Radiative decays of resonances in lattice QCD

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Zakopane, June 21, 2015

AA, V. Bernard, U.-G. Meißner, A. Rusetsky, Nucl. Phys. B 886 (2014)

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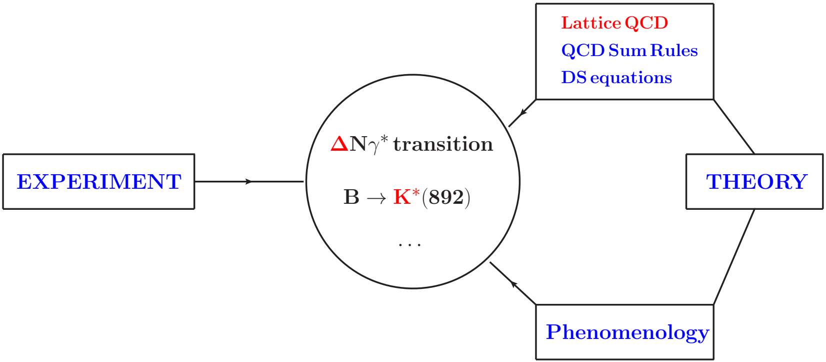
Outline

- Introduction: the $\Delta N\gamma^*$ transition
- Motivation
- Goals, results
- The $B \to K^*\gamma^*$: work in progress
- Outlook

Introduction

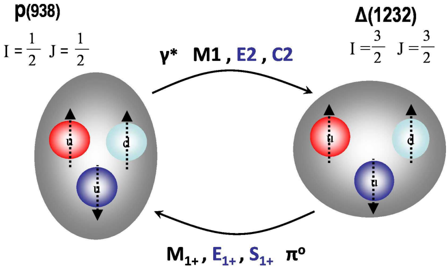


- $\gamma^* N \to \pi N$ near the $\Delta(1232)$: study of the hadron deformation
- $B \to K^* \gamma^*$, $B \to K^* l^+ l^-$ with $K^*(892) \to K\pi$: sensitive to NP

$\gamma^* N \rightarrow \Delta$

$$\gamma^* p \rightarrow \Delta^+(1232) \rightarrow p\pi^0 \quad (66\%)$$
$$\gamma^* p \rightarrow \Delta^+(1232) \rightarrow n\pi^+ \quad (33\%)$$
$$\gamma^* p \rightarrow \Delta^+(1232) \rightarrow p\gamma \quad (0.56\%)$$

$\hookrightarrow$ study of the de-excitation radiation pattern



Spherical $\Rightarrow$ **M1**

Deformed $\Rightarrow$ **M1 , E2 , C2**

C. Alexandrou et al., Rev. Mod. Phys. **84** (2012) 1231

- The electromagnetic transition matrix element:

$$\langle \Delta(P,\lambda)|J_\mu(0)|N(Q,\epsilon)\rangle = \left(\frac{2}{3}\right)^{1/2} \bar{u}^\sigma(P,\lambda)\mathcal{O}_{\sigma\mu}u(Q,\epsilon)\,,$$

with the Lorentz-structure

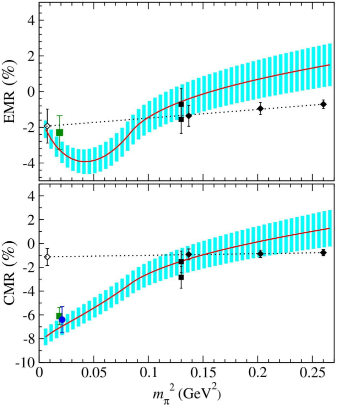
$$\mathcal{O}_{\sigma\mu} = G_M(Q^2)K^{M1}_{\sigma\mu} + G_E(Q^2)K^{E2}_{\sigma\mu} + G_C(Q^2)K^{C2}_{\sigma\mu}$$

