54 Cracow School of Theoretical Physics (Zakopane June 2014)

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QCD meets experiment
June 12-20, 2014 Zakopane, Poland
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References

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- **Polyakov-Nambu–Jona-Lasinio:** hep-ph/0410053; AIP Conf.Proc. 756 (2005) 436-438, Phys.Rev. D74 (2006) 065005, Rom.Rep.Phys. 58 (2006) 081-086. PoS JHW2005 (2006) 025. AIP Conf.Proc. 892 (2007) 444-447. Eur.Phys.J. A31 (2007) 553-556.
- **Dim-2 Condensates:**JHEP 0601 (2006) 073, Phys.Rev. D75 (2007) 105019. Indian J.Phys. 85 (2011) 1191-1196. Nucl.Phys.Proc.Suppl. 186 (2009) 256-259. Phys.Rev. D81 (2010) 096009.
- **Hadron Resonance Gas for Polyakov loop:** Phys.Rev.Lett. 109 (2012) 151601. arXiv:1207.4875 [hep-ph]. arXiv:1207.7287 [hep-ph].
- **Polyakov loop Spectroscopy:** Phys.Rev. D89 (2014) 076006 109.

Outline

- Quarks and gluons at finite temperature
- Insights from Gluodynamics
- Coupling quarks with Polyakov loops
- Polyakov loops spectroscopy
- Hadron Resonance Gas from Chiral Quark Models
- Conclusions

QUARKS AND GLUONS AT FINITE TEMPERATURE

QCD at finite temperature

QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f \overline{q}^a_f (i\gamma_\mu D_\mu - m_f) q^a_f \, ;$$

Partition function

$$\begin{aligned} Z_{\rm QCD} &= {\rm Tr} e^{-H/T} = \sum_n e^{-E_n/T} \\ &= \int \mathcal{D} A_{\mu,a} \exp\left[-\frac{1}{4} \int d^4x (G^a_{\mu\nu})^2 \right] {\rm Det}(i\gamma_\mu D_\mu - m_f) \end{aligned}$$

Boundary conditions and Matsubara frequencies

$$q(\vec{x},\beta) = -q(\vec{x},0) \qquad A_\mu(\vec{x},\beta) = A_\mu(\vec{x},0) \qquad \beta = 1/T$$

$$\int \frac{dp_0}{2\pi} f(p_0) \to T \sum_n f(w_n)$$

$$w_n = (2n+1)\pi T \qquad w_n = 2n\pi T$$

Thermodynamic relations

- Statistical mechanics of non-interacting particles

$$\log Z = V \eta g_i \int \frac{d^3 p}{(2\pi)^3} \log \left[1 + \eta e^{-E_p/T} \right] \qquad E_p = \sqrt{p^2 + m^2}$$

$\eta = -1$ for bosons ; $\eta = -1$ for fermions ; g_i-number of species

$$F = -T \log Z \qquad P = -T \frac{\partial F}{\partial V}$$

$$S = -\frac{\partial (TF)}{\partial T} \qquad E = F + TS$$

- High temperature limit $\to$ Free gas of gluons and quarks

$$p \equiv \frac{P}{V} = \left[2(N_c^2 - 1) + 4 N_c N_f \frac{7}{8} \right] \frac{\pi^2}{90} T^4$$

Interaction measure (trace anomaly)

$$\Delta \equiv \frac{\epsilon - 3p}{T^4} \to 0 \qquad (T \to \infty)$$

Thermodynamic relations

- Low temperature limit (large N_c) gas of hadrons and glueballs

$$p = \sum_i \eta g_i \int \frac{d^3p}{(2\pi)^3} \log\left[1 + \eta e^{-E_p/T}\right]$$

Level density. Hagedorn spectrum for mesons and baryons (Broniowski+Florkowski)

$$\rho(m) = \sum_i g_i \delta(m - m_i) \to A_M e^{m/T_{H,M}} + A_H e^{m/T_{H,M}} \qquad m < 1.5 - 2\mathrm{GeV}$$

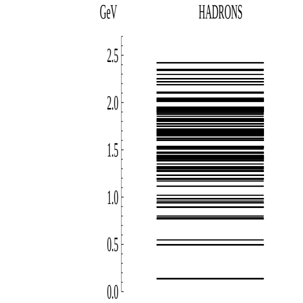
Interaction measure

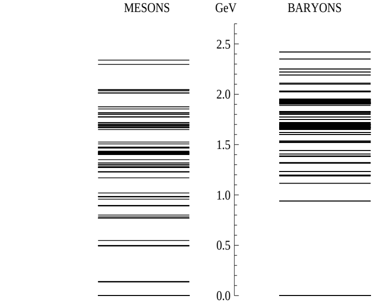
$$\Delta_{HRG} \equiv \frac{\epsilon - 3p}{T^4} \to \sum_i \Delta_i = \Delta_\pi + \ldots \qquad (T \to 0)$$

Minimal Hagedorn temperature

$$\Delta_{HRG} \to \frac{A}{T - T_{H,Min}}$$

The states in the Particle Data Group (PDG) book

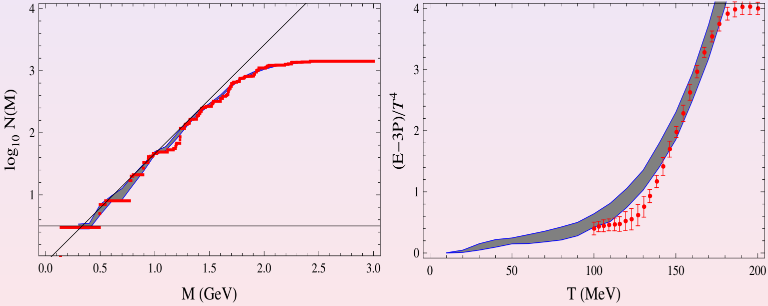




- No data (is a compilation)
- No particles (resonances)
- No book
- Which particles enter PDG ?

