Latest developments in anisotropic hydrodynamics

Outline

- Hydrodynamics in heavy ions collisions
- Expansion around an anisotropic background
- Leading order for the previous formulation
- Newest results

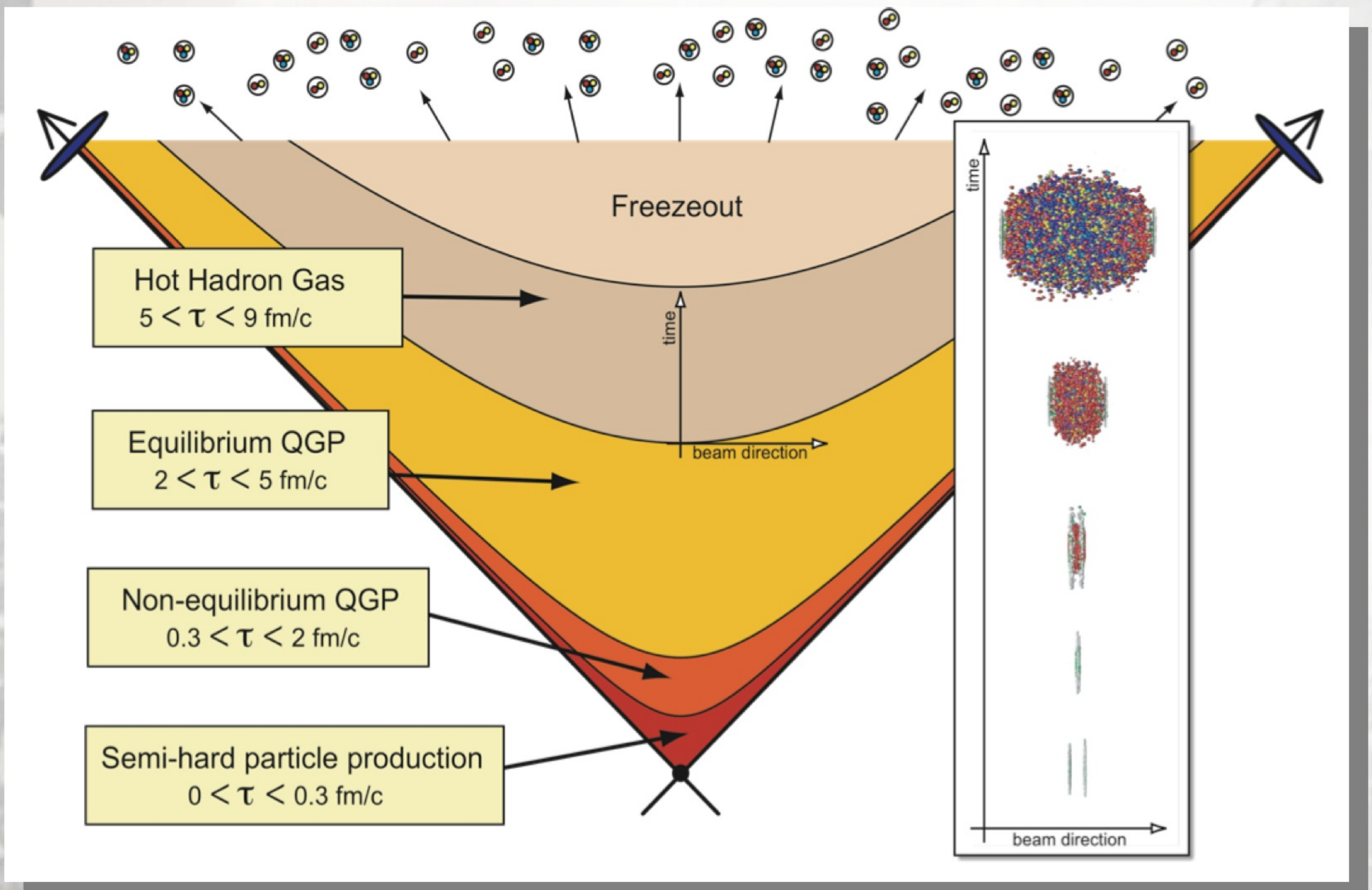
55. Cracow school of **theoretical physics**

Leonardo Tinti

Hydrodynamics

Hydrodynamic modeling of heavy ion collisions (small viscosity)

Large gradients → *viscous corrections*



Strong longitudinal expansion,

pressure anisotropy (also AdS/CFT)

Large momentum anisotropy from microscopic models (pQCCD, CGC)

