Hadron spectroscopy from lattice QCD

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UNIVERSITY OF CAMBRIDGE

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Outline

- Lecture 1 – Lattice QCD and some applications
- Lecture 2 – Hadron spectroscopy
- Lecture 3 – Resonances, scattering, etc

Lecture 1

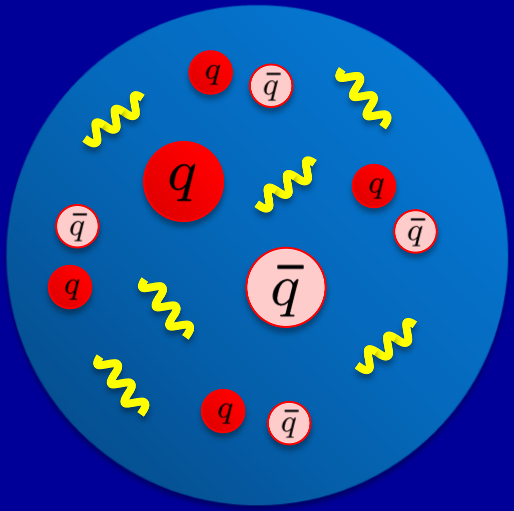
- Why lattice quantum chromodynamics?
- Introduction to lattice QCD
- Some applications

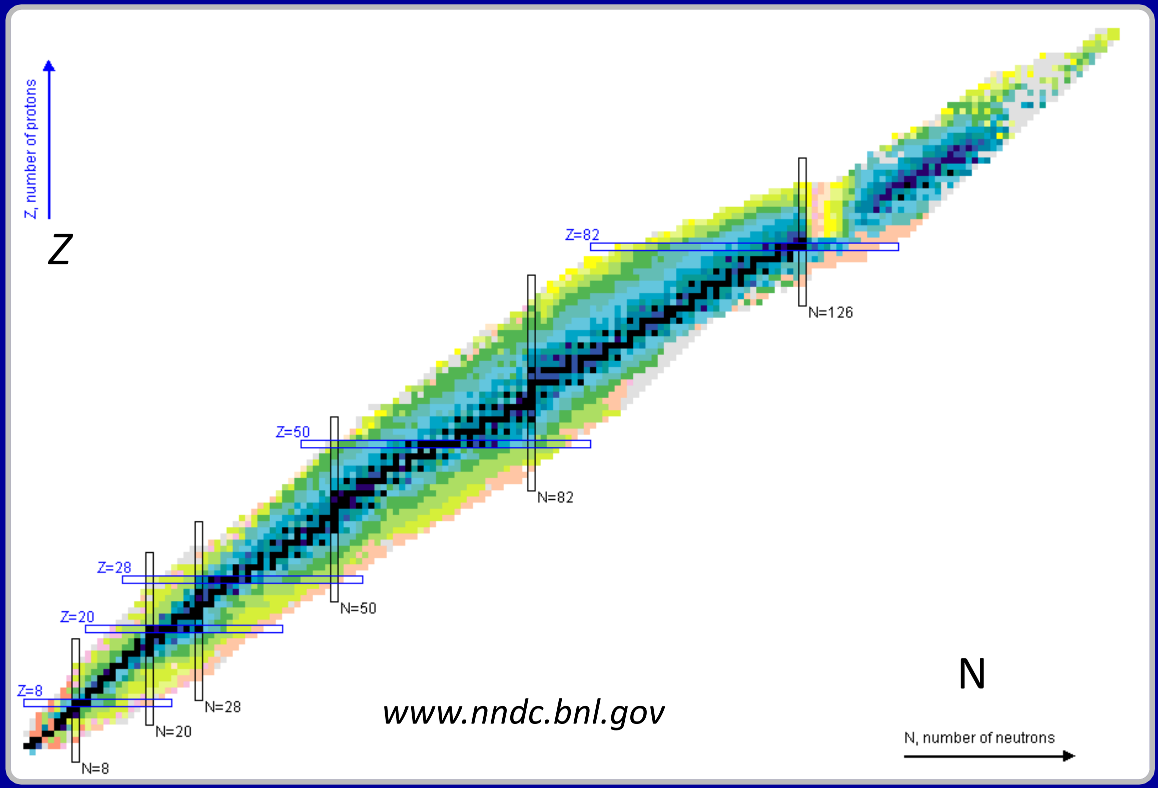
General references

- "*Lattice Gauge Theories, An Introduction*", Heinz Rothe (World Scientific, Lecture Notes in Physics, 4th edn. 2012)
- "*Lattice Methods for Quantum Chromodynamics*", Thomas Degrand and Carleton DeTar (World Scientific, 2006)
- "*Quantum Chromodynamics on the Lattice: An Introductory Presentation*", Christof Gattringer and Christian Lang (Springer, Lecture Notes in Physics, 2009, also available as an e-book)
- "*Quantum fields on the lattice*", I. Monvay and G. Münster (CUP, 1994)
- Reviews from the annual International Symposium on Lattice Field Theory, http://www.bnl.gov/lattice2014/ and proceedings, http://pos.sissa.it/cgi-bin/reader/family.cgi?code=lattice
- INT Summer School on Lattice QCD for Nuclear Physics (2012) http://www.int.washington.edu/PROGRAMS/12-2c/

The strong interaction

- Binds quarks → hadrons: mesons and baryons (protons, neutrons, ...)
- Binds protons and neutrons → nuclei
- Responsible for most of mass of conventional matter (~99% of proton mass)





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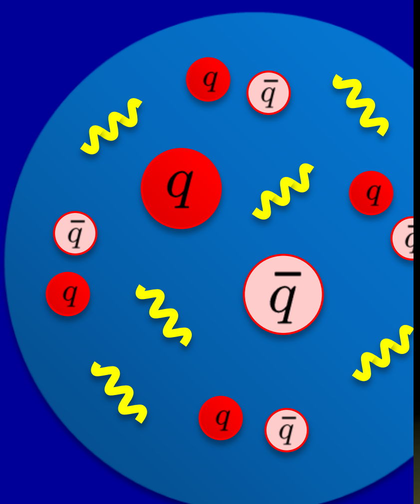

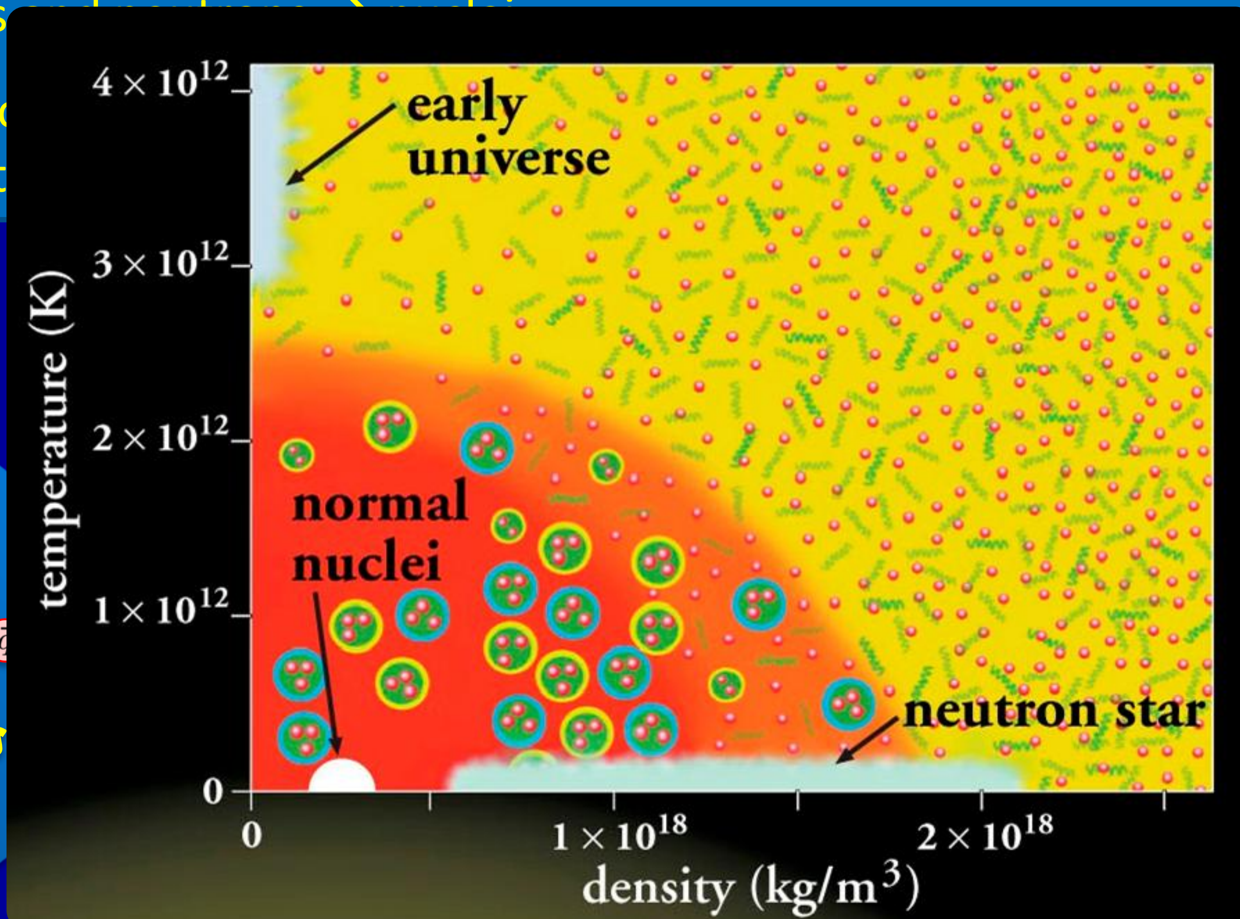

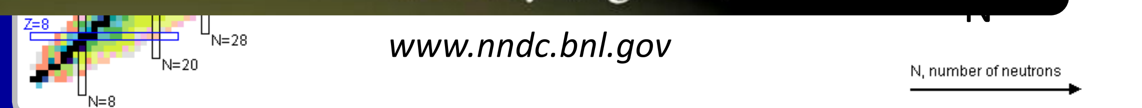

Quantum Chromodynamics

$$\mathcal{L} = \sum_q \bar{\psi}_{q,a} \left(i\delta_{ab}\gamma^\mu \partial_\mu - g\ \gamma^\mu t^C_{ab} A^C_\mu - m_q \delta_{ab} \right) \psi_{q,b} - \frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu}$$

$$F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g\ f_{ABC} A^B_\mu A^C_\nu$$

SU(3) gauge field theory;
quarks and gluons

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

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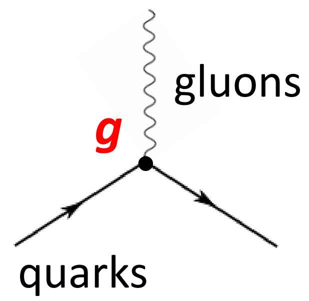

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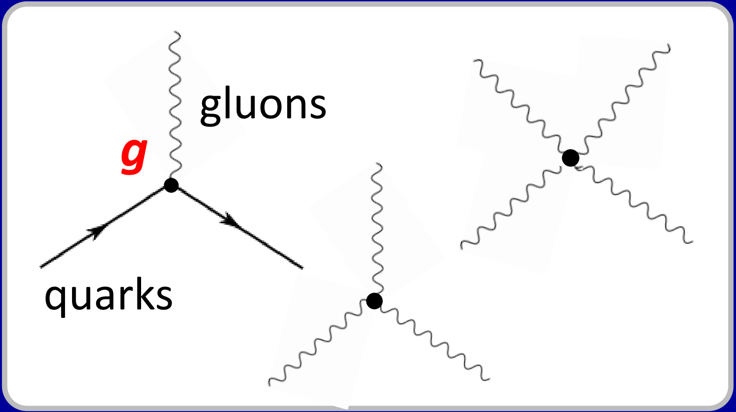
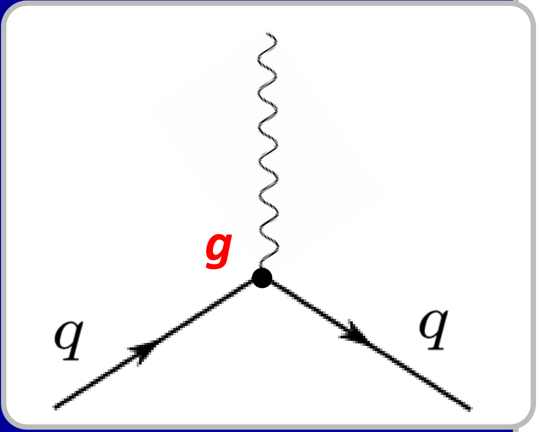
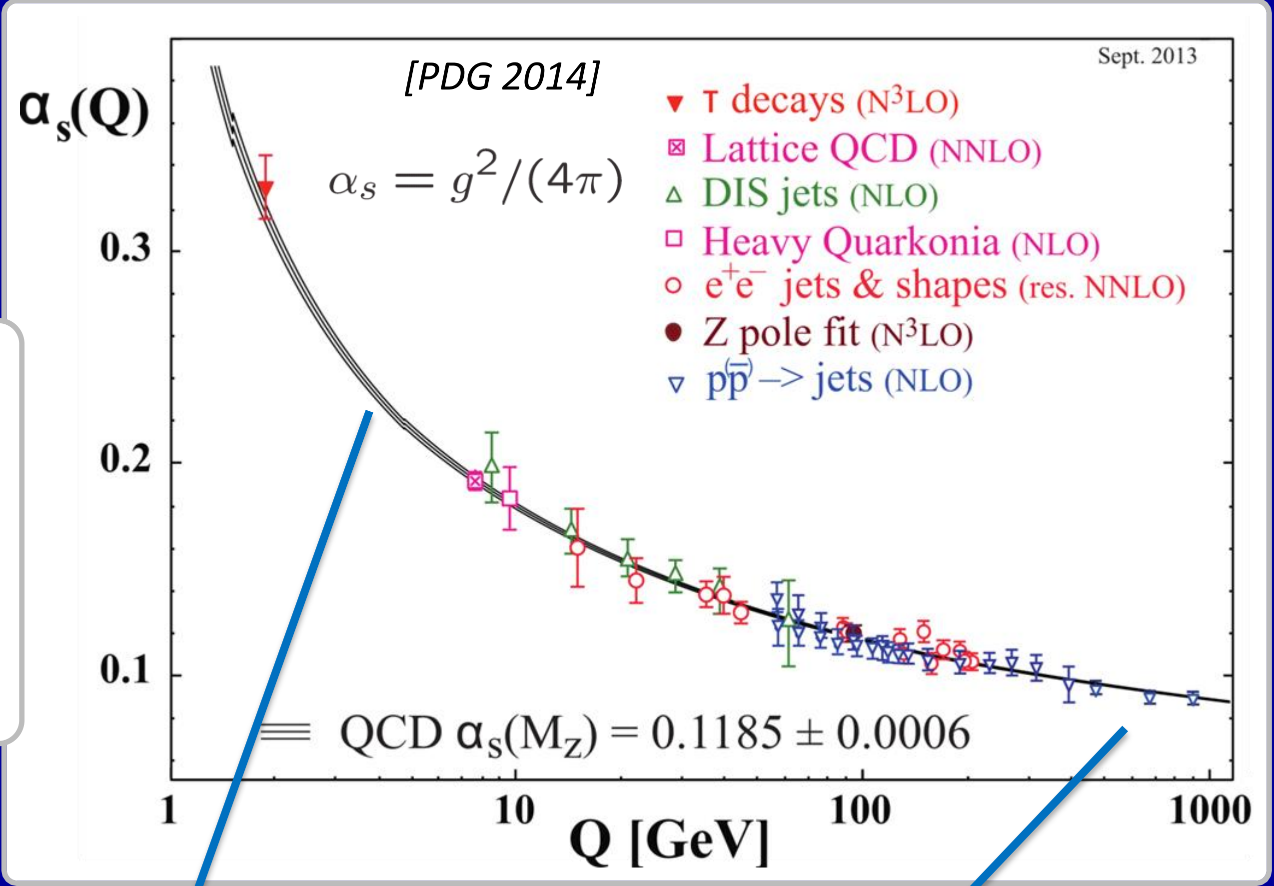


Quantum Chromodynamics

Running coupling constant



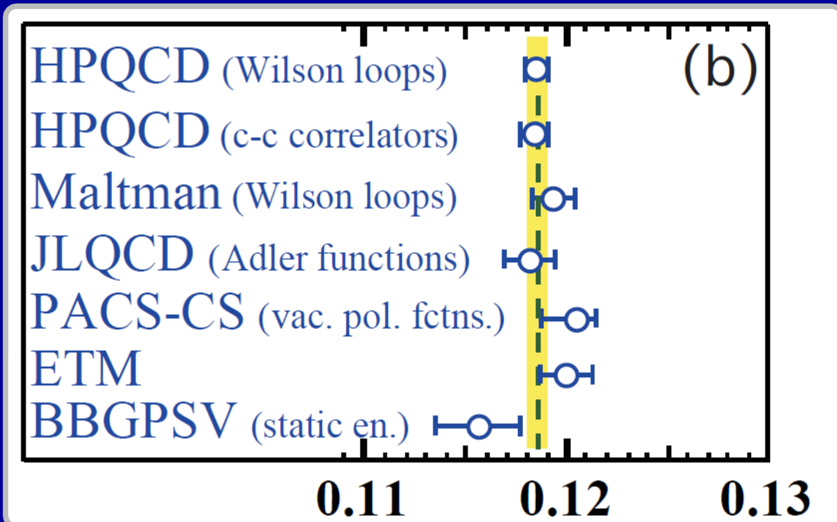


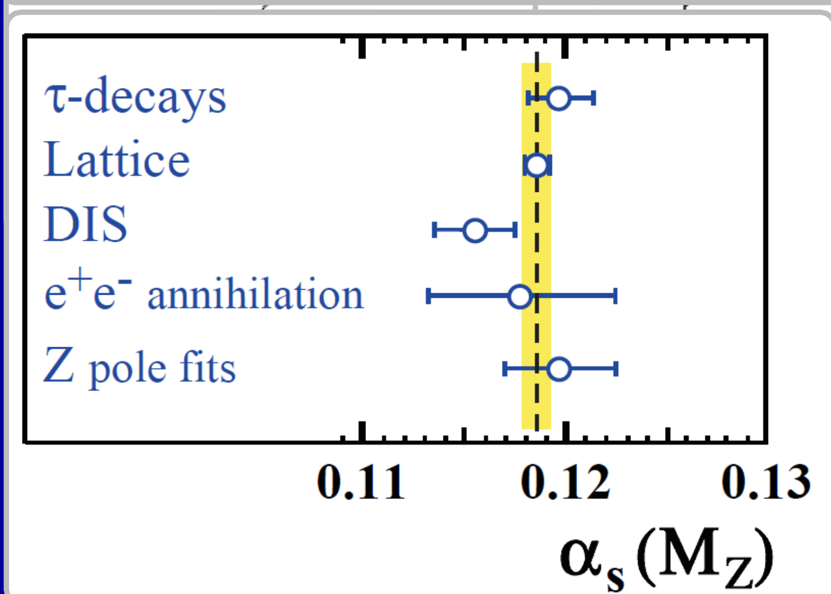
Large at low energies – can't make perturbative expansion

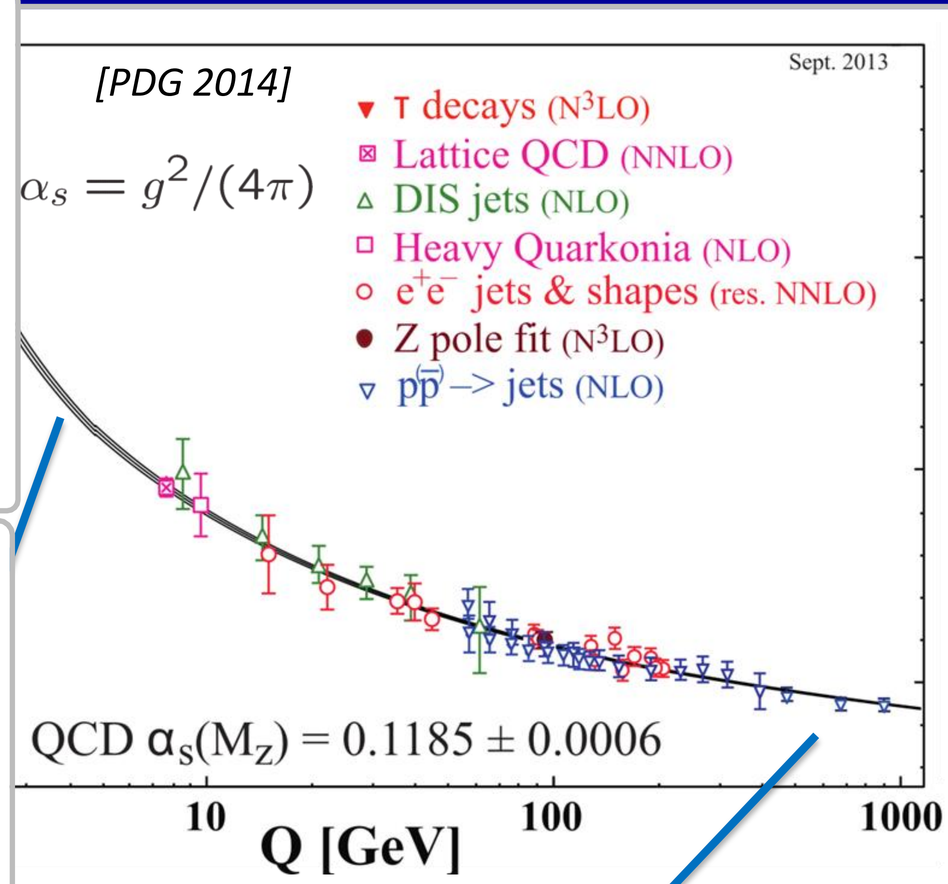
Asymptotic freedom

Quantum Chromodynamics

Running coupling constant







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Asymptotic freedom

Why lattice QCD?

- Non-perturbative regime:
  - *Confinement* of quarks into hadrons
  - Masses of hadrons (spectra), widths, transitions, ...
  - Nuclei
  - ...
- Models, effective field theories (EFTs), ...
  - Based on some symmetry properties, (expected) physics of QCD, approximation in some regime.
  - In general not derived from QCD
  - May be only approach (currently) applicable to some problems
  - Can be useful for getting insight into physics (complementary)
- **Lattice QCD – numerical non-perturbative calculations in QCD**

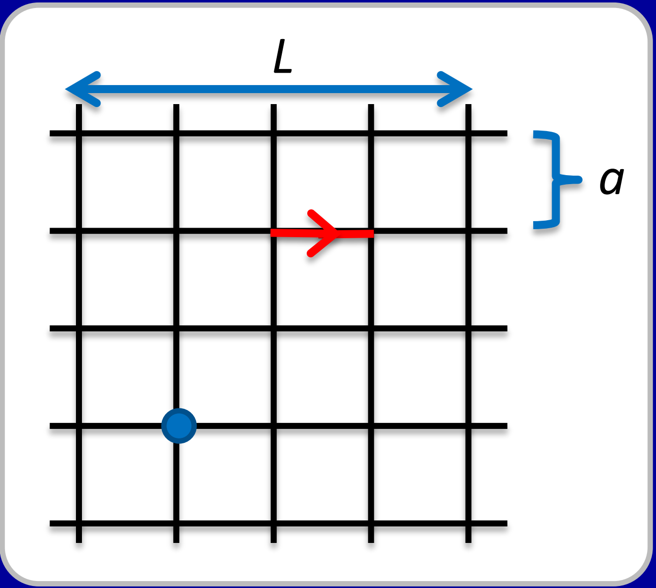
QCD on a lattice

Discretise theory on a 4d grid (spacing = *a*) – UV regulator

Finite volume (L^3 x T) → finite no. of d.o.f.

Quantised momenta

$\vec{p} = \frac{2\pi}{L_s}(n_x, n_y, n_z)$ for spatial periodic BCs



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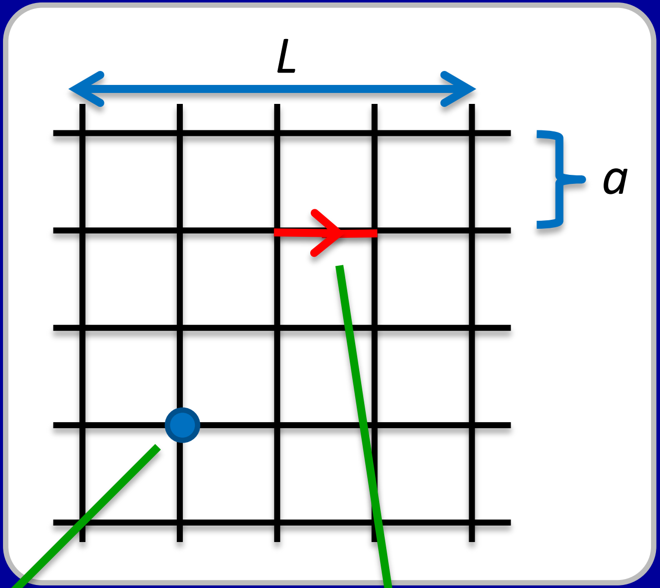

Gauge fields on links;
U is an element of SU(3)

Quark fields on lattice sites

$$A_\mu(x) \to U_{x,\mu} = e^{-aA_{x,\mu}}$$

$$\psi(x) \to \psi_x$$

$$\frac{\partial}{\partial x}\psi(x) \to \frac{1}{2a}\left(\psi_{x+1} - \psi_{x-1}\right)$$

QCD on a lattice

Path integral formulation (continuum)

– Integrate over all field configurations (infinite number)

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A e^{i\mathcal{S}[\psi,\bar{\psi},A]} \qquad S = \int d^4x \mathcal{L}[\psi, \bar{\psi}, A]$$

Observable:

$$\left\langle f[\psi, \bar{\psi}, A] \right\rangle = Z^{-1} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \; f[\psi, \bar{\psi}, A] \; e^{i\mathcal{S}[\psi,\bar{\psi},A]}$$

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Finite lattice – finite num. of quark and gluon fields to integrate over

Euclidean time: *t* → - i *t*

oscillating phase → decaying exponential

– amenable to numerical computation

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U e^{-\tilde{S}[\psi,\bar{\psi},U]}$$

c.f. statistical physics $\quad \mathrm{Tr}\,\exp(-\beta H)$

QCD on a lattice

Many possible discretisations which all → continuum QCD as *a* → 0.
'Improved' actions reduce discretisation effects, e.g. O(*a*).

Generic Euclidean action (gauge invariant):

$$\tilde{S} = \sum_{q,x,y} \bar{\psi}_{q,x} \; Q_{x,y}[U] \; \psi_{q,y} \; + \; \tilde{S}_{gauge}[U]$$

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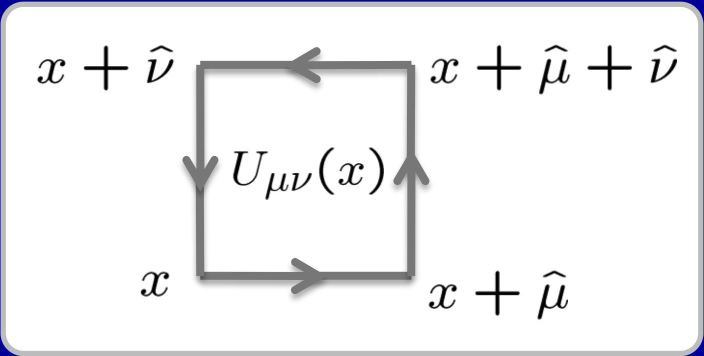
Gauge fields, e.g. $\tilde{S}_{gauge} = \frac{2}{g^2} \sum_x \sum_{\mu<\nu} \mathrm{ReTr}\left[1 - U_{\mu\nu}(x)\right]$



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'Naive' fermions:

$$Q_{x,y}[U] = a^4 \sum_\mu \frac{1}{2a} \gamma_\mu \left(U_\mu(x)\delta_{x,y+\hat{\mu}} - U_\mu(x-\mu)\delta_{x,y-\hat{\mu}} \right) + m_q \delta_{x,y}$$

Technical problems with this...
Various solutions each with advantages and disadvantages

QCD on a lattice

$$< \bar{\psi}(x_1)\psi(x_0) >= Z^{-1} \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}U \; \bar{\psi}(x_1)\psi(x_0) \; e^{-\tilde{S}[\psi,\bar{\psi},U]}$$

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Gauge fields (bosons) – complex matrices

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Fermion fields – anticommuting 'Grassmann' numbers

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Bilinear in fermion fields

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$$= Z^{-1} \int \mathcal{D}U \; Q^{-1}_{x_1 x_0}[U] \; \det(Q[U]) \; e^{-\tilde{\mathcal{S}}'[U]}$$

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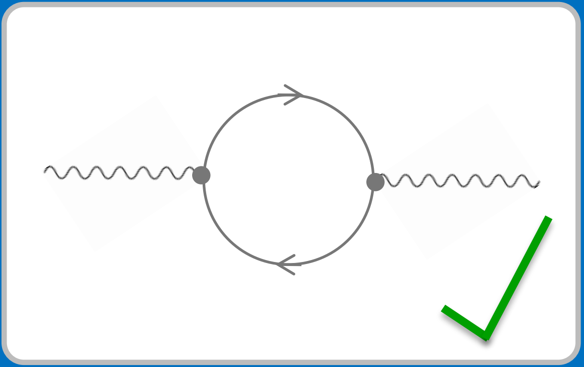
Fermion det. – nonlocal function of U
→ very computationally expensive

Q^{-1} and det(Q) more expensive for small m_π

Historically, quenched approx: set det[Q] = 1
– don't include quark loops



Now most calculations are dynamical ('unquenched')
– include det[Q]

QCD on a lattice

$$\int \mathcal{D}U \;\; f[U] \; \det(Q[U]) \;\, e^{-\tilde{S}'[U]}$$

Use **Importance Sampling Monte Carlo** to evaluate numerically

Dominated by field cfgs of *U* where this is **large**

Sample integral with prob. $\sim \left[\det(Q[U]) \;\, e^{-\tilde{S}'[U]}\right]$

det(*Q*[*U*]) must be re-calculated for each *U* – expensive

Sample integral a finite number of times (num. of cfgs.)
→ mean and statistical uncertainty

QCD on a lattice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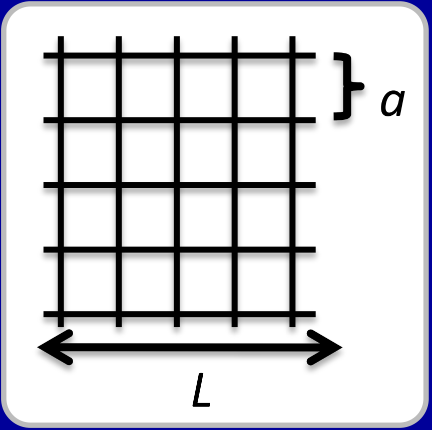
Lattice → QCD

- Continuum limit: lattice spacing, $a \rightarrow 0$
  L = const, so $N = L/a \rightarrow \infty$
- Volume, L >> physical size of problem
  e.g. $L\, m_\pi >> 1$
- Pion mass, $m_\pi \rightarrow$ physical m_π



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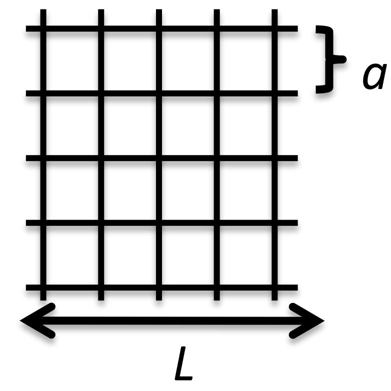


Setting the scale (determine *a* in physical units)

- Every dimensional quantity measured in terms of a
- 'Set the scale' by comparing with a physical observable calculated on the lattice to experimental value
- E.g. static quark potential, Ω baryon mass, ...

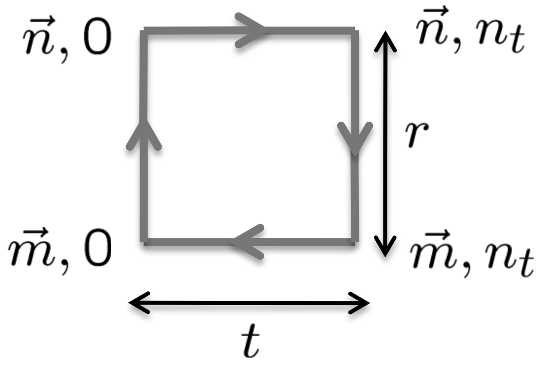
Set **bare quark masses (m_q) in action** by comparing lattice computations of hadron masses with experimental masses

Some applications

Static potential from lattice QCD

Potential between two infinitely heavy quarks (static colour sources)

$$W(\vec{m}, \vec{n}, n_t = t/a) \qquad \mathrm{Tr}[UUUU]$$



$$\langle W \rangle \propto e^{-tV(r)} \left(1 + \mathcal{O}(e^{-t\Delta_E})\right)$$

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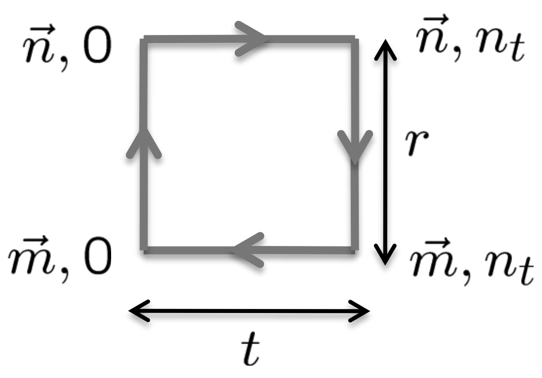

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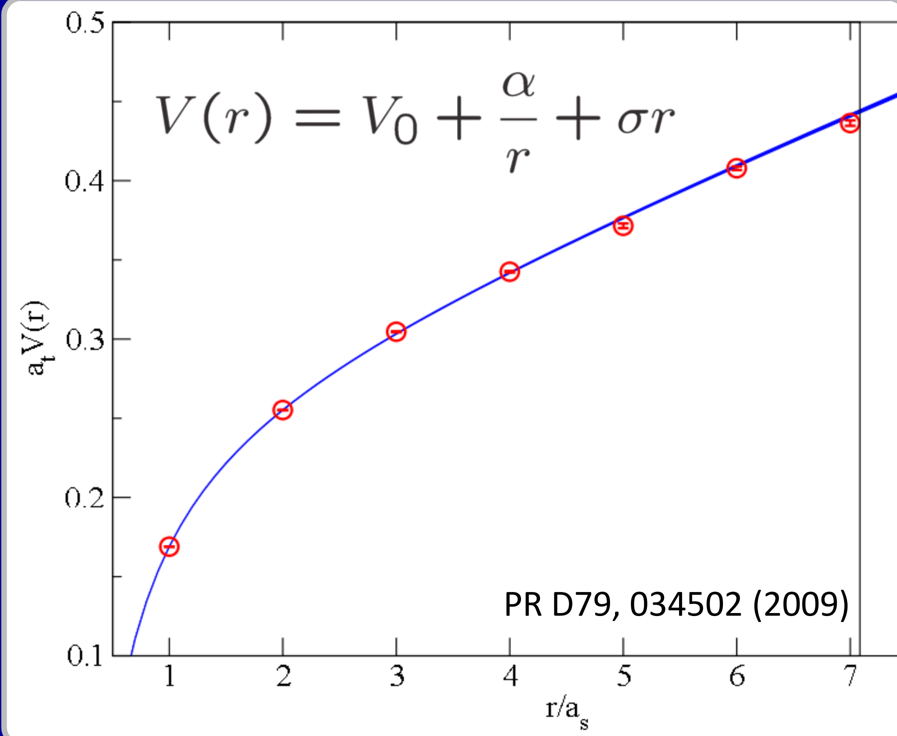

Compare length scale with experimental charmonium and bottomonium spectra

Spectroscopy on the lattice

Spectroscopy on the lattice

Calculate energies and matrix elements ("overlaps", Z) from 2-point correlation functions of hadron interpolating fields "operators"

$$\bar{\psi}\Gamma\psi \qquad \epsilon^{abc}\psi_a\psi_b\psi_c \qquad +\overleftrightarrow{D}_i$$

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

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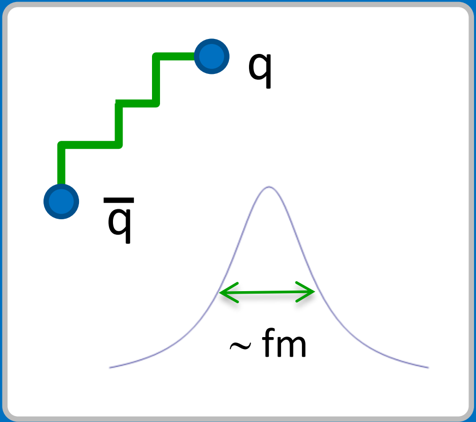
- $\bar{\psi}(\vec{x})\Gamma\psi(\vec{x})$ is local but hadrons are extended objects ~ 1 fm.
- Improve overlap onto states of interest (reduce overlap with UV modes) by **spatially smearing quark fields**.

$\psi(\vec{x},t) = \sum_{\vec{y}} F(\vec{x},\vec{y},U(t))\psi(\vec{y},t)$



Diagrams

$$< \bar{\psi}(x_1)\psi(x_1)\bar{\psi}(x_0)\psi(x_0) > =$$

$$Z^{-1}\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}U\ \bar{\psi}(x_1)\psi(x_1)\bar{\psi}(x_0)\psi(x_0)\ e^{-\tilde{S}[\psi,\bar{\psi},U]}$$

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Wick's theorem: contract in all possible ways

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Wick's theorem: contract in all possible ways

$$\rightarrow \quad -Q^{-1}_{x_1x_0}[U]Q^{-1}_{x_0x_1}[U] + Q^{-1}_{x_1x_1}[U]Q^{-1}_{x_0x_0}[U]$$

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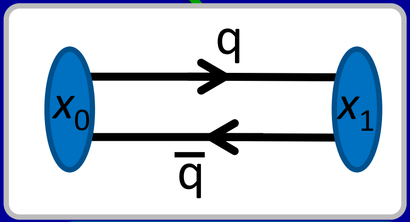
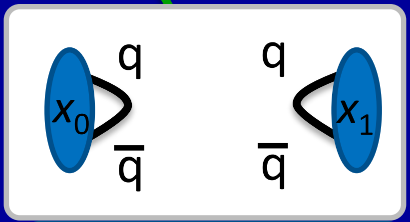
$$\rightarrow \quad -Q^{-1}_{x_1x_0}[U]Q^{-1}_{x_0x_1}[U] + Q^{-1}_{x_1x_1}[U]Q^{-1}_{x_0x_0}[U]$$

Diagrammatically:



'Connected'



'Disconnected'

Diagrams

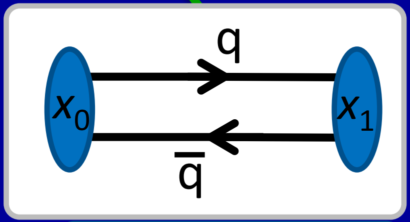
$$< \bar{\psi}(x_1)\psi(x_1)\bar{\psi}(x_0)\psi(x_0) > =$$

$$Z^{-1} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \, \bar{\psi}(x_1)\psi(x_1)\bar{\psi}(x_0)\psi(x_0) \, e^{-\tilde{S}[\psi,\bar{\psi},U]}$$

Wick's theorem: contract in all possible ways

$$\rightarrow \quad - Q^{-1}_{x_1 x_0}[U] Q^{-1}_{x_0 x_1}[U] + Q^{-1}_{x_1 x_1}[U] Q^{-1}_{x_0 x_0}[U]$$

Diagrammatically:



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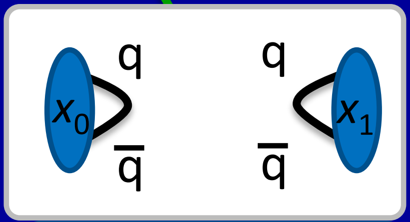

'Disconnected'

N.B. these are **not** perturbation theory diagrams

Spectroscopy on the lattice

Calculate energies and matrix elements ("overlaps", Z) from 2-point correlation functions of meson interpolating fields "operators"

$\bar{\psi}\Gamma\psi \qquad \epsilon^{abc}\psi_a\psi_b\psi_c \qquad +\overleftrightarrow{D}_i$

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

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$$
\begin{aligned}
C_{ij}(t) &= \left\langle 0 \left| \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0) \right| 0 \right\rangle \\
&= \sum_n 1/(2E_n) \left\langle 0 \left| O_i(t) \right| n \right\rangle \left\langle n \left| O_j^\dagger(0) \right| 0 \right\rangle \\
&= \sum_n \frac{e^{-E_n\ t}}{2\ E_n} \left\langle 0 \left| O_i(0) \right| n \right\rangle \left\langle n \left| O_j^\dagger(0) \right| 0 \right\rangle
\end{aligned}
$$

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$$Z_i^{(n)} \equiv < 0|O_i|n >$$

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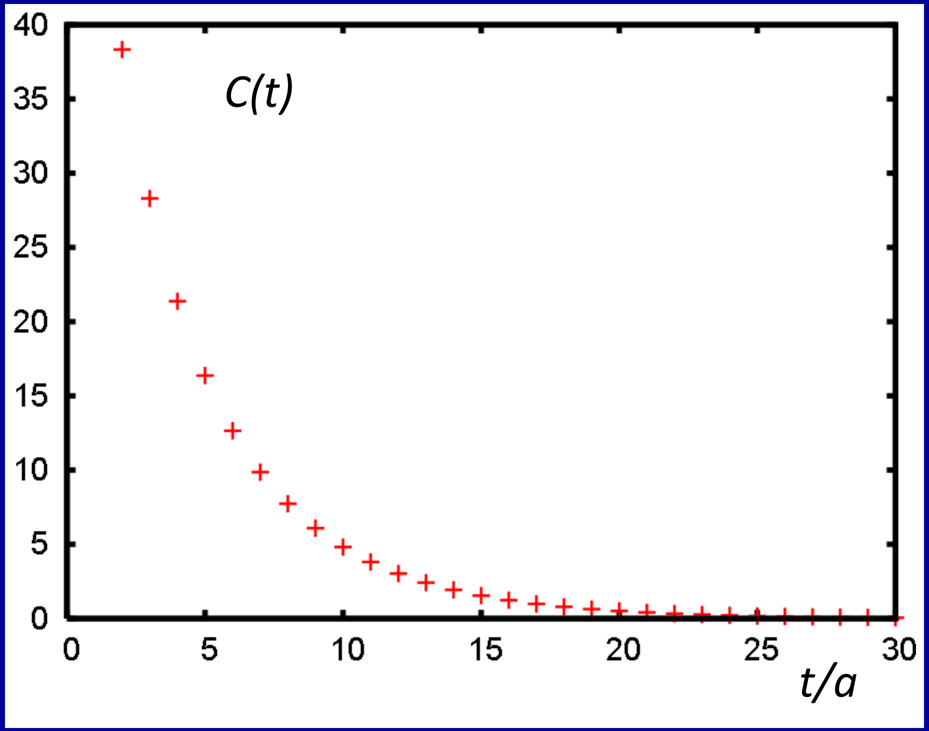
$$\xrightarrow[t\to\infty]{} \frac{Z_i^{(0)} Z_j^{(0)*}}{2E_0} e^{-E_0 t}$$

$$Z_i^{(n)} \equiv < 0|O_i|n >$$

Correlators

$$C(t) =< 0|\bar{\psi}(t)\gamma_i\psi(t) \cdot \bar{\psi}(0)\gamma_i\psi(0))|0 >$$

