# Flavour Physics – Lecture 2

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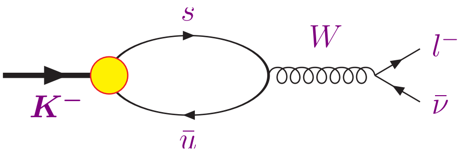
## Determination of $V_{us}$ - $K_{\ell 2}$ Decays






- All QCD effects are contained in a single constant, $f_K$, the kaon's *(leptonic) decay constant.*

$$\langle 0|\bar{s}\gamma^\mu\gamma^5 u\,|K(p)\rangle = if_K\,p^\mu\ . \qquad\qquad (f_\pi \simeq 132\,\mathrm{MeV})$$

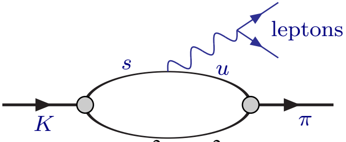
$$\frac{\Gamma(K\to\mu\bar{\nu}(\gamma))}{\Gamma(\pi\to\mu\bar{\nu}(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{f_K^2}{f_\pi^2}\,\frac{m_K\left(1-\frac{m_\mu^2}{m_K^2}\right)}{m_\pi\left(1-\frac{m_\mu^2}{m_\pi^2}\right)}\times 0.9930(35)$$

- From the experimental ratio of the widths we get:

$$\frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{f_K^2}{f_\pi^2} = 0.07602(23)_{\mathrm{exp}}(27)_{\mathrm{RC}}\,, \qquad \text{PDG2006}$$

so that a precise determination of $f_K/f_\pi$ will yield $V_{us}/V_{ud}$ .

- Every collaboration calculates $f_K$ and $f_\pi$.




$$\langle \pi(p_\pi)\,|\bar{s}\gamma_\mu u\,|\,K(p_K)\rangle = f_0(q^2)\,\frac{M_K^2-M_\pi^2}{q^2}q_\mu + f_+(q^2)\left[(p_\pi+p_K)_\mu - \frac{M_K^2-M_\pi^2}{q^2}q_\mu\right]$$

where $q \equiv p_K - p_\pi$.

$$\Gamma_{K\to\pi\ell\nu} = C_K^2\,\frac{G_F^2 m_K^5}{192\pi^3}\,I\,S_{\rm EW}[1+2\Delta_{\rm SU(2)}+\Delta_{EM}]\,|V_{us}|^2\,|f_+(0)|^2$$

From the experimental measurement of the width we get:

$$|V_{us}|f_+(0) = 0.2169(9)\,, \qquad \text{PDG2006}$$

so that a precise determination of $f_+(0)$ will yield $V_{us}$.

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- We have the two precise results:

$$\left|\frac{V_{us}f_K}{V_{ud}f_\pi}\right| = 0.27599(59) \quad \text{and} \quad |V_{us}f_+(0)| = 0.21661(47)$$

Flavianet – arXiv:0801.1817

- We can view these as two equation for the four unknowns $f_K/f_\pi$, $f_+(0)$, $V_{us}$ and $V_{ud}$ .
- Within the Standard Model we also have the unitarity constraint:

$$|V_{ud}|^2 + |V_{us}|^2 \sout{+ |V_{ub}|^2} = 1$$

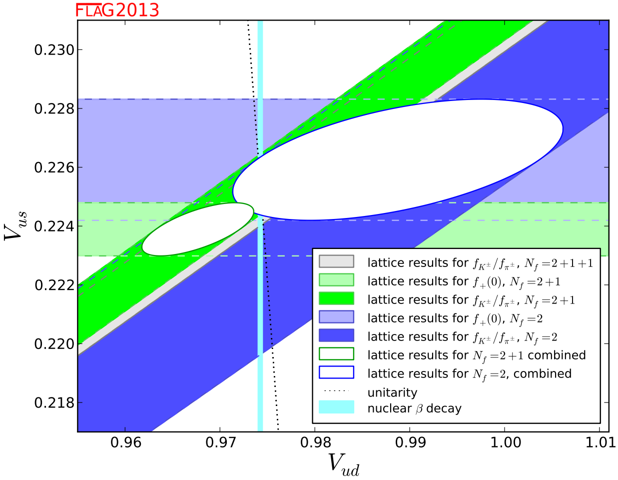
- Thus we now have 3 equations for four unknowns.
- There has been considerable work recently in updating the determination of $V_{ud}$ based on 20 different superallowed transitions. Hardy and Towner, arXiV:0812.1202

$$|V_{ud}| = 0.97425(22)\,.$$

- If we accept this value then we are able to determine the remaining 3 unknowns:

$$|V_{us}| = 0.22544(95), \quad f_+(0) = 0.9608(46), \quad \frac{f_K}{f_\pi} = 1.1927(59)\,.$$

# $V_{us}$ from Lattice Simulations




Flavianet Lattice Averaging Group - arXiv:1310.8555v2

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Lattice results are consistent with the unitarity of the CKM Matrix

- For $N_f = 2+1$ simulations FLAG quotes the following current values:

$$\frac{f_K}{f_\pi} = 1.192(5) \quad \text{and} \quad f_+(0) = 0.9677(23)(33)\,.$$

- Taking the experimental results for $K_{\ell 2}$ and $K_{\ell 3}$ decays and dividing by the $N_f = 2+1$ lattice values of $f_K/f_\pi$ and $f_+(0)$ gives:

$$V_{ud}^2 + V_{us}^2 = 0.987(10)\,.$$

- If we combine the experimental results with the value of $V_{ud}$ and the lattice values of $f_+(0)$ or $f_K/f_\pi$ we find:

$$V_{ud}^2 + V_{us}^2 = 0.9993(5) \quad \text{or} \quad V_{ud}^2 + V_{us}^2 = 1.0000(6)\,.$$

- Very significant test of universality of coupling of "W"-like bosons to quarks and leptons.
- Private view: At such level of precision, I believe that there may still be chiral and continuum effects to control fully.

