2. Aspects of EWSB

- Naturalness
- Vacuum stability
- Custodial Symmetry
- Equivalence Theorem
- Unitarity

Don Quixote and the Windmills

Beautiful Discovery

Boson, $\quad J = 0$

Fermions = Matter ; Bosons = Forces

- **Fundamental Boson:** **New interaction which is not gauge**
- **Composite Boson:** **New underlying dynamics**

If New Physics exists at Λ_{NP}

$$\delta M_H^2 \sim \frac{g^2}{(4\pi)^2}\, \Lambda_{\mathrm{NP}}^2 \, \log\left(\frac{\Lambda_{\mathrm{NP}}^2}{M_H^2}\right)$$

Which symmetry keeps M_H away from Λ_{NP}?

- Fermions: Chiral Symmetry
- Gauge Bosons: Gauge Symmetry
- Scalar Bosons: Supersymmetry, Scale/Conformal Symmetry . . . ?

Symmetries & Mass Scales

Fermions: $\psi_{L,R} \longrightarrow e^{i\alpha_{L,R}}\, \psi_{L,R}$ **Chiral symmetry**

$$\mathcal{L}_\psi = \bar{\psi}\,(i\partial\!\!\!/ - m_\psi)\,\psi = \bar{\psi}_L i\partial\!\!\!/ \psi_L + \bar{\psi}_R i\partial\!\!\!/ \psi_R - m_\psi \left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\right)$$

Symmetry recovered at $m_\psi = 0$ ➡ $\delta m_\psi \propto m_\psi$

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Vectors: $A_\mu \longrightarrow A_\mu + \partial_\mu\theta$ **Gauge symmetry**

$$\mathcal{L}_A = -\frac{1}{4}\, F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\, m_A^2\, A_\mu A^\mu$$

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Scalars: $\mathcal{L}_\phi = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\,m_\phi^2\,\phi^2$ **Any symmetry?**

No additional symmetry at $m_\phi = 0$ ➡ $\delta m_\phi^2 \propto M^2$ $(M = \text{any scale})$

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Pseudo-Goldstone Boson

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Pseudo-Goldstone Boson

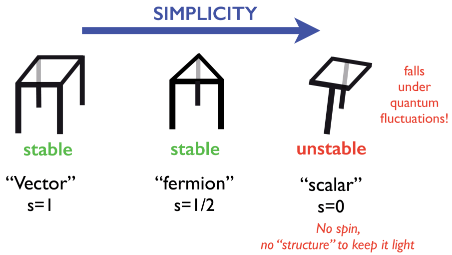
- **Scale symmetry:** $x \;\longrightarrow\; x/\lambda \quad , \quad \phi(x) \;\longrightarrow\; \lambda\,\phi(x/\lambda)$

$$\mathbf{M = 0} \quad , \quad \forall\, \mathbf{M}$$

Conformal Invariance. Dilaton

Which symmetry keeps M_H away from Λ_{NP} ?

A. Pomarol



Quantum Stability

Vectors/fermions
protected by
gauge/chiral symmetries

Proposed Solutions

SUSY / Composite Higgs

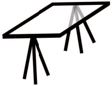
1) Keep the Higgs elementary, but protect it by symmetries: **Supersymmetry**

Higgs (boson) ⟷ Higgsino (fermion)

2) The Higgs is not elementary: **Composite Higgs**

Higgs made of fermions

(as a pion in strong interactions)



Spin, Mass & Degrees of Freedom

	$J = 1$	$J = \frac{1}{2}$	$J = 0$
$M = 0$	2 d.o.f. Trans. Pol.	2 d.o.f. ψ_L	1 d.o.f.
$M \neq 0$	3 d.o.f. Trans & Long.	4 d.o.f. ψ_L, ψ_R	1 d.o.f.

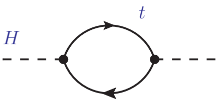
Vector $(2 \neq 3)$ and fermion masses are safe $(2 \neq 4)$

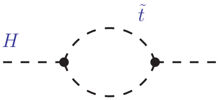
Scalar masses not protected (continuous $m \to 0$ limit)

Higgs Self-energy: **The top is the largest SM contribution**

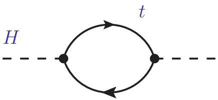
δM_H^2 stabilized through new-physics contributions