Hydrodynamics

From quantum field theory

$$T^{\mu\nu} = \langle \widehat{T}^{\mu\nu} \rangle$$

$$\partial_\mu T^{\mu\nu} = \langle \partial_\mu \widehat{T}^{\mu\nu} \rangle$$

and translation invariance

$$\partial_\mu T^{\mu\nu} = 0$$

Hydrodynamics

Perfect fluid

$$T^{\mu\nu} = (\varepsilon + P)U^\mu U^\nu - g^{\mu\nu}P$$

From four-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Continuity and Euler equations

$$U^\mu \partial_\mu \varepsilon + (\varepsilon + P)\partial_\mu U^\mu = 0 \quad \rightarrow \quad \partial_t \rho + \nabla_{\mathbf{x}} \cdot (\rho \mathbf{v}) = 0$$

$$(\varepsilon + P)A^\mu - \nabla^\mu P = 0 \rightarrow \rho\,\mathbf{a} + \nabla_{\mathbf{x}} P = \rho\,\mathbf{a} + \nabla_{\mathbf{x}} \cdot T|_{\text{pf}} = 0$$

Local equilibrium?

Gradient expansion?

$$T^{00}, T^{0i} \qquad \Leftrightarrow \qquad \varepsilon, u^{\mu}$$

Shear pressure corrections

$$\pi^{\mu\nu} \simeq 2\eta\sigma^{\mu\nu} + \cdots$$

Instabilities, causality violation

From kinetic theory to hydrodynamics

Relativistic Boltzmann equation:

$$p^\mu \, \partial_\mu f(x,p) = -\mathcal{C}[f]$$

First moment:

$$\int dP \, p^\mu p^\nu \, \partial_\mu f = \partial_\mu T^{\mu\nu} =$$

$$= -\int dP \, p^\nu \mathcal{C}[f] \left[\, = 0 \right]$$

- S. R. De Groot, W. A. van Leeuwen, Ch. G. van Weert, *Relativistic kinetic theory*, North Holland (1980).

From kinetic theory to hydrodynamics

Ansatz for the relativistic Boltzmann distribution

Perfect fluid:

$$f \simeq f_{\text{eq.}} = k \exp\left[-\frac{p \cdot U(x)}{T(x)}\right]$$

$$T^{\mu\nu} = k \int dP \, p^\mu p^\nu \exp\left[-\frac{p \cdot U}{T}\right] = (\varepsilon + P) \, U^\mu U^\nu - g^{\mu\nu} P$$

$$T_{\text{L.R.F.}} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

Dissipative hydrodynamics

$$f = f_{\mathrm{eq.}} + \delta f \quad \Rightarrow \quad T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq.}} + \delta T^{\mu\nu}$$

$\delta f \Rightarrow \delta T^{\mu\nu}$ treated as, small, perturbations

Landau frame, massless particles

$$\delta T^{\mu\nu} = \pi^{\mu\nu}$$

$$U^{\mu}\pi_{\mu\nu} = 0 \qquad g_{\mu\nu}\pi^{\mu\nu} = 0$$

Four equations, five more degrees of freedom!

Entropy current and entropy source

Obtaining the remaining equations from the second principle of thermodynamics

$$\mathcal{S}^\mu = \mathcal{S}^\mu_{\text{eq.}} + \delta\mathcal{S}^\mu \simeq \frac{p}{T}U^\mu + \frac{1}{T}T^{\mu\nu}U_\nu - \frac{\tau_\pi}{4\eta T}\pi^{\alpha\beta}\pi_{\alpha\beta}U^\mu$$

$$\partial_\mu \mathcal{S}^\mu \geq 0$$

$$\partial_\mu \mathcal{S}^\mu \simeq \frac{1}{T}\pi^{\mu\nu}\left[\sigma_{\mu\nu} - \frac{\tau_\pi}{2\eta}\Delta^\alpha_\mu\Delta^\beta_\nu D\pi_{\alpha\beta} - \frac{1}{2}T\pi_{\mu\nu}\partial\cdot\left(\frac{\tau_\pi}{2\eta T}U\right)\right]$$

$$\pi^{\mu\nu}\pi_{\mu\nu} \geq 0$$

$$\tau_\pi\Delta^\alpha_\mu\Delta^\beta_\nu D\pi_{\alpha\beta} + \pi_{\mu\nu} = 2\eta\sigma_{\mu\nu} - \pi_{\mu\nu}T\eta\,\partial\cdot\left(\frac{\tau_\pi}{2\eta T}U\right)$$

Example, Bjorken flow

0+1 dimensions: boost invariant in the longitudinal direction, homogeneous in the transverse plane

$$U = (\cosh \eta_{\|}, 0, 0, \sinh \eta_{\|}) \qquad \eta_{\|} = \frac{1}{2} \ln\left(\frac{t+z}{t-z}\right) \qquad \tau = \sqrt{t^2 - z^2}$$