The physical resonance spectrum and the half-width rule

- Resonances have a *mass spectrum* (what is the mass?)
- The **half-width rule**: $\Delta M_R = \Gamma_R/2$ or $\Delta M_R^2 = M_R \Gamma_R$

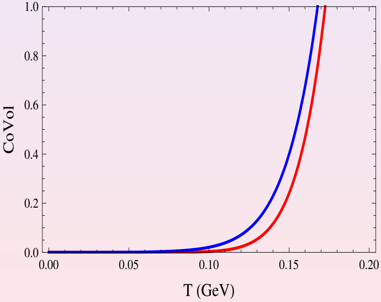


Excluded Volume constraint

$$\sum V_i N_i \leq V \qquad \sum_i V_i \int \frac{d^3p}{(2\pi)^3} \frac{g_i}{e^{E_i(p)/T} \pm 1} \leq 1$$

MIT bag model

$$V_i = M_i/(4B) \qquad B = (0.166 GeV)^4$$



When hadrons overlapp, excluded volume corrections are important

Symmetries in QCD

Colour gauge invariance

$$q(x) \to e^{i\sum_a (\lambda_a)^c \alpha_a(x)} q(x) \equiv g(x) q(x)$$
$$A_\mu^g(x) = g^{-1}(x)\partial_\mu g(x) + g^{-1}(x) A_\mu(x) g(x)$$

Only periodic gauge transformations are allowed:

$$g(\vec{x}, x_0 + \beta) = g(\vec{x}, x_0)\,, \qquad \beta = 1/T\,.$$

In the static gauge $\partial_0 A_0 = 0$

$$g(x_0) = e^{i2\pi x_0 \lambda/\beta}\,, \quad \text{where} \quad \lambda = \mathrm{diag}(n_1, \cdots, n_{N_c})\,, \quad \mathrm{Tr}\lambda = 0\,.$$

Large Gauge Invariance: $\Rightarrow$ periodicity in A_0 with period $2\pi/\beta$

$$A_0 \to A_0 + 2\pi T \mathrm{diag}(n_j) \qquad \text{Gribov copies}$$

Explicitly Broken in perturbation theory (non-perturbative finite temperature gluons)

Symmetries in QCD

In the limit of massless quarks ($m_f = 0$),

- Invariant under scale

$$(\mathbf{x} \longrightarrow \lambda \mathbf{x})$$

Broken by quantum corrections regularization (Trace anomaly)

$$\epsilon - 3p = \frac{\beta(g)}{2g} \langle (G^a_{\mu\nu})^2 \rangle \neq 0 \,,$$

- Chiral **Left** $\leftrightarrow$ **Right** transformations.

$$q(x) \to e^{i \sum_a (\lambda_a)^f \alpha_a} q(x) \qquad q(x) \to e^{i \sum_a (\lambda_a)^f \alpha_a \gamma_5} q(x)$$

Broken by chiral condensate in the vacuum

$$\langle \bar{q} q \rangle \neq 0$$

INSIGHTS FROM GLUODYNAMICS

Symmetries in QCD

Gluodynamics: In the limit of heavy quarks ($m_f \to \infty$)

$$Z \to \int \mathcal{D}A_{\mu,a} \exp\left[-\frac{1}{4}\int d^4x (G^a_{\mu\nu})^2\right] \mathrm{Det}(-m_f)$$

Larger symmetry ('t Hooft) Center Symmetry $\mathbb{Z}(N_c)$

$$g(\vec{x}, x_0 + \beta) = z\, g(\vec{x}, x_0)\,, \qquad z^{N_c} = 1, \quad (z \in \mathbb{Z}(N_c))\,.$$

$$g(x_0) = e^{i2\pi x_0 \lambda/(N_c \beta)}\,, \qquad A_0 \to A_0 + \frac{2\pi T}{N_c}\mathrm{diag}(n_j)$$

The Poyakov loop

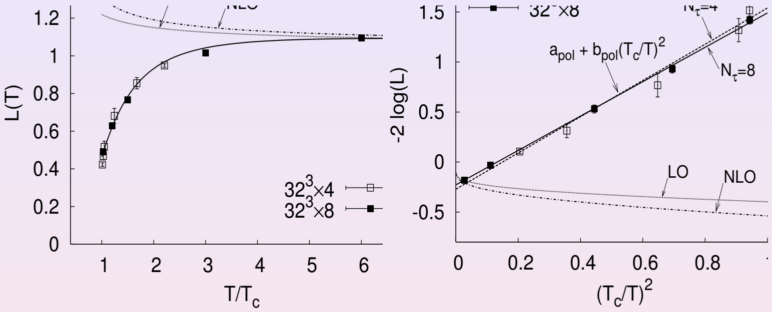
$$L_T = \frac{1}{N_c}\langle \mathrm{tr}_c e^{iA_0/T}\rangle = e^{-F_q/T} = e^{i2\pi/N_c} L_T = 0$$

$F_q = \infty$ means CONFINEMENT
At high temperatures $A_0/T << 1$

$$L_T = 1 - \frac{\langle \mathrm{tr}_c A_0^2\rangle}{2N_c T^2} + \cdots = e^{-\frac{\langle \mathrm{tr}_c A_0^2\rangle}{2N_c T^2} + \cdots}$$

In full QCD $L_T = \mathcal{O}(e^{-m_q/T}) \neq 0 \ll 1$

Power temperature corrections in the Polyakov loop



$$-2\log(L) = a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,, \quad a_{\rm NP} = (1.81 \pm 0.13)\,T_c^2\,, \qquad 1.03\,T_c < T < 6\,T_c\,.$$

Perturbative result fails to reproduce lattice data in this regime.

