \delta^{3\times 2} (C \cdot \widetilde{\lambda}) \delta^{2\times 3} (\lambda \cdot C^{\perp})$$

$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{\alpha_5} & \frac{3}{0} & \frac{4}{0} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & 0 & \alpha_7 & \alpha_6 \alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

$$(12): c_2 \mapsto c_2 + \alpha_5 c_1$$

$$f_4 \{6 & 7 & 3 & 5 & 8 & 10\} (12)$$

$$f_5 \{7 & 6 & 3 & 5 & 8 & 10\} (45)$$

$$f_6 \{7 & 6 & 3 & 8 & 5 & 10\} (24)$$

$$f_7 \{7 & 8 & 3 & 6 & 5 & 10\} (46)$$

$$f_4$$
 {6 7 3 5 8 10}(12)
 f_5 {7 6 3 5 8 10}(45)
 f_6 {7 6 3 8 5 10}(24)
 f_7 {7 8 3 6 5 10}(24)
 f_8 {7 8 3 10 5 6}

There are many ways to decompose a permutation into transpositions—e.g., always choose the first transposition $\tau \equiv (a \ b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$

$$f_3 = \frac{d\alpha_4}{\alpha_4} \cdots \frac{d\alpha_8}{\alpha_8} \delta^{3\times4} (C \cdot \widetilde{\eta}) \delta^{3\times2} (C \cdot \widetilde{\lambda}) \delta^{2\times3} (\lambda \cdot C^{\perp})$$

$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{\alpha_5} & \frac{3}{0} & \frac{4}{\alpha_4\alpha_5} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & 0 & (\alpha_4 + \alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

$$(24): c_4 \mapsto c_4 + \alpha_4 c_2$$

$$f_3$$
 {6 5 3 7 8 10}(24)
 f_4 {6 7 3 5 8 10}(12)
 f_5 {7 6 3 5 8 10}(45)
 f_6 {7 6 3 8 5 10}(24)
 f_7 {7 8 3 6 5 10}(24)
 f_8 {7 8 3 10 5 6}

There are many ways to decompose a permutation into transpositions—e.g., always choose the first transposition $\tau \equiv (a \ b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$

$$f_2 = \frac{d\alpha_3}{\alpha_3} \cdots \frac{d\alpha_8}{\alpha_8} \delta^{3\times 4} (C \cdot \widetilde{\eta}) \delta^{3\times 2} (C \cdot \widetilde{\lambda}) \delta^{2\times 3} (\lambda \cdot C^{\perp})$$

$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{(\alpha_3 + \alpha_5)} & \frac{3}{0} & \frac{4}{\alpha_4 \alpha_5} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & 0 & (\alpha_4 + \alpha_7) & \alpha_6 & \alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

$$(12): c_2 \mapsto c_2 + \alpha_3 c_1$$

$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$

$$f_1 = \frac{d\alpha_2}{\alpha_2} \cdots \frac{d\alpha_8}{\alpha_8} \delta^{3\times 4} (C \cdot \widetilde{\eta}) \delta^{3\times 2} (C \cdot \widetilde{\lambda}) \delta^{2\times 3} (\lambda \cdot C^{\perp})$$

$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{(\alpha_3 + \alpha_5)} & \frac{3}{\alpha_2(\alpha_3 + \alpha_5)} & \frac{4}{\alpha_4\alpha_5} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & \alpha_2 & (\alpha_4 + \alpha_7)\alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

$$(23): c_3 \mapsto c_3 + \alpha_2 c_2$$

	'B							tion
		1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	au
)	f_1	{5	3	6	7	8	10]	(23)
	f_3	{5 {6	5	3	7	8	10	(12) $ (24)$
	f_4 f_5	{ 6 {7	7 6	3	5 5	8	10 10	(12)
	f_6 f_7	{7 {7	6 8	3	8	55	10] 10]	{(23) (12) (24) (12) (45) (24) (46)
	£	(7	0	2	10	5	6	(40)

$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$

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$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{(\alpha_1 + \alpha_3 + \alpha_5)} & \frac{3}{\alpha_2(\alpha_3 + \alpha_5)} & \frac{4}{\alpha_4\alpha_5} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & \alpha_2 & (\alpha_4 + \alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

$$(12): c_2 \mapsto c_2 + \alpha_1 c_1$$

	'Brid	ge'	De	ecoi	mp	osition
	1	2	3	4	5	6 τ
	$f_0 \left\{ \begin{array}{l} \downarrow \\ 3 \end{array} \right.$	5	$\overset{\downarrow}{6}$	$\overset{\downarrow}{7}$	8	$ \downarrow \tau $ $ 10} $ $ 10 \rbrace (12) $
-)	$f_1 \ \{ 5 \}$		6	7		
	<i>f</i> ₂ { 5	6	3	7	8	10 (2.3) (1.2)
	f_3 {6	5	3	7		$\{0\}_{(2,4)}$
	f_4 {6	7	3	5		$\{10\}_{(1,2)}$
	f_5 {7	6	3	5		$10\}_{(45)}^{(12)}$
	$f_6 \{7$	6	3	8	5	$\frac{10}{10}$ (24)
	$f_7 \{ 7 \}$	8		6	5	10 (46)
	$f_8 \{7$	8	3	10	5	6 } ()

$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$

$$f_0 = \frac{d\alpha_1}{\alpha_1} \cdots \frac{d\alpha_8}{\alpha_8} \delta^{3\times 4} (C \cdot \widetilde{\eta}) \delta^{3\times 2} (C \cdot \widetilde{\lambda}) \delta^{2\times 3} (\lambda \cdot C^{\perp})$$

$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{(\alpha_1 + \alpha_3 + \alpha_5)} & \frac{3}{\alpha_2(\alpha_3 + \alpha_5)} & \frac{4}{\alpha_4\alpha_5} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & \alpha_2 & (\alpha_4 + \alpha_7)\alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

$f_{0} \begin{cases} 3 & 5 & 6 & 7 & 8 & 10 \end{cases} (1) \\ f_{1} \begin{cases} 5 & 3 & 6 & 7 & 8 & 10 \end{cases} (2) \\ f_{2} \begin{cases} 5 & 6 & 3 & 7 & 8 & 10 \end{cases} (2) \\ f_{3} \begin{cases} 6 & 5 & 3 & 7 & 8 & 10 \end{cases} (2) \\ f_{4} \begin{cases} 6 & 7 & 3 & 5 & 8 & 10 \end{cases} (1) \\ f_{5} \begin{cases} 7 & 6 & 3 & 5 & 8 & 10 \end{cases} (2) \\ f_{6} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (2) \\ f_{6} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (2) \\ f_{6} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (2) \\ f_{6} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (3) \\ f_{6} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & $		'Brid	ge'	De	ecoi	mp	oosition
f_3 {6 5 3 7 8 10}(2) f_4 {6 7 3 5 8 10}(1) f_5 {7 6 3 5 8 10}(4) f_6 {7 6 3 8 5 10}(2)	^L)	$f_0 \begin{cases} 1 \\ \downarrow \\ f_0 \end{cases} \begin{cases} 3 \\ f_1 \end{cases} \{ 5 $	2 ↓ 5 3	3 ↓ 6		5 ↓ 8 8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10\}\\ 10\}\\ (12)\\ (23) \end{array} $
$f_6 \{7 \ 6 \ 3 \ 8 \ 5 \ 10\}_{(2)}^{(4)}$		<i>f</i> ₃ {6 <i>f</i> ₄ {6	7	_		8 8	$10\}_{(1,2)}^{(2,4)}$
$f_7 \{7 \ 8 \ 3 \ 6 \ 5 \ 10\} $ $(4 \ f_8 \{7 \ 8 \ 3 \ 10 \ 5 \ 6\} $		$f_6 \ \{7\}$ $f_7 \ \{7\}$	6 8	3 3		5 5	10 $\{(45)$ $\{(24)\}$ $\{(46)\}$

$$f_0 = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \frac{d\alpha_4}{\alpha_4} \frac{d\alpha_5}{\alpha_5} \frac{d\alpha_6}{\alpha_6} \frac{d\alpha_7}{\alpha_7} \frac{d\alpha_8}{\alpha_8} f_8$$

$$f_0 = \frac{d\alpha_1}{\alpha_1} \cdots \frac{d\alpha_8}{\alpha_8} \delta^{3\times 4} (C \cdot \widetilde{\eta}) \delta^{3\times 2} (C \cdot \widetilde{\lambda}) \delta^{2\times 3} (\lambda \cdot C^{\perp})$$

$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{(\alpha_1 + \alpha_3 + \alpha_5)} & \frac{3}{\alpha_2(\alpha_3 + \alpha_5)} & \frac{4}{\alpha_4\alpha_5} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & \alpha_2 & (\alpha_4 + \alpha_7)\alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

$f_{0} \begin{cases} 3 & 5 & 6 & 7 & 8 & 10 \end{cases} (1) \\ f_{1} \begin{cases} 5 & 3 & 6 & 7 & 8 & 10 \end{cases} (2) \\ f_{2} \begin{cases} 5 & 6 & 3 & 7 & 8 & 10 \end{cases} (2) \\ f_{3} \begin{cases} 6 & 5 & 3 & 7 & 8 & 10 \end{cases} (2) \\ f_{4} \begin{cases} 6 & 7 & 3 & 5 & 8 & 10 \end{cases} (1) \\ f_{5} \begin{cases} 7 & 6 & 3 & 5 & 8 & 10 \end{cases} (2) \\ f_{6} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (2) \\ f_{6} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (2) \\ f_{6} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (2) \\ f_{6} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (3) \\ f_{6} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 5 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 & 10 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & 3 & 8 \end{cases} (4) \\ f_{7} \begin{cases} 7 & 6 & $		'Brid	ge'	De	ecoi	mp	oosition
f_3 {6 5 3 7 8 10}(2) f_4 {6 7 3 5 8 10}(1) f_5 {7 6 3 5 8 10}(4) f_6 {7 6 3 8 5 10}(2)	^L)	$f_0 \begin{cases} 1 \\ \downarrow \\ f_0 \end{cases} \begin{cases} 3 \\ f_1 \end{cases} \{ 5 $	2 ↓ 5 3	3 ↓ 6		5 ↓ 8 8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10\}\\ 10\}\\ (12)\\ (23) \end{array} $
$f_6 \{7 \ 6 \ 3 \ 8 \ 5 \ 10\}_{(2)}^{(4)}$		<i>f</i> ₃ {6 <i>f</i> ₄ {6	7	_		8 8	$10\}_{(1,2)}^{(2,4)}$
$f_7 \{7 \ 8 \ 3 \ 6 \ 5 \ 10\} $ $(4 \ f_8 \{7 \ 8 \ 3 \ 10 \ 5 \ 6\} $		$f_6 \ \{7\}$ $f_7 \ \{7\}$	6 8	3 3		5 5	10 $\{(45)$ $\{(24)\}$ $\{(46)\}$

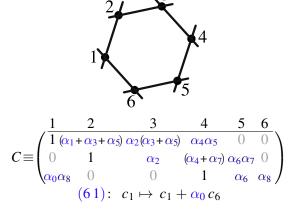
$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{(\alpha_1 + \alpha_3 + \alpha_5)} & \frac{3}{\alpha_2(\alpha_3 + \alpha_5)} & \frac{4}{\alpha_4\alpha_5} & \frac{5}{0} & 0\\ 0 & 1 & \alpha_2 & (\alpha_4 + \alpha_7) & \alpha_6\alpha_7 & 0\\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

'Brid	lge'	De	ecoi	mŗ	osition
1	2	3	4	5	6 τ
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	6	↓ 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {5		6	7	8	10 (23) (23)
<i>f</i> ₂ { 5	6	3	7	8	$10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7		$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5		$10\}_{(12)}^{(24)}$
f_5 {7	6	3	5		10 (45)
$f_6 \{ 7 \}$	6	3	8	5	$10\}_{(24)}$
$f_7 \{ 7 \}$	8	3	6	5	10 (46)
$f_8 \{7$	8	3	10	5	6 }

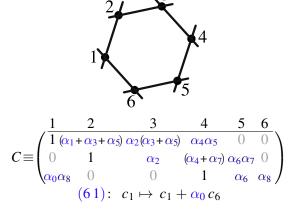
$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{(\alpha_1 + \alpha_3 + \alpha_5)} & \frac{3}{\alpha_2(\alpha_3 + \alpha_5)} & \frac{4}{\alpha_4\alpha_5} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & \alpha_2 & (\alpha_4 + \alpha_7) & \alpha_6\alpha_7 & 0 \\ \alpha_0\alpha_8 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

$$(61): c_1 \mapsto c_1 + \alpha_0 c_6$$

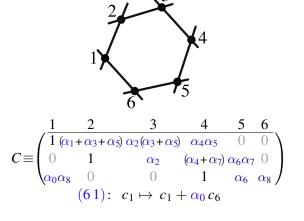
'Bridg	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	→ 5	♦	→ 7	♦	$ \begin{array}{c} \downarrow & \tau \\ 10\} \\ 10\} \\ 10\} \\ (23) \\ 10\} \\ (12) \end{array} $
$f_1 \{ 5 \}$	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f ₂ {5	6	3	7	8	$\frac{10}{10}$ (12)
<i>f</i> ₃ { 6	5	3	7	8	$10\}_{(2.4)}^{(1.2)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(4.5)}$
f_6 {7	6	3	8	5	$10\}_{(24)}^{(43)}$
f_7 {7	8		6	5	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ 6 \end{array} $
f_8 {7	8	3	10	5	6



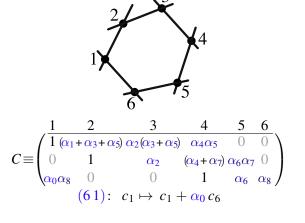
'B	rid	ge'	De	ecoi	mŗ	osition
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	{5	6	3	7	8	10 (23) 10 (12)
f_3	{6	5	3	7	8	$\frac{10}{(24)}$
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	1/21/1
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



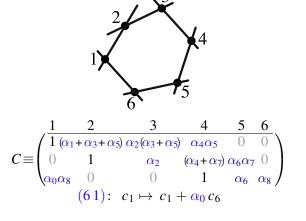
'B	rid	ge'	De	ecoi	mţ	osition
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	{5	6	3	7	8	10 (23) 10 (12)
f_3	{6	5	3	7	8	$\frac{10}{(24)}$
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	1/21/1
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



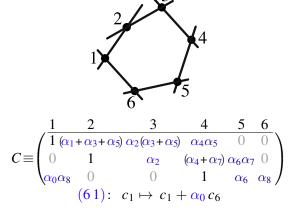
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4 ↓	5	6 τ			
f_0	$\left\{\stackrel{\star}{3}\right\}$	5	6	[*] 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $			
	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $			
f_3	{6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$			
f_4	6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $			
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6			



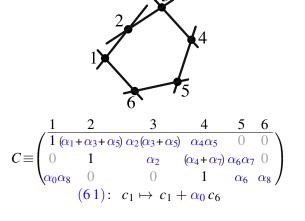
'B	'Bridge' Decomposition								
	1 ↓	2 ↓	3		5	6 ↓ τ			
f_0	${3 \choose 3}$	5	↓ 6	$\overset{\downarrow}{7}$	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$			
f_1		3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{10}$ (24)			
f_4	6	7	3	5	8	$10\}(12)$			
f_5	{7	6	3	5	8	$10\}(45)$			
f_6	{7	6	3	8	5	10 (24)			
f_7	{7	8	3	6	5	$\frac{10}{6}$ (46)			
f_8	{7	8	3	10	5	6			



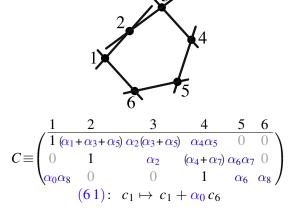
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4	5	6 τ			
f_0	${3 \choose 3}$	5	$\overset{\downarrow}{6}$	→ 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $			
f_1		3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$\frac{10}{10}$ (12)			
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



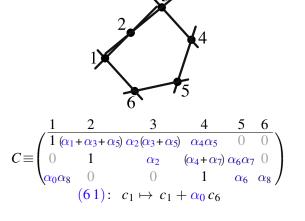
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ			
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$			
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$			
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	1/21/1			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



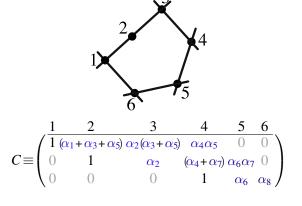
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ			
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$			
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$			
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	1/21/1			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



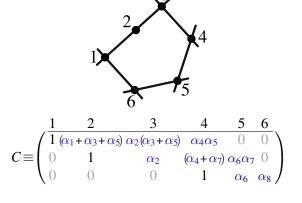
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ			
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$			
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$			
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	1/21/1			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



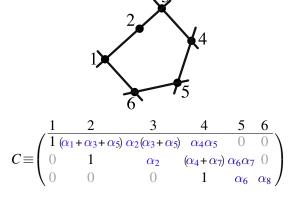
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ			
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$			
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$			
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	1/21/1			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