H. F. Jones and M. D. Scadron, Ann. Phys. **81** (1973) 1

$$\mathrm{EMR} \equiv \frac{\mathrm{Im}E^{3/2}_{1+}}{\mathrm{Im}M^{3/2}_{1+}} = -\frac{G_E(Q^2)}{G_M(Q^2)}, \quad \mathrm{CMR} \equiv \frac{\mathrm{Im}L^{3/2}_{1+}}{\mathrm{Im}M^{3/2}_{1+}} = -\frac{|\vec{Q}|}{2m_\Delta}\frac{G_C(Q^2)}{G_M(Q^2)}$$

$\hookrightarrow$ extracted from experiment   calculated on the lattice $\hookleftarrow$

The values of EMR and CMR



V. Pascalutsa, M. Vanderhaeghen, Phys. Rev. D **73** (2006) 034003

- The Δ is treated as a stable particle!

↪ *Note*: the chiral extrapolation is not reliable for $m_\pi \gtrsim 300$ MeV!

Motivation

The largest conceptual question as we enter the chiral regime in full QCD, is how to fully incorporate the physical effect of the decay of the Δ into a pion and nucleon on the transition form factors.

C. Alexandrou et al., Phys. Rev. D **77** (2008) 085012

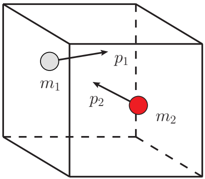
Methods

- How to study resonances/bound states on the lattice? $\rightarrow$ Lüscher approach
- How to find electroweak process amplitude/form factors? $\rightarrow$ Lellouch-Lüscher method
- How to build a bridge: two particle spectrum $\leftrightarrow$ scattering sector? $\rightarrow$ non-relativistic EFT/Bethe-Salpeter equation

Lüscher approach

- scattering phase shift $\leftrightarrow$ finite volume energy spectrum

M. Lüscher, Nucl. Phys. B **354** (1991) 531.



$\cot\delta_0(p) = -\cot\phi(q)$ (Lüscher equation)

$\phi(q)$ – known function

$q = \frac{pL}{2\pi}, \quad p^2 = \lambda(s, m_1^2, m_2^2)/4s$

$p_R \cot\delta_0(p_R) = -\frac{1}{a_0} + \frac{1}{2} r_0 p_R^2 + \cdots = -ip_R$

▷ lattice data ⇒Lüscher equation ⇒ scattering phase

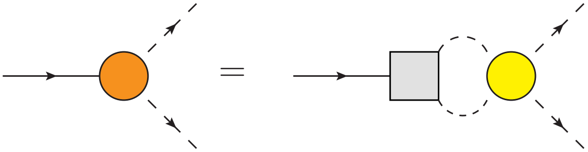
▷ fit effective-range expansion parameters

▷ analytic continuation to the resonance position p_R

Lellouch-Lüscher method

- Seminal work on $K \to \pi\pi$ by

L. Lellouch and M. Lüscher, Commun.Math.Phys. **219**, 31 (2001)



$$|\langle \pi\pi | H_{\rm weak} | K \rangle|_{\rm finite\ volume} \propto |A(K \to \pi\pi)| \times \left(\frac{p^2}{\delta'(p) + \phi'(q)} \right)^{-1/2}$$

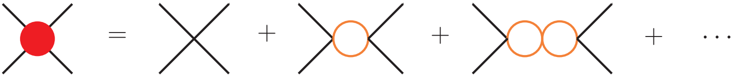
$\hookrightarrow$ *Note*: both methods are valid only

▷ for sufficiently big volumes $L > 2R_{\rm int}$ $(m_\pi L \gtrsim 4)$

▷ below 3(4)-particle threshold (low 3-momenta)

The framework: non-relativistic EFT

- Ideally suited for the problem we study:
  - ▷ the total number of heavy particles is conserved
  - ▷ manifestly Lorentz-invariant formulation is possible
  - ▷ the theory is matched to the full QFT (e.g., ChPT)