Trace Anomaly

Partition function (gluodynamics $m_f \to \infty$) $\bar{A}_\mu = gA_\mu$

$$Z = \int \mathcal{D}\bar{A}_{\mu,a} \exp\left[-\frac{1}{4g^2}\int d^4x (\bar{G}^a_{\mu\nu})^2\right]$$

$$\frac{\partial \log Z}{\partial g} = \frac{1}{2g^3}\left\langle \int d^4x (\bar{G}^a_{\mu\nu})^2 \right\rangle = \frac{1}{2g}\frac{V}{T}\langle (G^a_{\mu\nu})^2\rangle$$

Free energy and Total Energy

$$F = -PV = -T\log Z \qquad \epsilon = \frac{E}{V} = \frac{T^2}{V}\frac{\partial \log Z}{\partial T} \tag{1}$$

$$\epsilon - 3P = T^5 \frac{\partial}{\partial T}\left(\frac{P}{T^4}\right) . \tag{2}$$

Renormalization scale μ

$$\frac{P}{T^4} = f(g(\mu), \log(\mu/2\pi T)). \tag{3}$$

$$\frac{\partial}{\partial \log T}\left(\frac{P}{T^4}\right) = \frac{\partial g}{\partial \log \mu}\frac{\partial}{\partial g}\left(\frac{P}{T^4}\right) \tag{4}$$

The trace anomaly

$$\epsilon - 3P = \frac{\beta(g)}{2g}\langle (G^a_{\mu\nu})^2 \rangle \,,$$

where we have introduced the beta function

$$\beta(g) = \mu \frac{dg}{d\mu} = -\frac{11 N_c}{48\pi^2} g^3 + \mathcal{O}(g^5)\,. \tag{5}$$

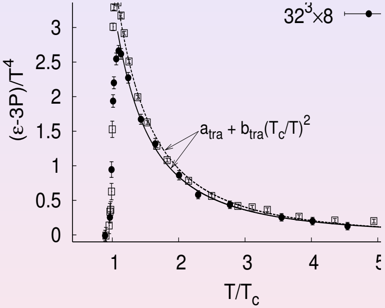
Perturbation theory to two loops (J.I.Kapusta, NPB148 (1979)):

$$\Delta \equiv \frac{\epsilon - 3p}{T^4} = \frac{N_c(N_c^2-1)}{1152\pi^2}\beta_0 g(T)^4 + \mathcal{O}(g^5)$$

where $1/g^2(\mu) = \beta_0 \log(\mu^2/\Lambda^2_{\rm QCD})$

Power temperature corrections from Lattice data

Trace Anomaly $N_c = 3, N_f = 0$
G. Boyd et al., Nucl. Phys. B469, 419 (1996).



$$\frac{\epsilon - 3P}{T^4} = a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,, \quad a_{\rm NP} = (3.46 \pm 0.13)\,T_c^2\,, \qquad 1.13\,T_c < T < 4.5\,T_c\,.$$

The fuzzy bag of Pisarski

Low temperature (confined) $\to$ glueball gas

$$P_{\text{glueball}}(T) =\sim e^{-M_G/T} \qquad M_G \gg T_c \to P_{\text{glueball}}(T_c) = 0$$

High temperature (deconfined) $\to$ free gluon gas

$$P_{\text{gluons}}(T) = \frac{b_0}{2} T^4 \qquad b_0 = \frac{(N_c^2 - 1)\pi^2}{45}$$

Pisarski's (temperature dependent) fuzzy bag , PTP 2006

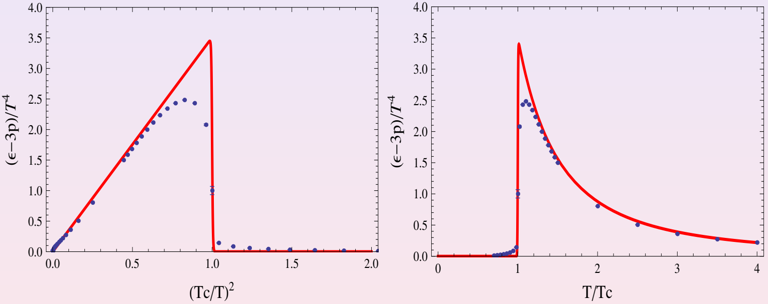
$$P(T) = P_{\text{gluons}}(T) - B_{\text{fuzzy}}(T), \qquad T > T_c\,, \qquad P(T_c) = P_{\text{glueballs}}(T_c) = 0$$

$$B_{\text{fuzzy}} = \frac{b_0}{2} T_c^2 T^2 \qquad \to \qquad P = \frac{b_0}{2}(T^4 - T^2 T_c^2)$$

Then

$$\Delta \equiv \frac{\epsilon - 3P}{T^4} = b_0 \left(\frac{T_c}{T}\right)^2 \qquad b_0 = 3.45(3.5 Fit!!!!) \tag{6}$$