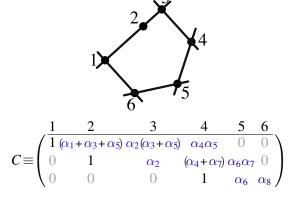
'B	rid	ge'	De	ecoi	mţ	osition
	1 ↓	2 ↓	3		5	6 ↓ τ
f_0	${3 \choose 3}$	5	↓ 6	→ 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
f_1		3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	5	6	3	7	8	10 (23) 10 (12)
f_3	6	5	3	7	8	$\frac{10}{10}$ (24)
f_4	6	7	3	5	8	$10\}(12)$
f_5	{7	6	3	5	8	$10\}(45)$
f_6	{7	6	3	8	5	10 (24)
f_7	{7	8	3	6	5	$\frac{10}{6}$ (46)
f_8	{7	8	3	10	5	6 } (40)



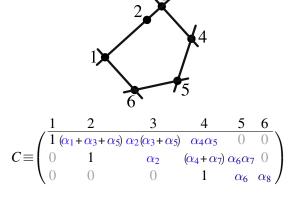
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{5$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$
<i>f</i> ₂ { 5	6	3	7	8	10 (23) 10 (12)
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(45)}$
f_6 {7	6	3	8	5	$10\}(2.4)$
f_7 {7	8	3	6	5	(4 0)
$f_8 \{7$	8	3	10	5	6 } (10)



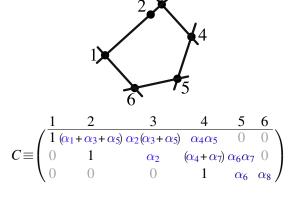
'B	rid	ge'	De	ecoi	mţ	osition
	1 ↓	2 ↓	3 ↓	4 ↓	5	6 ↓ τ
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$		8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	{5	6	3	7	8	10 (23) 10 (12)
f_3	{6	5	3	7	8	$\frac{10}{(24)}$
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	1/71/1
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



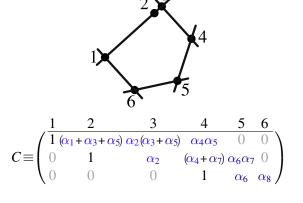
'B	rid	ge'	De	ecoi	mţ	osition
	1 ↓	2 ↓	3 ↓	4 ↓	5	6 ↓ τ
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$		8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	{5	6	3	7	8	10 (23) 10 (12)
f_3	{6	5	3	7	8	$\frac{10}{(24)}$
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	1/71/1
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



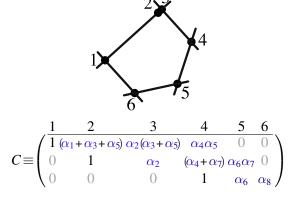
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{5$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$
<i>f</i> ₂ { 5	6	3	7	8	10 (23) 10 (12)
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(45)}$
f_6 {7	6	3	8	5	$10\}(2.4)$
f_7 {7	8	3	6	5	(4 0)
$f_8 \{7$	8	3	10	5	6 } (10)



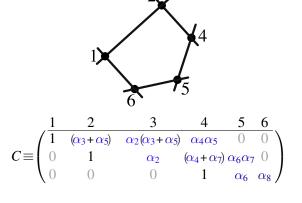
'Br	id	ge'	Dε	ecoi	mŗ	oosition
	1	2	3	4	5	6
f_0 {	3	5	↓ 6	$\overset{\downarrow}{7}$	♦	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {			6	7	8	10 (23)
f_2 {	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {	6	5	3	7		$10\}_{(24)}$
f_4 {	6	7	3	5		$10\}_{(1,2)}$
f_5 {	7	6	3	5	8	$10\}_{(4.5)}$
f_6 {		6	3	8	5	$10\}_{(24)}$
f_7 {		8		6	5	(46)
f_8 {	7	8	3	10	5	6 } (10)



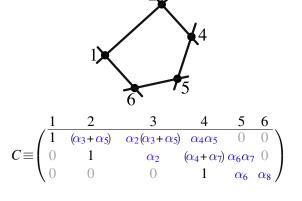
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{5$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$
<i>f</i> ₂ { 5	6	3	7	8	10 (23) 10 (12)
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(45)}$
f_6 {7	6	3	8	5	$10\}(2.4)$
f_7 {7	8	3	6	5	(4 0)
$f_8 \{7$	8	3	10	5	6 } (10)



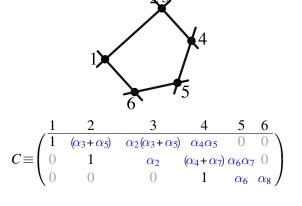
'B	rid	ge'	De	eco:	mŗ	osition
	1	2 ↓	3 ↓	4 ↓	5	6 τ
f_0	${3 \choose 3}$	5	6	[*] 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	{5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $
f_3	{6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$
f_4	<u>{</u> 6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6



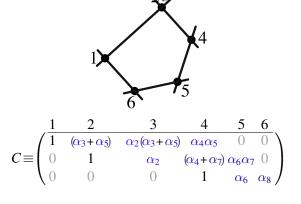
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{5$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$
<i>f</i> ₂ { 5	6	3	7	8	10 (23) 10 (12)
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(45)}$
f_6 {7	6	3	8	5	$10\}(2.4)$
f_7 {7	8	3	6	5	(4 0)
$f_8 \{7$	8	3	10	5	6 } (10)



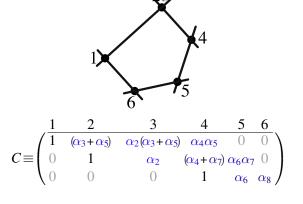
'Bridg	ge'	De	eco	mp	osition
1	2	3	4	5	6 τ
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	$\overset{\downarrow}{6}$	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
f_1 {5		6	7	8	10 (23)
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {6	5	3	7		$10\}_{(24)}$
f_4 {6	7	3	5		$\{10\}_{(1,2)}$
f_5 {7	6	3	5		$\frac{10}{(4.5)}$
f_6 {7	6	3	8	5	$10\}_{(24)}$
$f_7 \{ 7 \}$	8	3	6	5	(46)
$f_8 \{7$	8	3	10	5	6 } (13)



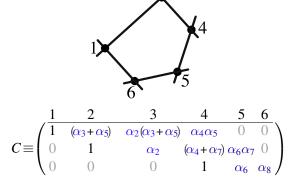
'Br	id	ge'	Dε	ecoi	mŗ	oosition
	1	2	3	4	5	6
f_0 {	→ 3	5	↓ 6	$\overset{\downarrow}{7}$	♦	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {			6	7	8	10 (23)
f_2 {	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {	6	5	3	7		$10\}_{(24)}$
f_4 {	6	7	3	5		$10\}_{(1,2)}$
f_5 {	7	6	3	5	8	$10\}_{(4.5)}$
f_6 {		6	3	8	5	$10\}_{(24)}$
f_7 {		8		6	5	(46)
f_8 {	7	8	3	10	5	6 } (10)



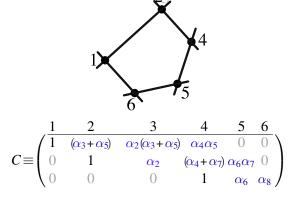
'B	rid	ge'	De	ecoi	mŗ	osition
	1 ↓	2 ↓	3		5	6 ↓ τ
f_0	${3 \choose 3}$	5	↓ 6	→ 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
f_1		3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	5	6	3	7	8	10 (23) 10 (12)
f_3	6	5	3	7	8	$\frac{10}{10}$ (24)
f_4	6	7	3	5	8	$10\}(12)$
f_5	{7	6	3	5	8	$10\}(45)$
f_6	{7	6	3	8	5	10 (24)
f_7	{7	8	3	6	5	$\frac{10}{6}$ (46)
f_8	{7	8	3	10	5	6 } (40)



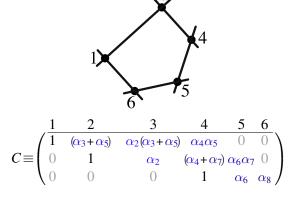
'Br	id	ge'	Dε	ecoi	mŗ	oosition
	1	2	3	4	5	6
f_0 {	→ 3	5	↓ 6	$\overset{\downarrow}{7}$	♦	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {			6	7	8	10 (23)
f_2 {	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {	6	5	3	7		$10\}_{(24)}$
f_4 {	6	7	3	5		$10\}_{(1,2)}$
f_5 {	7	6	3	5	8	$10\}_{(4.5)}$
f_6 {		6	3	8	5	$10\}_{(24)}$
f_7 {		8		6	5	(46)
f_8 {	7	8	3	10	5	6 } (10)



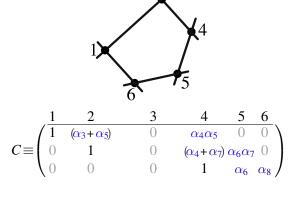
'B	rid	ge'	De	eco:	mŗ	osition
	1	2	3 ↓	4 ↓	5	6 τ
f_0	${3 \choose 3}$	5	6	$\overset{\star}{7}$	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	5	6	3	7	8	10 (23) 10 (12)
f_3	{6	5	3	7	8	$\frac{10}{(24)}$
f_4	6	7	3	5	8	$\frac{10}{10}$ (12)
f_5	{7	6	3	5	8	$\frac{10}{10}$ (45)
f_6	{7	6	3	8	5	$\frac{10}{10}$ (24)
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



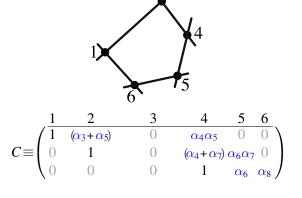
'B	rid	ge'	De	ecoi	mŗ	osition
	1 ↓	2 ↓	3		5	6 ↓ τ
f_0	${3 \choose 3}$	5	↓ 6	→ 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
f_1		3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	5	6	3	7	8	10 (23) 10 (12)
f_3	6	5	3	7	8	$\frac{10}{10}$ (24)
f_4	6	7	3	5	8	$10\}(12)$
f_5	{7	6	3	5	8	$10\}(45)$
f_6	{7	6	3	8	5	10 (24)
f_7	{7	8	3	6	5	$\frac{10}{6}$ (46)
f_8	{7	8	3	10	5	6 } (40)



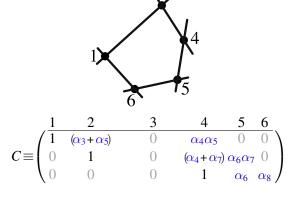
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{5$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$
<i>f</i> ₂ { 5	6	3	7	8	10 (23) 10 (12)
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(45)}$
f_6 {7	6	3	8	5	$10\}(2.4)$
f_7 {7	8	3	6	5	(4 0)
$f_8 \{7$	8	3	10	5	6 } (10)



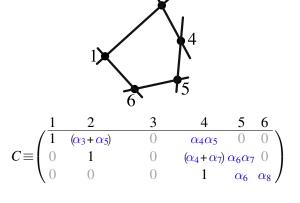
'Brio	lge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{l} \downarrow \\ 3 \end{array} \right.$. ↓ 5	$\overset{\downarrow}{6}$	↓ 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \end{array}$
$f_1 \{ 5 \}$		6	7		10 (12)
$f_2 \{ 5 \}$	6	3	7	8	10 (23) 10 (12)
f_3 {6	5	3	7		$10\}_{(24)}$
f_4 {6	7	3	5		$10\}_{(1,2)}$
$f_5 \{ 7 \}$		3	5	8	$\frac{10}{(4.5)}$
$f_6 \{ 7 \}$		3	8	5	$10\}_{(24)}$
$f_7 \{ 7 \}$	8		6	5	$10\}(21)$
$f_8 \{ 7 \}$	8	3	10	5	6 } ()



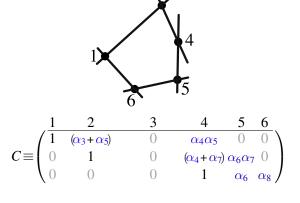
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{5$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$
<i>f</i> ₂ { 5	6	3	7	8	10 (23) 10 (12)
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(45)}$
f_6 {7	6	3	8	5	$10\}(2.4)$
f_7 {7	8	3	6	5	(4 0)
$f_8 \{7$	8	3	10	5	6 } (10)



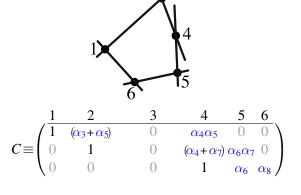
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	♦	→ 7	♦	$ \begin{array}{c} \downarrow & \tau \\ 10\} \\ 10\} \\ 10\} \\ (23) \\ 10\} \\ (12) \end{array} $
$f_1 \{ 5 \}$	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f ₂ {5	6	3	7	8	$\frac{10}{10}$ (12)
<i>f</i> ₃ {6	5	3	7	8	$10\}_{(2.4)}^{(1.2)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(4.5)}$
f_6 {7	6	3	8	5	$10\}_{(24)}^{(43)}$
f_7 {7	8		6	5	$10\}_{(46)}^{(24)}$
f_8 {7	8	3	10	5	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ 6 \end{array} $



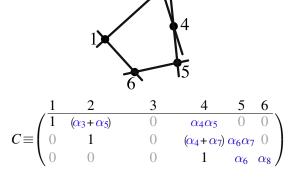
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{ 5 \}$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$
<i>f</i> ₂ { 5	6	3	7	8	10 (23) 10 (12)
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(45)}$
f_6 {7	6	3	8	5	$10\}(2.4)$
f_7 {7	8	3	6	5	(4 0)
$f_8 \{7$	8	3	10	5	6 } (10)



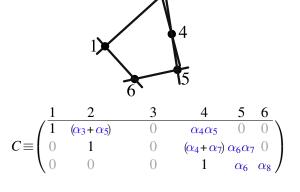
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	♦	→ 7	♦	$ \begin{array}{c} \downarrow & \tau \\ 10\} \\ 10\} \\ 10\} \\ (23) \\ 10\} \\ (12) \end{array} $
$f_1 \{ 5 \}$	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f ₂ {5	6	3	7	8	$\frac{10}{10}$ (12)
<i>f</i> ₃ {6	5	3	7	8	$10\}_{(2.4)}^{(1.2)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(4.5)}$
f_6 {7	6	3	8	5	$10\}_{(24)}^{(43)}$
f_7 {7	8		6	5	$10\}_{(46)}^{(24)}$
f_8 {7	8	3	10	5	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ 6 \end{array} $



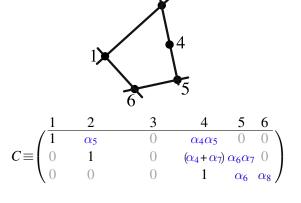
'Bri	dge	' De	eco	mŗ	osition
	1 2	3	4	5	6
f_0 {	\downarrow	$\overset{\downarrow}{6}$	$\stackrel{\downarrow}{7}$	♦	$\downarrow \tau$ 10 $\rbrace_{(1,2)}$
f_1 {:	5 3	6	7		101(12)
f_2 {		3	7		$10^{(23)}$
f_3 {		3	7	8	$10^{10}(12)$ $10^{10}(12)$
f_4 {		3	5	8	$10^{10}(24)$ $10^{10}(12)$
f_5 {	7 6	3	5	8	10 (12) 10 (45)
f_6 {		3	8	5	$10\}_{(24)}^{(43)}$
f_7 {	7 8	3	6	5	$10\}_{(46)}^{(24)}$
f_8 {	7 8	3	10	5	6 }(40)



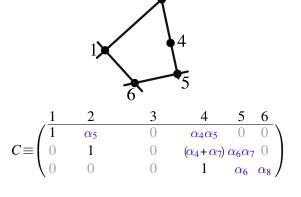
'Bridg	ge'	De	ecoi	mŗ	osition
1	2	3		5	6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	↓ 6	$\overset{\downarrow}{7}$	♦	$ \downarrow \tau $ 10} 10)(12)
f_1 {5		6	7	8	$10\}_{(2,3)}^{(1,2)}$
f ₂ { 5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5		$\frac{10}{(1.2)}$
$f_5 \ \{7$	6	3	5		$10\}_{(4.5)}$
f_6 {7	6	3	8		$10\}_{(24)}$
f_7 {7	8	3	6	5	10 (46)
f_8 {7	8	3	10	5	6 }(40)



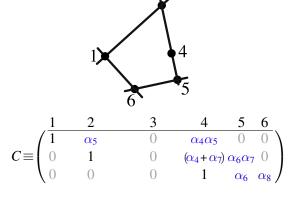
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{ 5 \}$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$
<i>f</i> ₂ { 5	6	3	7	8	10 (23) 10 (12)
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(45)}$
f_6 {7	6	3	8	5	$10\}(2.4)$
f_7 {7	8	3	6	5	(4 0)
$f_8 \{7$	8	3	10	5	6 } (10)



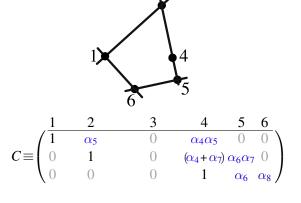
'Brid	ge'	De	eco	mŗ	osition
1					6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	↓	↓ 7	γ	$\downarrow \tau$
$f_1 \{ 5 \}$		6	7	8	$\frac{10}{10}$ (12)
f_2 {5	6	3	7	8	10 $\{(23)$ $\{(23)\}$ $\{(12)\}$
f_3 {6	5	3	7		101(12)
f_4 {6	7	3	5	8	$10^{10}(24)$ $10^{10}(12)$
f_5 {7	6	3	5	8	10 (12) 10 (45)
f_6 {7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7 {7	8	3	6	5	10 (24) 10 (46)
f_8 {7	8	3	10	5	6 } (40)



