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Asymptotic freedom of gluons in the Fock space

Stanisław D. Głazek

Faculty of Physics, University of Warsaw

Asymptotic freedom of gluons is described in terms of a family of scale-dependent renormalized Hamiltonian operators acting in the Fock space. The Hamiltonians are obtained by applying the renormalization group procedure for effective particles (RG-PEP) to quantum $SU(3)$ Yang-Mills theory. The RGPEP is a general method for solving quantum field theories in the Minkowski space-time.

S.D.Głazek, *Dynamics of effective gluons*, Phys. Rev. D **63**, 116006 (2001).

S.D.Głazek, *Perturbative formulae for relativistic interactions of effective particles*, Acta Phys. Pol. B43, 1843, 20p (2012).

M.Gómez-Rocha, S.D.Głazek, *Asymptotic freedom in the front-form Hamiltonian for quantum chromodynamics of gluons*, arXiv:1505.06688.

stglazek@fuw.edu.pl

Outline:

explanation of asymptotic freedom of gluons in the Fock space

as an example of application of

the renormalization group procedure for effective particles (**RGPEP**)

as a new computational tool in quantum field theory

stglazek@fuw.edu.pl

In popular approaches to QCD, one way or the other,

one assumes that there exists some H_{QCD}.

$$H_{QCD} = ?$$

simplify

$$H_{YM} = ?$$

stglazek@fuw.edu.pl

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$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{2}\,\mathrm{tr}\,F^{\mu\nu}F_{\mu\nu} \\
F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu] \\
\mathcal{T}^{\mu\nu} &= -F^{a\mu\alpha}\partial^\nu A^a_\alpha + g^{\mu\nu} F^{a\alpha\beta} F^a_{\alpha\beta}/4 \\
P^\nu &= \int d\sigma_\mu \, \mathcal{T}^{\mu\nu}
\end{aligned}
$$

stglazek@fuw.edu.pl

$P^\nu = \int_\Sigma d\sigma_\mu \, \mathcal{T}^{\mu\nu}$ requires a choice of a hyperplane Σ

$$d\sigma^\mu \, = \, (1,0,0,0)^\mu \; d^3x \qquad \mathcal{T}^{00} \qquad H \, = \, P^0$$

$$d\sigma^\mu \, = \, (1,0,0,-1)^\mu \; \frac{1}{2} \, dx^- d^2x^\perp \qquad \mathcal{T}^{+-} \qquad H \, = \, P^-$$

$$A^\pm \, = \, A^0 \pm A^3 \qquad A^\perp \, = \, (A^1, A^2)$$

instant hyperplane $\leftrightarrow$ 6-dimensional invariance group

front hyperplane $\leftrightarrow$ 7-dimensional invariance group

stglazek@fuw.edu.pl

$$\mathcal{H}_{YM} = \mathcal{H}_{A^2} + \mathcal{H}_{A^3} + \mathcal{H}_{A^4}$$

$$\mathcal{H}_{A^3} = g\ i\partial_\alpha A^a_\beta\ [A^\alpha, A^\beta]^a$$

$$\text{quantization} \quad \hat{A}^\mu = \sum_{\sigma c} \int_k \left[t^c \varepsilon^\mu_{k\sigma} a_{k\sigma c} e^{-ikx} + t^c \varepsilon^{\mu *}_{k\sigma} a^\dagger_{k\sigma c} e^{ikx} \right]_{on\ \Sigma}$$

$$\left[a_{k\sigma c}, a^\dagger_{k'\sigma' c'} \right] = k^+ \tilde{\delta}(k - k')\ \delta^{\sigma\sigma'}\ \delta^{cc'}$$

$$\hat{H}_{YM} = \frac{1}{2} \int dx^- d^2 x^\perp\ :\mathcal{H}_{YM}(\hat{A}):$$

Is it it? No, it is not.

