Green's function approach to the investigation of the features of hadron cross sections close to nucleon-antinucleon threshold

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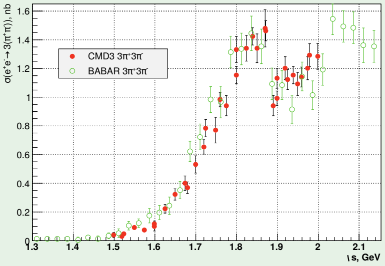
Outline

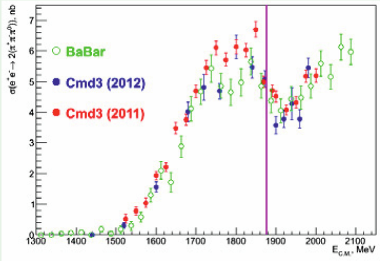
- The motivation: some features of the cross sections which have no explanation.
- $e^+e^- \to N\bar{N}$ cross section with the final state interaction taken into account.
- Green's function of $N\bar{N}$ system and the total cross section of e^+e^- annihilation.
- A simple potential model qualitatively describing some of the features. Further steps to improve the model.

Motivation: $e^+e^- \to 6\pi$

$e^+e^- \to 3\left(\pi^+\pi^-\right)$ cross section

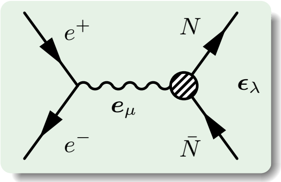


$e^+e^- \to 2\left(\pi^+\pi^-\pi^0\right)$ cross section



These features are expected to be the consequences of interaction of nucleons in the intermediate state.

Amplitude of the process



There are indications that the process of $N\bar{N}$ production can be divided into two regions.

$$T^I_{\lambda\mu} = \frac{4\pi\alpha}{Q^2} \cdot \sqrt{2}\boldsymbol{\epsilon}^*_\lambda \left\{ \mathcal{G}^I_s(Q^2)\boldsymbol{e}_\mu + \mathcal{G}^I_d(Q^2) \frac{\boldsymbol{k}^2\boldsymbol{e}_\mu - 3\left(\boldsymbol{k}\cdot\boldsymbol{e}_\mu\right)\boldsymbol{k}}{6M^2} \right\}$$

$$\mathcal{G}^I_s = \mathcal{F}^I_1(Q^2) + \mathcal{F}^I_2(Q^2) + \frac{\beta^2}{6}\left[\mathcal{F}^I_2(Q^2) - \mathcal{F}^I_1(Q^2)\right]$$

$$\mathcal{G}^I_d = \mathcal{F}^I_1(Q^2) - \mathcal{F}^I_2(Q^2)$$

$\mathcal{F}^I_i$ are the Dirac form factors of the nucleon with the effects of final-state interaction.

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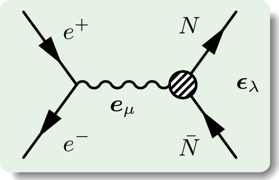

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$\mathcal{F}^I_i$ are the Dirac form factors of the nucleon with the effects of final-state interaction.

$$T^I_{\lambda\mu} = \frac{4\pi\alpha}{Q^2} \cdot \sqrt{2} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \boldsymbol{\Phi}^{I(-)*}_{\boldsymbol{k}\lambda}(\boldsymbol{p}) \left\{ G^I_s \boldsymbol{e}_\mu + G^I_d \frac{\boldsymbol{p}^2\boldsymbol{e}_\mu - 3\left(\boldsymbol{p}\cdot\boldsymbol{e}_\mu\right)\boldsymbol{p}}{6M^2} \right\}$$

$\boldsymbol{\Phi}^{I(-)}_{\boldsymbol{k}\lambda}(\boldsymbol{p})$ is the Fourier transform of the wave function $\boldsymbol{\Psi}^{I(-)}_{\boldsymbol{k}\lambda}(\boldsymbol{r})$ of $N\bar{N}$ pair.