- $K \to \pi\pi$ decays are a very important class of processes for standard model phenomenology.
- Bose Symmetry $\Rightarrow$ the two-pion state has isospin 0 or 2.
- Among the interesting issues are the origin of the $\Delta I = 1/2$ rule (Re$A_0$/Re$A_2 \simeq 22.5$) and an understanding of the experimental value of $\varepsilon'/\varepsilon$, the parameter which was the first experimental evidence of direct CP-violation.
- The evaluation of $K \to \pi\pi$ matrix elements requires an extension of the standard computations of $\langle 0\,|\,O(0)\,|\,h\rangle$ and $\langle h_2\,|\,O(0)\,|\,h_1\rangle$ matrix elements with a single hadron in the initial and/or final state.

# Effective Hamiltonian for $K \to \pi\pi$ Decays

$$\mathscr{H}_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)] Q_i \,, \text{ where } \tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \text{ and}$$

Current – Current Operators

$$Q_1 = (\bar{s}d)_L(\bar{u}u)_L \qquad Q_2 = (\bar{s}^i d^j)_L(\bar{u}^j u^i)_L$$

QCD Penguin Operators

$$Q_3 = (\bar{s}d)_L \textstyle\sum_{q=u,d,s}(\bar{q}q)_L \qquad Q_4 = (\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s}(\bar{q}^j q^i)_L$$

$$Q_5 = (\bar{s}d)_L \textstyle\sum_{q=u,d,s}(\bar{q}q)_R \qquad Q_6 = (\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s}(\bar{q}^j q^i)_R$$

Electroweak Penguin Operators

$$Q_7 = \tfrac{3}{2}(\bar{s}d)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}q)_L \qquad Q_8 = \tfrac{3}{2}(\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}^j q^i)_L$$

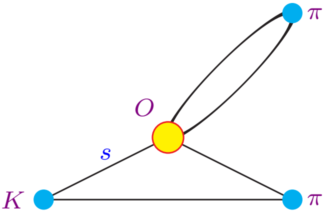
$$Q_9 = \tfrac{3}{2}(\bar{s}d)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}q)_R \qquad Q_{10} = \tfrac{3}{2}(\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}^j q^i)_R$$

This 10 operator basis is very natural but over-complete:

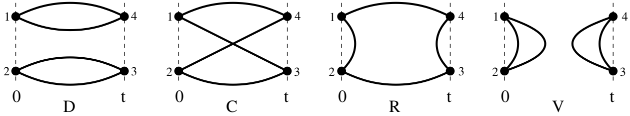
$$\begin{aligned} Q_{10} - Q_9 &= Q_4 - Q_3 \\ Q_4 - Q_3 &= Q_2 - Q_1 \\ 2Q_9 &= 3Q_1 - Q_3 \,. \end{aligned}$$

The original material on this topic is taken from the following RBC-UKQCD papers:

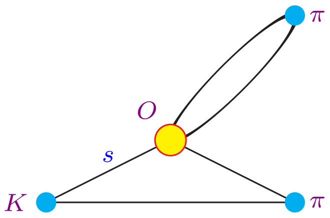
1. "$K$ to $\pi\pi$ Decay amplitudes from Lattice QCD,"
T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Lehner, Q.Liu, R.D.Mawhinney, C.T.Sachrajda, A.Soni, C.Sturm, H.Yin and R.Zhou,
Phys. Rev. D **84** (2011) 114503 (arXiv:1106.2714 [hep-lat]).
2. "The $K \to (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD,"
T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, R.D.Mawhinney, C.T.Sachrajda, A.Soni and C.Sturm,
Phys. Rev. Lett. 108 (2012) 141601, (arXiv:1111.1699 [hep-lat]).
3. "Lattice Determination of the $K \to (\pi\pi)_{I=2}$ Decay Amplitude $A_2$,"
T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, R.D.Mawhinney, C.T.Sachrajda, A.Soni and C.Sturm,
Phys.Rev. D86 (2012) 074513, (arXiv:1206.5142 [hep-lat]).
4. "Emerging understanding of the $\Delta I = 1/2$ rule from Lattice QCD,"
P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Janowski, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, C.T.Sachrajda, A.Soni and D.Zhang,
Phys. Rev. Lett. 110 (2013) 152001, (arXiv:1212.1474 [hep-lat]).




- We need to evaluate correlation functions as in the diagram above.
- In order to divide by $\langle 0 | J_\pi J_\pi | \pi\pi \rangle$, we also need to evaluate the two-pion correlation functions.




- For I=2 $\pi\pi$ states the correlation function is proportional to D-C.