Bubble-chain diagrams

$$T \propto 1 + cJ + c^2 J^2 + \cdots = \frac{1}{1 - cJ}$$

- A bridge: finite volume spectrum $\leftrightarrow$ scattering sector

G. Colangelo, J. Gasser, B. Kubis, A. Rusetsky, Phys. Lett. B **638** (2006) 187

J. Gasser, B. Kubis, A. Rusetsky, Nucl. Phys. B **850** (2011) 96

Goals, results

- Previous work:

  D. Hoja, U.-G. Meißner, A. Rusetsky, JHEP 1004 (2010) 050

  V. Bernard, D. Hoja, U.-G. Meißner, A. Rusetsky, JHEP 1209 (2012) 023

  $\hookrightarrow$ scalar resonance form factor in the external scalar field (analog: $\Delta\Delta\gamma^*$)

- The $\Delta N\gamma^*$ transition:

  ▷ inclusion of *spin*;

  ▷ generalization to *transition* form factors.

Kinematics

- The Δ is at rest $\mathbf{P} = 0$ and nucleon momentum $\mathbf{Q}$ along 3-axis
- rotational symmetry $\to$ cubic symmetry: choose some irrep(s)
- Irreps G_1, G_2: no $S - P$–wave mixing (the P_{31} wave is small)

$$p \cot \delta(p) = \frac{2}{\sqrt{\pi} L} \left\{ \hat{Z}_{00}(1; q^2) \pm \frac{1}{\sqrt{5} q^2} \hat{Z}_{20}(1; q^2) \right\} \equiv -p \cot \phi(q), \qquad q = \frac{pL}{2\pi}$$

$\hookrightarrow$ Lüscher eq. for the P_{33} wave: M. Göckeler, Phys. Rev. D **86** (2012) 094513

- *Note*: in practice, the case $\mathbf{P} \neq 0$ could be preferable

$\hookrightarrow$ must be considered separately

Spin

- Introduce $\mathcal{O}^\mu(\mathbf{x},t)$ - the Δ field, $\psi(\mathbf{x},t)$ - the nucleon field
- Spin-projecting local operators:

$$
\begin{aligned}
G_2 \qquad \mathcal{O}_{3/2}(t) &= \sum_{\mathbf{x}} \frac{1}{2}(1+\Sigma_3)\frac{1}{2}(1+\gamma_4)\frac{1}{\sqrt{2}}\left(\mathcal{O}^1(\mathbf{x},t) - i\Sigma_3\mathcal{O}^2(\mathbf{x},t)\right), \\
G_2 \qquad \mathcal{O}_{1/2}(t) &= \sum_{\mathbf{x}} \frac{1}{2}(1-\Sigma_3)\frac{1}{2}(1+\gamma_4)\frac{1}{\sqrt{2}}\left(\mathcal{O}^1(\mathbf{x},t) + i\Sigma_3\mathcal{O}^2(\mathbf{x},t)\right), \\
G_1 \qquad \tilde{\mathcal{O}}_{1/2}(t) &= \sum_{\mathbf{x}} \frac{1}{2}(1+\Sigma_3)\frac{1}{2}(1+\gamma_4)\mathcal{O}^3(\mathbf{x},t), \\
\bar{\psi}^{\mathbf{Q}}_{\pm 1/2}(t) &= \sum_{\mathbf{x}} e^{i\mathbf{Q}\mathbf{x}}\bar{\psi}(\mathbf{x},t)\frac{1}{2}(1\pm\Sigma_3)\frac{1}{2}(1+\gamma_4), \qquad \Sigma_3 = \mathrm{diag}(\sigma_3,\sigma_3)
\end{aligned}
$$

$\Rightarrow$ three $\Delta N\gamma^*$ form factors are separately projected out

$\hookrightarrow$ similar procedure can be applied to fields with other spin

Extraction of the form factors

- on the real energy axis $\rightarrow$ model- and process-dependent

I. G. Aznauryan, V. D. Burkert and T. -S. H. Lee, arXiv:0810.099

D. Drechsel,O. Hanstein,S.S. Kamalov,L. Tiator, Nucl. Phys. A 645 (1999) 145

- at the resonance pole $\rightarrow$ *process-independent* $\Rightarrow$ favourable!

