# Dark Matter Candidates

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- and does not couple to elm radiation

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Possible loophole: primordial black holes; not easy to make in sufficient quantity sufficiently early.

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- Precise “Planck” determination of DM density hinges on assumption of “standard cosmology”, including assumption of nearly scale–invariant primordial spectrum of density perturbations: almost assumes inflation!
- Evidence for $\Omega_{\rm DM} \gtrsim 0.2$ much more robust than that! (Does, however, assume standard law of gravitation.)
- No known model of gravity can explain early structure formation w/o introducing some sort of Dark Matter!

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$\Longrightarrow$ Use theoretical "prejudice" as guideline: Model should be simple and/or should solve some other problem!

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- Proliferation of WIMP candidates in recent years

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  - For WIMPs: Order of magnitude of $\Omega_{\rm DM}$ is understood; $\Omega_{\rm baryon}$ isn't

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- Early Universe was dominated by radiation! (Except in some extreme 'quintessence' or 'brane cosmology' models.)

# Thermal DM production

Let $\chi$ be a generic DM particle, $n_\chi$ its number density (unit: GeV$^3$). Assume $\chi = \bar{\chi}$, i.e. $\chi\chi \leftrightarrow$SM particles is possible, but single production of $\chi$ is forbidden by some symmetry.

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Evolution of $n_\chi$ determined by Boltzmann equation:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\rm ann} v \rangle \left( n_\chi^2 - n_{\chi,\,{\rm eq}}^2 \right)$$

$H = \dot{R}/R$ : Hubble parameter
$\langle \ldots \rangle$ : Thermal averaging
$\sigma_{\rm ann} = \sigma(\chi\chi \rightarrow {\rm SM\ particles})$
$v$ : relative velocity between $\chi$'s in their cms
$n_{\chi,\,{\rm eq}}$ : $\chi$ density in full equilibrium

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Check: creation and annihilation balance iff $n_\chi = n_{\chi,\,\rm eq}$.

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If interactions are negligible: $Y_\chi \to$ const., i.e. $\chi$ density in *co–moving* volume is unchanged ($\chi$ has decoupled)

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For $T \gtrsim 200$ MeV: $10 \lesssim \frac{4\pi\sqrt{g_*}}{\sqrt{90}} \lesssim 20$ (SM, MSSM)

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- For $T_R < m_\chi$: Easiest to satisfy for $T \simeq T_R$ (see: WIMP at low $T_R$).

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1) $n_\chi \simeq g_\chi \left( \frac{m_\chi T}{2\pi} \right)^{3/2} e^{-x}$

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2) Most of the time: can expand cross section in $\chi$ velocity:

$$\sigma_{\rm ann} v = a + bv^2 + \ldots \implies \langle \sigma_{\rm ann} v \rangle = a + 6\frac{b}{x} + \ldots$$

# Example 1: WIMP

Decouple (freeze out) at temperature $T \ll m_\chi$ (see below).
(N.B. Means $\chi$ makes *cold* DM!)

$\chi$'s are non–relativistic: two consequences

1) $n_\chi \simeq g_\chi \left(\frac{m_\chi T}{2\pi}\right)^{3/2} e^{-x}$

$$\langle \sigma_{\rm ann} v \rangle \simeq \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv\, v^2 (\sigma_{\rm ann} v) e^{-xv^2/4}$$

2) Most of the time: can expand cross section in $\chi$ velocity:

$$\sigma_{\rm ann} v = a + bv^2 + \ldots \implies \langle \sigma_{\rm ann} v \rangle = a + 6\frac{b}{x} + \ldots$$

Typically, $a, b \lesssim \frac{\alpha^2}{m_\chi^2}$, $\alpha^2 \sim 10^{-3}$, unless $a$ is suppressed by some symmetry; e.g. for $\tilde\chi\tilde\chi \to f\bar f:\ a \propto m_f^2$.

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Example: $\hat{a} = 0,\ \hat{b} = 10^{-4} \Longrightarrow$ need $T_R \simeq 0.04 m_\chi$

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For $T < T_F$: WIMP production negligible, only annihilation relevant in Boltzmann equation.

# Thermal WIMP: solution of Boltzmann eq.

$$\Omega_\chi h^2 \simeq \frac{8.7 \cdot 10^{-11}\ \mathrm{GeV}^{-2}}{\sqrt{g_*} J(x_F)}$$

$J(x_F) = \int_{x_F}^{\infty} dx \langle \sigma v \rangle / x^2$ “annihilation integral”.

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Non-relativistic expansion: $J(x_F) = \frac{a}{x_F} + \frac{3b}{x_F^2} \ldots$