- In the physical decay, in the centre-of-mass frame, $E_{\pi\pi} = m_K$.
- In lattice calculations, in order to eliminate excited states we do not integrate over time, and so, in general, energy is not conserved.
- In the centre-of-mass frame the ground-state is the two-pion state with $E_{\pi\pi} \simeq 2m_\pi$.
- Therefore the correlation function is dominated by the unphysical transition of a kaon at rest into two pions at rest. Maiani-Testa Problem
- The Lellouch-Lüscher solution is to tune the volume so that one of the excited states corresponds to $E_{\pi\pi} = m_K$. (Loss of precision.) hep-lat/0003023

- The operators whose matrix elements have to be calculated are:

$$
\begin{aligned}
O^{3/2}_{(27,1)} &= (\bar{s}^i d^i)_L \{(\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L\} + (\bar{s}^i u^i)_L (\bar{u}^j d^j)_L \\
O^{3/2}_7 &= (\bar{s}^i d^i)_L \{(\bar{u}^j u^j)_R - (\bar{d}^j d^j)_R\} + (\bar{s}^i u^i)_L (\bar{u}^j d^j)_R \\
O^{3/2}_8 &= (\bar{s}^i d^j)_L \{(\bar{u}^j u^i)_R - (\bar{d}^j d^i)_R\} + (\bar{s}^i u^j)_L (\bar{u}^j d^i)_R
\end{aligned}
$$

- It is convenient to use the Wigner-Eckart Theorem: (Notation - $O^{\Delta I}_{\Delta I_z}$)

$$
{}_{I=2}\langle \pi^+(p_1)\pi^0(p_2)\,|O^{3/2}_{1/2}|K^+\rangle = \frac{\sqrt{3}}{2}\langle \pi^+(p_1)\pi^+(p_2)\,|O^{3/2}_{3/2}|K^+\rangle\,,
$$

where
  - $O^{3/2}_{3/2}$ has the flavour structure $(\bar{s}d)\,(\bar{u}d)$.
  - $O^{3/2}_{1/2}$ has the flavour structure $(\bar{s}d)\,(\,(\bar{u}u) - (\bar{d}d)\,) + (\bar{s}u)\,(\bar{u}d)$.

- We can then use antiperiodic boundary conditions for the $u$-quark say, so that the $\pi\pi$ ground-state is $\langle \pi^+(\pi/L)\,\pi^+(-\pi/L)\,|$. C-h Kim, Ph.D. Thesis
  - • Do not have to isolate an excited state. •
  - Size ($L$) needed for physical $K \to \pi\pi$ decay halved.

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- The main theoretical ingredients of the *infrared* problem with two-pions in the s-wave are understood.
- Two-pion quantization condition in a finite-volume

$$\delta(q^*)+\phi^P(q^*)=n\pi\,,$$

where $E^2=4(m_\pi^2+q^{*\,2})$, $\delta$ is the s-wave $\pi\pi$ phase shift and $\phi^P$ is a kinematic function.
M.Lüscher, 1986, 1991, $\cdots$ .

- The relation between the physical $K\to\pi\pi$ amplitude $A$ and the finite-volume matrix element $M$

$$|A|^2=8\pi V^2\,\frac{m_K E^2}{q^{*\,2}}\left\{\delta'(q^*)+\phi^{P\,\prime}(q^*)\right\}|M|^2\,,$$

where $\prime$ denotes differentiation w.r.t. $q^*$ .

L.Lellouch and M.Lüscher, hep-lat/0003023; C.h.Kim, CTS and S.Sharpe, hep-lat/0507006;
N.H.Christ, C.h.Kim and T.Yamazaki hep-lat/0507009

- Computation of $K\to(\pi\pi)_{I=2}$ matrix elements does not require the subtraction of power divergences or the evaluation of disconnected diagrams.
- In 2011-2012, we evaluated the $\Delta I=3/2$ $K\to\pi\pi$ matrix elements for the first time and at physical kinematics.

# $K \to (\pi\pi)_{I=2}$ decay amplitudes (Cont.)

- The calculations were performed on a $32^3 \times 64 \times 32$ ($L = 4.58$ fm, $a^{-1} = 0.14$ fm lattice using Domain Wall Fermions and the IDSDR gauge action.

| Systematic Error Budget | Re$A_2$ | Im$A_2$ |
|---|---|---|
| lattice artefacts | 15% | 15% |
| finite-volume corrections | 6.0% | 6.5% |
| partial quenching | 3.5% | 1.7% |
| renormalization | 1.8% | 5.6% |
| unphysical kinematics | 0.4% | 0.8% |
| derivative of the phase shift | 0.97% | 0.97% |
| Wilson coefficients | 6.6% | 6.6% |
| Total | 18% | 19% |

- The dominant error is due to lattice artefacts and the fact that out lattice is coarse. This will be eliminated when the calculation is repeated at a second lattice spacing.
- The 15% estimate, intended to be conservative, is obtained by
  1. Studying the dependence on $a$ of quantities which have been calculated at several lattice spacings.
  2. In particular by determining the $a$ dependence of $B_K$, which is also given by the matrix element of a (27,1) operator.