Trace Anomaly $N_c = 3, N_f = 0$
JHEP Wuppertal 2012



$$\Delta(T) = \frac{(N_c^2 - 1)\pi^2}{45} \left(\frac{T_c}{T}\right)^2 \theta(T - T_c)$$

Static energies and Casimir scaling

The interaction between heavy sources A and B in perturbation theory

$$V_{AB}(r) = \lambda_A \cdot \lambda_B \frac{\alpha_S}{r}$$

The interaction between heavy sources on the lattice

$$E_{AB}(r) = \lambda_A \cdot \lambda_B \left[\frac{\alpha_S}{r} + \sigma r\right]$$

Casimir scaling requires that the ratio between the fundamental $Q\bar{Q} \equiv \mathbf{3} \times \bar{\mathbf{3}}$ and adjoint $GG \equiv \mathbf{8} \times \mathbf{8}$ colour sources are

$$V_{Q\bar{Q}}(r) = \sigma_F r - \frac{4\alpha_s}{3r} + \dots \quad (7)$$

$$V_{GG}(r) = \sigma_A r - \frac{3\alpha_s}{r} + \dots \quad (8)$$

$$\frac{\sigma_A}{\sigma_F} = \frac{9}{4} \quad (9)$$

Polyakov loop correlators in higher representations

Polyakov line in the fundamental representation

$$\Omega_F(\vec{x}) = e^{iA_0(\vec{x})/T} \qquad A_0 = \sum_{a=1}^{N_c^2-1} \lambda_a A_0^a$$

The interaction between heavy sources A and B

$$\langle \mathrm{Tr}_F \Omega(\vec{x}_1) \mathrm{Tr}_F \Omega(\vec{x}_2)^\dagger \rangle \to e^{-\sigma_F |\vec{x}_1 - \vec{x}_2|/T}$$

In general (Casimir scaling)

$$\langle \mathrm{Tr}_R \Omega(\vec{x}_1) \mathrm{Tr}_R \Omega(\vec{x}_2)^\dagger \rangle \to e^{-\sigma_R |\vec{x}_1 - \vec{x}_2|/T} \qquad \sigma_R = (C_R/C_F)\sigma_F$$

Glueball spectrum

Two masless spin-1 particles in CM system. Salpeter equation for the mass operator

$$\hat{M} = 2p + \sigma_A r \qquad \sigma_A = \frac{9}{4}\sigma$$

Uncertainty principle for the ground state $pr \sim 1$

$$M_0 = \min\left[\frac{2}{r} + \sigma_A r\right] = 2\sqrt{2\sigma_A} = 3.4\sqrt{\sigma}$$

WKB spectrum for excited states. Bohr-Sommerfeld quantization condition (L=0)

$$2\int_0^a dr p_r = 2(n+\alpha)\pi \quad \to M_n^2 = 4\pi\sigma_A(n+\alpha)$$

Glueball spectrum of two gluons

$$(2p + \sigma_A r)\psi_n = M_n \psi_n$$

Harmonic oscillator wave functions

$$R_{nl}(r) = \frac{u_{nl}(r)}{r} = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{r^2}{2b^2}} \left(\frac{r}{b}\right)^l \sqrt{\frac{(n-1)!2^{l+n+1}}{b^3(2l+2(n-1)+1)!!}} L_{n-1}^{l+\frac{1}{2}}\left(\frac{r^2}{b^2}\right)$$

$L_{n-1}^{l+\frac{1}{2}}(x)$ are asociated Laguerre polynomials.

$$-u''_{nl}(r) + \left[\frac{r^2}{b^4} + \frac{l(l+1)}{r^2}\right] u_{nl}(r) = \frac{1}{b^2}(2l + 4n - 1) u_{nl}(r)$$

Normalization

$$\int_0^\infty dr r^2 R_{nl}(r)^2 = \int_0^\infty dr u_{nl}(r)^2 = 1$$

where b has dimensions of length. The single-particle energies are

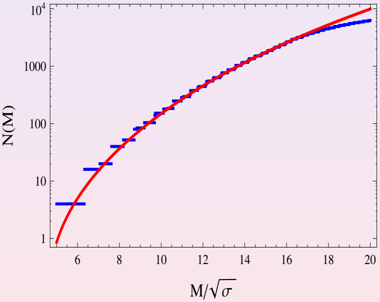
$$\epsilon_{nl} = \frac{1}{2Mb^2}(4n + 2l - 1) = \omega\,(2n + l - 1/2)$$

where the oscillator frequency is $\omega = 1/(Mb^2)$.

At large masses a derivative expansion at long distances

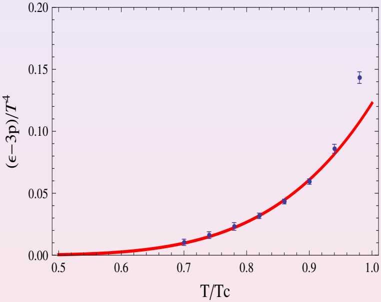
$$N_{2g}(M) \to g^2 \int \frac{d^3x d^3p}{(2\pi)^3} \theta(M - H(p,r)) + \mathcal{O}(\nabla H)$$

$$= \frac{g^2 M^6}{720\pi\sigma_A^3} + \frac{\alpha_s g^2 M^4}{16\pi\sigma_A^2} + \frac{9\alpha_s^2 g^2 M^2}{8\pi\sigma_A} - \frac{g^2 M^2}{9\pi\sigma_A} + \dots$$