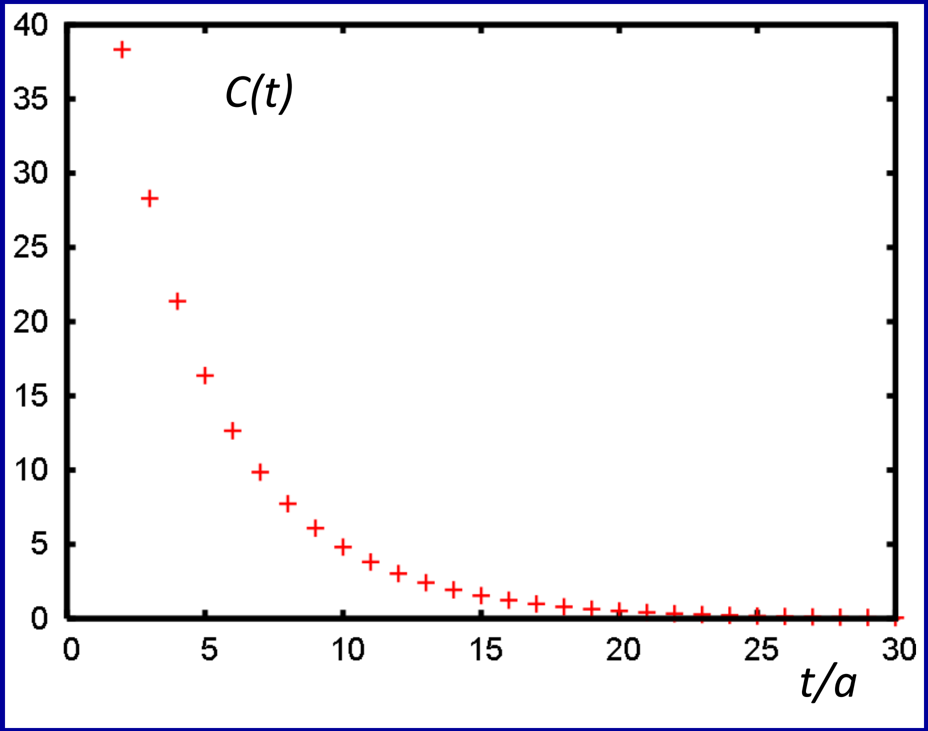


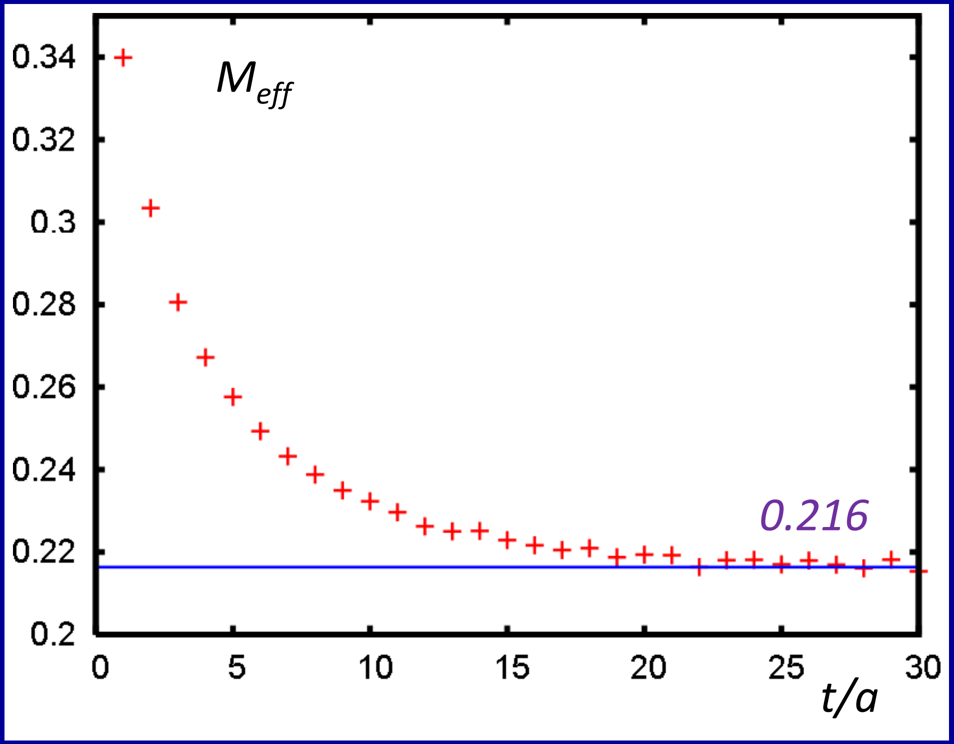
$$C(t) \sim e^{-E_0 t} + \ldots$$

Correlators

$C(t) =< 0|\bar{\psi}(t)\gamma_i\psi(t) \cdot \bar{\psi}(0)\gamma_i\psi(0))|0 >$

$M_{eff}(t) = -\ln\left[C(t+dt)/C(t)\right]/dt$



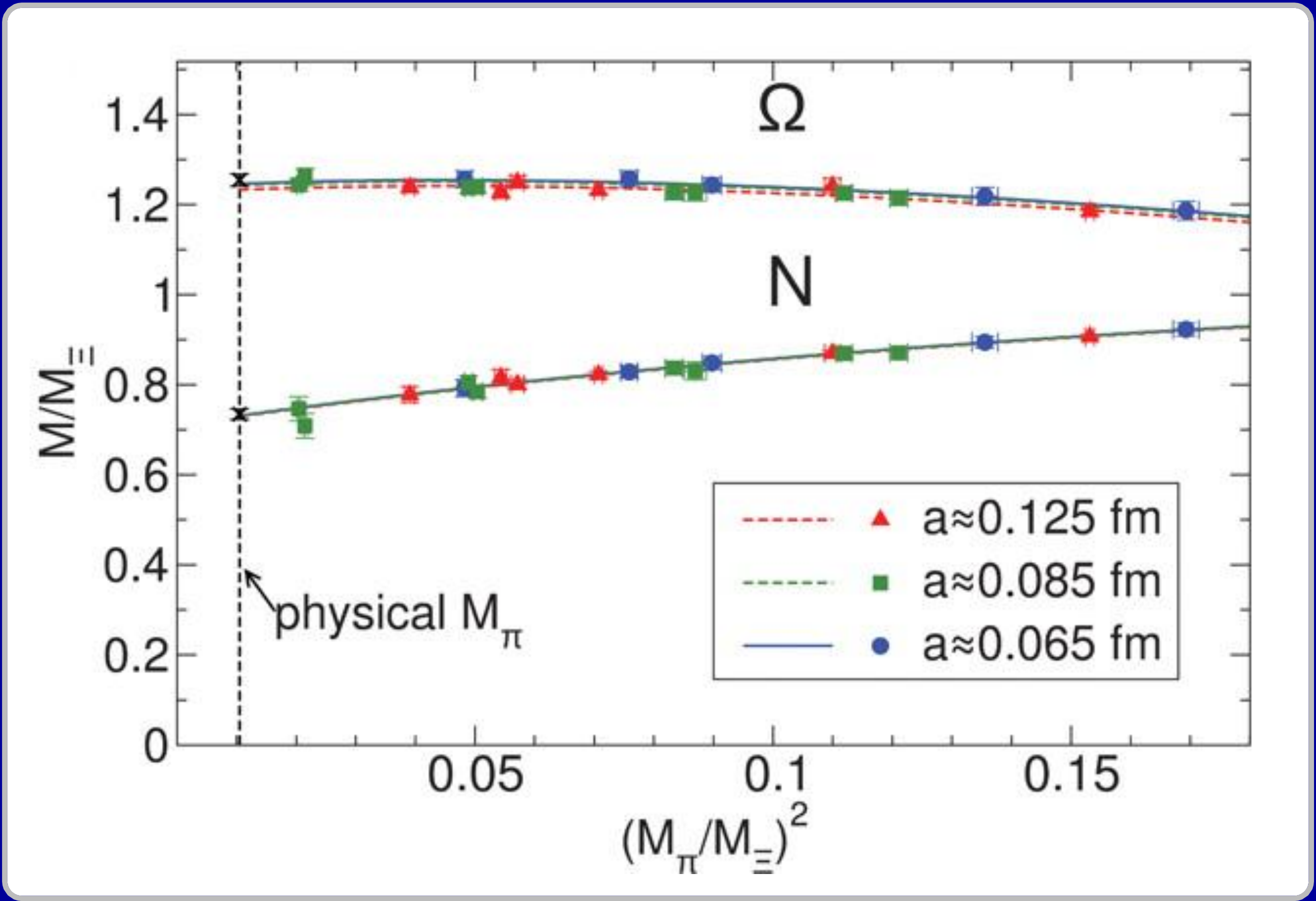


$C(t) \sim e^{-E_0 t} + \ldots$

$M_{eff}(t) = E_0 + \ldots$

Low-lying spectrum of hadrons

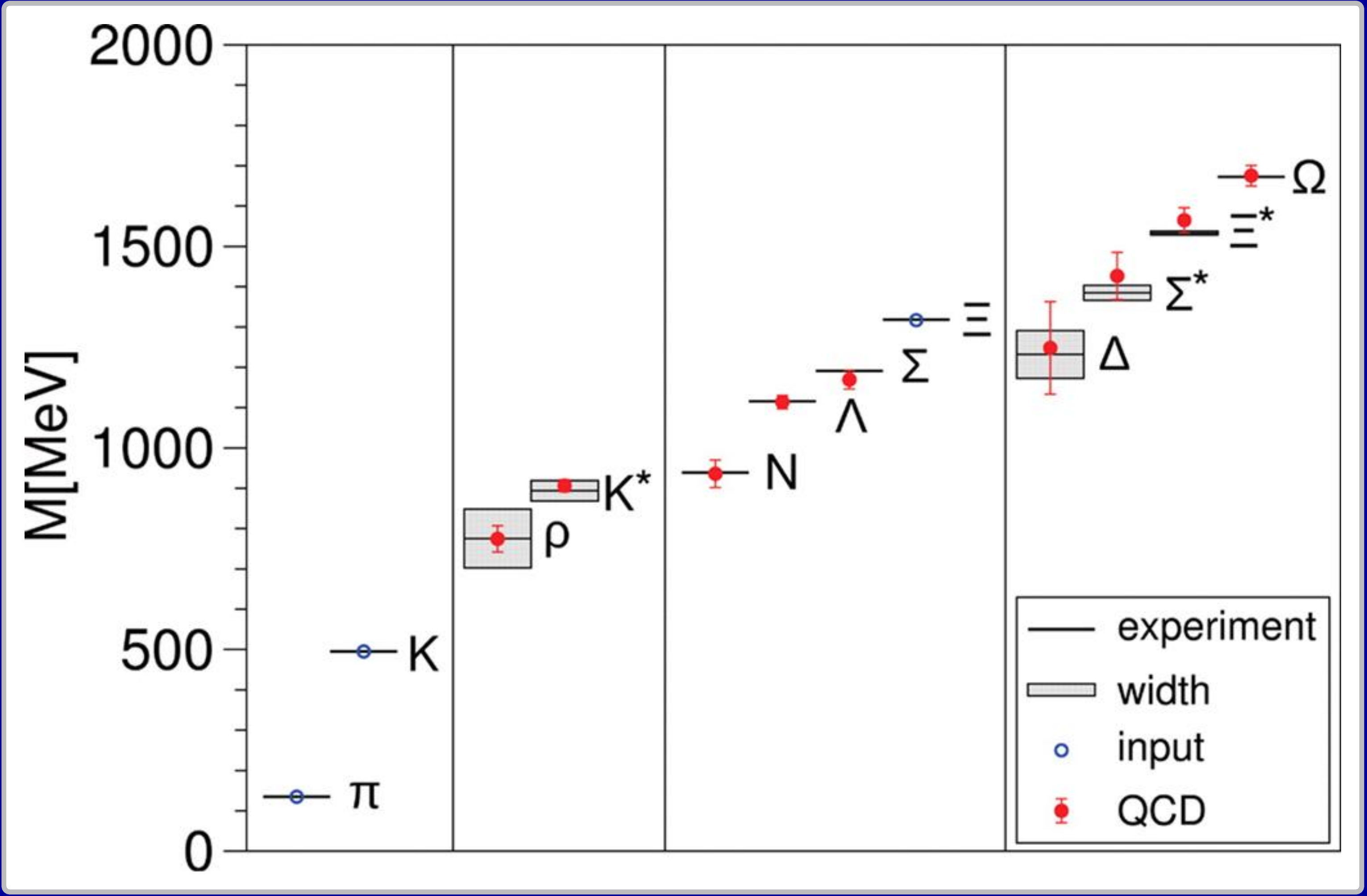
BMW Collaboration, Durr et al, Science 322, 1224 (2008)



Use only smeared local operators (e.g. γ_i). Set scale using M_Ξ
Nucleons & isovector mesons – only connected diagrams

Low-lying spectrum of hadrons

BMW Collaboration, Durr et al, Science 322, 1224 (2008)



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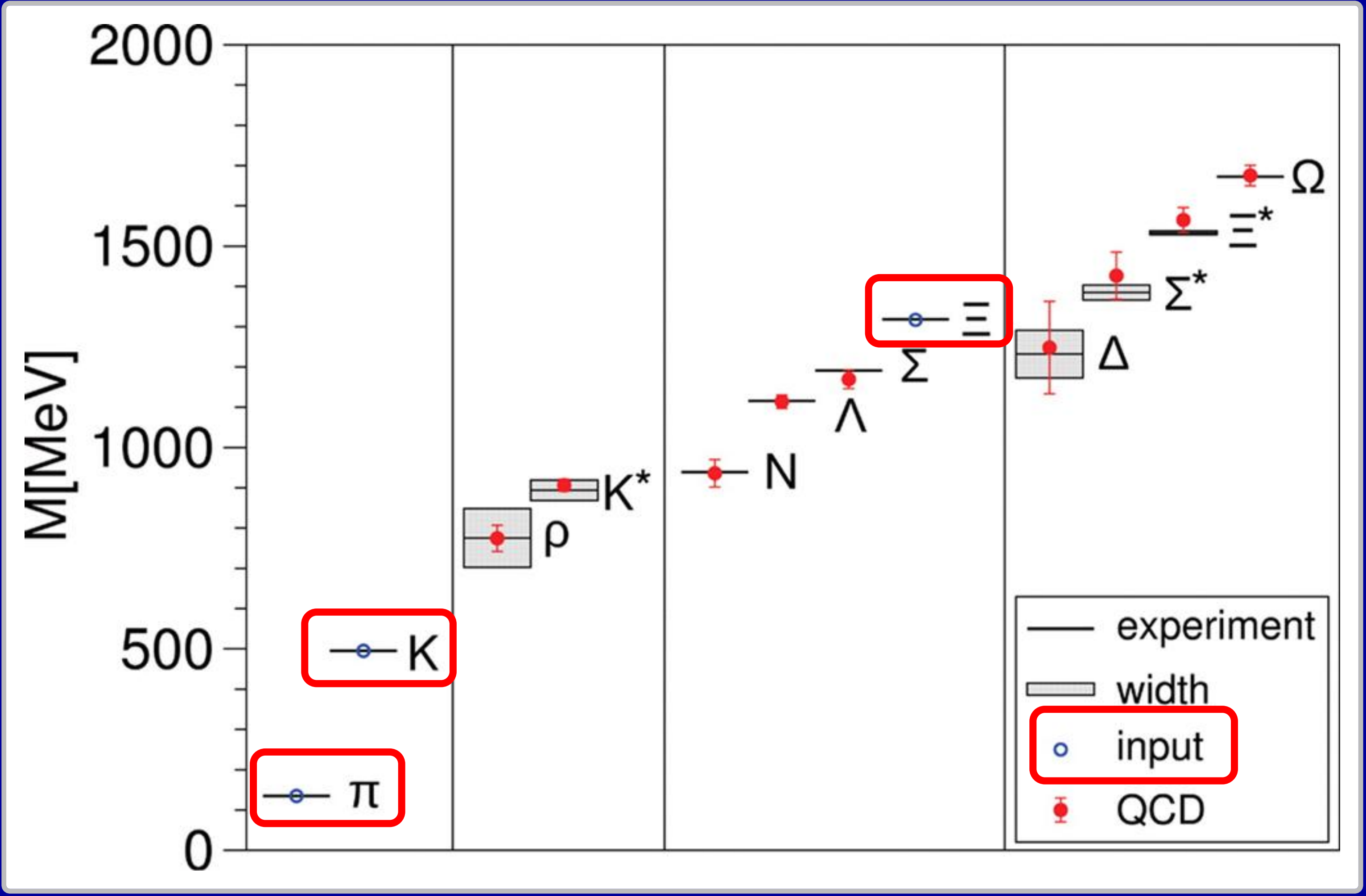
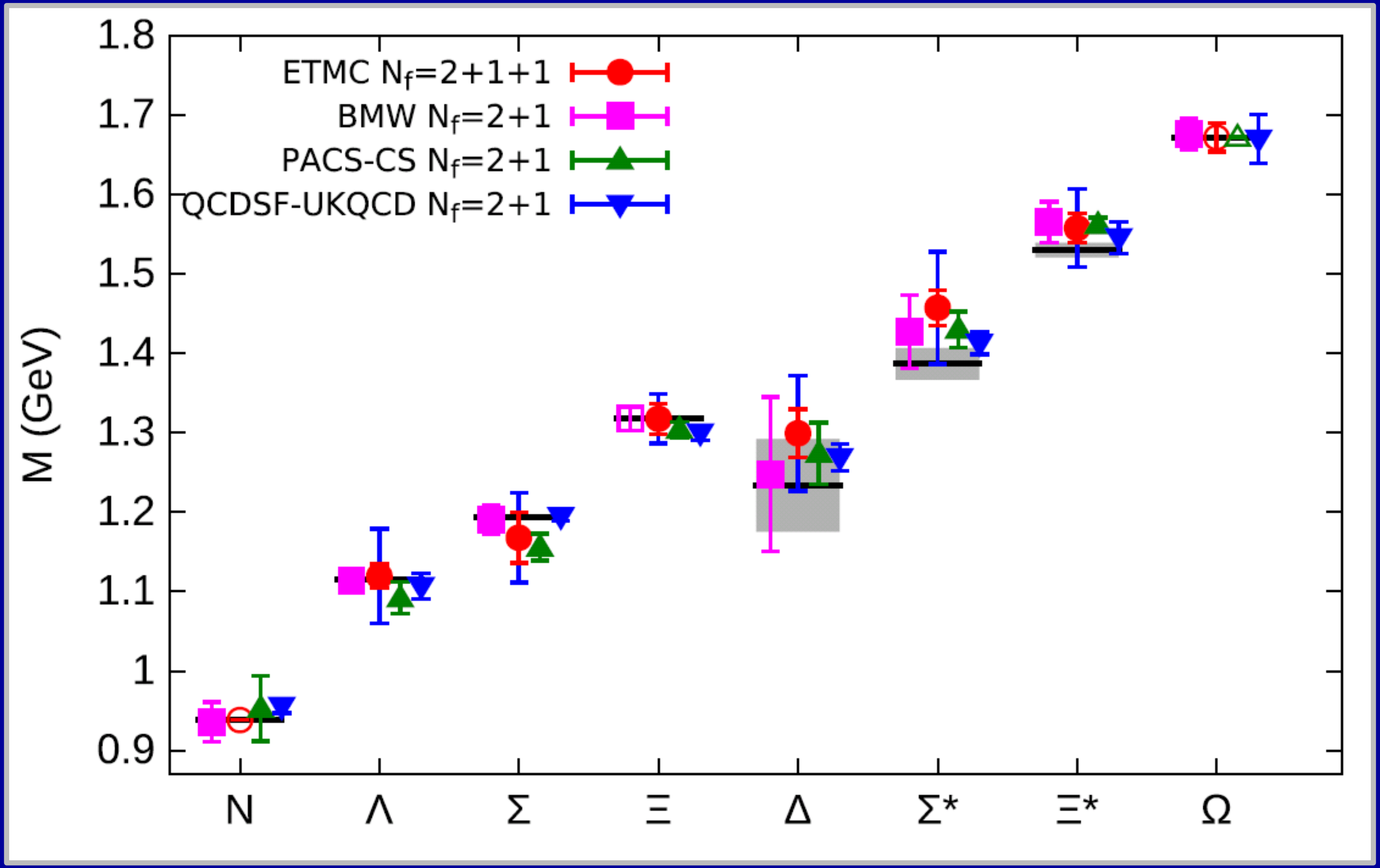


Light baryons – comparison

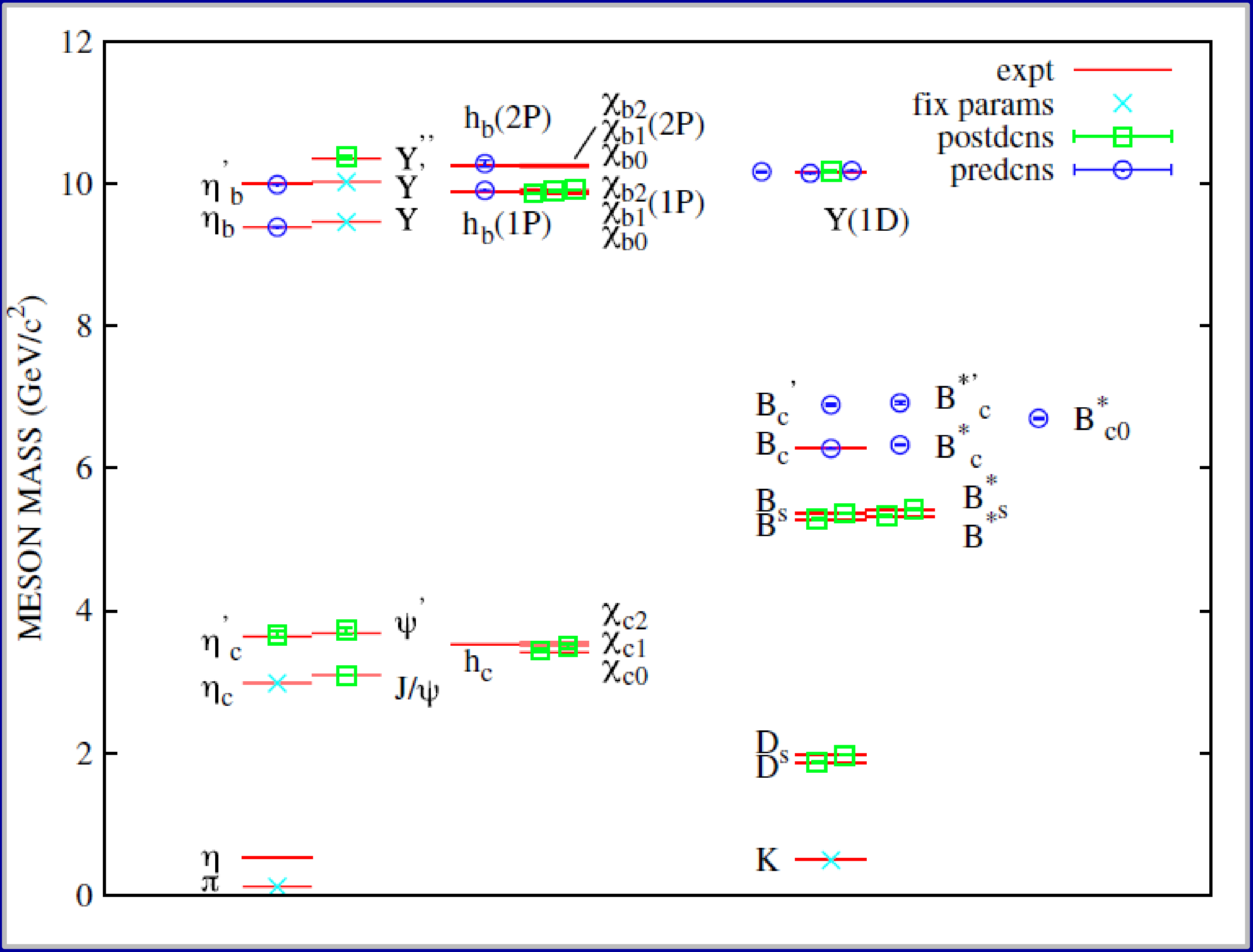
Alexandrou et al, PR D90, 074501 (2014)



Agreement between results from different lattice actions (extrapolated to continuum limit and physical m_π)

Quarkonia and heavy-light mesons

Dowdall et al (HPQCD)
[PR D86, 094510 (2012)]



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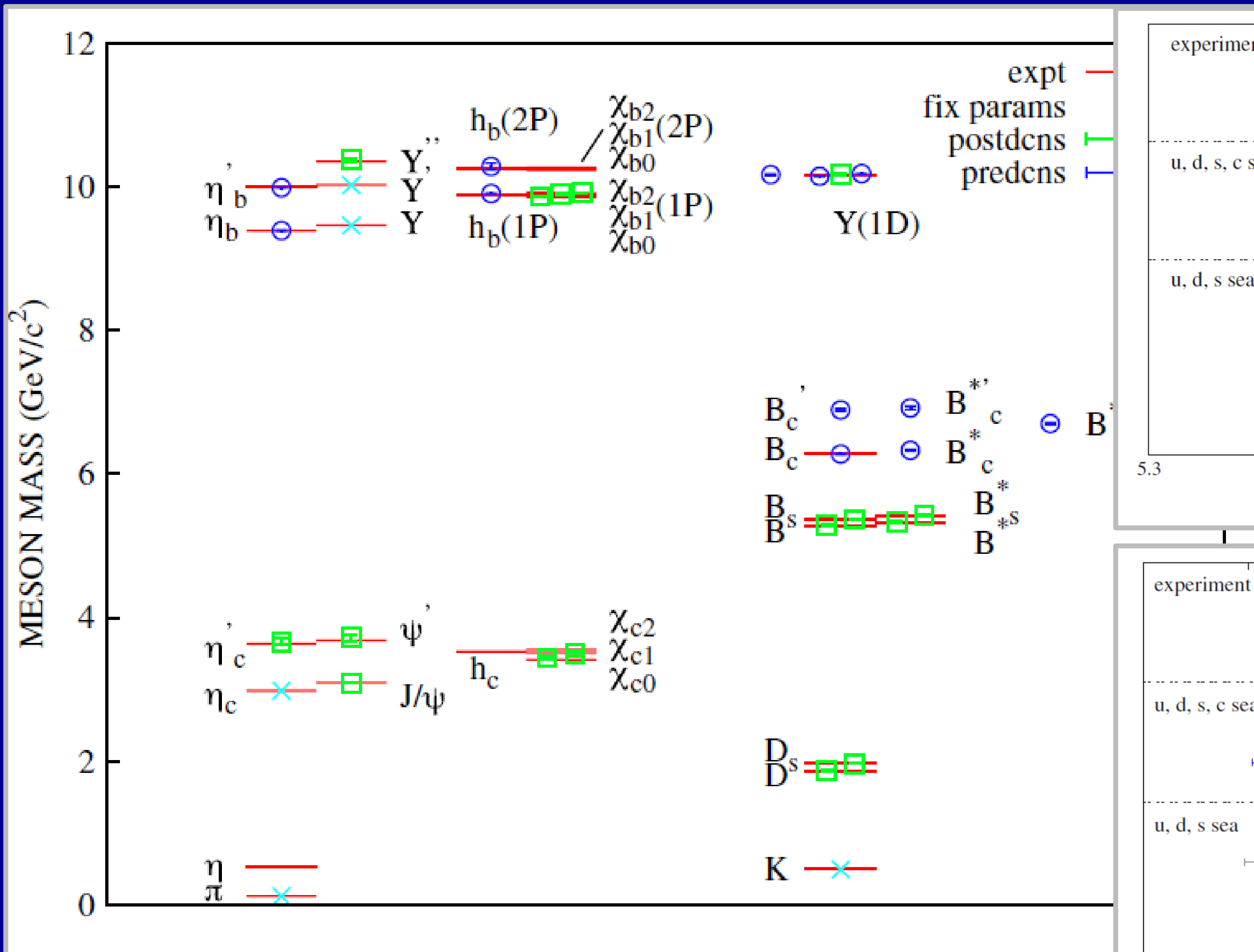

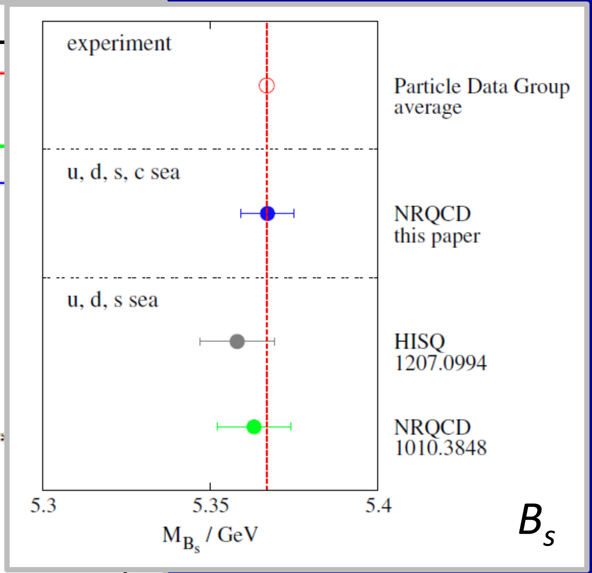

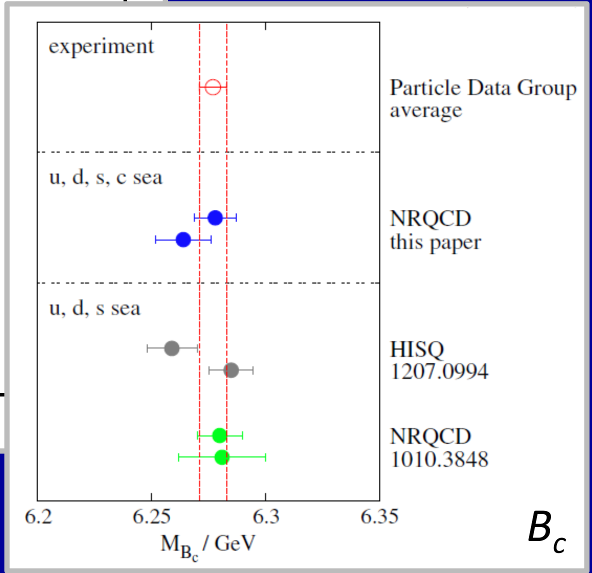

More applications

- Hadron form factors, radiative transitions
- Hadron structure, TMDs, etc
- Decay constants
- Weak matrix elements, flavour physics
  → SM tests and BSM constraints
- Nuclear physics / nuclei
- QCD at finite temperature and density
- Other field theories, BSM physics
- …

Summary of lecture 1

- Why lattice quantum chromodynamics?
- A (very brief) introduction to lattice QCD
- Applications: static potential and some 'simple' hadron spectroscopy

Next time

- Excited hadron spectroscopy

Hadron spectroscopy from lattice QCD

Lecture 2

- Background on hadron spectroscopy
- Excited hadron spectroscopy in lattice QCD
  - Mesons
  - Baryons

(I won't review all lattice calculations of hadron spectra)

Hadron spectroscopy

Masses and other properties of hadrons probe the non-perturbative regime of QCD.

- Relevant degrees of freedom?
- Confinement?
- Role of gluons?

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Experiments

LHCb

ATLAS CMS

CLAS12 GlueX

+ others at 12 GeV JLab

BESIII KLOE2

PANDA

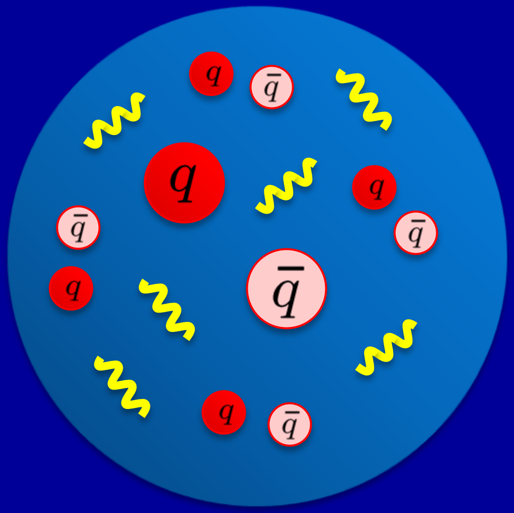
+ others at GSI

COMPASS

Belle II

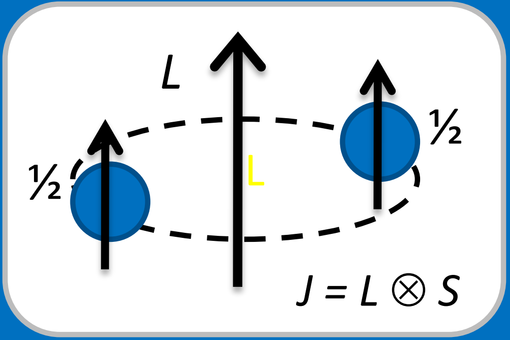
ELSA MAMI J-PARC Spring-8

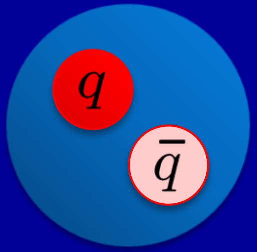
Hadron spectroscopy – mesons



Hadron spectroscopy – mesons

Quark-antiquark pair: $n\ {}^{2S+1}L_J$



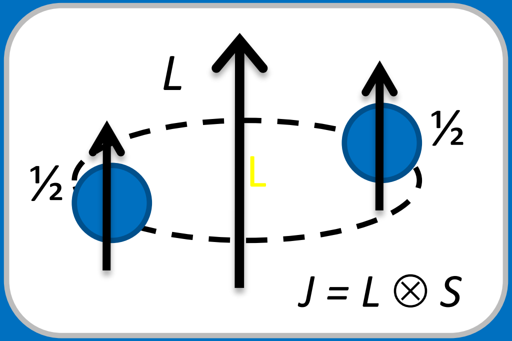


Hadron spectroscopy – mesons

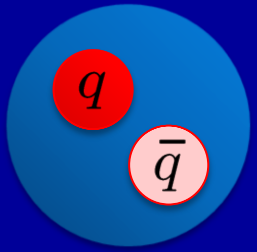
Quark-antiquark pair: $n\ ^{2S+1}L_J$

Parity: $P = (-1)^{(L+1)}$

Charge Conj Sym: $C = (-1)^{(L+S)}$



$J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}, 1^{+-}, 2^{--}, 2^{++}, 2^{-+}, \ldots$



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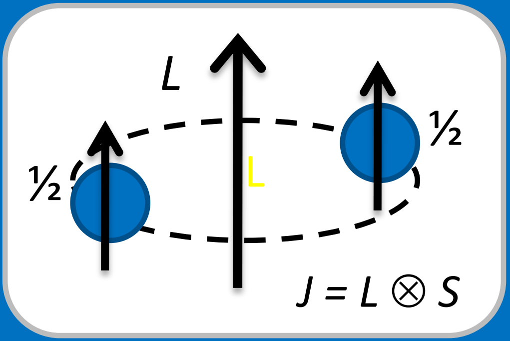


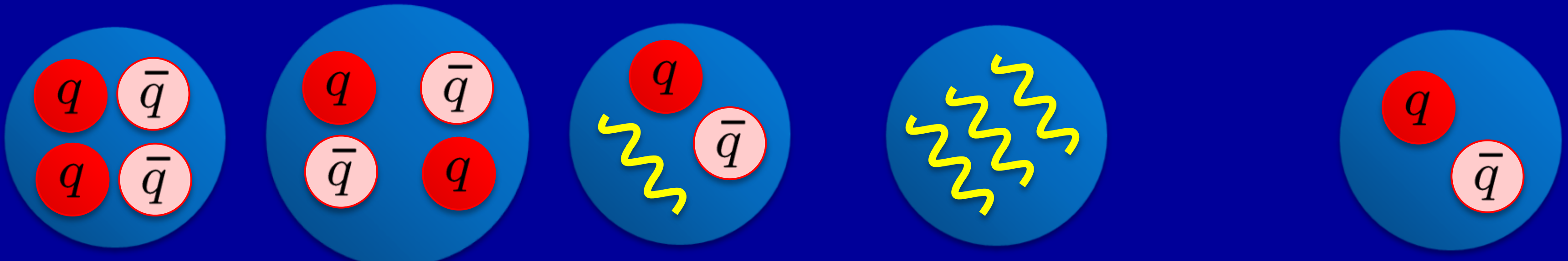
$J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}, 1^{+-}, 2^{--}, 2^{++}, 2^{-+}, \ldots$

Exotic J^{PC} **(0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, ...)** or flavour quantum numbers – can't just be a $q\bar{q}$ pair

E.g. multiquark systems (tetraquarks, molecular mesons)

Hybrid mesons (gluonic field excited)

Glueballs



Hadron spectroscopy – mesons

$m_u = m_d$ – isospin sym.

$$I_z = \begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix} \quad \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

Flavours of mesons

Light mesons u, d, s (anti)quarks $\gtrsim 135$ MeV

Isovectors (I = 1) e.g. π, ρ, a_1 $\quad \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$

Isoscalars (I = 0) e.g. η, η', ω, ϕ $\quad \ell\bar{\ell} \equiv \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) \quad s\bar{s}$

Kaons (I = 1/2) e.g. K, K* $\quad u\bar{s} \quad d\bar{s}$

Hadron spectroscopy – mesons

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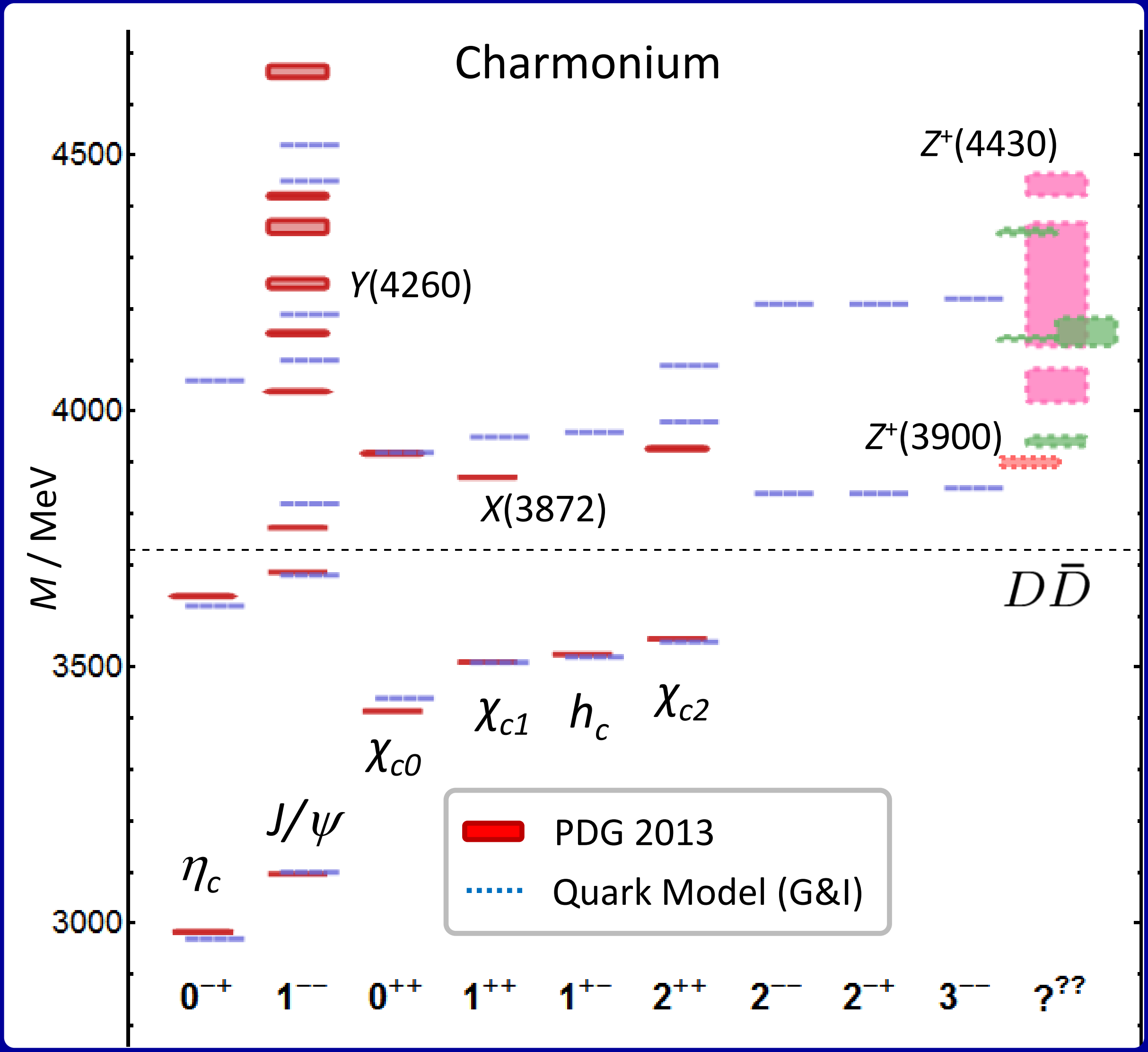
Kaons (I = 1/2) e.g. K, K* $\quad u\bar{s} \quad d\bar{s}$

Charmonium e.g. J/ψ $\quad c\bar{c} \quad \gtrsim 3000\ \mathrm{MeV}$

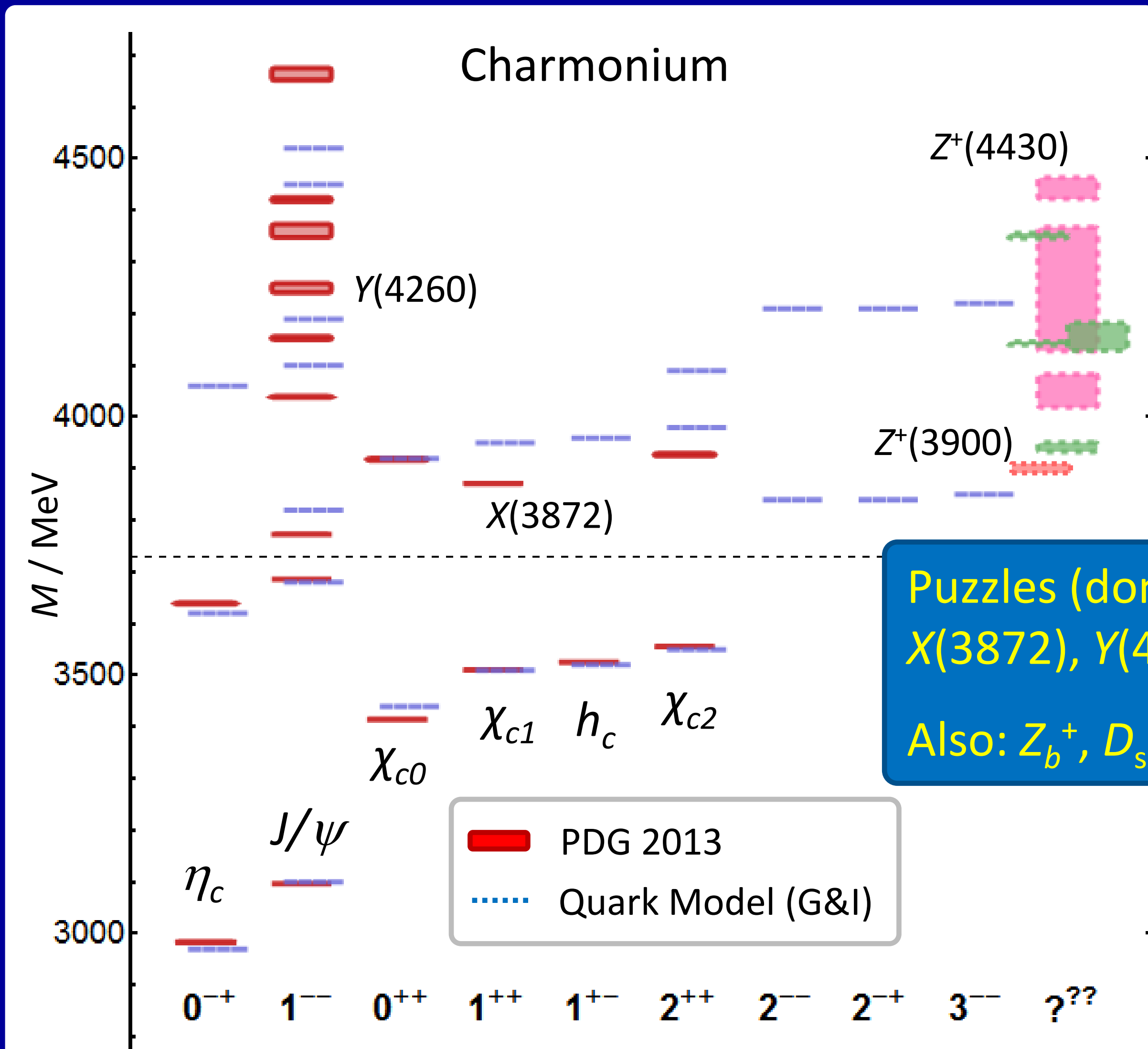
Bottomonium e.g. ϒ $\quad b\bar{b} \quad \gtrsim 9400\ \mathrm{MeV}$

D, D_s, *B*, ...

Hadron spectroscopy – mesons



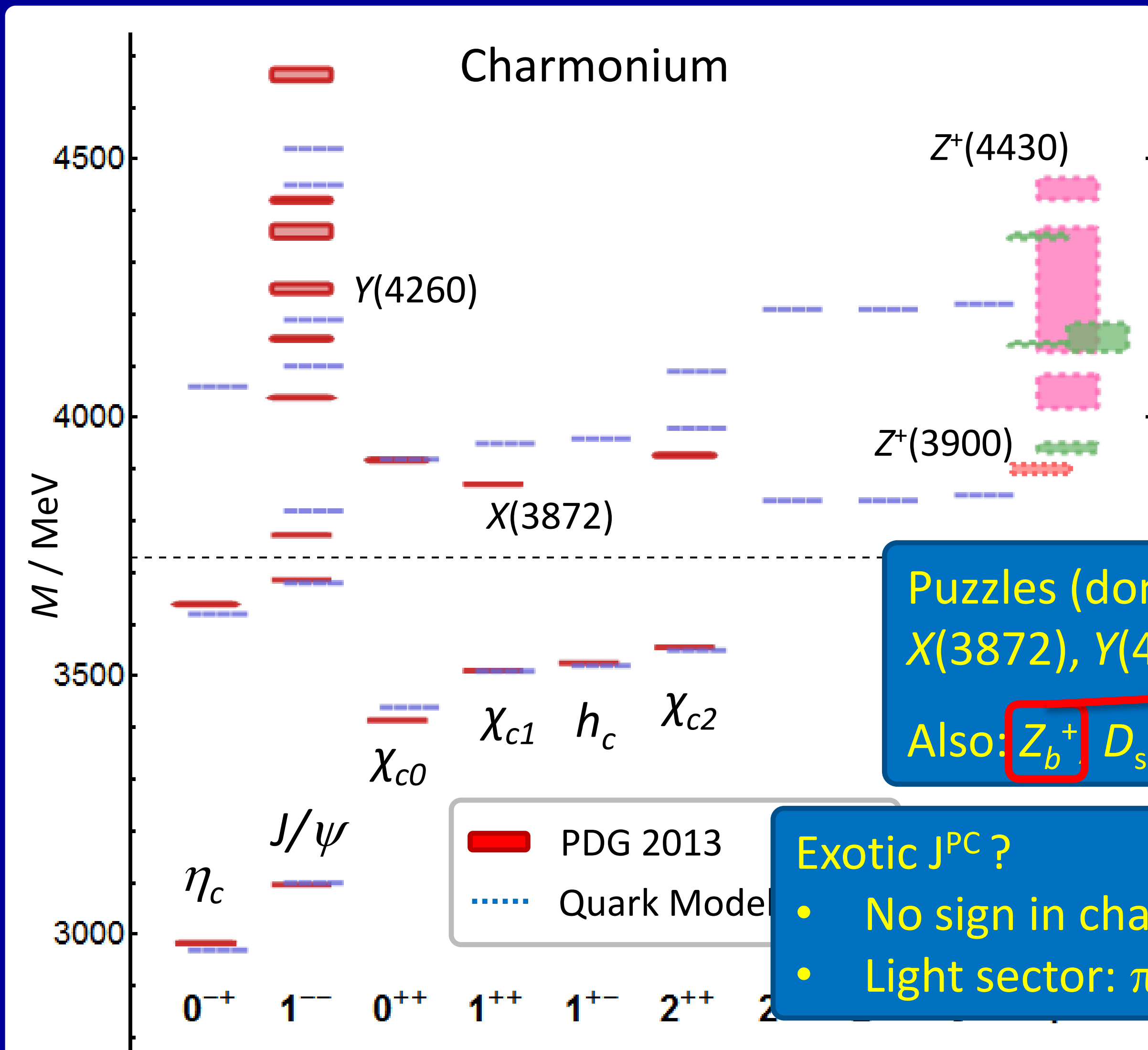
Hadron spectroscopy – mesons



Puzzles (don't fit expected pattern):
$X(3872)$, $Y(4260)$, $Z^+(4430)$, $Z_c^+(3900)$,

Also: Z_b^+, $D_s(2317)$, light scalars, ...

Hadron spectroscopy – mesons



Flavour exotics

Puzzles (don't fit expected pattern):
$X(3872)$, $Y(4260)$, $Z^+(4430)$, $Z_c^+(3900)$,

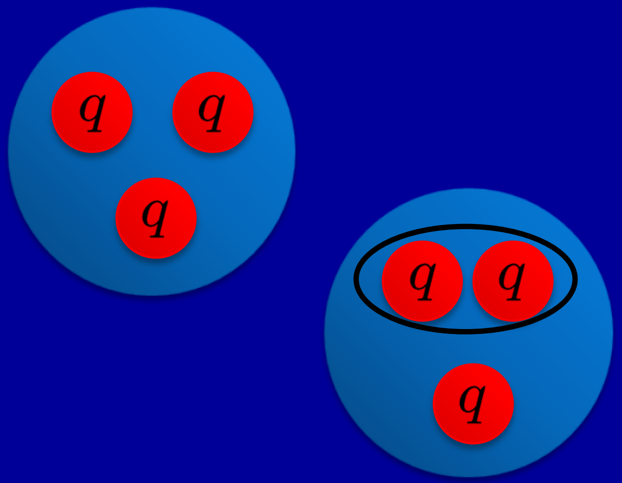
Also: Z_b^+, $D_s(2317)$, light scalars, ...

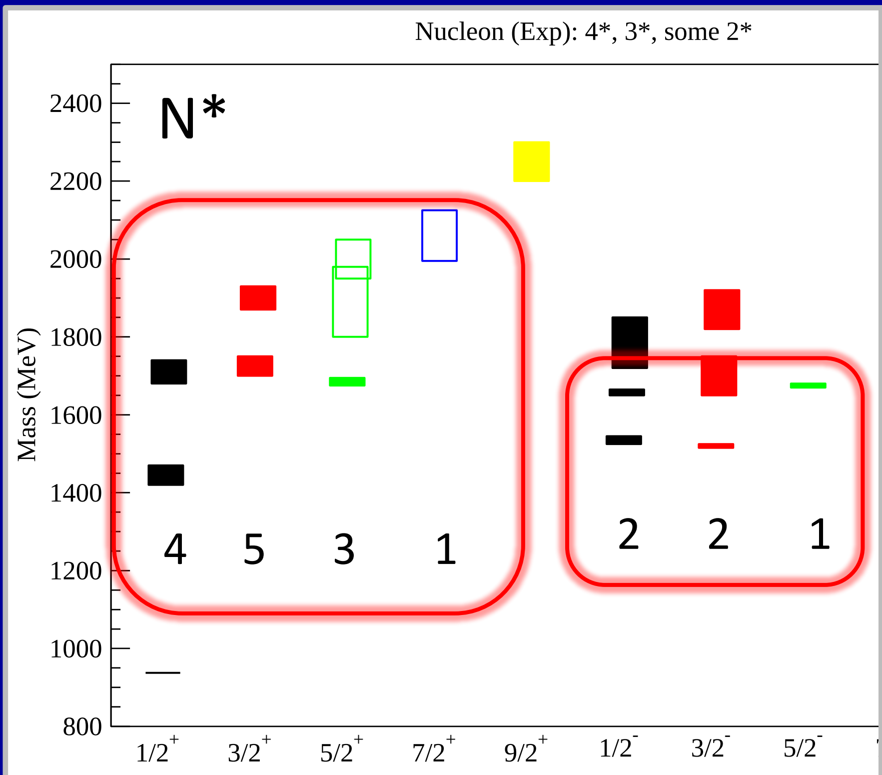
Exotic J^{PC} ?

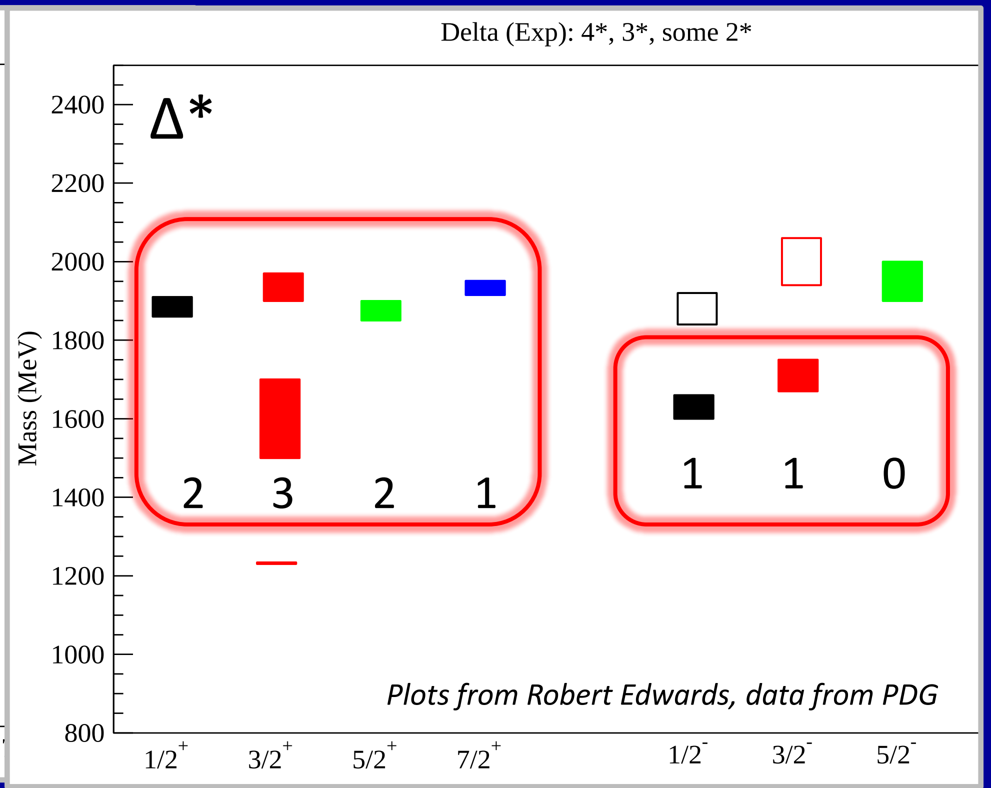
- No sign in charmonium or bottomonium
- Light sector: $\pi_1(1600)$ (1^{-+}) needs conf.

