Baryons at large N_c

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... is based on the papers with D.Diakonov, A.Vladimirov, M.Polyakov, P.Pobylitsa, K.Göeke, M. Praszalowicz and some others

Impressions of newcomer

Common wisdom: **Theory is understood if its spectrum is understood.**

- Many efforts were invested into the spectrum of particles in the strong interactions. In particular, **baryons**.

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Question of poor QCD theorist: What information about underlying QCD is coded in these data? (Too many numbers!)

Reduce the number of numbers! $SU(3)$-multiplets

Chiral limit:

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- Not easy to identify — still many of members of multiplets are **not known**.
  1. As many **octets** as number of N.
  2. As many **decuplets** as number of Δ.
  3. To say nothing about exotics!

Low Lying baryons — $SU(3)$-multiplets.

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We will accept:

- We stop at masses ≤ 2000 **MeV**.
- We follow multiplets suggested by **V.Guzey& M.Polyakov** (2005). **Advantage** — they checked
  1. Gell-Mann-Okubo formula
  2. relation for widthes
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  The quality of $SU(3)$ is **good**
- We do not take baryons with high spin — **different mechanism**

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Parity "+"			Parity "-"		
56 (L=0)	$(8, \frac{1}{2})$	1155	70(L=1)	$(1, \frac{1}{2})$	1405
	$(10, \frac{3}{2})$	1382		$(1, \frac{1}{2})$	1520
56 (L=0)	$(8, \frac{1}{2})$	1630		$(8, \frac{1}{2})$	1615
	$(10, \frac{1}{2})$	1732		$(8, \frac{3}{2})$	1680
?	$(8, \frac{1}{2})$	1845		$(8, \frac{1}{2})$	1710
	$(8, \frac{3}{2})$	1865		$(10, \frac{1}{2})$	1758
	$(8, \frac{5}{2})$	1868		$(8, \frac{5}{2})$	1802
	$(10, \frac{1}{2})$	2060		$(10, \frac{3}{2})$	1850
	$(10, \frac{5}{2})$	2071		$(1, \frac{1}{2})$	1890
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(56 and 70 are $SU(6)$-multiplets). We omit higher spin and mass multiplets.

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  2. Quark **confinement**
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- There is an **infinite number of mesons** with
  1. **Masses** $m_{meson} \sim O(1)$.
  2. **Widths** $= O(1)$ (mesons are stable)
  3. **Interaction of mesons** $= O(1/N_c)$ (mesons do not interact)

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Baryons are different

- **Theorem (Witten)**
  Baryons can be described as solitons of meson field, in the **mean field approximation**. Fluctuations are **suppressed by** N_c. **Masses** $M_{baryon} \sim O(N_c)$.
- **Widths** $= O(1)$ (baryons, in general, have *finite* width)
- **Interaction of baryons** $= O(N_c)$ (interaction is *large*)
- Mesons at large N_c remain **quantum** particles. **Baryons** are **heavy semiclassical** objects. Theory of baryons, in a sense, *is simpler*

The **main questions** are

- **What are relevant degrees of freedom?**
- What is the *effective Lagrangian?*

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Low energy images of QCD

Answers:

- **Quark model**: Nucleon is a bound state of N_c **constituent** quarks bound by some potential. D.o.F. — **quarks**

The most **popular** and old. 99% of discussion of baryon properties in the literature implies quark model. In the limit $N_c \to \infty$ reduce to *mean field potential* theory. Find levels in the **mean potential** and fill by quarks.

- **Skyrme model**: Low energy QCD $\sim$ Effective chiral lagrangian. D.o.F. — **pions**.

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\mathrm{L}_\mu \mathrm{L}_\mu + \frac{1}{32e^2}\mathrm{Tr}[L_\mu, L_\nu] + \textbf{WZW}$$

with

$$L_\mu = iU^+\partial_\mu U, \qquad U = \exp[i\frac{\pi^a\lambda^a}{F_\pi^2}]$$

Baryon $==$**soliton** of π-meson field.