Trace anomaly

$$\Delta^{2g}_{\text{glueball}}(T) = \sum_n \frac{1}{2\pi^2 k} K_1\left(\frac{kM_n}{T}\right)\left(\frac{M_n}{T}\right)^3$$



$$\frac{T_c}{\sqrt{\sigma}} = 0.736385 \qquad \text{Lattice} \quad 0.629(3)$$

Trace anomaly (WKB)

$$\Delta(T) = \sum_{k=1}^{\infty} \int dM \frac{\partial N(M)}{\partial M} \frac{1}{2k\pi^2} \left(\frac{M}{T}\right)^3 K_1\left(k\frac{M}{T}\right)$$

Large M expansion $\rightarrow$ Large T expansion

$$N(M) = \sum_n a_n M^n$$

$$\int_0^{\infty} nM^{n-1} \left(\frac{M}{T}\right)^3 \frac{1}{2\pi^2 k} K_1(kM/T) = \frac{2^n n k^{-n-4} T^n \Gamma\left(\frac{n}{2}+1\right)\Gamma\left(\frac{n}{2}+2\right)}{\pi^2}$$

$$\Delta(T) = \sum_n a_n \frac{2^n n T^n \zeta(n+4)\Gamma\left(\frac{n}{2}+1\right)\Gamma\left(\frac{n}{2}+2\right)}{\pi^2}$$

$$\Delta_{2g}(T) = \frac{2048\pi^8}{3465} a_6 T^6 + \frac{128\pi^6}{1575} a_4 T^4 + \frac{128\pi^6}{1575} a_2 T^2$$

Multigluon states

$$H_n = \sum_{i=1}^{N} p_i + \sum_{i<j} \sigma_A |\vec{x}_i - \vec{x}_j| \tag{10}$$

In the CM system

$$N_n(M) \sim \int \prod_{i=1}^{N} \frac{d^3x_i d^3p_i}{(2\pi)^3} \theta(M - H_n)\delta(\sum_i \vec{x}_i)\delta(\sum_i \vec{p}_i) \sim \left(\frac{M^2}{\sigma_A}\right)^{6n-6} \tag{11}$$

$$\Delta_{ng}(T) \sim \left(\frac{T^2}{\sigma_A}\right)^{6n-6} \tag{12}$$

Scale separation between 2g-WKB and 3g glueballs

$$\Delta_{3g}(T) \sim e^{-M_{3g}/T} << \Delta_{2g}(T) \tag{13}$$

Gluelump spectrum

One masless spin-1 particle and one gluon source (infinitely heavy) in CM system. Salpeter equation for the mass operator

$$\hat{\Delta} = p + \sigma_A r \qquad \rightarrow \quad M_{\rm gluelump} = M_{\rm glueball}/\sqrt{2}$$

The smallest mass gap is the gluelump not the glueball !
The partition function

$$Z_{\rm gluelumps}(T) = Z_{\rm glueballs}(T/\sqrt{2})/g$$

Quark-Hadron duality for the Polyakov loop at low temperatures

$$\langle \Omega_8 \rangle_T \sim Z_{\rm gluelumps}(T) = \sum_n e^{-\Delta_n/T} \neq 0 \quad (T < T_c)$$

Higher representations in the gauge group, multigluon states ...

COUPLING QUARKS AND GLUONS WITH POLYAKOV LOOPS

Lattice results in full QCD

The chiral-deconfinement cross over is a unique prediction of lattice QCD

- Order parameter of chiral symmetry breaking ($m_q = 0$)
  Quark condensate $SU(N_f) \otimes SU(N_f) \to SU_V(N_f)$

$$\langle \bar{q}q \rangle \neq 0 \quad T < T_c \qquad \langle \bar{q}q \rangle = 0 \quad T > T_c$$

- Order parameter of deconfinement ($m_q = \infty$)
  Polyakov loop: Center symmetry $Z(N_c)$ broken

$$L_T = \frac{1}{N_c} \langle \mathrm{tr}_c e^{iA_0/T} \rangle = 0 \quad T < T_c \qquad L_T = \frac{1}{N_c} \langle \mathrm{tr}_c e^{iA_0/T} \rangle = 1 \quad T > T_c$$

- In the real world m_q is finite but inflexion points nearly coincide (accidental)

$$\frac{d^2}{dT^2} L_T = 0 \qquad \frac{d^2}{dT^2} \langle \bar{q}q \rangle_T = 0$$

  For about the same $T_c = 155(10)$

Temperature ranges

Momentum scale $p \sim 2\pi T$ (thermal wavelength)

$$T_c = 150\mathrm{MeV} \to p = 1000\mathrm{MeV}$$

- Low temperatures $\to$ Chiral Perturbation theory (ChPT) (pions dominate)
$$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = 1 - \frac{T^2}{8f_\pi^2} - \frac{T^4}{384 f_\pi^4} + \dots$$
$$L(T) = 0$$
- Intermediate temperatures $\to$ Hadron Resonance Gas (HRG) (treats $\pi\pi$ interactions as a ρ-resonance, large N_c physics)
- Phase transition (renormalization)
- Above the phase transition (condensates, dim-red)
- Not too high (hard thermal loops) $T \geq 2T_c$
- High temperature $\to$ Perturbation Theory (pQCD)