Our results for the amplitude $A_2$ are:

$$\mathrm{Re}A_2 = (1.381 \pm 0.046_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}})\, 10^{-8}\,\mathrm{GeV}$$
$$\mathrm{Im}A_2 = -(6.54 \pm 0.46_{\mathrm{stat}} \pm 1.20_{\mathrm{syst}})\, 10^{-13}\,\mathrm{GeV}.$$

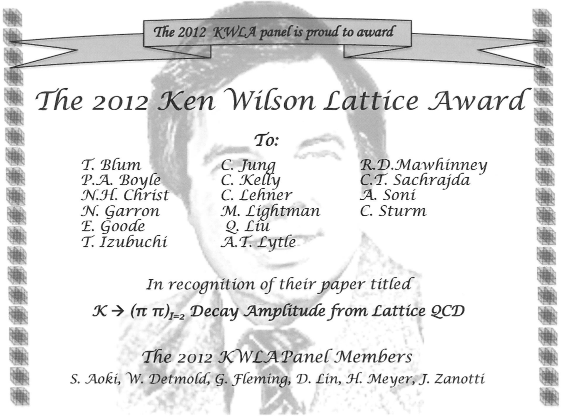
- The result for Re $A_2$ agrees well with the experimental value of $1.479(4) \times 10^{-8}$ GeV obtained from $K^+$ decays and $1.573(57) \times 10^{-8}$ GeV obtained from $K_S$ decays .
- Im $A_2$ is unknown so that our result provides its first direct determination.
- For the phase of $A_2$ we find Im $A_2$/Re$A_2 = -4.42(31)_{\mathrm{stat}}(89)_{\mathrm{syst}}\, 10^{-5}$.
- Combining our result for Im $A_2$ with the experimental results for Re $A_2$, Re $A_0 = 3.3201(18) \cdot 10^{-7}$ GeV and $\varepsilon'/\varepsilon$ we obtain:

$$\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = -1.61(19)_{\mathrm{stat}}(20)_{\mathrm{syst}} \times 10^{-4}\,.$$

(Of course, we wish to confirm this directly.)

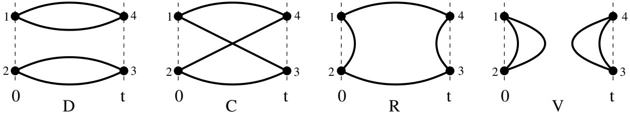
$$\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\sqrt{2}\,|\varepsilon|}{\omega}\frac{\varepsilon'}{\varepsilon}$$

$$-1.61(19)_{\mathrm{stat}}(20)_{\mathrm{syst}} \times 10^{-4} = -4.42(31)_{\mathrm{stat}}(89)_{\mathrm{syst}} \times 10^{-5} - 1.16(18) \times 10^{-4}\,.$$




- For this work we received the 2012 Ken Wilson Lattice award at Lattice 2012.
- Criteria: The paper must be important research beyond the existing state of the art. $\cdots$

- The $I = 0$ final state has vacuum quantum numbers.
- Vacuum contribution must be subtracted; disconnected diagrams require statistical cancelations to obtain the $e^{-2m_\pi t}$ behaviour.
- Consider first the two-pion correlation functions, which are an important ingredient in the evaluation of $K \to \pi\pi$ amplitudes.

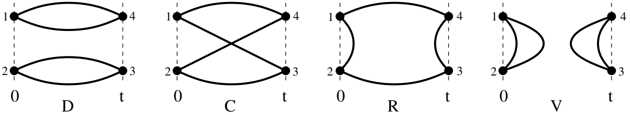



- For I=2 $\pi\pi$ states the correlation function is proportional to D-C.
- For I=0 $\pi\pi$ states the correlation function is proportional to 2D+C-6R+3V.

The major practical difficulty is to subtract the vacuum contribution with sufficient precision.

- In the paper we report on high-statistics experiments on a $16^3 \times 32$ lattice, $a^{-1} = 1.73$ GeV, $m_\pi = 420$ MeV, with the propagators evaluated from each time-slice.

# Diagrams contributing to two-pion correlation functions

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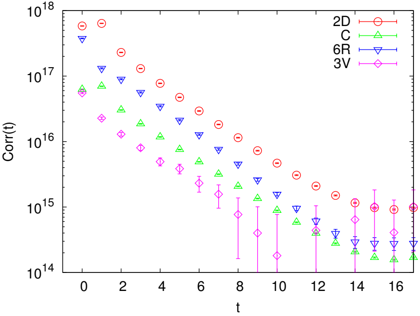
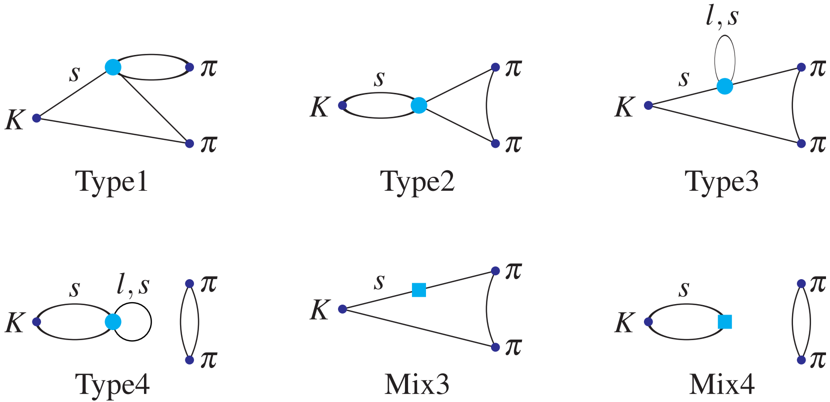



RBC/UKQCD, Qi Liu – Lattice 2010




- There are 48 different contractions and we classify the contributions into the 6 different types illustrated above.
- Mix3 and Mix4 are needed to subtract the power divergences which are proportional to matrix elements of $\bar{s}\gamma_5 d$ .

