Introduction to the Physics of Saturation

Yuri Kovchegov
The Ohio State University

Outline

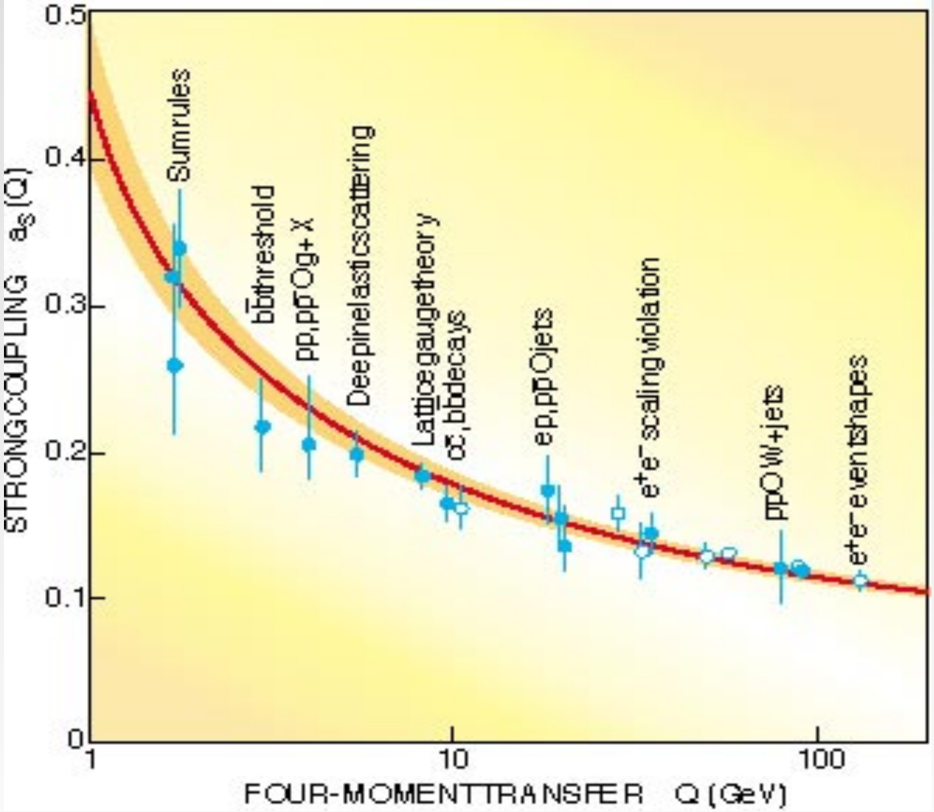
- General concepts
- Classical gluon fields, parton saturation
- Quantum (small-x) evolution
  - Linear BFKL evolution
  - Non-linear BK and JIMWLK evolution
- Recent progress (selected topics)
- Phenomenology: DIS, Heavy Ions

General Concepts

Running of QCD Coupling Constant

⇨ QCD coupling constant $\alpha_S = \dfrac{g^2}{4\pi}$ changes with the momentum scale involved in the interaction

$$\alpha_S = \alpha_S(Q)$$



Asymptotic Freedom!

Gross and Wilczek,
Politzer, ca '73

Physics Nobel Prize 2004!

For short distances x < 0.2 fm, or, equivalently, large momenta k > 1 GeV the QCD coupling is small $\alpha_S << 1$ and interactions are weak.

A Question

- Can we understand, qualitatively or even quantitatively, the structure of hadrons and their interactions in High Energy Collisions?
  - What are the total cross sections?
  - What are the multiplicities and production cross sections?
  - Diffractive cross sections.
  - Particle correlations.

What sets the scale of running QCD coupling in high energy collisions?

- "String theorist": $\alpha_S = \alpha_S\left(\sqrt{s}\right) << 1$

  (not even wrong)

- Pessimist: $\alpha_S = \alpha_S\left(\Lambda_{QCD}\right) \sim 1$ we simply can not tackle high energy scattering in QCD.

- pQCD expert: only study high-p_T particles such that

$$\alpha_S = \alpha_S\left(p_T\right) << 1$$

  But: what about total cross section? bulk of particles?

What sets the scale of running QCD coupling in high energy collisions?

- Saturation physics is based on the existence of a large internal momentum scale Q_S which grows with both energy s and nuclear atomic number A

$$Q_S^2 \sim A^{1/3} \, s^{\lambda}$$

such that

$$\alpha_S = \alpha_S\left(Q_S\right) << 1$$

and we can calculate total cross sections, particle spectra and multiplicities, etc, from first principles.