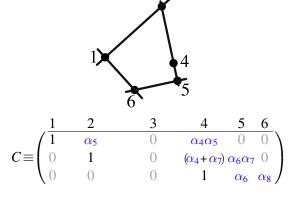
'Br	id	ge'	Dε	ecoi	mŗ	oosition
	1	2	3	4	5	6
f_0 {	→ 3	5	↓ 6	$\overset{\downarrow}{7}$	♦	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {			6	7	8	10 (23)
f_2 {	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {	6	5	3	7		$10\}_{(24)}$
f_4 {	6	7	3	5		$10\}_{(1,2)}$
f_5 {	7	6	3	5	8	$10\}_{(4.5)}$
f_6 {		6	3	8	5	$10\}_{(24)}$
f_7 {		8		6	5	(46)
f_8 {	7	8	3	10	5	6 } (10)



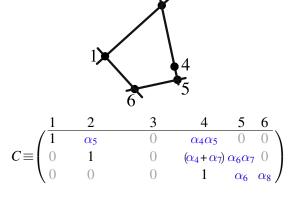
'Bridg	ge'	De	ecoi	mŗ	osition
1	2	3		5	6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	↓ 6	$\overset{\downarrow}{7}$	♦	$ \downarrow \tau $ 10} 10)(12)
f_1 {5		6	7	8	$10\}_{(2,3)}^{(1,2)}$
f ₂ { 5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5		$\frac{10}{(1.2)}$
$f_5 \ \{7$	6	3	5		$10\}_{(4.5)}$
f_6 {7	6	3	8		$10\}_{(24)}$
f_7 {7	8	3	6	5	10 (46)
f_8 {7	8	3	10	5	6 }(40)



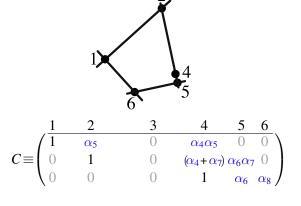
'Brid	ge'	De	eco	mŗ	osition
1					6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	↓	↓ 7	γ	$\downarrow \tau$
$f_1 \{ 5 \}$		6	7	8	$\frac{10}{10}$ (12)
f_2 {5	6	3	7	8	10 $\{(23)$ $\{(23)\}$ $\{(12)\}$
f_3 {6	5	3	7		101(12)
f_4 {6	7	3	5	8	$10^{10}(24)$ $10^{10}(12)$
f_5 {7	6	3	5	8	10 (12) 10 (45)
f_6 {7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7 {7	8	3	6	5	10 (24) 10 (46)
f_8 {7	8	3	10	5	6 } (40)



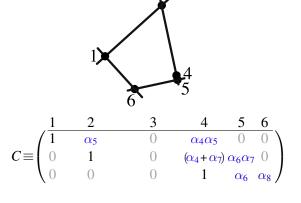
'Brid	'Bridge' Decomposition									
1	2	3	4	5	6 τ					
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	$\overset{\downarrow}{6}$	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $					
f_1 {5		6	7	8	10 (23)					
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)					
f_3 {6	5	3	7		$10\}_{(24)}$					
f_4 {6	7	3	5		$\frac{10}{10}$ (12)					
f_5 {7	6	3	5		$10\}_{(4.5)}$					
$f_6 \{7$	6	3	8	5	$10\}_{(24)}$					
$f_7 \{ 7 \}$	8	3	6	5	(46)					
$f_8 \ \{7$	8	3	10	5	6 }					



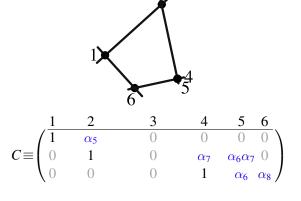
'Brid	ge'	De	eco	mŗ	osit	ion
1	2	3	4	5	6	_
$f_0 \left\{\stackrel{\downarrow}{3}\right\}$	↓ 5	♦	↓ 7	γ	↓ 10}	au
$f_1 \{5$		6	7	8	10)	(12)
f_2 {5	6	3	7		10}	(23)
f_3 {6	5	3	7		10}	(12)
f_4 {6	7	3	5	8	10}	(24)
f_5 {7	6	3	5	8	10)	$\begin{array}{c} (12) \\ (45) \end{array}$
f_6 {7	6	3	8	5	10}	(24)
f_7 {7	8	3	6	5	10	(46)
f_8 {7	8	3	10	5	6 }	(+0)



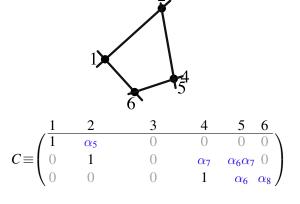
'Brid	'Bridge' Decomposition									
1	2	3	4	5	6					
$f_0 \left\{\stackrel{\downarrow}{3}\right\}$	5	6	† 7	8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10\} \\ 10\} \\ \end{array} $					
$f_1 \{ 5 \}$	3	6	7	_	-~ J/ ^ 2\					
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)					
f_3 {6	5	3	7		$10\}_{(24)}$					
f_4 { 6	7	3	5		$\{10\}_{(1,2)}$					
f_5 {7	6	3	5	8	$10\}_{(4.5)}$					
f_6 {7	6	3	8	5	$10\}_{(24)}$					
f_7 {7	8	3	6	5	1461					
f_8 {7	8	3	10	5	6					



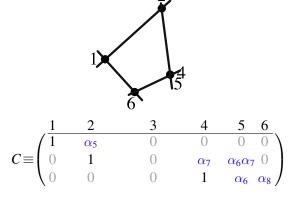
'Brid	ge'	De	eco	mŗ	osition
1					6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	↓	↓ 7	γ	$\downarrow \tau$
$f_1 \{ 5 \}$		6	7	8	$\frac{10}{10}$ (12)
f_2 {5	6	3	7	8	10 $\{(23)$ $\{(23)\}$ $\{(12)\}$
f_3 {6	5	3	7		101(12)
f_4 {6	7	3	5	8	$10^{10}(24)$ $10^{10}(12)$
f_5 {7	6	3	5	8	10 (12) 10 (45)
f_6 {7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7 {7	8	3	6	5	10 (24) 10 (46)
f_8 {7	8	3	10	5	6 } (40)



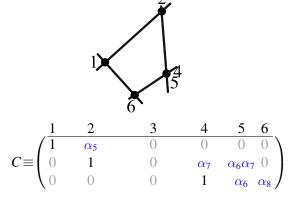
'Brid	'Bridge' Decomposition									
1	2	3	4	5	6 τ					
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	$\overset{\downarrow}{6}$	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $					
f_1 {5		6	7	8	10 (23)					
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)					
f_3 {6	5	3	7		$10\}_{(24)}$					
f_4 {6	7	3	5		$\frac{10}{10}$ (12)					
f_5 {7	6	3	5		$10\}_{(4.5)}$					
$f_6 \{7$	6	3	8	5	$10\}_{(24)}$					
$f_7 \{ 7 \}$	8	3	6	5	(46)					
$f_8 \ \{7$	8	3	10	5	6 }					



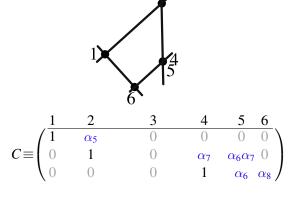
'Brid	'Bridge' Decomposition									
1	2	3	4	5	6					
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	♦	→ 7	♦	$ \begin{array}{c} \downarrow & \tau \\ 10\} \\ 10\} \\ 10\} \\ (23) \\ 10\} \\ (12) \end{array} $					
$f_1 \{ 5 \}$	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$					
f ₂ {5	6	3	7	8	$\frac{10}{10}$ (12)					
<i>f</i> ₃ {6	5	3	7	8	$10\}_{(2.4)}^{(1.2)}$					
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $					
f_5 {7	6	3	5	8	$10\}_{(4.5)}$					
f_6 {7	6	3	8	5	$10\}_{(24)}^{(43)}$					
f_7 {7	8		6	5	$10\}_{(46)}^{(24)}$					
f_8 {7	8	3	10	5	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ 6 \end{array} $					



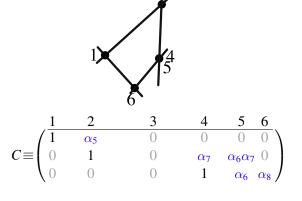
'Brid	'Bridge' Decomposition									
1	2	3	4	5	6					
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	♦	→ 7	♦	$ \begin{array}{c} \downarrow & \tau \\ 10\} \\ 10\} \\ 10\} \\ (23) \\ 10\} \\ (12) \end{array} $					
$f_1 \{ 5 \}$	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$					
f ₂ {5	6	3	7	8	$\frac{10}{10}$ (12)					
f_3 {6	5	3	7	8	$10\}_{(2.4)}^{(1.2)}$					
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $					
f_5 {7	6	3	5	8	$10\}_{(4.5)}$					
f_6 {7	6	3	8	5	$10\}_{(24)}^{(43)}$					
f_7 {7	8		6	5	$10\}_{(46)}^{(24)}$					
f_8 {7	8	3	10	5	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ 6 \end{array} $					



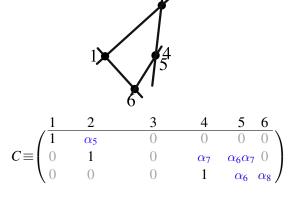
'Brid	'Bridge' Decomposition									
1	2	3	4	5	6					
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	♦	→ 7	♦	$ \begin{array}{c} \downarrow & \tau \\ 10\} \\ 10\} \\ 10\} \\ (23) \\ 10\} \\ (12) \end{array} $					
$f_1 \{ 5 \}$	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$					
f ₂ {5	6	3	7	8	$\frac{10}{10}$ (12)					
f_3 {6	5	3	7	8	$10\}_{(2.4)}^{(1.2)}$					
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $					
f_5 {7	6	3	5	8	$10\}_{(4.5)}$					
f_6 {7	6	3	8	5	$10\}_{(24)}^{(43)}$					
f_7 {7	8		6	5	$10\}_{(46)}^{(24)}$					
f_8 {7	8	3	10	5	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ 6 \end{array} $					



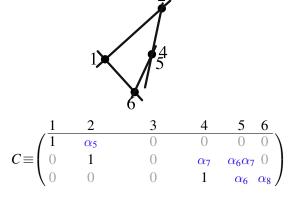
'Brid	ge'	De	eco	mŗ	osition
1					6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	↓	↓ 7	γ	$\downarrow \tau$
$f_1 \{ 5 \}$		6	7	8	$\frac{10}{10}$ (12)
f_2 {5	6	3	7	8	10 $\{(23)$ $\{(23)\}$ $\{(12)\}$
f_3 {6	5	3	7		101(12)
f_4 {6	7	3	5	8	$10^{10}(24)$ $10^{10}(12)$
f_5 {7	6	3	5	8	10 (12) 10 (45)
f_6 {7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7 {7	8	3	6	5	10 (24) 10 (46)
f_8 {7	8	3	10	5	6 } (40)



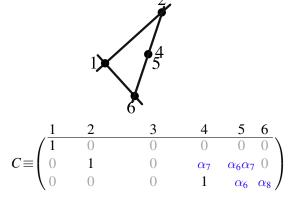
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{ 5 \}$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$
<i>f</i> ₂ { 5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(45)}$
f_6 {7	6	3	8	5	$10\}(2.4)$
f_7 {7	8	3	6	5	(4 0)
$f_8 \{7$	8	3	10	5	6 } (10)



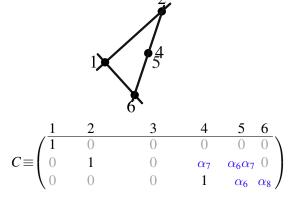
'Brid	ge'	De	eco	mŗ	osit	ion
1	2	3	4	5	6	_
$f_0 \left\{\stackrel{\downarrow}{3}\right\}$	↓ 5	♦	↓ 7	γ	↓ 10}	au
$f_1 \{5$		6	7	8	10)	(12)
f_2 {5	6	3	7		10}	(23)
f_3 {6	5	3	7		10}	(12)
f_4 {6	7	3	5	8	10}	(24)
f_5 {7	6	3	5	8	10)	$\begin{array}{c} (12) \\ (45) \end{array}$
f_6 {7	6	3	8	5	10}	(24)
f_7 {7	8	3	6	5	10	(46)
f_8 {7	8	3	10	5	6 }	(+0)



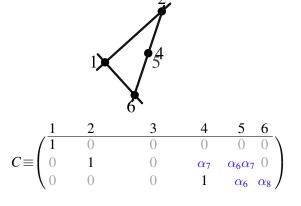
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{ 5 \}$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$
<i>f</i> ₂ { 5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(45)}$
f_6 {7	6	3	8	5	$10\}(2.4)$
f_7 {7	8	3	6	5	(4 0)
$f_8 \{7$	8	3	10	5	6 } (10)



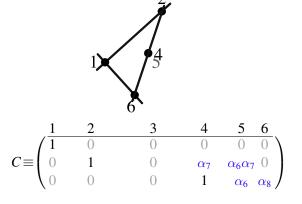
'Brid	ge'	De	eco	mp	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	$\overset{\downarrow}{6}$	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
f_1 {5		6	7	8	10 (23)
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {6	5	3	7		$10\}_{(24)}$
f_4 {6	7	3	5		$\frac{10}{10}$ (12)
f_5 {7	6	3	5		$10\}_{(4.5)}$
$f_6 \{7$	6	3	8	5	$10\}_{(24)}$
$f_7 \{ 7 \}$	8	3	6	5	(46)
$f_8 \ \{7$	8	3	10	5	6 }



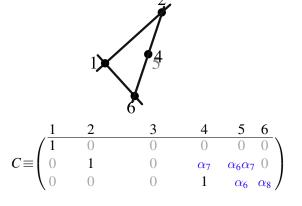
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{\stackrel{\downarrow}{3}\right\}$	5	6	† 7	8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10\} \\ 10\} \\ \end{array} $
$f_1 \{ 5 \}$	3	6	7	_	-~ J/ ^ 2\
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {6	5	3	7		$10\}_{(24)}$
f_4 { 6	7	3	5		$\{10\}_{(1,2)}$
f_5 {7	6	3	5	8	$10\}_{(4.5)}$
f_6 {7	6	3	8	5	$10\}_{(24)}$
f_7 {7	8	3	6	5	100/(46)
f_8 {7	8	3	10	5	6



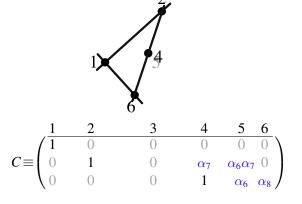
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{\stackrel{\downarrow}{3}\right\}$	5	6	† 7	8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10\} \\ 10\} \\ \end{array} $
$f_1 \{ 5 \}$	3	6	7	_	-~ J/ ^ 2\
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {6	5	3	7		$10\}_{(24)}$
f_4 { 6	7	3	5		$\{10\}_{(1,2)}$
f_5 {7	6	3	5	8	$10\}_{(4.5)}$
f_6 {7	6	3	8	5	$10\}_{(24)}$
f_7 {7	8	3	6	5	100/(46)
f_8 {7	8	3	10	5	6



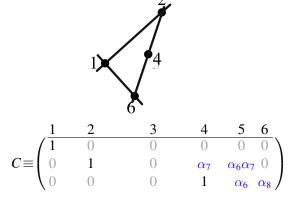
'Brid	ge'	De	eco	mŗ	osition
1					6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	↓	↓ 7	γ	$\downarrow \tau$
$f_1 \{ 5 \}$		6	7	8	$\frac{10}{10}$ (12)
f_2 {5	6	3	7	8	10 $\{(23)$ $\{(23)\}$ $\{(12)\}$
f_3 {6	5	3	7		101(12)
f_4 {6	7	3	5	8	$10^{10}(24)$ $10^{10}(12)$
f_5 {7	6	3	5	8	10 (12) 10 (45)
f_6 {7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7 {7	8	3	6	5	10 (24) 10 (46)
f_8 {7	8	3	10	5	6 } (40)



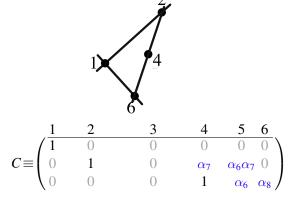
'Brid	ge'	De	eco	mp	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	$\overset{\downarrow}{6}$	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
f_1 {5		6	7	8	10 (23)
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {6	5	3	7		$10\}_{(24)}$
f_4 {6	7	3	5		$\frac{10}{10}$ (12)
f_5 {7	6	3	5		$10\}_{(4.5)}$
$f_6 \{7$	6	3	8	5	$10\}_{(24)}$
$f_7 \{ 7 \}$	8	3	6	5	(46)
$f_8 \ \{7$	8	3	10	5	6 }



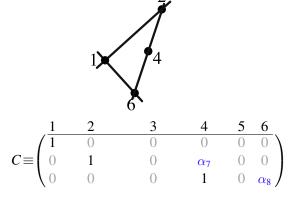
'Brid	ge'	De	eco	mŗ	osition
1					6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	↓	↓ 7	γ	$\downarrow \tau$
$f_1 \{ 5 \}$		6	7	8	$\frac{10}{10}$ (12)
f_2 {5	6	3	7	8	10 $\{(23)$ $\{(23)\}$ $\{(12)\}$
f_3 {6	5	3	7		101(12)
f_4 {6	7	3	5	8	$10^{10}(24)$ $10^{10}(12)$
f_5 {7	6	3	5	8	10 (12) 10 (45)
f_6 {7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7 {7	8	3	6	5	10 (24) 10 (46)
f_8 {7	8	3	10	5	6 } (40)