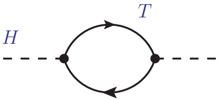
- **SUSY:** stop loops



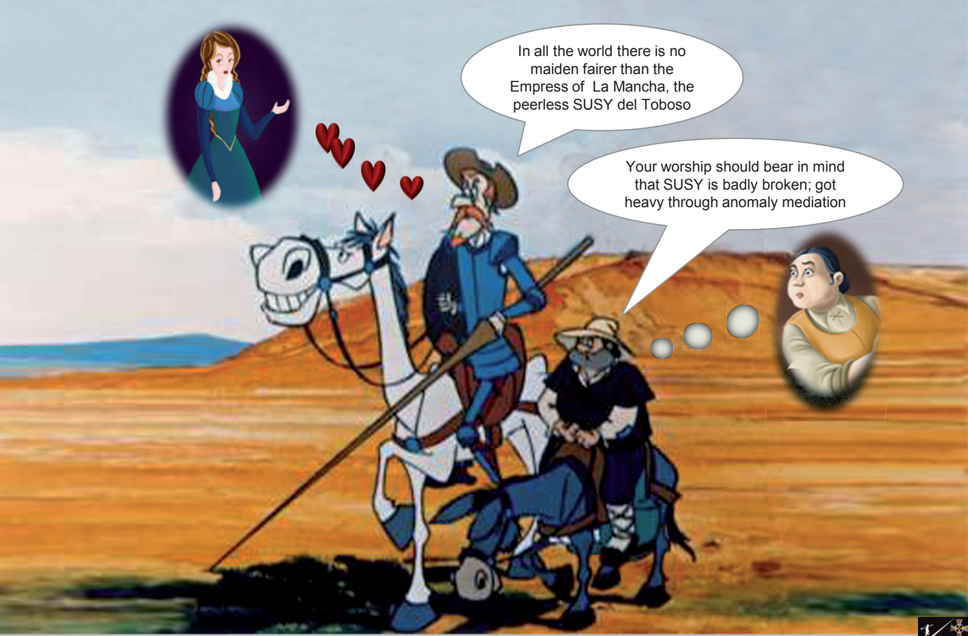


- **Composite Higgs:** fermionic top partners

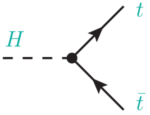




Desperately Seeking SUSY (Dulcinea)



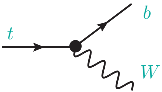
The Heaviest Mass Scale



$$y_t = \frac{\sqrt{2}}{v}\, m_t = 2^{3/4} G_F^{1/2} m_t \approx 1 \qquad (0.995)$$

The top quark:

- **Sensitive probe of Electroweak Symmetry Breaking**
- **Non-perturbative (strong) dynamics?**
- **Very different from other quarks:** $y_b = 0.025\,,\; y_c = 0.007\;\ldots$
- **Is it really a SM quark?**



So far, we only know the decay $\mathbf{t \to W^+ b}$

$$|\mathbf{V_{tb}}| > \mathbf{0.92} \qquad (95\%\ \text{CL})$$

Top Mass

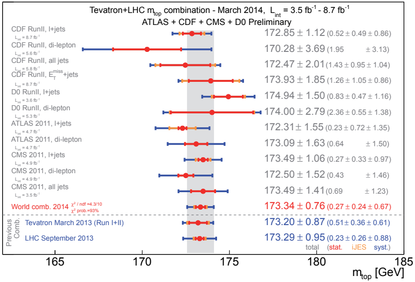
- **Monte Carlo mass:**

$$\mathbf{m_t^{MC} = (173.34 \pm 0.76)\ GeV}$$

Lacks a proper QCD definition

$$\Delta m_t^{\rm th} = |m_t^{\rm pole} - m_t^{\rm MC}| \approx \mathcal{O}(1\ {\rm GeV})$$