$$\boldsymbol{\Psi}^{I(-)*}_{\boldsymbol{k}\lambda}(\boldsymbol{r})\hat{H} = \frac{k^2}{M}\boldsymbol{\Psi}^{I(-)*}_{\boldsymbol{k}\lambda}(\boldsymbol{r}), \qquad \hat{H} = \frac{\boldsymbol{p}^2}{M} + V_{N\bar{N}}$$

$$\boldsymbol{\Psi}^{I(-)*}_{\boldsymbol{k}\lambda}(\boldsymbol{r}) \sim \boldsymbol{\epsilon}_\lambda e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + f_{\lambda\lambda'}\frac{e^{-ikr}}{r}\boldsymbol{\epsilon}_{\lambda'}$$

Amplitude of the process

After partial wave expansion $J=1$ wave function has form

$$\boldsymbol{\psi}^{I}_{\boldsymbol{k}\lambda}(\boldsymbol{r}) = \left[u^{I*}_{1R}(r)\boldsymbol{\epsilon}_{\lambda} + w^{I*}_{1R}(r)\sqrt{4\pi}\boldsymbol{Y}^{2}_{1\lambda}(\boldsymbol{n})\right] + \sqrt{5}C^{1\lambda}_{20,1\lambda}\left[u^{I*}_{2R}(r)\boldsymbol{\epsilon}_{\lambda} + w^{I*}_{2R}(r)\sqrt{4\pi}\boldsymbol{Y}^{2}_{1\lambda}(\boldsymbol{n})\right]$$

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$$\frac{p_r^2}{M}\chi_n + \mathcal{V}\chi_n = 2E\chi_n\,, \qquad \chi_n = \begin{pmatrix} u^I_n \\ w^I_n \end{pmatrix} \quad \begin{matrix} \longleftarrow L = 0 \\ \longleftarrow L = 2 \end{matrix}$$

$$\mathcal{V} = V^I_0(r)\delta_{L0} + V^I_2(r)\delta_{L2} + V^I_3(r)S_{12} = \begin{pmatrix} V^I_0 & -2\sqrt{2}\,V^I_3 \\ -2\sqrt{2}\,V^I_3 & V^I_2 - 2V^I_3 \end{pmatrix}$$

$$u^I_{1R}(r) \sim \frac{1}{2ikr}\Big[S^I_{11}\,e^{ikr} - e^{-ikr}\Big] \qquad u^I_{2R}(r) \sim \frac{1}{2ikr}S^I_{21}\,e^{ikr}$$

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$$\mathcal{G}^I_s = G^I_s u^I_{1R}(0) + \frac{5G^I_d}{\sqrt{2}M^2}\lim_{r\to 0}\left(\frac{w^I_{1R}(r)}{r^2}\right)$$

$$\mathcal{G}^I_d = \frac{6G^I_s}{\sqrt{2}\beta^2}u^I_{2R}(0) + 15G^I_d\lim_{r\to 0}\left(\frac{w^I_{2R}(r)}{k^2r^2}\right)$$

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Cross section of $N\bar{N}$ production

The differential cross section of $N\bar{N}$ production can be written in terms of electromagnetic Sachs form factors:

$$\frac{d\sigma^I}{d\Omega} = \frac{\beta\alpha^2}{4Q^2}\left[\left|G_M^I(Q^2)\right|^2(1+\cos^2\theta) + \frac{4M^2}{Q^2}\left|G_E^I(Q^2)\right|^2\sin^2\theta\right]$$

$$G_M^I = \mathcal{G}_s^I + \frac{\beta^2}{6}\mathcal{G}_d^I = G_s^I\left[u_{1R}^I(0) + \frac{1}{\sqrt{2}}u_{2R}^I(0)\right]$$

$$\frac{2M}{Q}G_E^I = \mathcal{G}_s^I - \frac{\beta^2}{3}\mathcal{G}_d^I = G_s^I\left[u_{1R}^I(0) - \sqrt{2}\,u_{2R}^I(0)\right]$$

In the non-relativistic approximation the ratio G_E^I/G_M^I is independent of G_s^I:

$$\frac{G_E^I}{G_M^I} = \frac{u_{1R}^I(0) - \sqrt{2}\,u_{2R}^I(0)}{u_{1R}^I(0) + \frac{1}{\sqrt{2}}u_{2R}^I(0)}$$

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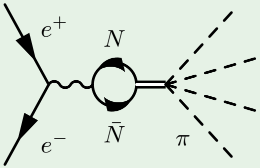
The contribution of the isospin I to the cross section of the $N\bar{N}$ pair production

$$\sigma^I = \frac{2\pi\beta\alpha^2}{Q^2}\left|G_s^I\right|^2\left[\left|u_{1R}^I(0)\right|^2 + \left|u_{2R}^I(0)\right|^2\right] \qquad \text{— "elastic" cross section}$$