Low energy images of QCD

- **Open moose model== AdS/QCD**:
  Generalization of Effective Chiral Lagrangian with **infinite number of mesons**: scalar, pseudoscalar and *vector*.

$$\mathcal{L} = \sum_{k=1}^{K+1} f_k^2 |D_\mu \boldsymbol{\Sigma}^{(\mathbf{k})}|^2 + \sum_{k=1}^{K} \frac{F_{\mu\nu}^2(\mathbf{A}^{(\mathbf{k})})}{2g_k^2}$$

where

$$D_\mu \Sigma^{(k)} = \partial_\mu \Sigma^{(k)} - iA_\mu^{(k)} \Sigma^{(k-1)} + i\Sigma^{(k-1)} A_\mu^{(k)}$$

$$A_\mu^{(0)} = A_\mu^{(K+1)} = 0, \qquad \Sigma = \Sigma^{(1)} \dots \Sigma^{(K+1)} = \exp[i \frac{\pi^{\mathbf{a}} \lambda^a}{F_\pi^2}]$$

has $\mathbf{SU(N_f)_R} \otimes \mathbf{SU(N_f)_L} \otimes [SU(N_f)_{local}]^K$ symmetry.
Very nice (**Son, Stefenov**): at $K \to \infty$

open moose theory $\longrightarrow$ $\mathbf{d = 5}$ YM theory + **dilaton** in **curved space**

Low energy images of QCD

$$S[A] = \frac{1}{2g_0^2}\mathrm{Tr}\int d^{\mathbf{5}}x\sqrt{g}e^{-2\phi}F_{\mu\nu}^2[\mathbf{A}]$$

$(a, b = 1, \dots 5)$ with metrics

$$ds^2 = -du^2 + \mathbf{e^{2w(u)}}\eta_{\mu\nu}dx^\mu dx^\nu$$

if we accept:

$$\mathbf{f^2}(u) = \frac{1}{g_0^2}e^{2w-2\phi}, \qquad \mathbf{g}^2(u) = g_0^2 e^{2\phi}$$

where $a\mathbf{f_k^2} = f^2(ka)$ and $a\mathbf{g_k^2} = g^2(u)$ — **meson coupling constants** and

$$\mathbf{\Sigma} = \int\left(i\int du\mathbf{A_5(x)}\right)$$

Low energy images of QCD

The model has **deep analogy** with **Ads/CFT correspondence**. Baryon is **soliton** of the model which is **instanton** in $d = 5$. D.o.F of the model are **pseudoscalar and vector** mesons. It is direct **generalization** of *Skyrme model with vector mesons*.

- **Quark-soliton model**
  is a marriage of quark model and meson models. In its original version relevant D.o.F are **π-mesons** and **quarks**.

Lagrangian

$$\mathcal{L} = \bar{\psi}(i\partial + iMU^{\gamma_5})\psi, U = e^{i\pi^a \lambda^a \gamma_5}$$

in such a form was *derived* as a low energy limit of **instanton vacuum** of QCD and **constituent mass** was calculated. **Instantons** lead to formation of **gluon** ($\langle \frac{G^2_{\mu\nu}}{32\pi^2} \rangle$) and **chiral** ($\langle \bar{\psi}\psi \rangle$) condensates (*spontaneous breakdown of chiral symmetry*). Quarks acquire momentum dependent mass $M(p^2)$ (and $M(0) = 345 MeV$).

Quark-soliton picture of baryon

Model has **no single parameter**.

Effective chiral Lagrangian

$$\mathcal{L}_{ECL} = \mathbf{N_c} \log \mathrm{Det}\left[i\hat{\partial} + iMU^{\gamma_5}\right]$$

contains all derivatives of pion field. Baryon can be said to be

Baryon is a bound state of N_c quarks in the self-consistent meson field

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Baryon is a soliton of effective chiral Lagrangian

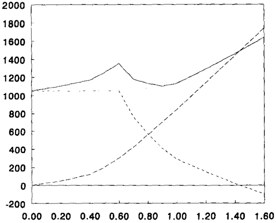
— similar to **Skyrme or Ads/QCD**. First language more suitable for small solitons, almost non-relativistic valence quarks second for large solitons — ultrarelativistic quarks.

Quark-soliton: how it works?

- pion field **symmetry** is assumed to be **hedgehog**: $\pi^a = n^a P(r)$ (can be proved)
- solve Dirac equation in pion field

$$[i\gamma_0\gamma_i\partial_i + iM\gamma_0 U_5^\gamma]\Psi_{\mathbf{n}} = \varepsilon_{\mathbf{n}}\Psi_{\mathbf{n}}$$

Nucleon mass



$$\mathcal{M}_N = N_c \mathbf{E_{val}} + N_c \mathbf{E_{field}}$$

$\mathbf{E_{valence}}$ is the energy of *discrete level*,

$$\mathbf{E_{field}} = \sum_{\mathbf{n}} \varepsilon_{\mathbf{n}}$$

— energy of **meson** self-consistent field

Quark-soliton: how it works?

One can completely go through **all** properties of ground state **octet** and **decuplet**

- *Static properties:* masses, **mass splittings** inside octet and decuplet and **splitting between** octet and decuplet, Σ-term.
- magnetic moments, e.m. formfactors, isotopic spliitings, axial constants, decay constants, transitional formfactors.
- high energy properties of nucleons: **parton distributions**, **generalized** parton distributions, **light cone** nucleon wave
- pion-nucleon, and **nucleon-nucleon** *interaction*

Every question has an answer obeying **general theorems**.Not true for **any** other model. Typical accuracy is 10-15%, sometimes less.