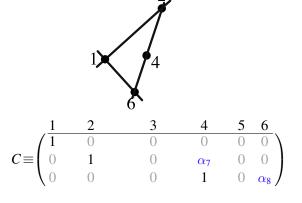
'Brid	lge'	De	ecoi	mŗ	osition
1	2	3	4	5	6 τ
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	6	↓ 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {5		6	7	8	10 (23) (23)
<i>f</i> ₂ { 5	6	3	7	8	$10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7		$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5		$10\}_{(12)}^{(24)}$
f_5 {7	6	3	5		10 (45)
$f_6 \{ 7 \}$	6	3	8	5	$10\}_{(24)}$
$f_7 \{ 7 \}$	8	3	6	5	10 (46)
$f_8 \{7$	8	3	10	5	6 }



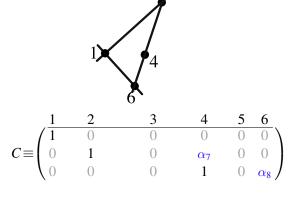
'Brid	lge'	De	ecoi	mŗ	osition
1	2	3	4	5	6 τ
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	6	↓ 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {5		6	7	8	10 (23) (23)
<i>f</i> ₂ { 5	6	3	7	8	$10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7		$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5		$10\}_{(12)}^{(24)}$
f_5 {7	6	3	5		10 (45)
$f_6 \{ 7 \}$	6	3	8	5	$10\}_{(24)}$
$f_7 \{ 7 \}$	8	3	6	5	10 (46)
$f_8 \{7$	8	3	10	5	6 }



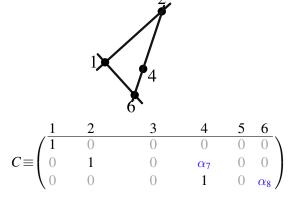
'Bridg	ge'	De	eco:	mŗ	osit	ion
1	2	3	4	5	6	_
$f_0 \left\{\stackrel{\downarrow}{3}\right\}$	↓ 5	♦	→ 7	♦	↓ 10}	τ
$f_1 \{ 5 \}$	3	6			10}	(12)
f_2 {5	6	3	7		10	(23)
f_3 {6	5	3	7	8	10	(12)
f_4 {6	7	3	5	8	10}	(24) (12)
f_5 {7	6	3	5	8	10	(45)
f_6 {7	6	3	8	5	10}	(24)
f_7 {7	8		6	5	10)	(46)
$f_8 \{7$	8	3	10	5	6 }	(.0)



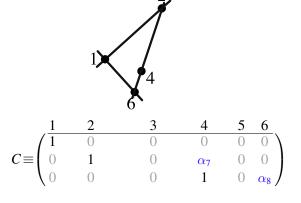
'Bridg	ge'	De	eco	mŗ	osit	ion
1	2	3		5	6	
$f_0 \left\{\stackrel{\downarrow}{3}\right\}$	↓ 5	↓ 6	↓ 7	♦	↓ 10}	au (12)
$f_1 \{ 5 \}$		6	7	8		(12) (23)
f ₂ {5	6	3	7	8	10	(23) (12)
f_3 {6	5	3	7		10}	(24)
f_4 {6	7	3	5		10}	(12)
f_5 {7	6	3	5		10}	(45)
$f_6 \{7\}$	6	3	8	5	10}	(24)
$f_7 \{ 7 \}$	8	3	6	5	10)	(46)
f_8 {/	8	3	10	5	6 }	



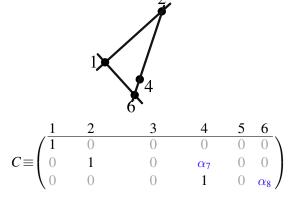
'Bridg	ge'	De	eco	mŗ	osition
1	2	3		5	6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	↓ 6		♦	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
$f_1 \{ 5 \}$		6	7	8	10 (23) (23)
f ₂ {5	6	3	7	8	10 (12) (12)
f_3 {6	5	3	7		10 ₍₂₄₎
f_4 {6	7	3	5		$\frac{10}{(1.2)}$
$f_5 \{7$	6	3	5	8	$\frac{10}{(4.5)}$
f_6 {7	6	3	8	5	$10\}_{(24)}$
$f_7 \{7$	8	3	6	5	$\frac{10}{(46)}$
$f_8 \{7$	8	3	10	5	6 } (10)



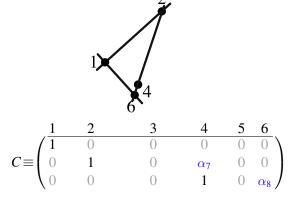
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	6	† 7	8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10 \\ 10 \end{array} $
$f_1 \{ 5 \}$	3	6	7	8	10 (23)
<i>f</i> ₂ { 5	6	3	7	8	$10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7		$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5		$10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5		$10\}_{(45)}^{(12)}$
f_6 {7	6	3	8	5	(2.4)
$f_7 \{7\}$	8		6	5	(46)
$f_8 \{ 7 \}$	8	3	10	5	6 }



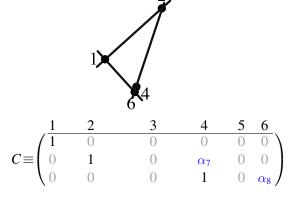
'Brid	ge'	De	eco	mp	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	$\overset{\downarrow}{6}$	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
f_1 {5		6	7	8	10 (23)
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {6	5	3	7		$10\}_{(24)}$
f_4 {6	7	3	5		$\frac{10}{10}$ (12)
f_5 {7	6	3	5		$10\}_{(4.5)}$
$f_6 \{7$	6	3	8	5	$10\}_{(24)}$
$f_7 \{ 7 \}$	8	3	6	5	(46)
$f_8 \ \{7$	8	3	10	5	6 }



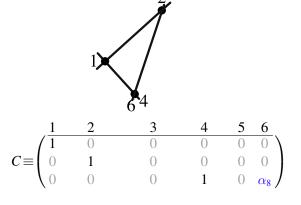
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{\stackrel{\downarrow}{3}\right\}$	5	6	† 7	8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10\} \\ 10\} \\ \end{array} $
$f_1 \{ 5 \}$	3	6	7	_	-~ J/ ^ 2\
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {6	5	3	7		$10\}_{(24)}$
f_4 { 6	7	3	5		$\{10\}_{(1,2)}$
f_5 {7	6	3	5	8	$10\}_{(4.5)}$
f_6 {7	6	3	8	5	$10\}_{(24)}$
f_7 {7	8	3	6	5	100/(46)
f_8 {7	8	3	10	5	6



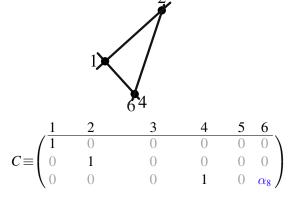
'Brid	lge'	De	ecoi	mŗ	osition
1	2	3	4	5	6 τ
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	6	↓ 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {5		6	7	8	10 (23) (23)
<i>f</i> ₂ { 5	6	3	7	8	$10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7		$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5		$10\}_{(12)}^{(24)}$
f_5 {7	6	3	5		10 (45)
$f_6 \{ 7 \}$	6	3	8	5	$10\}_{(24)}$
$f_7 \{ 7 \}$	8	3	6	5	10 (46)
$f_8 \{7$	8	3	10	5	6 }



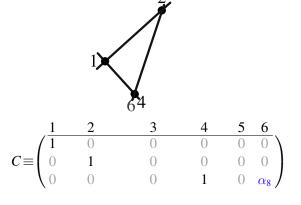
'Brid	ge'	De	eco	mp	osition
1	2	3	4	5	6 τ
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	$\overset{\downarrow}{6}$	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
f_1 {5		6	7	8	10 (23)
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {6	5	3	7		$10\}_{(24)}$
f_4 {6	7	3	5		$\frac{10}{10}$ (12)
f_5 {7	6	3	5		$10\}_{(4.5)}$
$f_6 \{7$	6	3	8	5	$10\}_{(24)}$
$f_7 \{ 7 \}$	8	3	6	5	(46)
$f_8 \ \{7$	8	3	10	5	6 }



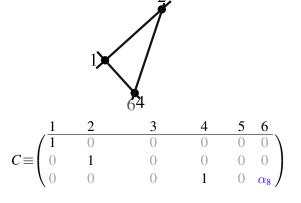
'Brid	lge'	De	ecoi	mŗ	osition
1	2	3	4	5	6 τ
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	6	↓ 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {5		6	7	8	10 (23) (23)
<i>f</i> ₂ { 5	6	3	7	8	$10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7		$10\}_{(24)}^{(12)}$
f_4 {6	7	3	5		$10\}_{(12)}^{(24)}$
f_5 {7	6	3	5		10 (45)
$f_6 \{ 7 \}$	6	3	8	5	$10\}_{(24)}$
$f_7 \{ 7 \}$	8	3	6	5	10 (46)
$f_8 \{7$	8	3	10	5	6 }



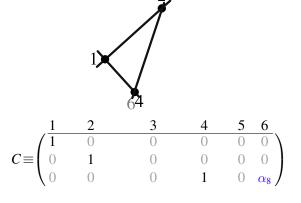
'Bridg	ge'	De	eco	mŗ	osit	ion
1	2	3		5	6	
$f_0 \left\{\stackrel{\downarrow}{3}\right\}$	↓ 5	↓ 6	↓ 7	♦	↓ 10}	au (12)
$f_1 \{ 5 \}$		6	7	8		(12) (23)
f ₂ {5	6	3	7	8	10	(23) (12)
f_3 {6	5	3	7		10}	(24)
f_4 {6	7	3	5		10}	(12)
f_5 {7	6	3	5		10}	(45)
$f_6 \{7\}$	6	3	8	5	10}	(24)
$f_7 \{ 7 \}$	8	3	6	5	10)	(46)
f_8 {/	8	3	10	5	6 }	



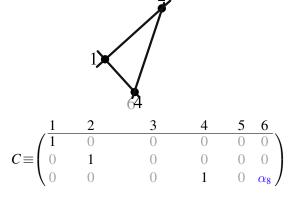
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{\stackrel{\downarrow}{3}\right\}$	5	6	† 7	8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10\} \\ 10\} \\ \end{array} $
$f_1 \{ 5 \}$	3	6	7	_	-~ J/ ^ 2\
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {6	5	3	7		$10\}_{(24)}$
f_4 { 6	7	3	5		$\{10\}_{(1,2)}$
f_5 {7	6	3	5	8	$10\}_{(4.5)}$
f_6 {7	6	3	8	5	$10\}_{(24)}$
f_7 {7	8	3	6	5	1461
f_8 {7	8	3	10	5	6



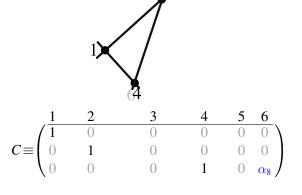
'Brid	ge'	De	eco	mŗ	osition
1					6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	↓	↓ 7	γ	$\downarrow \tau$
$f_1 \{ 5 \}$		6	7	8	$\frac{10}{10}$ (12)
f_2 {5	6	3	7	8	10 $\{(23)$ $\{(23)\}$ $\{(12)\}$
f_3 {6	5	3	7		101(12)
f_4 {6	7	3	5	8	$10^{10}(24)$ $10^{10}(12)$
f_5 {7	6	3	5	8	10 (12) 10 (45)
f_6 {7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7 {7	8	3	6	5	10 (24) 10 (46)
f_8 {7	8	3	10	5	6 } (40)



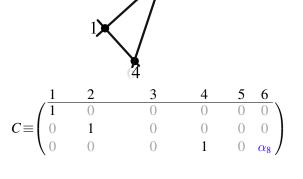
'Brid	'Bridge' Decomposition								
1	2	3	4	5	6 τ				
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	$\overset{\downarrow}{6}$	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $				
f_1 {5		6	7	8	10 (23)				
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)				
f_3 {6	5	3	7		$10\}_{(24)}$				
f_4 {6	7	3	5		$\frac{10}{10}$ (12)				
f_5 {7	6	3	5		$10\}_{(4.5)}$				
$f_6 \{7$	6	3	8	5	$10\}_{(24)}$				
$f_7 \{ 7 \}$	8	3	6	5	(46)				
$f_8 \ \{7$	8	3	10	5	6 }				



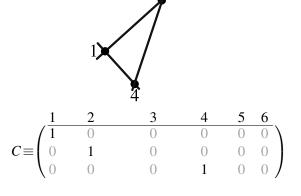
'Bri	'Bridge' Decomposition								
	1 2	3	4	5	6				
f_0 {	\downarrow	$\overset{\downarrow}{6}$	$\stackrel{\downarrow}{7}$	♦	$\downarrow \tau$ 10 $\rbrace_{(1,2)}$				
f_1 {:	5 3	6	7		101(12)				
f_2 {		3	7		$10^{(23)}$				
f_3 {		3	7	8	$10^{10}(12)$ $10^{10}(12)$				
f_4 {		3	5	8	$10^{10}(24)$ $10^{10}(12)$				
f_5 {	7 6	3	5	8	10 (12) 10 (45)				
f_6 {		3	8	5	$10\}_{(24)}^{(43)}$				
f_7 {	7 8	3	6	5	$10\}_{(46)}^{(24)}$				
f_8 {	7 8	3	10	5	6 }(40)				



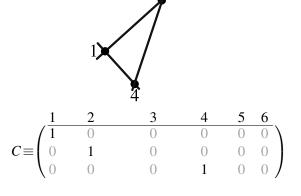
'Br	'Bridge' Decomposition								
	1 ↓	2 ↓	3 ↓	4 ↓	5	6 ↓ τ			
f_0 {	* [3	5	6		8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$			
f_1 {		3	6	7	8	10 (23)			
f_2 {	5	6	3	7	8	$\frac{10}{10}$ (12)			
f_3 {	6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$			
f_4 {	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$			
f_5 {	[7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$			
f_6 {	[7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$			
f_7 {	[7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8 {	[7	8	3	10	5	6			



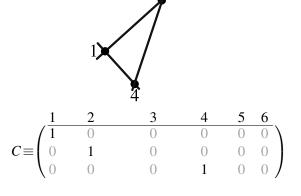
'Brid	'Bridge' Decomposition								
1	2	3	4	5	6 τ				
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	6	↓ 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $				
f_1 {5		6	7	8	10 (23) (23)				
<i>f</i> ₂ { 5	6	3	7	8	$10\}_{(1,2)}^{(2,3)}$				
f_3 {6	5	3	7		$10\}_{(24)}^{(12)}$				
f_4 {6	7	3	5		$10\}_{(12)}^{(24)}$				
f_5 {7	6	3	5		10 (45)				
$f_6 \{ 7 \}$	6	3	8	5	$10\}_{(24)}$				
$f_7 \{ 7 \}$	8	3	6	5	10 (46)				
$f_8 \{7$	8	3	10	5	6 }				



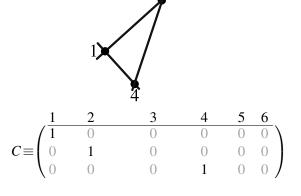
'Bri	dge	' De	eco:	mŗ	osit	ion
	1 2		4		6	au
f_0 {	$\stackrel{\downarrow}{3} \stackrel{\downarrow}{5}$	6	↓ 7	8	↓ 10}	(12)
f_1 {:			7	8	10}	(23)
f_2 {	5 6	3	7	8	10}	(23)
f_3 {	6 5	3	7	8	10}	(12)
f_4 {	6 7	3	5	8	10}	(24) (12)
f_5 {	7 6	3	5	8	10}	
f_6 {	7 6	3	8	5	10}	(45)
f_7 {	7 8	3	6	5	10}	(24)
f_8 {	7 8	3	10	5	6 }	(40)



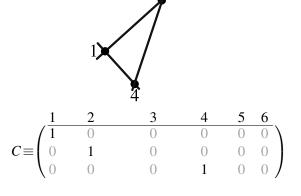
'Bri	dge	' De	eco:	mŗ	osit	ion
	1 2		4		6	au
f_0 {	$\stackrel{\downarrow}{3} \stackrel{\downarrow}{5}$	6	↓ 7	8	↓ 10}	(12)
f_1 {:			7	8	10}	(23)
f_2 {	5 6	3	7	8	10}	(23)
f_3 {	6 5	3	7	8	10}	(12)
f_4 {	6 7	3	5	8	10}	(24) (12)
f_5 {	7 6	3	5	8	10}	
f_6 {	7 6	3	8	5	10}	(45)
f_7 {	7 8	3	6	5	10}	(24)
f_8 {	7 8	3	10	5	6 }	(40)



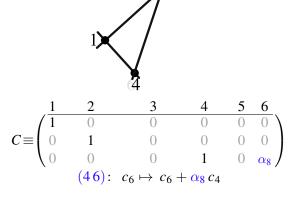
'Bri	dge	' De	eco:	mŗ	osit	ion
	1 2		4		6	au
f_0 {	$\stackrel{\downarrow}{3} \stackrel{\downarrow}{5}$	6	↓ 7	8	↓ 10}	(12)
f_1 {:			7	8	10}	(23)
f_2 {	5 6	3	7	8	10}	(23)
f_3 {	6 5	3	7	8	10}	(12)
f_4 {	6 7	3	5	8	10}	(24) (12)
f_5 {	7 6	3	5	8	10}	
f_6 {	7 6	3	8	5	10}	(45)
f_7 {	7 8	3	6	5	10}	(24)
f_8 {	7 8	3	10	5	6 }	(40)



