Constraining new physics with SM effective potential

Marek Lewicki

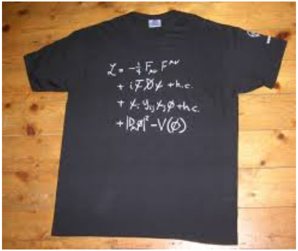
Institute of Theoretical Physics, Faculty of Physics, University of Warsaw

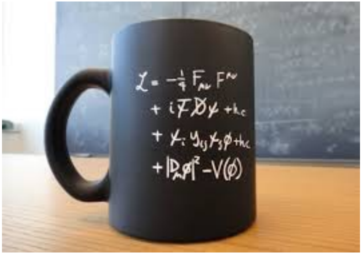
55 Cracow School of Theoretical Physics, 25 June 2015, Zakopane

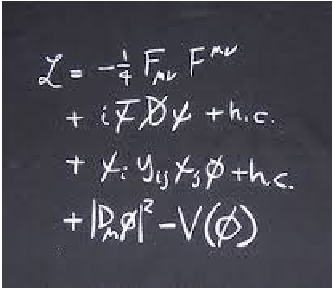
based on:
Z. Lalak, P. Olszewski and ML, JHEP **1405**, 119 (2014) arXiv:1402.3826
Z. Lalak, P. Olszewski and ML, arXiv:1505.05505.

The project "International PhD Studies in Fundamental Problems of Quantum Gravity and Quantum Field Theory" is realized within the MPD programme of Foundation for Polish Science, cofinanced from European Union, Regional Development Fund

Standard Model



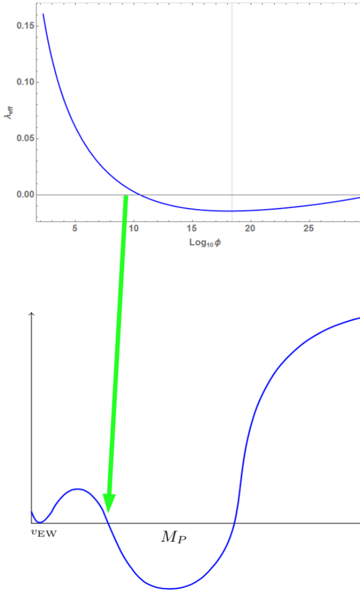




$$V_{SM}^{1-loop} = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$
$$+\sum_i \frac{n_i}{64\pi^2} M_i^4 \left[\ln\left(\frac{M_i^2}{\mu^2}\right) - C_i\right]$$

- For large field values
  $m^2 << \phi^2 \rightarrow M_i \propto \phi$
- Expansion under control if
  $\mu = \phi$

$$V_{SM}(\phi) \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4$$



SM Metastability

- Effective potential

$$\lambda_{eff} = \Bigg\{ \lambda + \frac{1}{(4\pi)^2} \Bigg[6 \left(\frac{g_2^2}{4}\right)^2 \left(\ln\left(\frac{g_2^2}{4}\right) - \frac{5}{6}\right)$$

$$+3 \left(\frac{g_1^2+g_2^2}{4}\right)^2 \left(\ln\left(\frac{g_1^2+g_2^2}{4}\right) - \frac{5}{6}\right)$$

$$-12 \left(\frac{y_t^2}{2}\right)^2 \left(\ln\left(\frac{y_t^2}{2}\right) - \frac{3}{2}\right)$$

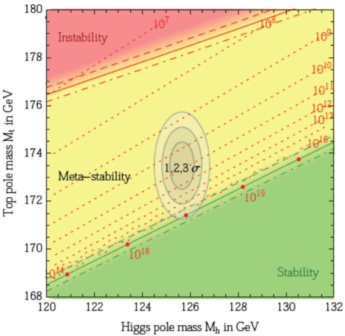
$$+ \left(\frac{3\lambda}{2}\right)^2 \left(\ln\left(\frac{3\lambda}{2}\right) - \frac{3}{2}\right) + 3\left(\frac{\lambda}{2}\right)^2 \left(\ln\left(\frac{\lambda}{2}\right) - \frac{3}{2}\right) \Bigg] \Bigg\}$$