  $\triangleright$ definition of the resonance matrix element:

$$\langle P, \mathrm{resonance}|J(0)|Q, \mathrm{stable}\rangle = \lim_{P^2\to s_R,\, Q^2\to M^2} Z_R^{-1/2} Z^{-1/2}(s_R-P^2)(M^2-Q^2)F(P,Q)$$

$\hookrightarrow$ analytic continuation to the pole is unavoidable

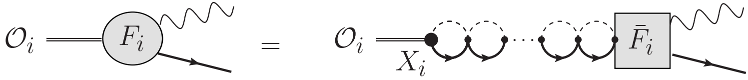
S. Mandelstam, Proc. Roy. Soc. Lond. A **233** (1955) 248.

- infinitely narrow resonances $\rightarrow$ identical results

$\hookrightarrow$ however, discrepancies in the results could be sizable:

R. L. Workman, L. Tiator and A. Sarantsev, Phys. Rev. C **87** (2013) 6, 068201

$\Delta N\gamma^*$ vertex: real axis



- ▷ The $F_i = F_i(p_n, |\mathbf{Q}|)$, i=1, 2, 3 → G_M, G_E, G_C form factors, are measured on *the lattice*
- ▷ The $\bar{F}_i(p_n, |\mathbf{Q}|)$ are *volume-independent* irreducible amplitudes

- The $\bar{F}_i$ are related to the $\gamma^* N \to \pi N$ multipole amplitudes

$$\mathcal{A}_i(p, |\mathbf{Q}|) = e^{i\delta(p)} \cos\delta(p)\, \bar{F}_i(p, |\mathbf{Q}|)$$

↪ Watson's theorem

$$\boxed{\mathcal{A}_i(p_n, |\mathbf{Q}|) = e^{i\delta(p_n)}\, V^{1/2} \left(\frac{1}{|\delta'(p_n) + \phi'(q_n)|} \frac{p_n^2}{2\pi} \right)^{-1/2} |F_i(p_n, |\mathbf{Q}|)|}$$

↪ LL equation for the photoproduction amplitude in the elastic region

- The $\mathcal{A}_i \leftrightarrow$ multipoles:

$$\tilde{\mathcal{A}}_{1/2} = -16\pi i E \sqrt{2} \frac{\sqrt{Q^2}}{|\mathbf{Q}|} S_{1+},$$
$$\mathcal{A}_{1/2} = -\frac{1}{2}(3E_{1+} + M_{1+})(-16\pi i E),$$
$$\mathcal{A}_{3/2} = \frac{\sqrt{3}}{2}(E_{1+} - M_{1+})(-16\pi i E).$$

- The narrow width approximation:

$$|\mathrm{Im}\,\mathcal{A}_i(p_A, |\mathbf{Q}|)| = \sqrt{\frac{8\pi}{p_A \Gamma}} |F_i^A(p_A, |\mathbf{Q}|)|,$$

$p = p_A$ - Breit-Wigner pole, $F_i^A(p_A, |\mathbf{Q}|) \to \Delta N \gamma^*$ form factors

I. G. Aznauryan, V. D. Burkert and T. -S. H. Lee, arXiv:0810.099

Complex plane

- The $\Delta N\gamma^*$ matrix elements, evaluated at the pole $p = p_R$:

$$\boxed{F_i^R(p_R, |\mathbf{Q}|) = Z_R^{1/2}\, \bar{F}_i(p_R, |\mathbf{Q}|)}$$

$$Z_R = \left(\frac{p_R}{8\pi E_R}\right)^2 \left(\frac{16\pi p_R^3 E_R^3}{w_{1R} w_{2R}(2p_R h'(p_R^2) + 3ip_R^2)}\right), \quad w_{iR} = \sqrt{m_i^2 + p_R^2}$$

$$p^3 \cot\delta(p) \doteq h(p^2) = -\frac{1}{a} + \frac{1}{2}\, rp^2 + \cdots, \quad h(p_R^2) = -ip_R^3$$