Hadron spectroscopy – baryons

- Missing states?
- 'Freezing' of degrees of freedom?
- Gluonic excitations?
- Flavour structure







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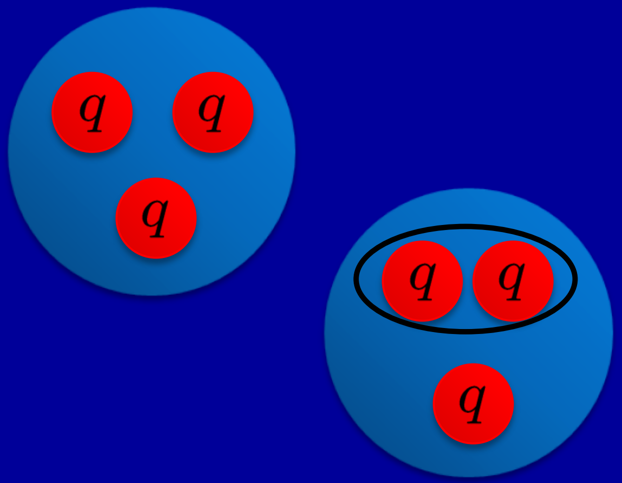

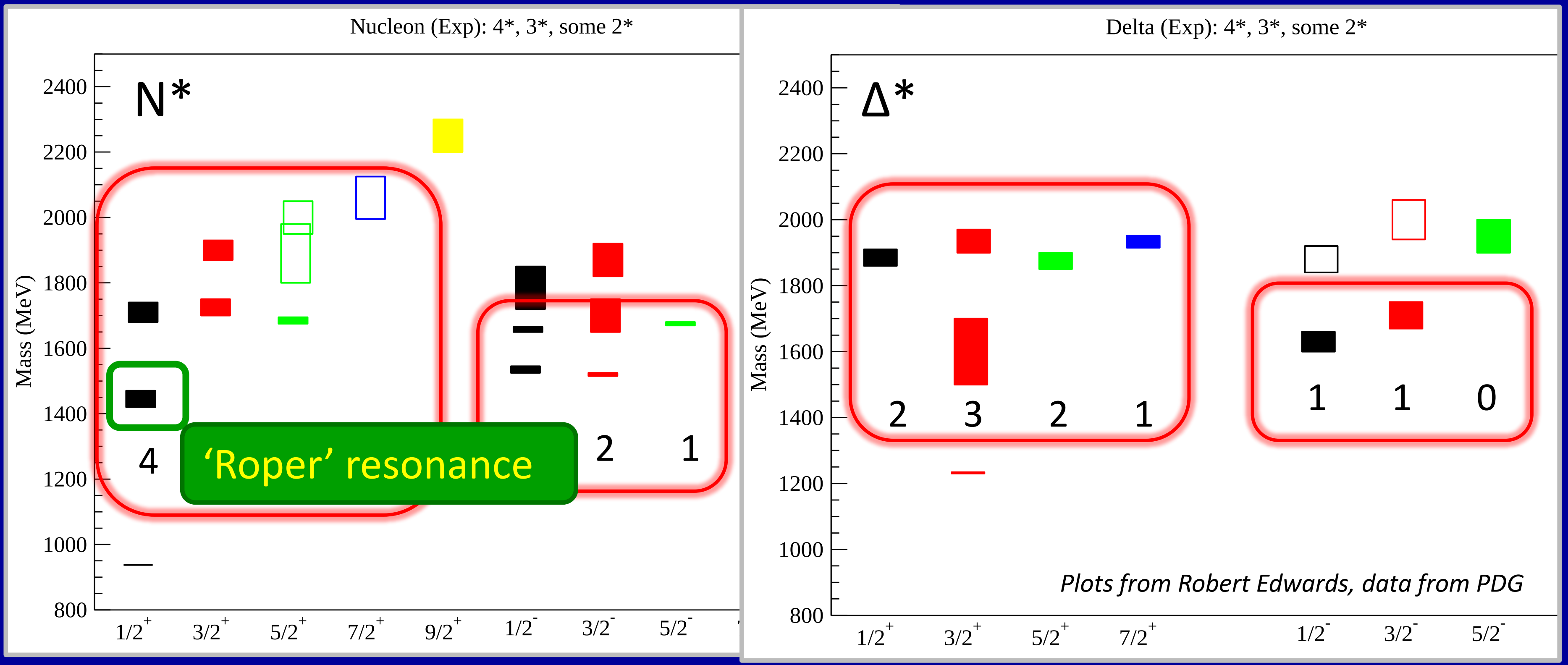

Hadron spectroscopy

Can we compute spectra of hadrons (including unconventional hadrons), understand these observations and address puzzles with first-principles calculations in QCD?
→ lattice QCD

Excited meson spectroscopy in LQCD – our approach

Energy eigenstates from:

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2 E_n} < 0|\mathcal{O}_i(0)|n >< n|\mathcal{O}_j^\dagger(0)|0 >$$

[Hadron Spectrum Collaboration, PR D80 054506, PRL 103 262001, PR D82 034508, D84 074508, D85 014507]

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Interpolating operators

$$O(t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\ \bar{\psi}(x) \left[\Gamma \times \overleftrightarrow{D} \times \overleftrightarrow{D} \ldots \right] \psi(x)$$

Ops have definite $J^{P(C)}$ in continuum (when **p** = **0**)

Here **p** = **0** and up to 3 derivatives included:

- many ops in each channel (up to ~ 26)
- different spin and angular structures, include ~ $[D_i, D_j]$

[Hadron Spectrum Collaboration, PR D80 054506, PRL 103 262001, PR D82 034508, D84 074508, D85 014507]

Variational method

Large basis of ops → matrix of correlators

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Generalised eigenvalue problem:

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

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Eigenvalues → energies

$$\lambda^{(n)}(t) \to e^{-E_n(t-t_0)}$$

($t >> t_0$)

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$$v_i^{(n)} \rightarrow Z_i^{(n)} \equiv < 0|\mathcal{O}_i|n >$$

Also → optimal linear combination of operators to overlap on to a state

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Also → optimal linear combination of operators to overlap on to a state

Var. method uses orthog of eigenvectors; don't just rely on separating energies

Reduced symmetry and spin

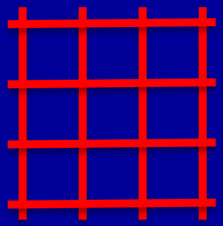
Continuum

Infinite number of *irreps*: J = 0, 1, 2, 3, 4, ...

Reduced symmetry and spin

Continuum

Infinite number of *irreps*: J = 0, 1, 2, 3, 4, ...



Finite cubic lattice

Broken sym: 3D rotation group → cubic group

Finite number of *irreps* Λ: A_1, A_2, T_1, T_2, E (+ others for half-integer spin)

Irrep	A_1	A_2	T_1	T_2	E
Dim	1	1	3	3	2

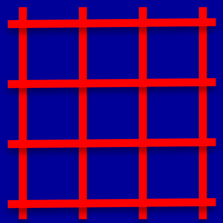
Cont. Spin	0	1	2	3	4	...
Irrep(s)	A_1	T_1	$T_2 + E$	$T_1 + T_2 + A_2$	$A_1 + T_1 + T_2 + E$	...

'Subduce' operators into lattice irreps (J → Λ):

Reduced symmetry and spin

Continuum

Infinite number of *irreps*: J = 0, 1, 2, 3, 4, ...



Finite cubic lattice

Broken sym: 3D rotation group → cubic group

Finite numbe[illegible] alf-integer spin)

Relevant symmetry reduced further for hadrons at non-zero momentum

Irrep	A_1	[illegible]	[illegible]	[illegible]	[illegible]
Dim	1	1	3	3	2

Cont. Spin	0	1	2	3	4	...
Irrep(s)	A_1	T_1	$T_2 + E$	$T_1 + T_2 + A_2$	$A_1 + T_1 + T_2 + E$	...

'Subduce' operators into lattice irreps (J → Λ):

Excited charmonia

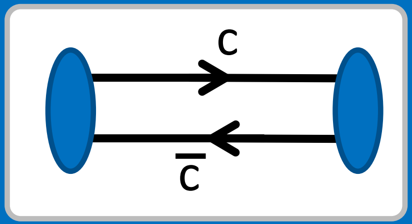
Hadron Spectrum Collaboration

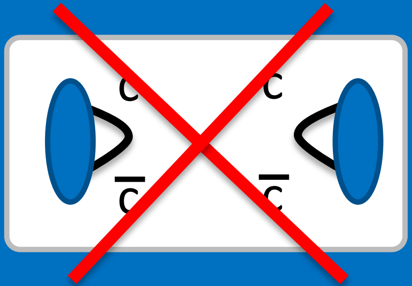
JHEP 07 (2012) 126 – Liu, Moir, Peardon, Ryan, CT, Vilaseca, Dudek, Edwards, Joó, Richards

Excited charmonia

Hadron Spectrum Collaboration

- **Dynamical** 'clover' *u*, *d* and *s* quarks, $m_u = m_d < m_s$ [$N_f = 2+1$]
- Relativistic *c* quark
- **Anisotropic** – finer in temporal dir ($a_s/a_t \approx 3.5$),
- $m_\pi \approx 400$ MeV (not physical m_π)
- One lattice spacing, $a_s \approx 0.12$ fm (no extrap. to contin. limit)
- Two volumes: 16^3, $\mathbf{24^3}$ ($L_s \approx 1.9$, 2.9 fm)
- Only compute **connected** contributions

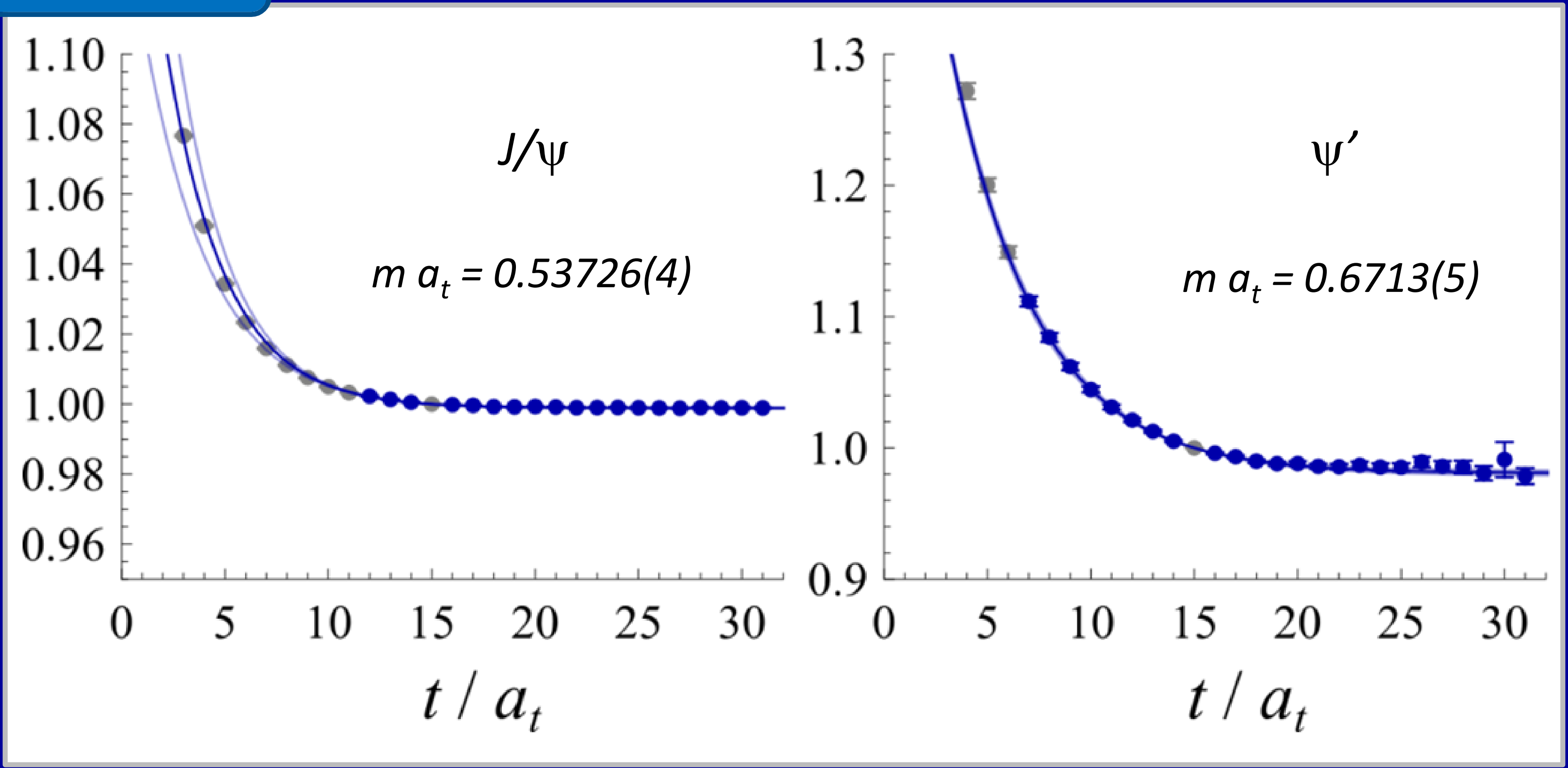




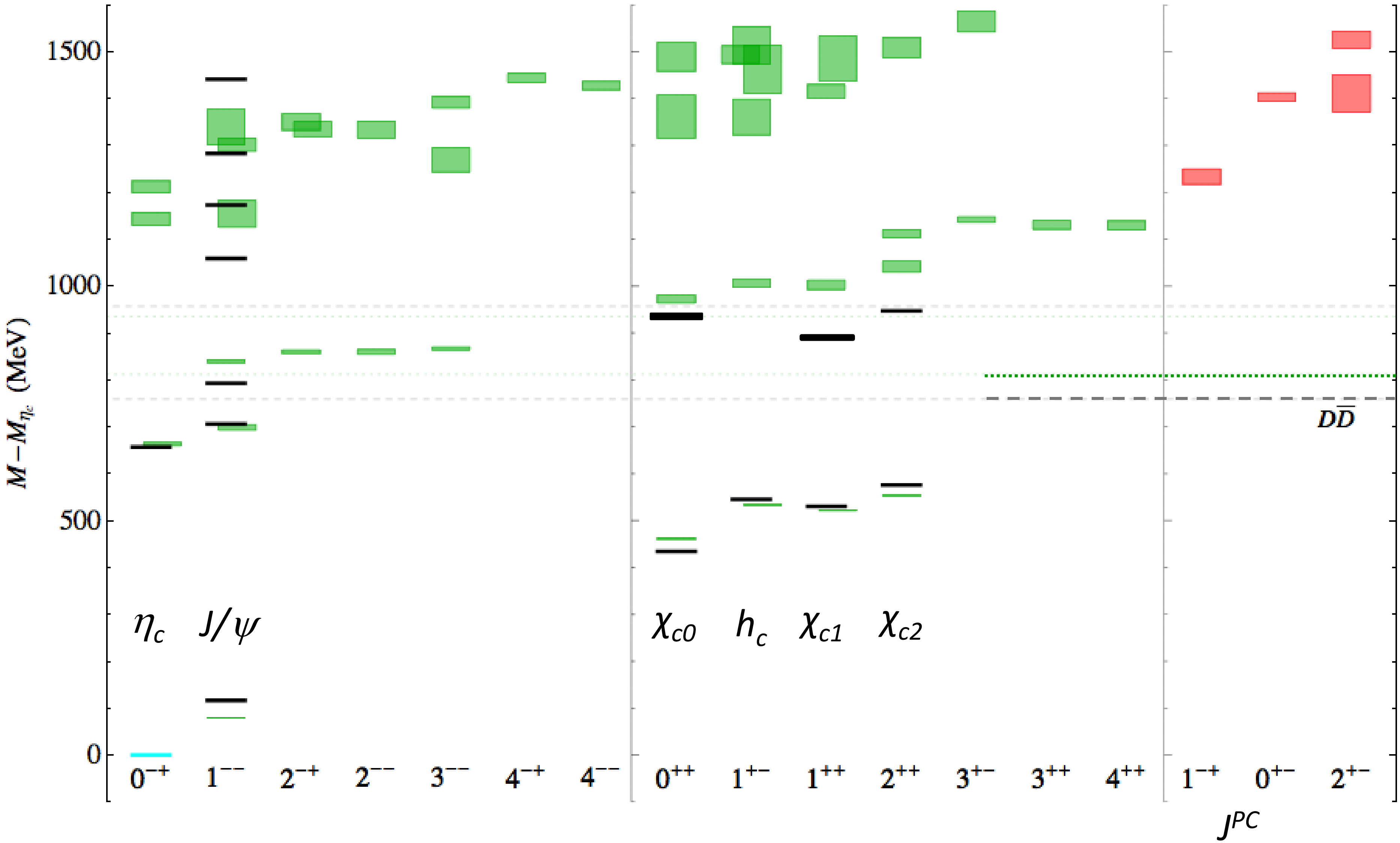
JHEP 07 (2012) 126 – Liu, Moir, Peardon, Ryan, CT, Vilaseca, Dudek, Edwards, Joó, Richards

Excited charmonia

$\lambda(t) \cdot e^{m(t-t_0)}$

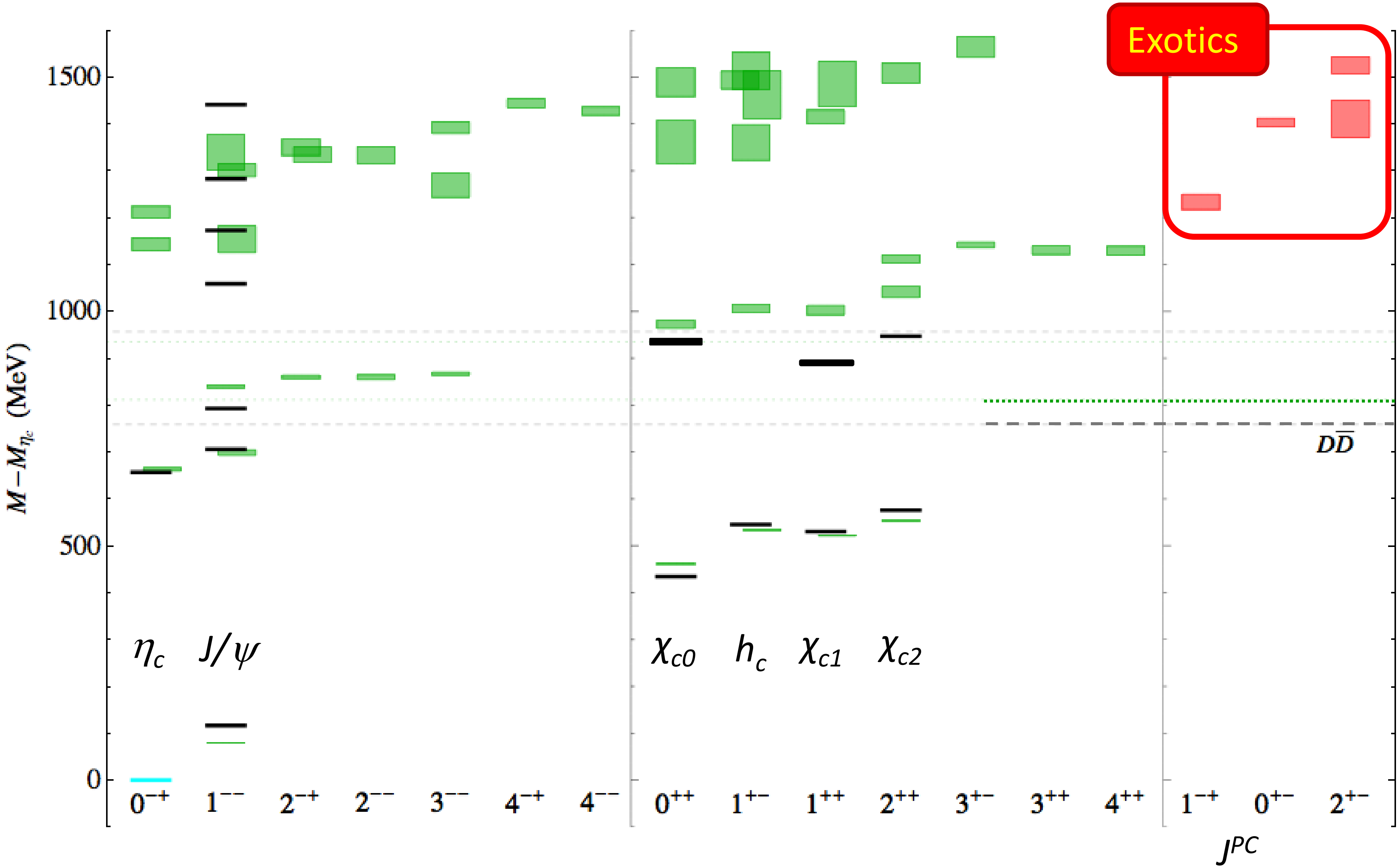


Excited charmonia



$M_\pi \approx 400$ MeV [HadSpec, JHEP 07 (2012) 126]

Excited charmonia



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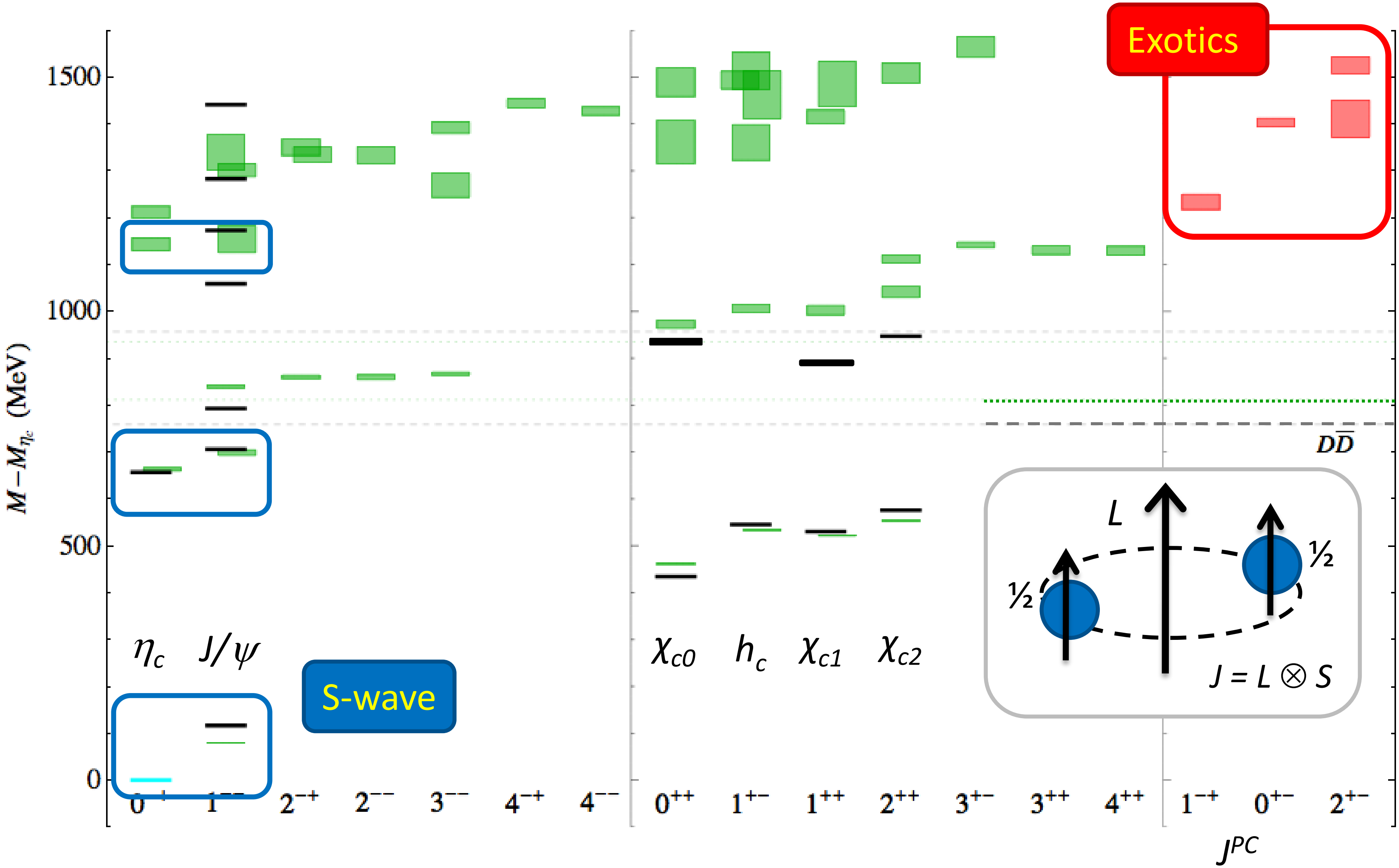

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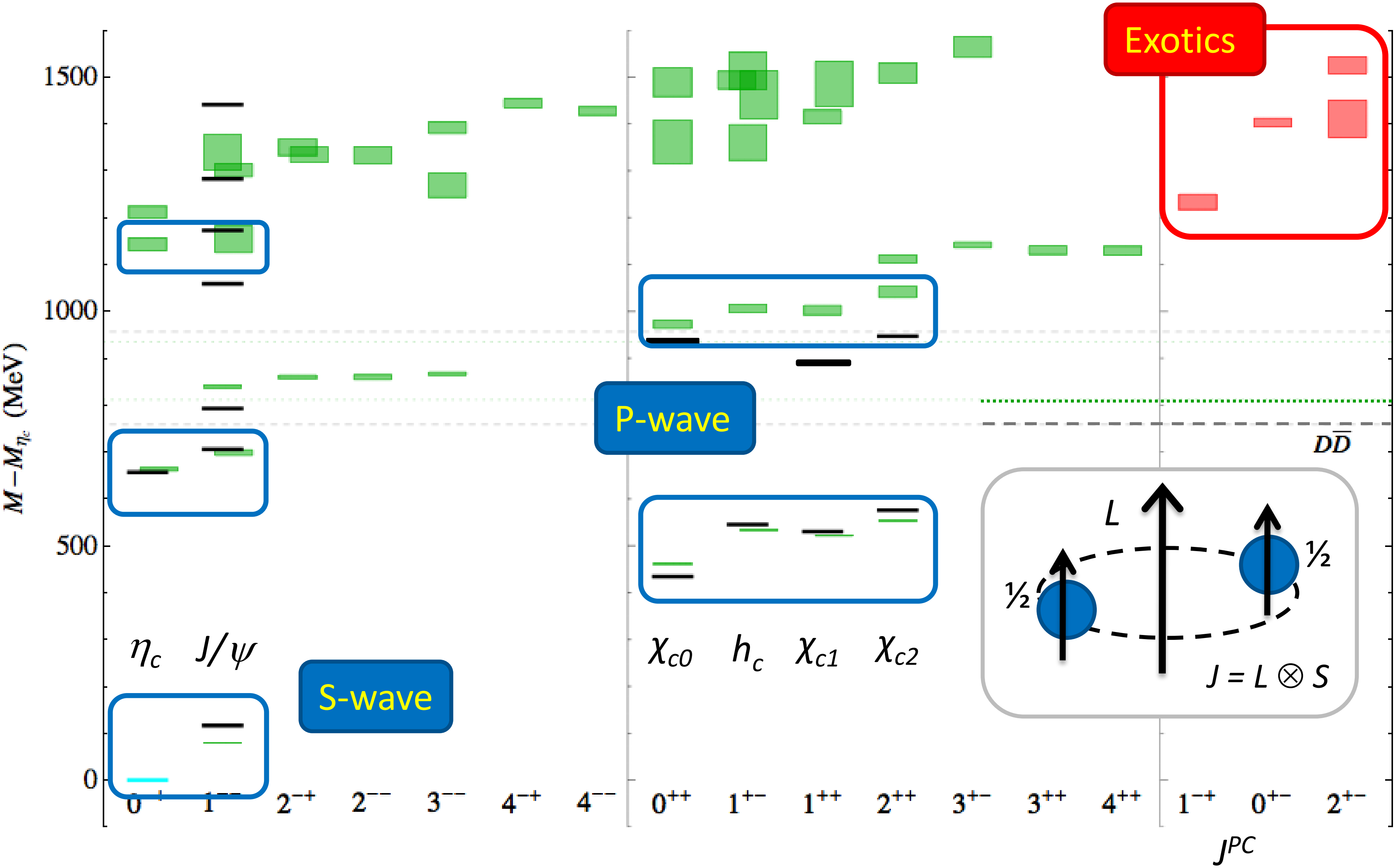

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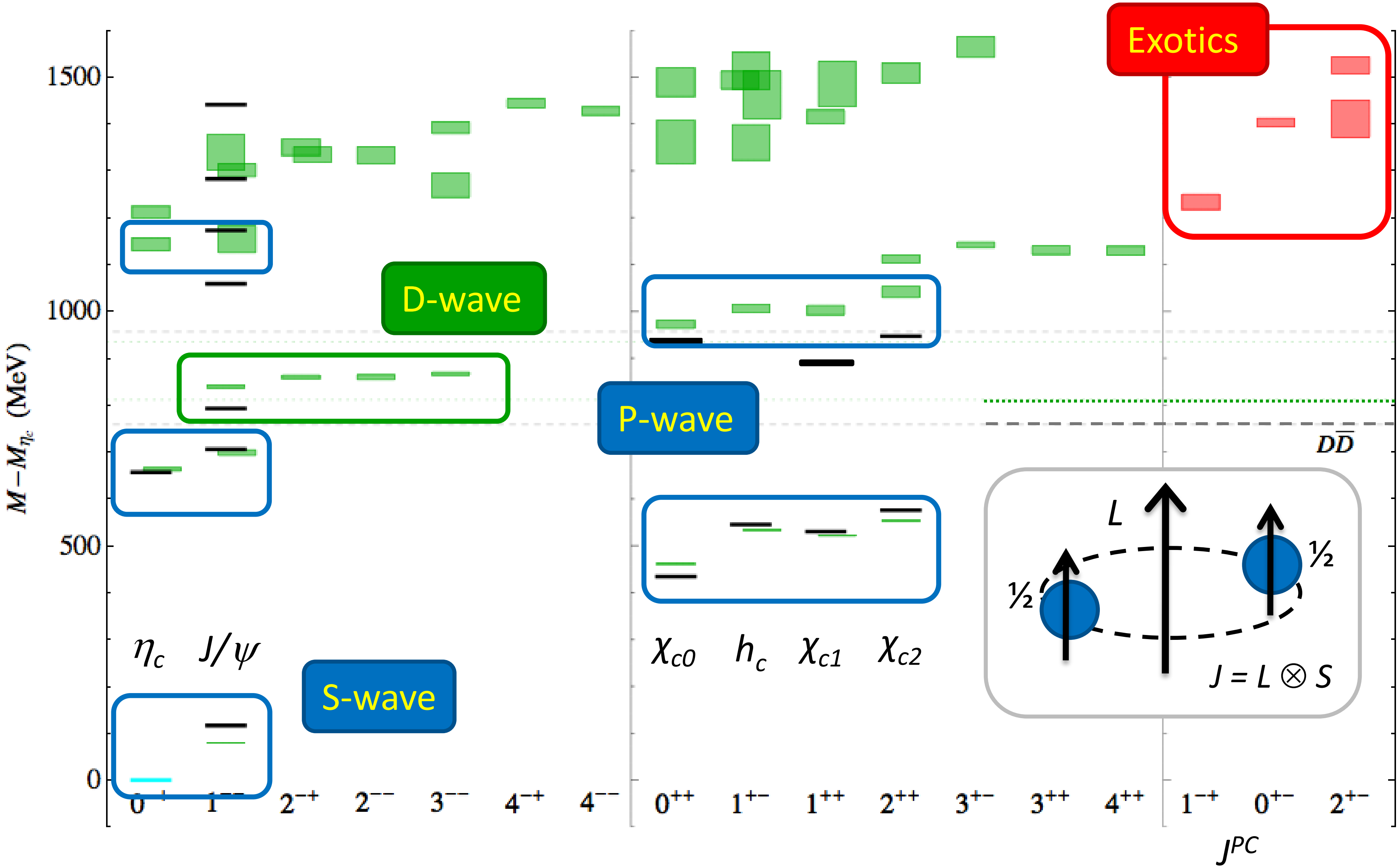

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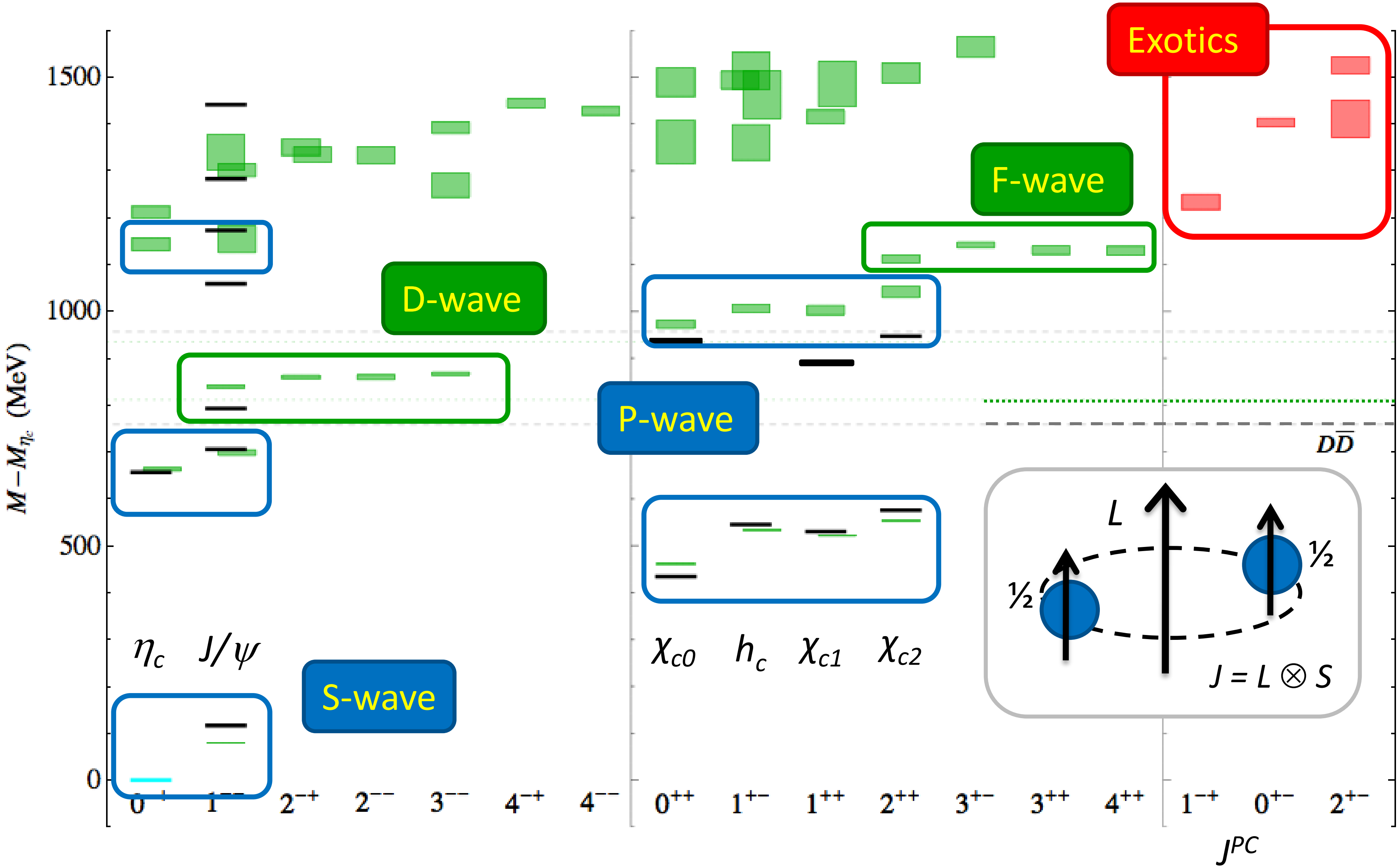

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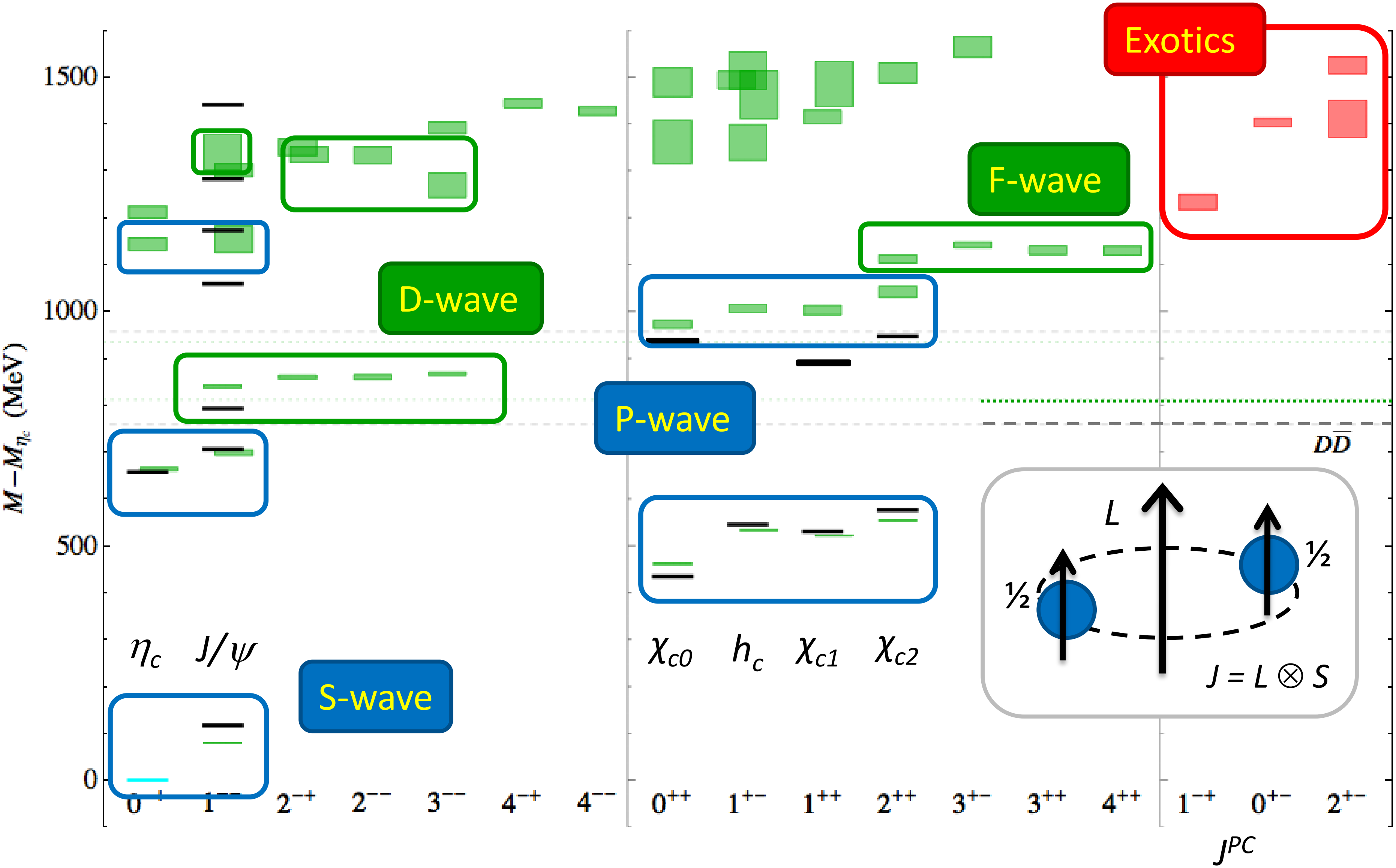

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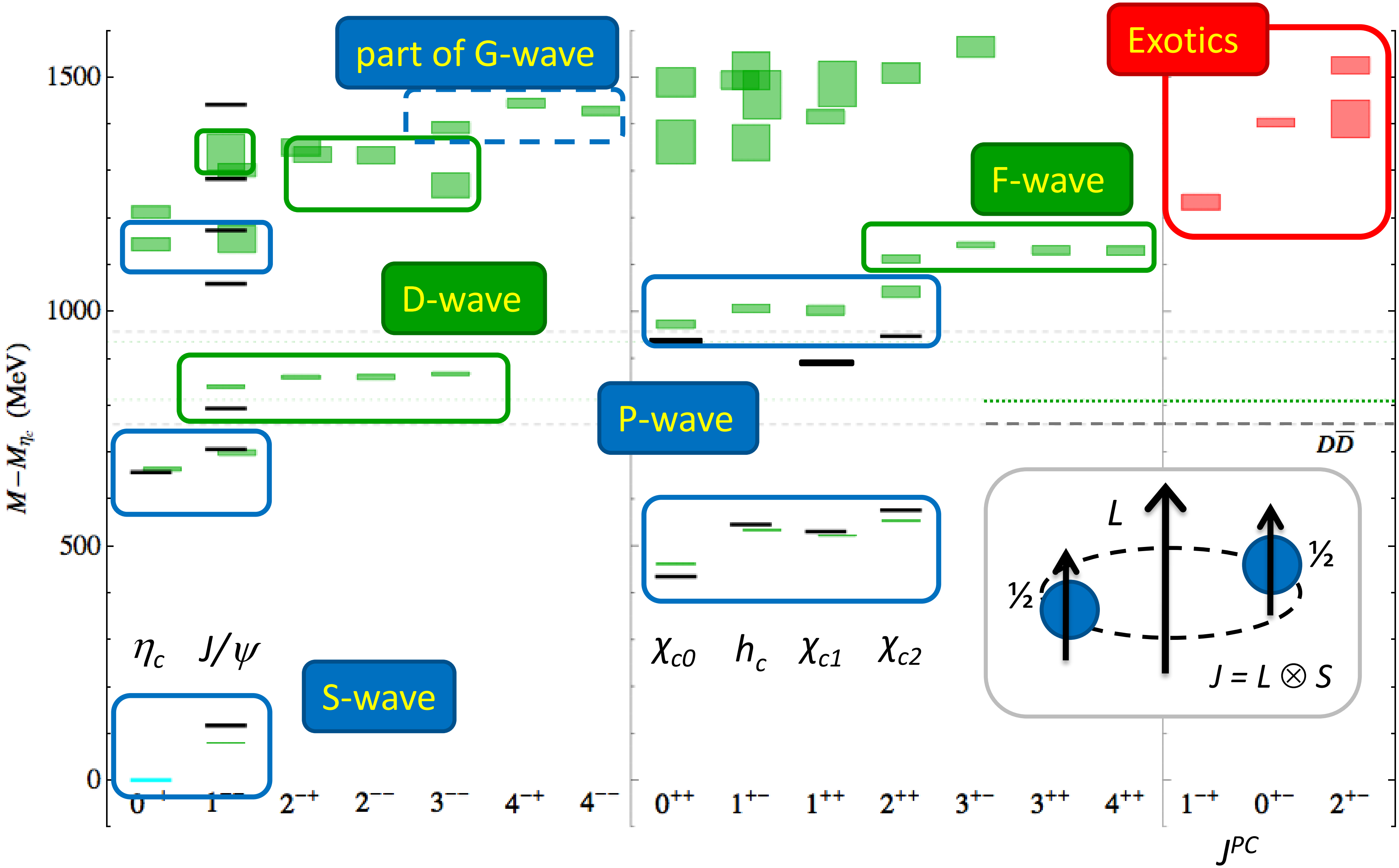

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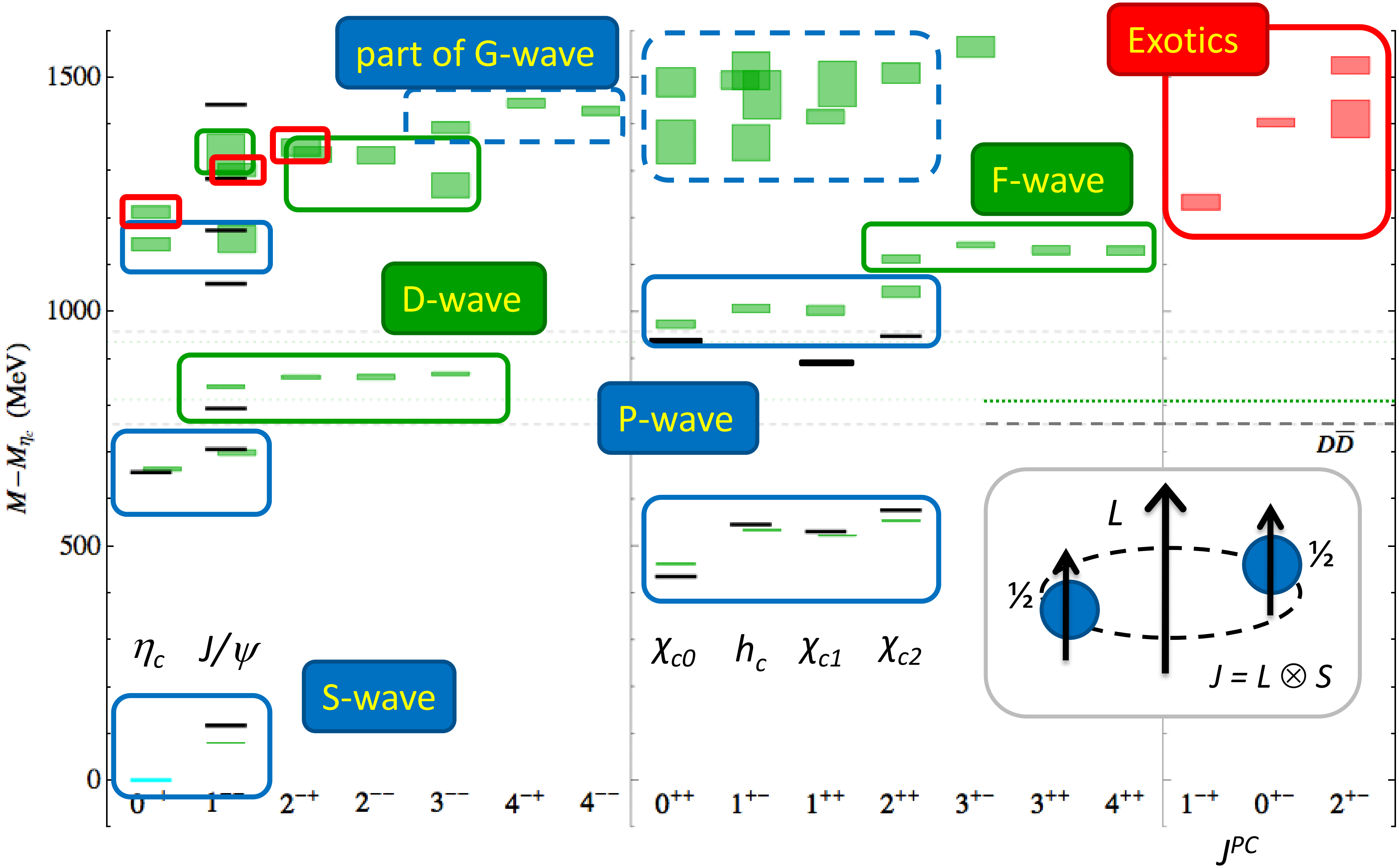

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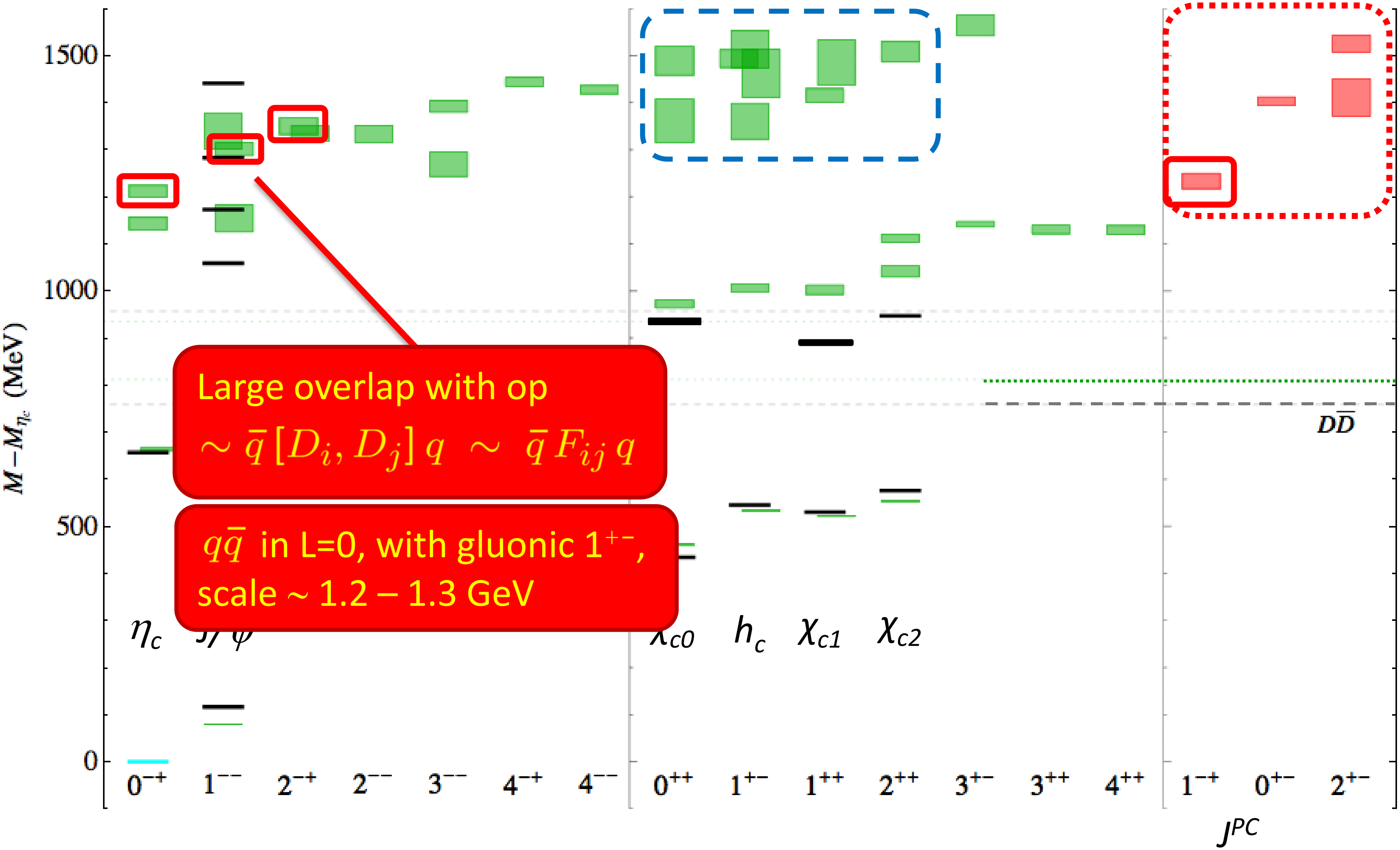