The main principle

- Saturation physics is based on the existence of a large internal transverse momentum scale Q_S which grows with both decreasing Bjorken x and with increasing nuclear atomic number A

$$Q_s^2 \sim A^{1/3} \left(\frac{1}{x}\right)^{\lambda}$$

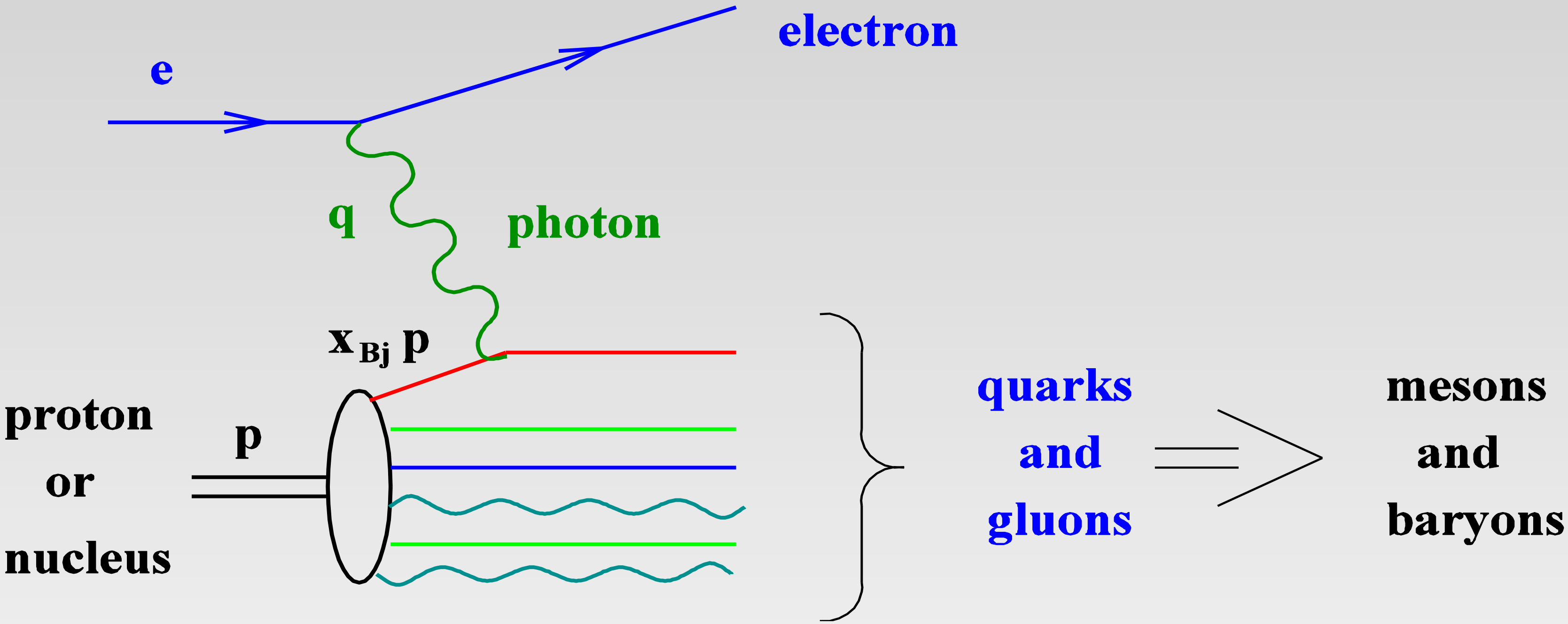
such that

$$\alpha_S = \alpha_S(Q_S) << 1$$

and we can use perturbation theory to calculate total cross sections, particle spectra and multiplicities, correlations, etc, from first principles.

Classical Fields

Kinematics of DIS



- Photon carries 4-momentum q_μ , its virtuality is

$$Q^2 = -q_\mu q^\mu$$

- Photon hits a quark in the proton carrying momentum $x_{Bj}p$ with p being the proton's momentum. Parameter x_{Bj} is called Bjorken x variable.