Bad news. Confinement

Chiral-soliton model lacks **confinement**. This was **inherited** from intanton vacuum which reproduce **breakdown of chiral symmetry** but *not confinement*.

Confinement is not easy to formulate in QCD **with light quarks**.

We are left with simplest

There are no other singularities in correlators of colorless currents than hadrons

Is there confinement in **effective chiral Lagrangian**?

- ECL is the expansion in derivatives of pion field. Confinement **is coded** in analytical properties of **all** meson amplitudes at **not small** momenta. It is a subtle requirement to the all higher derivatives terms of ECL.

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Vast majority of baryon properties has nothing to do with confinement, they are determined by breakdown of chiral symmetry

Drawbacks of other pictures

Quark model

- Not a consistent **quantum field theory**. In fact, consistent only in the *non-relativictic limit*
- Completely ignores breakdown of **chiral** symmetry. Do not consider **pions** as essential low energy D.o.F.
- **Wrong symmetry** of mean field (central symmetric). Wrong leading approximation (**SU(6)**)

Skyrme model & AdS/QFT

- Ignore the concept of **constituent** quark and idea of **two scales** which proved to be correct in the past. All components of Fock w.f. are equally important
- allow to consider only **restricted set** of **quark** observables
- no explanation (?) for excited resonances with $\delta M \sim O(1)$ (discrete non-zero modes).

Baryon resonances

What changes when consider higher resonances?

- Account for **higher** mesons — scalar, vector, etc
- Include some **meson** Lagrangian (e.g. to **imitate confinement**

Main question

Is this resonance **excitation** of **nucleon** (distance $O(1)$) or it is related to another **mean field**?

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Answer

It is not necessary to change the **soliton field** to describe baryons *below 2* GeV. Those **20 muliplets** can be viewed as **1-quark excitations** of nucleon. However, soliton field is definitely deformed in **high spin baryons** (*centrifugal forces*. It forms a kind of **rotating string**.

Mean field and Dirac equation

In the **mean field approximation** the given is viewed as N_c constituent quarks sitting on **1-particle levels** — $\boxed{\mathcal{H}\Psi = \varepsilon\psi}$ in the **general meson** field:

$$\mathcal{H} = \gamma^0(-\mathbf{i}\partial_\mu\gamma_\mu + \mathbf{S(x)} + \mathbf{P(x)i}\gamma^{\mathbf{5}} + V_\mu(x)\gamma^\mu + A_\mu\gamma^\mu\gamma^5 + T_{\mu\nu}(x)\frac{i}{2}[\gamma^\mu, \gamma^\nu])$$

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- u, d**-quarks, isovectors**
  1. Pseudoscalar isoscalar field: $P^a(x) = n^a P_0(r)$
  2. Vector isovector, space components $V_i^a(x) = \varepsilon_{aij} n^j P_1$
  3. Axial isovector, space components $A_i^a(x) = \delta_{ai} P_2(r) + n_a n_i P_3(r)$
  4. Tensor, isovector, space components $T_{ij}^a(x) = \varepsilon_{aij} P_4(r) + \varepsilon_{bij} n_a n_b P_3(r)$

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Mean Field and Dirac equation

- **u, d-quarks, isoscalars**
  1. scalar isoscalar $P(x) = n^a Q_0(r)$
  2. vector isoscalar, time component $V_0(x) = Q_1(r)$
  3. tensor isoscalar, mixed component $T_{0i}(x) = n_i Q_2(r)$
- **s-quarks**
  1. scalar isoscalar $P(x) = n^a R_0(r)$
  2. vector isoscalar, time component $V_0(x) = R_1(r)$
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12 radial functions required for general mean field Solve Dirac equation — find **one-quark levels** $\boxed{\varepsilon_i}$.