# Results from exploratory simulation at unphysical kinematics



- These results are for the $K \to \pi\pi$ (almost) on-shell amplitudes with 420 MeV pions at rest: RBC/UKQCD arXiv:1106.2714

| | | | |
|---|---|---|---|
| Re $A_0$ | $(3.80 \pm 0.82)\,10^{-7}$ GeV | Im $A_0$ | $-(2.5 \pm 2.2)\,10^{-11}$ GeV |
| Re $A_2$ | $(4.911 \pm 0.031)\,10^{-8}$ GeV | Im $A_2$ | $-(5.502 \pm 0.0040)\,10^{-13}$ GeV |

- This was an exploratory exercise in which we are learning how to do the calculation.
- We, along with the rest of the world, continue to develop techniques with the aim of enhancing the signal for disconnected diagrams.
- The exploratory results for $K \to (\pi\pi)_{I=0}$ decays are very encouraging.
- For $(\pi\pi)_I = 0$ states the Wigner-Eckart theorem and the use of antiperiodic boundary conditions for the $d$-quark does not help.

  C.Sachrajda and G.Villadoro hep-lat/0411033

  We are currently developing and testing the use of G-parity boundary conditions.

  C.-h Kim, hep-lat/0311003

  $\Rightarrow$ a quantitative understanding of the $\Delta I = 1/2$ rule and the value of $\varepsilon'/\varepsilon$.

- The evaluation of disconnected diagram has allowed us to study the $\eta$ and $\eta'$ mesons and their mixing. RBC-UKQCD – arXiV:1002.2999

## Emerging understanding of the $\Delta I = 1/2$ Rule

arXiv:1212.1474

- In his thesis Qi Liu extended the above study to the $24^3 \times 64$ ensembles.
  - Larger $T \Rightarrow$ suppression of around-the-world effects.
  - Two-pion sources separated in time $\Rightarrow$ better plateaus.
  - Faster algorithm for the inversions.

  1. $16^3 \times 32$ ensembles; 877 MeV kaon decaying into two 422 MeV pions at rest:

  $$\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2} = 9.1 \pm 2.1\,.$$

  2. $24^3 \times 64$ ensembles; 662 MeV kaon decaying into two 329 MeV pions at rest:

  $$\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2} = 12.0 \pm 1.7\,.$$

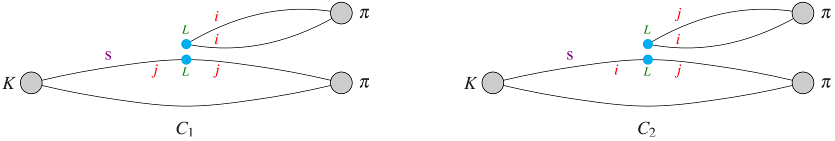
- Whilst both these results are obtained at unphysical kinematics and are different from the physical value of 22.5, it is nevertheless interesting to understand the origin of these enhancements.
- 99% of the contribution to the real part of $A_0$ and $A_2$ come from the matrix elements of the current-current operators.

# Contributions from Individual Matrix Elements

| i | $Q_i^{\text{lat}}$ [GeV] | $Q_i^{\overline{\text{MS}}\text{-NDR}}$ [GeV] |
|---|---|---|
| 1 | 8.1(4.6) $10^{-8}$ | 6.6(3.1) $10^{-8}$ |
| 2 | 2.5(0.6) $10^{-7}$ | 2.6(0.5) $10^{-7}$ |
| 3 | -0.6(1.0) $10^{-8}$ | 5.4(6.7) $10^{-10}$ |
| 4 | – | 2.3(2.1) $10^{-9}$ |
| 5 | -1.2(0.5) $10^{-9}$ | 4.0(2.6) $10^{-10}$ |
| 6 | 4.7(1.7) $10^{-9}$ | -7.0(2.4) $10^{-9}$ |
| 7 | 1.5(0.1) $10^{-10}$ | 6.3(0.5) $10^{-11}$ |
| 8 | -4.7(0.2) $10^{-10}$ | -3.9(0.1) $10^{-10}$ |
| 9 | – | 2.0(0.6) $10^{-14}$ |
| 10 | – | 1.6(0.5) $10^{-11}$ |
| Re$A_0$ | 3.2(0.5) $10^{-7}$ | 3.2(0.5) $10^{-7}$ |

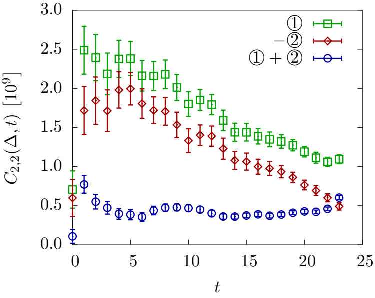
- Contributions from each operator to Re$A_0$ for $m_K = 662$ MeV and $m_\pi = 329$ MeV. The second column contains the contributions from the 7 linearly independent lattice operators with $1/a = 1.73(3)$ GeV and the third column those in the 10-operator basis in the $\overline{\text{MS}}$-NDR scheme at $\mu = 2.15$ GeV. Numbers in parentheses represent the statistical errors.