stglazek@fuw.edu.pl

Key example

$$\hat{H}_{A^3} = \int_\Sigma \; g \; : i\partial_\alpha \hat{A}^a_\beta [\hat{A}^\alpha, \hat{A}^\beta]^a :$$

$$\hat{A} \sim a + a^\dagger \qquad : \hat{A}^3 : \quad \sim \quad a^{\dagger 3} + a^{\dagger 2} a + a^\dagger a^2 + a^3$$

$$a_k |0\rangle = 0 \qquad a^\dagger_{k_1} a^\dagger_{k_2} a^\dagger_{k_3} |0\rangle = \, ? \qquad e^{-i\hat{H}t/\hbar} |0\rangle = \, ?$$

$$\Delta E^{(2)}_{|0\rangle} = \sum_{|3g\rangle} \frac{|\langle 3g|\hat{H}|0\rangle|^2}{-E_{3g}} = -\infty$$

$$\Delta E^{(2)}_{|g\rangle} = \sum_{|2g\rangle} \frac{|\langle 2g|\hat{H}|g\rangle|^2}{E_g - E_{2g}} = -\infty \qquad \text{etc.}$$

.

$$\hat{H}|G\,P\rangle = \frac{M_G^2 + P^{\perp\,2}}{P^+}\,|G\,P\rangle$$

$$P^2 = P^+P^- - P^{\perp\,2} = M_G^2$$

$$|G\,P\rangle = |gg\,P\rangle + |ggg\,P\rangle + |gggg\,P\rangle + ...$$

$$(P^+\hat{H} - P^{\perp\,2})|G\,P\rangle = M_G^2\,|G\,P\rangle$$

infinitely large range of momenta (locality)

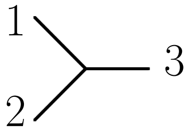
inifintely many components (products of more than 2 fileds)

Regularization

example $\hat{H}_{A^3} \to \hat{H}_{A^3\,R}$

$$\hat{H}_{A^3} = \sum_{123} \int [123] \delta(1+2-3) \left[g\, Y_{123}\, a_1^\dagger a_2^\dagger a_3 + g\, Y_{123}^*\, a_3^\dagger a_2 a_1 \right]$$

$$\to \hat{H}_{A^3\,R} = \sum_{123} \int [123] \delta(1+2-3)\; R\; \left[g\, Y_{123}\, a_1^\dagger a_2^\dagger a_3 + g\, Y_{123}^*\, a_3^\dagger a_2 a_1 \right]$$



$$x_1 = p_1^+/p_3^+ = x \qquad k_1 = p_1 - x_1 p_3 = \kappa$$

$$x_2 = p_2^+/p_3^+ = 1 - x \qquad k_2 = p_2 - x_2 p_3 = -\kappa$$

$$r(x_i, k_i^\perp) \;=\; x_i^\delta \; e^{-k_i^{\perp 2}/\Delta^2} \qquad i = 1, 2, 3 \qquad x_3 = 1 \qquad k_3^\perp = 0^\perp$$

$$R = r(x_1, k_1^\perp) r(x_2, k_2^\perp) r(1, 0^\perp) = x^\delta (1-x)^\delta e^{-2\kappa^2/\Delta^2}$$

$$Y_{123} = i f^{c_1 c_2 c_3} \left[\varepsilon_1^* \varepsilon_2^* \cdot \varepsilon_3 \kappa - \varepsilon_1^* \varepsilon_3 \cdot \varepsilon_2^* \kappa \frac{1}{x_2} - \varepsilon_2^* \varepsilon_3 \cdot \varepsilon_1^* \kappa \frac{1}{x_1} \right]$$

stglazek@fuw.edu.pl

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$$\hat{H}_R |G\,P\rangle = \frac{M_G^2 + P^{\perp\,2}}{P^+}\,|G\,P\rangle$$