Green's function



The contribution of $N\bar{N}$ pair to hadron cross section

$$\sigma^I_{tot} = -\frac{2\pi\alpha^2}{M^2Q^2}\left|G^I_s\right|^2 \mathrm{Sp}\left[\mathrm{Im}\,\mathcal{D}(0,0|E)\right] \qquad \text{— "total"}$$

$$\left(\frac{p_r^2}{M} + \mathcal{V} - 2E\right)\mathcal{D}(r, r'|E) = -\frac{1}{rr'}\delta(r - r')$$

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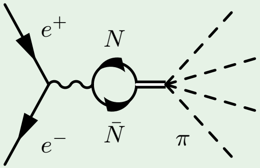

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The Green's function with tensor potential taken into account:

$$\mathcal{D}(r,r'|E) = -Mk\sum_{n=1,2}\left[\vartheta\left(r'-r\right)\chi_{nR}(r)\chi^T_{nN}(r') + \vartheta\left(r-r'\right)\chi_{nN}(r)\chi^T_{nR}(r')\right]$$

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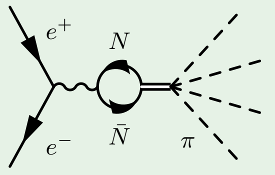

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Valid below $N\bar{N}$ threshold

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$N\bar{N}$ potentials

We want a nucleon-antinucleon potential that describes the following data:

- $N\bar{N}$ scattering including elastic scattering, charge-exchange ($p\bar{p} \leftrightarrow n\bar{n}$) and annihilation ($N\bar{N} \to \mathrm{mesons}$) cross sections

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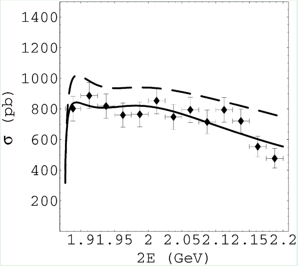
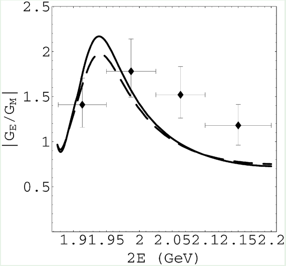
Paris $N\bar{N}$ potential

V.F. Dmitriev, A.I. Milstein, Phys. Lett. B. **658** (2007) 13





But not total cross section

Highly simplified potential model

- Only $^3S_1$ and $^3D_1$ partial waves are considered.
- Isospin dependence of the potential is neglected leading to the absence of charge-exchange reaction.
- Long-range pion-exchange potential

$$V_\pi(r) = f_\pi^2 \frac{m_{\pi_0}^2}{m_{\pi^\pm}^2} \left[\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + S_{12} \left(1 + \frac{3}{m_{\pi_0} r} + \frac{3}{(m_{\pi_0} r)^2} \right) \right] \frac{e^{-m_{\pi_0} r}}{3r}$$

- Short-range potential well

$$V_i(r) = U_i \, \theta\,(a_i - r) - i\, W_i \, \theta\,(b_i - r)\,, \qquad i = 0,\, 2,\, 3$$

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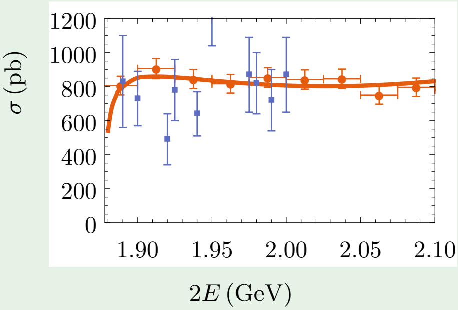
Fit to reproduce the scattering S-matrix, the cross section of $N\bar{N}$ production and the ratio of electromagnetic form factors:

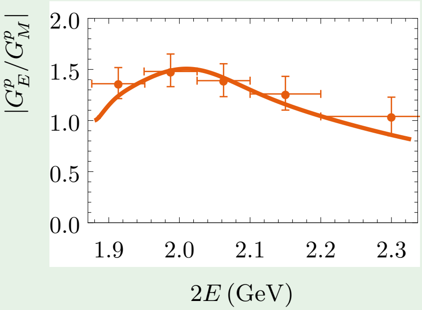
	V_0	V_2	V_3
U (MeV)	-164	9,2	-238
a (fm)	0,59	2,8	0,87
W (MeV)	494	8,9	$-42{,}8$
b (fm)	0,56	2,8	1,17