# Emerging understanding of the $\Delta I = 1/2$ Rule (Cont.)

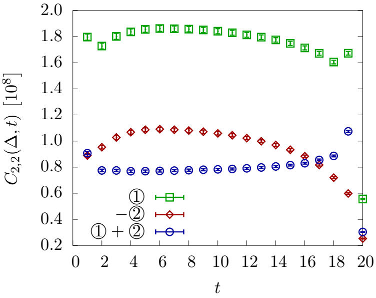



- Re $A_2$ is proportional to $C_1 + C_2$.
- The contribution to Re $A_0$ from $Q_2$ is proportional to $2C_1 - C_2$ and that from $Q_1$ is proportional to $C_1 - 2C_2$ with the same sign.
- Colour counting might suggest that $C_2 \simeq \frac{1}{3} C_1$.
  - Much continuum phenomenology has been done in the vacuum insertion hypothesis.
- We find instead that $C_2 \approx -C_1$ so that $A_2$ is significantly suppressed!
- $A_2$ has a larger kinematic dependence than $A_0$.
- **We believe that the strong suppression of Re $A_2$ and the (less-strong) enhancement of Re $A_0$ is a major factor in the $\Delta I = 1/2$ rule.**
  - Of course before claiming a quantitative understanding of the $\Delta I = 1/2$ rule we need to compute Re $A_0$ at physical kinematics and reproduce the experimental value of 22.5.

# Evidence for the Suppression of Re $A_2$




Physical Kinematics




$m_\pi \simeq$ 330 MeV at threshold.

- Notation ⓘ $\equiv C_i,\ i = 1,2.$

- Tadeusz Janowski et al. are completing a paper updating our results of $A_2$, computed at two finer lattice spacings and at physical quark masses. (Errors are significantly reduced.)
- Development and testing of $G$-parity boundary conditions with the primary aim of computing the $K \to (\pi\pi)_{I=0}$ decay amplitude $A_0$.
- Evaluation of long-distance effects in $\Delta M_K$ and $\varepsilon_K$.
- Beginning to perform the exploratory work to study the rare kaon decays $K \to \pi\ell^+\ell^-$ and $K \to \pi\nu\bar{\nu}$.
- These last two quantities are an extension of lattice calculations to matrix elements of the form:

$$\int d^4x \int d^4y \, \langle h_2 \, | \, T\{O_1(x)\, O_2(y)\} \, | \, h_1 \rangle \, .$$

- The $b$-quark is light-enough to be produced copiously and heavy enough to have a huge number of possible decay channels.
- In addition to the lattice systematics already discussed, we now have to deal with the fact that $m_b a \gtrsim 1$.
- Most approaches rely on effective theories and invest a considerable effort in matching the effective theory to QCD.
  - Heavy Quark Effective Theory (expansion in $\frac{\Lambda_{\text{QCD}}}{m_B}$).
  - Nonrelativistic QCD (expansion in the quark's velocity).
  - Relativistic Heavy Quarks ("Fermilab Approach" and extensions).

  A. El Khadra, A. Kronfeld and P. Mackenzie, hep-lat/9604004
- Some groups also extrapolate results from the charm to the bottom region, using scaling laws where applicable and possibly using results in the static limit.
- There are far fewer calculations in heavy-quark physics, so less opportunity to check for consistency of different approaches.
  This is not a criticism of those who have done the calculations but of those of us who have not!
- Unfortunately we do not know (yet?) how to compute non-leptonic $B$-decays ($B \to \pi\pi$, $B \to \pi K$ etc).

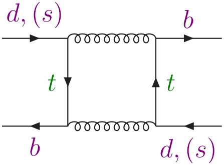
- For many years the experimental upper bound for this FCNC decay has been several orders of magnitude above the SM prediction.
- Most extensions of the SM give loop corrections which enhance the width and hence this was viewed as a good channel for the discovery of new physics.
- The LHC experiments observed this decay in 2012 and recent results are

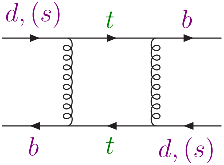
$$\begin{aligned} \mathrm{Br}(B_s \to \mu^+\mu^-)_{\mathrm{LHCb}} &= 2.9^{+1.1}_{-1.0} \times 10^{-9} && \mathrm{LHCb, arXiv:1307.5024} \\ \mathrm{Br}(B_s \to \mu^+\mu^-)_{\mathrm{CMS}} &= 3.0^{+1.0}_{-0.9} \times 10^{-9} && \mathrm{CMS, arXiv:1307.5025} \\ \mathrm{Br}(B_s \to \mu^+\mu^-)_{\mathrm{Combined}} &= (2.9 \pm 0.7) \times 10^{-9} && \mathrm{LHCb+CMS,\ Conf.\ Presentation} \end{aligned}$$

- Unfortunately(!?), these results are fully consistent with the Standard Model e.g.

$$\begin{aligned} \mathrm{Br}(B_s \to \mu^+\mu^-) &= (3.65 \pm 0.23) \times 10^{-9} && \mathrm{Bobeth\ et\ al.,\ arXiv:1311.0903} \\ \mathrm{Br}(B_s \to \mu^+\mu^-) &= (3.23 \pm 0.27) \times 10^{-9} && \mathrm{Buras\ et\ al.,\ arXiv:1208.0934} \end{aligned}$$

- Lattice input is the evaluation of $f_{B_s}$.
- For the corresponding branching fraction of $B_d$ decays, the combined result is $(3.6^{+1.6}_{-1.4}) \times 10^{-10}$ compared to the theoretical prediction of $(1.06 \pm 0.09) \times 10^{-10}$.