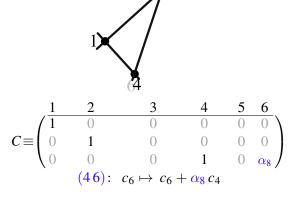
'Bri	dge	' De	eco:	mŗ	osit	ion
	1 2		4		6	au
f_0 {	$\stackrel{\downarrow}{3} \stackrel{\downarrow}{5}$	6	↓ 7	8	↓ 10}	(12)
f_1 {:			7	8	10}	(23)
f_2 {	5 6	3	7	8	10}	(23)
f_3 {	6 5	3	7	8	10}	(12)
f_4 {	6 7	3	5	8	10}	(24) (12)
f_5 {	7 6	3	5	8	10}	
f_6 {	7 6	3	8	5	10}	(45)
f_7 {	7 8	3	6	5	10}	(24)
f_8 {	7 8	3	10	5	6 }	(40)



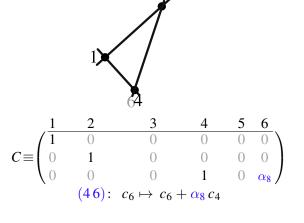
'Bri	dge	' De	eco:	mŗ	osit	ion
	1 2		4		6	au
f_0 {	$\stackrel{\downarrow}{3} \stackrel{\downarrow}{5}$	6	↓ 7	8	↓ 10}	(12)
f_1 {:			7	8	10}	(23)
f_2 {	5 6	3	7	8	10}	(23)
f_3 {	6 5	3	7	8	10}	(12)
f_4 {	6 7	3	5	8	10}	(24) (12)
f_5 {	7 6	3	5	8	10}	
f_6 {	7 6	3	8	5	10}	(45)
f_7 {	7 8	3	6	5	10}	(24)
f_8 {	7 8	3	10	5	6 }	(40)



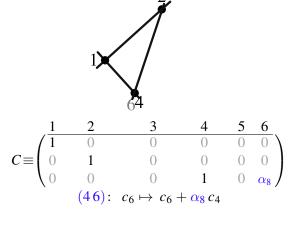
'Bridg	ge'	De	ecoi	mŗ	osit	ion
1	2	3	4	5	6	au
<i>t</i> o 3.3	↓ 5	6	†	8	[↓] 10}	au (12)
$f_1 \{ 5 \}$	3	6	7	8	10}	(12) (23)
<i>f</i> ₂ { 5	6	3	7		10}	(23)
f_3 {6	5	3	7		10}	(24)
f_4 { 6	7	3	5		$10\}$	(27)
f_5 {7	6	3	5	8	$10\}$	(45)
f_6 {7	6	3	8	5	10}	(1 3) (24)
f_7 {7	8	3	6	5	10)	(44) (46)
f_8 {7	8	3	10	5	6 }	(+0)



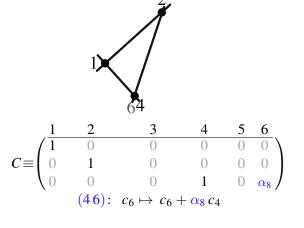
'Brid	ge'	De	eco	mp	osition
f_0 {3	2 ↓ 5	3 ↓ 6	4 ↓ 7	5 ↓ 8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10 \\ 10 \\ 10 \\ 10 \end{array} $
f_1 {5 f_2 {5	3	6 3	7 7	8	10 (23) (23) (12)
f_3 {6 f_4 {6	5 7	3	7 5	8	10 $\{(24)$ $\{(12)\}$ $\{$
$f_5 \ \{7 \ f_6 \ \{7 \ f_6 \ \}\}$	6	3	5	8	$10\}_{(45)}^{(12)}$
f_7 {7 f_8 {7	8	3 3	6 10		$10\}_{(46)}^{(24)}$



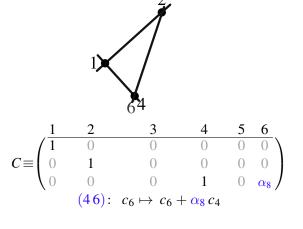
'Brio	lge'	De	ecoi	mp	osition
f_0 {3		3 ↓ 6	4 ↓ 7	8	$\downarrow \tau$ 10 ₁₀
$f_1 \ \{5\}$ $f_2 \ \{5\}$ $f_3 \ \{6\}$	6	633	7 7 7	8	$ \begin{array}{c} 10 \\ 10 \\ 10 \\ 10 \end{array} $ $ \begin{array}{c} (23) \\ 10 \end{array} $
$f_4 \ \{ 6 \ f_5 \ \{ 7 \ \} \}$	7 6	3	5 5	8 8	$ \begin{array}{c} 10 \\ (12) \\ 10 \\ (45) \end{array} $
$f_6 \ \{7\}$ $f_7 \ \{7\}$ $f_8 \ \{7\}$		3 3 3	8 6 10	555	10 $\{(24)\}$ $\{(46)\}$



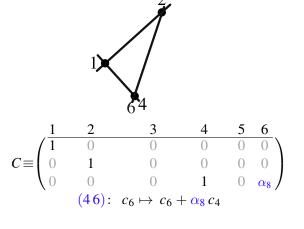
'Brid	ge'	De	ecoi	mp	osition
1 ↓ f ₀ {3	2 ↓ 5	3 ↓ 6	4 ↓ 7	5 ↓ 8	6 ↓ τ 10} _(1.2)
f_1 {5 f_2 {5	3	6 3	7 7	8	10 $\{(23)$ $\{(23)\}$ $\{(23)\}$ $\{(23)\}$ $\{(23)\}$ $\{(23)\}$ $\{(23)\}$ $\{(23)\}$ $\{(23)\}$ $\{(23)\}$
f_3 {6 f_4 {6	5	3	7 5	8	$ \begin{array}{c} (12) \\ 10 \\ (24) \\ 10 \\ (12) \end{array} $
$f_5 \ \{7 \ f_6 \ \{7 \ \}\}$	6	3	5	8 5	$ \begin{array}{c} 10 \\ 10 \\ 10 \\ 45 \end{array} $
$f_{7} \{7\}$	8	3 3	6 10	5 5	10 $\begin{cases} (24) \\ (46) \end{cases}$



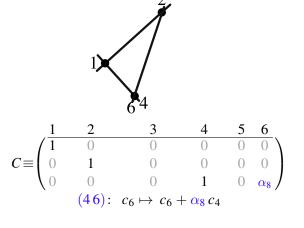
'Brio	'Bridge' Decomposition								
1	. 2	3	4			au			
$f_0 \left\{ 3 \right\}$	√5	6	$\overset{\downarrow}{7}$	8	↓ 10}	τ (12)			
$f_1 $ {5	3		7		10	(23)			
f_2 {5		3	7		10}	(12)			
$f_3 \{ \epsilon \}$		3	7		10}	(24)			
$f_4 \left\{ e \right\}$		3	5		10	(12)			
$f_5 \ \{7, f_6 \ \}$		3	5 8	5	10 }	(45)			
$f_7 \{ 7 \}$		J	6	5	,	(24)			
f_8 {7	8	3	10	5	-)	(46)			



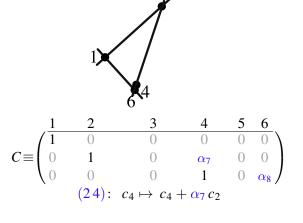
'Brio	dge'	De	eco:	mŗ	osit	tion
1	2	3	4	5	6	au
f_0 {	↓5	$\overset{\downarrow}{6}$	↓ 7	8	10}	
$f_1 \{ 5 \}$		6	7	8	10}	(23)
f_2 {5	6	3	7	8	10}	(23)
$f_3 \{ \epsilon$	5	3	7	8	10}	(24)
f_4 {	7	3	5	8	10}	_ ` /
f_5 {7	6	3	5	8	10}	(12) (45)
f_6 {7	6	3	8	5	10}	(24)
f_7 {7	8	3	6	5	10}	(44)
f_8 {7	8	3	10	5	6 }	(40)



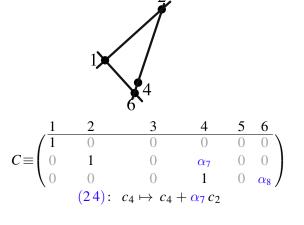
'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	$\overset{\downarrow}{6}$	₇	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5	3	6	7	8	$\frac{10}{10}$ (23)
<i>f</i> ₂ { 5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$\{10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10^{10}(12)$ $10^{10}(12)$
f_6 {7	6	3	8	5	$10 \ (45) \ (10) \ (24)$
f_7 {7	8	3	6	5	$10^{10}(24)$ $10^{10}(46)$
f_8 {7	8	3	10	5	6 } (4 0)



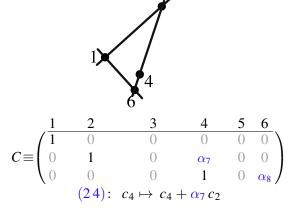
'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	$\overset{\downarrow}{6}$	₇	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5	3	6	7	8	$\frac{10}{10}$ (23)
<i>f</i> ₂ { 5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$\{10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10^{10}(12)$ $10^{10}(12)$
f_6 {7	6	3	8	5	$10 \ (45) \ (10) \ (24)$
f_7 {7	8	3	6	5	$10^{10}(24)$ $10^{10}(46)$
f_8 {7	8	3	10	5	6 } (4 0)



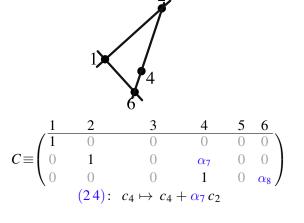
'Brid	ge'	De	eco	mŗ	osit	ion
1	2	3	4			au
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	↓ 5	$\overset{\downarrow}{6}$	†	8	↓ 10}	au (12)
f_1 {5	3	6	7		10}	(23)
<i>f</i> ₂ { 5	6	3	7		10}	(23)
<i>f</i> ₃ { 6	5	3	7		10}	(24)
f_4 {6	7	3	5		10}	(12)
$f_5 \{ 7 \}$	6	3	5		10}	(45)
$f_6 \{ 7 \}$	6	3	8	5	- 0	(24)
$f_7 \{ 7 \}$	8	3	6	5		(46)
J8 {/	8	3	10	5	0 }	



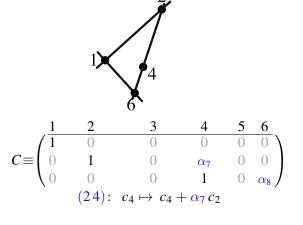
'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	$\overset{\downarrow}{6}$	₇	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5	3	6	7	8	$\frac{10}{10}$ (23)
<i>f</i> ₂ { 5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$\{10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10^{10}(12)$ $10^{10}(12)$
f_6 {7	6	3	8	5	$10 \ (45) \ (10) \ (24)$
f_7 {7	8	3	6	5	$10^{10}(24)$ $10^{10}(46)$
f_8 {7	8	3	10	5	6 } (4 0)



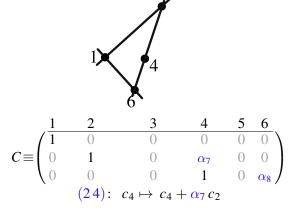
'Brid	ge'	De	eco	mŗ	osit	ion
1	2	3	4			au
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	↓ 5	$\overset{\downarrow}{6}$	†	8	↓ 10}	au (12)
f_1 {5	3	6	7		10}	(23)
<i>f</i> ₂ { 5	6	3	7		10}	(23)
<i>f</i> ₃ { 6	5	3	7		10}	(24)
f_4 {6	7	3	5		10}	(12)
$f_5 \{ 7 \}$	6	3	5		10}	(45)
$f_6 \{ 7 \}$	6	3	8	5	- 0	(24)
$f_7 \{ 7 \}$	8	3	6	5		(46)
J8 {/	8	3	10	5	0 }	



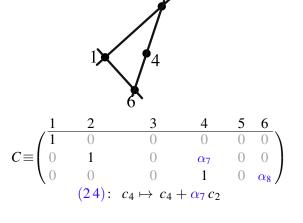
'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	$\overset{\downarrow}{6}$	₇	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5	3	6	7	8	$\frac{10}{10}$ (23)
<i>f</i> ₂ { 5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$\{10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10^{10}(12)$ $10^{10}(12)$
f_6 {7	6	3	8	5	$10 \ (45) \ (10) \ (24)$
f_7 {7	8	3	6	5	$10^{10}(24)$ $10^{10}(46)$
f_8 {7	8	3	10	5	6 } (4 0)



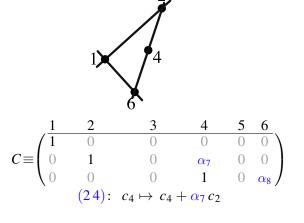
'Brid	ge'	De	eco	mŗ	osit	ion
1	2	3	4			au
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	↓ 5	$\overset{\downarrow}{6}$	†	8	↓ 10}	au (12)
f_1 {5	3	6	7		10}	(23)
<i>f</i> ₂ { 5	6	3	7		10}	(23)
<i>f</i> ₃ { 6	5	3	7		10}	(24)
f_4 {6	7	3	5		10}	(12)
$f_5 \{ 7 \}$	6	3	5		10}	(45)
$f_6 \{ 7 \}$	6	3	8	5	- 0	(24)
$f_7 \{ 7 \}$	8	3	6	5		(46)
J8 {/	8	3	10	5	0 }	



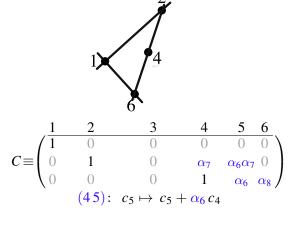
'Brid	ge'	De	eco	mŗ	osit	ion
1	2	3	4			au
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	↓ 5	$\overset{\downarrow}{6}$	†	8	↓ 10}	au (12)
f_1 {5	3	6	7		10}	(23)
<i>f</i> ₂ { 5	6	3	7		10}	(23)
<i>f</i> ₃ { 6	5	3	7		10}	(24)
f_4 {6	7	3	5		10}	(12)
$f_5 \{ 7 \}$	6	3	5		10}	(45)
$f_6 \{ 7 \}$	6	3	8	5	- 0	(24)
$f_7 \{ 7 \}$	8	3	6	5		(46)
J8 {/	8	3	10	5	0 }	



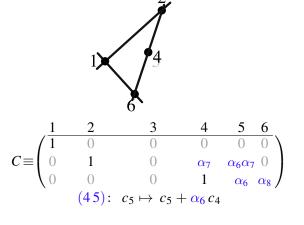
'Brid	ge'	De	eco	mŗ	osit	ion
1	2	3	4			au
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	↓ 5	$\overset{\downarrow}{6}$	†	8	↓ 10}	au (12)
f_1 {5	3	6	7		10}	(23)
<i>f</i> ₂ { 5	6	3	7		10}	(23)
<i>f</i> ₃ { 6	5	3	7		10}	(24)
f_4 {6	7	3	5		10}	(12)
$f_5 \{ 7 \}$	6	3	5		10}	(45)
$f_6 \{ 7 \}$	6	3	8	5	- 0	(24)
$f_7 \{ 7 \}$	8	3	6	5		(46)
J8 {/	8	3	10	5	0 }	



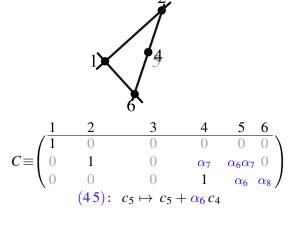
'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	$\overset{\downarrow}{6}$	₇	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5	3	6	7	8	$\frac{10}{10}$ (23)
<i>f</i> ₂ { 5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$\{10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10^{10}(12)$ $10^{10}(12)$
f_6 {7	6	3	8	5	$10 \ (45) \ (10) \ (24)$
f_7 {7	8	3	6	5	$10^{10}(24)$ $10^{10}(46)$
f_8 {7	8	3	10	5	6 } (4 0)



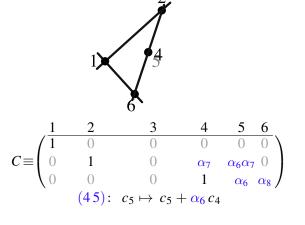
'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	$\overset{\downarrow}{6}$	₇	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5	3	6	7	8	$\frac{10}{10}$ (23)
<i>f</i> ₂ { 5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$\{10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10^{10}(12)$ $10^{10}(12)$
f_6 {7	6	3	8	5	$10 \ (45) \ (10) \ (24)$
f_7 {7	8	3	6	5	$10^{10}(24)$ $10^{10}(46)$
f_8 {7	8	3	10	5	6 } (4 0)



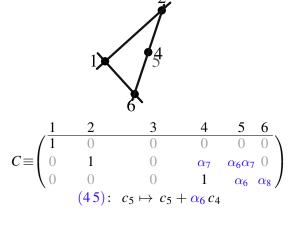
'Bri	'Bridge' Decomposition								
	1 2		4		6	au			
f_0 {	$\stackrel{\downarrow}{3} \stackrel{\downarrow}{5}$	6	↓ 7	8	↓ 10}	(12)			
f_1 {:			7	8	10}	(23)			
f_2 {	5 6	3	7	8	10}	(23)			
f_3 {	6 5	3	7	8	10}	(12)			
f_4 {	6 7	3	5	8	10}	(24) (12)			
f_5 {	7 6	3	5	8	10}				
f_6 {	7 6	3	8	5	10}	(45)			
f_7 {	7 8	3	6	5	10}	(24)			
f_8 {	7 8	3	10	5	6 }	(40)			