Baryon mass is

$$\mathcal{M} = \mathbf{N_c}\varepsilon_0 + \mathcal{E}_{\mathbf{meson}}(\mathbf{meanfield}) \sim \mathcal{O}(\mathbf{N_c})$$

Mean Field and Dirac equation

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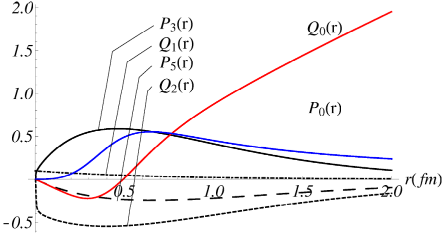
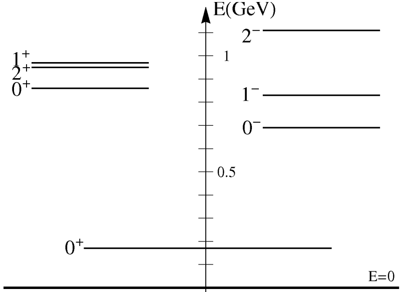
A typical example of Mean Field





Confining “potential”. Quark spectrum — **discrete**

“Grand spin”:

$$\vec{K} = \vec{j} + \vec{t}$$

— conserved. Parity — conserved.

$$K^P$$

Nature as one particle excitations in the mean field

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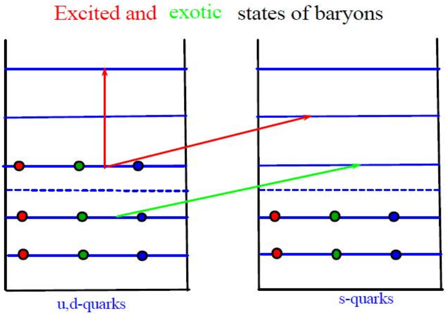

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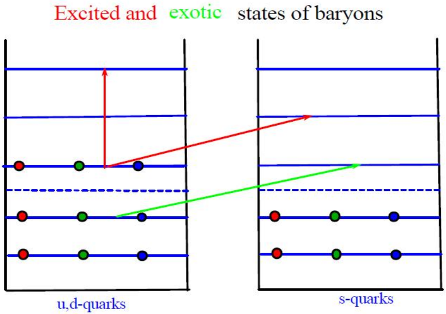
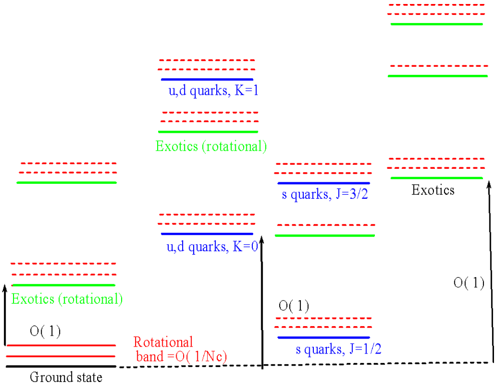


Multiplets at large N_c

Different type of excitations: in sector of u, d-**quarks** and **separately** in sector of **s-quarks** and also *Gamov-Teller* type **exotics**:



Every excitation accompanied by **rotational band** $\sim O(1/N_c)$ and **rotational exotics** $\sim O(1)$

Quark Nuclear Physics

We see that theory of baryons turns in **quark nuclear physics**. Concentrate on **model independent** quantities (not to use real dynamics).
Nevertheless, even in this case quark chiral soliton **does not** reduce to N_c limit for baryons of **Dashen, Manohar et al.** Requiring that πN has amplitude $O(1)$ not $O(N_c)$ they derived a kind of **current algebra** in **solitonic sector** of chiral Lagrangian. (**strict** $N_c \to \infty$ limit) Soliton approach **redproduces all** relations of this **"current algebra"** but provides essentially **more** relations.

Quark Nuclear Physics

Strict $N_c \to \infty$

1. $N_c \to \infty$ **justifies mean field approximation** but — mean field can be valid for **other** reasons.
2. $N_c \to \infty$ leads to ugly mutiplets — what **corresponds** to particles in nature?
3. *Calculation* of any quantity consists of **dynamics** (matrix elements)and **kinematics**. First should be done in mean field and **coincides** with $N_c \to \infty$, second - calculated for $N_c = 3$. Meanwhile Clebsch-Gordans change **strongly** from $N_c = \infty$ to 3

General theory of rotations at large N_c

The wave function of quarks in the mean field approximation:

$$\Phi = \prod_{\mathbf{occupied}} \int d^3x_n \phi^{(n)}(x) \psi^+(\mathbf{x_n})$$

ϕ— solutions of the Dirac eq in the **mean field**.

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$$\tilde{\phi}^{\alpha\mathbf{i}} = \mathbf{R}^{\alpha}_{\alpha'}\mathbf{S}^{\mathbf{i}}_{\mathbf{i}'}\phi^{\alpha'\mathbf{i}'}(\mathbf{O}x) \qquad \tilde{\pi}(x) = \mathbf{R}^+\pi(\mathbf{O}x)\mathbf{R}$$

where $\mathbf{R}^{\alpha}_{\alpha'}$ is a **flavor** and $\mathbf{S}^{\mathbf{i}}_{\mathbf{i}'}$ is **spin** rotation ($\mathbf{O}$ is matrix of adjoint space rotation) is also **solution of soliton problem** with the same energy.