- For the SU(3)-breaking parameter $\xi$, FLAG take the result of the FNAL/MILC collaboration as currently the best result: FNAL/MILC, arXiv:1205.7013

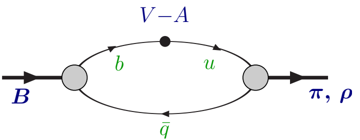
$$\xi^2 \equiv \frac{\langle \bar{B}^0_s \,|\, (\bar{b}\gamma^\mu(1-\gamma^5)s)(\bar{b}\gamma_\mu(1-\gamma^5)s)\,|B^0_s\rangle}{\langle \bar{B}^0 \,|\, (\bar{b}\gamma^\mu(1-\gamma^5)d)(\bar{b}\gamma_\mu(1-\gamma^5)d)\,|B^0\rangle} = 1.268(63)\,.$$

- Combining this result with experimental values of $\Delta m_d$ and $\Delta m_s \Rightarrow$

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.216 \pm 0.011\,.$$

FNAL/MILC, arXiv:1205.7013

- For generic BSM theories, there are 5 $\Delta B = 2$ operators (and 5 $\Delta S = 2$ operators for neutral kaon mixing) whose matrix elements can be computed in a similar way.




QCD effects are contained in form factors e.g. for $B \to \pi$ decays:

$$\langle \pi(p_\pi)\,|\bar{b}\gamma_\mu u\,|B(p_B)\rangle = f_0(q^2)\,\frac{m_B^2 - m_\pi^2}{q^2}\,q_\mu + f_+(q^2)\left[(p_\pi + p_B)_\mu - \frac{m_B^2 - m_\pi^2}{q^2}\,q_\mu\right]$$

where $q \equiv p_B - p_\pi$.

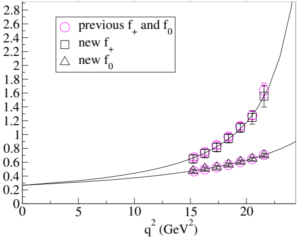
- For $B$-decays, in order to avoid lattice artefacts, the momentum of the $\pi$ or $\rho$ is limited $\Rightarrow$ get results only at large values of $q^2$.
- Thus $V_{ub}$ can only be obtained directly by combining the lattice results with a subset of the experimental data:

$$\Delta\zeta(q_1^2, q_2^2) = \frac{1}{|V_{ub}|^2}\int_{q_1^2}^{q_2^2} dq^2\ \frac{d\Gamma}{dq^2}\,.$$

- The lattice results can be combined with theoretically motivated parametrisations for the form factors, including perhaps constrains from analyticity and other general properties of field theory, to extend the range of the predictions. (Not discussed here.)

The (peer-reviewed) published values for the form factors are relatively old:

| Collaboration | Reference | $\Delta\zeta$ ps$^{-1}$ |
|---|---|---|
| FNAL/MILC | arXiv:0811.3640 | $2.21^{+0.47}_{-0.42}$ |
| HPQCD | hep-lat/0601021 | 2.07(41)(39) |



HPQCD

- The two collaborations use overlapping sets of rooted staggered ensembles, but different treatments of the heavy quarks (HPQCD use NRQCD and FNAL/MILC use the FNAL approach). Assuming (conservatively) a 100% correlation FLAG quote

$$\Delta\zeta(16\,\text{GeV}^2, q^2_{\text{max}}) = 2.16(50)\,\text{ps}^{-1}$$

- FLAG, perform a detailed analysis, finding a preferred parametrization and quote

$$\text{Lattice + BABAR} \qquad |V_{ub}| = 3.37(21)\times 10^{-3}$$
$$\text{Lattice + Belle} \qquad |V_{ub}| = 3.47(22)\times 10^{-3}.$$

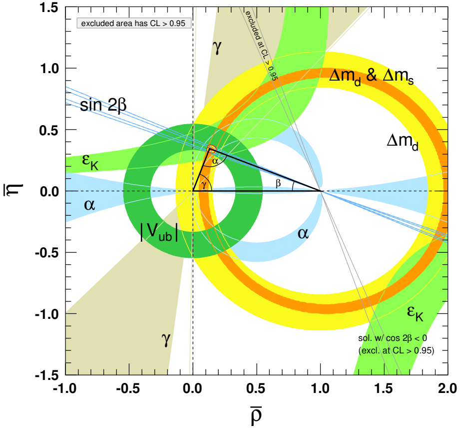
- FLAG, perform a detailed analysis, finding a preferred parametrization and quote

$$\begin{aligned} \text{Lattice + BaBar} \qquad & |V_{ub}| = 3.37(21) \times 10^{-3} \\ \text{Lattice + Belle} \qquad & |V_{ub}| = 3.47(22) \times 10^{-3}. \end{aligned}$$

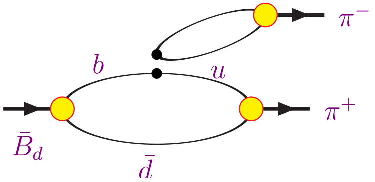
- Assuming (not assuming) unitarity PDG quote $|V_{ub}| = 3.51^{+0.15}_{-0.14} \times 10^{-3}$ ($|V_{ub}| = (4.15 \pm 0.49) \times 10^{-3}$).
- The issue is the tension with the inclusive determination $|V_{ub}| = (4.41 \pm 0.15^{+0.15}_{-0.19}) \times 10^{-3}$. This has very different systematics and cannot be studied in lattice simulations.
- The evaluation of $f_+(q^2)$ and $f_0(q^2)$ and the subsequent determination of $V_{ub}$ is clearly a major priority for lattice simulations and is now a priority of several collaborations.