Gradients of the four velocity are proportional to $1/\tau$

In the Navier-Stokes limit

$$\pi_{\mu\nu} \simeq 2\eta\sigma_{\mu\nu}$$

therefore

$$\frac{P_{\|}}{P_{\perp}} = \frac{P_{\text{eq.}} + \pi_{ZZ}}{P_{\text{eq.}} + \pi_{XX}} \simeq \frac{3T\tau - 16\bar{\eta}}{3T\tau + 8\bar{\eta}}$$

Anisotropic hydrodynamics

Reorganization of the hydrodynamic expansion

$$f = f_{\rm eq.} + \delta f$$

around an anisotropic background instead of the local equilibrium

$$f = f_{\rm aniso.} + \delta \tilde{f}$$

“Romatschke-Strickland” form:

$$f_{\rm aniso.} = k \exp\left[-\frac{\sqrt{\left(p\cdot U(x)\right)^2 + \xi(x)\left(p\cdot Z(x)\right)^2}}{\Lambda(x)} \right]$$

0+1 dimensions

Boost invariant in the longitudinal direction, homogeneous in the transverse plane

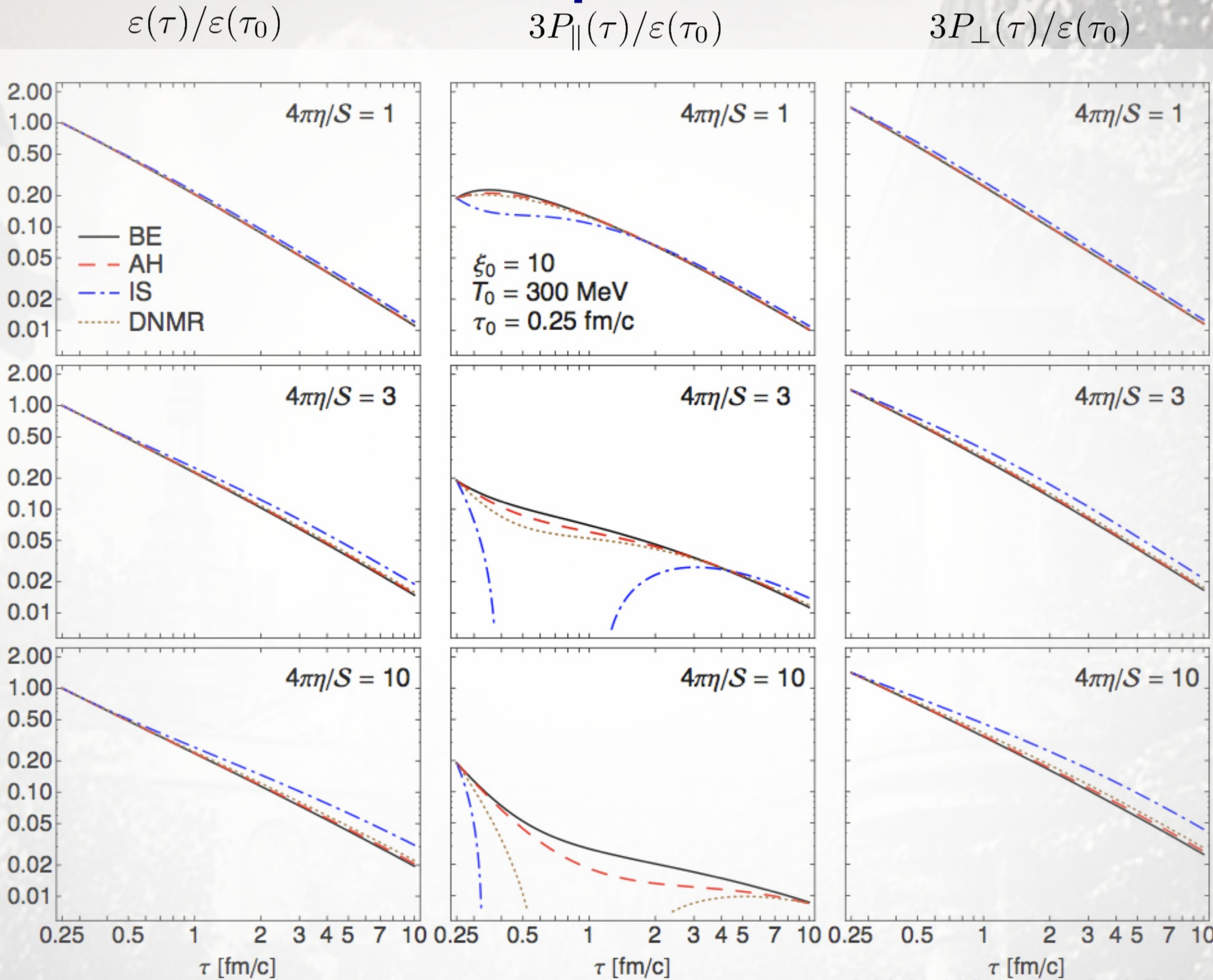
Exact solutions for the Boltzmann equation with the collisional kernel treated in relaxation time approximation

$$\mathcal{C}[f] = (p \cdot U)\,\frac{f - f_{\rm eq.}}{\tau_{\rm eq.}}$$

Test for viscous and anisotropic hydrodynamics!