Large overlap with op
$\sim \bar{q}\,[D_i, D_j]\,q \;\sim\; \bar{q}\,F_{ij}\,q$

$q\bar{q}$ in L=0, with gluonic 1^{+-},
scale ~ 1.2 – 1.3 GeV

$M_\pi \approx 400$ MeV [HadSpec, JHEP 07 (2012) 126]

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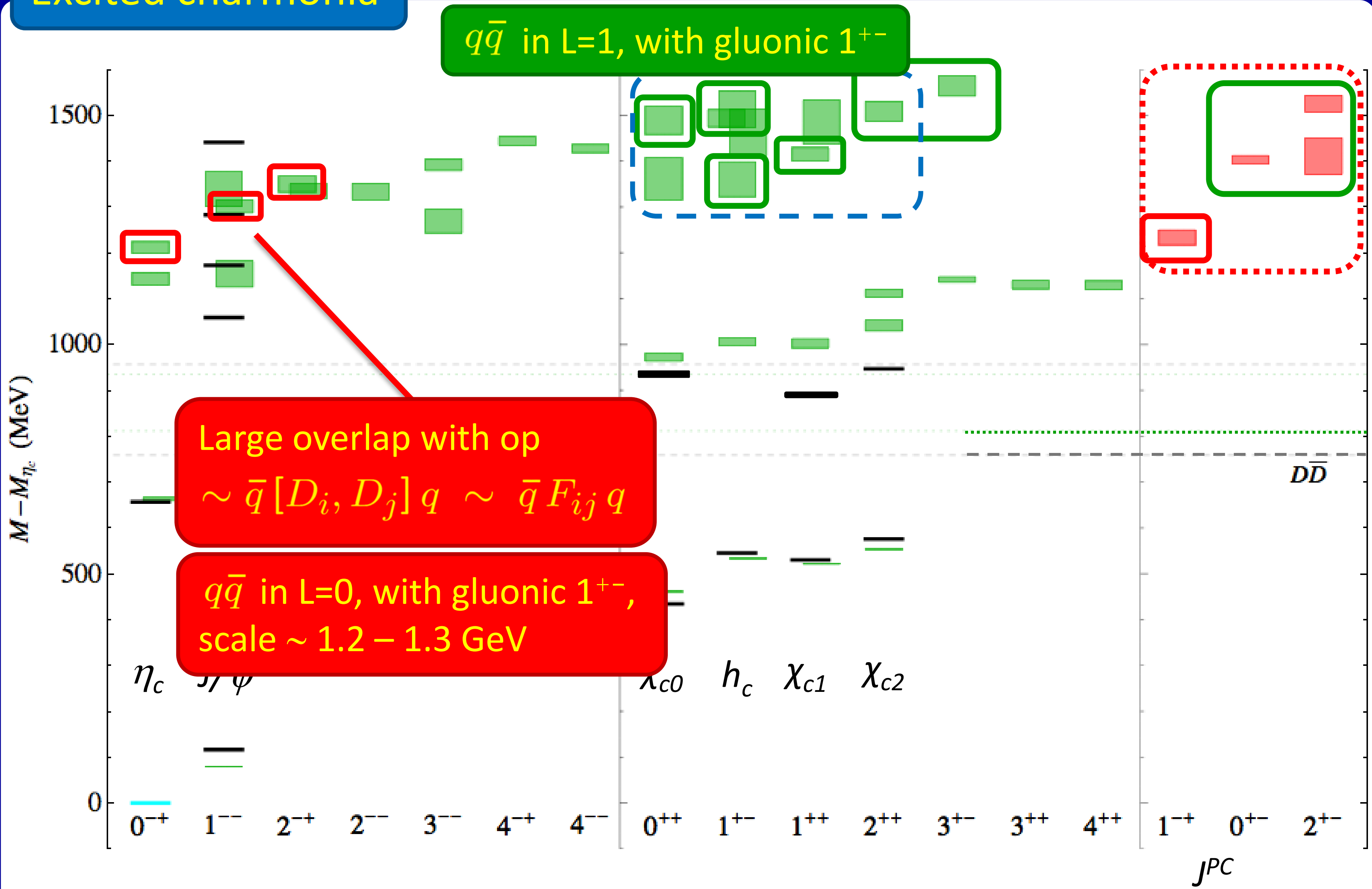

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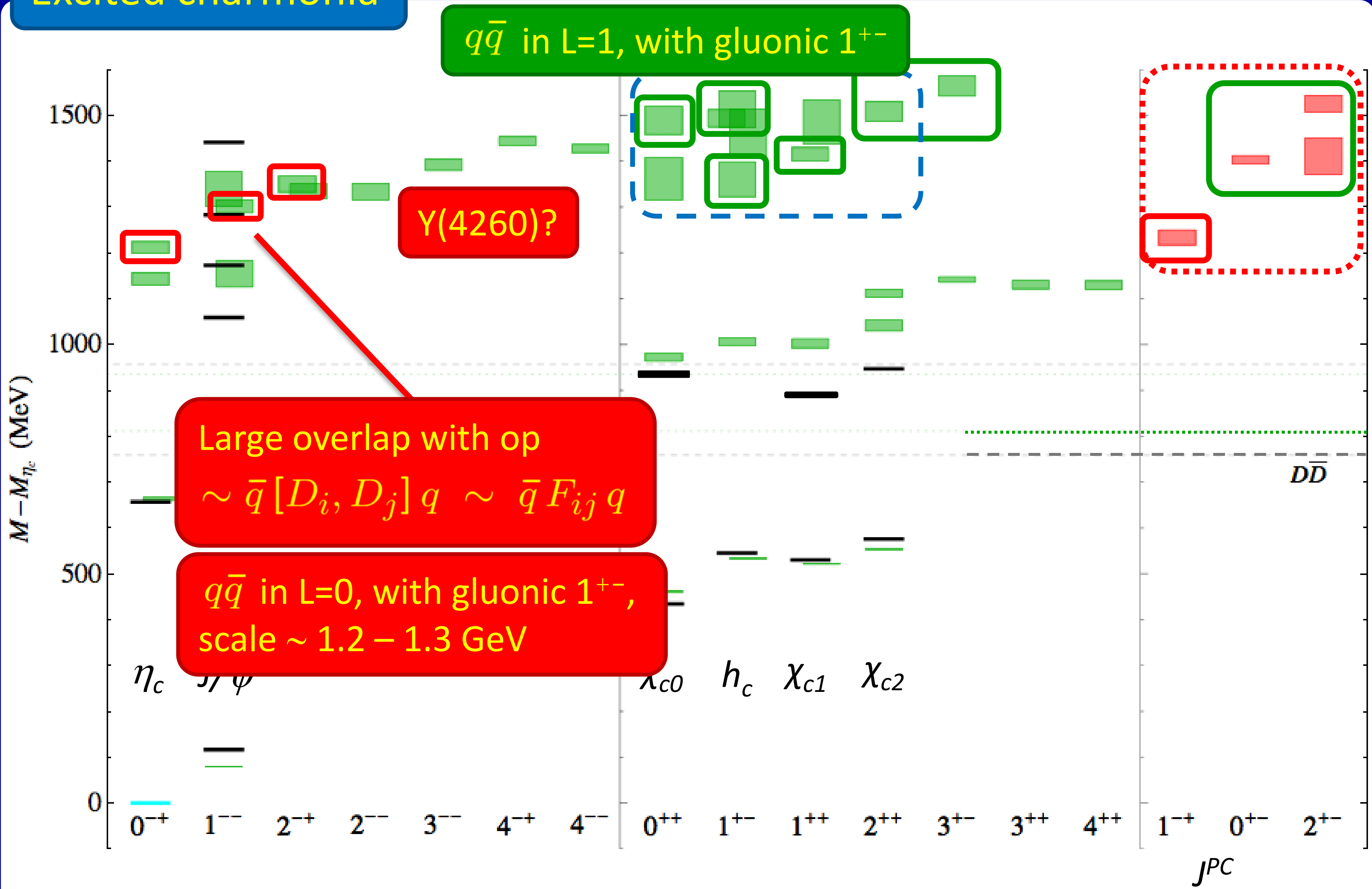

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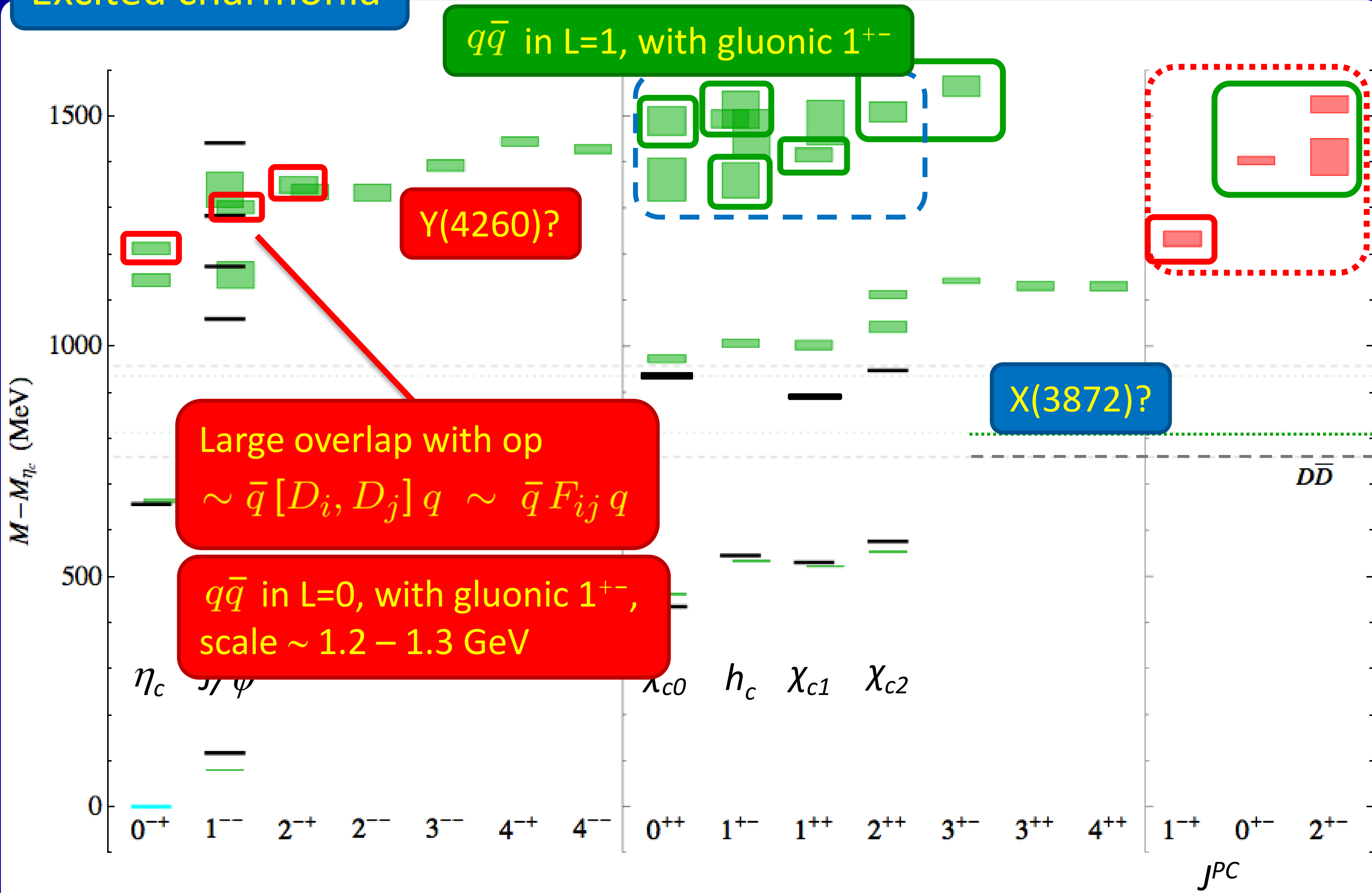

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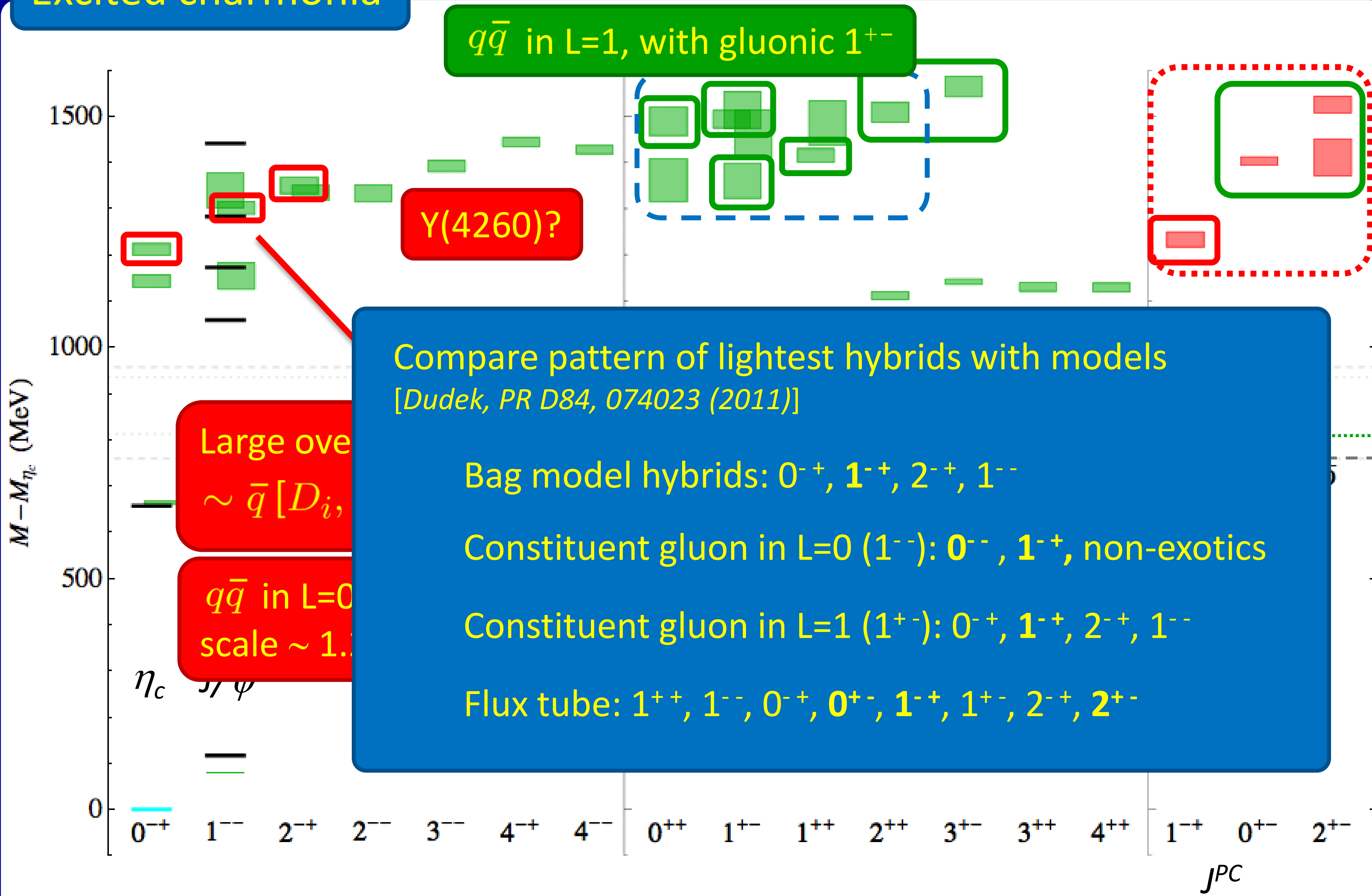


Compare pattern of lightest hybrids with models
[Dudek, PR D84, 074023 (2011)]

Bag model hybrids: 0^{-+}, $\mathbf{1^{-+}}$, 2^{-+}, 1^{--}

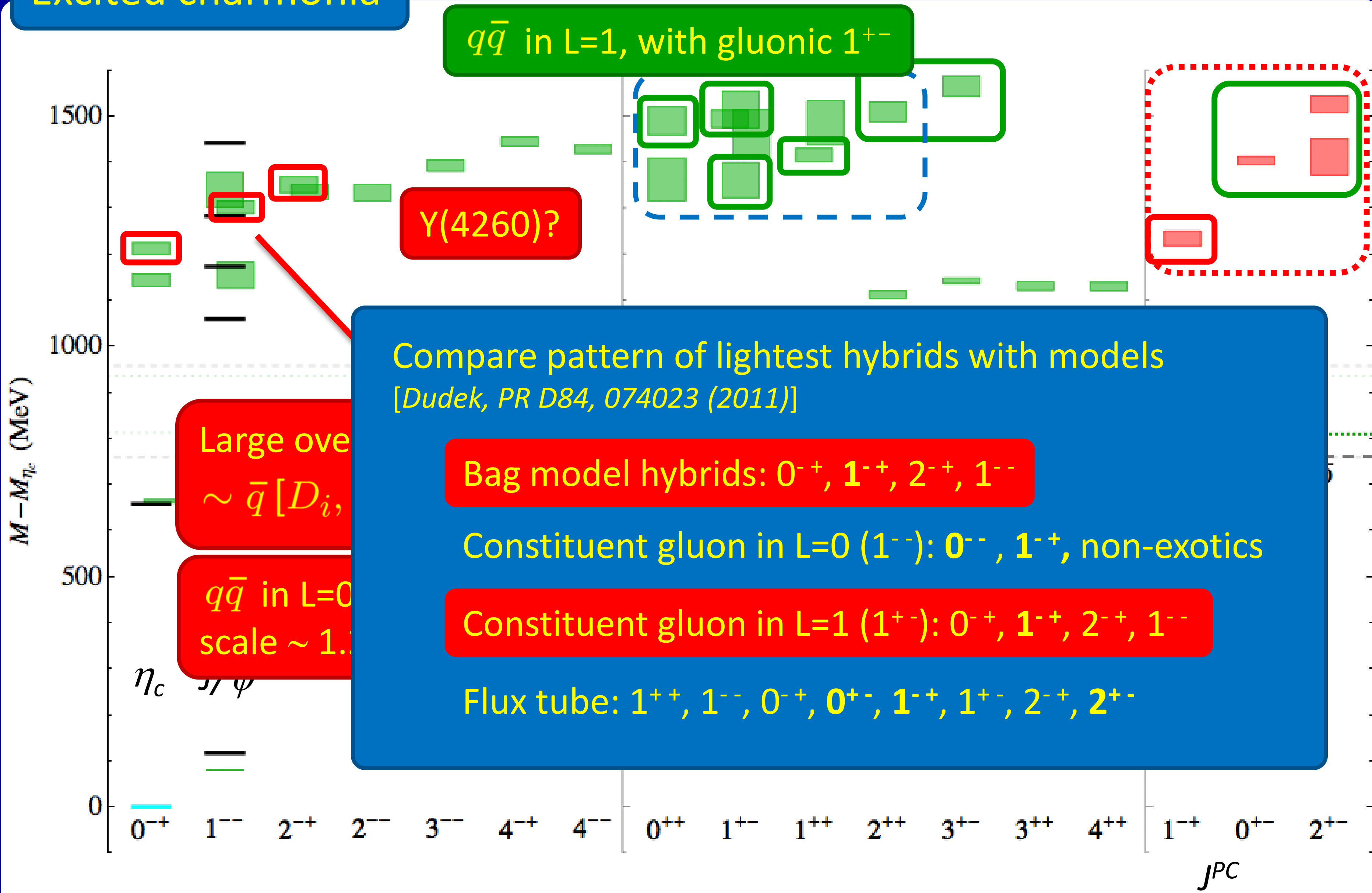
Constituent gluon in L=0 (1^{--}): $\mathbf{0^{--}}$, $\mathbf{1^{-+}}$, non-exotics

Constituent gluon in L=1 (1^{+-}): 0^{-+}, $\mathbf{1^{-+}}$, 2^{-+}, 1^{--}

Flux tube: 1^{++}, 1^{--}, 0^{-+}, $\mathbf{0^{+-}}$, $\mathbf{1^{-+}}$, 1^{+-}, 2^{-+}, $\mathbf{2^{+-}}$

$M_\pi \approx 400$ MeV [HadSpec, JHEP 07 (2012) 126]

Excited charmonia



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Flux tube: 1^{++}, 1^{--}, 0^{-+}, $\mathbf{0^{+-}}$, $\mathbf{1^{-+}}$, 1^{+-}, 2^{-+}, $\mathbf{2^{+-}}$

$M_\pi \approx 400$ MeV [HadSpec, JHEP 07 (2012) 126]

Light mesons

u and *d* quarks are degenerate – isospin symmetry

$$I_z = \begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix} \quad \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

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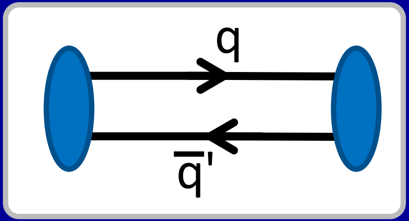
Isovectors (I = 1) e.g. π, ρ, a_1 – only connected contributions

$$\frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$



Isoscalars (I = 0) e.g. η, η', ω, ϕ

Light isoscalar mesons

Isoscalars (I = 0) e.g. $\eta, \eta', \omega, \phi$

QCD annihilation dynamics

Light isoscalar mesons

Isoscalars (I = 0) e.g. η, η', ω, ϕ

QCD annihilation dynamics

$m_s = m_u = m_d$ [SU(3) sym]
– eigenstates are octet, singlet

$$\frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

$$\frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

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$m_s \neq m_u = m_d$ → mixing

'Ideal mixing'

$$\ell\bar{\ell} \equiv \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$$

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$$\alpha = 54.7^\circ$$

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$$s\bar{s}$$

$$\alpha = 0$$

In general

$$\begin{aligned} |a\rangle &= \cos\alpha\left|\ell\bar{\ell}\right\rangle - \sin\alpha\,|s\bar{s}\rangle \\ |b\rangle &= \sin\alpha\left|\ell\bar{\ell}\right\rangle + \cos\alpha\,|s\bar{s}\rangle \end{aligned}$$

Light isoscalar mesons

Isoscalars (I = 0) e.g. $\eta, \eta', \omega, \phi$

QCD annihilation dynamics

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$\frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$ $\frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$

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$$
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\end{aligned}
$$

Experimentally ω, ϕ (1^{--}) and $f_2(1270)$, $f_2'(1525)$ (2^{++}) – close to 'ideal'

η, η' (0^{-+}) – closer to octet-singlet

Light isoscalar mesons

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$$\alpha = 54.7^\circ$$

$m_s \neq m_u = m_d$ → mixing

'Ideal mixing'

$$\ell\bar{\ell} \equiv \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) \qquad s\bar{s} \qquad \alpha = 0$$

In general

$$\begin{aligned} |a\rangle &= \cos\alpha \left|\ell\bar{\ell}\right\rangle - \sin\alpha \left|s\bar{s}\right\rangle \\ |b\rangle &= \sin\alpha \left|\ell\bar{\ell}\right\rangle + \cos\alpha \left|s\bar{s}\right\rangle \end{aligned}$$

Can also mix with glueballs

Experimentally

ω, ϕ (1^{--}) and $f_2(1270)$, $f_2'(1525)$ (2^{++}) – close to 'ideal'

η, η' (0^{-+}) – closer to octet-singlet

Light isoscalar mesons

Light isoscalar mesons

Operator basis doubled in size c.f. isovectors:

$$O^{\ell} \sim \frac{1}{\sqrt{2}}\left(\bar{u}\mathbf{\Gamma}u + \bar{d}\mathbf{\Gamma}d\right)$$

$$O^{s} \sim \bar{s}\mathbf{\Gamma}s$$

No glueball ops for now

Light isoscalar mesons

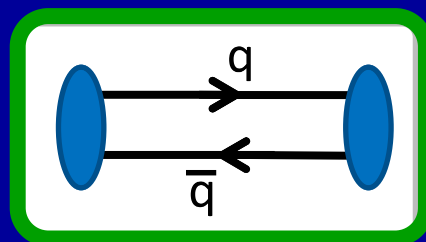
Operator basis doubled in size c.f. isovectors:

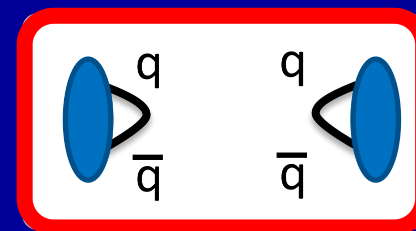
$$O^\ell \sim \frac{1}{\sqrt{2}}\left(\bar{u}\Gamma u + \bar{d}\Gamma d\right)$$

$$O^s \sim \bar{s}\Gamma s$$

No glueball ops for now

Connected and **disconnected** contrib. required



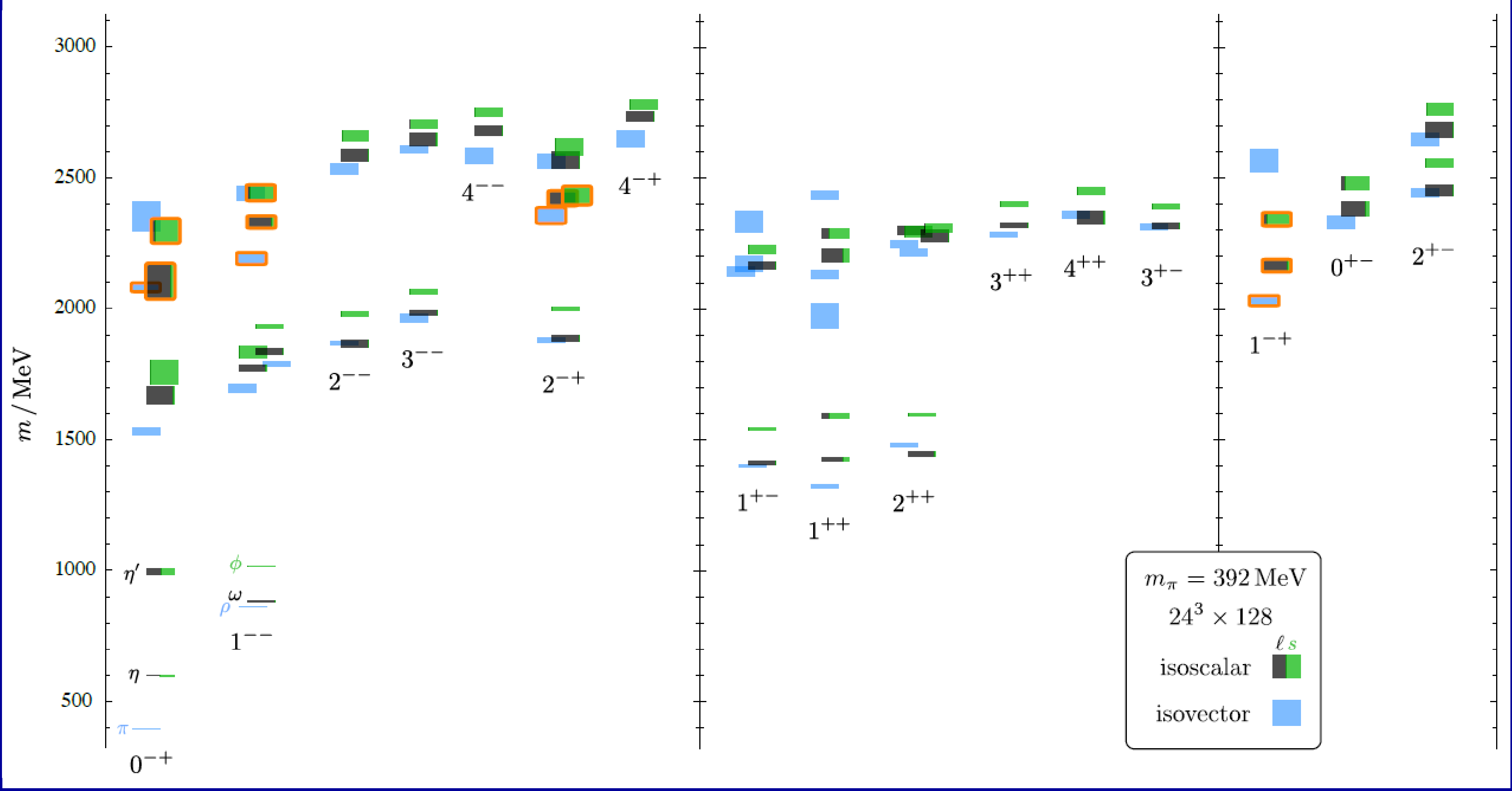


$$C^{q'q}_{AB}(t',t) = \langle 0|\mathcal{O}^{q'}_A(t')\mathcal{O}^{q\dagger}_B(t)|0\rangle$$

$$C = \begin{pmatrix} -\mathcal{C}^{\ell\ell} + 2\,\mathcal{D}^{\ell\ell} & \sqrt{2}\,\mathcal{D}^{\ell s} \\ \sqrt{2}\,\mathcal{D}^{s\ell} & -\mathcal{C}^{ss} + \mathcal{D}^{ss} \end{pmatrix}$$

Same lattice setup as before

Light mesons (isospin = 0 and 1)



Dudek, Edwards, Guo, CT (HadSpec), PR D88, 094505 (2013)

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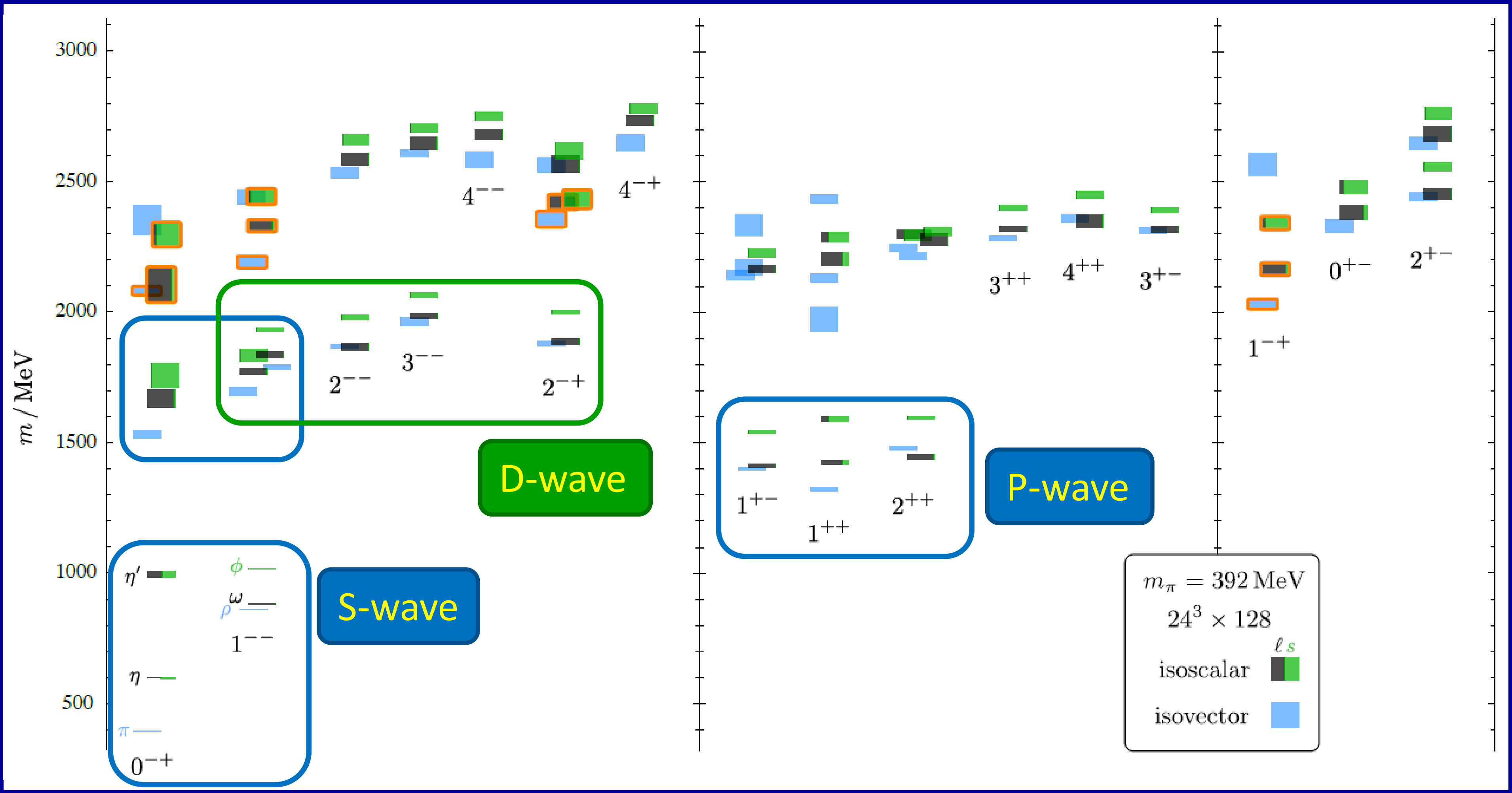

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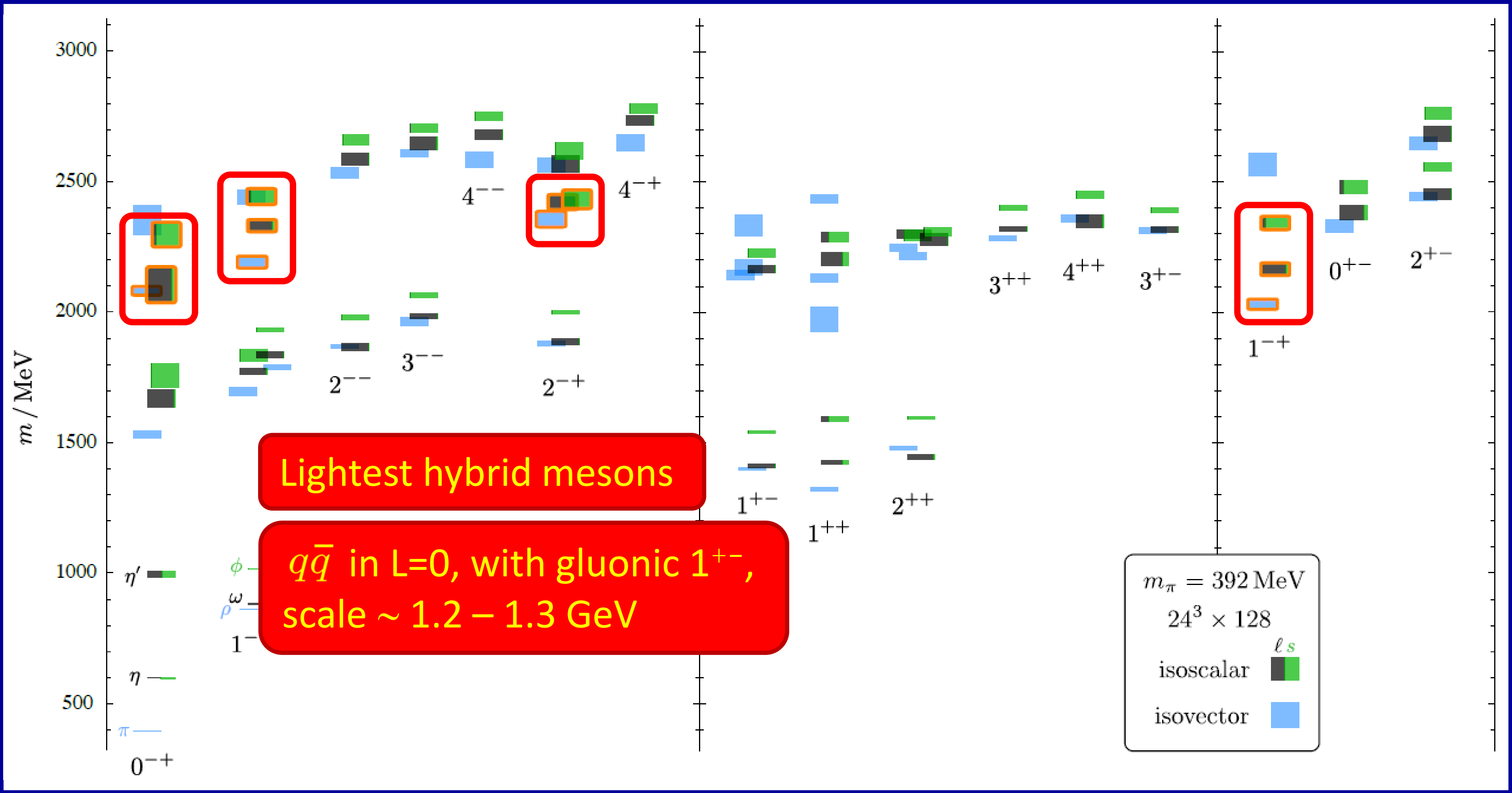
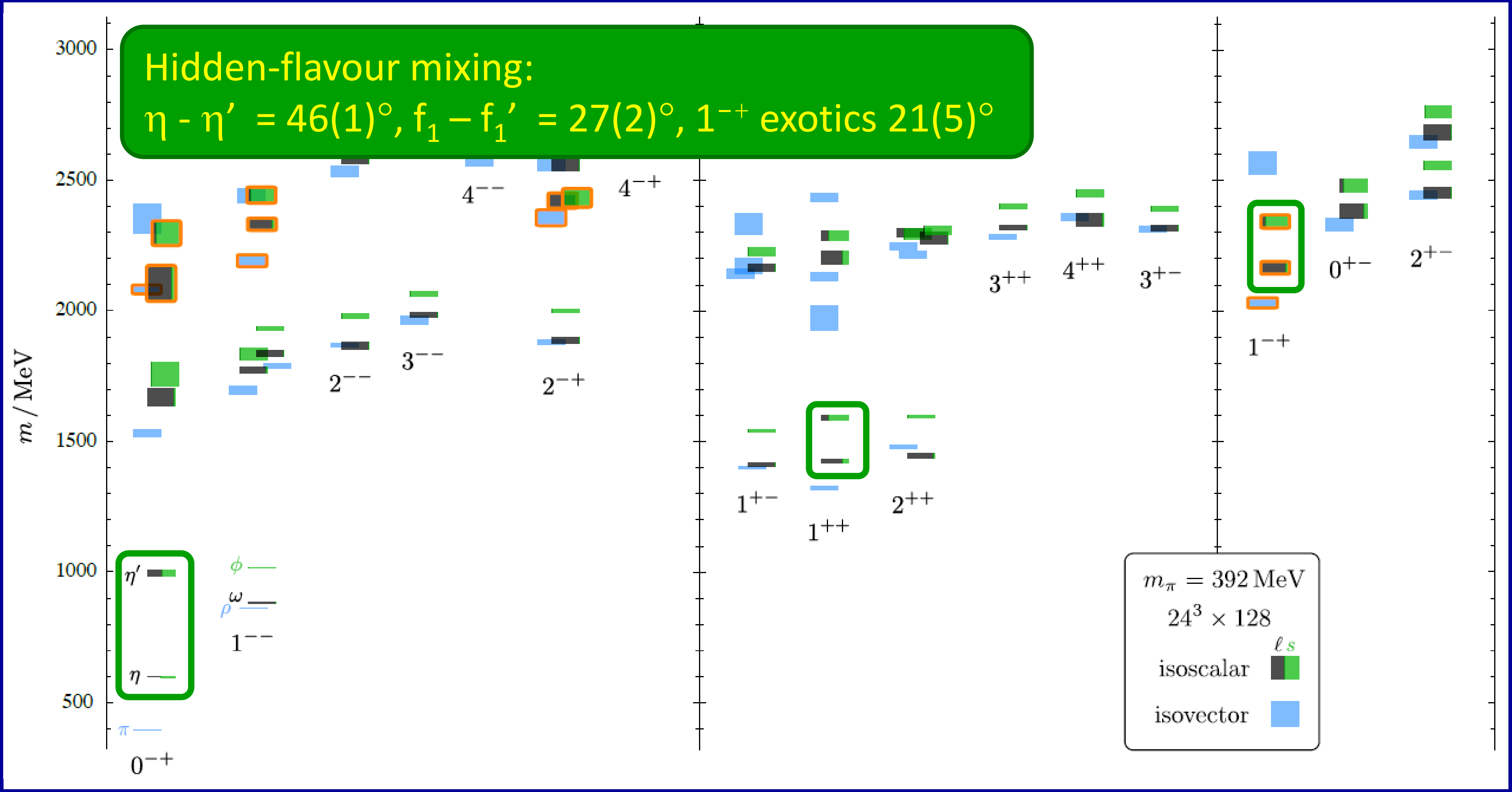


Dudek, Edwards, Guo, CT (HadSpec), PR D88, 094505 (2013)

Light mesons (isospin = 0 and 1)

Hidden-flavour mixing:

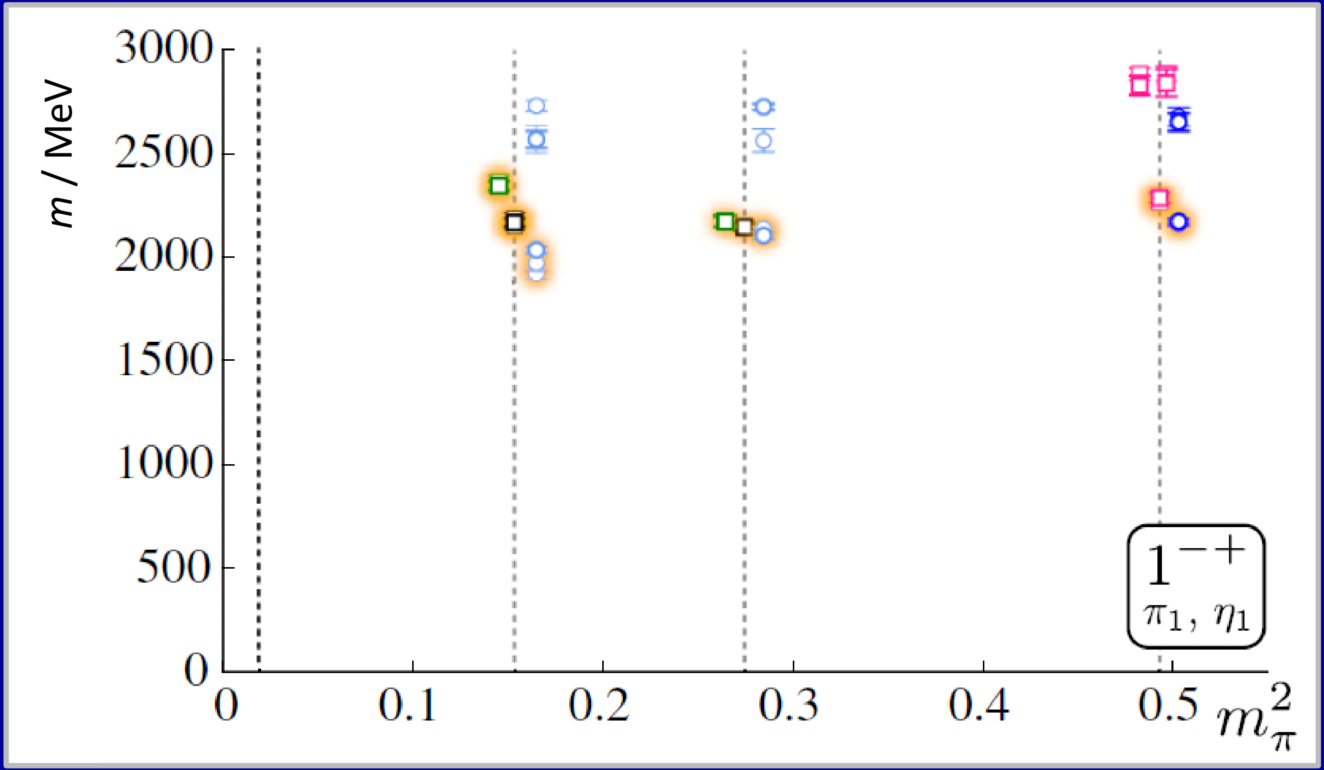
$\eta - \eta'$ = 46(1)°, $f_1 - f_1'$ = 27(2)°, 1^{-+} exotics 21(5)°



Dudek, Edwards, Guo, CT (HadSpec), PR D88, 094505 (2013)

Light mesons (isospin = 0 and 1)

Volume and m_π dependence



Dudek, Edwards, Guo, CT (HadSpec), PR D88, 094505 (2013)

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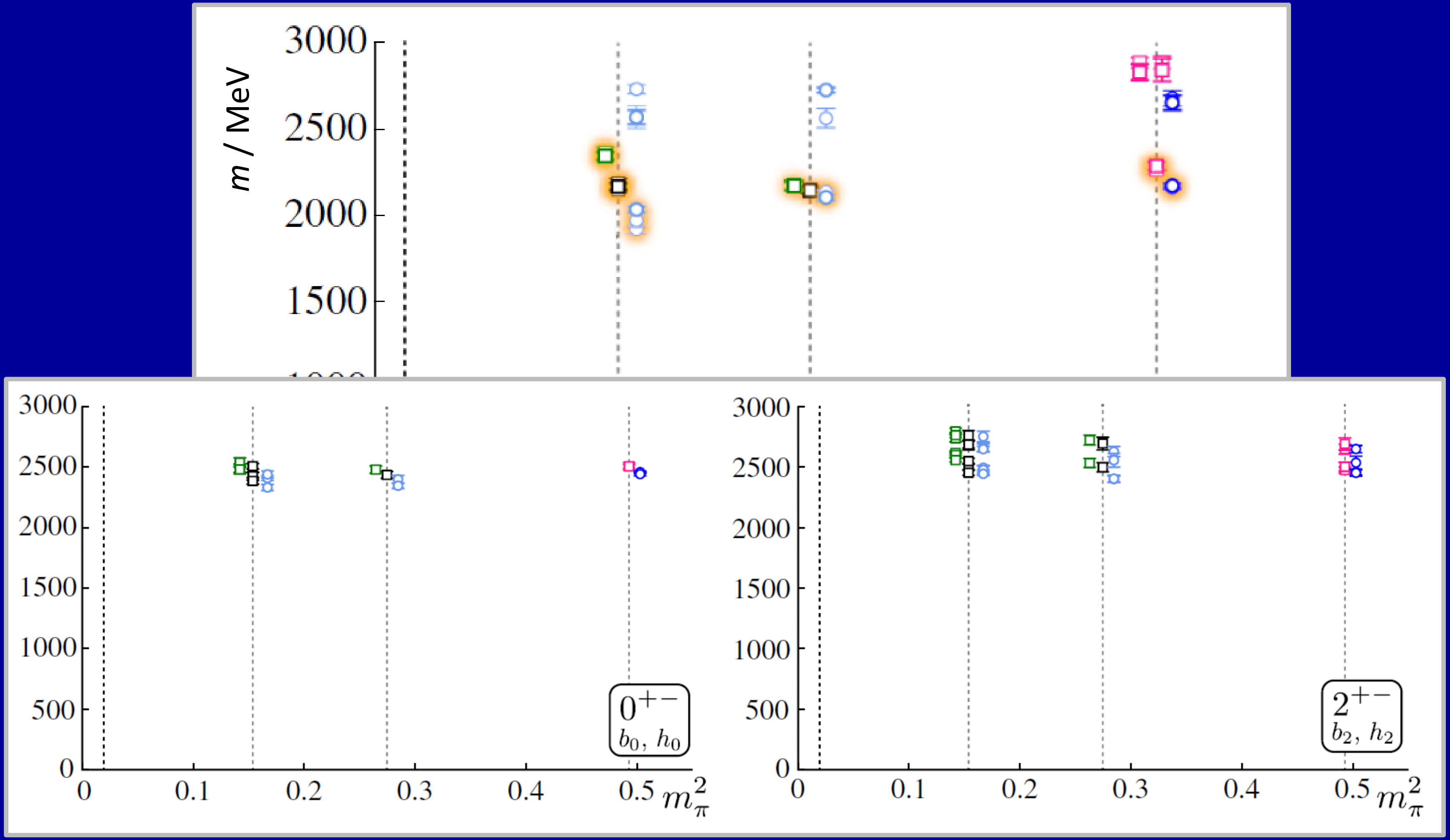
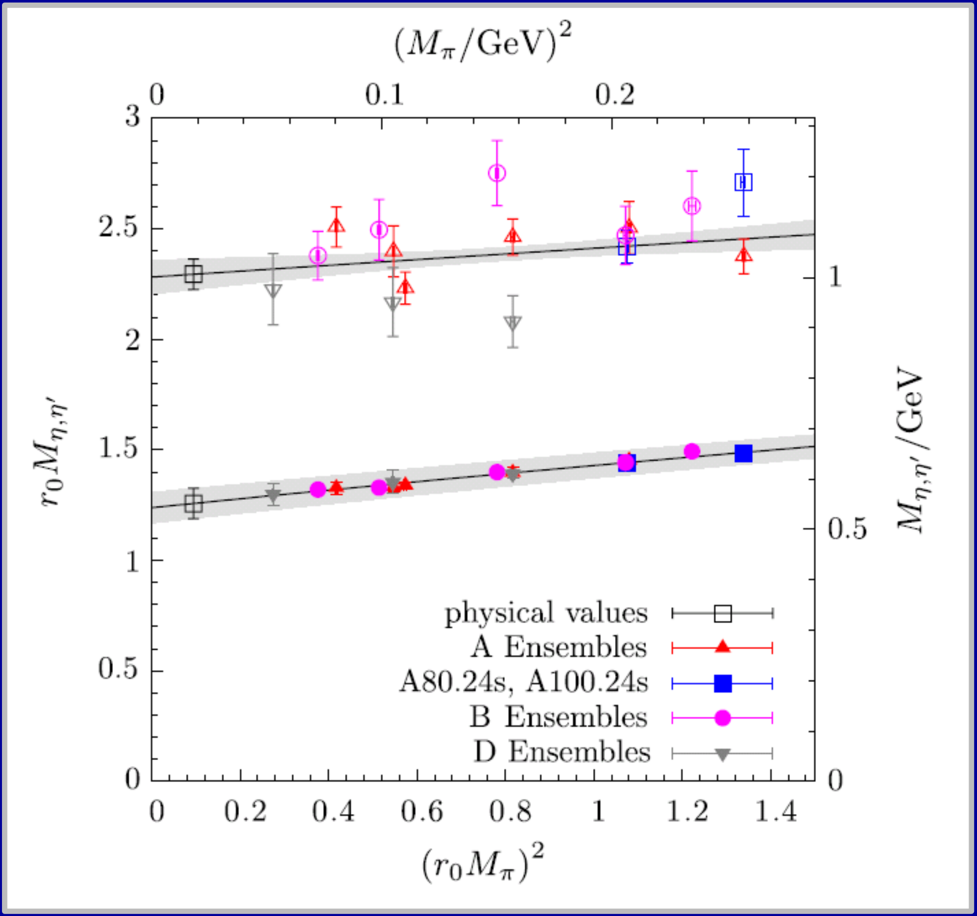
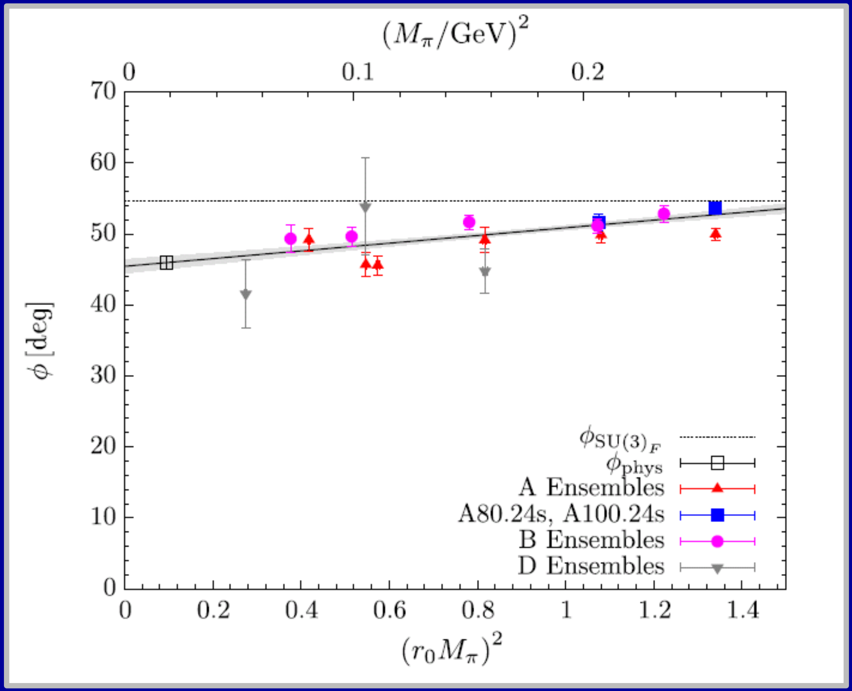