Chiral Quark Models at Finite T

- Chiral Quark Models $\longrightarrow$ Dynamics of QCD at low energies (low temperatures).
- Chiral Perturbation Theory $\longrightarrow$ Suppose the non-vanishing of chiral condensate. It cannot describe the QCD phase transition.
- Ogilvie and Meissinger PLB (1995) K. Fukushima, PLB591, 277 (2004). W. Weise et al. PRD73, 014019 (2006), N. Scoccola, D. G. Dumm (2008), S.K. Ghosh et al. PRD73, 114007 (2006), Minimal coupling of Polyakov loop (analogy with chemical potential). **Mean field approximation**.
- E.Megías, E.Ruiz Arriola and L.L.Salcedo, **PRD74**: 065005 (2006). **Quantum and local polyakov loop**

Minimal coupling of the Polyakov loop

Constituent Quark model:

$$\mathcal{L}_{\rm QC} = \overline{q}\, \mathbf{D}\, q\,, \qquad \mathbf{D} = \partial\!\!\!/ + V\!\!\!/^{f} + A\!\!\!/^{f} + MU^{\gamma_5} + \hat{m}_0$$

Consider the minimal coupling of the gluons in the model:

$$V_\mu^f \longrightarrow V_\mu^f + g V_\mu^c\,, \quad V_\mu^c = \delta_{\mu 0} V_0^c$$

Covariant derivative expansion (E. Megías et al. PLB563(2003), PRD69(2004), Oswald and Dyakonov PRD (2004)).

$$\mathcal{L}(x) = \sum_n \mathrm{tr}[f_n(\Omega(x))\mathcal{O}_n(x)]\,, \qquad \Omega(\vec{x}, x_0) = \mathbb{P}\, e^{i \int_{x_0}^{x_0+\beta} dx_0' V_0^c(\vec{x}, x_0')}$$

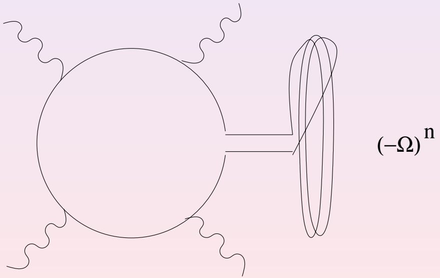
Ω enters in: $\hat{\omega}_{\mathbf{n}} = \mathbf{2}\pi\mathbf{T}(\mathbf{n} + \mathbf{1}/\mathbf{2} + \hat{\nu})\,, \qquad \Omega = e^{i2\pi\hat{\nu}}$

The rule to pass from $T = 0$ to $T \neq 0$ is:

$$\tilde{F}(x;x) \to \sum_{n=-\infty}^{\infty} (-\Omega(\vec{x}))^n \tilde{F}(\vec{x}, x_0 + n\beta; \vec{x}, x_0)\,.$$

The quark condensate writes:

$$\langle \overline{q} q \rangle^* = \sum_n \frac{1}{N_c} \langle \mathrm{tr}_c (-\Omega)^n \rangle \langle \overline{q}(n\beta) q(0) \rangle\,.$$



Peierls-Yoccoz projection on color singlets

- We introduce a colour source (Polyakov loop).
- We obtain the projection onto the color neutral states by integrating over the A_0 field.
- In Quenched approximation: Group integration in SU(N_c).

$$\langle \mathrm{tr}_c(-\Omega)^n \rangle \equiv \int_{\mathrm{SU}(N_c)} D\Omega \, \mathrm{tr}_c(-\Omega)^n = \begin{cases} N_c \,, & n = 0 \\ -1 \,, & n = \pm N_c \\ 0 \,, & \text{otherwise} \end{cases}$$

There is only contribution from $n = 0, \pm N_c$.

$$\langle \bar{\mathbf{q}}\mathbf{q} \rangle^* \overset{\text{Low T}}{\sim} \langle \bar{q}q \rangle + 4 \left(\frac{MT}{2\pi N_c} \right)^{3/2} e^{-N_c M/T} \, .$$

The N_c suppression is consistent with ChPT.

- Beyond the Quenched approximation:

$$Z = \int DUD\Omega \, e^{-\Gamma_G[\Omega]} \, e^{-\Gamma_Q[U,\Omega]}$$

For any observable: $\langle \mathcal{O} \rangle^* = \frac{1}{Z} \int DUD\Omega \, e^{-\Gamma_G[\Omega]} \, e^{-\Gamma_Q[U,\Omega]} \, \mathcal{O}$.

1. $\int$ **DU**: Saddle point approximation.
2. $\int$ **D**Ω:
   - Analytically $\longrightarrow$ Expand the exponents and compute correlation functions of Polyakov loops:

$$\int D\Omega \, \mathrm{tr}_c \Omega(\vec{x}) \, \mathrm{tr}_c \Omega^{-1}(\vec{y}) = e^{-\sigma|\vec{x}-\vec{y}|/T}$$

   - Numerically $\longrightarrow$ Consider the Polyakov gauge.