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$\mathbf{S}^{\mathbf{i}}_{\mathbf{i}'}$ and $\mathbf{R}^{\alpha}_{\alpha'}$ are collective coordinates.

General theory of rotations at large N_c

Effective Lagrangian:

$$\mathcal{S} = i\mathbf{N_c}\mathrm{Sp}_{\mathbf{occup}} \log\left\{ i\frac{\partial}{\partial \mathbf{t}} + i\gamma^0\gamma^i\partial_i + MU^{\gamma_5}[\pi] \right\}$$

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For rotated soliton:

$$\mathcal{S} = i\mathbf{N_c}\mathrm{Sp}_{\mathbf{occup}} \log\left\{ i\frac{\partial}{\partial \mathbf{t}} + i\gamma^0\gamma^i\partial_i + MU^{\gamma_5}[\pi] - \tilde{\mathbf{\Omega}}^{\mathbf{a}}\mathbf{t}^{\mathbf{a}} - \tilde{\omega}^{\mathbf{a}}\mathbf{j}^{\mathbf{a}} \right\}$$

$\tilde{\mathbf{\Omega}}^{\mathbf{a}}$ is flavor and $\tilde{\omega}^{\mathbf{a}}$ space frequencies in **body-fixed frame**, $\mathbf{t}^{\mathbf{a}}$ — **flavor**, $\mathbf{j}^{\mathbf{a}}$ — **total one particle angular momentum**,

$$\tilde{\mathbf{\Omega}}^{\mathbf{a}} = -\mathbf{i}\mathrm{Tr}\left[\mathbf{t}^{\mathbf{a}}\mathbf{R}^{+}\dot{\mathbf{R}}\right], \qquad \tilde{\omega}^{\mathbf{a}} = -\mathbf{i}\mathrm{Tr}\left[\sigma^{\mathbf{a}}\mathbf{S}^{+}\dot{\mathbf{S}}\right]$$

$$\mathbf{j}^{\mathbf{a}} = \mathbf{s}^{\mathbf{a}} + \mathbf{l}^{\mathbf{a}} = \mathbf{s}^{\mathbf{a}} + \mathbf{i}\varepsilon_{\mathbf{ikl}}\mathbf{x}^{\mathbf{k}}\frac{\partial}{\partial \mathbf{x}^{\mathbf{l}}}$$

General theory of rotations at large N_c

Theory of rotations is exac‘tly the same as for nucleus (**spherical top**) but

1. For nucleus group is $O(3)$. For baryon group is $O(3) \otimes SU(3)_{flavor}$
2. **Two** quantum numbers in the body-fixed frame (internal quantum numbers) $\widetilde{T}$ (flavor) and $\widetilde{J}$ (space)
3. **Hedgehog symmetry** leads to condition $\widetilde{T} = -\widetilde{J}$
4. As mean field acts only in $SU(2)$ **subgroup** of $SU(3)$ **Wess-Zumino-Witten** quantization rule appears $\widetilde{Y} = N_c/3$

General theory of rotations at large N_c

$$\mathcal{M} = \mathcal{M}_0 + \Delta\varepsilon_{lev} + \frac{1}{2I_1}\left[a_K \tilde{J}(\tilde{J}+1) + (1-a_K)\tilde{T}(\tilde{T}+1) - \right.$$

$$\left. - a_K(1-a_K)K(K+1)\right] + \frac{(1+X)(2+3\tilde{Y})}{2I_2}$$

1. I_1, I_2 moments of inertia. In the *cranking* approximation:

$$I_1, I_2 = N_c \sum \frac{< n|\lambda_a|m >< m|\lambda_b|n >}{\varepsilon_n - \varepsilon_m} +$$

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(Inglis formula).**m** - occupied, **n** - free. **Thouless-Valatin** term describes **mixing of rotations and other fluctuations**. Mainly **do not appear** in relativistic theory.

General theory of rotations at large N_c

- I_1 for λ^a with $a = 1, 2, 3$ **controls rotational band** of usual excitations (splitting is $O(1/N_c)$
- I_2 for λ^a with $a = 4, 5, 6, 7$ **controls exotical states** (splitting is $O(1)$). X — **exoticness** (minimal number of **additional** $q\bar{q}$ to create this mutiplet)

For **ground state** quantization rules (N_c quarks on the level $K = 0$) lead to $(8, 1/2)$ and $(10, 3/2)$ states **only!**

Theory of rotations for one particle excitations

What is different for one particle excitations:

- Change of w.f. of **one quark** of N_c **does not change** mean field. Only corrections $O(1/N_c)$ to the energy (linear terms **absent**).