Dudek, Edwards, Guo, CT (HadSpec), PR D88, 094505 (2013)

Light mesons (isospin = 0)





Twisted mass quarks [N_f = 2+1+1]

Extrapolate in a and m_π:

η: 551 ± 8 ± 6 MeV,

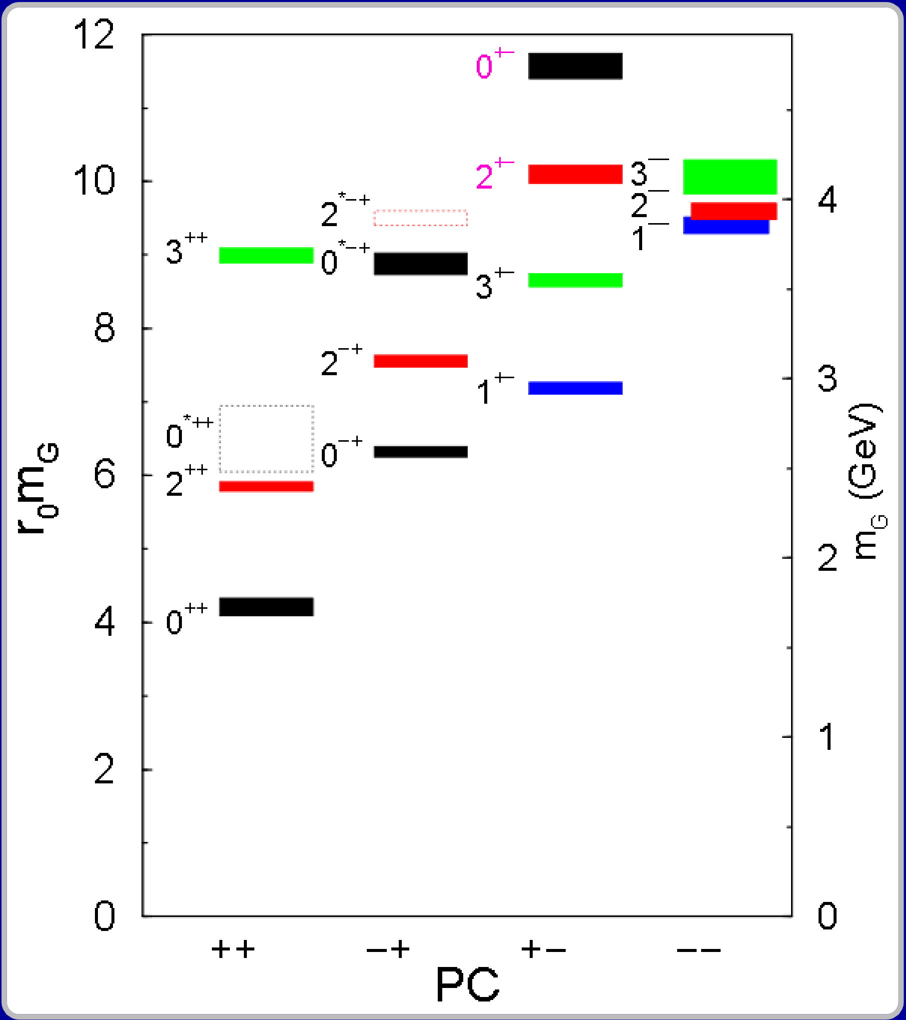
η′: 1006 ± 54 ± 38 $^{+61}$ MeV

ϕ: 46 ± 1 ± 3 °

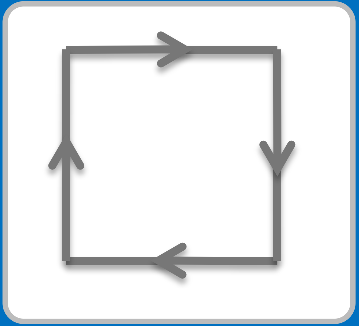
[c.f. HadSpec 46(1)° @ $m_\pi \approx$ 400 MeV]

Michael, Ottnad, Urbach (ETM), PRL 111, 181602 (2013)

Glueballs in pure gauge theory (SU(3) Yang-Mills)



- No fermion fields – computationally much less expensive
- Operators are closed loops of links, with different spatial symmetries



Morningstar and Peardon, PR D60, 034509 (1999)

Baryons

Excited baryon spectroscopy – our approach

Energy eigenstates from:

$$C_{ij}(t) = \langle 0 \,|\, \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \,|\, 0 \rangle$$

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2 E_n} < 0|\mathcal{O}_i(0)|n >< n|\mathcal{O}_j^\dagger(0)|0 >$$

Interpolating operators

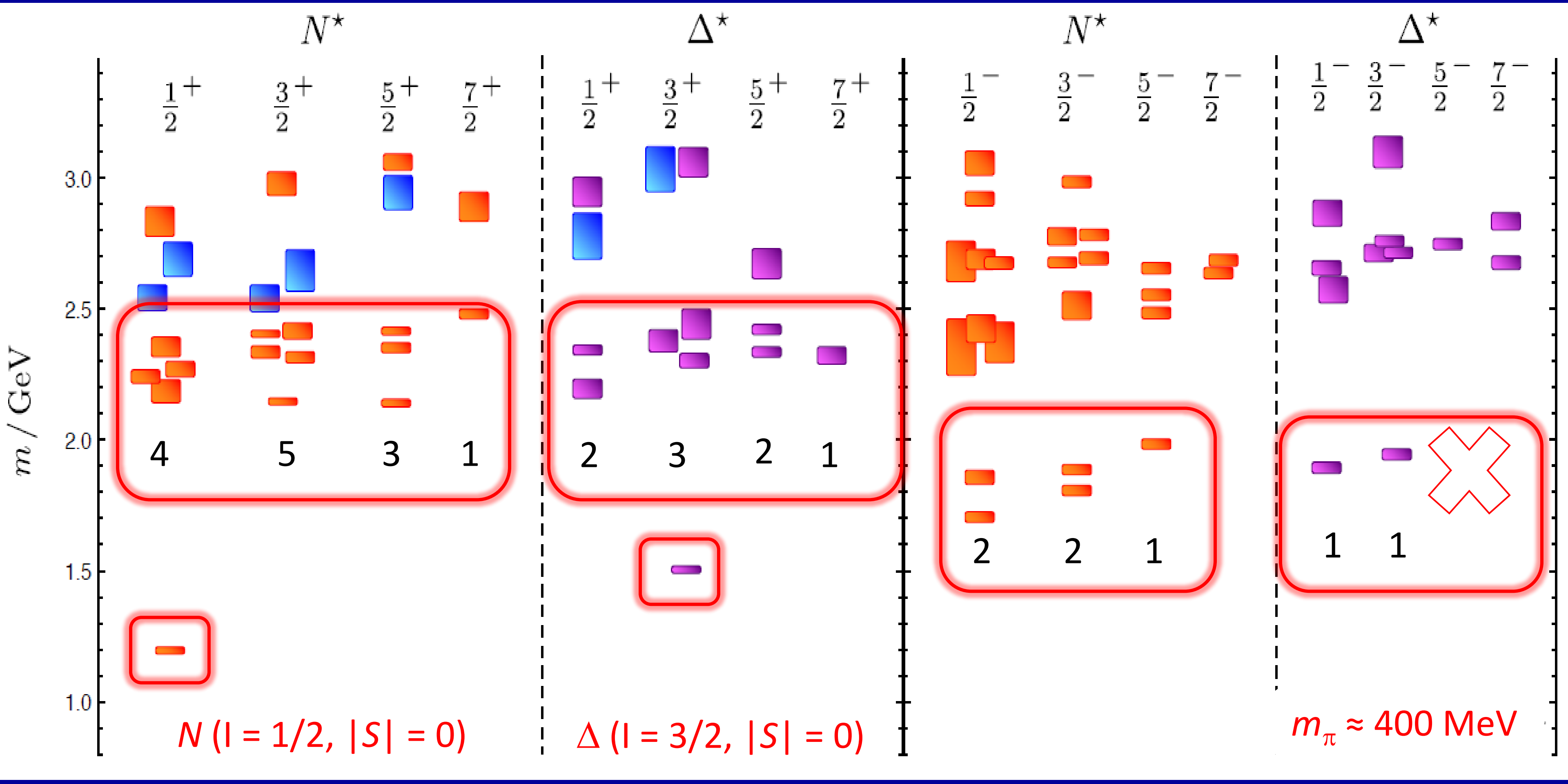
$$\langle 1, m_1; 1, m_2 | L, m_l \rangle \langle L, m_l; S, m_s | J, m_J \rangle \overrightarrow{D}_{m_1} \overrightarrow{D}_{m_2} [\psi\psi\psi]_{S,m_s}$$

Up to 2 derivs: $1 \otimes 1 \otimes S \rightarrow \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$

- Again, many ops in each channel with different spin and angular structures
- Same lattice setup as before but only one volume (16^3 , $L_s \approx 1.9$ fm)

N and Δ baryons

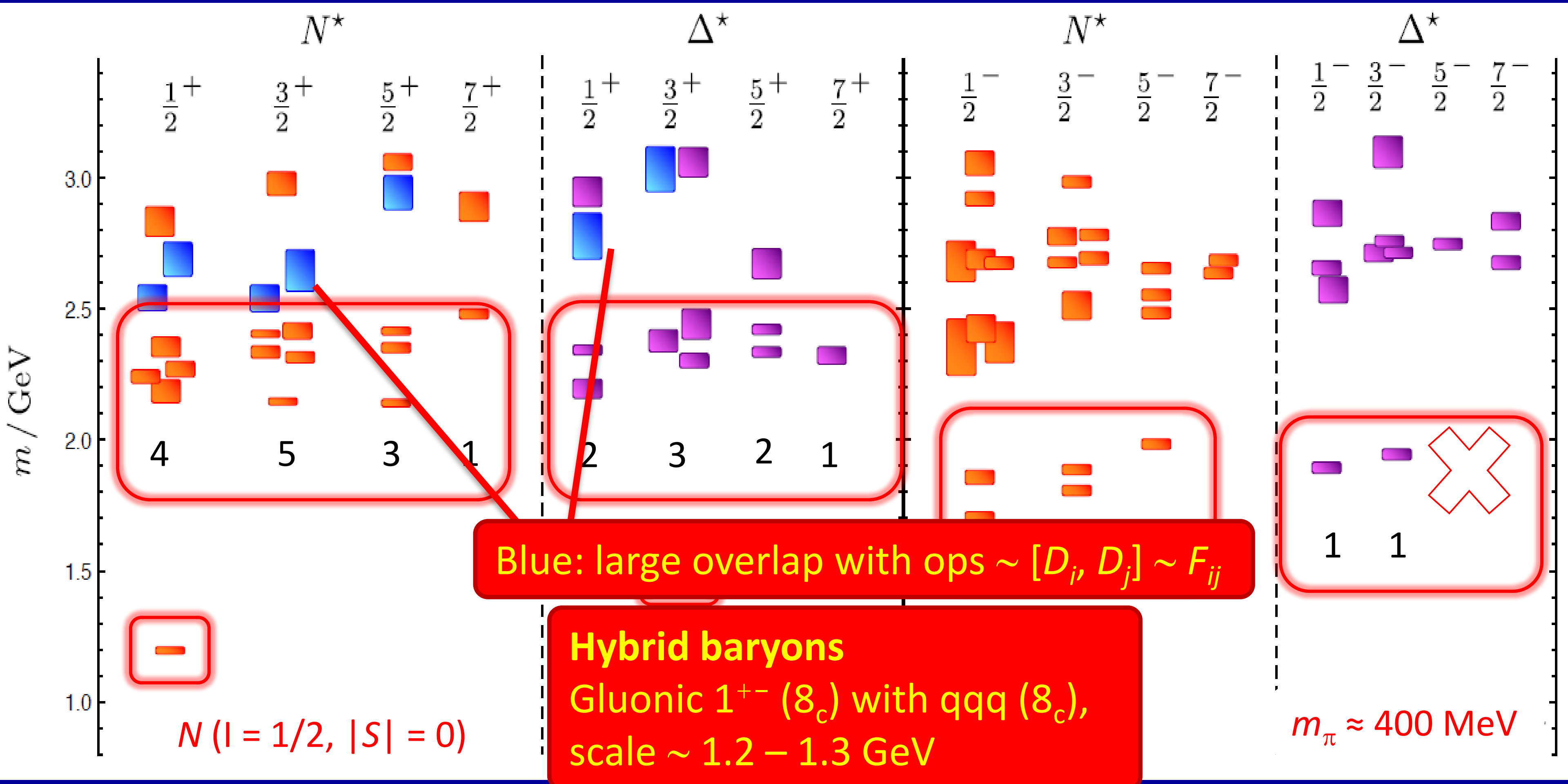
[HadSpec, PR D84 074508, PR D85 054016]



Counting in lowest bands as expected in non. rel. quark model, SU(6) x O(3) (flavour x spin x space), no ‘freezing of d.o.f.’

N and Δ baryons

[HadSpec, PR D84 074508, PR D85 054016]



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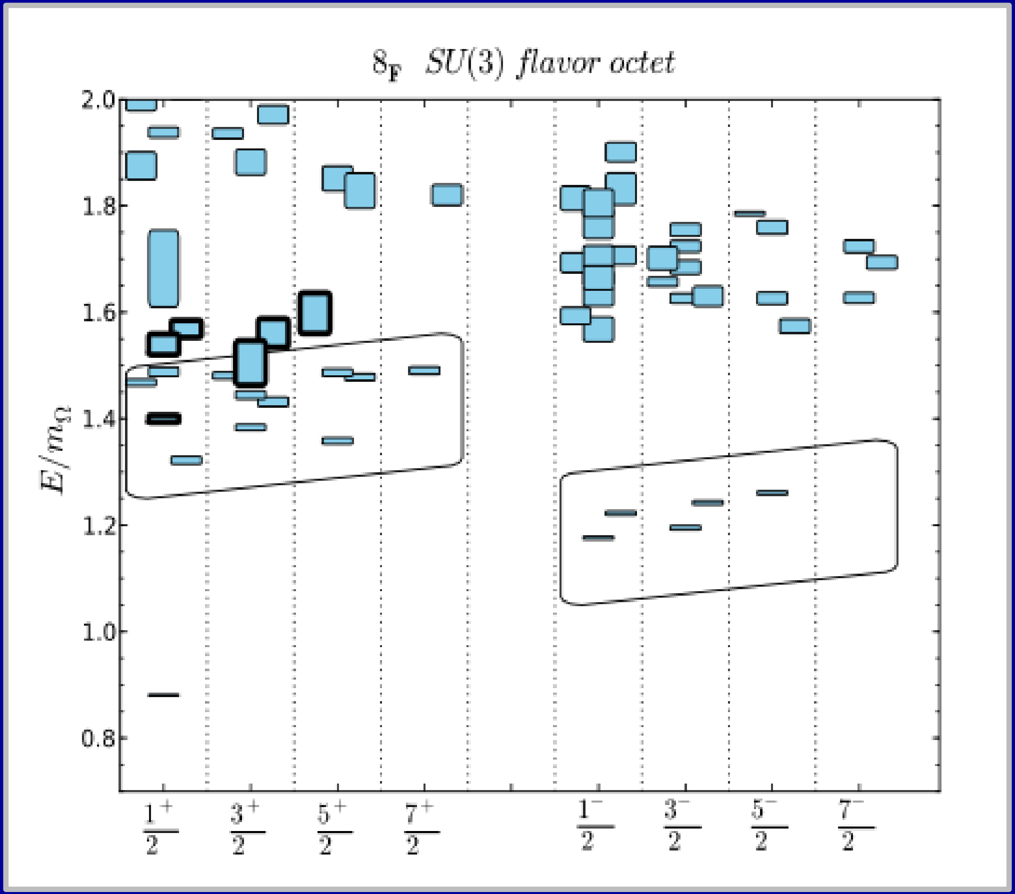
Flavour structure of excited baryons

[HadSpec, PR D87, 054506 (2013)]

$m_s = m_u = m_d$ SU(3) flavour symmetry, $M_\pi \approx 700$ MeV, qqq $\rightarrow 1_F \oplus 8_F \oplus 10_F$



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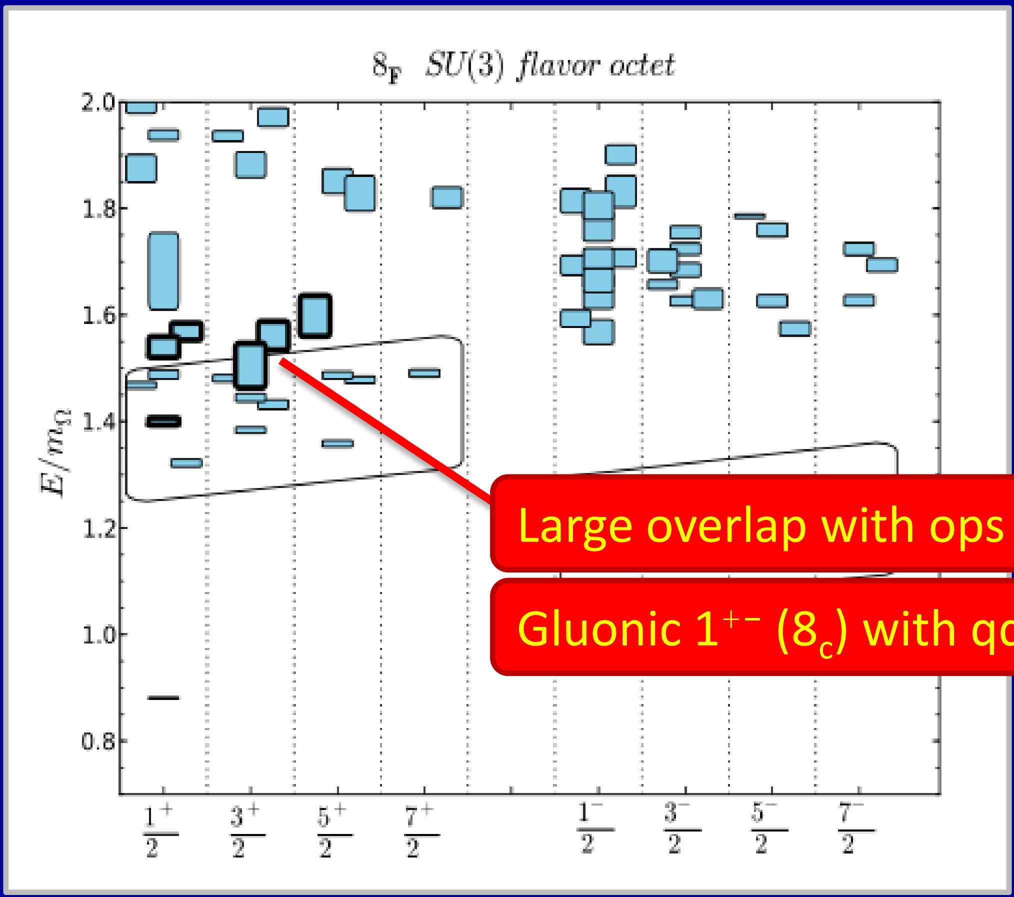


- Again, multiplicities in lowest bands as expected in non. rel. quark model SU(6) x O(3)
- No 'freezing' of d.o.f.

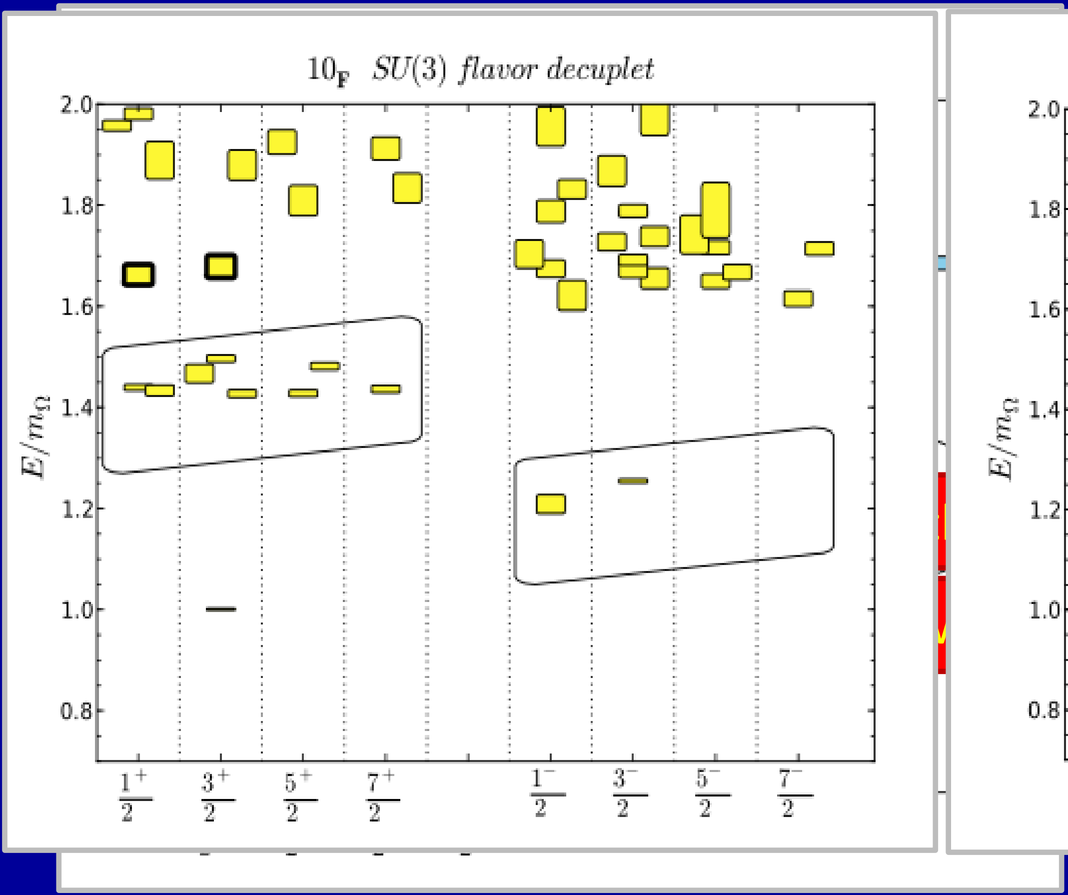
Large overlap with ops ~ $[D_i, D_j] \sim F_{ij}$

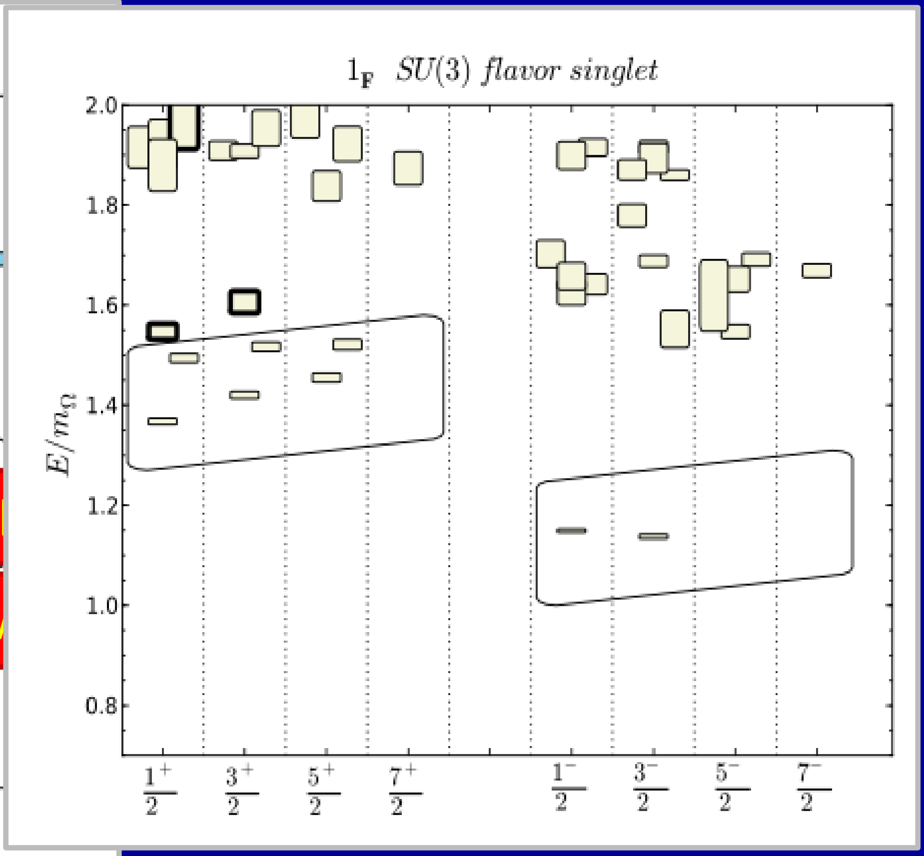
Gluonic 1^{+-} (8_c) with qqq (8_c), scale ~ 1.2 – 1.3 GeV

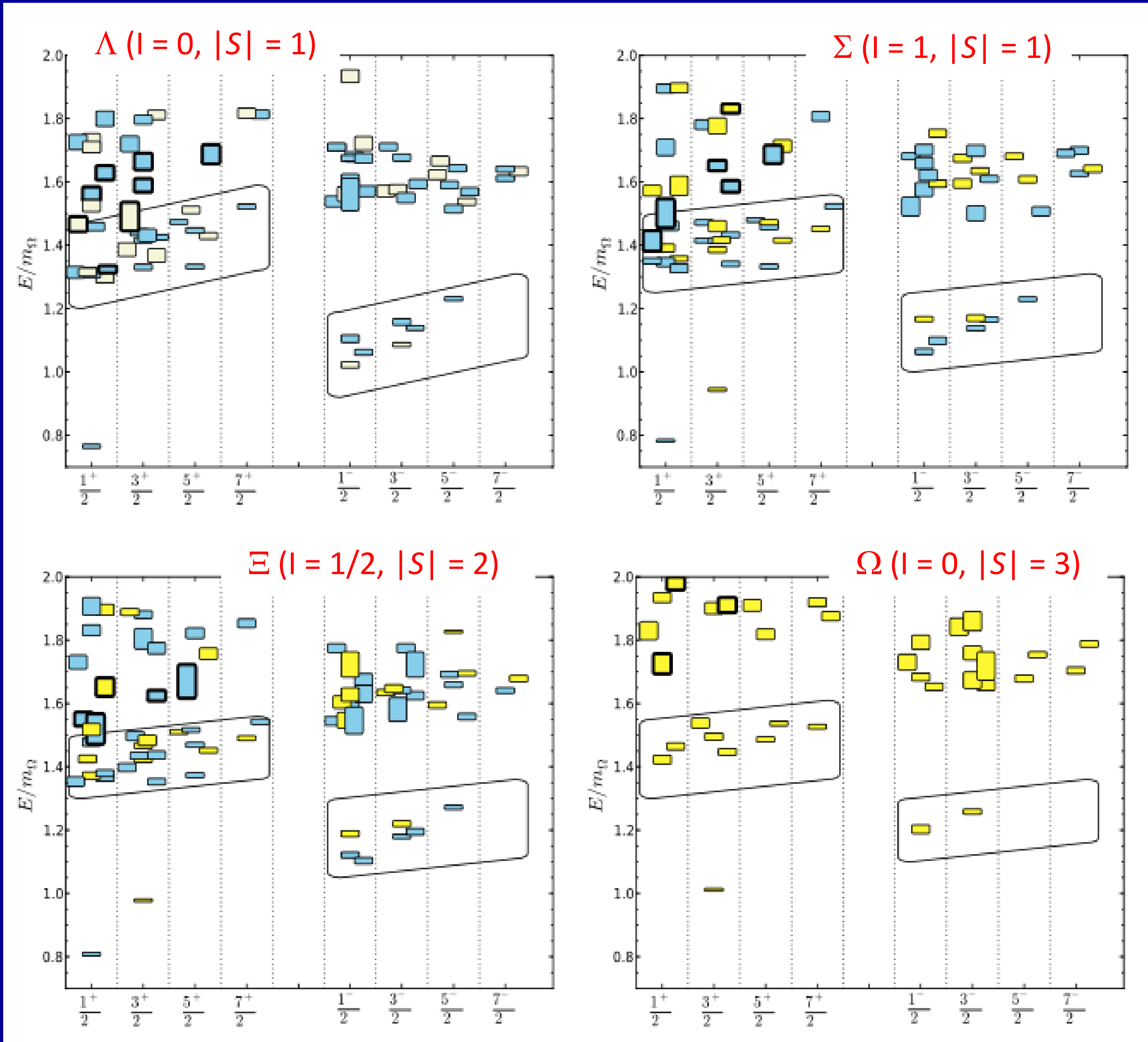
Flavour structure of excited baryons

[HadSpec, PR D87, 054506 (2013)]

$m_s = m_u = m_d$ SU(3) flavour symmetry, $M_\pi \approx 700$ MeV, qqq $\rightarrow 1_F \oplus 8_F \oplus 10_F$





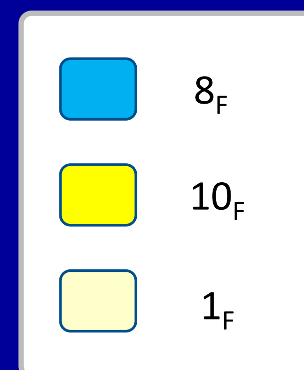


[HadSpec, PR D87, 054506 (2013)]

$m_s > m_u = m_d$

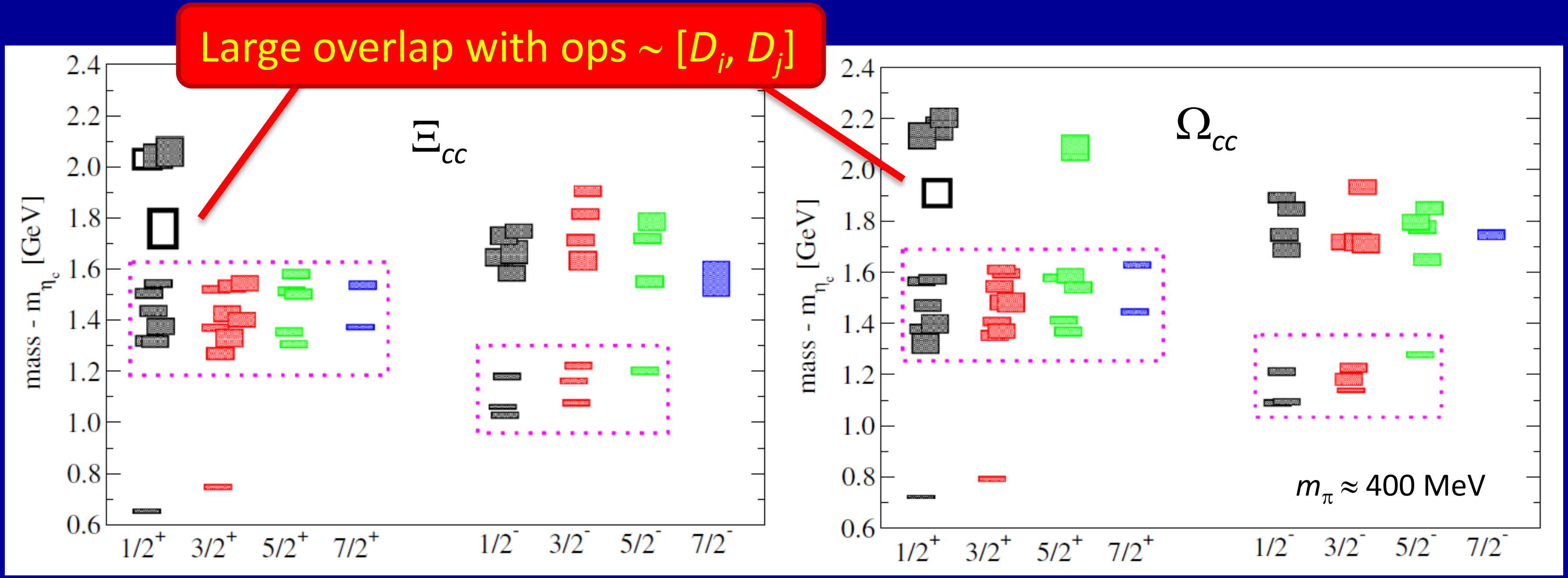
Broken SU(3) flav. sym.

$m_\pi \approx 400$ MeV

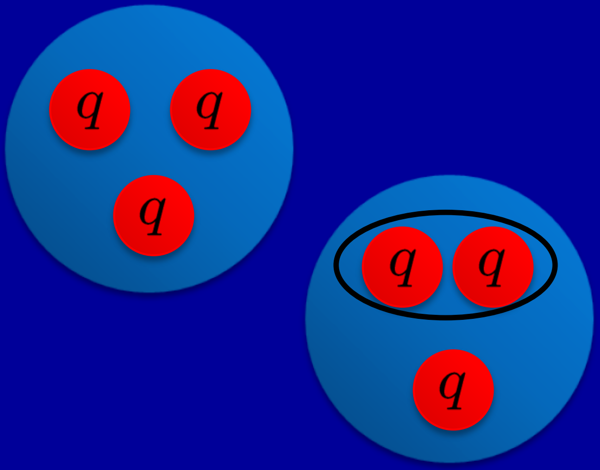


Excited charm (cc) baryons

Padmanath et al (HadSpec), PR D91, 094502 (2015)

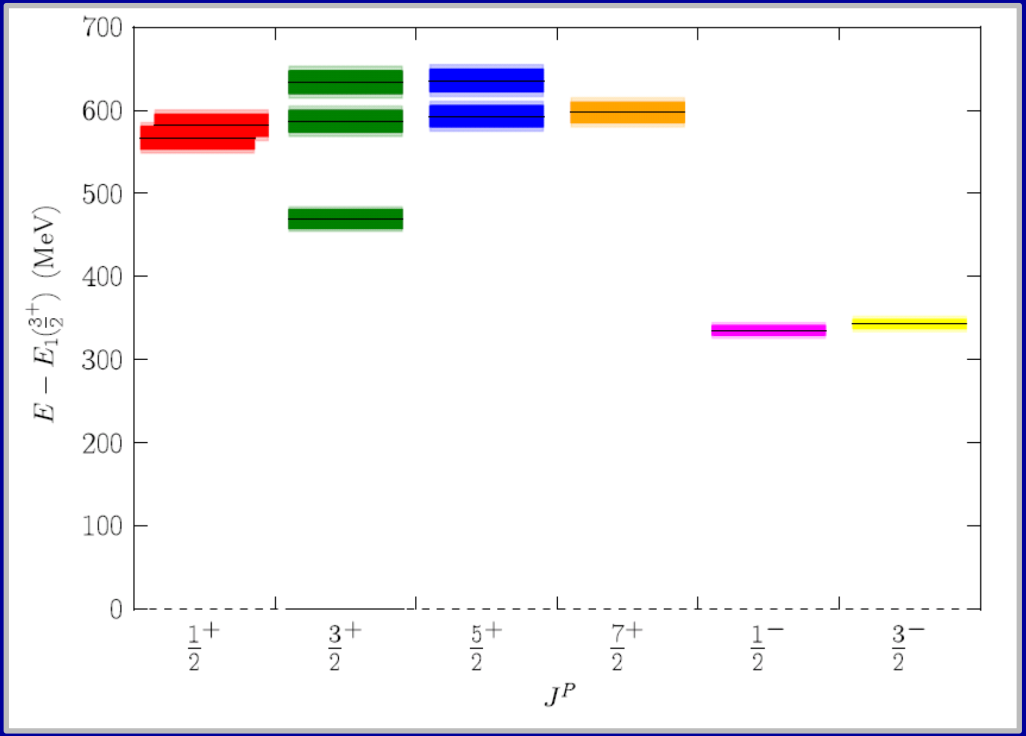


- Again, pattern in lowest bands consistent with non. rel. quark model
- Spectra don't support quark-diquark picture
- Also triply-charmed (ccc) baryons



Excited bottom (bbb) baryons

Meinel, PR D85, 114510 (2012)



- NRQCD action for *b* quark
- Dynamical domain-wall *u*,*d* and *s* quarks
- Two different lattice spacings
- A number of m_π – extrapolate to physical m_π

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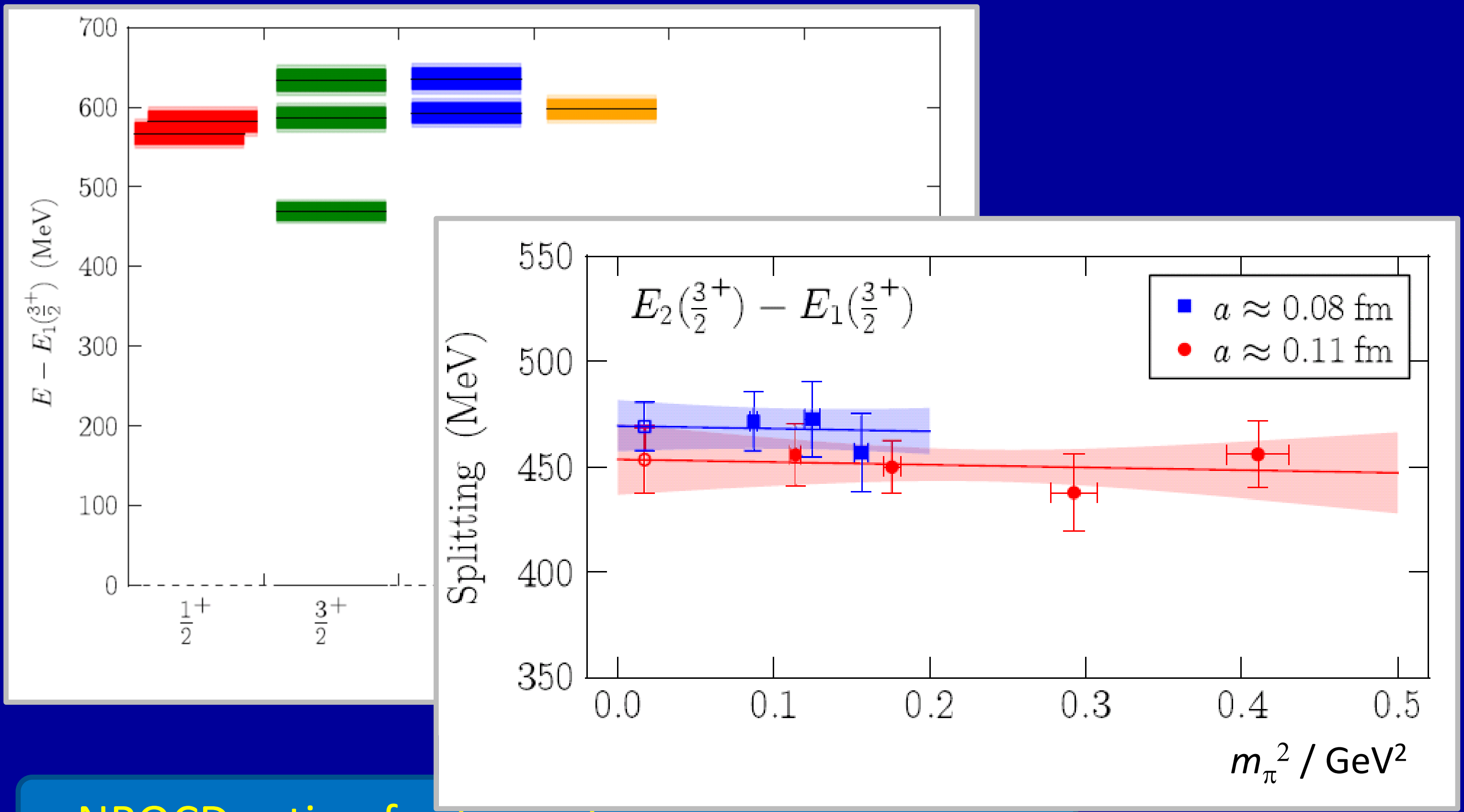

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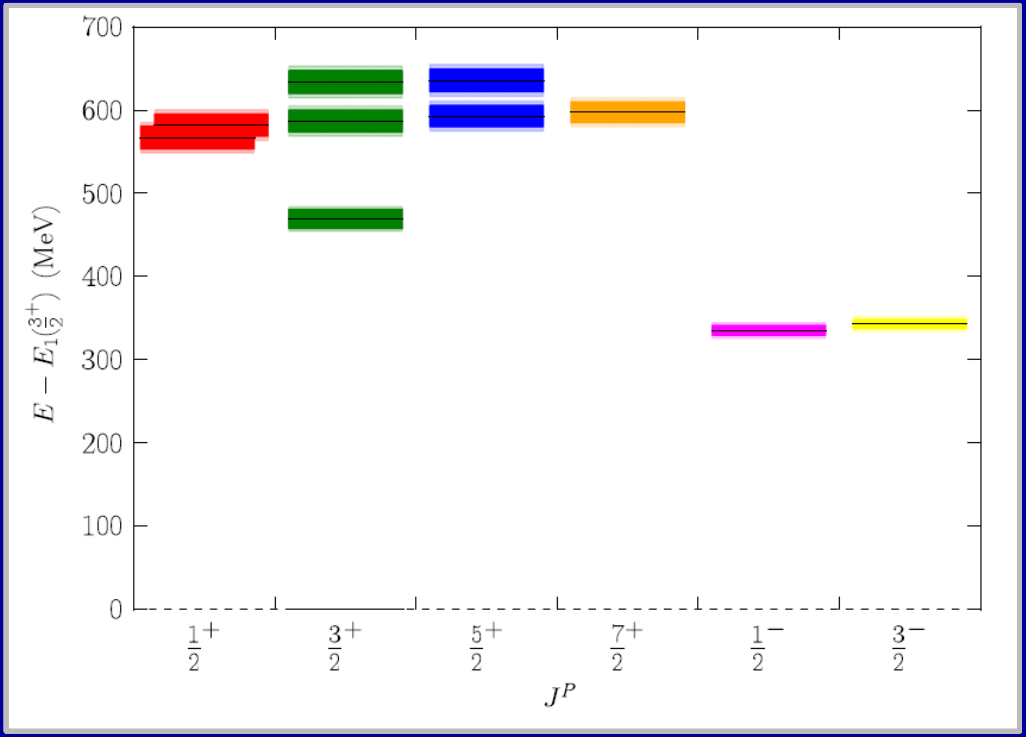

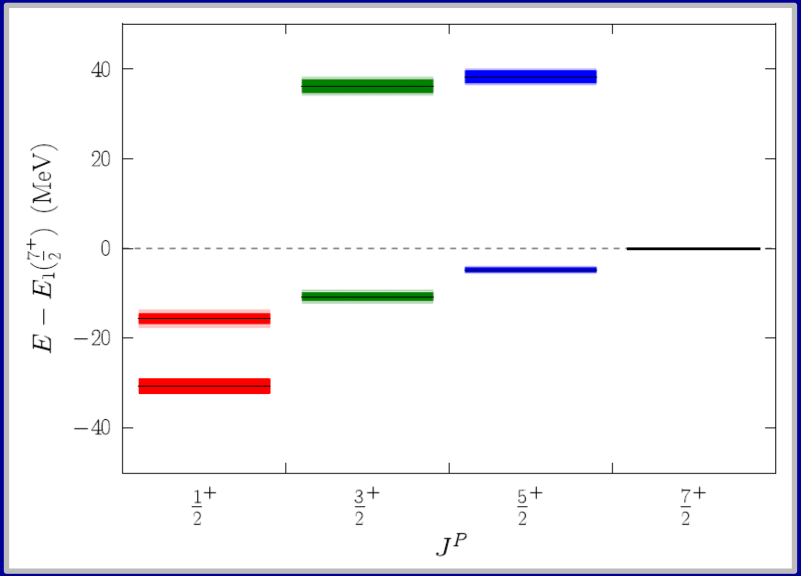

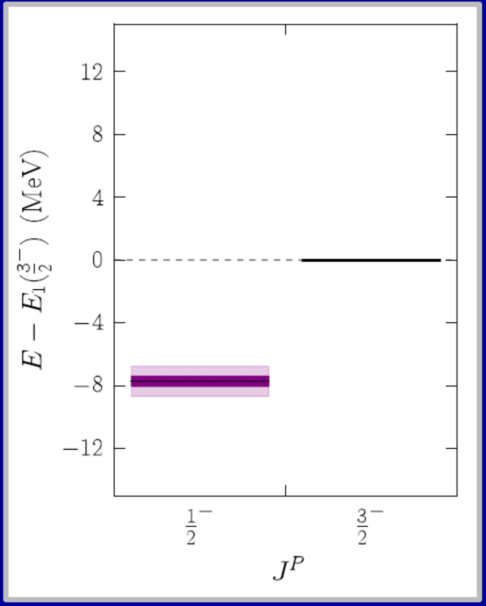

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Summary of lecture 2

- Background on hadron spectroscopy
- Excited hadron spectroscopy in lattice QCD
  - Mesons
  - Baryons

But so far we've neglected the fact that many of these hadrons are above the threshold for strong decay…