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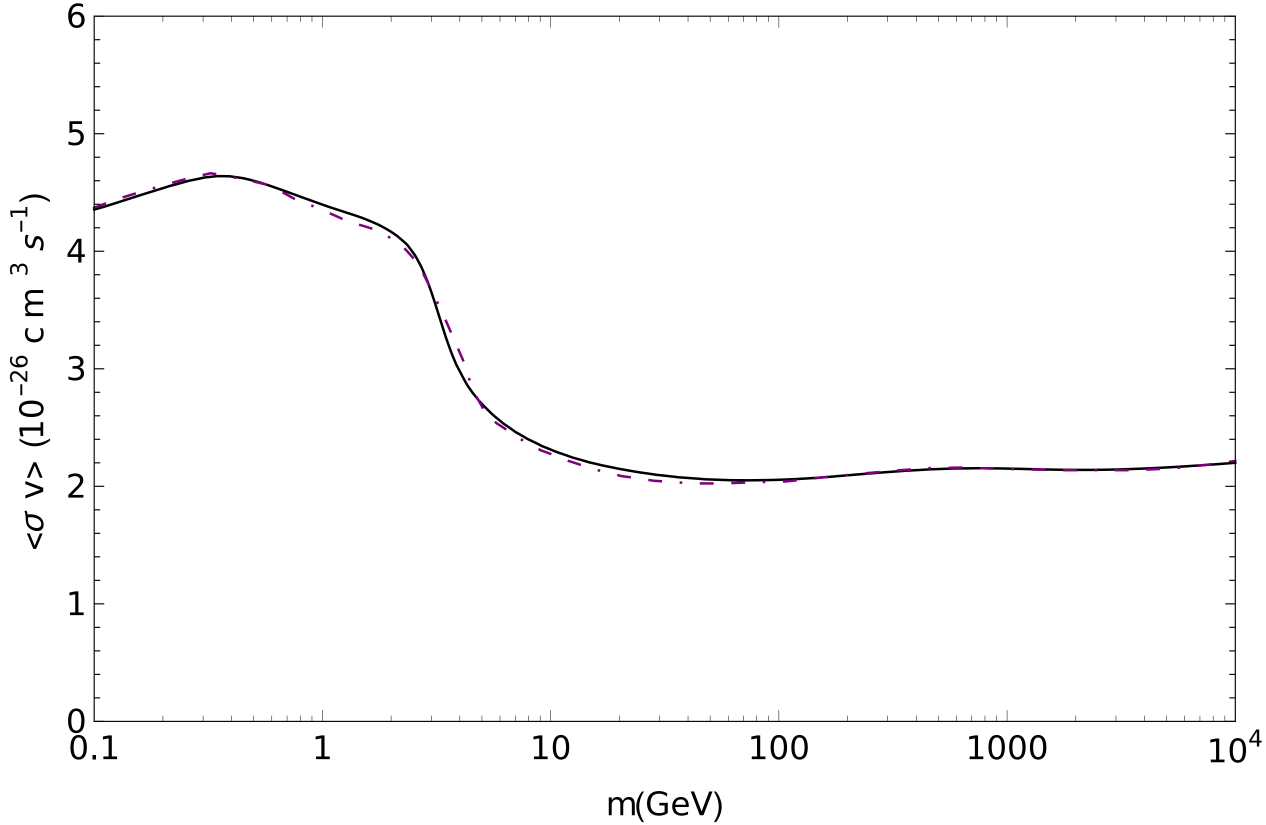
- Density scales like inverse of annihilation cross section: The stronger the WIMPs annihilate, the fewer are left.
- Smooth transition to previous case ($T_R < T_F$): MD, Iminniyaz, Kakizaki, hep-ph/0603165

# Recent numerical analysis ($b = 0$)

MD, F. Hajkarim, E. Rossi Schmitz, arXiv:1503.03513



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$\sigma(\tilde\chi\tilde\chi'),\ \sigma(\tilde\chi'\tilde\chi') \gg \sigma(\tilde\chi\tilde\chi)$ possible!

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Hall et al., arXiv:0911.1120

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- Final relic density proportional to cross section, independent of FIMP mass
- Needs small couplings: $\alpha^2 < \frac{m_\chi}{M_P}$

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# Thermal Gravitino Dark Matter

Each gravitino coupling gives factor $\frac{m_{\text{sparticle}} s}{m_{\tilde{G}} M_P}$ in cross section, if $m_{\tilde{G}} \ll \sqrt{s}, m_{\text{sparticle}}$

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$\tilde{G}$ annihilation can be ignored; write Boltzmann eq. for $\tilde{Y}_{\tilde{G}} \equiv n_{\tilde{G}} / n_\gamma$:

$$\frac{d\tilde{Y}_{\tilde{G}}}{dT} = - \frac{n_\gamma \sigma_{\tilde{G}}}{4 T H(T)}$$

# Gravitino DM (cont.'d)

Solution of Boltzmann eq.:

$\tilde{Y}_{\tilde{G},0} = \frac{4n_\gamma(T_R)\sigma_{\tilde{G}}}{4H(T_R)} \propto T_R$ (assuming $\tilde{Y}_{\tilde{G}}(T_R) = 0$)

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In general, have to add $\Omega_{\text{NLSP}} \frac{m_{\tilde{G}}}{m_{\text{NLSP}}}$ from (late) decays of NLSPs. (BBN!)

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Inflatons are non–relativistic when they decay.

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Energy conserved during $\phi$ decay

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# DM Production from Inflaton Decay (cont.'d)

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If $\chi$ production and annihilation at $T < T_R$ is negligible, universe evolves adiabatically:

$$\Longrightarrow \Omega_\chi h^2 = 2.1 \cdot 10^8 \frac{m_\chi}{m_\phi} \frac{T_R}{1~\mathrm{GeV}} B(\phi \to \chi)$$

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- Can be most important production mechanism for superheavy Dark Matter ($m_\chi \sim 10^{12}$ GeV) in chaotic inflation ($m_\phi \sim 10^{13}$ GeV); for LSP if $T_R \lesssim 0.03 m_\chi$; ...

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- Production from inflaton decay: $\Omega_\chi h^2 \propto \frac{m_\chi T_R B(\phi \rightarrow \chi)}{m_\phi}$.
- Only the thermal WIMP scenario can be tested using collider data and results from WIMP search experiments. Other scenarios can only be tested with additional input to constrain cosmology ($T_R, \dots$).