- **W Florkowski, R Ryblewski and M Strickland, Phys. Rev. C88, 024903 (2013)**

Some plots


$\varepsilon(\tau)/\varepsilon(\tau_0)$
$3P_{\parallel}(\tau)/\varepsilon(\tau_0)$
$3P_{\perp}(\tau)/\varepsilon(\tau_0)$
4πη/S = 1
4πη/S = 3
4πη/S = 10
BE
AH
IS
DNMR
ξ0 = 10
T0 = 300 MeV
τ0 = 0.25 fm/c
2.00
1.00
0.50
0.20
0.10
0.05
0.02
0.01
0.25
0.5
1
2
3
4
5
7
10
τ [fm/c]

Higher dimensions

Radial flow, pressure asymmetries in the transverse plane

Non trivial transverse dynamics is important to explain collective behavior like the elliptic flow

The Romatschke-Strickland form has only one anisotropy parameter

Possible solutions

Next to leading order

- **D Bazow, U W Heinz, M Strickland, Phys.Rev. C 90 054910 (2014)**
- **D Bazow, U W Heinz, M Martinez, Phys.Rev. C 91 064903 (2015)**

or **Improve the leading order**

First step, cylindrically symmetric radial flow

For a conformal system

$$f_{\mathrm{aniso.}} = k \exp\left[-\frac{\sqrt{(1+\xi_X)\,(p\cdot X)^2 + (1+\xi_Y)\,(p\cdot Y)^2 + (1+\xi_Z)\,(p\cdot Z)^2}}{\lambda(x)} \right]$$

Pressure not isotropic in the transverse plane

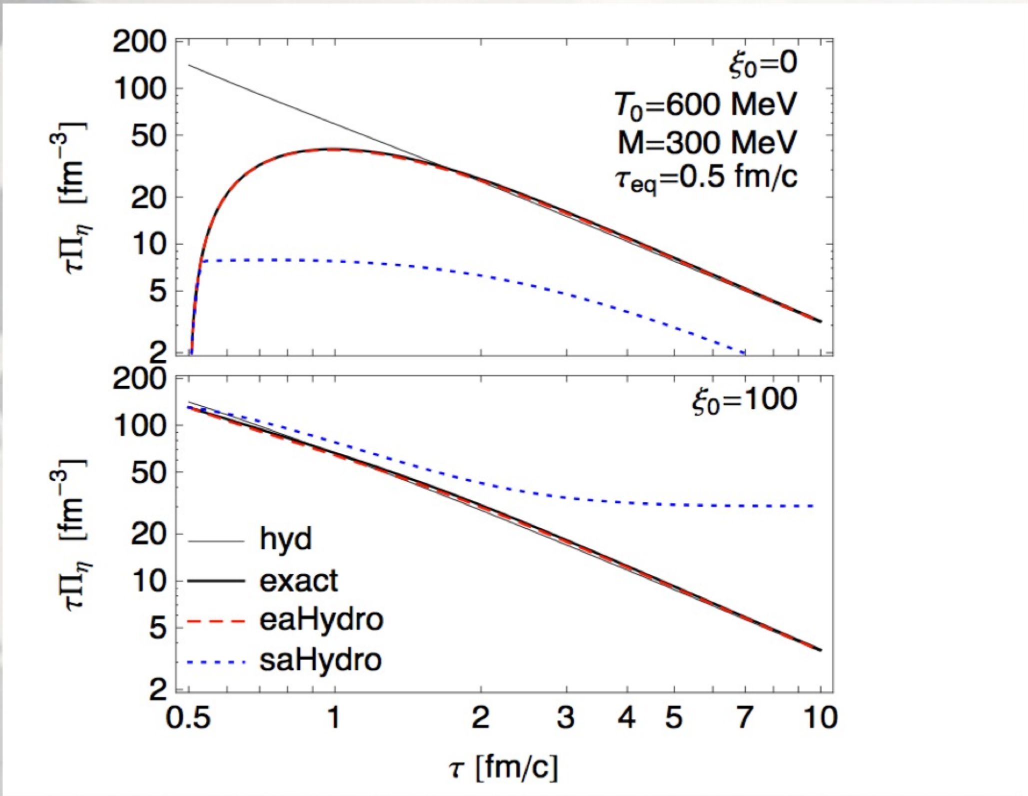
$$\sum_I \xi_I = 0$$

Dynamical equations from the second moment of the Boltzmann equation and four-momentum conservation