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What is different for one particle excitations:

- For levels in u, d-sector, $\mathbf{K} = \mathbf{j} + \mathbf{t}$ is conserved. Quantization rules are

$$\widetilde{\mathbf{J}} + \widetilde{\mathbf{T}} = \mathbf{K}, \qquad \widetilde{Y} = \frac{N_c}{3}$$

- For levels in s-sector, $\mathbf{S} = \mathbf{l} + \mathbf{s}$ is conserved. Quantization rules are

$$\widetilde{\mathbf{J}} + \widetilde{\mathbf{T}} = \mathbf{S}, \qquad \widetilde{Y} = \frac{N_c - 3}{3}$$

- Coefficients a_K describes **mixing of j and t** for **given level**

$$< n|\mathbf{t}|n >= a_K < n|\mathbf{K}|n >, \qquad < n|\mathbf{j}|n >= (1 - a_K) < n|\mathbf{K}|n >$$

  they are known if w.f. of level are known.
- analogously for a_S

Quark model at $N_c \to \infty$

At $N_c \to \infty$ the lowest multiplet of the $SU(6)$ has dimension

$$dim_{SU(6)} = \frac{1}{120}(N_c+1)(N_c+2)(N_c+3)(N_c+4)(N_c+5)$$

It becomes **56** at $N_c = 3$. In terms of **quark soliton picture** these states are contained in the **rotational band** of the ground state with (exoticness $X = 0$, $\tilde{Y} = N_c/3$): $J = \tilde{T}$ **and** $\frac{1}{2} < J < \frac{N_c}{2}$.

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$$\sum dim(2J+1) dim_{SU(3)} = dim_{SU(6)}$$

Next multiplet is:

$$dim_{SU(6)} = \frac{1}{24}(N_c-1)(N_c+1)(N_c+2)(N_c+3)(N_c+4)$$

It is **70** at $N_c = 3$.

Quark model at $N_c \to \infty$

It can be checked that it can be divided in the following series of $SU(3)$-**multiplets**:

- series with $\tilde{Y} = N_c/3$, **exoticness** $X = 0$ with
  1. $\tilde{T} = J - 1$ ($\tilde{J} = \frac{3}{2} \dots \frac{N_c}{2}$),
  2. $\tilde{T} = J$ ($\tilde{J} = \frac{1}{2} \dots \frac{N_c}{2} - 1$),
  3. $\tilde{T} = J + 1$ ($\tilde{J} = \frac{1}{2} \dots \frac{N_c}{2} - 1$),

  Obtained quantization for $K = 1$ in u, d-quarks.
- 2 series $\tilde{Y} = (N_c - 3)/3$, **exoticness** $X = 0$ with
  1. $\tilde{T} = J + 1/2$ ($\tilde{J} = \frac{1}{2} \dots \frac{N_c}{2} - 2$),
  2. $\tilde{T} = J - 1/2$ ($\tilde{J} = \frac{1}{2} \dots \frac{N_c}{2} - 1$),

  Obtained quantization in the sector of s-**quarks with** $j = 1/2$

$SU(6)$-**multiplets unite different levels** with splittings $O(1)$

Quark model at $N_c \to \infty$

Quark model also includes **orbital angular momentum**(and "principal quantum number"). The complete symmetry of quark model is $SU(6) \otimes SO(3)$. The contents of few multiplets is

$$\{\mathbf{56}, \mathbf{0}\} = \mathbf{K} = \mathbf{0}, \text{ground level}$$

$$\{\mathbf{70}, \mathbf{1}\} = \begin{cases} \mathbf{K} = \mathbf{0}, & u, d - quarks \\ \mathbf{K} = \mathbf{1}, & u, d - quarks \\ \mathbf{K} = \mathbf{2}, & u, d - quarks \\ \mathbf{J} = \frac{1}{2}, & s - quarks \\ \mathbf{J} = \frac{3}{2}, & s - quarks \end{cases}$$

(Orbital momentum adds to the $K = 1$ or $J = 1/2$). Hence, this quark multiplet contains **5 levels in the soliton picture** splitted by $O(1)$. (*Cohen&Lebed, 2003*, large N_c approach)

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Spurios states are those **forbidden by Pauli principle**

Spurious states. Skyrme example

Three **spinless** and **flavorless** quarks in the **oscillator** potential.

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Mean field wave functions.