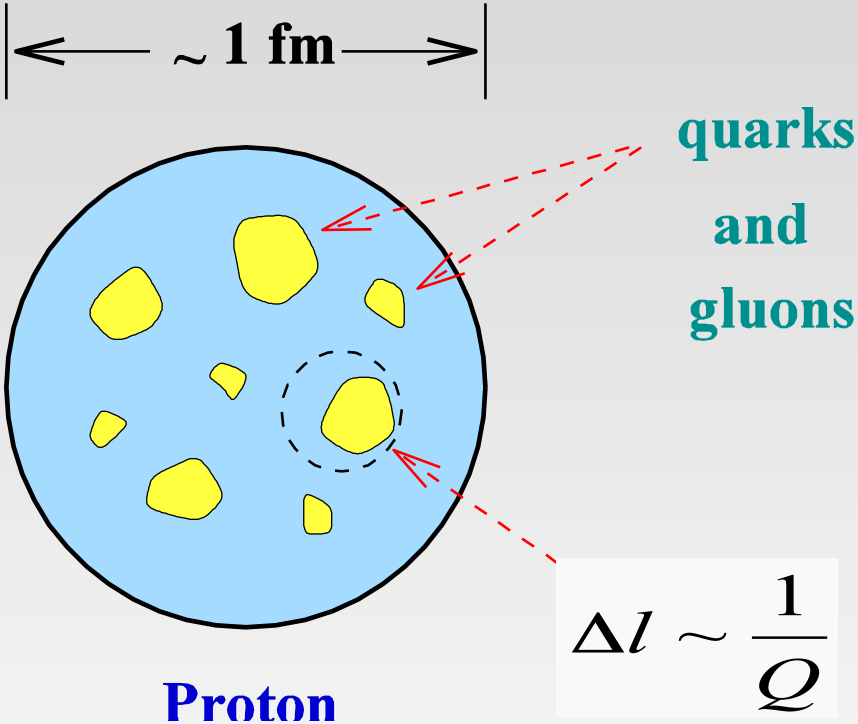
Physical Meaning of Q

Uncertainty principle teaches us that

$$\Delta p \Delta l \approx \hbar$$

which means that the photon probes the proton at the distances of the order (ħ=1)

$$\Delta l \sim \frac{1}{Q}$$



Large Momentum Q = Short Distances Probed

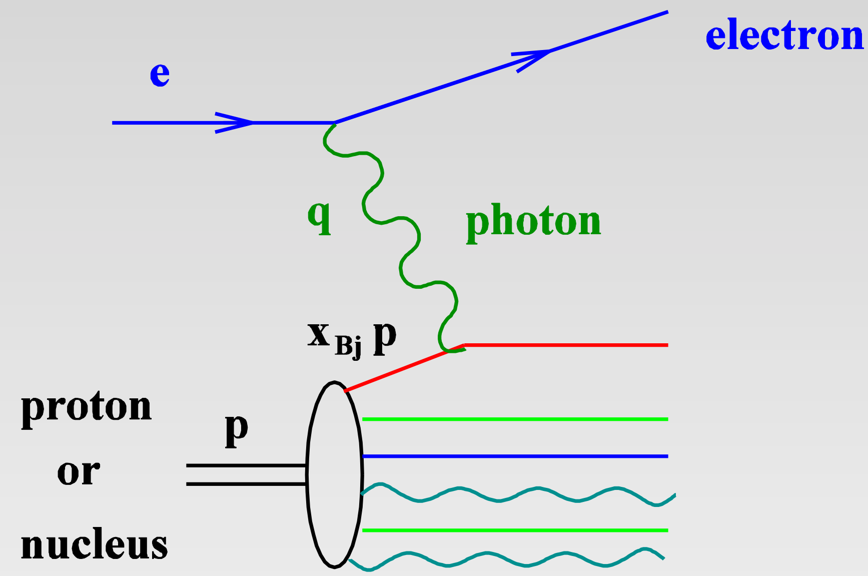
Physical Meaning of Bjorken x

In the rest frame of the electron the momentum of the struck quark is equal to some typical hadronic scale m:

$$x_{Bj} p \approx m$$

Then the energy of the collision

$$E \sim p \sim \frac{1}{x_{Bj}}$$

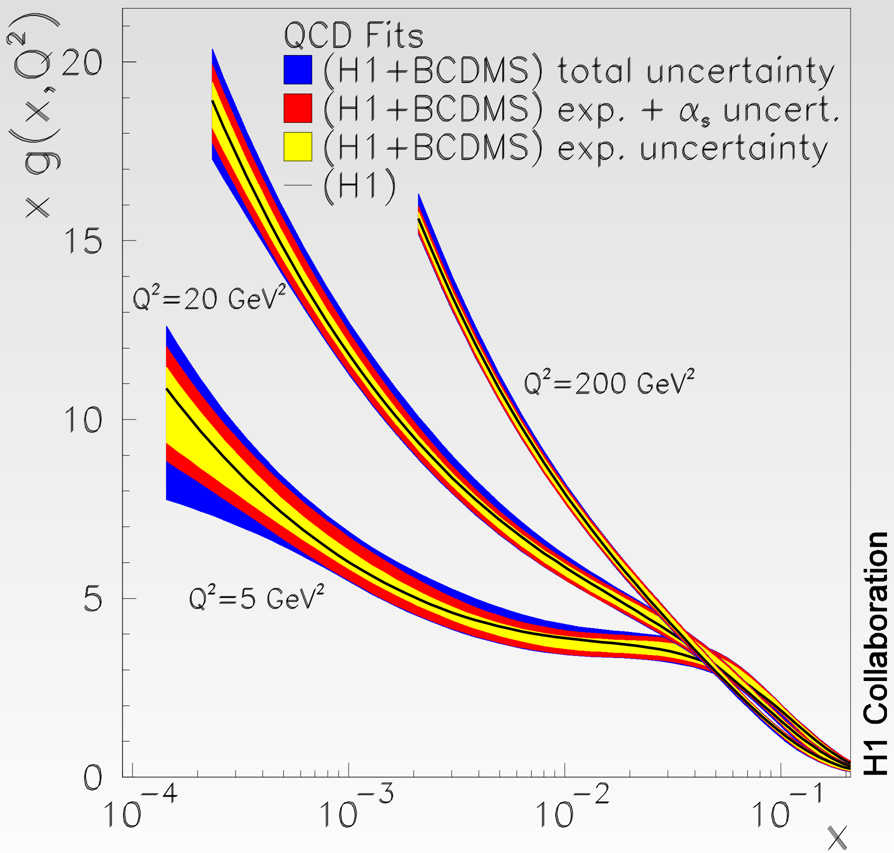


High Energy = Small x

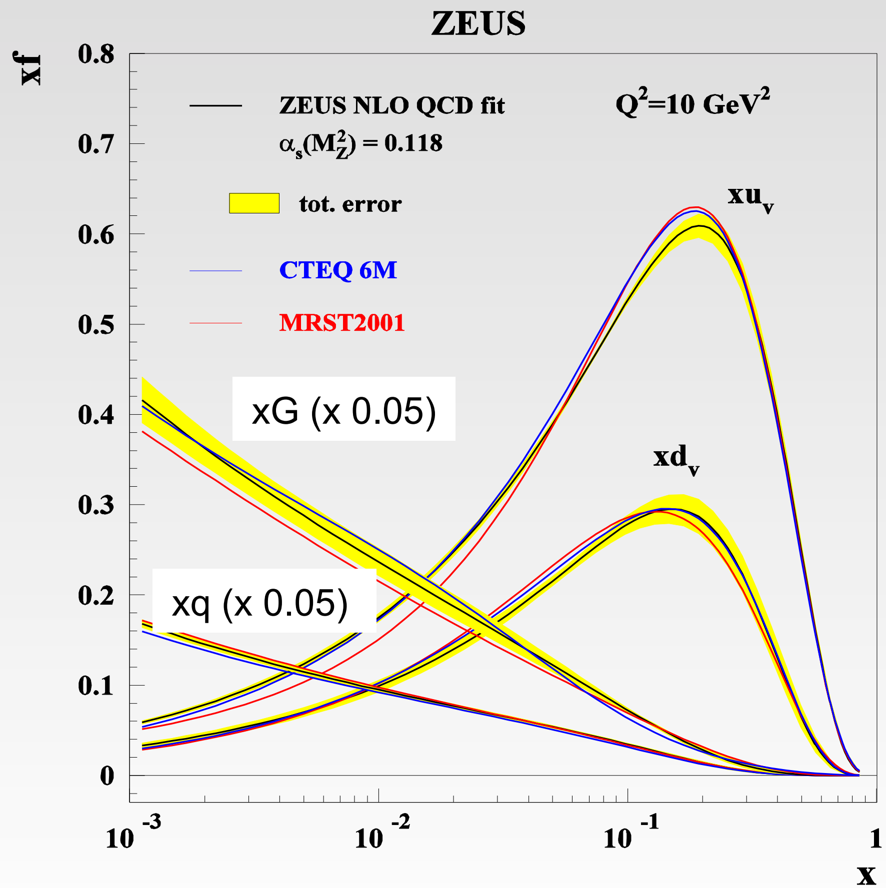
What have we learned at HERA?