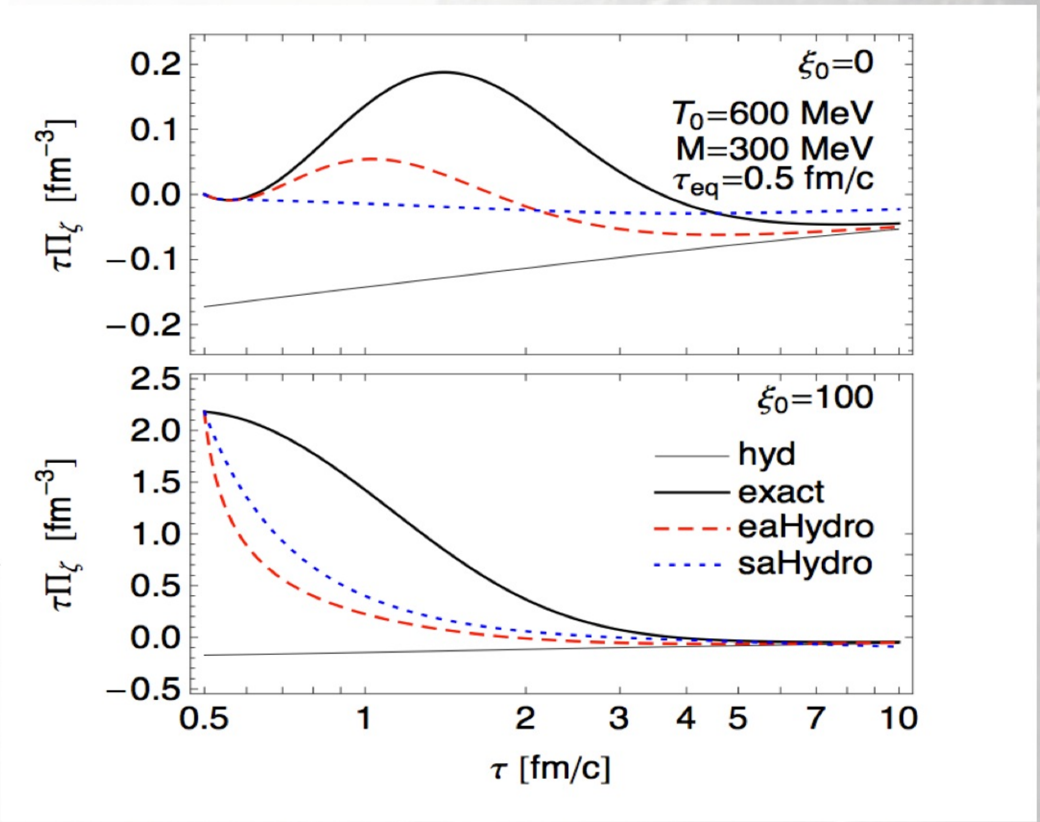
- **L Tinti and W Florkowski , Phys. Rev. C 89, 034907 (2014)**

Important even without radial expansion

- W Florkowski, R Ryblewski, M Strickland, L Tinti, Phys. Rev. C 89, 054909 (2014)



Much better agreement with the exact solution



but not for bulk viscosity...

Bulk degree of freedom (M Nopoush, R Ryblewski, M Strickland, Phys. Rev. C 90, 014908 (2014))

Gubser Flow (M Nopoush, R Ryblewski, M Strickland, Phys. Rev. D 91, 045007 (2015))

(3+1)-dimensional framework

No symmetry constraints from boost invariance or cylindrical symmetry

Generalized "Romatschke-Strickland" form

$$f = k \exp\left(-\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)$$

Dynamical equations from the second moment of the Boltzmann equation, the zeroth moment, and four-momentum conservation

- **L Tinti, arXiv:1411.7268**

Generalizing in this way the leading order comes with the price of a slightly reduced agreement with the exact solutions...

- **L Tinti, R Ryblewski, W Florkowski, M Strickland,arXiv:1505.06456**

It is not necessary to take the equations from the moments of the Boltzmann equation!