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Next time

- Scattering, resonances, etc

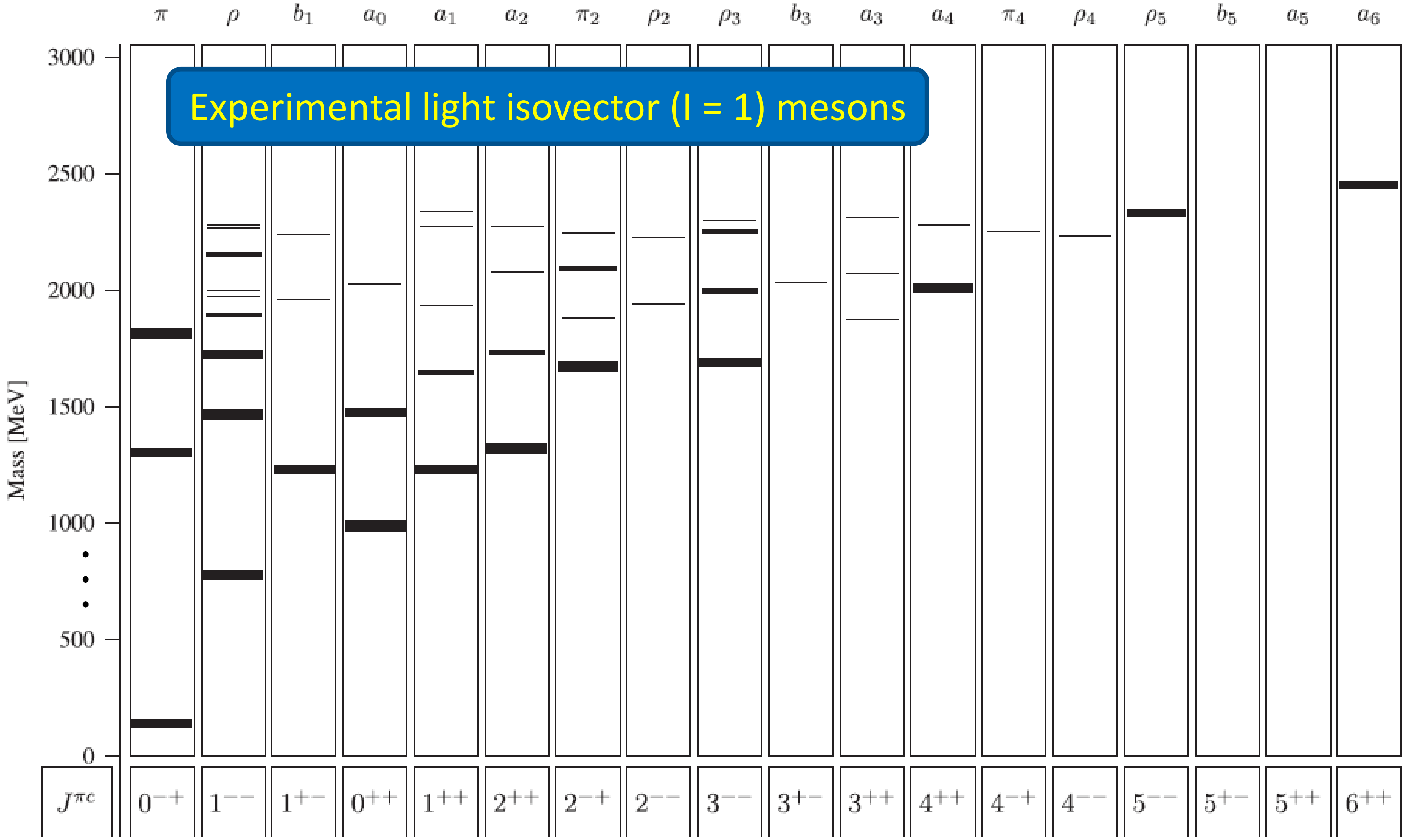
Hadron spectroscopy from lattice QCD

Lecture 3

- Scattering, resonances, etc in lattice QCD
- Some examples:
  - The ρ resonance in elastic $\pi\pi$ scattering
  - Coupled-channel K π, K η scattering
  - Some D_s mesons and charmonia

(I won't review all lattice calculations)

Hadron Spectroscopy



Based on Klempt & Zaitsev, Phys Rep 454, 1 (2007)

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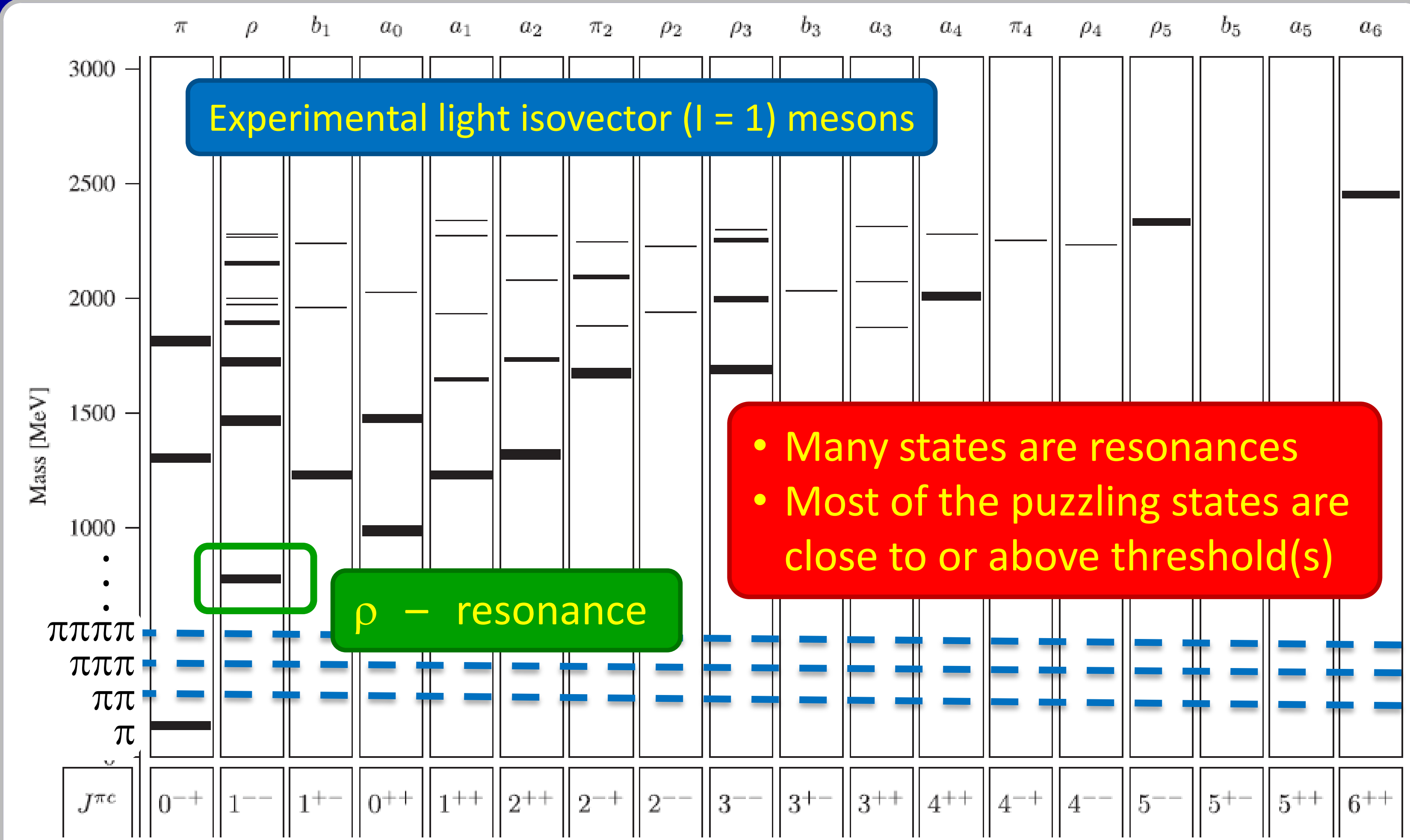
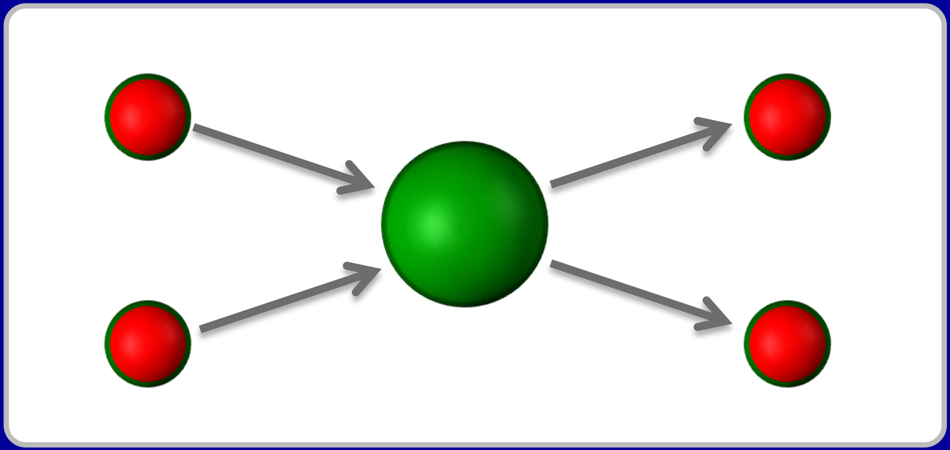


Based on Klempt & Zaitsev, Phys Rep 454, 1 (2007)

Scattering in LQCD

Maiani & Testa (1990): scattering matrix elements cannot be extracted from infinite-volume Euclidean-space correlation functions (except at threshold).



Scattering in LQCD

Two hadrons: **non-interacting**

$$E_{AB} = \sqrt{m_A^2 + \vec{k}_A^2} + \sqrt{m_B^2 + \vec{k}_B^2}$$

Infinite volume

Continuous spectrum

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Infinite volume

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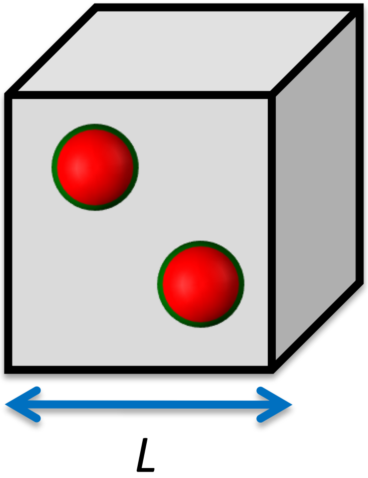
Finite volume

Discrete spectrum



$$\vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

periodic b.c.s (torus)

Scattering in LQCD

Two hadrons: **interacting**

Infinite volume

Continuous spectrum

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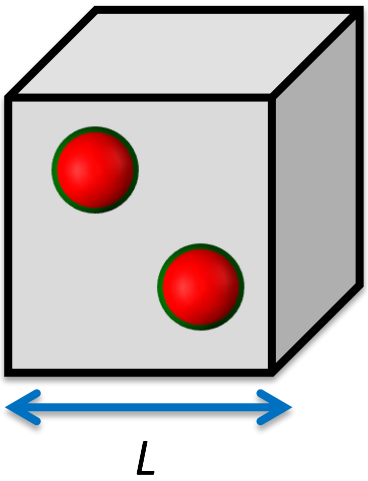
Discrete spectrum



$$\vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$$

periodic b.c.s (torus)

c.f. 1-dim: $k = \frac{2\pi}{L}n + \frac{2}{L}\delta(k)$

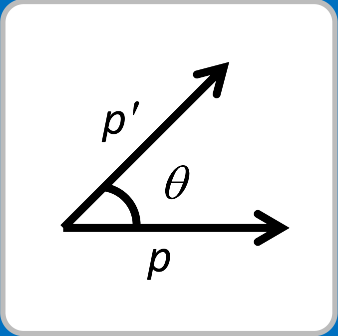
scattering phase shift

Scattering in an infinite volume – reminder

Scattering amplitude, f

$$\langle \vec{p'} | (S-1) | \vec{p} \rangle = \frac{i}{2\pi m} \delta(E' - E) f(E, \theta)$$

$$\frac{d\sigma}{d\Omega} = |f(E,\theta)|^2$$



Partial-wave expansion: $$f(E,\theta) = \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) f_l(E)$$

$$\sigma_l = 4\pi(2l+1)|f_l|^2$$

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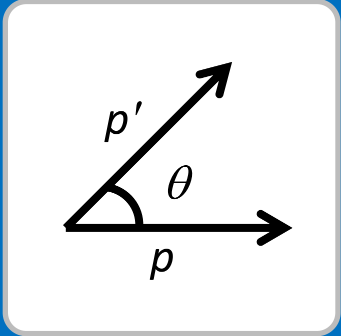

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Elastic scattering and the **phase shift,** $\delta_\ell(E)$

$$\langle E', l', m' | S | E, l, m \rangle = \delta(E' - E) \delta_{l'l} \delta_{m'm} e^{2i\delta_l(E)}$$

Scattering – 'Lüscher method'

Lüscher, NP B354, 531 (1991);
extended by many others

$$\det\left[\delta_{ij}\delta_{\ell\ell'}\delta_{nn'} + i\rho_i\; t_{ij}^{(l)}\;\left(\delta_{\ell\ell'}\delta_{nn'} + i\mathcal{M}^{\vec{P},\Lambda}_{\ell n;\ell' n'}(q_i^2)\right)\right] = 0$$

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i, *j* label channels
e.g. Kπ, Kη

$$\rho_i = \frac{2k_{\mathrm{cm},i}}{E_{\mathrm{cm}}}$$

$$\vec{q} \equiv \vec{k}_{\mathrm{cm}} L/2\pi$$

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Infinite-volume scattering *t*-matrix $s_{ij}^{(\ell)} = \delta_{ij} + 2i\sqrt{\rho_i \rho_k}\, t_{ij}^{(\ell)}$ Here $t^{(\ell)} \propto f_\ell$

$\vec{P} = \text{overall mom.}$

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effect of finite vol.

Reduced symmetry → ℓ mix

[all ℓ that subduce to a given lattice irrep (Λ) mix]

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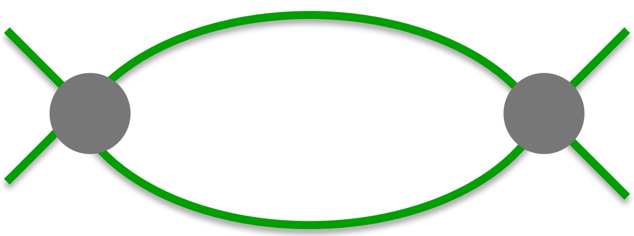
effect of finite vol.



Integrals over momenta in loops
→ sums over momenta
Difference ~ $1/L^3$ effects

Ignore $\sim e^{-ML}$ effects

Reduced symmetry → ℓ mix

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effect of finite vol.

Reduced symmetry → ℓ mix

[all ℓ that subduce to a given lattice irrep (Λ) mix]

Given *t*(*E*): solns → finite-vol. spec. {*E**}

We need: spectrum → *t*-matrix

Elastic scattering

$$t^{(\ell)} = \frac{1}{\rho} e^{i\delta_\ell} \sin \delta_\ell$$

$$\det \left[\delta_{\ell\ell'}\delta_{nn'} + i\rho_i \; t^{(l)} \; \left(\delta_{\ell\ell'}\delta_{nn'} + i\mathcal{M}^{\vec{P},\Lambda}_{\ell n;\ell' n'}(q_i^2) \right) \right] = 0$$

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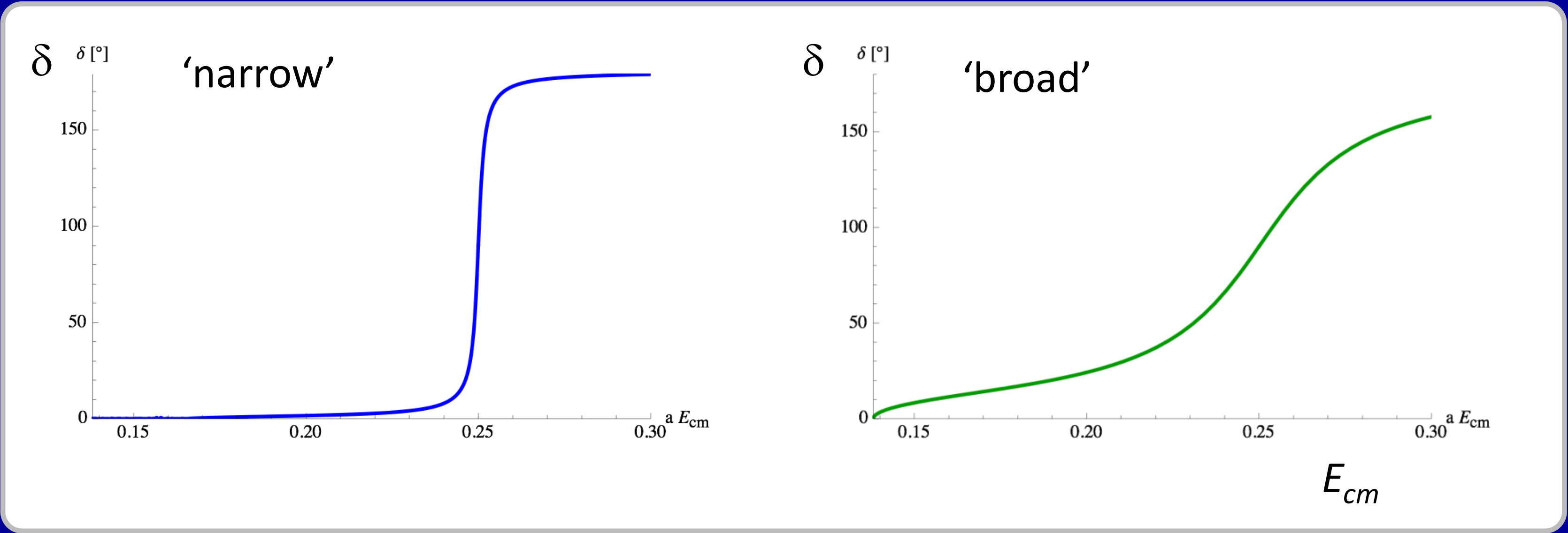
E.g. Relativistic Breit-Wigner param. (m_R, g_R) for an isolated resonance

$$t^{(\ell)} = \frac{1}{\rho(s)} \frac{\sqrt{s}\,\Gamma_\ell(s)}{m_R^2 - s - i\sqrt{s}\,\Gamma_\ell(s)} \qquad \Gamma_\ell(s) = \frac{g_R^2}{6\pi} \frac{k_{\text{cm}}^{2\ell+1}}{s\, m_R^{2(\ell-1)}} \qquad s = E_{\text{cm}}^2$$



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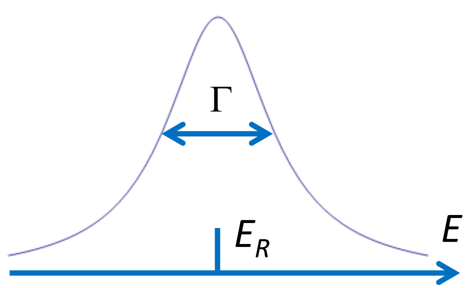

$$\sigma_l(E) \propto \sin^2 \delta_l(E) = \frac{(\Gamma/2)^2}{(E - E_R)^2 + (\Gamma/2)^2}$$

Elastic scattering

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If assume only lowest ℓ relevant [near threshold $t \sim k^{2\ell}$]
→ can solve equ. for each energy level {E^*} → phase shift $\delta(E)$

Alternatively parameterise $t(E)$ and fit {E^*_{lat}} to {E^*_{param}}

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Alternatively parameterise $t(E)$ and fit $\{E^*_{\text{lat}}\}$ to $\{E^*_{\text{param}}\}$

Need many (multi-hadron) energy levels

- Single and multi-hadron ops
- Non-zero P_{cm}, different box sizes and shapes, twisted b.c.s, ...

Map out phase shift → resonance parameters etc

The ρ resonance in $\pi\pi$ scattering

$\mathrm{BR}(\rho \to \pi\pi) \sim 100\%$

P-wave $\pi\pi$
($J^{PC} = 1^{--}$, I = 1)

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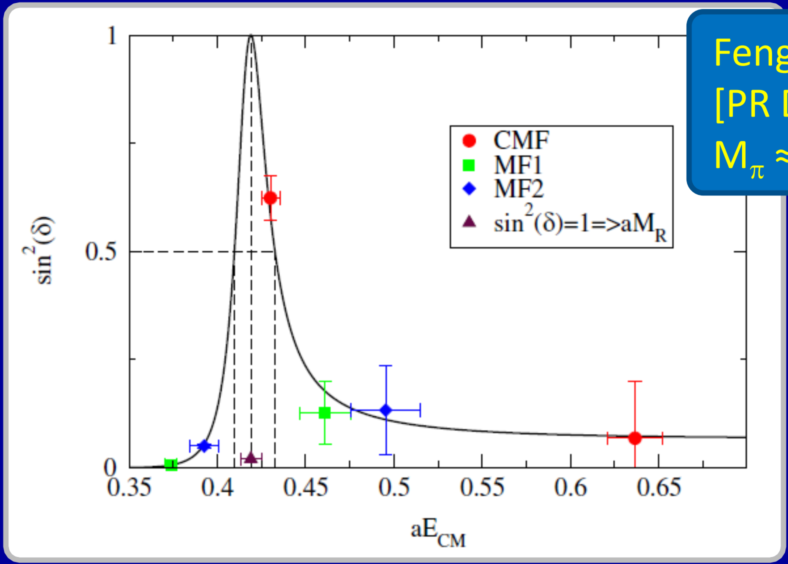
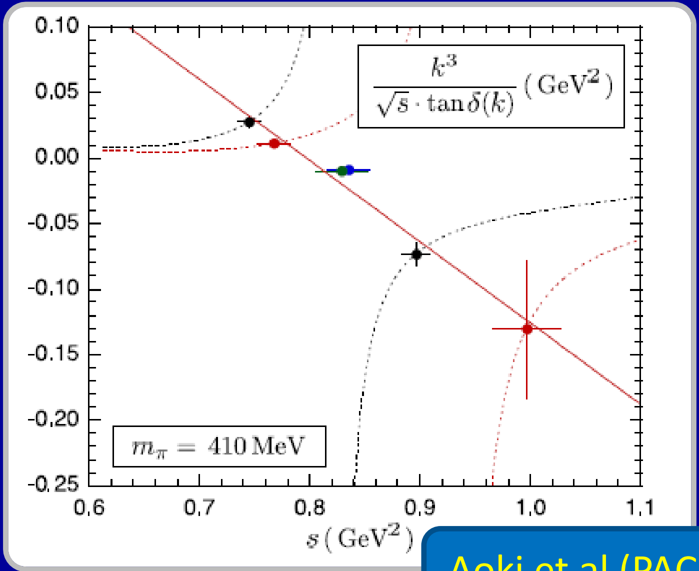
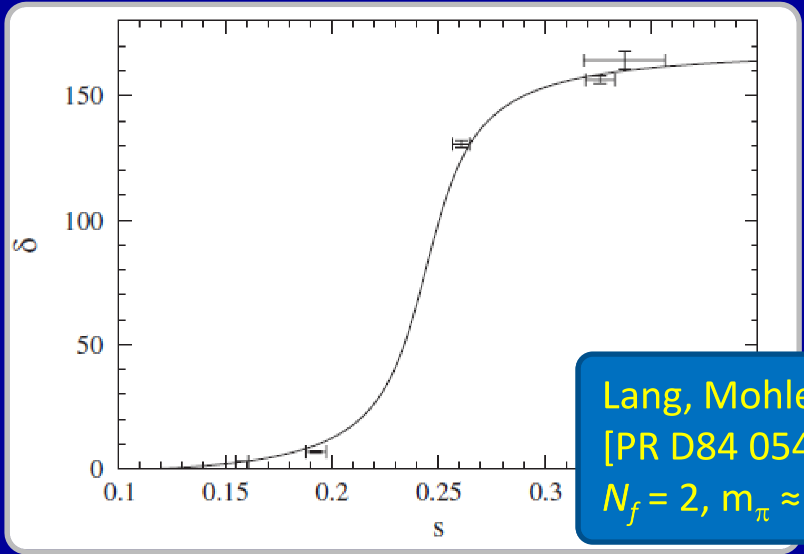


Feng, Jansen, Renner (ETMC),
[PR D83 094505 (2011)] $N_f = 2$,
$M_\pi \approx$ 480, 420, 330, 290 MeV



Aoki et al (PACS-CS),
[PR D84 094505 (2011)]
N_f = 2+1, $m_\pi \approx$ 410, 300 MeV



Lang, Mohler, Prelovsek, Vidmar,
[PR D84 054503 (2011)]
N_f = 2, $m_\pi \approx$ 266 MeV

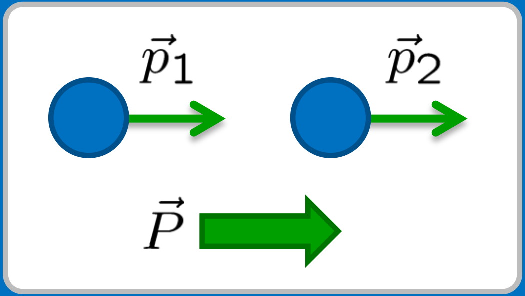
The ρ resonance in $\pi\pi$ scattering

$$C_{ij}(t) = < 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0 >$$

Use many 'single-meson' ops. $\sim \bar{\psi}\Gamma D \ldots \psi$

and ππ ops. $\mathcal{O}(\vec{P}) \sim \sum_{\hat{p}_1,\ \hat{p}_2} \mathcal{C}_\Lambda(\vec{P}, \vec{p}_1, \vec{p}_2)\ \mathcal{O}_\pi(\vec{p}_1)\ \mathcal{O}_\pi(\vec{p}_2)$

for various different **P** and Λ



$M_\pi \approx 400$ MeV,
3 volumes ($L \approx 2 - 3$ fm, $m_\pi L \approx 4$ - 6),
$a_s \approx 0.12$ fm, $a_s/a_t \approx 3.5$

Dudek, Edwards, CT (HadSpec), PR D87, 034505 (2013)

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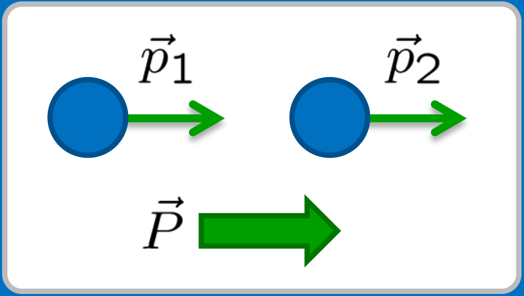


$M_\pi \approx 400$ MeV,
3 volumes ($L \approx 2 - 3$ fm, $m_\pi L \approx 4 - 6$),
$a_s \approx 0.12$ fm, $a_s/a_t \approx 3.5$

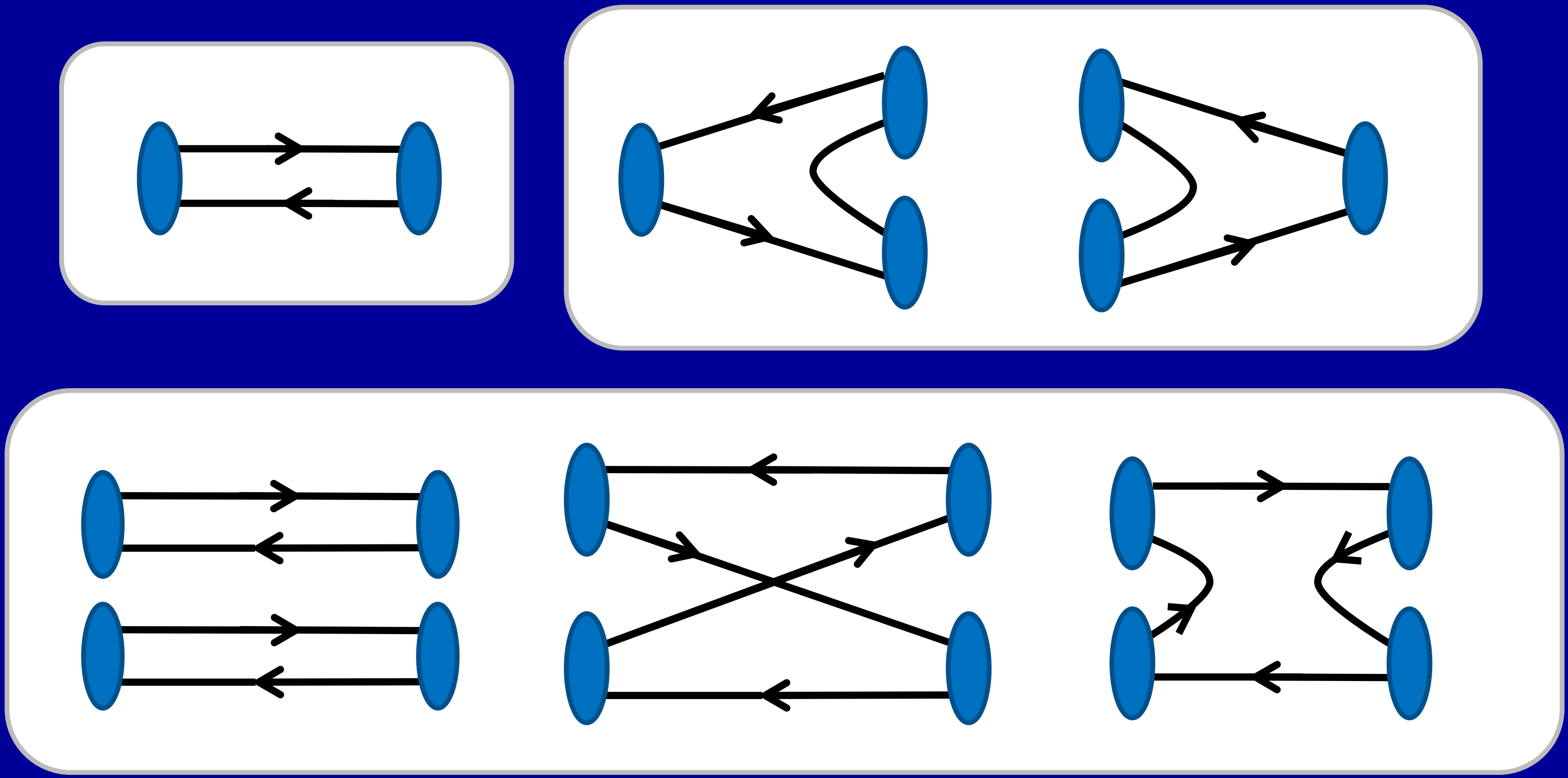
The $l = 1$ partial wave can mix with $l = 3$ and higher.

Find no significant signal for $\delta_{l=3}$ and so assume $\delta_{l>3} \approx 0$ in this energy range.

Dudek, Edwards, CT (HadSpec), PR D87, 034505 (2013)

$\pi\pi$ I=1 – diagrams

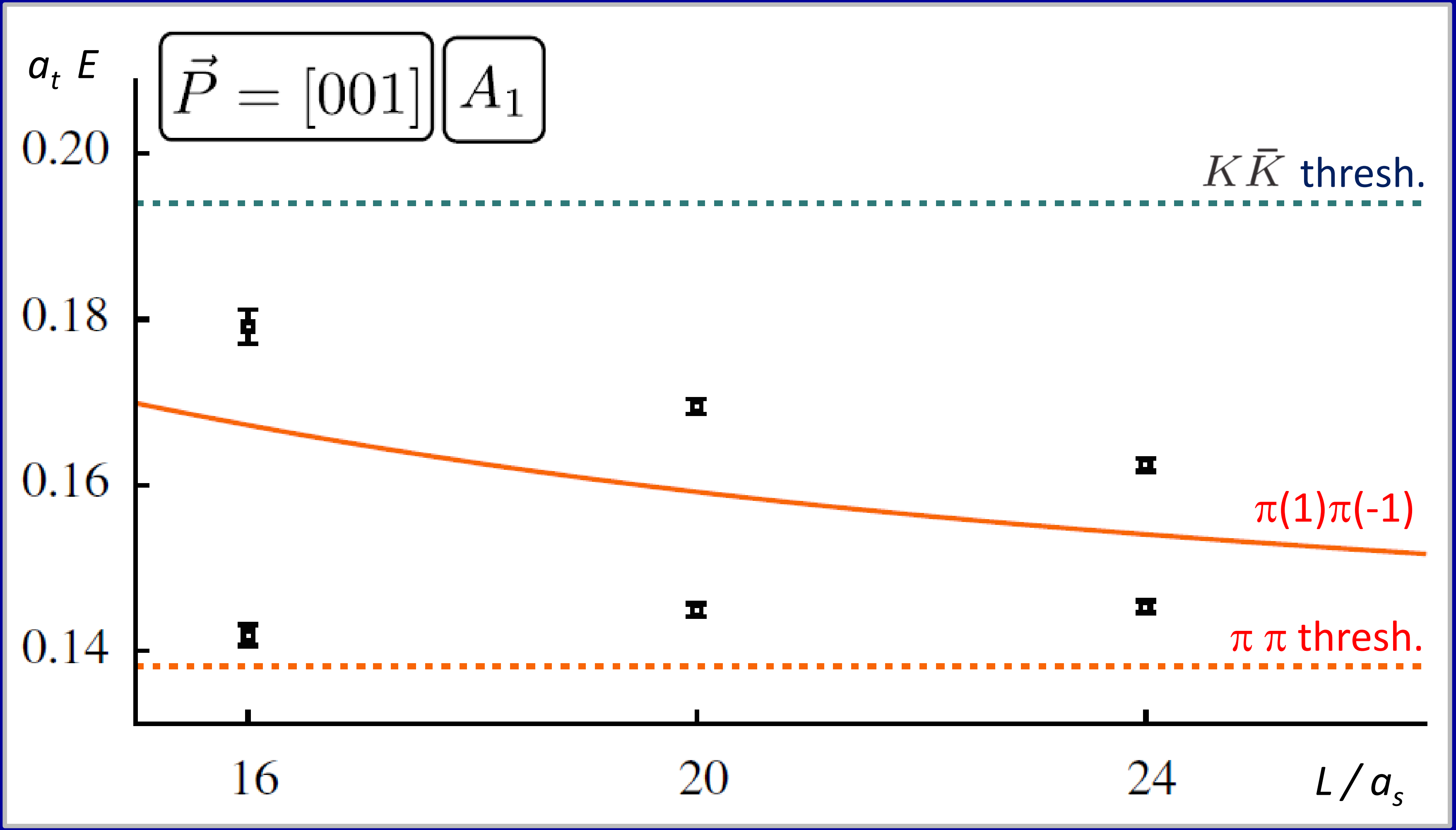
[PR D87, 034505]



+ similar diagrams

ππ I=1 – spectra

[PR D87, 034505]



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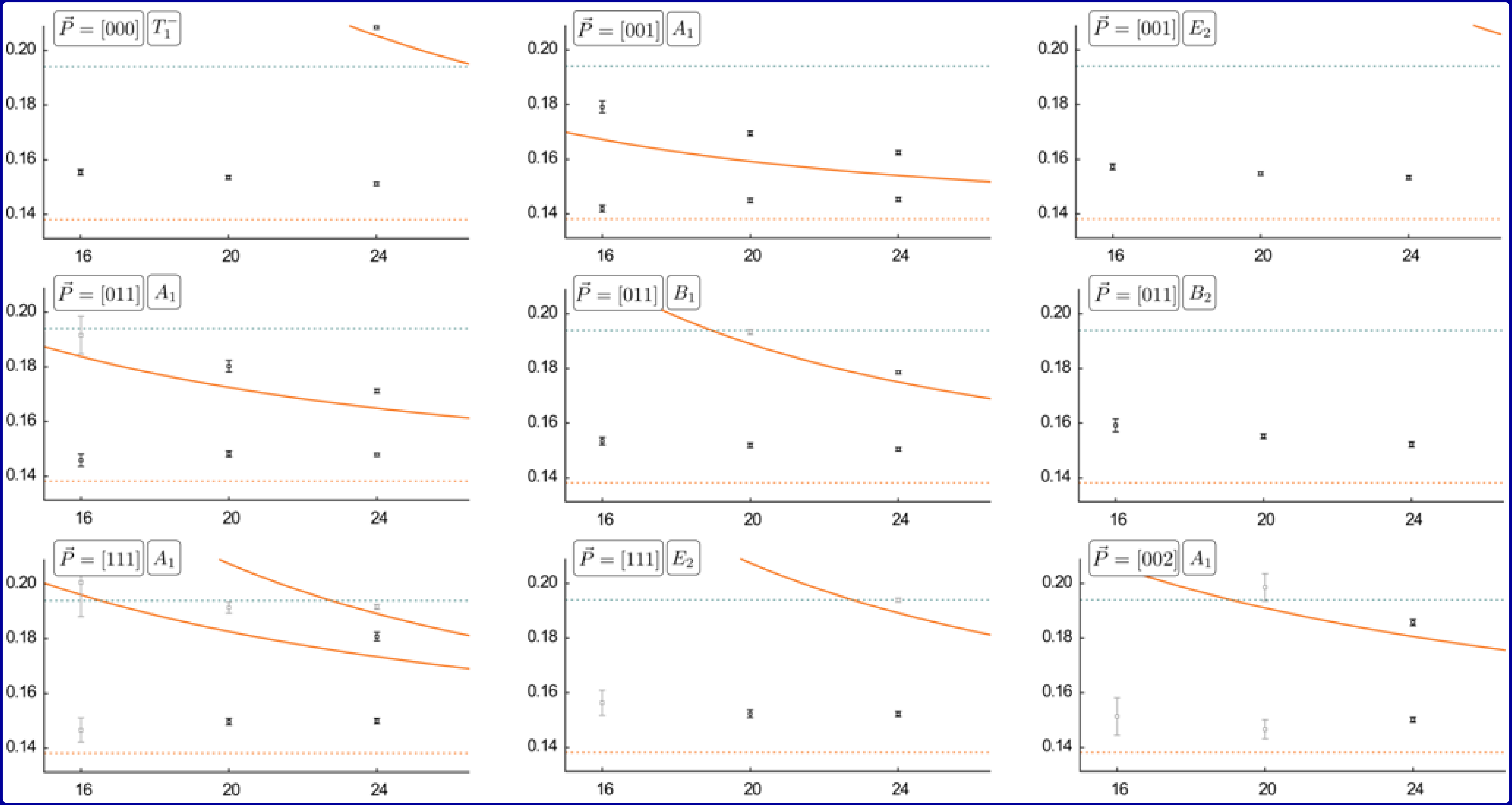
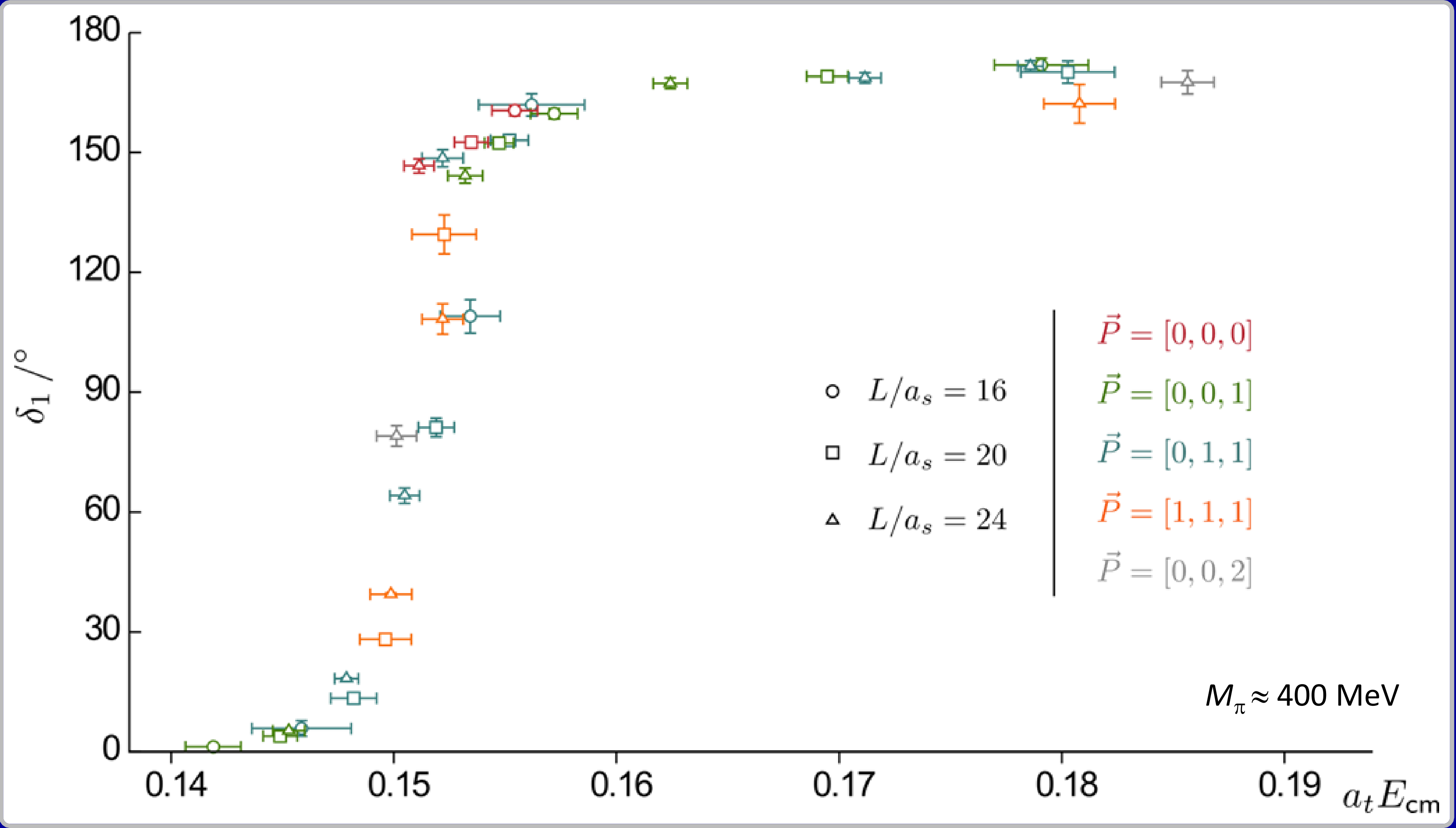


The ρ resonance in $\pi\pi$ scattering

[PR D87, 034505]



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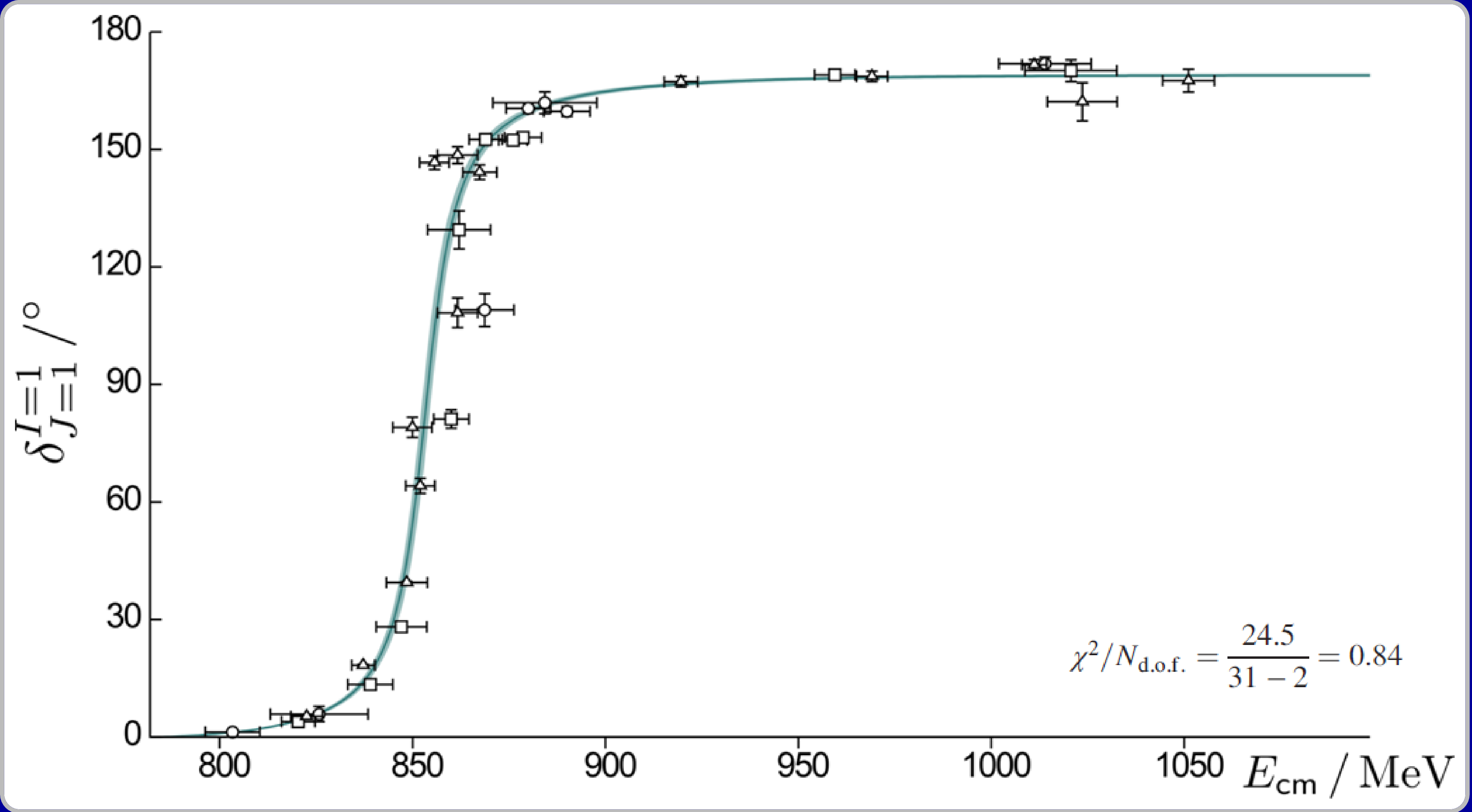
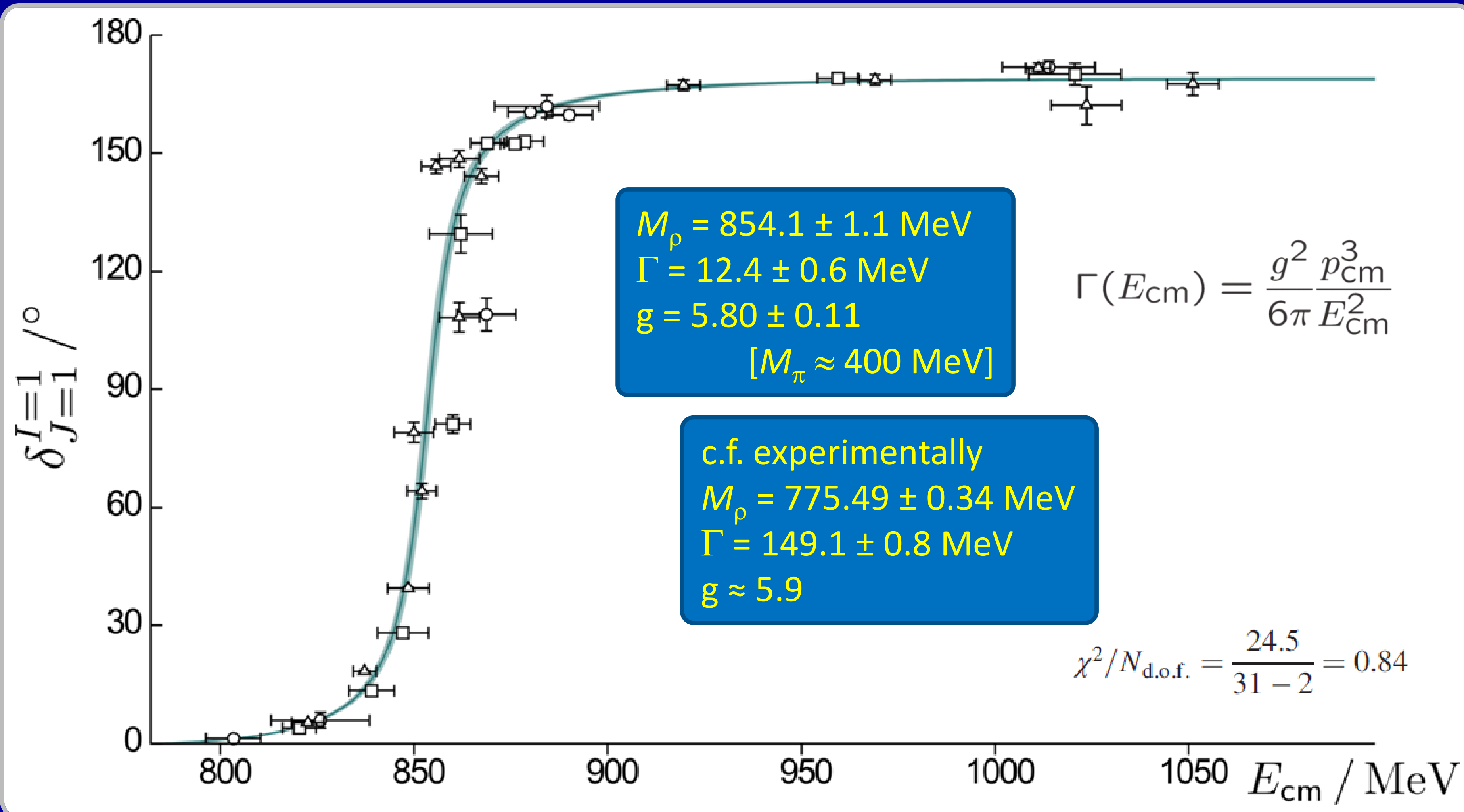
[PR D87, 034505]

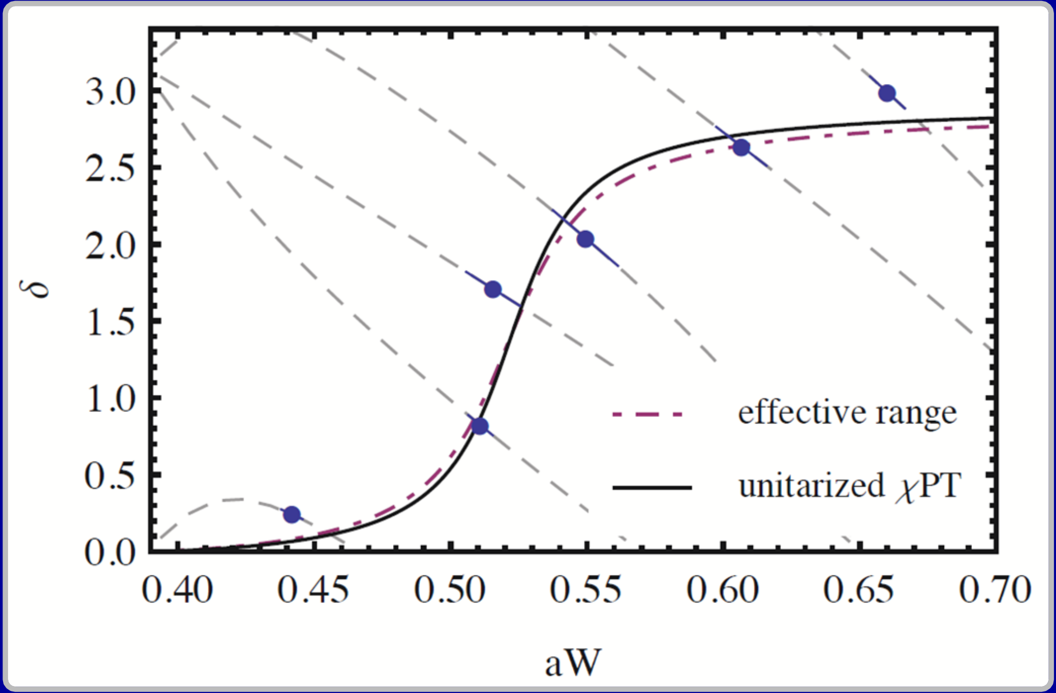


The ρ resonance in $\pi\pi$ scattering

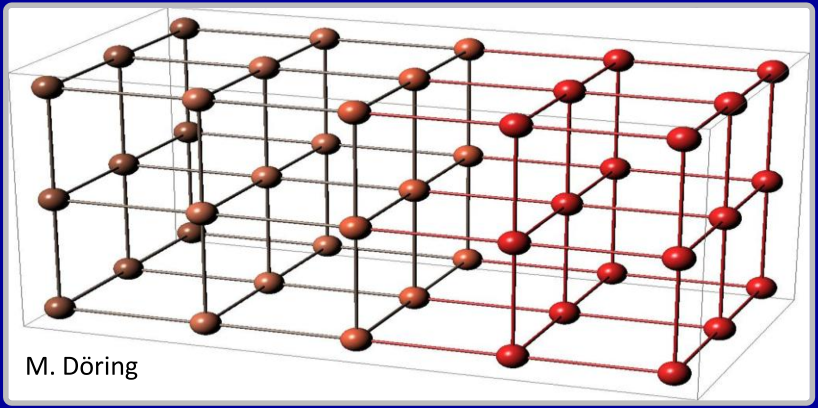
[PR D87, 034505]



The ρ resonance – other calcs.