Hoang-Stewart 0808.0222



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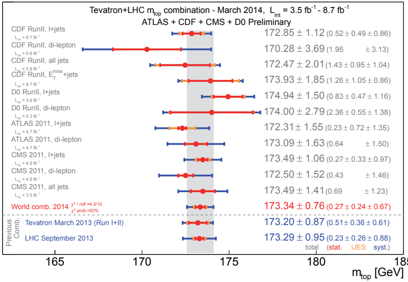
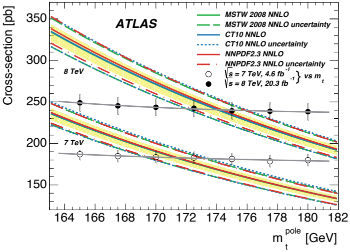
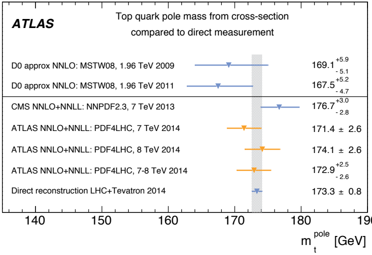
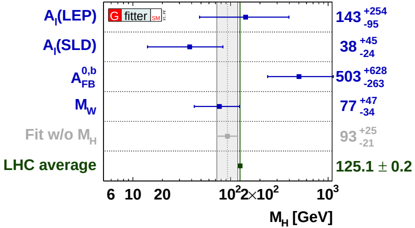


- **Well-defined mass:** $\sigma_{t\bar{t}}$ **NNLO + NNLL** Czakon et al., Bärnreuther et al., Cacciari et al.

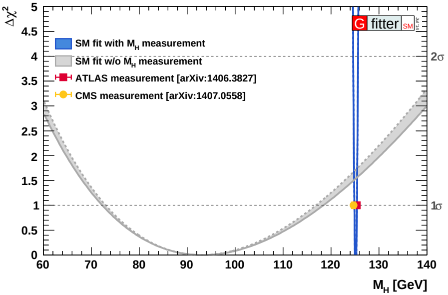


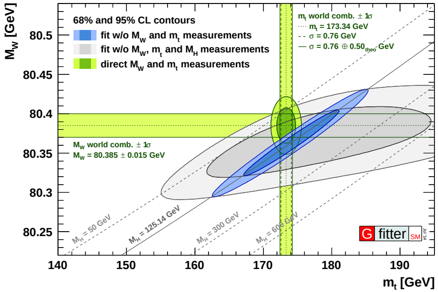


SM Higgs

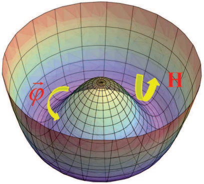


Favoured by EW precision tests





SM Higgs Potential



$$\Phi(x) \;=\; \exp\left\{\frac{i}{v}\,\vec{\sigma}\,\vec{\varphi}(x)\right\}\;\frac{1}{\sqrt{2}}\left[\begin{array}{c} 0 \\ v + H(x)\end{array}\right]$$

$$V(\Phi) + \frac{\lambda}{4}\,v^4 \;=\; \lambda\left(|\Phi|^2 - \frac{v^2}{2}\right)^2 \;=\; \frac{1}{2}\,M_H^2\,H^2 \,+\, \frac{M_H^2}{2v}\,H^3 \,+\, \frac{M_H^2}{8v^2}\,H^4$$

$$v \;=\; \left(\sqrt{2}\,G_F\right)^{-1/2} \;=\; 246\ \mathrm{GeV}$$

$$M_H = (125.09 \pm 0.24)\ \mathrm{GeV} \quad \Longrightarrow \quad \lambda \;=\; \frac{M_H^2}{2v^2} \;=\; 0.13$$

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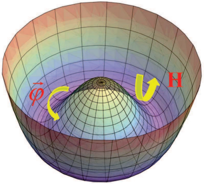

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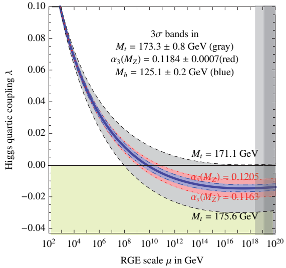
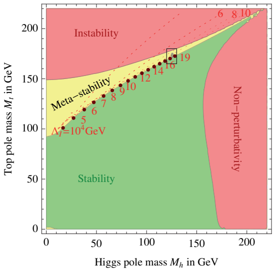
$$\frac{M_H^2}{2v^2} \; = \; \lambda(\mu) \; + \; \frac{2y_t^2}{(4\pi)^2} \; \left[\lambda + 3\left(y_t^2 - \lambda\right)\, \log\left(\mu/m_t\right)\right] + \cdots$$

$y_t \; = \; \sqrt{2}\, m_t/v \; \approx \; 1$

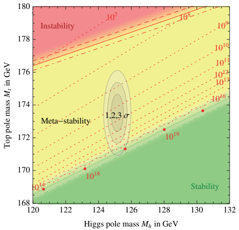
Vacuum Stability: $\lambda(\Lambda) \geq 0$