Kinetic theory already provides exact equations for the pressure corrections

$$D\pi^{\langle\mu\nu\rangle} + \mathcal{C}_{-1}^{\langle\mu\nu\rangle} = -\Delta^{\mu\nu}_{\rho\sigma}\nabla_\alpha \int dP\, \frac{p^\rho p^\sigma p^\alpha\, f}{(p\cdot U)} - \left(\sigma_{\rho\sigma} + \frac{1}{3}\,\theta\,\Delta_{\rho\sigma}\right)\int dP\, \frac{p^{\langle\mu}p^{\nu\rangle}p^\rho p^\sigma\, f}{(p\cdot U)^2}$$

$$D\Pi - \frac{1}{3}\Delta_{\mu\nu}\mathcal{C}_{-1}^{\mu\nu} = -D\mathcal{P}_{\mathrm{eq.}} + \frac{1}{3}\Delta_{\mu\nu}\nabla_\rho \int dP\, \frac{p^\mu p^\nu p^\rho\, f}{(p\cdot U)} + \frac{1}{3}\left(\sigma_{\rho\sigma} + \frac{1}{3}\,\theta\,\Delta_{\rho\sigma}\right)\int dP\, \frac{(p\cdot\Delta\cdot p)p^\rho p^\sigma\, f}{(p\cdot U)^2}$$

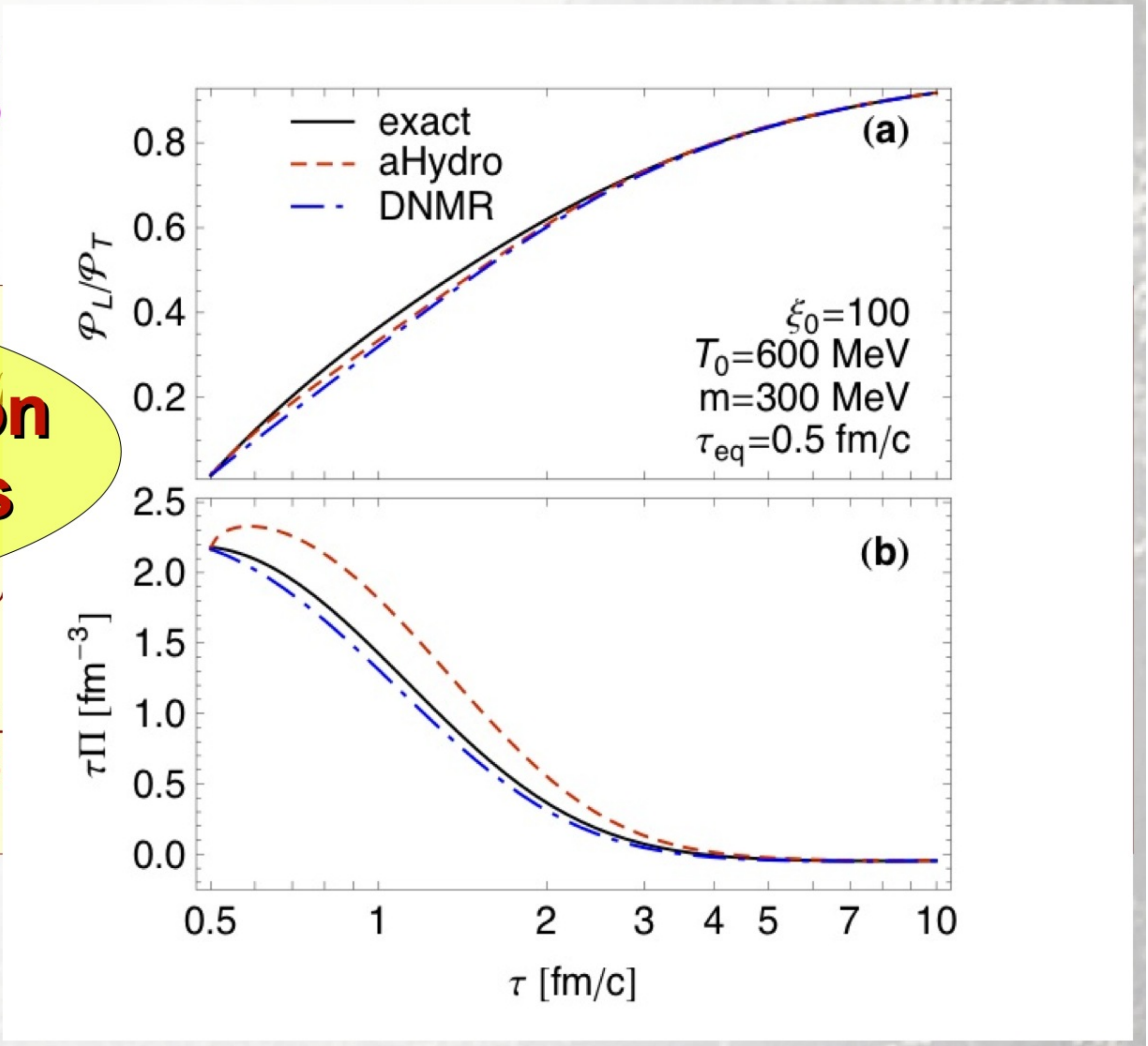
It is not necessary to take the equations from the moments of the Boltzmann equation!