Analytical results in the Unquenched Theory

In the NJL model with Polyakov loop:

$$\langle \bar{q}q \rangle^* \overset{\text{Low T}}{\sim} \langle \bar{q}q \rangle + \frac{N_f V}{\pi^3}(MT)^3 e^{-2M/T} + \mathcal{O}(e^{-N_c M/T})$$

$$L \equiv \left\langle \frac{1}{N_c}\mathrm{tr}_c \Omega \right\rangle \overset{\text{Low T}}{\sim} \frac{N_f}{N_c}\frac{V}{T}\sqrt{\frac{M^3 T^5}{2\pi^3}} e^{-M/T} + \mathcal{O}(e^{-2M/T})$$

L-small because Spontaneous Chiral Symmetry Breaking Taking into account the quark binding effects:

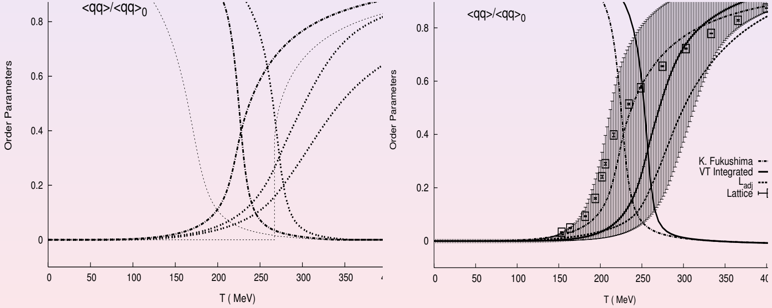
$$\mathcal{O}_q^* = \mathcal{O}_q + \sum_{m_\pi} \mathcal{O}_{m_\pi} \frac{1}{N_c} e^{-m_\pi/T} + \sum_B \mathcal{O}_B e^{-M_B/T} + \dots$$

Phase transition using confinenement domains

$$\langle \mathrm{tr}_c \Omega(\vec{x}) \, \mathrm{tr}_c \Omega^{-1}(y) \rangle_{S_G} = e^{-\sigma |\vec{x}-y|/T} ,$$

We take Ω x-independent in a volume

$$V_\sigma = \int d^3 x e^{-\sigma r/T} = \frac{8\pi T^3}{\sigma^3}$$



Polyakov “cooling” : The condensate does not change at low temperatures.

POLYAKOV LOOP SPECTROSCOPY

Quark-Hadron Duality at Finite Temperature

Partition function for N_f-flavours

$$Z_{\rm HRG}(N_f) \equiv \int D\Omega \, e^{-S(N_f)} \qquad S(N_f) = S_q(N_f) + S_G$$

Quark contribution

$$S_q(N_f) = -2N_f \int \frac{d^3x d^3p}{(2\pi)^3} \left({\rm tr}_c \log \left[1 + \Omega(x) \, e^{-E_p/T} \right] + {\rm c.c.} \right)$$

One extra HEAVY QUARK (not anti-quark) with flavour a

$$S_q(N_f+1) - S_q(N_f) \quad = \quad -2 \log(1 + \Omega_{aa} e^{-E_h/T}) \approx -2 e^{-m_H/T} \Omega_{aa}$$

$$\frac{1}{N_c} \langle {\rm tr}_c \Omega \rangle = \lim_{m_H \to \infty} \frac{1}{2} \left[\frac{Z_{\rm HRG}(N_f+1)}{Z_{\rm HRG}(N_f)} - 1 \right] e^{m_H/T} = \frac{1}{2N_c} \sum_\alpha g_\alpha e^{-\Delta_\alpha/T}$$

$$\Delta_\alpha = \lim_{m_H \to \infty} (M_{H,\alpha} - m_H)$$

Polyakov loop in the HRG model

$$L(T) \sim \frac{1}{2N_C} \sum_{\alpha} g_\alpha e^{-\Delta_\alpha / T}$$

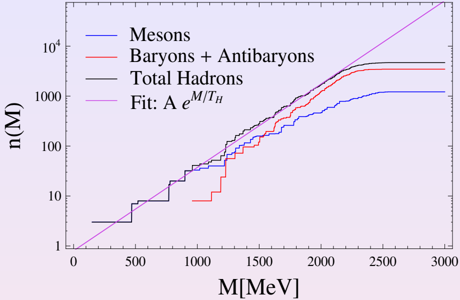
From the PDG

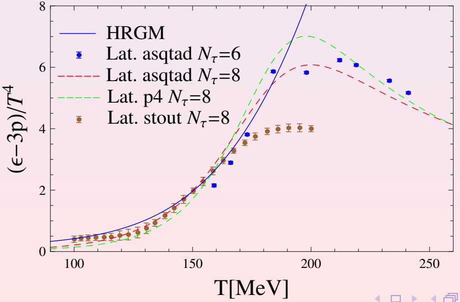
$$\begin{aligned} M_K - m_s \equiv \Delta_s &= 396(24)\,, \\ M_D - m_c \equiv \Delta_c &= 603(81)\,, \\ M_B - m_b \equiv \Delta_b &= 1040(130)\,. \end{aligned} \tag{14}$$

We use single charm states

- Mesons $D^0, D^+, D_s^+, D^{*0}, D^{*+}, D_s^{*+}$. A
- Baryons $\Sigma_c^0, \Sigma_c^+, \Sigma_c^{++}, \Xi_c^+, \Xi_c^0, \Omega_c^0, \Lambda_c^+$
- Many $\bar{q}q$ and qqq states needed $\to$ Relativized-Quark-Model (Isgur) ; MIT Bag Model
- Possibility of discernig exotics $\bar{q}q\bar{q}q$, $\bar{q}qqqq$ from data at finite temperature !!