Degrassi et al, 1205.6497, 1307.3536
Buttazzo et al, 1307.3536





$\Lambda = M_{\rm Planck}$ ➡ $M_H > (129.6 \pm 1.5)\ \text{GeV}$



Assumes SM valid all the way up to $\Lambda \leq M_{\rm Planck}$

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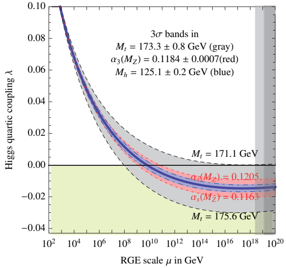

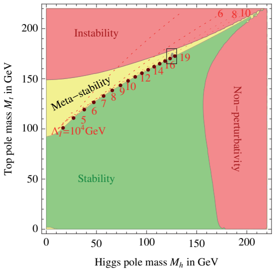

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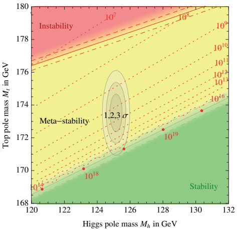
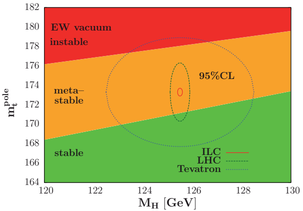


Alekhin et al, 1207.0980



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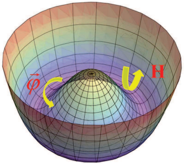

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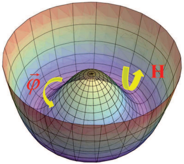

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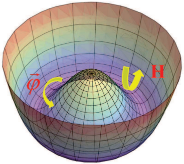

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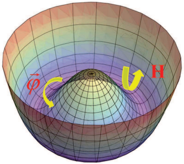

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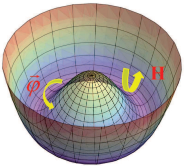

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Same Goldstone Lagrangian as QCD pions:

$$f_\pi \to v \qquad , \qquad \vec{\pi} \to \vec{\varphi} \to W_L^\pm, Z_L$$

EFFECTIVE LAGRANGIAN: $\mathcal{L}(U) = \sum_n \mathcal{L}_{2n}$

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Parity ➡ even dimension ; $U\,U^\dagger = 1$ ➡ $2n \geq 2$

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$$\mathcal{L}_2 = \frac{f^2}{4}\,\mathrm{Tr}\left(\partial_\mu U^\dagger\, \partial^\mu U\right)$$

Derivative Coupling

EFFECTIVE LAGRANGIAN: $\mathcal{L}(U) = \sum_n \mathcal{L}_{2n}$

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- $\mathbf{SU(2)_L \otimes SU(2)_R}$ **invariant**

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$$\mathcal{L}_2 = \frac{f^2}{4} \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)$$

Derivative Coupling

Goldstones become free at zero momenta

Electroweak Symmetry Breaking

$$\mathcal{L}_2 = \frac{v^2}{4}\,\mathrm{Tr}\left(D_\mu U^\dagger D^\mu U\right) \quad \xrightarrow{U=1} \quad \mathcal{L}_2 = M_W^2\, W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2\, Z_\mu Z^\mu$$

$$\mathbf{M_W} = \mathbf{M_Z} \cos\theta_\mathbf{W} = \frac{1}{2}\,\mathbf{g}\,\mathbf{v}$$

$$D^\mu U = \partial^\mu U - i\,\hat{W}^\mu U + i\,U\,\hat{B}^\mu \quad , \quad D^\mu U^\dagger = \partial^\mu U^\dagger + i\,U^\dagger \hat{W}^\mu - i\,\hat{B}^\mu U^\dagger$$

$$\hat{W}^\mu = -\tfrac{g}{2}\,\vec{\sigma}\cdot\vec{W}^\mu \quad , \quad \hat{B}^\mu = -\tfrac{g'}{2}\,\sigma_3\,B^\mu$$

(explicit symmetry breaking)

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- **EW Goldstones are responsible for $M_{W,Z}$** **(not the Higgs!)**
- **QCD pions also generate small W, Z masses:** $\delta_\pi M_W = \frac{1}{2}\, g\, f_\pi$