# PDG2012 Unitarity Triangle





- A huge amount of information has been obtained about decay rates and CP-asymmetries for $B \to M_1 M_2$ decays (over 100 channels).
- With just a few exceptions (e.g. CP-asymmetry in $B \to J/\Psi K_s$) our ability to deduce fundamental information about CKM matrix elements is limited by our inability to quantify the non-perturbative strong interaction effects.
- Most approaches were based on **Naive Factorization:**




$$\langle \pi^+ \pi^- \,|\, (\bar{u}b)_{V-A}\, (\bar{d}u)_{V-A} \,|\, \bar{B}_d \rangle \to \langle \pi^- \,|\, (\bar{d}u)_{V-A} \,|\, 0 \rangle \, \langle \pi^+ \,|\, (\bar{u}b)_{V-A} \,|\, \bar{B}_d \rangle$$

- $\langle \pi^- \,|\, (\bar{d}u)_{V-A} \,|\, 0 \rangle$ is known ($f_\pi$).
- $\langle \pi^+ \,|\, (\bar{u}b)_{V-A} \,|\, \bar{B}_d \rangle$ is known in principle ($F_0^{B\to\pi}(m_\pi^2)$).
- No rescattering in the final state. No strong phase-shifts.
- $\mu$ dependence does not match on the two sides.

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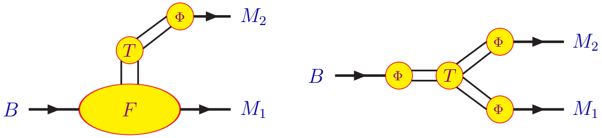
- In 1999 we realized that in the limit $m_b \to \infty$, the long distance effects *factorise* into simpler universal quantities:

M.Beneke, G.Buchalla, M.Neubert, CTS, (BBNS)




$$\langle M_1, M_2 \,|\, O_i \,|\, B \rangle \;=\; \sum_j F_j^{B\to M_1}(m_2^2) \int_0^1 du\, T_{ij}^{I}(u) \Phi_{M_2}(u) + \;(M_1 \leftrightarrow M_2)$$

$$+ \int_0^1 d\xi\, du\, dv\, T_i^{II}(\xi, u, v)\, \Phi_B(\xi)\, \Phi_{M_1}(v)\, \Phi_{M_2}(u)$$

## Implications of Factorization

- The significance and usefulness of the factorization formula stems from the fact that the non-perturbative quantities which appear on the RHS are much simpler than the original matrix elements which appear on the LHS.
  They either reflect universal properties of a single meson state (the light-cone distribution amplitudes) or refer to a $B \to$ meson transition matrix element of a local current (form-factor).
- Conventional (naive) factorization is recovered as a rigorous prediction in the infinite quark-mass limit (i.e. neglecting $O(\alpha_s)$ and $O(\Lambda_{\rm QCD}/m_b)$ corrections).
- Perturbative corrections to naive factorization can be computed systematically. The results are, in general, non-universal (i.e. process dependent).
- All strong interaction phases are generated perturbatively in the heavy quark limit.
- The factorization formulae are valid up to $O(\Lambda_{\rm QCD}/m_b)$ corrections.
- Many observables of interest for $CP$-violation become accessible. The precision of the calculations is limited by our knowledge of the wave-functions and of the power corrections.
- For a comprehensive study of 96 PP and PV decay modes see

Beneke and Neubert, hep-ph/0308039.

- The main limitation of the factorization framework is due to the fact that $m_b$ is not so large, so that CKM and chiral enhancements to non-factorizable $O(\Lambda_{\text{QCD}}/m_b)$ terms are important.
- At present we do not know how to begin computing $B \to M_1 M_2$ matrix elements!
  - Many intermediate states contribute.
- What can lattice simulations contribute to the factorization formula:
  - Parton distribution amplitudes of light mesons (at least the low moments) $\surd$.
  - $B \to M$ form-factors $\surd$.
  - Parton distribution amplitudes of $B$-meson $X$.
- I now briefly explain why we have not been able to compute $\phi^B$ or its moments.

$$\phi^B_{\alpha\beta}(\tilde{k}_+) = \int dz_- \, e^{i\tilde{k}_+ z_-} \, \langle 0 \,|\, \bar{u}_\beta(z)[z,0]b_\alpha(0) \,|\, B \rangle\big|_{z^+, z_\perp = 0}$$

- $\phi^B$ is convoluted with the perturbative hard-scattering amplitude $T_i^{II} \Rightarrow$ we need

$$\frac{\sqrt{2}}{\lambda_b} = \int_0^\infty \frac{d\tilde{k}_+}{\tilde{k}_+} \, \phi^B_+(\tilde{k}_+) \, .$$

  (In higher orders of perturbation theory factors containing $\log(\tilde{k}_+)$ appear.)
- At large $\tilde{k}_+$, $\phi^B(\tilde{k}_+) \sim 1/\tilde{k}_+$, but the convolution is finite.
- Positive moments of $\phi^B(\tilde{k}_+)$, which can be written in terms of local operators, diverge as powers of $1/a \Rightarrow$ need a technique to subtract these divergences with sufficient precision.
- We need new theoretical ideas for the lattice to contribute to $B \to M_1 M_2$ decays.

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- Precision flavour physics is a complementary approach to the large $p_\perp$ studies at the LHC in exploring the limits of the standard model.
- The hugely improved precision of Lattice QCD simulations is making this approach truly viable.
- In addition to the improved precision in the evaluation of "standard" quantities, it is important to continue extending the range of physical quantities which can be studied.
- There is a huge amount of work to be done!