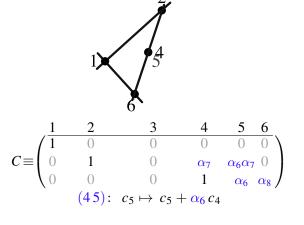
'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	$\overset{\downarrow}{6}$	₇	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5	3	6	7	8	$\frac{10}{10}$ (23)
<i>f</i> ₂ { 5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$\{10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10^{10}(12)$ $10^{10}(12)$
f_6 {7	6	3	8	5	$10 \ (45) \ (10) \ (24)$
f_7 {7	8	3	6	5	$10^{10}(24)$ $10^{10}(46)$
f_8 {7	8	3	10	5	6 } (4 0)



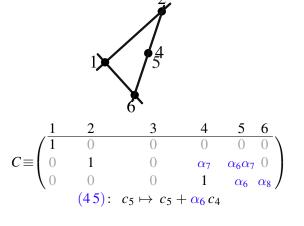
'Brio	dge'	De	eco	mŗ	osit	ion
1	. 2	3	4	5	6	au
f_0 {	\$ 5	$\overset{\downarrow}{6}$	$\check{7}$	8	↓ 10}	(12)
f_1 {5	3	6	7	8	$10\}$	(23)
f_2 {5	6	3	7	8	$10\}$	(23) (13)
f_3 { ϵ	5	3	7	8	10	(12) (24)
f_4 { ϵ	7	3	5	8	10	(44) (1 2)
f_5 {7	6	3	5	8	10	(12) (45)
f_6 {7	6	3	8	5	$10\}$	(43)
f_7 {7	8	3	6	5	10	(44) (46)
f_8 {7	8	3	10	5	6 }	(+0)



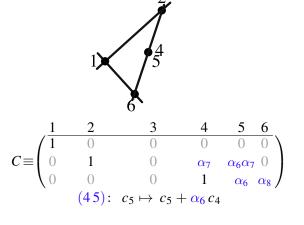
'Brid	'Bridge' Decomposition								
1	2	3	4	5	6	au			
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	6	↓ 7	8	10}				
f_1 {5		6	7	8	10}	(23)			
$f_2 \{ 5 \}$	6	3	7	8	10}	(23)			
f_3 {6	5	3	7	8	10}	(24)			
f_4 {6	7	3	5	8	10}				
f_5 {7	6	3	5	8	10}	(12)			
f_6 {7	6	3	8	5	10}	(45)			
f_7 {7	8	3	6	5	10}	(24)			
f_8 {7	8	3	10	5	6 }	(40)			



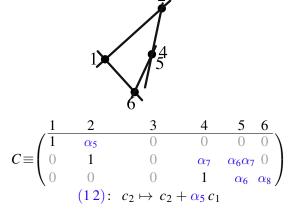
'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	$\overset{\downarrow}{6}$	₇	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5	3	6	7	8	$\frac{10}{10}$ (23)
<i>f</i> ₂ { 5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$\{10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10^{10}(12)$ $10^{10}(12)$
f_6 {7	6	3	8	5	$10 \ (45) \ (10) \ (24)$
f_7 {7	8	3	6	5	$10^{10}(24)$ $10^{10}(46)$
f_8 {7	8	3	10	5	6 } (4 0)



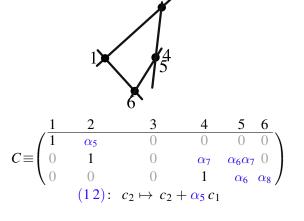
'Br	id	ge'	Dε	ecoi	mp	osition
	1	2	3	4	5	6 ↓ τ
f_0 {	3	↓ 5	6	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
f_1 {		3	6	7	8	10 (23)
f_2 {	5	6	3	7	8	10 (23) (12)
f_3 {	6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$
f_4 {	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5 {	7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$
f_6 {	7	6	3	8	5	$10\}_{(24)}^{(43)}$
f_7 {	7	8	3	6	5	$10\}_{(46)}^{(24)}$
f_8 {	7	8	3	10	5	6



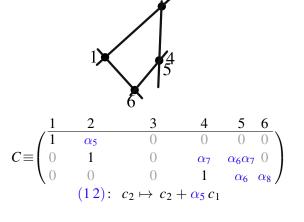
'Brid	lge'	De	eco:	mŗ	osition
1	2	3	4 ↓		6 ↓ τ
$f_0 \left\{ \stackrel{Y}{3} \right\}$	↓ 5	$\overset{\downarrow}{6}$	$\check{7}$	8	$\frac{10}{10}$ (12)
<i>f</i> ₁ {5	3	6	7	8	$\frac{10}{(23)}$
f ₂ {5	6	3	7	8	$10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	10 (24)
f_4 {6	7	3	5	8	$\frac{10}{10}$ (12)
f_5 {7	6	3	5	8	$\frac{10}{10}$ (45)
f_6 {7	6	3	8	5	10 \\ \ \ \ \ \ \
f_7 {7	8	3	6	5	10 (24) (46)
f_8 {7	8	3	10	5	6 } (40)



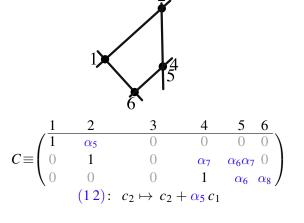
'Brid	'Bridge' Decomposition								
1	2	3	4			au			
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	↓ 5	$\overset{\downarrow}{6}$	†	8	↓ 10}	au (12)			
f_1 {5	3	6	7		10}	(23)			
<i>f</i> ₂ { 5	6	3	7		10}	(23)			
<i>f</i> ₃ { 6	5	3	7		10}	(24)			
f_4 {6	7	3	5		10}	(12)			
$f_5 \{ 7 \}$	6	3	5		10}	(45)			
$f_6 \{ 7 \}$	6	3	8	5	- 0	(24)			
$f_7 \{ 7 \}$	8	3	6	5		(46)			
J8 {/	8	3	10	5	0 }				



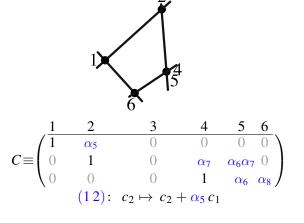
'Brid	'Bridge' Decomposition								
1	2	3	4			au			
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	↓ 5	$\overset{\downarrow}{6}$	†	8	↓ 10}	au (12)			
f_1 {5	3	6	7		10}	(23)			
<i>f</i> ₂ { 5	6	3	7		10}	(23)			
<i>f</i> ₃ { 6	5	3	7		10}	(24)			
f_4 {6	7	3	5		10}	(12)			
$f_5 \{ 7 \}$	6	3	5		10}	(45)			
$f_6 \{ 7 \}$	6	3	8	5	- 0	(24)			
$f_7 \{ 7 \}$	8	3	6	5		(46)			
J8 {/	8	3	10	5	0 }				



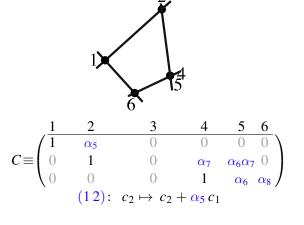
'Brid	ge'	De	eco	mp	osition
f_0 {3	2 ↓ 5	3 ↓ 6	4 ↓ 7	5 ↓ 8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10 \\ 10 \\ 10 \\ 10 \end{array} $
f_1 {5 f_2 {5	3	6 3	7 7	8	10 (23) (23) (12)
f_3 {6 f_4 {6	5 7	3	7 5	8	10 $\{(24)$ $\{(12)\}$ $\{$
$f_5 \ \{7 \ f_6 \ \{7 \ f_6 \ \}\}$	6	3	5	8	$10\}_{(45)}^{(12)}$
f_7 {7 f_8 {7	8	3 3	6 10		$10\}_{(46)}^{(24)}$



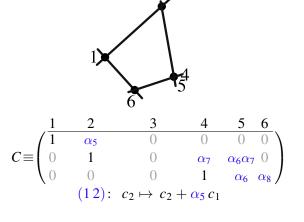
'Brid	ge'	De	eco:	mŗ	osit	ion
1	2	3	4	5	6	au
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	6	↓ 7	8	10}	
f_1 {5		6	7	8	10}	(23)
$f_2 \{ 5 \}$	6	3	7	8	10}	(23)
f_3 {6	5	3	7	8	10}	(24)
f_4 {6	7	3	5	8	10}	
f_5 {7	6	3	5	8	10}	(12)
f_6 {7	6	3	8	5	10}	(45)
f_7 {7	8	3	6	5	10}	(24)
f_8 {7	8	3	10	5	6 }	(40)



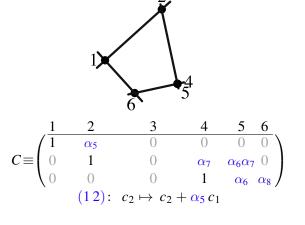
'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	$\overset{\downarrow}{6}$	₇	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5	3	6	7	8	$\frac{10}{10}$ (23)
<i>f</i> ₂ { 5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$\{10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10^{10}(12)$ $10^{10}(12)$
f_6 {7	6	3	8	5	$10 \ (45) \ (10) \ (24)$
f_7 {7	8	3	6	5	$10^{10}(24)$ $10^{10}(46)$
f_8 {7	8	3	10	5	6 } (4 0)



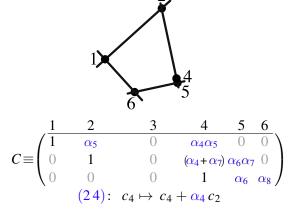
'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	$\overset{\downarrow}{6}$	₇	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5	3	6	7	8	$\frac{10}{10}$ (23)
<i>f</i> ₂ { 5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$\{10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10^{10}(12)$ $10^{10}(12)$
f_6 {7	6	3	8	5	$10 \ (45) \ (10) \ (24)$
f_7 {7	8	3	6	5	$10^{10}(24)$ $10^{10}(46)$
f_8 {7	8	3	10	5	6 } (4 0)



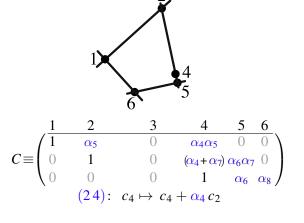
'Bridg	ge'	De	ecoi	mŗ	osition
f_0 $\begin{cases} 1 \\ \downarrow \end{cases}$	2 ↓ 5 3	3 ↓ 6 6	4 ↓	5 ↓ 8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10 \\ 10 \\ 10 \end{array} $
f ₂ {5 f ₃ {6	6 5	3	7 7	8 8	$ \begin{array}{c} 10 \\ (12) \\ 10 \\ (24) \end{array} $
f ₄ {6 f ₅ {7 f ₆ {7	7 6 6	3 3 3	5 5 8	8	$ \begin{array}{c} 10 \\ (12) \\ 10 \\ (45) \\ 10 \\ (24) \end{array} $
$f_7 \ \{7 \ f_8 \ \{7 \ $	8	3	6 10	5 5	$10\}_{(46)}^{(24)}$



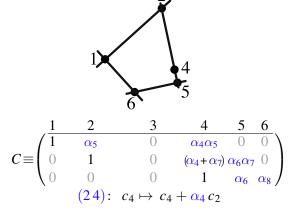
'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{\stackrel{\star}{3}\right\}$	↓ 5	$\overset{\downarrow}{6}$	₇	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5	3	6	7	8	$\frac{10}{10}$ (23)
<i>f</i> ₂ { 5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
f_3 {6	5	3	7	8	$\{10\}_{(2,4)}^{(1,2)}$
f_4 {6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10^{10}(12)$ $10^{10}(12)$
f_6 {7	6	3	8	5	$10 \ (45) \ (10) \ (24)$
f_7 {7	8	3	6	5	$10^{10}(24)$ $10^{10}(46)$
f_8 {7	8	3	10	5	6 } (4 0)



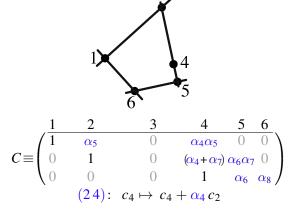
'Brid	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right\}$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{5$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$
<i>f</i> ₂ { 5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 { 6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(45)}$
f_6 {7	6	3	8	5	$10\}(2.4)$
f_7 {7	8	3	6	5	(4 0)
$f_8 \ \{7$	8	3	10	5	6 } (10)



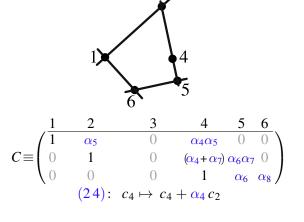
'Br	id	ge'	Dε	eco ₁	mŗ	osition
	1	2	3	4	5	6
f_0 {	→ 3	5	↓ 6	$\stackrel{\downarrow}{7}$	♦	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {			6	7	8	10 (23)
f_2 {	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {	6	5	3	7		$10\}_{(24)}$
f_4 {		7	3	5		$10\}_{(1,2)}$
f_5 {	7	6	3	5	8	$10\}_{(4.5)}$
f_6 {		6	3	8	5	$10\}_{(24)}$
f_7 {		8		6	5	(46)
f_8 {	7	8	3	10	5	6 } (10)



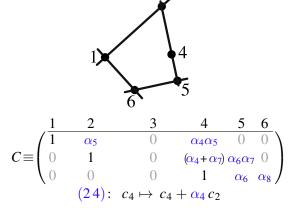
'Brio	dge'	De	eco	mp	osition
$f_0 \ \{ 5 \ f_1 \ \{ 5 \ f_2 \ \{ 5 \ f_2 \ \{ 5 \ f_2 \ \} \} \}$	2 3 5 5 3	3 ↓ 6 6 3	4 ↓	5 ↓ 8 8	$ \begin{array}{c} 6 \\ \downarrow \tau \\ 10\} \\ 10\} \\ 10\} \\ (23) \\ 10\} \\ (12) \end{array} $
$f_3 \ \{ \epsilon \}$	5 5	3	7 5	8	$10\}_{(24)}^{(12)}$
$f_5 \ \{7\}$ $f_6 \ \{7\}$ $f_7 \ \{7\}$	7 6 7 6 7 8	3 3	5 8 6	855	$ \begin{array}{c} 10 \\ (45) \\ 10 \\ (24) \\ 10 \\ (46) \end{array} $
$f_8 \{ 7 \}$	7 8	3	10	5	(40)



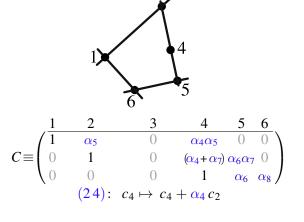
'B	rid	ge'	De	ecoi	mţ	osition
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	{5	6	3	7	8	10 (23) 10 (12)
f_3	{6	5	3	7	8	$\frac{10}{(24)}$
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	1/21/1
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



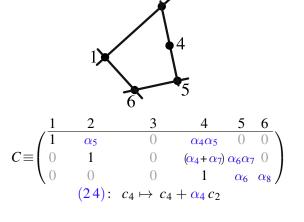
'Bı	rid	ge'	De	ecoi	mp	osition
	1	2 ↓	3	4 ↓	5	6 ↓ τ
f_0	${3 \choose 3}$	5	6	[*] 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
f_1		3	6	7	8	10 (23)
f_2	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3	{6	5	3	7	8	$10\} (24)$
f_4	6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



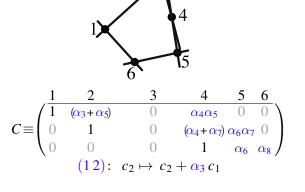
'Bı	rid	ge'	De	ecoi	mp	osition
	1	2 ↓	3	4 ↓	5	6 ↓ τ
f_0	${3 \choose 3}$	5	6	[*] 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
f_1		3	6	7	8	10 (23)
f_2	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3	{6	5	3	7	8	$10\} (24)$
f_4	6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



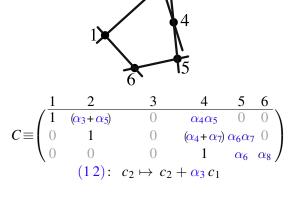
'Bı	rid	ge'	De	ecoi	mp	osition
	1	2 ↓	3	4 ↓	5	6 ↓ τ
f_0	${3 \choose 3}$	5	6	[*] 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
f_1		3	6	7	8	10 (23)
f_2	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3	{6	5	3	7	8	$10\} (24)$
f_4	6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



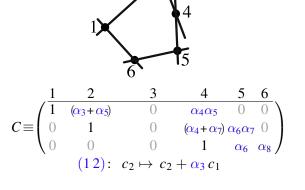
'Bı	rid	ge'	De	ecoi	mp	osition
	1	2 ↓	3	4 ↓	5	6 ↓ τ
f_0	${3 \choose 3}$	5	6	[*] 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
f_1		3	6	7	8	10 (23)
f_2	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3	{6	5	3	7	8	$10\} (24)$
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



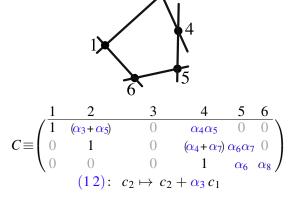
'Bı	id	ge'	Dε	ecoi	mţ	osition
	1 ↓	2 ↓	3	4	5	6 ↓ τ
f_0	3	5	6	[*] 7	8	$\frac{10}{10}$ (12)
f_1		3	6	7	8	$10\} (23)$
f_2	[5	6	3	7	8	$10\}_{(12)}^{(23)}$
f_3	[6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6