- $L = 0: \ \varphi_{\mathbf{0}}(\mathbf{r})$
- $L = 1: \ \varphi_1(r)_m = \tilde{\mathbf{x}}_{\mathbf{m}}\varphi_{\mathbf{0}}(\mathbf{r})$

Mean field wave function with **one quark excited to $L = 1$**is a product:

$$\Phi^1_{m.f.}(x_1, x_2, x_3) = \varphi_{\mathbf{1}}(\mathbf{r_1})\varphi_{\mathbf{0}}(\mathbf{r_2})\varphi_{\mathbf{0}}(\mathbf{r_3})$$

One has to take into account color and **symmetrize**

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$$\Phi_1(x_1, x_2, x_3) = \varepsilon^{c_1 c_2 c_3} Sym_{\mathbf{r_1},\mathbf{r_2},\mathbf{r_3}} \Phi^1_{m.f.}(x_1, x_2, x_3) =$$

$$= \varepsilon^{c_1 c_2 c_3}(\tilde{\mathbf{x}}_\mathbf{1} + \tilde{\mathbf{x}}_\mathbf{2} + \tilde{\mathbf{x}}_\mathbf{3})\varphi_0(\mathbf{r_1})\varphi_0(\mathbf{r_2})\varphi_0(\mathbf{r_3}) = \tilde{\mathbf{x}}_\mathbf{c}\Phi_0(x_1, x_2, x_3)$$

Spurious state!

Spurious states.

- If potential **not oscillator** it is not easy to judge what is spurious. We **deform** potential to oscillator and **trace** what state **becomes spurious**.

Spurious states.

- If potential **not oscillator** it is not easy to judge what is spurious. We **deform** potential to oscillator and **trace** what state **becomes spurious**.
- For **rotation**: we deform potential to **scalar, spherically symmetric** (**quark model** limit) and trace what states from **rotational bands** becoming **spurious**. Wave function obtained by **projection**.

Parity minus.

Following the **experience of quark model** we **assume** that following levels are present with **parity "-"**:

- **s-quark** excitation with $J = 1/2$. **No band.** $\{1, 1/2\}$
- **s-quark** excitation with $J = 3/2$. **No band.** $\{1, 3/2\}$
- **u,d-quark** excitation with $K = 0$. $\{8, 1/2\}$, $\{10, 3/2\}$
- **u,d-quark** with $K = 1$. $\{8, 1/2\}$, $\{8, 3/2\}$, $\{10, 1/2\}$, $\{10, 3/2\}$, $\{10, 5/2\}$
- **u,d-quark** with $K = 2$. $\{8, 3/2\}$, $\{8, 5/2\}$, $\{10, 1/2\}$, $\{10, 3/2\}$, $\{10, 5/2\}$,$\{10, 7/2\}$

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Spurious states labeled by green.

Parity minus.

- Singlet $\Lambda(1405)$ and $\Lambda(1520)$ belong to (**determine energy of levels**)
- $\{8, 1/2\}(1615)$ and $\{10, 3/2\}(1850)$ belong to $K = 0$. Predict mass of decuplet:

$$\mathcal{M}_1 0 = \mathcal{M}_8 + \frac{3}{2I_1} = \mathbf{1847}$$

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$$\mathcal{M}_10 = \mathcal{M}_8 + \frac{3}{2I_1} = \mathbf{1847}$$

- $\{8,1/2\}(1710)$ and $\{8,3/2\}(1890)$ and $\{10,1/2\}(1758)$ belong to $K=1$. Determine **energy** ε and coefficient c_K. **Predict decuplet mass** $= 1762$ **Mev**

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- $\{8,3/2\}(1680)$ and $\{8,5/2\}(1802)$ belong to $K=2$. Determine energy and c_K

No more particles!

Parity plus.

The situation in parity "+" is worse. The identification is based again on $K = 0, 1, 2$ levels

- $\{8, 1/2\}(1605)$ and $\{10, 3/2\}(1732)$ belong to $K = 0$
- $\{8, 3/2\}(1865)$ and $\{8, 5/2\}(1873)$ and $\{10, 3/2\}(2087)$ and $\{10, 5/2\}(2031)$ and $\{10, 7/2\}(2038)$ belong to $K = 2$ State $\{10, 1/2\}$ **is spurious**
- $\{8, 1/2\}(1846)$ belongs to $K = 1$. No other states are known except few nucleons and $\Delta's$.