- Running of λ

$$\frac{d\lambda}{d\ln\mu} = \frac{1}{(4\pi)^2} \Bigg[24\lambda^2 - 6{y_t}^4 - 6y_b^4 - 2y_\tau^4$$

$$+2\lambda\left(6{y_t}^2 + 6y_b^2 + 2y_\tau^2 - \frac{9}{2}g_2^2 - \frac{9}{10}g_1^2\right)$$

$$+\frac{9}{8}g_2^4 + \frac{27}{200}g_1^4 \frac{9}{20}g_1^2 g_2^2 \Bigg]$$

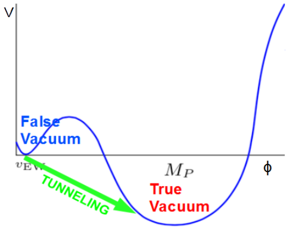


D. Buttazzo, et al. [arXiv:1307.3536].
G. Degrassi, et al. [arXiv:1205.6497].

Tunneling

- Vacuum decay proceeds through nucleation of true vacuum bubbles within false vacuum.

S. R. Coleman, Phys. Rev. D **15** (1977) 2929.
C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D **16** (1977) 1762.

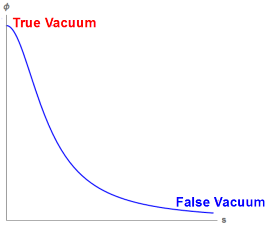


- Bubble: $O(4)$ symmetric solution of euclidean EOM:

$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi}, \quad s = \sqrt{\tau^2 + \vec{x}^2}.$$

with

- $\dot{\phi}(s = 0) = 0$ at the true vacuum
- $\phi(s = \infty) = \phi_{min}$ at the false vacuum



Tunneling

- Decay probability dp of a volume d^3x

$$dp = dt d^3x \frac{S_E^2}{4\pi^2} \left| \frac{det'[-\partial^2 + V''(\phi)]}{det[-\partial^2 + V''(\phi_0)]} \right|^{-1/2} e^{-S_E}.$$

- Action of the bounce solution

$$S_E = 2\pi^2 \int dss^3 \left(\frac{1}{2}\dot{\phi}^2(s) + V(\phi(s)) \right).$$

- Simplifying:
  - prefactor replaced with width of the barrier $\propto \phi^4(s=0)$
  - volume of the universe approximated by $T_U^3 = (10^{10}\mathrm{yr})^3$

Expected lifetime of the false vacuum ($p(\tau) = 1$):

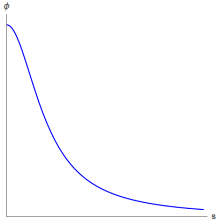
$$\frac{\tau}{T_U} = \frac{1}{\phi_0^4 T_U^4} e^{S_E}$$

Standard Model

- Anlytical solution for quartic potential
  (with $\lambda = \text{const} < 0$):

$$V(\phi) = \frac{\lambda}{4}\phi^4 \quad \Longrightarrow \quad S_E = \frac{8\pi^2}{3|\lambda|}$$

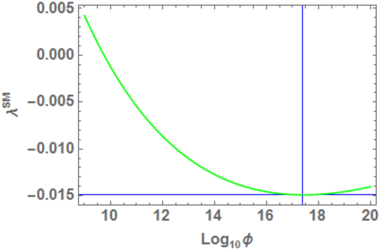
K. M. Lee and E. J. Weinberg, Nucl. Phys. B **267** (1986) 181.



- Approximating SM with a constant λ:

$$\frac{\tau}{T_U} = \frac{1}{\phi^4(\lambda_{min})\,T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}} \approx 10^{596},$$

we obtain a lower bound for ϕ that minimizes $\lambda_{eff}(\phi)$.