Kinetic theory already pro
for the pressure

Very successful application to viscous hydrodynamics

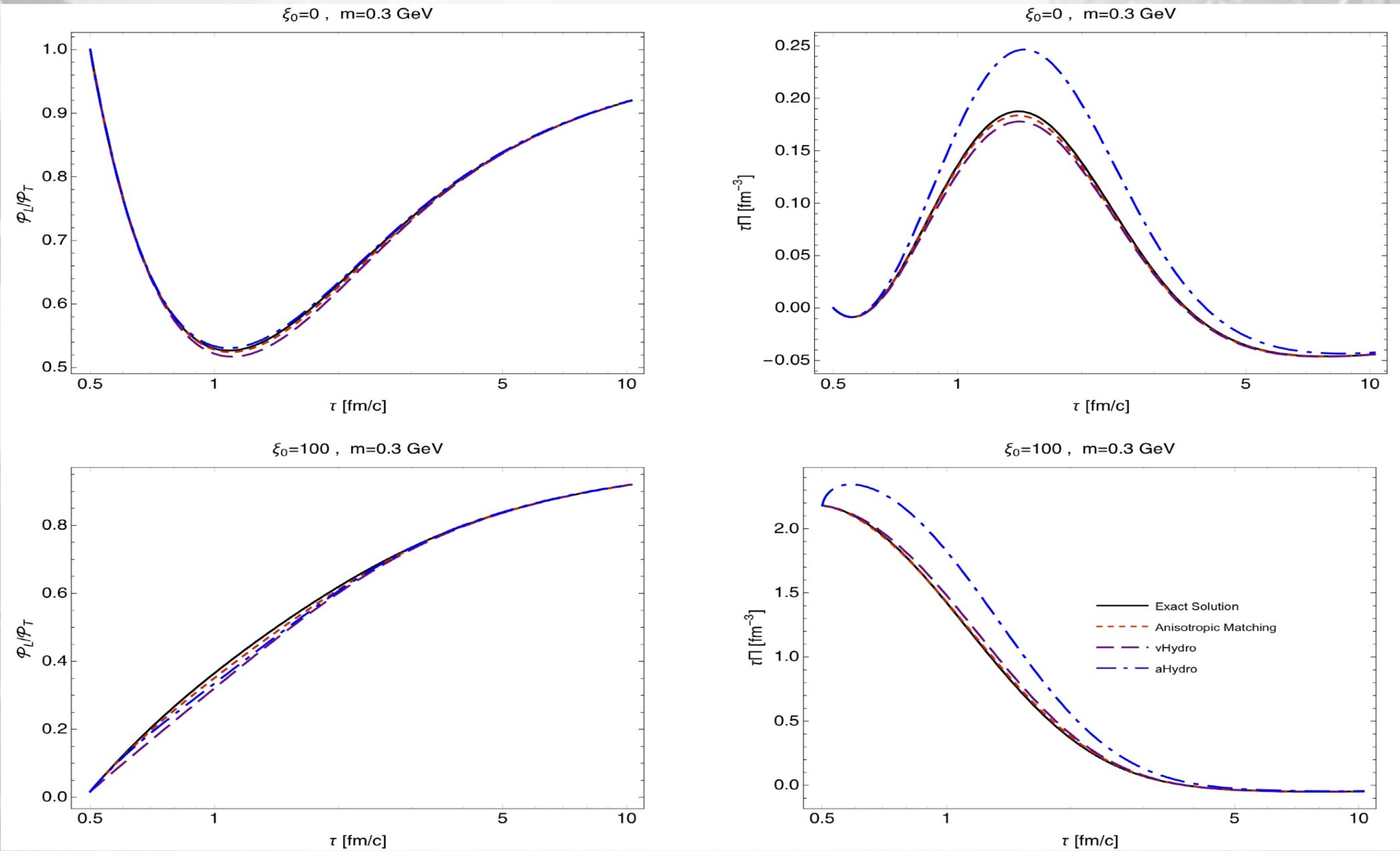
$$D\Pi - \frac{1}{3}\Delta_{\mu\nu}\mathcal{C}^{\mu\nu}_{-1} = -D\mathcal{P}_{\rm eq.} + \frac{1}{3}\Delta_{\mu\nu}$$

$$+\frac{1}{3}\left(\sigma_{\rho\sigma} + \frac{1}{3}\right.$$



- G S Denicol, W Florkowski, R R, M Strickland,Phys.Rev. C 90 044905 (2014)

New! Very successful application to anisotropic hydrodynamics too!