Results of the fit

$e^+e^- \to p\bar{p}$ cross section



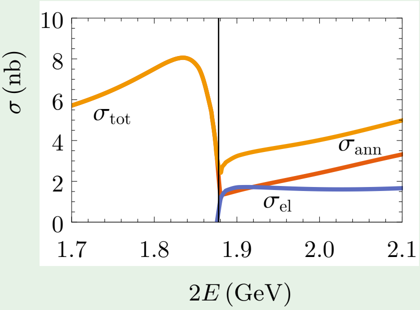
$|G_E^p/G_M^p|$



It is possible to describe the data with quite good accuracy.

Predictions of the model

Total, elastic and inelastic $N\bar{N}$ contributions to hadron cross section



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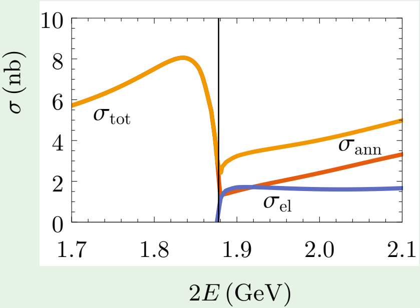
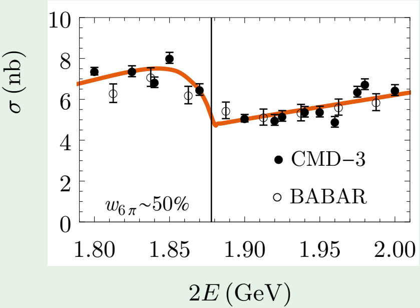


Fit to the cross section of 6π production (smooth function plus $\sigma_{ann} \cdot w_{6\pi}$)



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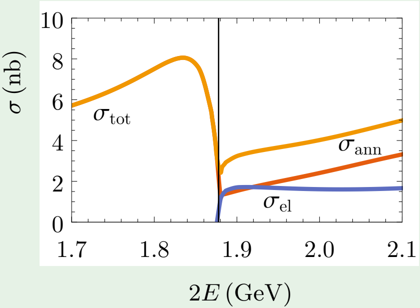

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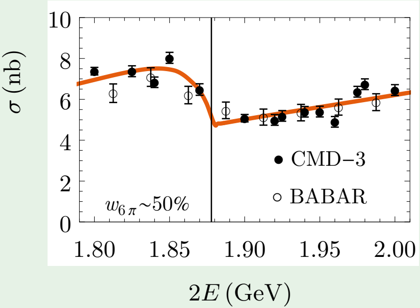

Further steps

- Consider the isospin dependence of the potential.
- Take the charge-exchange reaction into account.

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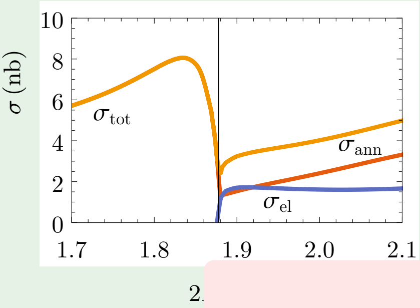

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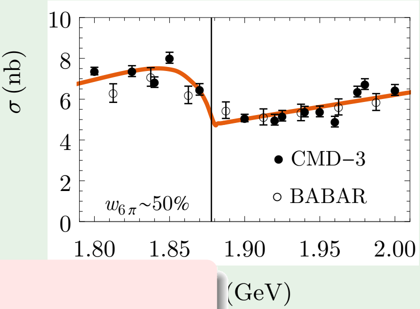

Further steps

- Consider the isc
- Take the charge

The work is in progress

Summary

- The formulas for the cross section of $N\bar{N}$ production in e^+e^- annihilation and the ratio of electromagnetic form factors have been obtained with the final-state interaction included.
- The expression for the Green's function has been derived with tensor forces taken into account.
- Attempts were made to describe all the data with one of the optical $N\bar{N}$ potentials.
- A very simple potential model qualitatively describing the features of the cross sections was proposed.
- The model is expected be improved in the nearest future.

Thank you for attention