Gauge Dependence

- tree-level potential with one-loop RGEs

$$V \approx \underbrace{\lambda(\mu) Z_h^2}_{\lambda_{eff}(\mu)} \frac{h^4}{4}, \quad Z_h^{\frac{1}{2}} = e^{\Gamma}, \quad \Gamma(\mu) = \int_{M_t}^{\mu} \gamma(\bar{\mu}) \ln \bar{\mu}$$

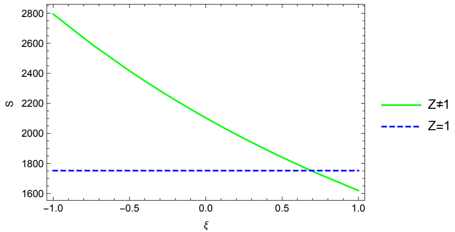
- RGE improvement induces gauge dependence

$$\gamma = \frac{1}{16\pi^2}\left(\frac{3}{20}\zeta g_1^2 + \frac{3}{4}\zeta g_2^2 + \frac{9}{4} g_2^2 + \frac{9}{20} g_1^2 - 3y_t^2 - 3y_b^2 - y_\tau^2\right)$$

L. Di Luzio and L. Mihaila, JHEP **1406** (2014) 079

- We need to normalize the field canonically

$$\mathcal{L} = \frac{1}{2}(\partial Z_h^{\frac{1}{2}} h)^2 + \frac{\lambda_{eff}}{4} Z_h^2 h^4 \xrightarrow{\tilde{h} = Z_h^{\frac{1}{2}} h} \mathcal{L} = \frac{1}{2}(\partial \tilde{h})^2 + \frac{\tilde{\lambda}_{eff}}{4} \tilde{h}^4 \Rightarrow \tilde{\lambda}_{eff} = \lambda_{eff}|_{Z_h=1}$$

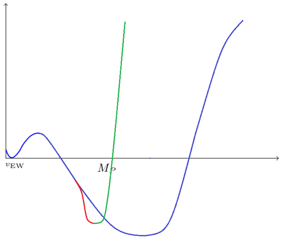


Effective potential with nonrenormalisable interactions

- Nonrenormalisable couplings modify the potential around the Planck scale ($\lambda_6 < 0$, $\lambda_8 > 0$):

$$V \approx \frac{\lambda_{eff}^{SM}(\phi)}{4}\phi^4 + \frac{\lambda_6}{6!}\frac{\phi^6}{M_p^2} + \frac{\lambda_8}{8!}\frac{\phi^8}{M_p^4},$$

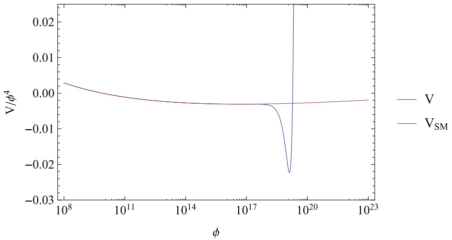
V. Branchina and E. Messina, Phys. Rev. Lett. **111** (2013) 241801.



- Simple quartic potential approximation:

$$\lambda_{eff}^{NEW}(\phi) = 4\frac{V}{\phi^4} =$$

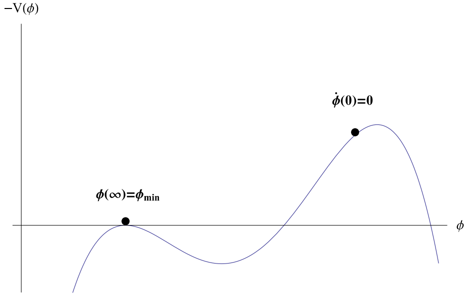
$$\lambda_{eff}^{SM}(\phi) + 4\frac{\lambda_6}{6!}\frac{\phi^2}{M_p^2} + 4\frac{\lambda_8}{8!}\frac{\phi^4}{M_p^4}.$$



Numerical calculations

$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi},$$

is an EOM of a particle in potential $-V(\phi)$ with a "time" dependent friction $\frac{3}{s}\dot{\phi}$.