Hadron Resonance Gas from Chiral Quark Models





FROM CHIRAL QUARK MODELS TO THE HADRON RESONANCE GAS

Quantization of multiquark states

- Multiquark states: Create/Anhiquilate a quark at point $\vec{x}$ and momentum p

$$\Omega(x) e^{-E_P/T} \qquad \Omega(x)^+ e^{-E_P/T}$$

- At low temperatures quark Boltzmann factor small $e^{-E_p/T} < 1$. The action becomes small

$$S_q[\Omega] = 2N_f \int \frac{d^3x d^3p}{(2\pi)^3} \left[\mathrm{tr}_c \Omega(x) + \mathrm{tr}_c \Omega(x) \right] e^{-E_p/T} + \dots$$

$$Z = \int D\Omega \, e^{-S[\Omega]} = \int D\Omega \left(1 - S[\Omega] + \frac{1}{2} S[\Omega]^2 + \dots \right)$$

- $\bar{q}q$ contribution

$$Z_{\bar{q}q} = (2N_f)^2 \int \frac{d^3x_1 d^3p_1}{(2\pi)^3} \int \frac{d^3x_2 d^3p_2}{(2\pi)^3} e^{-E_1/T} e^{-E_2/T} \underbrace{\langle \mathrm{tr}_c \Omega(\vec{x}_1) \mathrm{tr}_c \Omega^\dagger(\vec{x}_2) \rangle}_{e^{-\sigma[\vec{x}_1 - \vec{x}_2|/T}}$$

$$= (2N_f)^2 \int \frac{d^3x_1 d^3p_1}{(2\pi)^3} \frac{d^3x_2 d^3p_2}{(2\pi)^3} e^{-H(x_1,p_1;x_2,p_2)/T}$$

$\bar{q}q$ Hamiltonian

$$H(x_1, p_1; x_2, p_2) = E_1 + E_2 + V_{12} \,.$$

- Quantization in the CM frame $p_1 = -p_2 \equiv p$

$$\left(2\sqrt{p^2 + M^2} + V_{q\bar{q}}(r) \right) \psi_n = M_n \psi_n \,.$$

- Boosting the CM to any frame with momentum P

$$Z_{\bar{q}q} \to \sum_n \int \frac{d^3R d^3P}{(2\pi)^3} e^{-E_n(P)/T}$$

A GAS OF NON INTERACTING MESONS ! (valid to $\bar{q}q\bar{q}q$)

Polyakov loop in the quark model

$$L_T = 2N_f \int \frac{d^3x\, d^3p}{(2\pi)^3} e^{-E_p/T} \frac{1}{N_c} \underbrace{\langle \mathrm{tr}_c \Omega(\vec{x}_0) \mathrm{tr}_c \Omega^\dagger(\vec{x}) \rangle}_{e^{-\sigma[\vec{x}_0 - \vec{x}|/T}} + \cdots$$

$$= \frac{2N_f}{N_c} \int \frac{d^3x\, d^3p}{(2\pi)^3} e^{-H(\vec{x},\vec{p})/T}$$

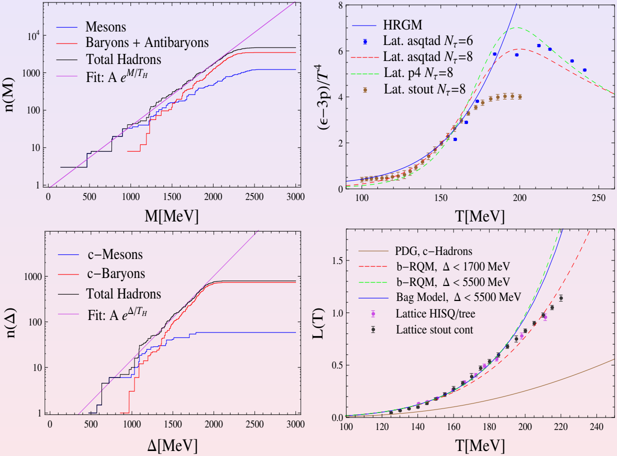
Heavy-light ground state system

$$(\sqrt{p^2 + m_q^2} + \sigma r)\psi_n = \Delta_n \psi_n$$

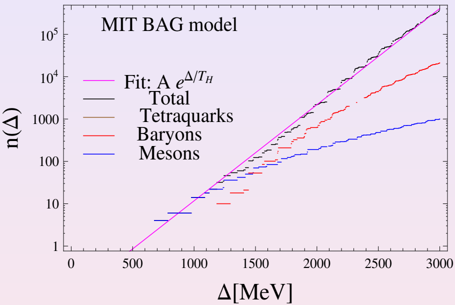
In the limit $m_q \to 0$ we make $p \sim 1/r$ and $\Delta \sim 2\sqrt{\sigma} \sim 900\mathrm{MeV}$

$$N_c L(T) \sim 2N_f e^{-\Delta_M/T} + (2N_f^2 + N_f) e^{-\Delta_B/T} + \cdots = 21 e^{-\bar{\Delta}/T} \qquad (N_f = 3)$$

Spectrum vs Thermodynamics



Hagedorn and The boostrap



Which are the complete set of states in the PDG ?
Should X,Y,Z's or the deuteron or ^{208}Pb enter as multiquark states ?

CONCLUSIONS

Conclusions:

- Quark Hadron Duality suggests that at low temperatures Hadrons can be considered as a complete basis of states in terms of a hadron resonance gas. The HRG works up to relatively large temperatures.
- PDG states incorporate currently just $q\bar{q}$ or qqq states which fit into the quark model. What states are needed when approaching the crossover from below ?
- Saturating at subcritical temperatures requires many hadronic states, so the excited spectrum involves relativistic effects even for heavy quarks.
- Polyakov loops in fundamental and higher representations allow to deduce multiquark quark states, gluelumps etc. containing one or several heavy quark states. This goes beyond the models and opens up the possibility of a Polyakov loop spectroscopy in cluding exotics.