Goldstone interactions are determined by the underlying symmetry

$$U(\vec{\varphi}\,) \equiv \exp\left\{ i\,\vec{\sigma}\cdot\frac{\vec{\varphi}}{v}\right\} \qquad , \qquad \vec{\sigma}\cdot\vec{\varphi} = \begin{pmatrix} \varphi^0 & \sqrt{2}\,\varphi^+ \\ \sqrt{2}\,\varphi^- & -\varphi^0 \end{pmatrix}$$

$$\begin{aligned} \frac{v^2}{4}\,\langle \partial_\mu U^\dagger \partial^\mu U\rangle &= \partial_\mu\varphi^-\partial^\mu\varphi^+ + \frac{1}{2}\,\partial_\mu\varphi^0\partial^\mu\varphi^0 \\ &+ \frac{1}{6v^2}\left\{\left(\varphi^+\overleftrightarrow{\partial}_\mu\,\varphi^-\right)\left(\varphi^+\overleftrightarrow{\partial}^\mu\varphi^-\right) + 2\left(\varphi^0\overleftrightarrow{\partial}_\mu\,\varphi^+\right)\left(\varphi^-\overleftrightarrow{\partial}^\mu\varphi^0\right)\right\} \\ &+ O\left(\varphi^6/v^4\right) \end{aligned}$$

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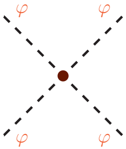

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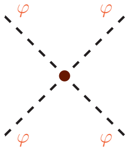

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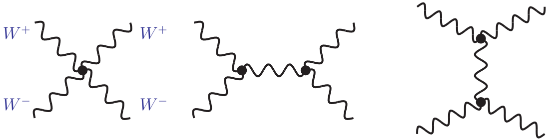
Non-Linear Lagrangian: $2\varphi \to 2\varphi,\ 4\varphi \cdots$ related

Equivalence Theorem



Cornwall–Levin–Tiktopoulos

Vayonakis

Lee–Quigg–Thacker

$$T(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{s+t}{v^2} + \mathcal{O}\left(\frac{M_W}{\sqrt{s}}\right)$$
$$= T(\varphi^+\varphi^- \to \varphi^+\varphi^-) + \mathcal{O}\left(\frac{M_W}{\sqrt{s}}\right)$$

The scattering amplitude grows with energy

Goldstone dynamics ⟷ **derivative interactions**

Tree-level violation of unitarity

Longitudinal Polarizations

$$k^\mu = \left(k^0, 0, 0, |\vec{k}|\right) \quad \longrightarrow \quad \epsilon_L^\mu(\vec{k}) = \frac{1}{M_W}\left(|\vec{k}|, 0, 0, k^0\right) = \frac{k^\mu}{M_W} + O\left(\frac{M_W}{|\vec{k}|}\right)$$

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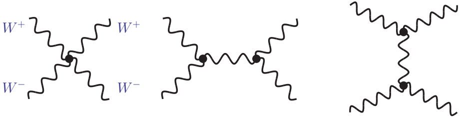
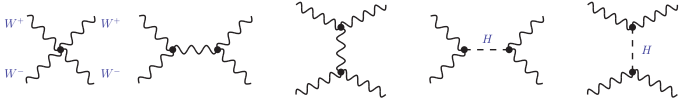


Gauge Cancelation

$$T(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right)$$

$$= T(\varphi^+ \varphi^- \to \varphi^+ \varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right)$$

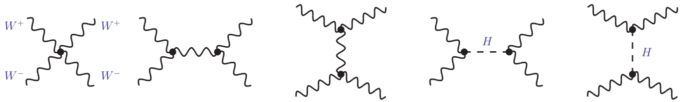
$W_L^+ W_L^- \to W_L^+ W_L^-$:



$$T_{\rm SM} = \frac{1}{v^2}\left\{s + t - \frac{s^2}{s-M_H^2} - \frac{t^2}{t-M_H^2}\right\} = -\frac{M_H^2}{v^2}\left\{\frac{s}{s-M_H^2} + \frac{t}{t-M_H^2}\right\}$$

Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

$W_L^+ W_L^- \to W_L^+ W_L^-$:



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Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

When $s \gg M_H^2$, $\quad T_{SM} \approx -\frac{2M_H^2}{v^2} \quad , \quad a_0 \equiv \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta \; T_{SM} \approx -\frac{M_H^2}{8\pi v^2}$

Unitarity:

Lee–Quigg–Thacker

$$|a_0| \le 1 \quad \Rightarrow \quad \mathbf{M_H < \sqrt{8\pi}v} \underbrace{\sqrt{2/3}}_{W^+W^-,\, ZZ,\, HH} \approx \mathbf{1\ TeV}$$