Numerical vs Analytical

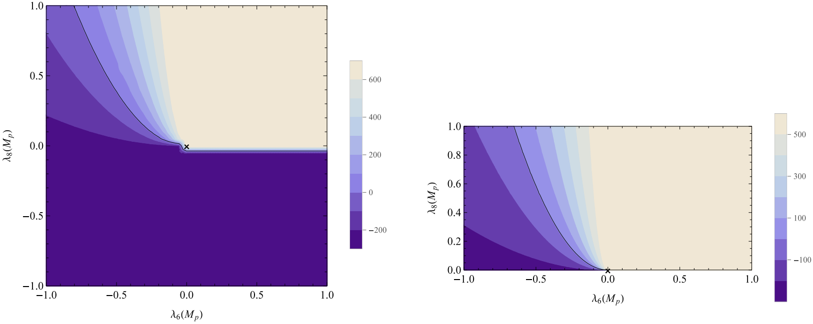


Figure: $\mathrm{Log}_{10}(\frac{\tau}{T_u})$ calculated numerically (left panel) and analytically (right panel).

RG improvement

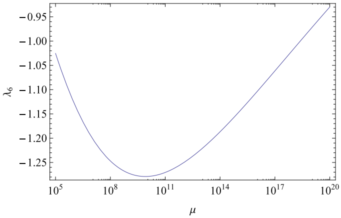
- Small correction to SM parameters

$$\Delta\beta_\lambda = \frac{\lambda_6}{16\pi^2}\frac{m^2}{M_p^2}.$$

- One-loop beta functions of new couplings

$$16\pi^2\beta_{\lambda_6} = \frac{10}{7}\lambda_8\frac{m^2}{M^2} - 6\lambda_6\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2 - 18\lambda\right),$$

$$16\pi^2\beta_{\lambda_8} = \frac{7}{5}28\lambda_6^2 - 8\lambda_8\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2 - \frac{45}{2}\lambda\right),$$



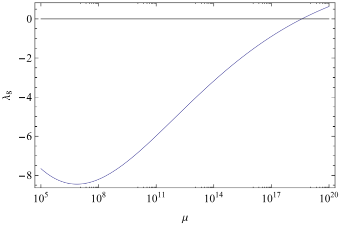


Figure: Example solution with $\lambda_6(M_p) = -1$ and $\lambda_8(M_p) = -0.1$.

Numerical vs Analytical with RG improvement

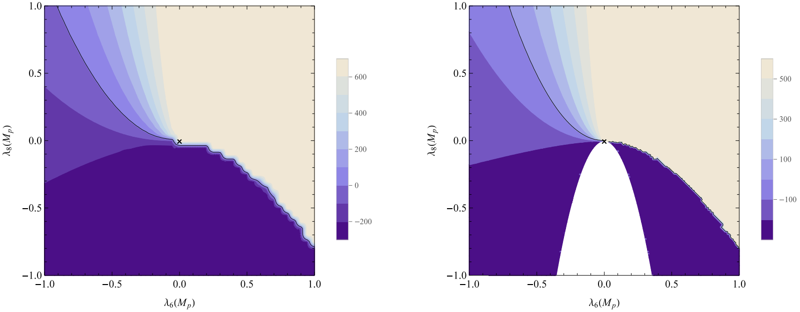


Figure: $\mathrm{Log}_{10}(\frac{\tau}{T_u})$ calculated numerically (left panel) and analytically (right panel).