Pelissier, Alexandru,
[PR D87 014503 (2013)]
$N_f = 2$, $M_\pi \approx 300$ MeV



πK, ηK (I=1/2) coupled-channel scattering

$J^P = 0^+$	κ, K_0*(1430), ...
$J^P = 1^-$	K*(892), ...
$J^P = 2^+$	K_2*(1430), ...

Isospin = 1/2
Strangeness = 1

πK, ηK (I=1/2) coupled-channel scattering

$J^P = 0^+$	κ, $K_0^*(1430)$, ...
$J^P = 1^-$	$K^*(892)$, ...
$J^P = 2^+$	$K_2^*(1430)$, ...

Isospin = 1/2
Strangeness = 1

$$C_{ij}(t) = < 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0 >$$

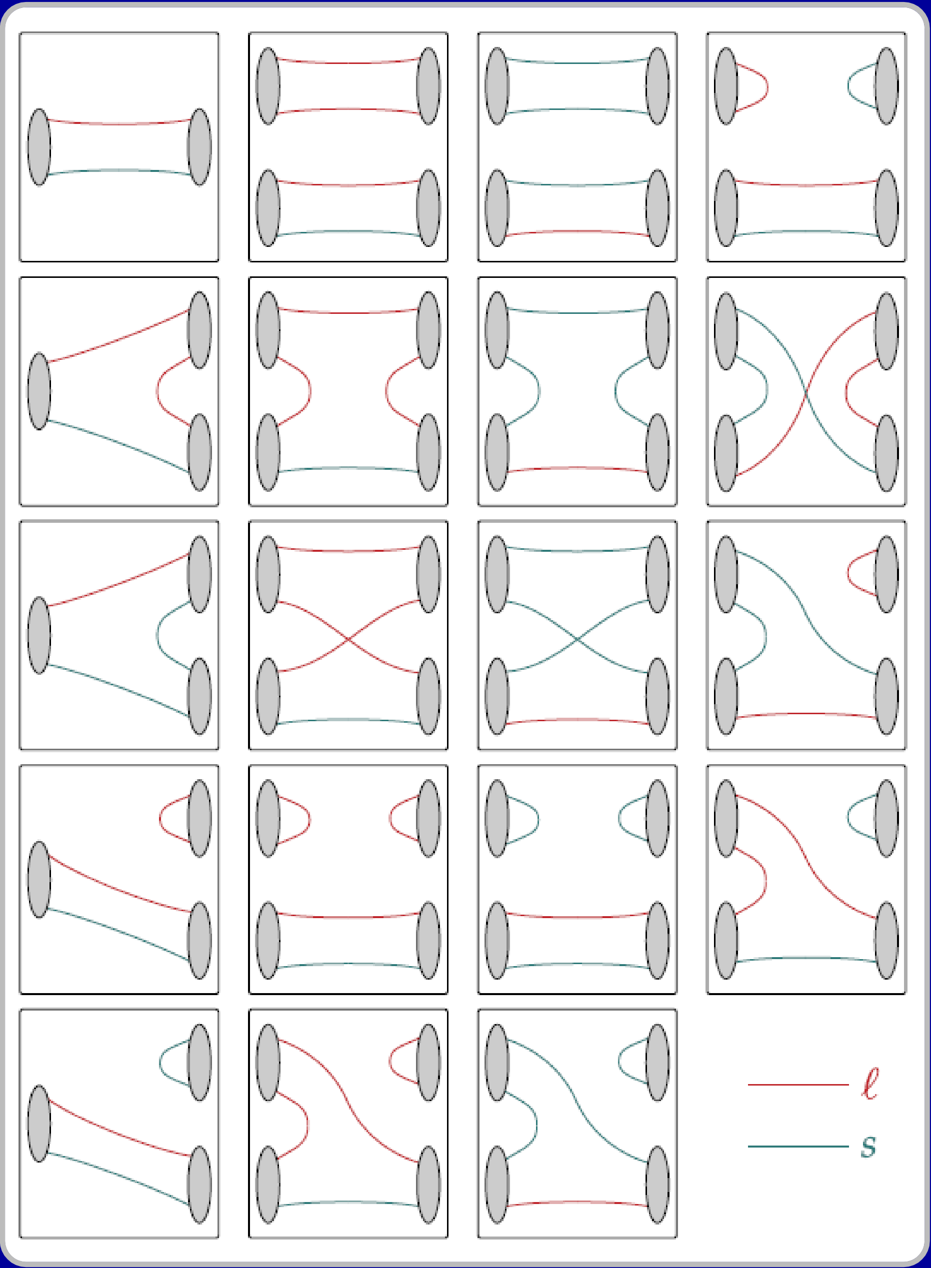
'single-meson' $\sim \bar{\psi}\Gamma D \ldots \psi$

+ πK + ηK ops.

M_π = 391 MeV, M_K = 549 MeV, M_η = 589 MeV; 3 volumes as before

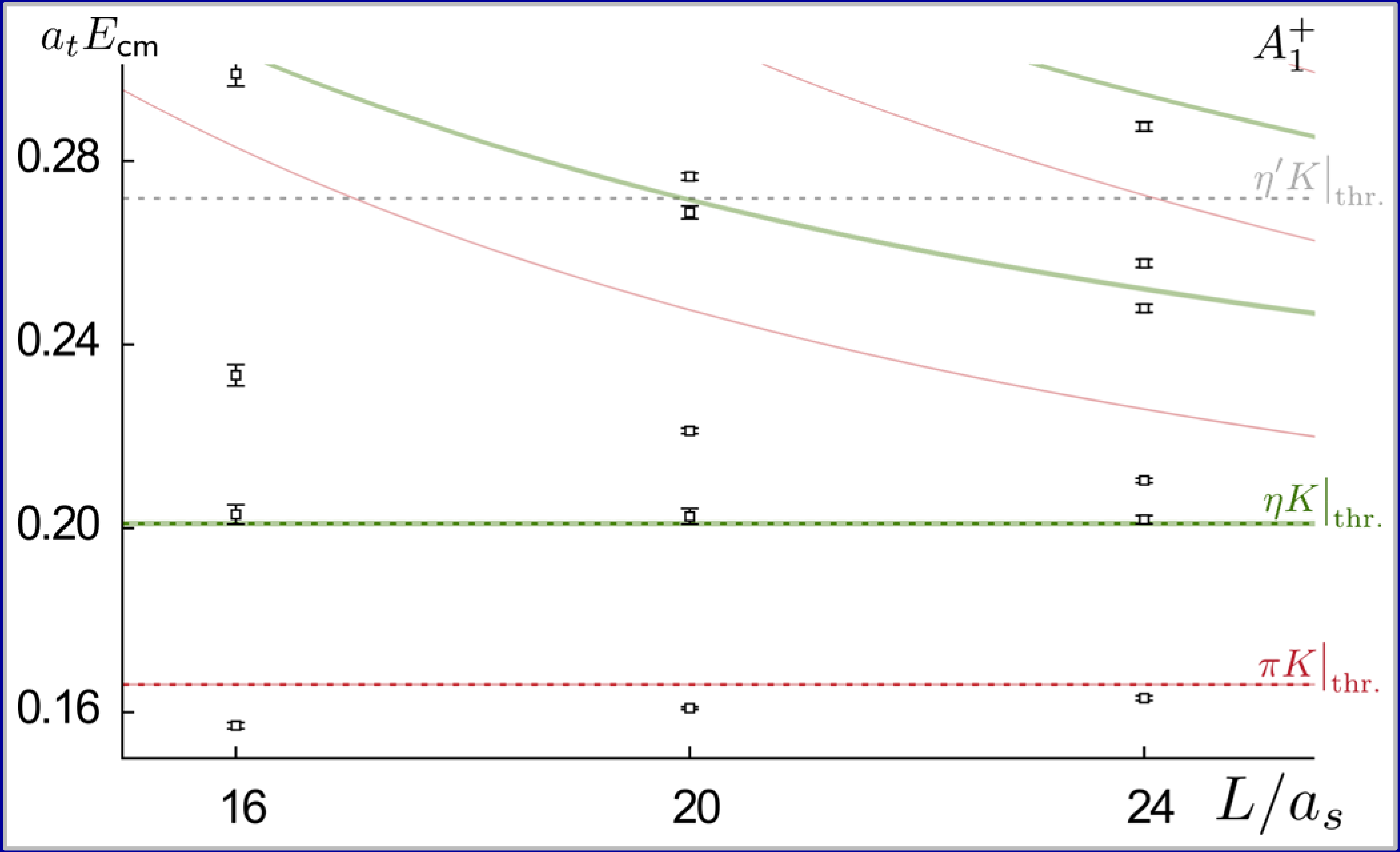
Wilson, Dudek, Edwards, CT (HadSpec), PRL 113, 182001; PR D91, 054008

πK, ηK (I=1/2) – diagrams



πK, ηK (I=1/2) spectra

P = [0,0,0] A_1^+



Neglect ℓ ≥ 4: only ℓ = 0 contributes

πK, ηK (I=1/2) coupled-channel scattering

Extension of Lüscher method to **inelastic scattering**:

relate finite vol. energy levels to infinite vol. scattering *t*-matrix.

Underdetermined problem

→ parameterize E_{cm} dependence of *t*-matrix and fit $\{E^*_{\text{lat}}\}$ to $\{E^*_{\text{param}}\}$

πK, ηK (I=1/2) coupled-channel scattering

Extension of Lüscher method to **inelastic scattering**:

relate finite vol. energy levels to infinite vol. scattering *t*-matrix.

Underdetermined problem

→ parameterize E_{cm} dependence of *t*-matrix and fit $\{E^*_{lat}\}$ to $\{E^*_{param}\}$

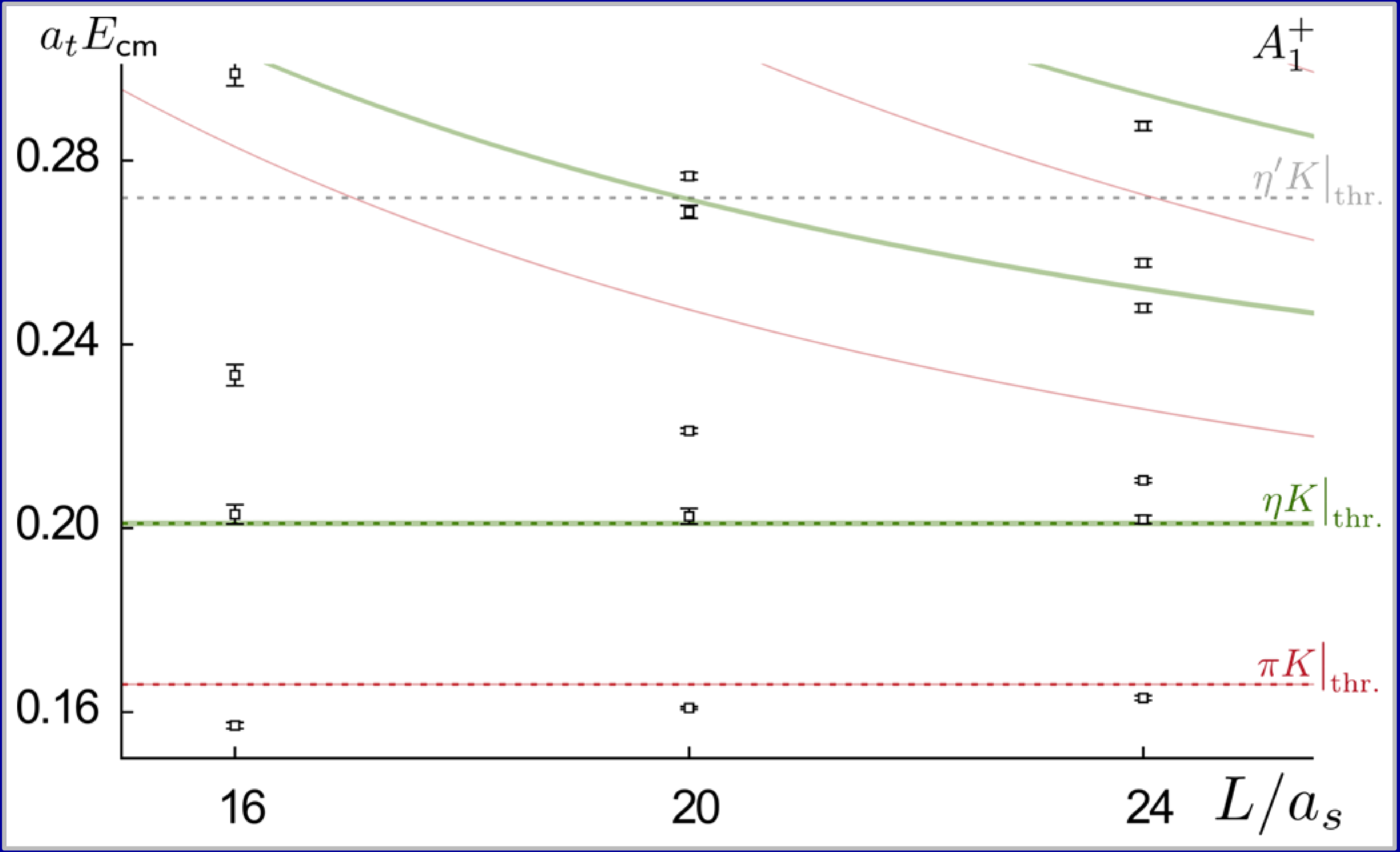
***K*-matrix param.** – respects unitarity (conserve prob.) and flexible

$$t_{ij}^{-1}(s) = \frac{1}{(2k_i)^\ell} K_{ij}^{-1}(s) \frac{1}{(2k_j)^\ell} + I_{ij}(s) \qquad \mathrm{Im} I_{ij} = -\delta_{ij}\rho_i(s)$$

Use various different params for *K* (see the paper for details)

πK, ηK (I=1/2) spectra

P = [0,0,0] A_1^+



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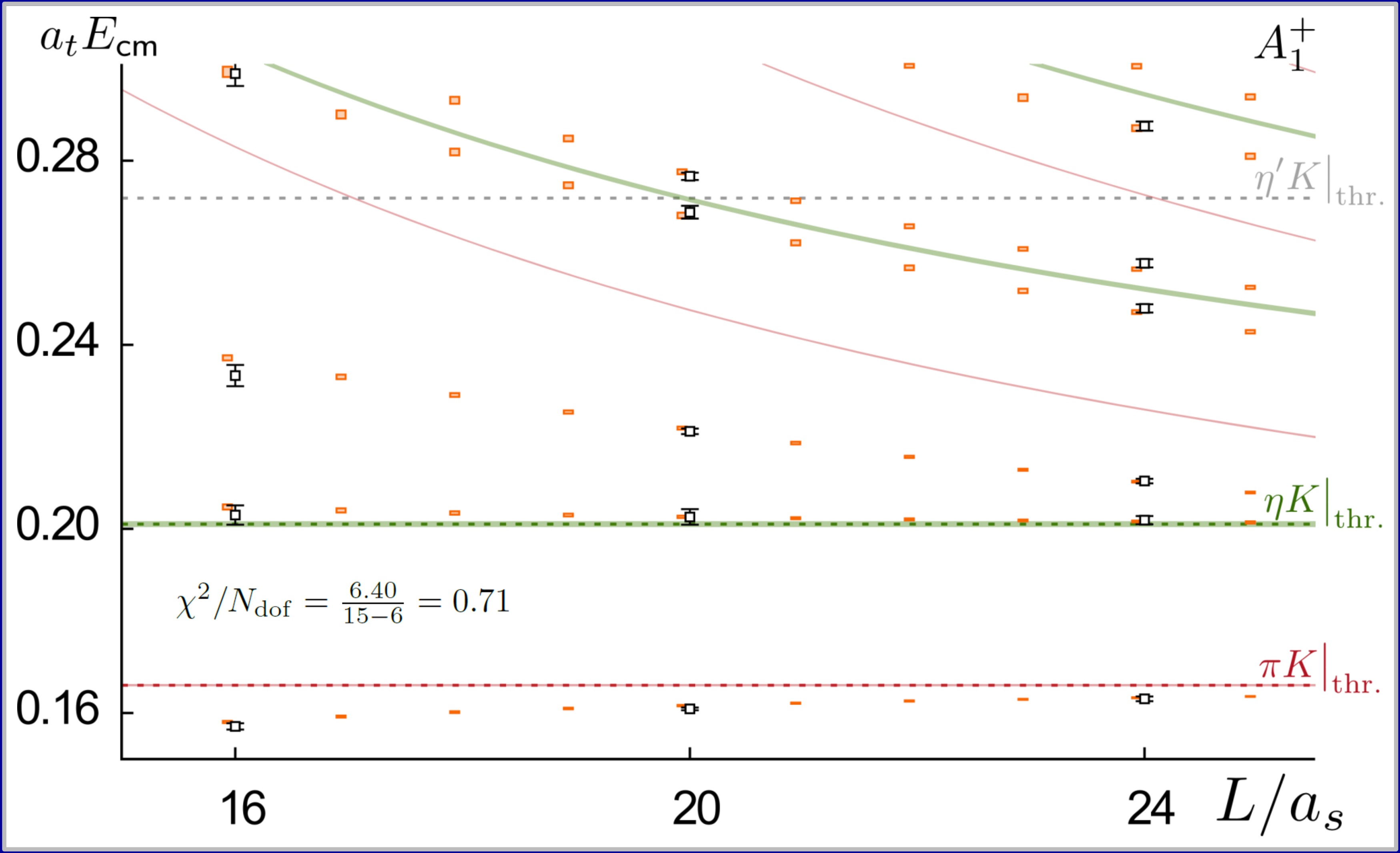
πK, ηK (I=1/2) spectra

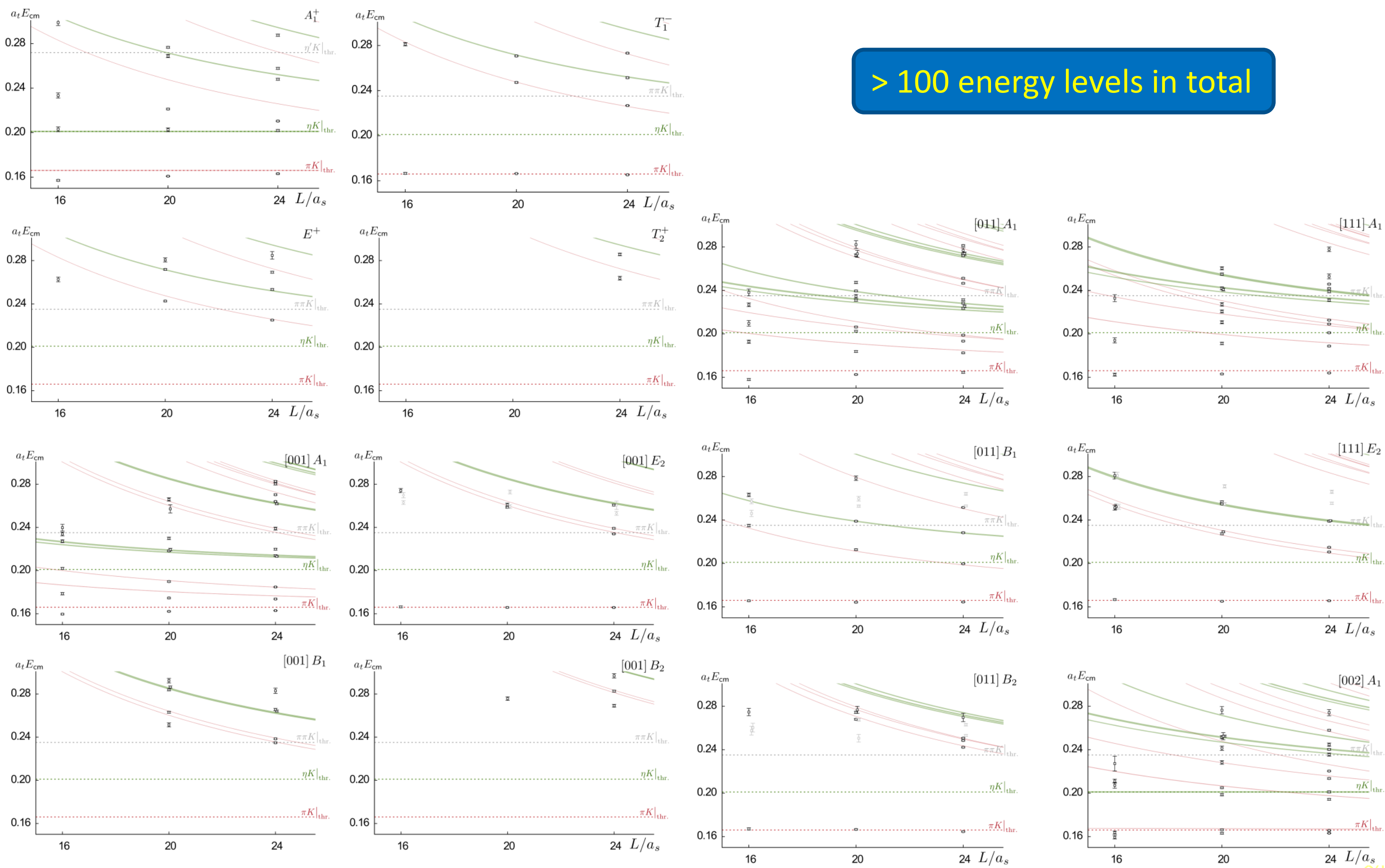
P = [0,0,0] A_1^+



Neglect ℓ ≥ 4: only ℓ = 0 contributes

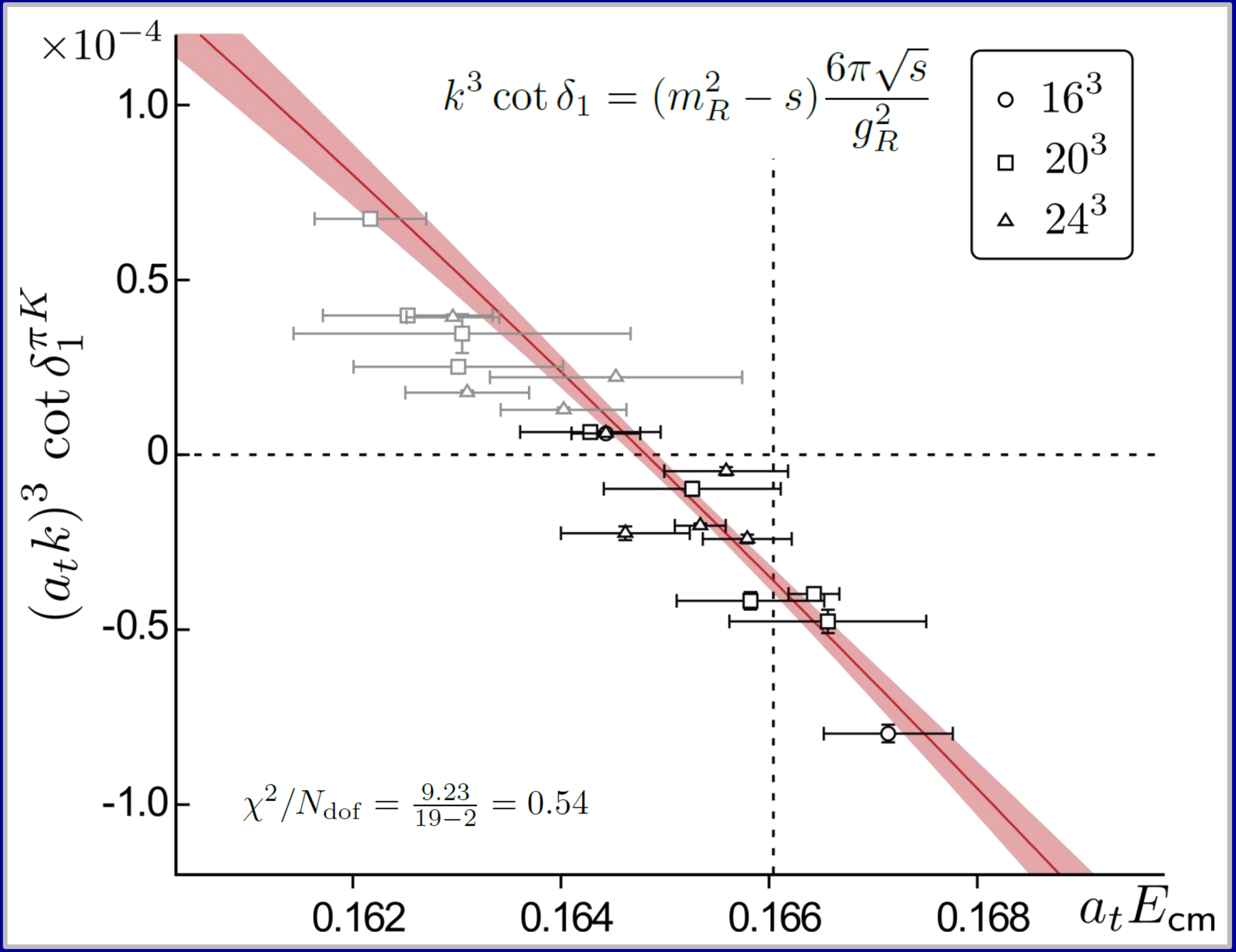
πK, ηK (I=1/2) spectra

> 100 energy levels in total



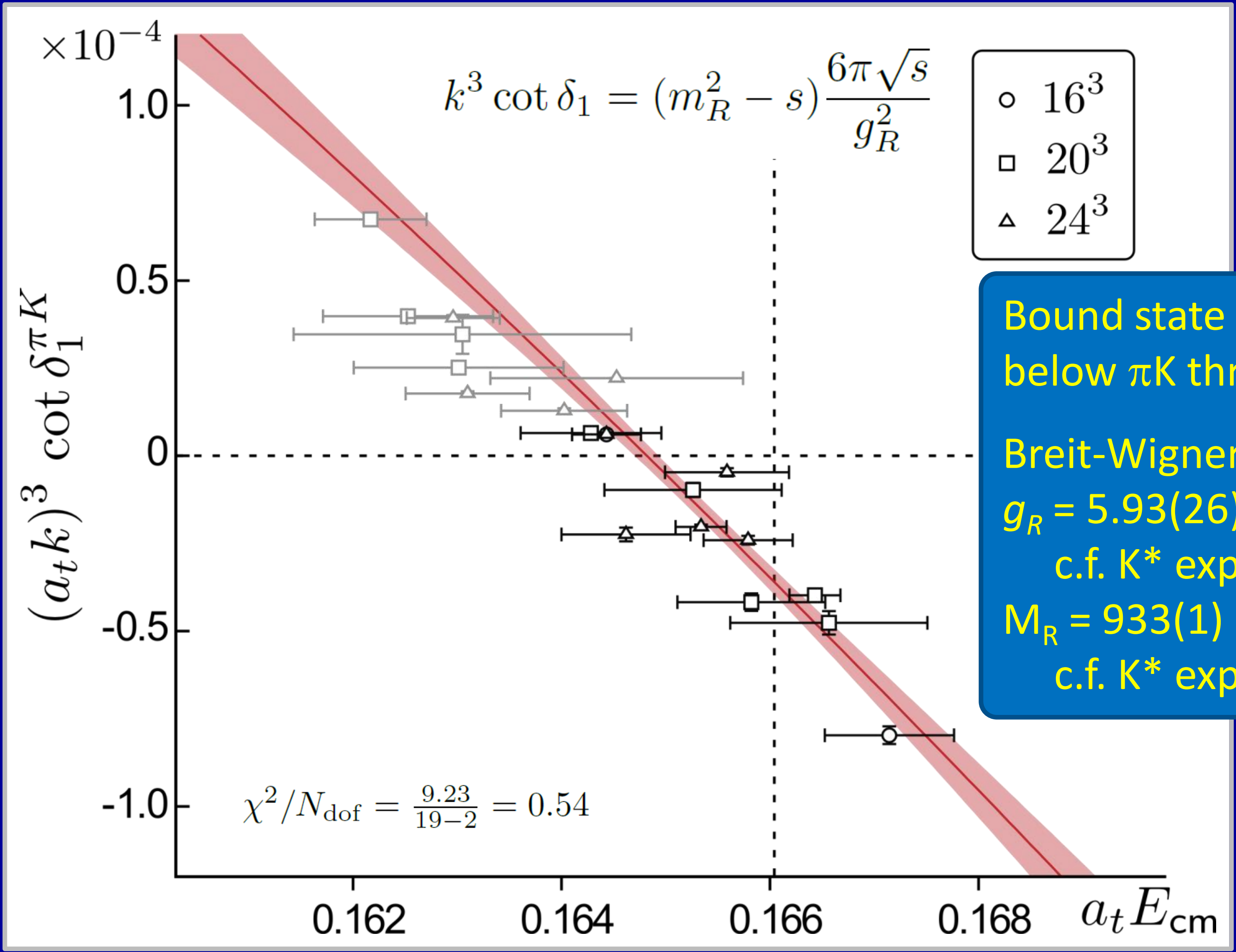
πK (I=1/2): P-wave near threshold

(well below ηK threshold)



πK (I=1/2): P-wave near threshold

(well below ηK threshold)



Bound state just below πK threshold

Breit-Wigner param.

g_R = 5.93(26)

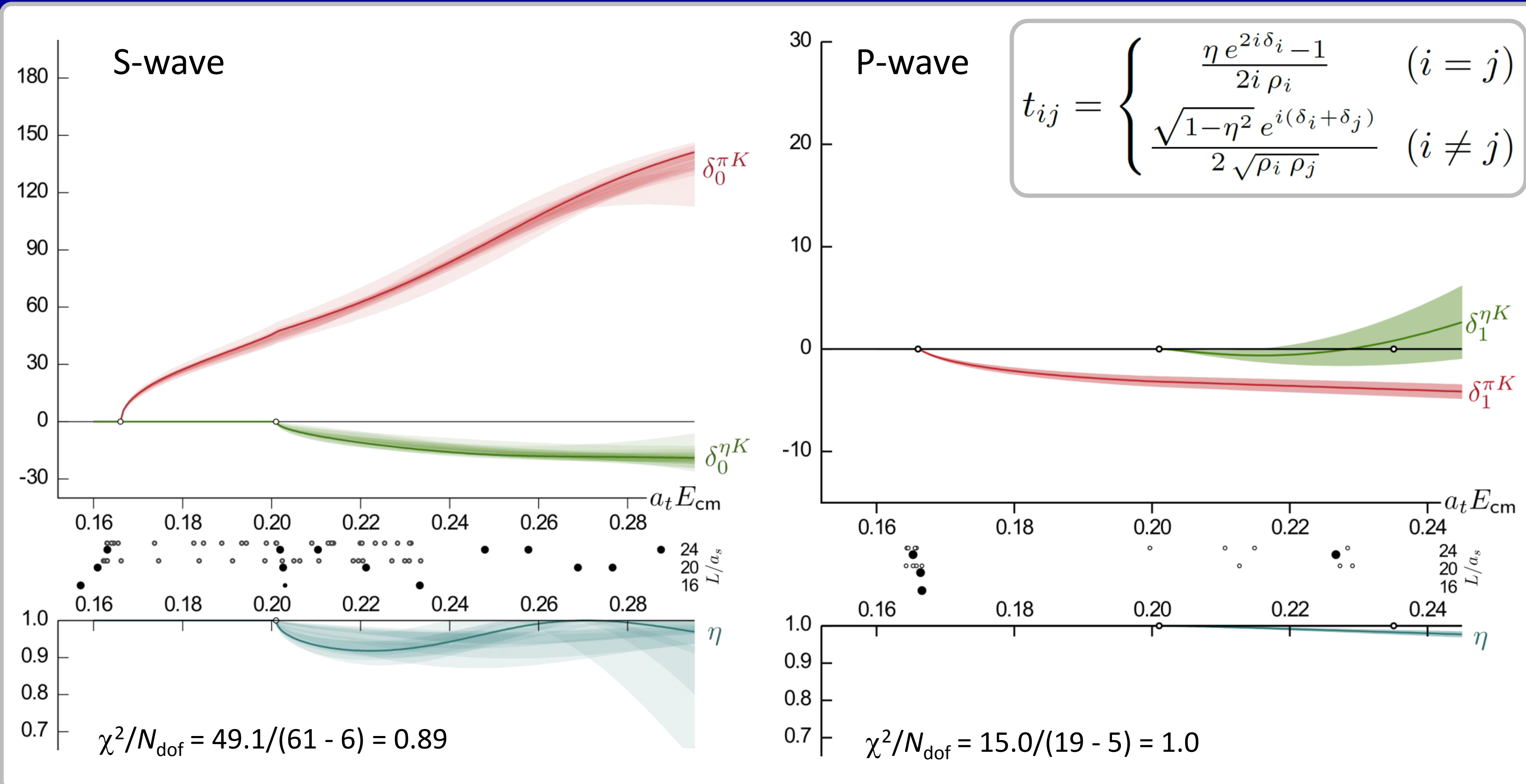
c.f. K* exp. = 5.52(16)

M_R = 933(1) MeV

c.f. K* exp. ≈ 892 MeV

πK, ηK (I=1/2): S & P-waves

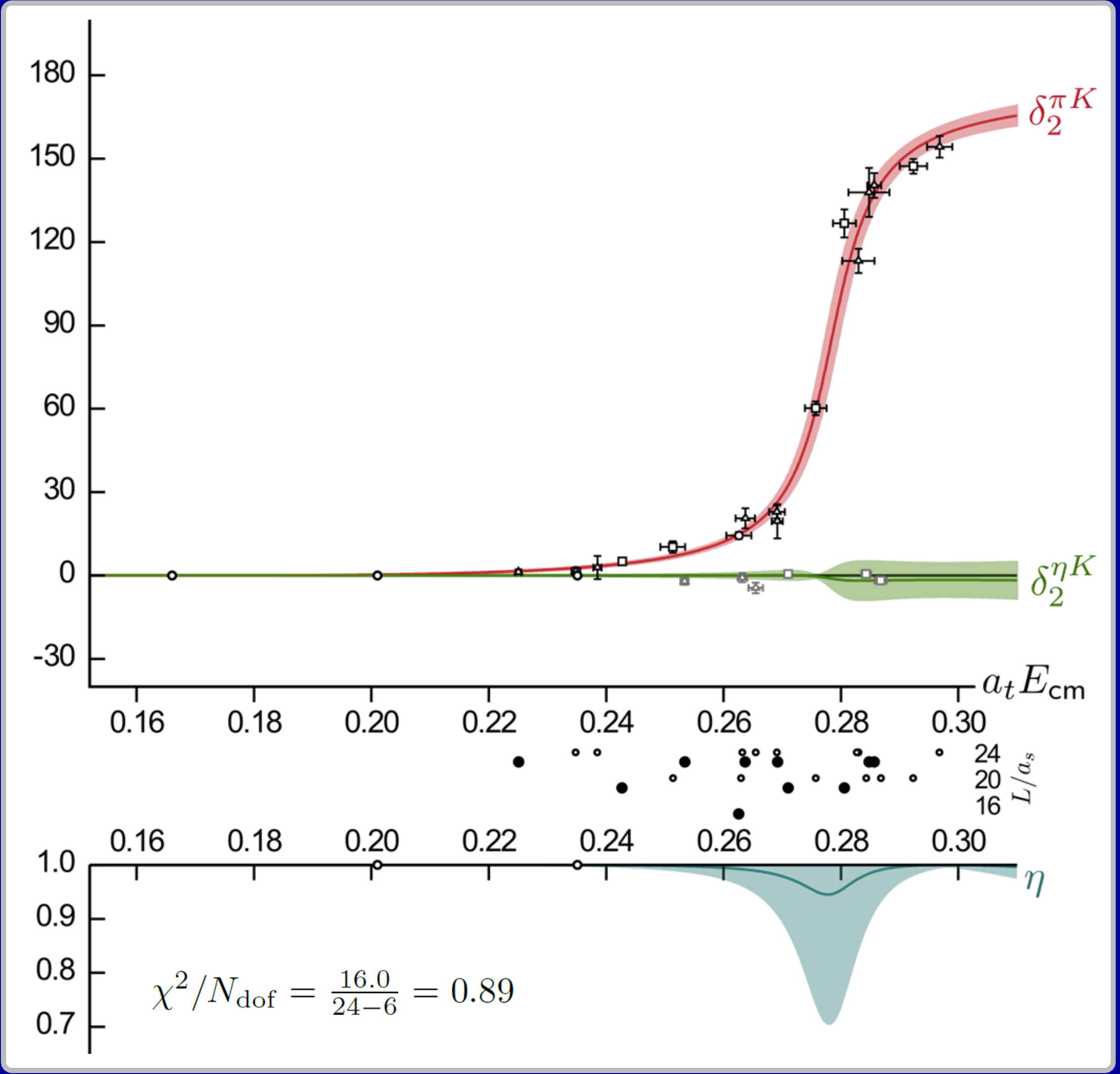
(73 energy levels)



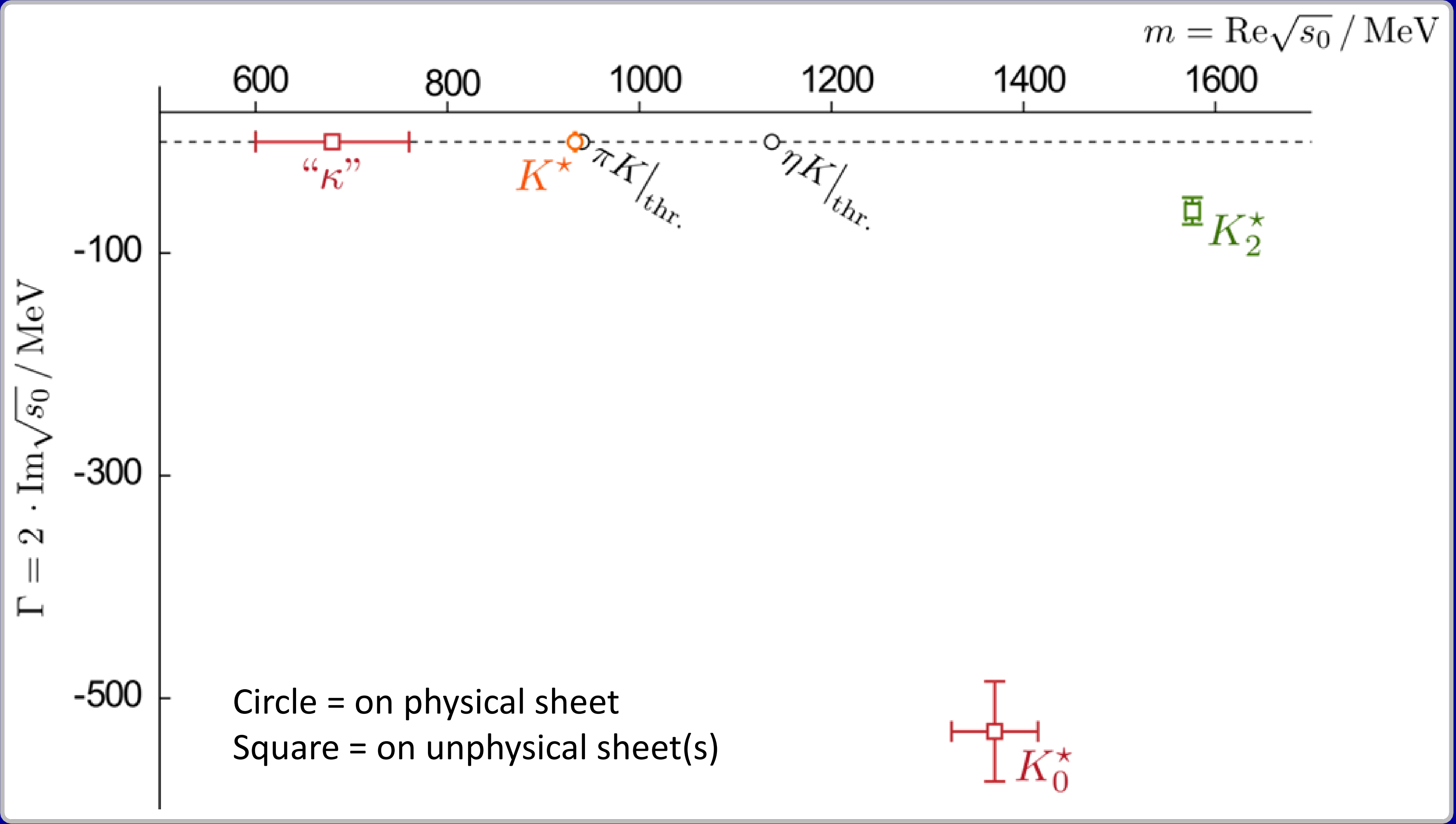
πK, ηK (I=1/2): D-wave

Assume ℓ ≥ 3 negligible

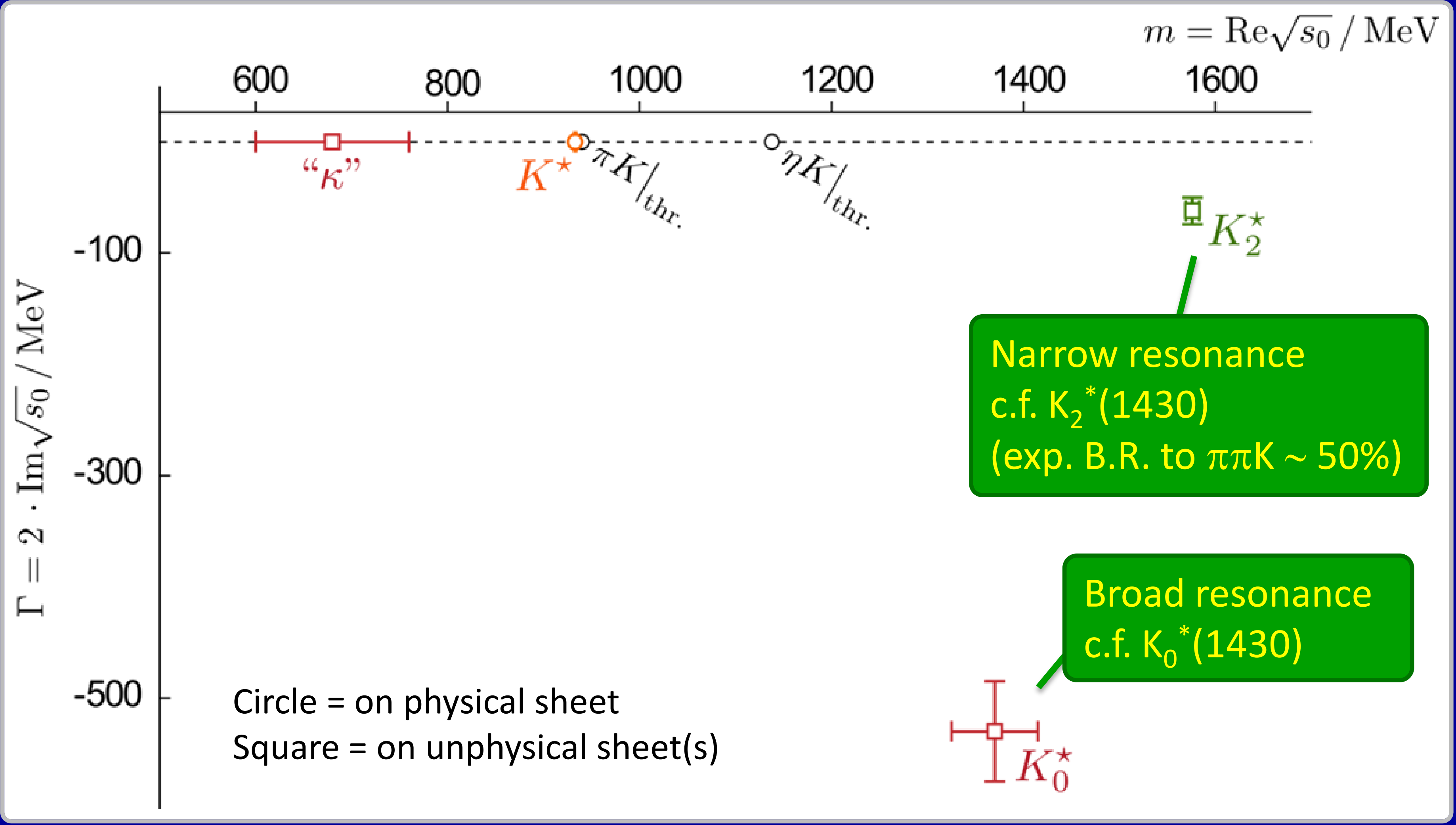
Up to πππK threshold; neglect coupling to ππK