$$|G\,P\rangle = |gg\,P\rangle + |ggg\,P\rangle + |gggg\,P\rangle + ...$$

$$(P^+ \hat{H}_R - P^{\perp\,2})|G\,P\rangle = M_G^2\,|G\,P\rangle$$

momenta limited by the regularization

need to remove effects of regularization $\rightarrow$

renormalization of Hamiltonians $\rightarrow$

scale-dependent effective theory, including bound states

bare gluons of size 0 $\qquad\qquad$ gluons of size s

$$\begin{bmatrix} \dots \\ |gggggg\rangle \\ |ggggg\rangle \\ |gggg\rangle \\ |ggg\rangle \\ |gg\rangle \end{bmatrix} = \begin{bmatrix} \dots \\ |ggggg\rangle \\ |gggg\rangle \\ |ggg\rangle \\ |gg\rangle \\ |g\rangle \end{bmatrix} \otimes \begin{bmatrix} \dots \\ |ggggg\rangle \\ |gggg\rangle \\ |ggg\rangle \\ |gg\rangle \\ |g\rangle \end{bmatrix} + \dots$$

$$|gg\rangle + |ggg\rangle + \dots = |g_s g_s\rangle + |g_s g_s g_s\rangle + \dots$$

stglazek@fuw.edu.pl

RGPEP $t = s^4$ drop hats

$$H_0(a_0) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, ..., i_n} c_0(i_1, ..., i_n)\ a^\dagger_{0i_1} \cdots a_{0i_n}$$

$$H_t(a_t) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, ..., i_n} c_t(i_1, ..., i_n)\ a^\dagger_{ti_1} \cdots a_{ti_n}$$

$$\boxed{c_0 = \text{initial condition, including } CT_R}$$

$$\boxed{c_t = ?}$$

stglazek@fuw.edu.pl

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\begin{aligned}
H_t(a_t) &= H_0(a_0) \\
a_t &= U_t\, a_0\, U_t^\dagger \\
H_t(a_0) &= U_t^\dagger H_0(a_0) U_t \\
H_t' &= \left[-U_t^\dagger U_t', H_t \right] = [G_t, H_t] \\
U_t &= T \exp\left(- \int_0^t d\tau\, G_\tau \right) \\
G_t &= [H_f, \tilde{H}_t]
\end{aligned}
$$

stglazek@fuw.edu.pl

RGPEP generator and non-perturbative QCD

$$G_t = [H_f, \tilde{H}_t]$$

$$H_f = \sum_i p_i^- \, a_{0i}^\dagger a_{0i} \qquad\qquad p_i^- = \frac{p_i^{\perp\,2}}{p_i^+}$$

$$H_t(a_0) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, \ldots, i_n} c_t(i_1, ..., i_n) \; a_{0i_1}^\dagger \cdots a_{0i_n}$$

$$\tilde{H}_t(a_0) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, \ldots, i_n} c_t(i_1, ..., i_n) \; \left(\sum_{k=1}^{n} p_{i_k}^+ / 2 \right)^2 \; a_{0i_1}^\dagger \cdots a_{0i_n}$$

stglazek@fuw.edu.pl

$$H'_t = \left[[H_f, \tilde{H}_t], H_t \right]$$

$$H_t = H_f + gH_{1t} + g^2 H_{2t} + g^3 H_{3t} + ...$$

$$H'_{1t} = \left[[H_f, \tilde{H}_{1t}], H_f \right]$$

$$H_{1t} = f_t \; H_{10}$$

$$f_t = e^{-t(\mathcal{M}_c^2 - \mathcal{M}_a^2)^2}$$

$$H_{A^3\,1t} = \sum_{123} \int [123] \delta(1+2-3) \; e^{-t\,\mathcal{M}_{12}^4} \; \left[Y_{123}\, a_1^\dagger a_2^\dagger a_3 + Y_{123}^*\, a_3^\dagger a_2 a_1 \right]$$