Comparison

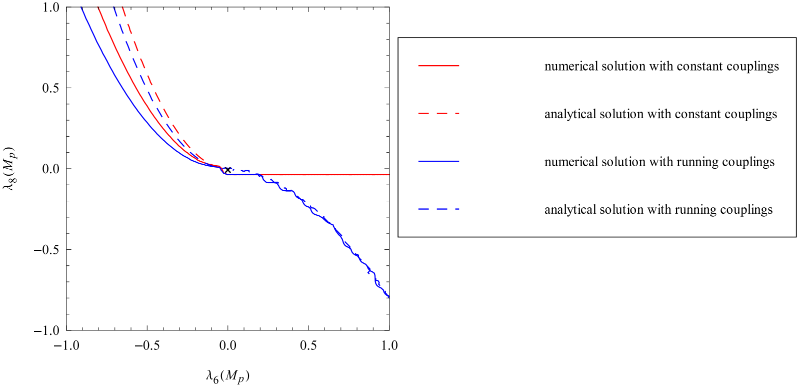
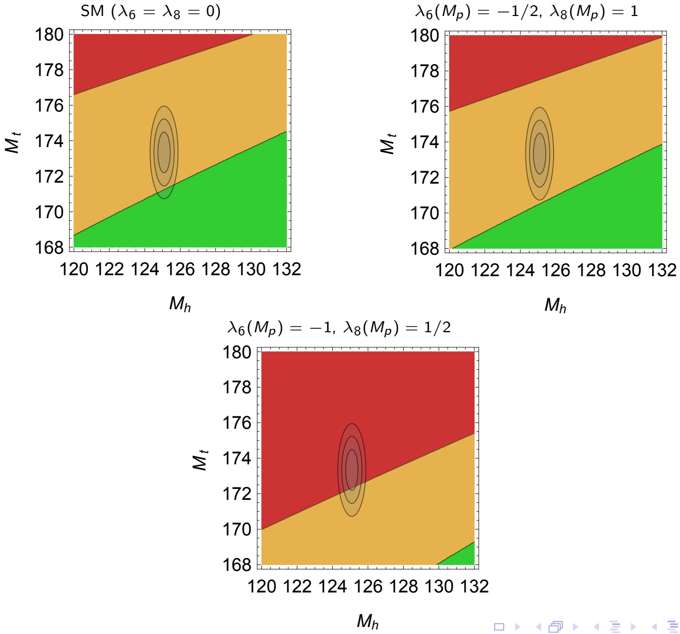


Figure: Metastability boundary ($\tau = T_u$) obtained using different methods.

SM phase diagram



Magnitude of the suppression scale

- Positive λ_6 and λ_8 $\rightarrow$ stabilizing the potential

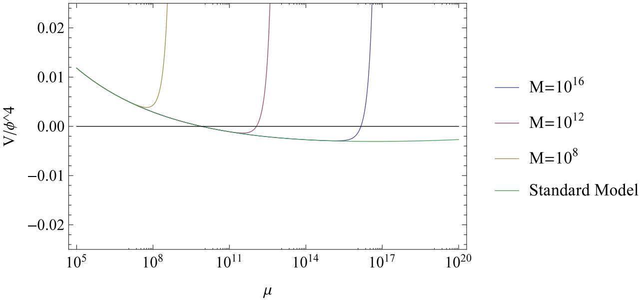


Figure: Scale dependence of λ_{eff}^{NEW} with $\lambda_6 = \lambda_8 = 1$ for different values of suppression scale M. The lifetimes are, respectively, $\log_{10}(\frac{\tau}{T_U}) = \infty, 1302, 641$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 596$.

- This method of probing new physics that result in absolute stability is gauge invariant which is not the case when using $M = \mu(\lambda = 0)$

A. Andreassen, W. Frost and M. D. Schwartz, Phys. Rev. Lett. **113** (2014) 24, 241801

- Positive λ_8 and negative λ_6 $\rightarrow$ New Minimum

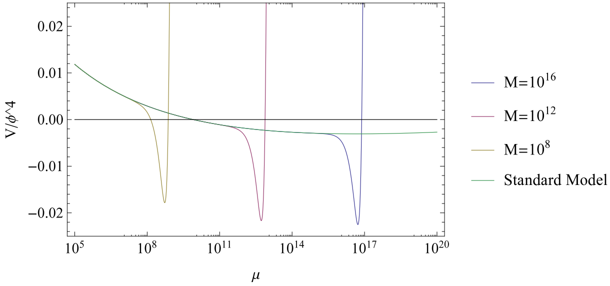


Figure: Scale dependence of λ_{eff}^{NEW} with $\lambda_6 = -1$ and $\lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$, are, respectively, $\log_{10}(\frac{\tau}{T_U}) = -45, -90, -110$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 596$.

Conclusions

- Gauge invariance can give important insights in to the vacuum lifetime calculation
- Analytical approximation of vacuum lifetime is qualitatively correct
- RG improvement stabilizes significant parts of the parameter space
- SM vacuum can be stabilized by new physics interactions only if they appear at low enough energy scale $\approx 10^{10} - 10^{11}$ GeV
- SM vacuum lifetime can be dramatically shortened by new physics at any scale