Identification of levels

quark levels	rotational bands	$(I_1)^{-1}$, MeV	$\tilde{a}_K$
$K^P = 0^+$, ground state	$(\mathbf{8}, 1/2^+, 1152)$ $(\mathbf{10}, 3/2^+, 1382)$	153	
$0^+ \to 0^+$ 482 MeV	$(\mathbf{8}, 1/2^+, 1608)$ $(\mathbf{10}, 3/2^+, 1732)$	83	
$0^+ \to 2^+$ 722 MeV	$(\mathbf{8}, 3/2^+, 1865)$ $(\mathbf{8}, 5/2^+, 1873)$ $(\mathbf{10}, 3/2^+, 2087)$ $(\mathbf{10}, 5/2^+, 2071)$ $(\mathbf{10}, 7/2^+, 2038)$	131	-0.050
$0^+ \to 1^+$ $\sim$780 MeV	$N(1/2^+, 1710)$ $N(3/2^+, 1900)$ $\Delta(1/2^+, 1910)$ $\Delta(3/2^+, \sim 1945)$? $\Delta(5/2^+, 2000)$		
$0^+ \to 1^-$ 468 MeV	$(\mathbf{8}, 1/2^-, 1592)$ $(\mathbf{8}, 3/2^-, 1673)$ $(\mathbf{10}, 1/2^-, 1758)$ $(\mathbf{10}, 3/2^-, 1850)$	171	0.336
$0^+ \to 0^-$ 563 MeV	$(\mathbf{8}, 1/2^-, 1716)$	155(fit)	
$0^+ \to 2^-$ 730 MeV	$(\mathbf{8}, 3/2^-, 1896)$ $(\mathbf{8}, 5/2^-, 1801)$	155(fit)	-0.244

Mass relations.

General formula

$$\mathcal{H}_{m_s} = \alpha \mathcal{D}^{(8)}_{88}(R) + \beta Y + \gamma \mathcal{D}^{(8)}_{8i}(R)\tilde{T}_i + \delta \mathcal{D}^{(8)}_{8i}(R)\hat{K}_i$$

where $\mathcal{D}$ $SU(3)$ **Wigner functions**.

$$\mathbf{K=0}: \quad \mu^{(10)}\left(\frac{3}{2}\right) = \mu_2^{(8)}\left(\frac{1}{2}\right)$$

$$\mathbf{K=2}: \quad 5\mu^{(10)}\left(\frac{7}{2}\right) + 7\mu^{(10)}\left(\frac{3}{2}\right) = 12\mu^{(10)}\left(\frac{5}{2}\right),$$

$$5\mu_2^{(8)}\left(\frac{3}{2}\right) + 9\mu^{(10)}\left(\frac{5}{2}\right) = 14\mu^{(10)}\left(\frac{3}{2}\right)$$

$$5\mu_2^{(8)}\left(\frac{5}{2}\right) + 11\mu^{(10)}\left(\frac{3}{2}\right) = 16\mu^{(10)}\left(\frac{5}{2}\right),$$

Mass relations.

$$\mathbf{K=1}: \quad 7\mu^{(10)}\left(\frac{1}{2}\right)+3\mu_2^{(8)}\left(\frac{3}{2}\right)=10\mu^{(10)}\left(\frac{3}{2}\right),$$

$$5\mu^{(10)}\left(\frac{3}{2}\right)+3\mu_2^{(8)}\left(\frac{1}{2}\right)=8\mu^{(10)}\left(\frac{1}{2}\right)$$

fulfilled with very good accuracy

Decays.

Levels.

At last we get our **QCD data**. They are **energy of levels**:

+	u,d	K=0	0
-	s	J=1/2	255
-	s	J=3/2	370
-	u,d	K=0	458
-	u, d	K=1	586
-	u, d	K=2	774
+	u,d	K=0	483
+	u, d	K=1	$\sim$ 800
+	u, d	K=2	715

These levels can be provided by Dirac equation!

Conclusions.

- **One-quark excitations** of **c**an explain all existing baryons. Baryons are unified as **rotational bands** around levels with splitting $O(1/N_c)$
- Energies of every rotational band depends on the **universal** moment of inertia I_1 and coefficients a_k **individual for this band**. These coefficients are known if wave function of the given level are known.
- One needs **8 levels** to describe all baryons below 2 GeV. They have **natural quantum numbers**. **Model independent** consequences of the soliton picture seem to work.

Conclusions.

- **Quark model** is the limiting case of the soliton picture corresponding to spherically symmetrical scalar potential. Many levels which are normally **splitted by** $O(1)$ are assumed to be degenerate in the quark model. The **problems of quark model** are related to this assumption. The **hedgehog** symmetry seems to be **better** assumption.
- It is possible to calculate m_s **corrections** to the baryon masses. This will give opportunity to include into the scheme multiplets with only one or two members known. Mass corrections are calculable if wave functions are known. However, there many **symmetry relations** for mass corrections (**Guadagnini type**) which relate mass splittings inside rotational band. They also can **help to check identification** of multiplets.

Conclusions.

- **QCD with light quarks** is rather **bad** object for studying the confinement. Properties of hadrons are determined **mainly** by spontaneous breakdown of *chiral symmetry*.
  One has to switch to QCD **at non-zero temperatures** to **make confinement explicit**