stglazek@fuw.edu.pl

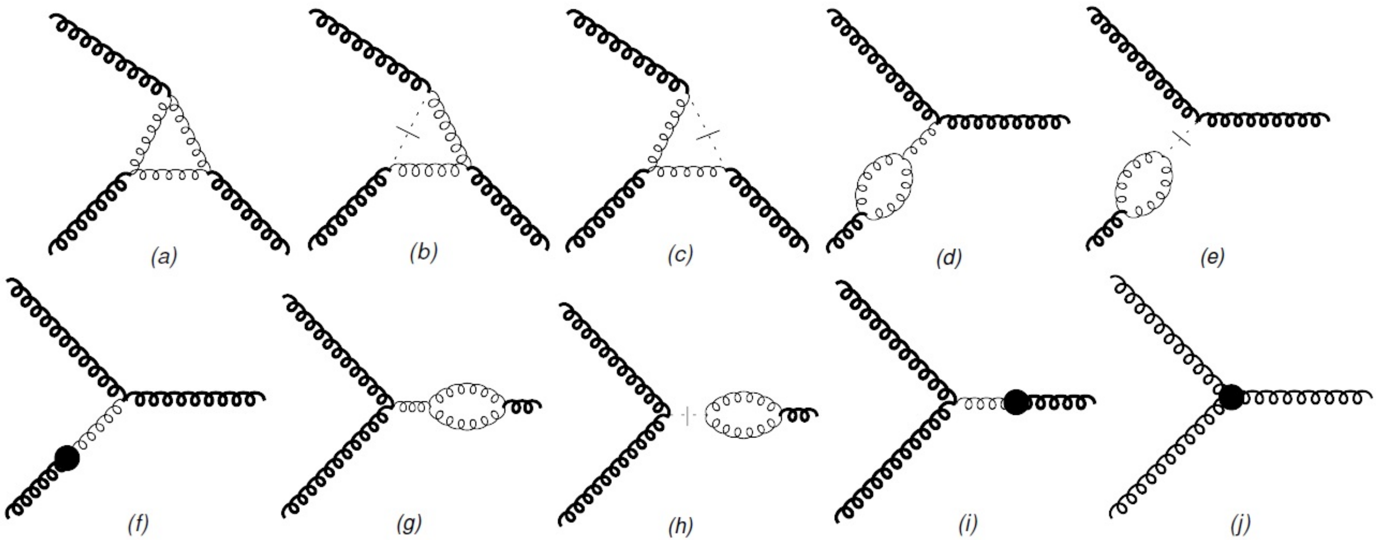

FIG. 1: Third-order contributions to the three-gluon vertex

$$H_{A^3\,(1+3)t} = \sum_{123} \int [123] \delta_{12.3} \; e^{-t\,\mathcal{M}_{12}^4} \; V_t(x, \kappa^\perp) \; \Big[Y_{123} \, a_1^\dagger a_2^\dagger a_3 + Y_{123}^* \, a_3^\dagger a_2 a_1 \Big]$$

$$g_t = V_t(x, 0^\perp)$$

$$g_t = g_0 + \frac{g_0^3}{48\pi^2} N_c \, 11 \, \ln \frac{s}{s_0} \qquad\qquad t = s^4$$

Hamiltonian $\beta(s)$ $\quad\leftrightarrow$ Gross-Wilczek-Politzer $\beta(\lambda)$

Minkowski s $\quad\leftrightarrow\quad$ $1/\lambda$ Euclid

stglazek@fuw.edu.pl

Hydrogen atom analogy

$$V_c = -\frac{\alpha_{atom}}{r}$$

$$\psi(\vec{r}\,) \sim e^{-\alpha_{atom}\mu|\vec{r}\,|}$$

$$\tilde{\psi}(\vec{k}\,) \sim \frac{1}{[\vec{k}^{\,2} + \alpha_{atom}^2\mu^2]^2}$$

$$\left[P^+\hat{H}_{QED}(s_{atom}) - P^{\perp\,2}\right]|\psi P\rangle = M_{atom}^2|\psi P\rangle$$

difference between QED and QCD

RGPEP(g_t^4) $\quad\rightarrow\quad$ hadrons

Conclusion

A. third-order RGPEP is just a beginning (4th order $\rightarrow$ breakthrough?)

B. simple generator $\rightarrow$ non-perturbative definition of QCD

C. two-fold universality:

1) Hamiltonian $\leftrightarrow$ $\overline{MS}$ Minkowski $s \leftrightarrow$ Euclid $1/\lambda$

2) different RGPEP generators

D. SM mass-generation issues $\sim$ QCD mass generation mechanism (?)

E. SM theory issues $\sim$ hierarchy, symmetry breaking, and lc (?)

stglazek@fuw.edu.pl