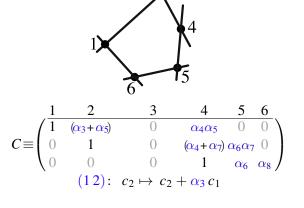
'Bridg	ge'	De	ecoi	mp	osition
1	2	3	4	5	6 τ
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right.$	5	$\overset{\downarrow}{6}$	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
f_1 {5		6	7	8	10 (23)
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {6	5	3	7	8	$\frac{10}{10}$ (24)
<i>f</i> ₄ { 6	7	3	5		$\{10\}_{(1,2)}$
f_5 {7	6	3	5	8	$\frac{10}{(45)}$
f_6 {7	6	3	8	5	$10\}_{(24)}$
f_7 {7	8	3	6	5	10 (46)
f_8 {7	8	3	10	5	6 }(40)



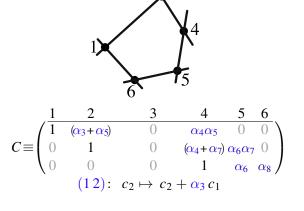
'B	rid	ge'	De	ecoi	mţ	osition
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	{5	6	3	7	8	10 (23) 10 (12)
f_3	{6	5	3	7	8	$\frac{10}{(24)}$
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	1/21/1
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



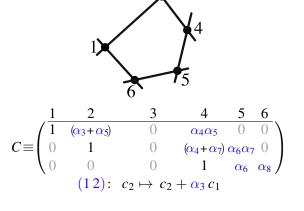
'B	rid	ge'	De	eco:	mŗ	osition
	1	2 ↓	3 ↓	4 ↓	5	6 τ
f_0	$\left\{\stackrel{\star}{3}\right\}$	5	6	[*] 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	{5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $
f_3	{6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$
f_4	6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6



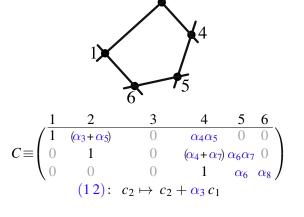
'Bı	id	ge'	Dε	ecoi	mţ	osition
	1 ↓	2 ↓	3	4	5	6 ↓ τ
f_0	3	5	6	[*] 7	8	$\frac{10}{10}$ (12)
f_1		3	6	7	8	$10\} (23)$
f_2	[5	6	3	7	8	$10\}_{(12)}^{(23)}$
f_3	[6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6



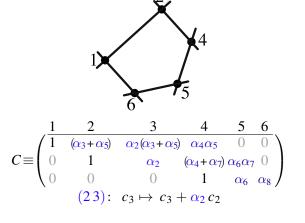
'Bridg	ge'	De	eco	mp	oosition
1	2	3	4	5	
$f_0 \left\{\stackrel{\downarrow}{3}\right\}$	5	6	↓ 7	8	$\downarrow \tau$ 10 $\rbrace_{(1,2)}$
f_1 {5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f ₂ {5	6	3	7	8	$\{10\}_{(1,2)}^{(2,3)}$
<i>f</i> ₃ { 6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$
<i>f</i> ₄ { 6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$
f_6 {7	6	3	8	5	$10\}_{(2.4)}^{(4.5)}$
f_7 {7	8	3	6	5	$10\}_{(46)}^{(24)}$
f_8 {7	8	3	10	5	6



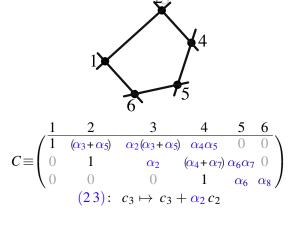
'Brid	ge'	De	ecoi	mŗ	osition
$f_0 \begin{cases} 3 \\ 4 \end{cases}$	2 ↓ 5	3 ↓ 6 6	4 ↓ 7	8	$\downarrow \tau$ 10 ₁₀
<i>f</i> ₁ {5 <i>f</i> ₂ {5 <i>f</i> ₃ {6	6 5	3	7 7 7	8 8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \\ 10 \\ (24) \end{array} $
f ₄ {6 f ₅ {7 f ₆ {7	7 6 6	3 3 3	5 5 8	8 8 5	$ \begin{array}{c} 10 \\ (12) \\ 10 \\ (45) \\ 10 \\ (24) \end{array} $
$f_7 \ \{7 \ f_8 \ \{7 \ $	8	3	6 10	5 5	$10\}_{(46)}^{(24)}$



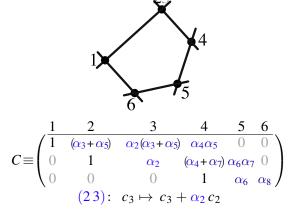
'Bridg	ge'	De	eco	mŗ	osition
1	2	3	4	5	6
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	→ 5	♦	→ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10\} \\ 10\} \\ 10\} \\ (23) \\ 10\} \\ (12) \end{array} $
$f_1 \{ 5 \}$	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f ₂ {5	6	3	7	8	$\frac{10}{10}$ (12)
<i>f</i> ₃ { 6	5	3	7	8	$10\}_{(2.4)}^{(1.2)}$
f_4 {6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $
f_5 {7	6	3	5	8	$10\}_{(4.5)}$
f_6 {7	6	3	8	5	$10\}_{(24)}^{(43)}$
f_7 {7	8		6	5	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ 6 \end{array} $
f_8 {7	8	3	10	5	6



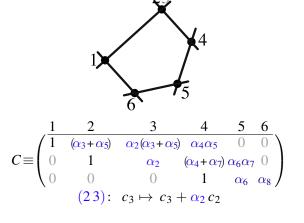
'B	rid	ge'	De	ecoi	mţ	osition
	1 ↓	2 ↓	3		5	6 ↓ τ
f_0	${3 \choose 3}$	5	↓ 6	$\overset{\downarrow}{7}$	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
	5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$
f_2	5	6	3	7	8	10 (23) 10 (12)
f_3	{6	5	3	7	8	$\frac{10}{10}$ (24)
f_4	6	7	3	5	8	$10\}(12)$
f_5	{7	6	3	5	8	$10\}(45)$
f_6	{7	6	3	8	5	10 (24)
f_7	{7	8	3	6	5	$\frac{10}{6}$ (46)
f_8	{7	8	3	10	5	6



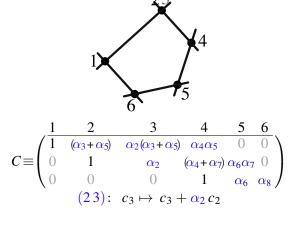
'Bric	lge'	De	eco	mŗ	oosition
1	2	3	4	5	6 τ
$f_0 \left\{ \begin{array}{l} \downarrow \\ 3 \end{array} \right.$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $
$f_1 \ \{ 5 \}$		6	7	8	10 (23)
f ₂ {5	6	3	7	8	$\frac{10}{(12)}$
f_3 {6	5	3	7		$10\}_{(24)}$
f_4 {6		3	5		$\frac{10}{(1.2)}$
f_5 {7	6	3	5	8	10 ₍₄₅₎
f_6 {7	6	3	8	5	$10\}_{(24)}$
f_7 {7		3	6	5	10 (46)
f_8 {7	8	3	10	5	6 }(40)



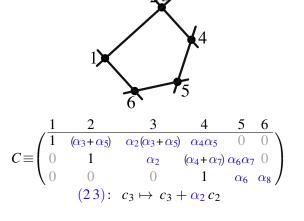
'Bı	rid	ge'	De	ecoi	mp	osition
	1	2 ↓	3	4 ↓	5	6 ↓ τ
f_0	${3 \choose 3}$	5	6	[*] 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$
f_1		3	6	7	8	10 (23)
f_2	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3	{6	5	3	7	8	$10\} (24)$
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$
f_8	{7	8	3	10	5	6 } (40)



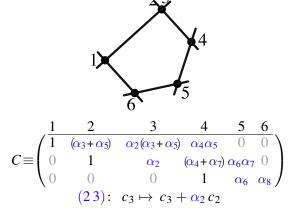
'Bı	'Bridge' Decomposition								
	1	2 ↓	3	4 ↓	5	6 ↓ τ			
f_0	${3 \choose 3}$	5	6	[*] 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$			
f_1		3	6	7	8	10 (23)			
f_2	5	6	3	7	8	$\frac{10}{10}$ (12)			
f_3	{6	5	3	7	8	$10\} (24)$			
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$			
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



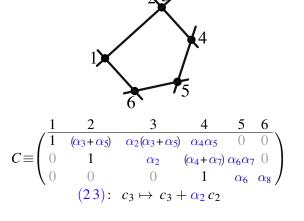
'B	'Bridge' Decomposition								
	1	2	3 ↓	4 ↓	5	6 τ			
f_0	₹ {3	5	$\overset{\star}{6}$	7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $			
	{5		6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	10 }(23) 10 }(12)			
f_3	{6	5	3	7	8	$10\}_{(2,4)}$			
f_4	{6	7	3	5	8	$\frac{10}{10}$ (12)			
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8		$10\}_{(2,4)}^{(4,3)}$			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	$\{6\}^{(40)}$			



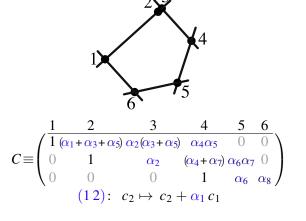
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4 ↓	5	6 τ			
f_0	$\left\{\stackrel{\star}{3}\right\}$	5	6	[*] 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $			
	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $			
f_3	{6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$			
f_4	6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $			
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6			



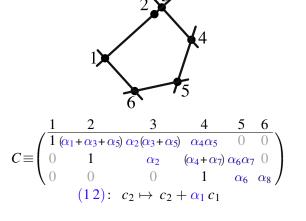
'B	'Bridge' Decomposition								
	1 ↓	2 ↓	3		5	6 ↓ τ			
f_0	${3 \choose 3}$	5	↓ 6	$\overset{\downarrow}{7}$	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$			
	5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{10}$ (24)			
f_4	6	7	3	5	8	$10\}(12)$			
f_5	{7	6	3	5	8	$10\}(45)$			
f_6	{7	6	3	8	5	10 (24)			
f_7	{7	8	3	6	5	$\frac{10}{6}$ (46)			
f_8	{7	8	3	10	5	6			



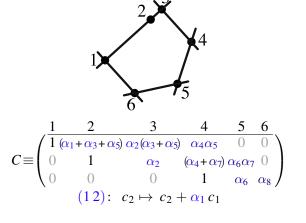
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4 ↓	5	6 τ			
f_0	$\left\{\stackrel{\star}{3}\right\}$	5	6	[*] 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $			
	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $			
f_3	{6	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$			
f_4	6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $			
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6			



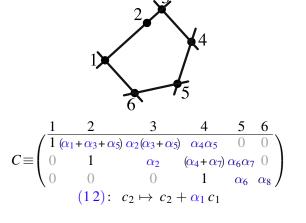
'B	'Bridge' Decomposition								
	1 ↓	2 ↓	3		5	6 ↓ τ			
f_0	${3 \choose 3}$	5	↓ 6	$\overset{\downarrow}{7}$	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$			
	5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{10}$ (24)			
f_4	6	7	3	5	8	$10\}(12)$			
f_5	{7	6	3	5	8	$10\}(45)$			
f_6	{7	6	3	8	5	10 (24)			
f_7	{7	8	3	6	5	$\frac{10}{6}$ (46)			
f_8	{7	8	3	10	5	6			



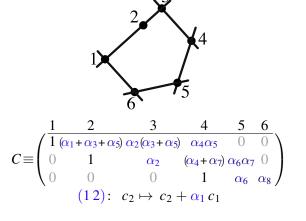
'Brid	'Bridge' Decomposition								
1	2	3	4	5	6				
$f_0 \left\{ \begin{array}{c} \downarrow \\ 3 \end{array} \right\}$	5	6	↓ 7	8	$ \begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \\ 10 \end{array} $				
$f_1 \ \{5$	3	6	7	8	$10\}_{(2,3)}^{(1,2)}$				
<i>f</i> ₂ { 5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $				
f_3 {6	5	3	7	8	$10\}_{(24)}^{(12)}$				
f_4 { 6	7	3	5	8	$ \begin{array}{c} 10 \\ (24) \\ 10 \\ (12) \end{array} $				
f_5 {7	6	3	5	8	$10\}_{(45)}$				
f_6 {7	6	3	8	5	$10\}(2.4)$				
f_7 {7	8	3	6	5	(4 0)				
$f_8 \ \{7$	8	3	10	5	6 } (10)				



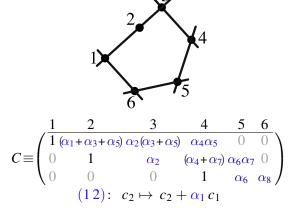
'B	'Bridge' Decomposition								
	1 ↓	2 ↓	3		5	6 ↓ τ			
f_0	${3 \choose 3}$	5	↓ 6	$\overset{\downarrow}{7}$	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$			
	5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{10}$ (24)			
f_4	6	7	3	5	8	$10\}(12)$			
f_5	{7	6	3	5	8	$10\}(45)$			
f_6	{7	6	3	8	5	10 (24)			
f_7	{7	8	3	6	5	$\frac{10}{6}$ (46)			
f_8	{7	8	3	10	5	6			



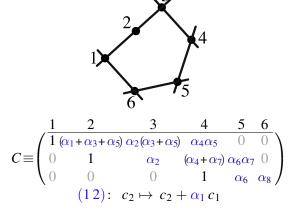
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4	5	6 τ			
f_0	${3 \choose 3}$	5	$\overset{\downarrow}{6}$	→ 7	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $			
f_1		3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	5	6	3	7	8	$ \begin{array}{c} 10 \\ (23) \\ 10 \\ (12) \end{array} $			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$\frac{10}{10}$ (12)			
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



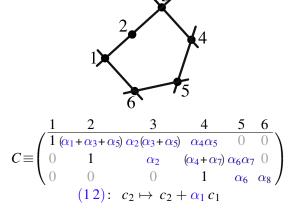
'B	'Bridge' Decomposition								
	1 ↓	2 ↓	3		5	6 ↓ τ			
f_0	${3 \choose 3}$	5	↓ 6	$\overset{\downarrow}{7}$	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$			
	5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{10}$ (24)			
f_4	6	7	3	5	8	$10\}(12)$			
f_5	{7	6	3	5	8	$10\}(45)$			
f_6	{7	6	3	8	5	10 (24)			
f_7	{7	8	3	6	5	$\frac{10}{6}$ (46)			
f_8	{7	8	3	10	5	6			



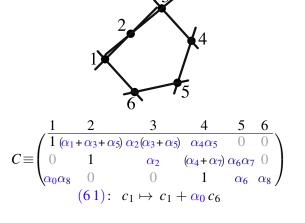
'Bri	'Bridge' Decomposition								
1	1 2	3	4		6 τ				
f_0 {	$\frac{1}{3}$ $\frac{1}{5}$	$\overset{\downarrow}{6}$		8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$				
f_1 {	5 3	6	7	8	$10\}_{(2,2)}^{(1,2)}$				
f_2 {	5 6	3	7	8	10 $\{(23)$ $\{(12)\}$ $\{(12)\}$				
f_3 {	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$				
f_4 {	5 7	3	5	8	$\frac{10}{10}$ (12)				
f_5 {	7 6	3	5	8	10 \\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \				
f_6 {	7 6	3	8	5	$10\}_{(2,4)}^{(4,5)}$				
f_7 {	7 8	3	6	5	10 (24) (46)				
f_8 {	7 8	3	10	5	6				



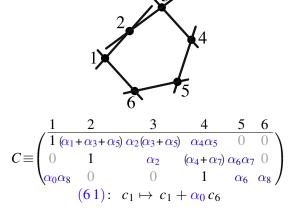
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ			
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$			
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$			
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	1/21/1			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



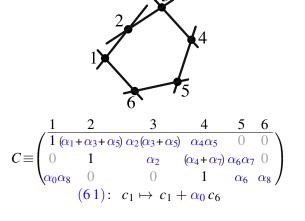
'B	'Bridge' Decomposition								
	1	2	3 ↓	4 ↓	5	6 τ			
f_0	${3 \choose 3}$	5	6	$\overset{\star}{7}$	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $			
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$\frac{10}{10}$ (12)			
f_5	{7	6	3	5	8	$\frac{10}{10}$ (45)			
f_6	{7	6	3	8	5	$\frac{10}{10}$ (24)			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



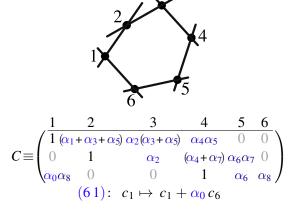
'B	'Bridge' Decomposition								
	1 ↓	2 ↓	3		5	6 ↓ τ			
f_0	${3 \choose 3}$	5	↓ 6	$\overset{\downarrow}{7}$	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$			
	5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{10}$ (24)			
f_4	6	7	3	5	8	$10\}(12)$			
f_5	{7	6	3	5	8	$10\}(45)$			
f_6	{7	6	3	8	5	10 (24)			
f_7	{7	8	3	6	5	$\frac{10}{6}$ (46)			
f_8	{7	8	3	10	5	6			



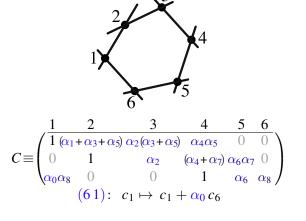
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ			
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$			
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$			
f_5	{7	6	3	5	8	$10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	1/21/1			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