▷ The narrow width approximation ($p_R \to p_n$):

$$\boxed{F_i^R(p_R, |\mathbf{Q}|) \to F_i^A(p_A, |\mathbf{Q}|) \quad \text{as} \quad p_R \to p_A\,!}$$

$$F_i^R(p_n, |\mathbf{Q}|) = V^{1/2}\left(\frac{E_n}{2w_{1n}w_{2n}}\right)^{1/2} F_i(p_n, |\mathbf{Q}|)$$

↪ proper normalization of states

Prescription on the lattice

- Measure the $F_i(p, |\mathbf{Q}|)$ at different values of p with $|\mathbf{Q}|$ fixed.
- Real energy: extract the multipole amplitudes (*see above*).
- Extraction of the matrix elements at the resonance pole:
  1. fit the functions $p^3 \cot\delta(p)\,\bar{F}_i(p, |\mathbf{Q}|)$

$$p^3 \cot\delta(p)\,\bar{F}_i(p, |\mathbf{Q}|) = A_i(|\mathbf{Q}|) + p^2 B_i(|\mathbf{Q}|) + \cdots$$

  2. evaluate the resonance matrix elements by substitution

$$F_i^R(p_R, |\mathbf{Q}|) = i\, p_R^{-3}\, Z_R^{1/2}\big(A_i(|\mathbf{Q}|) + p_R^2 B_i(|\mathbf{Q}|) + \cdots\big)$$

$\hookrightarrow$ *Note*: no additional computation to find the $F_i^R(p_R, |\mathbf{Q}|)$

$B \to K^*\gamma^*$: work in progress

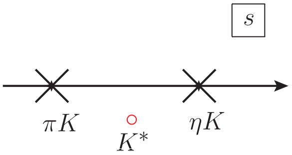
- Recent lattice measurement:

R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, Phys. Rev. D **89**, 094501 (2014)

$\hookrightarrow$ consistent with LCSR determinations

- Related work: multichannel LL equation, kinematics

R. A. Briceño et al., Phys.Rev. D **91** (2015) 3, 034501



- The 2-channel Lellouch-Lüscher formula is reproduced
- The form factor at the pole is extracted in the 2-channel problem
- The infinitely narrow resonance limit is considered

Outlook

- Complete the work on the $B \to K^* \gamma^*$ process
- Application to the electromagnetic form factors of the $\Lambda(1405)$-resonance

B. J. Menadue et al., PoS LATTICE **2013**, 280 (2014)

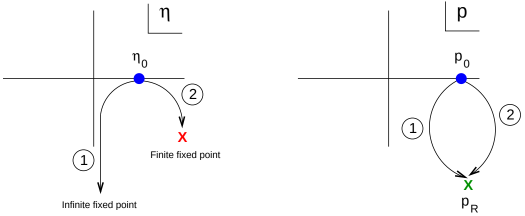
Thank you!

Finite fixed points ($\Delta\Delta\gamma^*$,...)