- <u>L Tinti</u>, soon to publish

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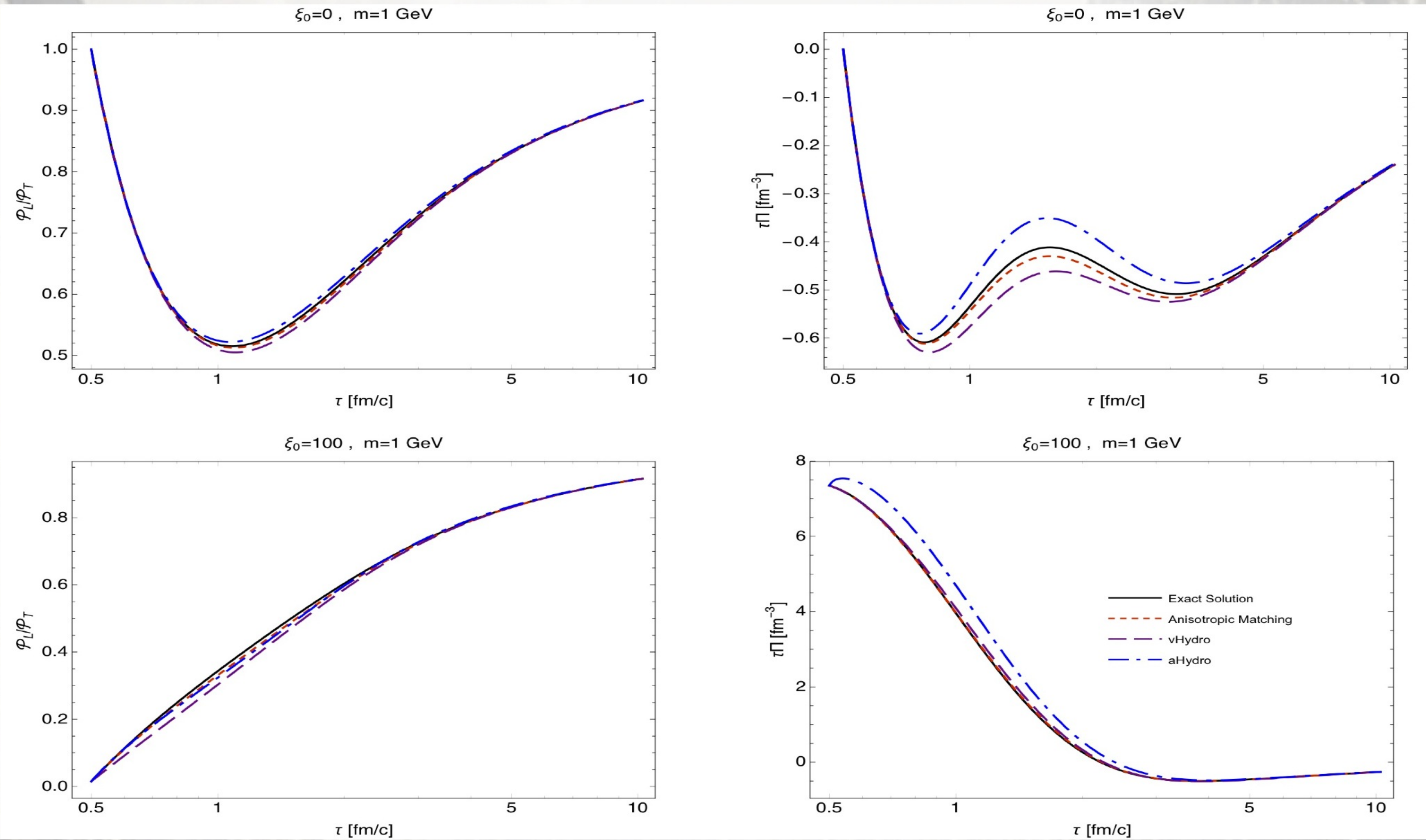

- <u>L Tinti</u>, soon to publish

Summary & outlook

- Anisotropic hydrodynamics is a reorganization of the hydrodynamic expansion, around an anisotropic distribution.

- Pressure anisotropies already at the leading order, treated in a non-perturbative manner.

- Generalized ansatz for the leading order, consistent with second order viscous hydrodynamics close to equilibrium (full 3+1 expansion).

- Striking agreement with the exact solutions of the Boltzmann equation in the one-dimensional expansion

- Is this agreement preserved in different situations (*e.g.* Gubser flow)?