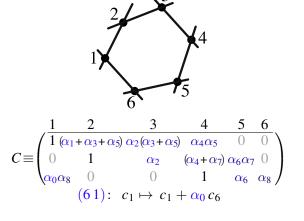
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ			
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$			
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$			
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	1/21/1			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



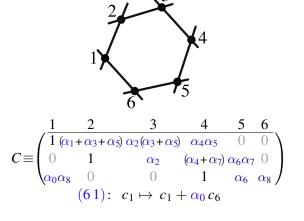
'Brio	'Bridge' Decomposition								
1	2	3	4	5	6				
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓5	$\stackrel{\downarrow}{6}$	↓ 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$				
$f_1 \ \{ 5 \}$		6	7		$10\}_{(2,3)}^{(1,2)}$				
f_2 {5	6	3	7		$10\}_{(1,2)}^{(2,3)}$				
f_3 { ϵ	5	3	7		$10\}_{(24)}^{(12)}$				
$f_4 \{ \epsilon$	7	3	5		$10\}_{(1,2)}^{(2,4)}$				
f_5 {7	6	3	5	8	$10\}_{(45)}^{(12)}$				
f_6 {7		3	8	5	$10\}_{(24)}^{(13)}$				
f_7 {7	8	3	6	5	$10\}_{(46)}^{(24)}$				
f_8 {7	8	3	10	5	6				



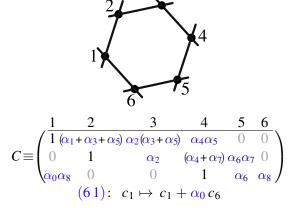
'B	'Bridge' Decomposition								
	1	2 ↓	3 ↓	4 ↓	5	6 ↓ τ			
f_0	$\{\stackrel{\star}{3}$	5	$\overset{\star}{6}$	7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$			
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	{5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$			
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	1/21/1			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



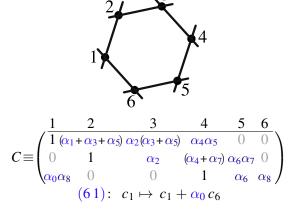
'Bri	'Bridge' Decomposition								
1	1 2	3	4		6 τ				
f_0 {	$\frac{1}{3}$ $\frac{1}{5}$	$\overset{\downarrow}{6}$		8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$				
f_1 {	5 3	6	7	8	$10\}_{(2,2)}^{(1,2)}$				
f_2 {	5 6	3	7	8	10 $\{(23)$ $\{(12)\}$ $\{(12)\}$				
f_3 {	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$				
f_4 {	5 7	3	5	8	$\frac{10}{10}$ (12)				
f_5 {	7 6	3	5	8	10 \\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \				
f_6 {	7 6	3	8	5	$10\}_{(2,4)}^{(4,5)}$				
f_7 {	7 8	3	6	5	10 (24) (46)				
f_8 {	7 8	3	10	5	6				



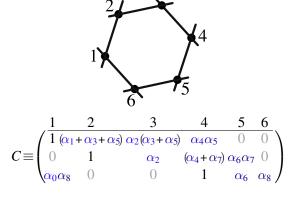
'Bri	'Bridge' Decomposition								
1	1 2	3	4		6 τ				
f_0 {	$\frac{1}{3}$ $\frac{1}{5}$	$\overset{\downarrow}{6}$		8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace \\ 10 \rbrace (12) \end{array}$				
f_1 {	5 3	6	7	8	$10\}_{(2,2)}^{(1,2)}$				
f_2 {	5 6	3	7	8	10 $\{(23)$ $\{(12)\}$ $\{(12)\}$				
f_3 {	5	3	7	8	$10\}_{(2,4)}^{(1,2)}$				
f_4 {	5 7	3	5	8	$\frac{10}{10}$ (12)				
f_5 {	7 6	3	5	8	10 \\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \				
f_6 {	7 6	3	8	5	$10\}_{(2,4)}^{(4,5)}$				
f_7 {	7 8	3	6	5	10 (24) (46)				
f_8 {	7 8	3	10	5	6				



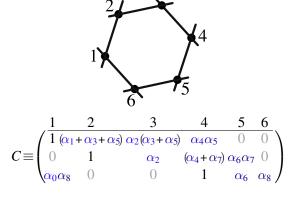
'Bı	'Bridge' Decomposition								
	1	2 ↓	3	4 ↓	5	6 ↓ τ			
f_0	${3 \choose 3}$	5	6	[*] 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \rbrace (12) \end{array}$			
f_1		3	6	7	8	10 (23)			
f_2	5	6	3	7	8	$\frac{10}{10}$ (12)			
f_3	{6	5	3	7	8	$10\} (24)$			
f_4	6	7	3	5	8	$\{10\}_{(1,2)}^{(2,4)}$			
f_5	{7	6	3	5	8	$\{10\}_{(4.5)}^{(1.2)}$			
f_6	{7	6	3	8	5	$10\}_{(2,4)}^{(4,3)}$			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



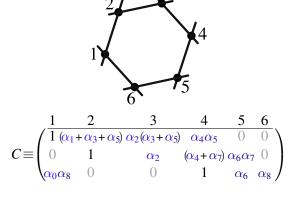
'B	'Bridge' Decomposition								
	1	2	3 ↓	4 ↓	5	6 τ			
f_0	${3 \choose 3}$	5	6	₇	8	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $			
f_1	{5	3	6	7	8	$10\}_{(2,2)}^{(1,2)}$			
f_2	5	6	3	7	8	10 (23) 10 (12)			
f_3	{6	5	3	7	8	$\frac{10}{(24)}$			
f_4	6	7	3	5	8	$\frac{10}{10}$ (12)			
f_5	{7	6	3	5	8	$\frac{10}{10}$ (45)			
f_6	{7	6	3	8	5	$\frac{10}{10}$ (24)			
f_7	{7	8	3	6	5	$10\}_{(4.6)}^{(2.4)}$			
f_8	{7	8	3	10	5	6 } (40)			



'Br	id	ge'	Dε	ecoi	mŗ	oosition
	1	2	3	4	5	6
f_0 {	→ 3	5	↓ 6	$\overset{\downarrow}{7}$	♦	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {			6	7	8	10 (23)
f_2 {	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {	6	5	3	7		$10\}_{(24)}$
f_4 {	6	7	3	5		$10\}_{(1,2)}$
f_5 {	7	6	3	5	8	$10\}_{(4.5)}$
f_6 {		6	3	8	5	$10\}_{(24)}$
f_7 {		8		6	5	(46)
f_8 {	7	8	3	10	5	6 } (10)

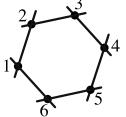


'Br	id	ge'	Dε	ecoi	mŗ	osition
	1	2	3	4	5	6
f_0 {	→ 3	5	↓ 6	$\overset{\downarrow}{7}$	♦	$ \downarrow \tau $ $ 10} $ $ 10\}(12) $
f_1 {			6	7	8	10 (23)
f_2 {	5	6	3	7	8	$\frac{10}{10}$ (12)
f_3 {	6	5	3	7		$10\}_{(24)}$
f_4 {	6	7	3	5		$10\}_{(1,2)}$
f_5 {	7	6	3	5	8	$10\}_{(4.5)}$
f_6 {		6	3	8	5	$10\}_{(24)}$
f_7 {		8		6	5	(46)
f_8 {	7	8	3	10	5	6 } (10)



'Brid	ge'	De	eco	mŗ	osit	ion
1	2				6	
f	<u></u>	†	\downarrow	V	10)	au
$f_0 \{3 \\ f_1 \} \{5 \}$	2	6	7	0	10}	(12)
			7	0	10)	(23) (12)
f_2 {5	6	3				
$f_3 \{ 6 \}$	5	3	7	8	10}	(24)
f_4 {6	1	3	5	8	10)	(12)
$f_5 $ {7	6	3	5	_	10}	(45)
$f_6 \{ 7 \}$	6	3	8	5	10}	(24)
$f_7 \{ 7 \}$		3	6			
J ₈ {/	8	3	10	5	6 }	.` ′

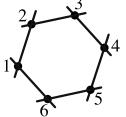
$$\mathcal{L}_{6,3} \equiv \frac{d\alpha_0}{\alpha_0} \cdots \frac{d\alpha_8}{\alpha_8}$$



$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{(\alpha_1 + \alpha_3 + \alpha_5)} & \frac{3}{\alpha_2(\alpha_3 + \alpha_5)} & \frac{4}{\alpha_4\alpha_5} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & \alpha_2 & (\alpha_4 + \alpha_7) & \alpha_6\alpha_7 & 0 \\ \alpha_0\alpha_8 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

'Brid	ge'	De	eco	mp	osition
1	2	3	4	5	6 ↓ τ
$f_0 \left\{ \stackrel{\checkmark}{3} \right\}$	5	ě	$\overset{\diamond}{7}$	8	$\frac{10}{10}$ (12)
	3	6	7	8	
<i>f</i> ₂ { 5	6	3	7	8	$10\}_{(1,2)}^{(2,3)}$
<i>f</i> ₃ { 6	5	3	7	8	$10\}_{(24)}^{(12)}$
f_4 { 6	7	3	5	8	$10\}_{(1,2)}^{(2,4)}$
f_5 {7	6	3	5	8	$10\}_{(45)}^{(12)}$
f_6 {7	6	3	8	5	$10\}(24)$
$f_7 \{7$	8	3	6	5	$10\}(46)$
$f_8 \{ 7 \}$	8	3	10	5	6 }

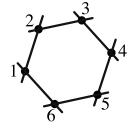
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$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{(\alpha_1 + \alpha_3 + \alpha_5)} & \frac{3}{\alpha_2(\alpha_3 + \alpha_5)} & \frac{4}{\alpha_4\alpha_5} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & \alpha_2 & (\alpha_4 + \alpha_7) & \alpha_6\alpha_7 & 0 \\ \alpha_0\alpha_8 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

'Brid	ge'	De	eco	mp	osition
1	2	3	4	5	6 ↓ τ
$f_0 \left\{ \stackrel{\checkmark}{3} \right\}$	5	ě	$\overset{\diamond}{7}$	8	$\frac{10}{10}$ (12)
	3	6	7	8	
<i>f</i> ₂ { 5	6	3	7	8	$10\}_{(1,2)}^{(2,3)}$
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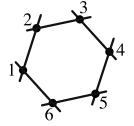
$$\mathcal{L}_{6,3} \equiv \frac{d\alpha_0}{\alpha_0} \cdots \frac{d\alpha_8}{\alpha_8} = \frac{d^{3\times 6}C}{\text{vol}(GL(3))} \frac{1}{(123)(234)(345)(456)(561)(612)}$$



$$C \equiv \begin{pmatrix} \frac{1}{1} & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{1} & (\alpha_1 + \alpha_3 + \alpha_5) & \alpha_2 (\alpha_3 + \alpha_5) & \alpha_4 \alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4 + \alpha_7) & \alpha_6 \alpha_7 & 0 \\ \alpha_0 \alpha_8 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

'Bridg	ge'	De	eco:	mŗ	osition
1	2	3	4		6 ↓ τ
$f_0 \left\{ \stackrel{\downarrow}{3} \right\}$	↓ 5	↓ 6	↓ 7	8	$\begin{array}{c} \downarrow & \tau \\ 10 \\ 10 \end{array}$
f_1 {5		6	7	8	10 (23) (23)
<i>f</i> ₂ { 5	6	3	7	8	$\frac{10}{10}$ (12)
<i>f</i> ₃ { 6	5	3	7		$10\}_{(24)}$
f_4 { 6	7	3	5		$10\}_{(1,2)}$
f_5 {7	6	3	5	8	$\frac{10}{(4.5)}$
f_6 {7	6	3	8	5	$\frac{10}{10}$ (24)
$f_7 \{ 7 \}$	8	3	6	5	$10\}_{(46)}^{(24)}$
$f_8 \{7$	8	3	10	5	6 } (10)

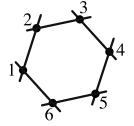
$$\mathcal{L}_{n,k} \equiv \frac{d\alpha_1}{\alpha_1} \cdots \frac{d\alpha_{k(n-k)}}{\alpha_{k(n-k)}} = \frac{d^{k \times n}C}{\operatorname{vol}(GL(k))} \frac{1}{(1 \cdots k)(2 \cdots k+1) \cdots (n \cdots k-1)}$$



$$C \equiv \begin{pmatrix} \frac{1}{1} & \frac{2}{(\alpha_1 + \alpha_3 + \alpha_5)} & \frac{3}{\alpha_2(\alpha_3 + \alpha_5)} & \frac{4}{\alpha_4\alpha_5} & \frac{5}{0} & \frac{6}{0} \\ 0 & 1 & \alpha_2 & (\alpha_4 + \alpha_7) & \alpha_6\alpha_7 & 0 \\ \alpha_0\alpha_8 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

'Brid	ge'	De	eco	mp	osition
1	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓ τ
$f_0 \{3$	5	6	7	8	$\{10\}$
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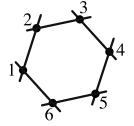
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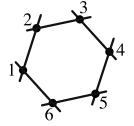
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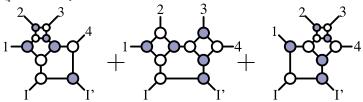
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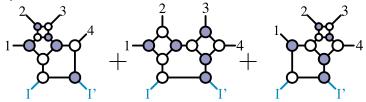
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$$=\sum_{L,R} \frac{1}{L} + \frac{A_{n+2}^{\ell-1}}{n}$$

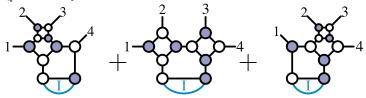
$$=\sum_{L,R} \frac{1}{1} \frac{1}{n} + \frac{1}{n} \frac{A_{n+2}^{\ell-1}}{n}$$



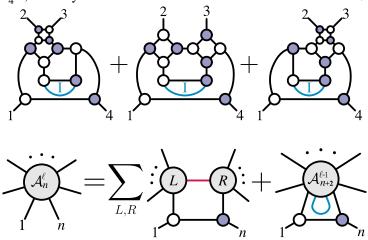
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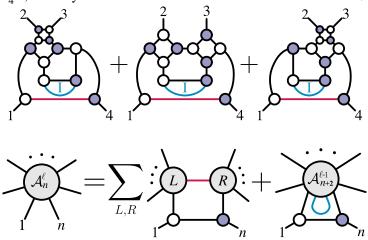


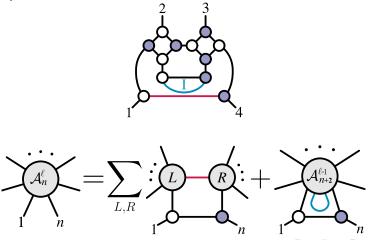
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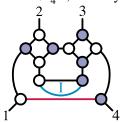


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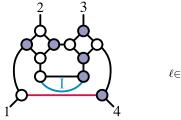






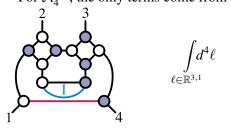


$$=\sum_{L,R} \frac{1}{1} \frac{1}{n} + \frac{1}{n} \frac{A^{\ell 1}_{n+2}}{n}$$



$$\int d^4\ell \ell \ell \in \mathbb{R}^{3,1}$$

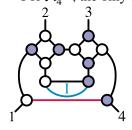
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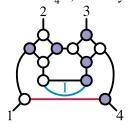
$$=\sum_{L,R} \frac{1}{1} \frac{1}{n} + \frac{A^{\ell-1}_{n+2}}{1}$$



$$\int_{\ell \in \mathbb{R}^{3,1}} d^4 \ell \quad \iff \quad \int_{\frac{1}{2}} \frac{d^2 \lambda_{\mathrm{I}} d^2 \widetilde{\lambda}_{\mathrm{I}}}{\mathrm{vol}(GL_1)} d\alpha \langle \mathrm{I1} \rangle [n\mathrm{I}]$$

$$\ell = (\lambda_{\mathrm{I}} \widetilde{\lambda}_{\mathrm{I}} + \alpha \lambda_{\mathrm{I}} \widetilde{\lambda}_{\mathrm{4}}) \in \mathbb{R}^{3,1}$$

$$\mathcal{A}_{4}^{(2),0} \times \int d\log\left(\frac{\ell^{2}}{(\ell-\ell^{*})^{2}}\right) d\log\left(\frac{(\ell+p_{1})^{2}}{(\ell-\ell^{*})^{2}}\right) d\log\left(\frac{(\ell+p_{1}+p_{2})^{2}}{(\ell-\ell^{*})^{2}}\right) d\log\left(\frac{(\ell-p_{4})^{2}}{(\ell-\ell^{*})^{2}}\right) d\log\left(\frac{(\ell-p_$$



$$\int_{\ell \in \mathbb{R}^{3,1}} d^4 \ell \quad \iff \quad \int_{\text{vol}(GL_1)} \frac{d^2 \lambda_{\text{I}} d^2 \widetilde{\lambda}_{\text{I}}}{\text{vol}(GL_1)} d\alpha \langle \text{I1} \rangle [n\text{I}]$$

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$$= \mathcal{A}_{4}^{(2),0} \times \int d^{4}\ell \frac{(p_{1}+p_{2})^{2}(p_{3}+p_{4})^{2}}{\ell^{2}(\ell+p_{1})^{2}(\ell+p_{1}+p_{2})^{2}(\ell-p_{4})^{2}}$$