- The corresponding integral I: V. Bernard, et. al., JHEP 1209 (2012) 023

$$I = I_\infty + \frac{1}{32\pi Ep}\left(1+\cot^2\delta(p)\right)\eta\phi'(\eta)\Big)\Big|_{p=p_n}, \quad \eta = \frac{pL}{2\pi}$$

- $p \to p_R$: $\quad Z_{00}(1;\eta_R^2) + i\pi^{3/2}\eta_R = 0 \to \exists\ \eta_R : |\mathrm{Im}\,\eta_R| < \infty$

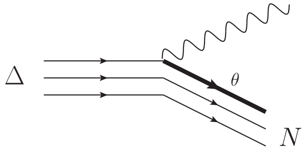


- Now, $\cot\phi(\eta)+i \ \propto\ \eta-\eta_R$ and $\phi(\eta)\propto \ln(\eta-\eta_R) \Rightarrow$

$$(\cot\phi(\eta)+i)\phi'(\eta) \to \mathrm{const} \neq 0$$

- Solution: perform the measurement at two energy levels

Kinematics



- The Δ is at rest $\mathbf{P} = 0$ and nucleon momentum $\mathbf{Q}$ along 3-axis.
- To perform the fit (*see below*): vary p, while $|\mathbf{Q}|$ fixed.
  - ▷ (partially) twisted boundary conditions;

↪ nucleon form factors: M. Göckeler et al., PoS LATTICE **2008** (2008) 138

  - ▷ asymmetric boxes $L \times L \times L'$. Alexandru, Döring
- *Note*: in practice, the case $\mathbf{P} \neq 0$ could be preferable.

Lattice correlators

- The two-point functions:

$$\begin{aligned}
\tilde{D}_{1/2}(t) &= \text{Tr}\,\langle 0|\tilde{\mathcal{O}}_{1/2}(t)\bar{\tilde{\mathcal{O}}}_{1/2}(0)|0\rangle\,, \\
D_{1/2}(t) &= \text{Tr}\,\langle 0|\mathcal{O}_{1/2}(t)\bar{\mathcal{O}}_{1/2}(0)|0\rangle\,, \\
D_{3/2}(t) &= \text{Tr}\,\langle 0|\mathcal{O}_{3/2}(t)\bar{\mathcal{O}}_{3/2}(0)|0\rangle\,, \\
D^{\pm}_{\mathbf{Q}}(t) &= \text{Tr}\,\langle 0|\psi^{\mathbf{Q}}_{\pm 1/2}(t)\bar{\psi}^{\mathbf{Q}}_{\pm 1/2}(0)|0\rangle.
\end{aligned}$$

- The three-point functions:

$$\begin{aligned}
\tilde{R}_{1/2}(t',t) &= \langle 0|\tilde{\mathcal{O}}_{1/2}(t')J^3(0)\bar{\psi}^{\mathbf{Q}}_{1/2}(t)|0\rangle\,, \\
R_{1/2}(t',t) &= \langle 0|\mathcal{O}_{1/2}(t')J^+(0)\bar{\psi}^{\mathbf{Q}}_{-1/2}(t)|0\rangle\,, \\
R_{3/2}(t',t) &= \langle 0|\mathcal{O}_{3/2}(t')J^+(0)\bar{\psi}^{\mathbf{Q}}_{1/2}(t)|0\rangle,
\end{aligned}$$

where

$$J^+(0) = \frac{1}{\sqrt{2}}\,(J^1(0) + iJ^2(0)).$$

$\Longrightarrow$

- In the limit $t' \to +\infty$, $t \to -\infty$:

$$\mathcal{N}\frac{\mathrm{Tr}\,(\tilde{R}_{1/2}(t',t))}{\tilde{D}_{1/2}(t'-t)}\left(\frac{D^+_{\mathbf{Q}}(t')\tilde{D}_{1/2}(-t)\tilde{D}_{1/2}(t'-t)}{\tilde{D}_{1/2}(t')D^+_{\mathbf{Q}}(-t)D^+_{\mathbf{Q}}(t'-t)}\right)^{1/2} \quad\to\quad \langle 1/2|J^3(0)|1/2\rangle\,,$$

$$-\mathcal{N}\frac{\mathrm{Tr}\,(R_{1/2}(t',t))}{D_{1/2}(t'-t)}\left(\frac{D^-_{\mathbf{Q}}(t')D_{1/2}(-t)D_{1/2}(t'-t)}{D_{1/2}(t')D^-_{\mathbf{Q}}(-t)D^-_{\mathbf{Q}}(t'-t)}\right)^{1/2} \quad\to\quad \langle 1/2|J^+(0)|-1/2\rangle\,,$$