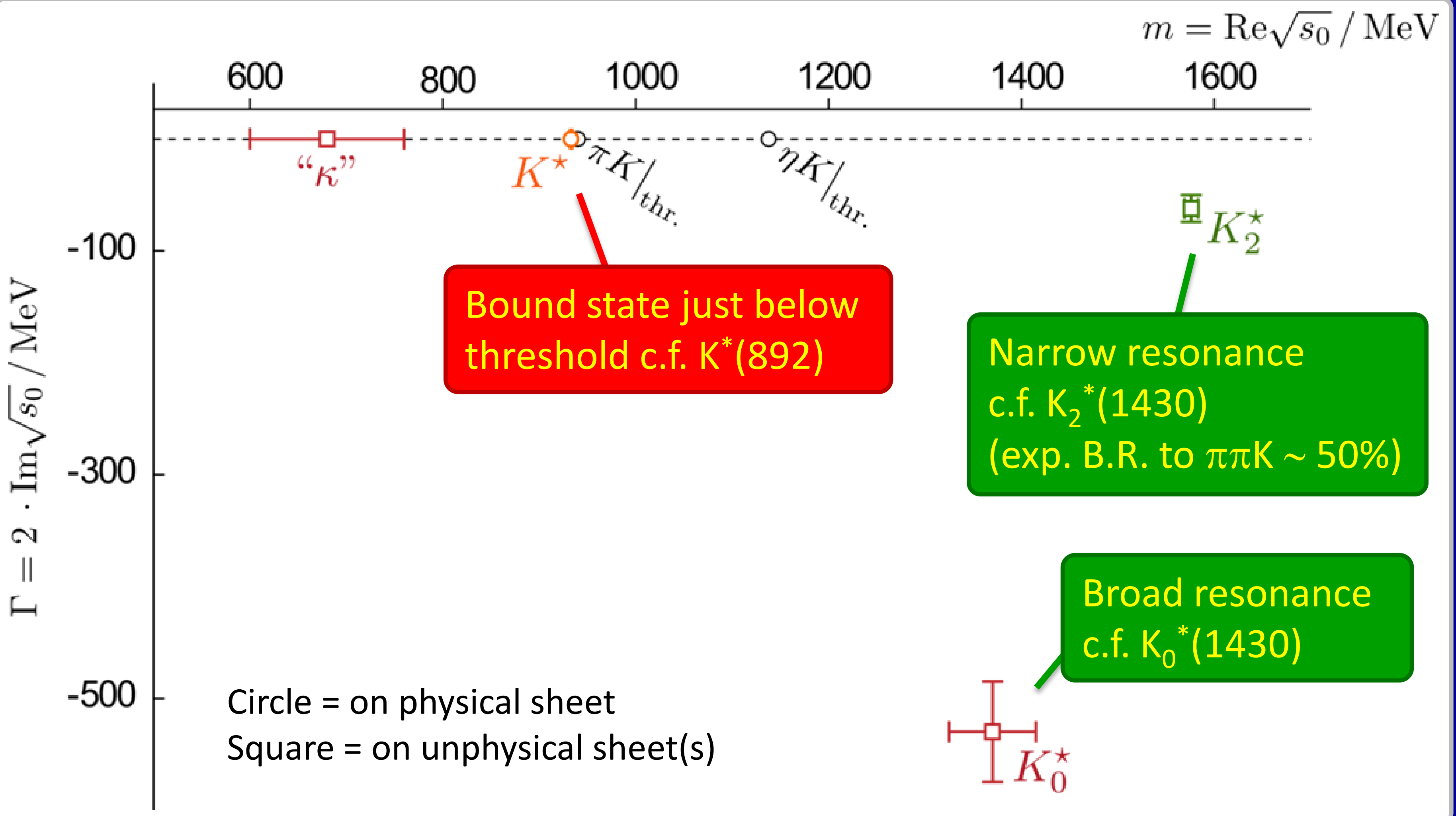
πK, ηK (I=1/2): *t*-matrix poles



πK, ηK (I=1/2): *t*-matrix poles



πK, ηK (I=1/2): *t*-matrix poles



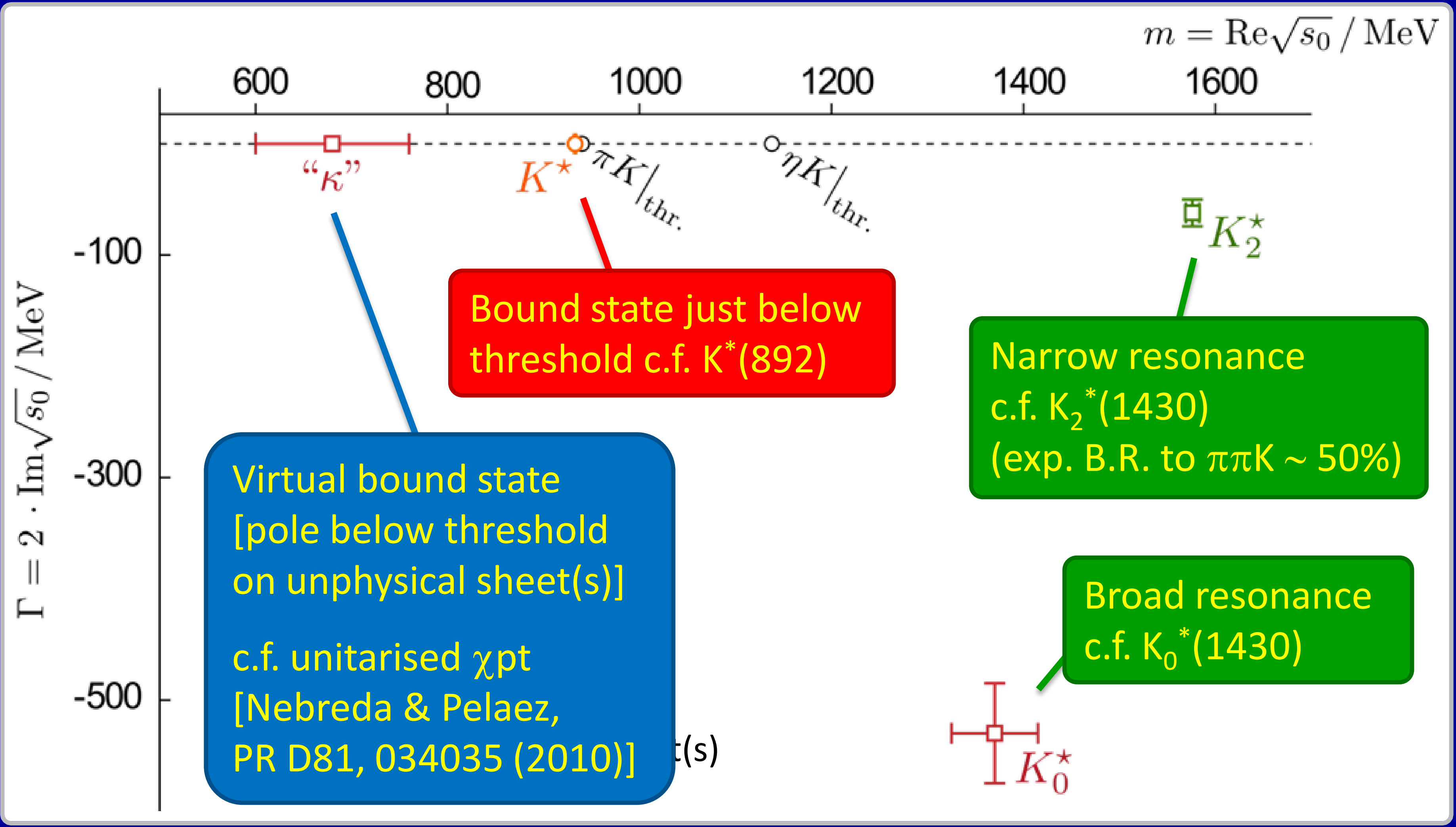
Bound state just below threshold c.f. $K^*(892)$

Narrow resonance c.f. $K_2^*(1430)$ (exp. B.R. to ππK ~ 50%)

Broad resonance c.f. $K_0^*(1430)$

Circle = on physical sheet
Square = on unphysical sheet(s)

πK, ηK (I=1/2): *t*-matrix poles



Some other recent lattice calculations

(not complete list)

First go at including multi-hadron operators in the **open-charm sector**:

- Mohler et al [PR D87, 034501 (2012)] – 0^+ $D\,\pi$ and 1^+ $D^*\,\pi$ resonances
- Mohler et al [PRL 111, 222001 (2013)] – 0^+ $D_s(2317)$ below $D\,K$ threshold
- Lang et al [PRD 90, 034510 (2014)] – 0^+ $D_s(2317)$ and 1^+ $D_{s1}(2460)$, $D_{s1}(2536)$

... **charmonium**:

- Ozaki, Sasaki [PR D87, 014506 (2013)] – no sign of Y(4140) in $J/\psi\ \varphi$
- Prelovsek & Leskovec [PRL 111, 192001 (2013)] – 1^{++} near/below DD^* – X(3872)?
- Prelovsek et al [PL B727, 172 ; PR D91, 014504 (2015)] – no sign of $Z^+(3900)$ in 1^{+-}
- Chen et al [PR D89, 094506 (2014)] – find 1^{++} I=1 $D\bar{D}^*$ is weakly repulsive

... **light and strange meson**:

- Lang et al [PR D86, 054508 (2012), PR D88, 054508 (2013)]
  – $K\,\pi$ in s-wave (0^+) and p-wave (1^-) including K^* resonances
- Lang et al [JHEP 04 (2014) 162] – channels relevant for $a_1(1260)$ & $b_1(1235)$

... **baryons:**

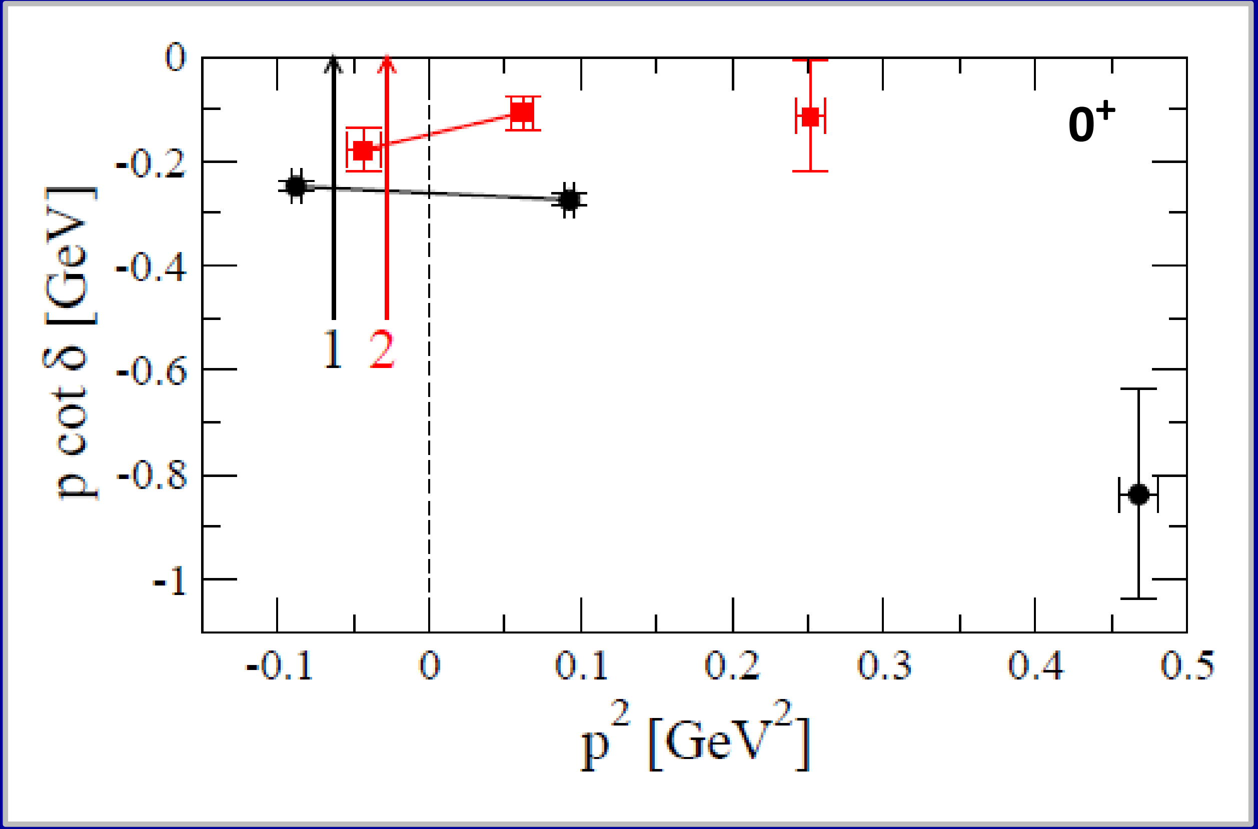
- Lang & Verduci [PR D87, 054502 (2013)] – $N\,\pi$ with $J^P=1/2^-$ I=1/2
- Alexandrou et al [PR D88, 031501 (2013)] – different approach for Δ ($3/2^+$ I=3/2)

- Also see reviews e.g. from Lattice 2014 or 2013

D_s mesons

Lang et al [PRD 90, 034510 (2014)]

$J^P = 0^+$ [relevant for $D_{s0}(2317)$]: 4 D_s + 3 DK ops



(1) Clover [N_f = 2] (Hasenfratz et al), m_π = 266 MeV, $m_\pi L \approx 2.7$, $a \approx 0.12$ fm [small volume]
(2) Clover [N_f = 2+1] (PACS-CS), m_π = 156 MeV, $M_\pi L \approx 2.3$, $a \approx 0.09$ fm [small volume]

D_s mesons

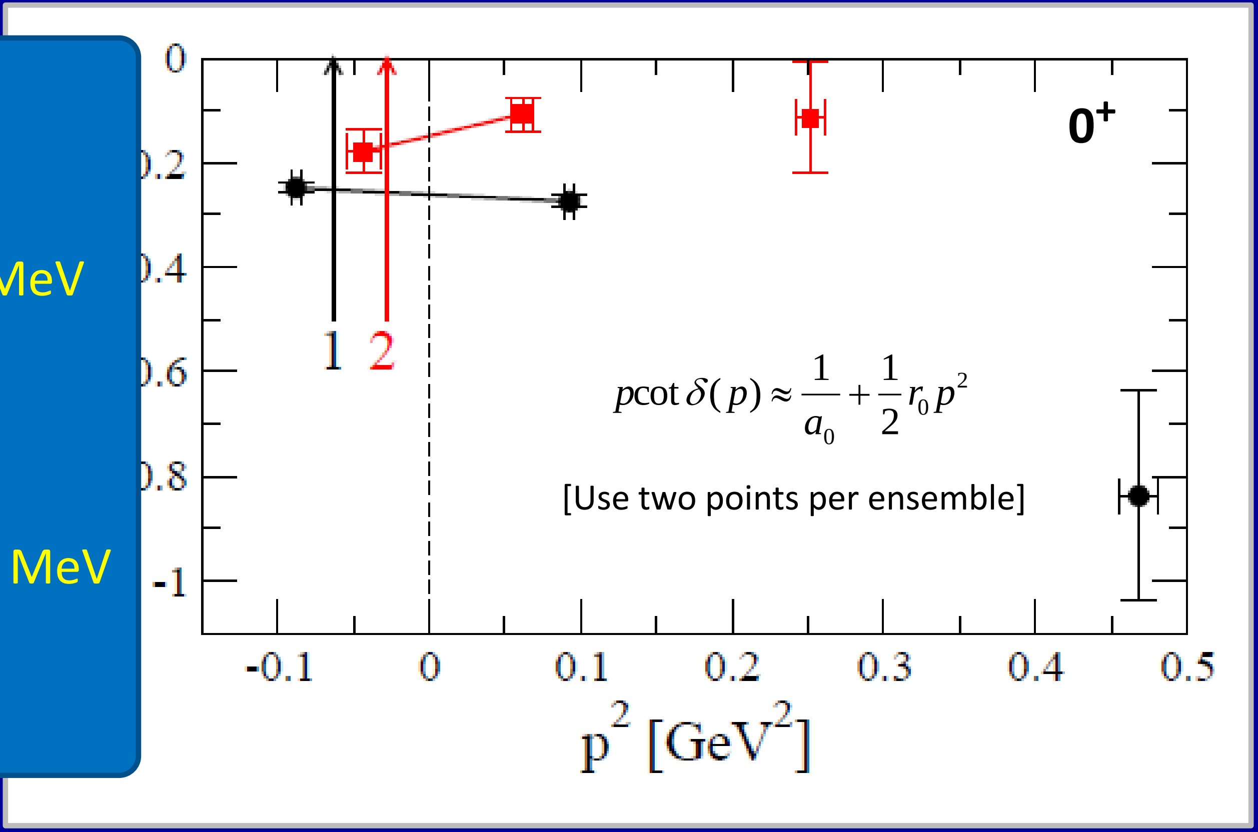
Lang et al [PRD 90, 034510 (2014)]

$J^P = 0^+$ [relevant for $D_{s0}(2317)$]: 4 D_s + 3 DK ops

(1) $a_0 = -0.756(25)$
$r_0 = -0.056(31)$
$m - (m_K + m_D)$
$= -78.9(5.4)(0.8)$ MeV

(2) $a_0 = -1.33(20)$
$r_0 = 0.27(17)$
$m - (m_K + m_D)$
$= -36.6(16.6)(0.5)$ MeV

c.f. $D_{s0}(2317)$ exp.
≈ -45.1 MeV



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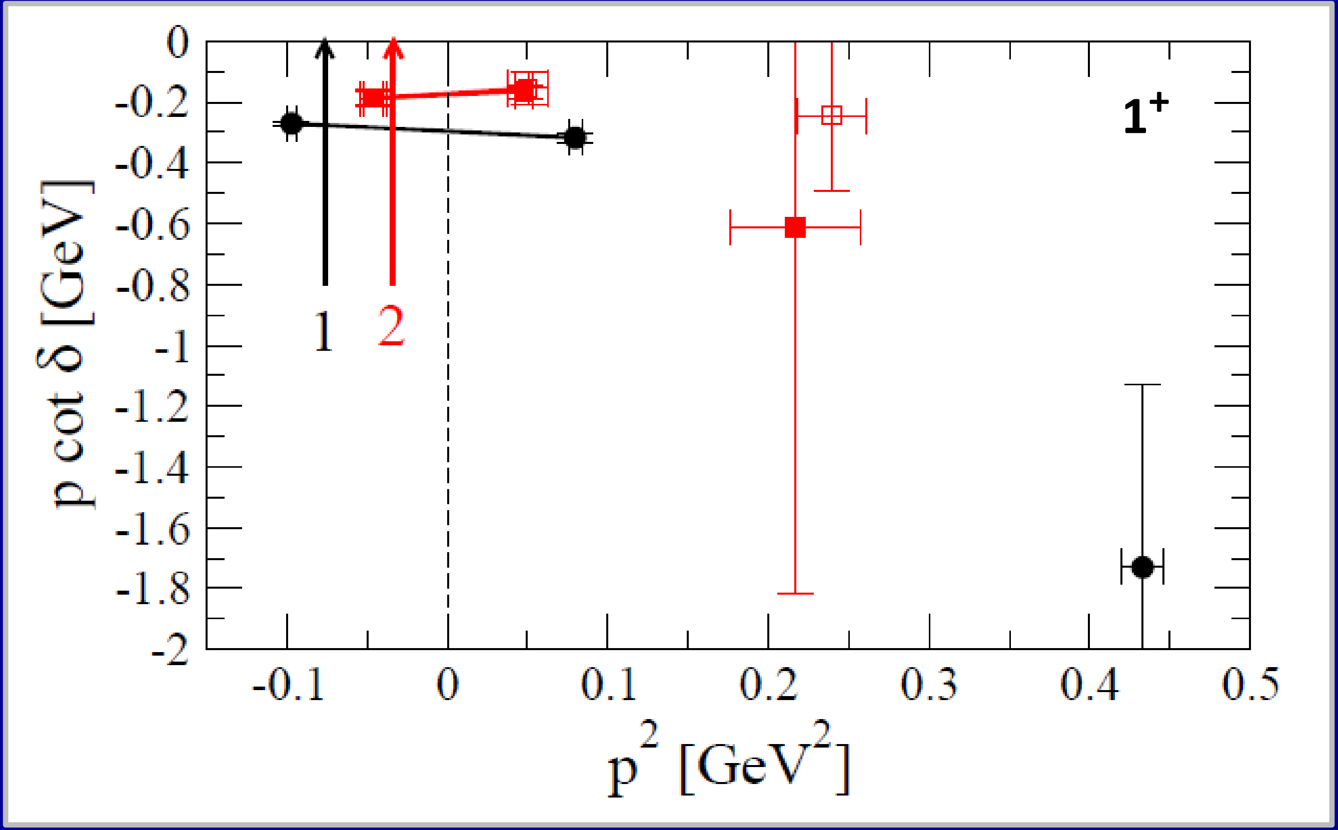
D_s mesons

Lang et al [PRD 90, 034510 (2014)]

$J^P = 1^+$ [relevant for $D_{s1}(2460)$, $D_{s1}(2536)$]: 8 D_s + 3 D^*K ops



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D_s mesons

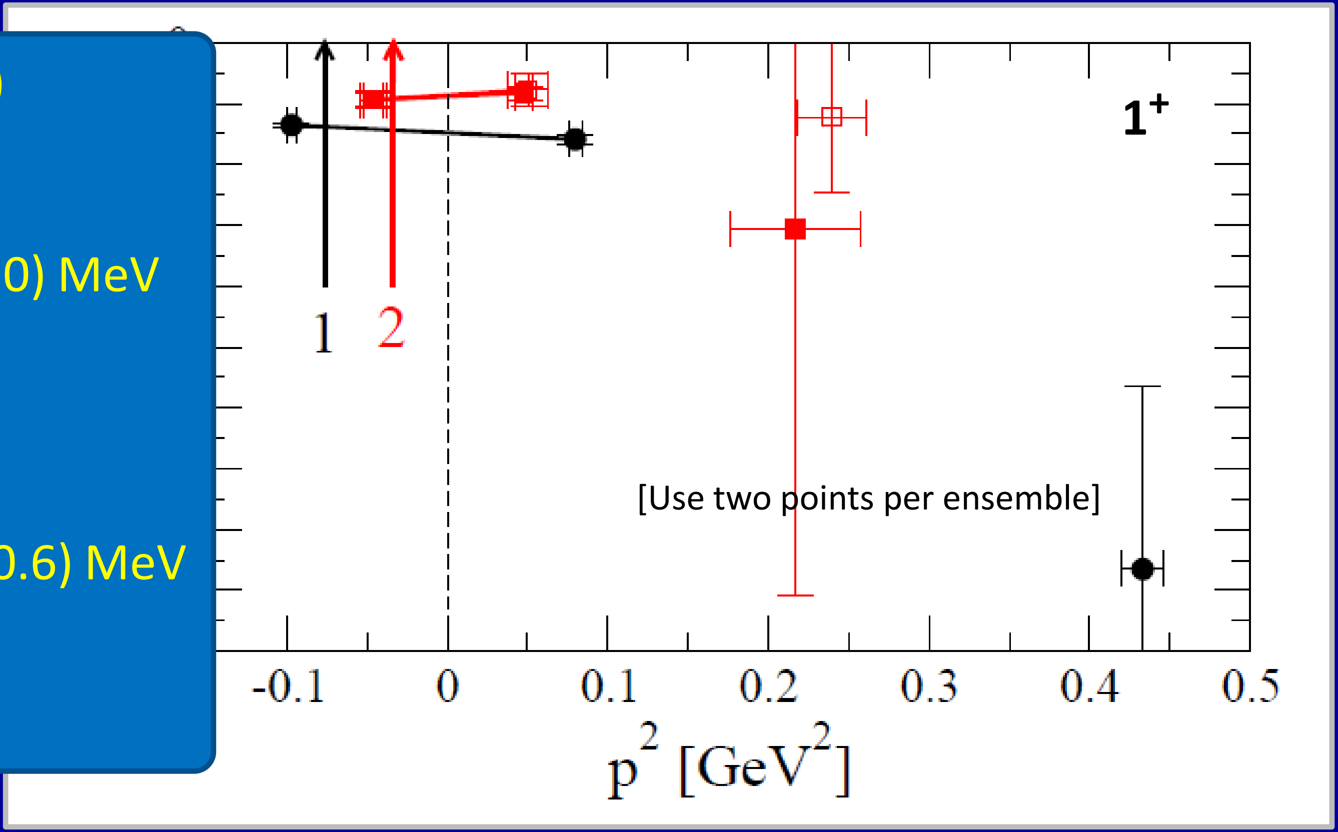
Lang et al [PRD 90, 034510 (2014)]

$J^P = 1^+$ [relevant for $D_{s1}(2460)$, $D_{s1}(2536)$]: 8 D_s + 3 D^*K ops

(1) $a_0 = -0.665(25)$
$r_0 = -0.106(37)$
$m - (m_K + m_{D^*})$
$= -93.2(4.7)(1.0)$ MeV

(2) $a_0 = -1.15(19)$
$r_0 = 0.13(22)$
$m - (m_K + m_{D^*})$
$= -43.2(13.8)(0.6)$ MeV

c.f. $D_{s1}(2460)$ exp.
≈ -44.7 MeV



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$r_0 = 0.13(22)$
$m - (m_K + m_{D^*})$
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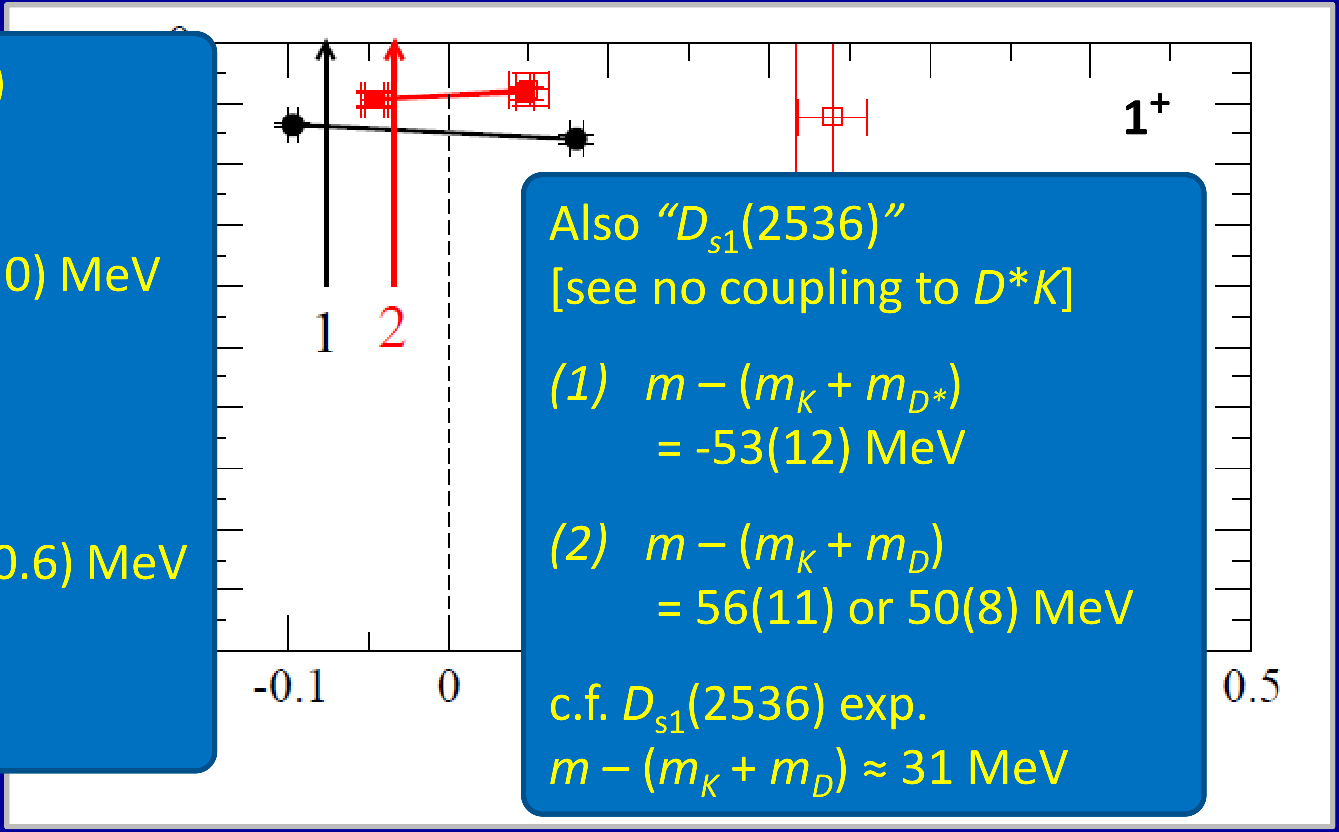
c.f. $D_{s1}(2460)$ exp.
≈ -44.7 MeV



Also "$D_{s1}(2536)$"
[see no coupling to D^*K]

(1) $m - (m_K + m_{D^*})$
$= -53(12)$ MeV

(2) $m - (m_K + m_D)$
$= 56(11)$ or $50(8)$ MeV

c.f. $D_{s1}(2536)$ exp.
$m - (m_K + m_D) \approx 31$ MeV

(1) Clover [$N_f = 2$] (Hasenfratz et al), $m_\pi = 266$ MeV, $m_\pi L \approx 2.7$, $a \approx 0.12$ fm [small volume]
(2) Clover [$N_f = 2+1$] (PACS-CS), $m_\pi = 156$ MeV, $M_\pi L \approx 2.3$, $a \approx 0.09$ fm [small volume]

Charmonium

Prelovsek, Leskovec [PRL 111, 192001 (2013)]

X(3872) [$J^{PC} = 1^{++}$] near/below D D* threshold

Look in I=0
(one vol, one $\boldsymbol{P}_{cm}$)

$c\bar{c}, D\bar{D}^*, J/\psi\,\omega$ ops

Clover [N_f = 2] (Hasenfratz et al), m_π = 266 MeV, $m_\pi L \approx 2.7$, $a \approx 0.12$ fm [small volume]

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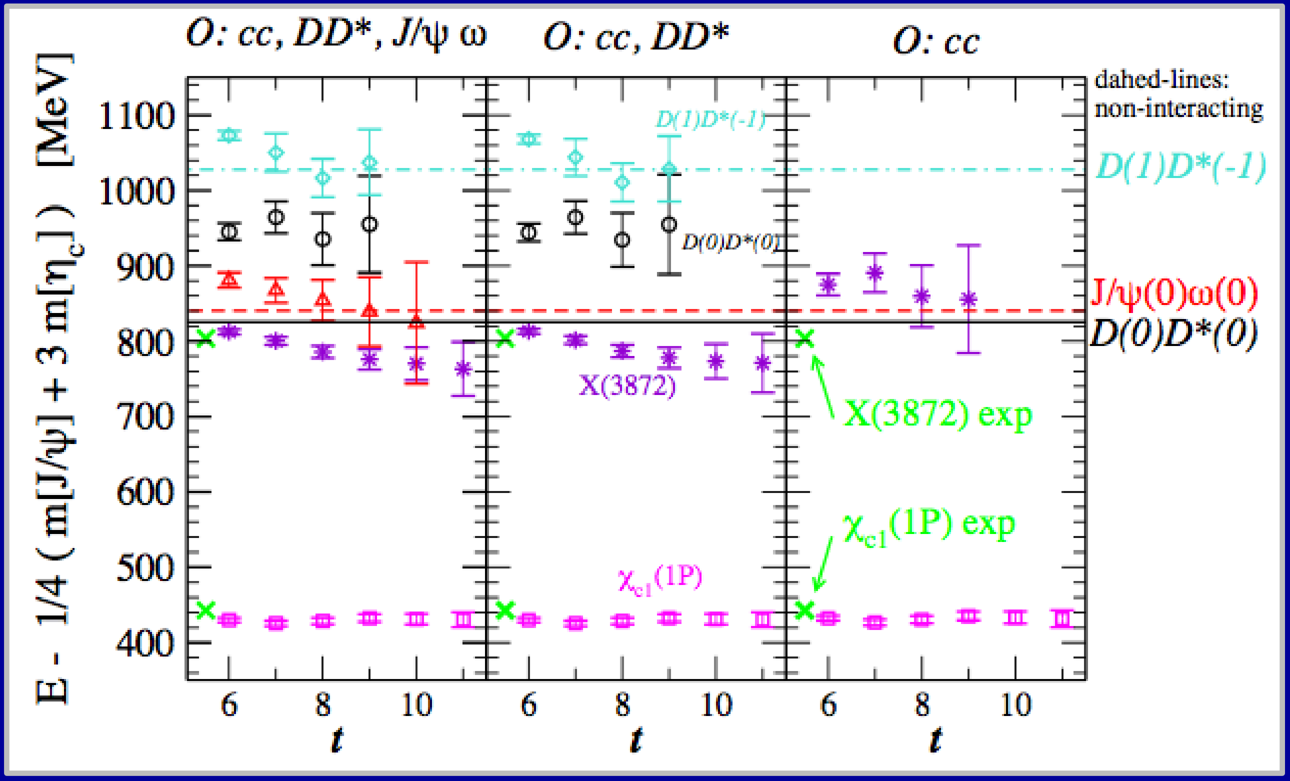

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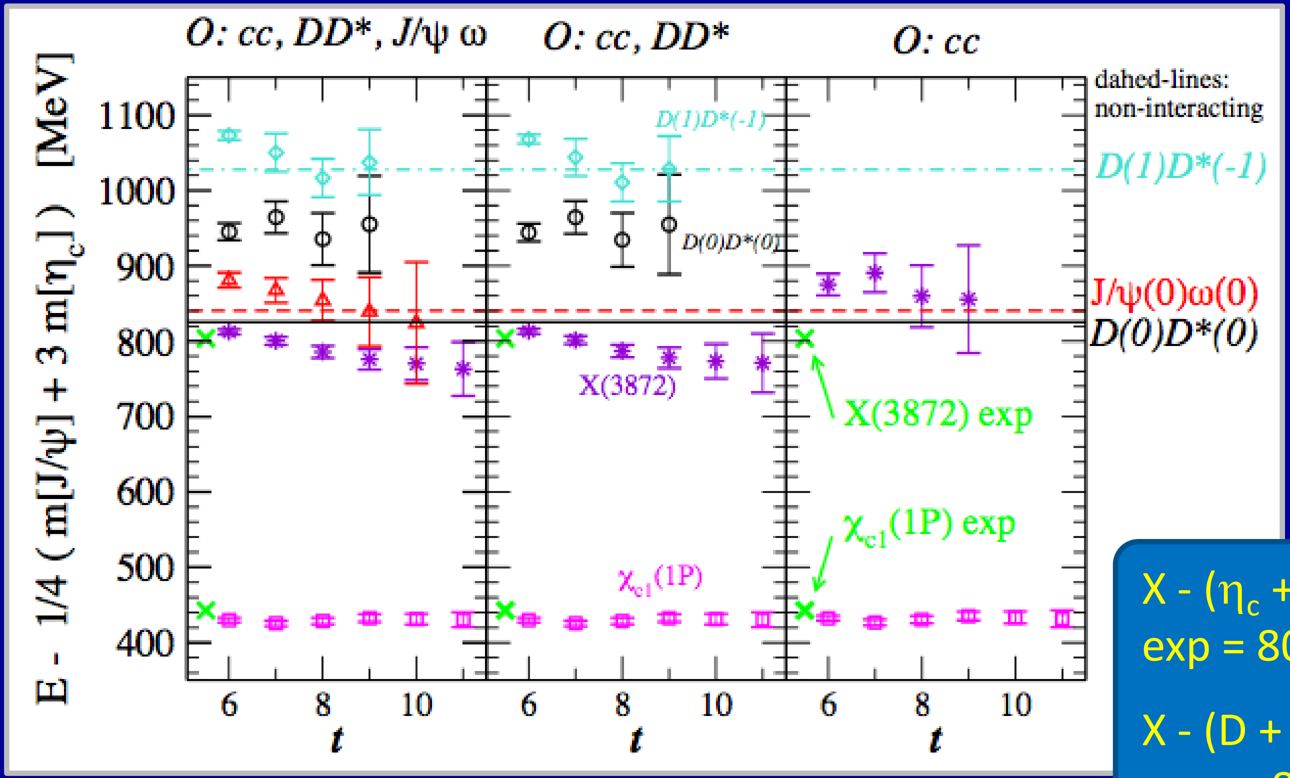

$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$a_0 = -1.7 \pm 0.4\text{fm}$

$r_0 = 0.5 \pm 0.1\text{fm}$

[2 points]

X - (η_c + 3 J/ψ)/4 = 815(7) MeV
exp = 803.1(2) MeV

X - (D + D*) = -11(7) MeV
exp = -8.2(3) MeV [$D^+ D^{*-}$]
= -0.2(3) MeV [$D^0 D^{*0}$]

Clover [N_f = 2] (Hasenfratz et al), m_π = 266 MeV, $m_\pi L \approx 2.7$, $a \approx 0.12$ fm [small volume]

Charmonium

Prelovsek, Leskovec [PRL 111, 192001 (2013)]

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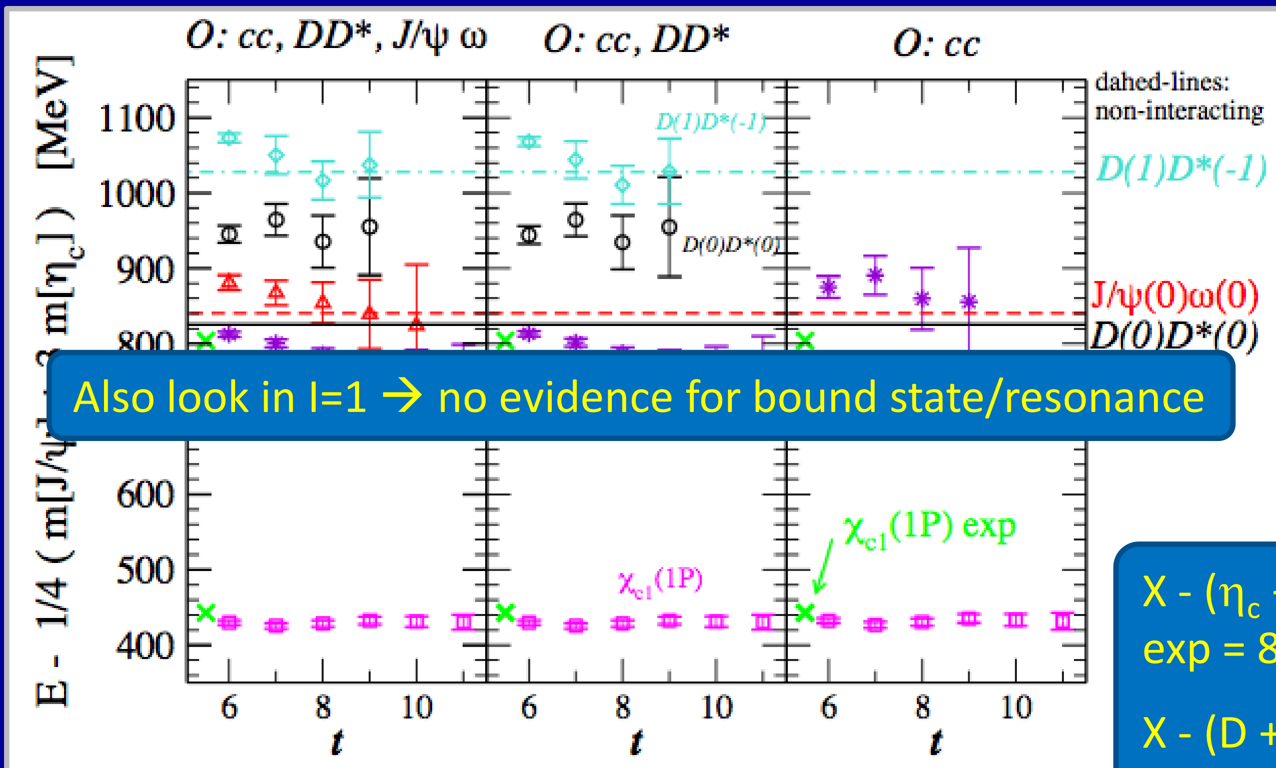
Look in I=0
(one vol, one $\boldsymbol{P}_{cm}$)

$c\bar{c}, D\bar{D}^*, J/\psi\,\omega$ ops



Also look in I=1 → no evidence for bound state/resonance

$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$$a_0 = -1.7 \pm 0.4\text{fm}$$

$$r_0 = 0.5 \pm 0.1\text{fm}$$

[2 points]

X - (η_c + 3 J/ψ)/4 = 815(7) MeV
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= -0.2(3) MeV [$D^0 D^{*0}$]

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Charmonium

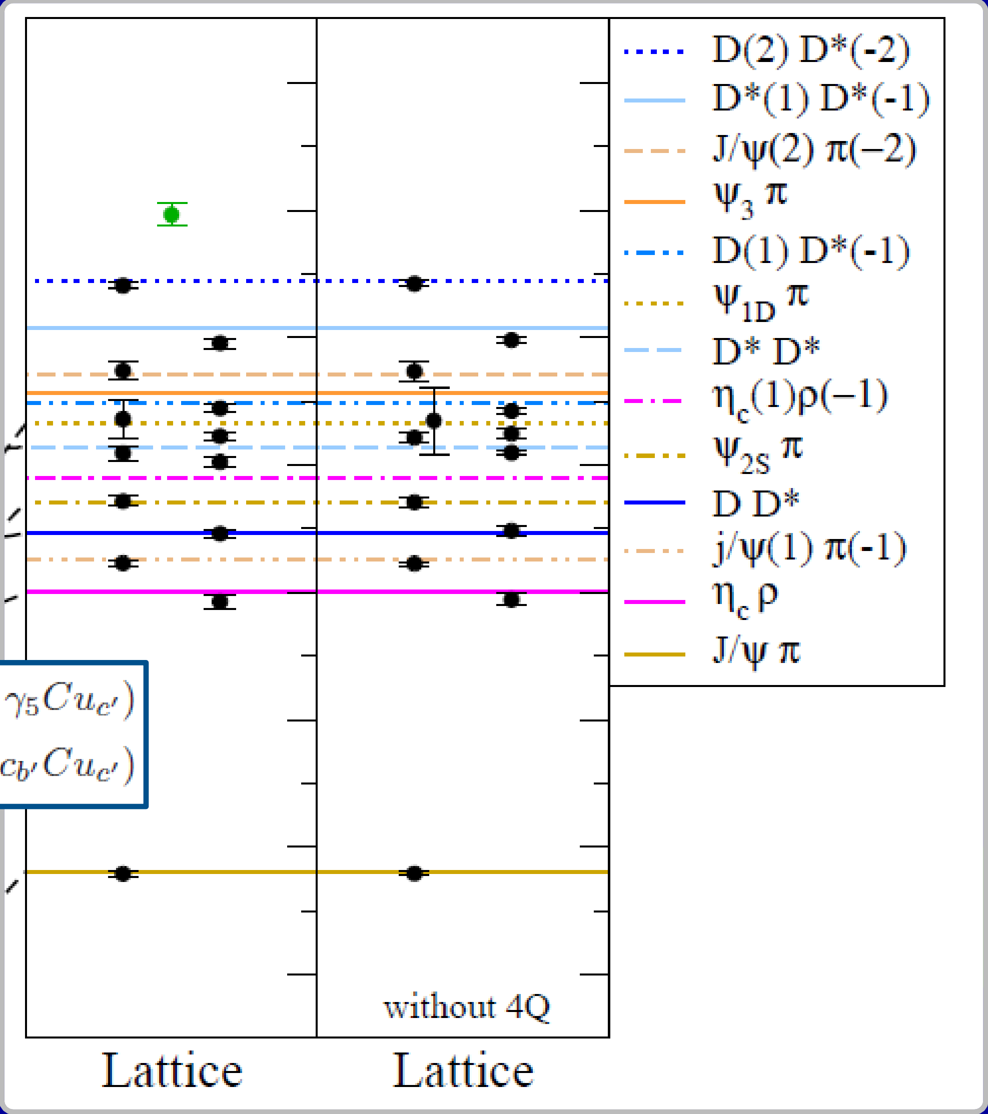
Prelovsek et al [PR D91, 014504 (2015)]

Various Z_c^+ structures in exp.
e.g. $Z_c^+(3900)$, $Z_c^+(4020)$, $Z_c^+(4200)$,
$J^{PC} = ?^{?-}$

Look in $J^{PC} = 1^{+-}$ I=1. Many two-meson and some '4-quark' ops

$D\bar{D}^*, J/\psi\,\pi, \eta_c\,\rho, D^*\bar{D}^*, \psi'\,\pi$

$$\mathcal{O}_1^{4q} \propto \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\gamma_5 \bar{d}_c \; c_{b'}\gamma_i C u_{c'} - \bar{c}_b C\gamma_i \bar{d}_c \; c_{b'}\gamma_5 C u_{c'})$$
$$\mathcal{O}_2^{4q} \propto \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C \bar{d}_c \; c_{b'}\gamma_i\gamma_5 C u_{c'} - \bar{c}_b C\gamma_i\gamma_5 \bar{d}_c \; c_{b'} C u_{c'})$$



Clover [$N_f = 2$] (Hasenfratz et al), m_π = 266 MeV, $m_\pi L \approx 2.7$, $a \approx 0.12$ fm [*small volume*]

Charmonium

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Various Z_c^+ structures in exp.
e.g. $Z_c^+(3900)$, $Z_c^+(4020)$, $Z_c^+(4200)$, $J^{PC} = ?^{?-}$

Look in $J^{PC} = 1^{+-}$ I=1. Many two-meson and some '4-quark' ops

$$D\bar{D}^*,\, J/\psi\,\pi,\, \eta_c\,\rho,\, D^*\bar{D}^*,\, \psi'\,\pi$$

$$\mathcal{O}_1^{4q} \propto \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\gamma_5 \bar{d}_c\; c_{b'}\gamma_i C u_{c'} - \bar{c}_b C\gamma_i \bar{d}_c\; c_{b'}\gamma_5 C u_{c'})$$
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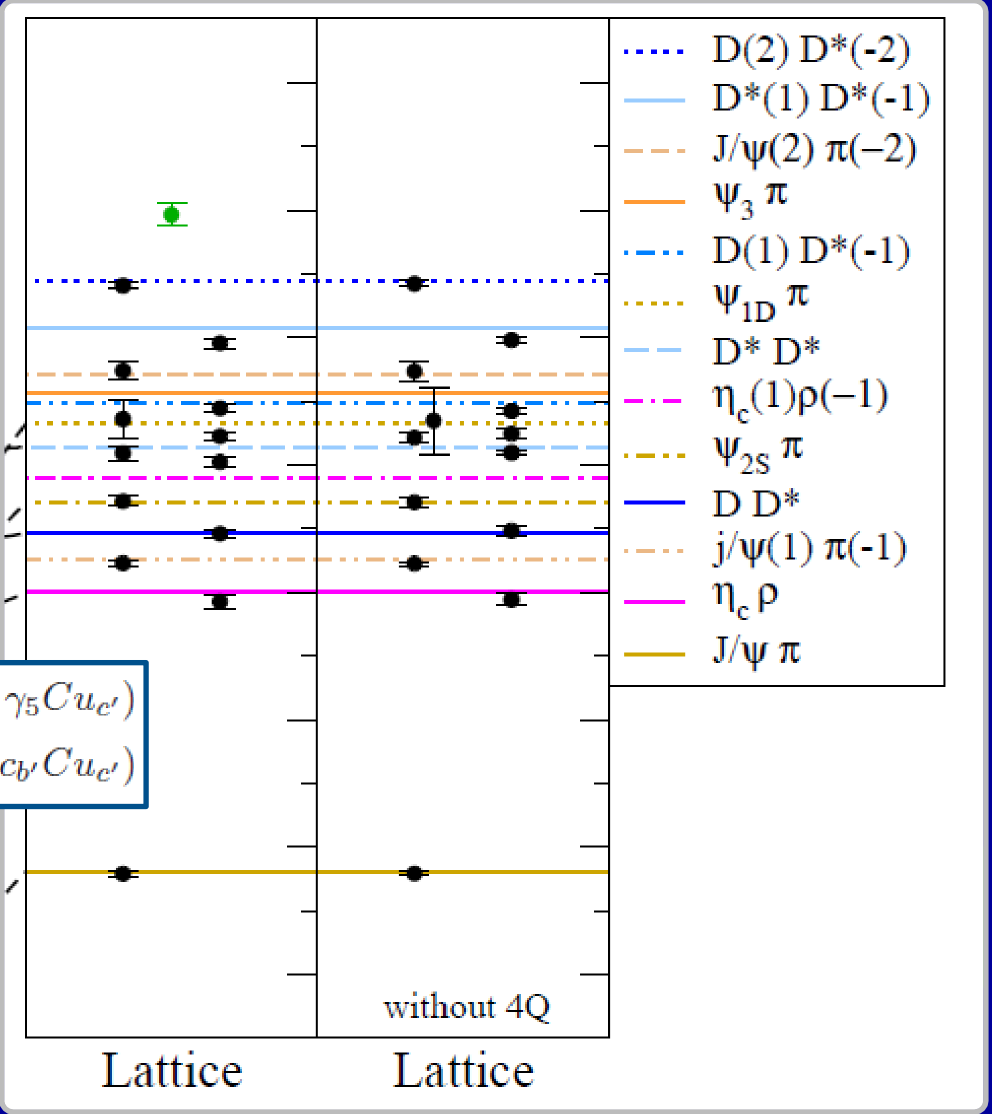
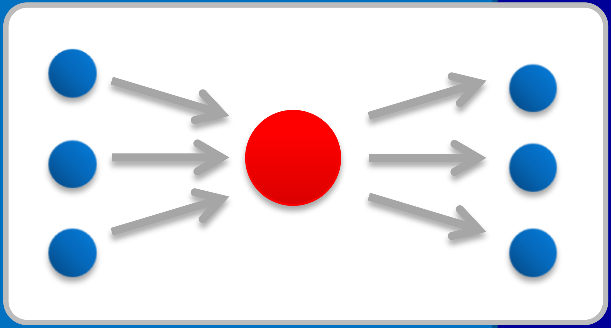


No sign of any Z_c^+ up to ~ 4.2 GeV
Only see non-interacting energies.

Clover [$N_f = 2$] (Hasenfratz et al), m_π = 266 MeV, $m_\pi L \approx 2.7$, $a \approx 0.12$ fm [*small volume*]

Scattering channels with 3 or more hadrons?

- **Much more complicated** than 2-hadron scattering.
- No straightforward analogue of the determinant equation.
- Theoretical work is ongoing, e.g.
  - Polejaeva, Rusetsky [EPJA 48, 67 (2012)]
  - Kreuzer, Griesshammer [EPJA 48, 93 (2012)]
  - Roca, Oset [PR D85, 054507 (2012)]
  - Briceno, Davoudi [PR D87, 094507 (2013)]
  - Hansen, Sharpe [PR D90, 116003 (2014)]
  - Meissner, Rios, Rusetsky [PRL 114, 091602 (2015)]
  - Hansen, Sharpe [1504.04248]
- No real applications yet.
- Another reason why calculating at physical m_π is challenging (particularly for light mesons): more >2 hadron channels open



Summary of lecture 3

- Scattering, resonances, etc in lattice QCD
- Some examples:
  - The ρ resonance in elastic ππ scattering
  - Coupled-channel K π, K η scattering
  - Some D_s mesons and charmonia

Conclusions

- **Significant progress** in computing spectra of (excited) hadrons using lattice QCD in last few years
  – improved algorithms, clever techniques, more powerful computers and novel use of technology (e.g. GPUs)
- I've aimed to give some idea of what goes into these lattice calculations, some highlights of results and some interpretation.
- Calculating properties of **unstable hadrons** is currently a very active area – only recently have we been able to do this in practice. There is still a lot to do here.
- Masses only get you so far. We can also compute **other properties of hadrons** that **probe their structure** using lattice QCD:
  e.g. form factors, transition amplitudes. Again, there is interesting work going on, but that's another set of lectures...