$$-\mathcal{N}\frac{\mathrm{Tr}\,(R_{3/2}(t',t))}{D_{3/2}(t'-t)}\left(\frac{D^+_{\mathbf{Q}}(t')D_{3/2}(-t)D_{3/2}(t'-t)}{D_{3/2}(t')D^+_{\mathbf{Q}}(-t)D^+_{\mathbf{Q}}(t'-t)}\right)^{1/2} \quad\to\quad \langle 3/2|J^+(0)|1/2\rangle,$$

where $\mathcal{N} = (4E\sqrt{m_N^2+\mathbf{Q}^2})^{1/2}$. $\qquad\qquad \|$

F_i, i=1, 2, 3

$\Rightarrow$ three $\Delta N\gamma^*$ form factors are separately projected out.

$\hookrightarrow$ similar procedure can be applied to fields with other spin

Real axis

- The $\bar{F}_i$ are related to the $\gamma^* N \to \pi N$ multipole amplitudes

$$\mathcal{A}_i(p,|\mathbf{Q}|) = e^{i\delta(p)} \cos\delta(p)\, \bar{F}_i(p,|\mathbf{Q}|)$$

$\hookrightarrow$ Watson's theorem

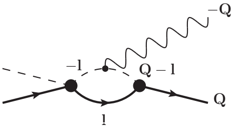
- $\mathcal{A}_i(p_A,|\mathbf{Q}|) = 0$ at $\delta(p_A) = \pi/2$? No!

$$\mathcal{A}_i(p,|\mathbf{Q}|) = \underbrace{\frac{e^{i\delta(p)}}{p^3}\sin\delta(p)}_{\text{potential}}\ \underbrace{p^3\cot\delta(p)}_{\text{zero at } p^2=p_A^2}\ \underbrace{\bar{F}_i(p,|\mathbf{Q}|)}_{\text{divereges at } p^2=p_A^2}$$

$$\mathcal{A}_i(p_n,|\mathbf{Q}|) = e^{i\delta(p_n)}\, V^{1/2}\left(\frac{1}{|\delta'(p_n)+\phi'(q_n)|}\frac{p_n^2}{2\pi}\right)^{-1/2} |F_i(p_n,|\mathbf{Q}|)|$$

$\hookrightarrow$ LL equation for the photoproduction amplitude in the elastic region

Analytic continuation



A potentially dangerous diagram in the $\bar{F}_i$

$$I = \frac{1}{V}\sum_{\mathbf{l}} \frac{1}{8w_1(\mathbf{l})w_2(-\mathbf{l})w_2(\mathbf{Q}-\mathbf{l})}\,\frac{1}{(w_1(\mathbf{l})+w_2(-\mathbf{l})-E_n)(w_1(\mathbf{l})+w_2(\mathbf{Q}-\mathbf{l})-Q^0)}$$

- After some transformations $\Rightarrow$

$$I = \underbrace{\frac{1}{V}\sum_{\mathbf{l}}\frac{1}{2E_n(\mathbf{l}^2-p_n^2)}}_{\propto\, p_n \cot\delta(p_n)}\;\frac{1}{2}\underbrace{\int_{-1}^{1} dy\,\frac{1}{2\hat{w}_2(\hat{w}_1+\hat{w}_2-Q^0)}}_{\text{polynomial in } p_n^2}$$

$$\hat{w}_1 = \sqrt{m_N^2+p_n^2}\,,\qquad \hat{w}_2 = \sqrt{M_\pi^2+p_n^2+\mathbf{Q}^2-2|\mathbf{Q}|p_n y}\,.$$

$\Rightarrow$ $p^2\, I$ is a low-energy polynomial in p^2