Distribution functions $xq(x,Q^2)$ and $xG(x,Q^2)$ count the number of quarks and gluons with sizes ≥ 1/Q and carrying the fraction x of the proton's momentum.

Gluons only

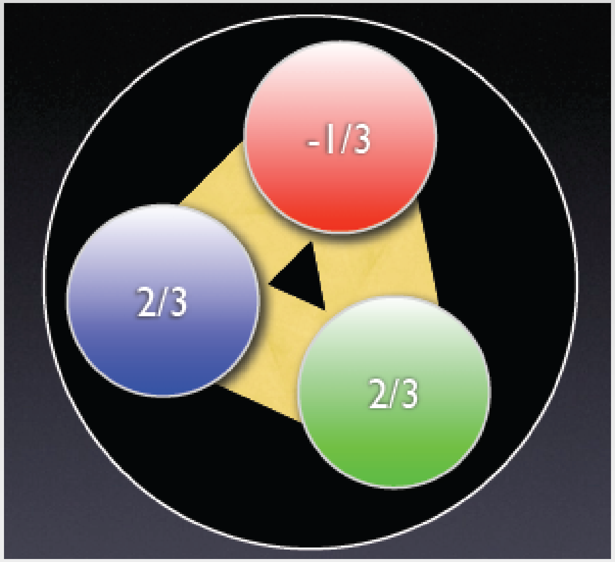


Gluons and Quarks



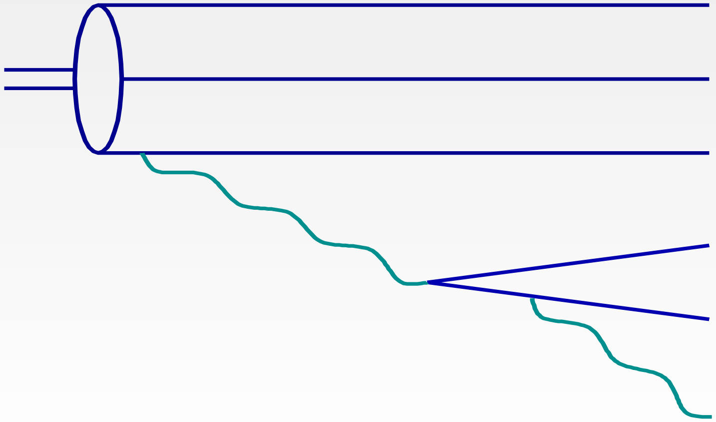
What have we learned at HERA?

⇨ There is a huge number of quarks, anti-quarks and gluons at small-x !



⇨ How do we reconcile this result with the picture of protons made up of three valence quarks?

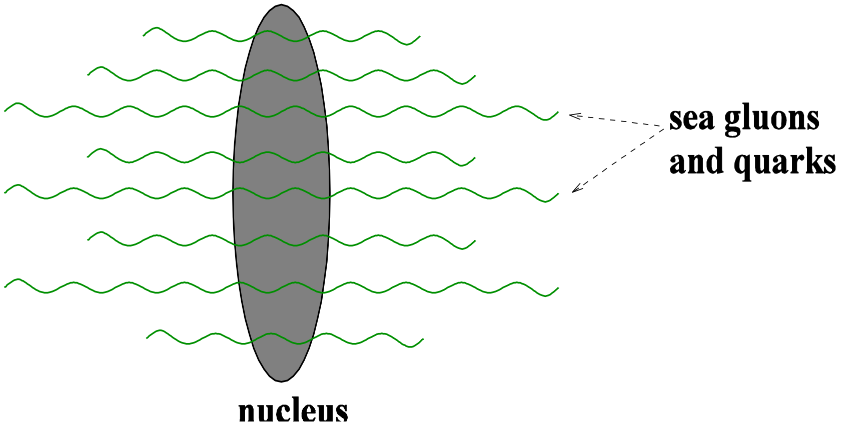
⇨ Qualitatively we understand that these extra quarks and gluons are emitted by the original three valence quarks in the proton.

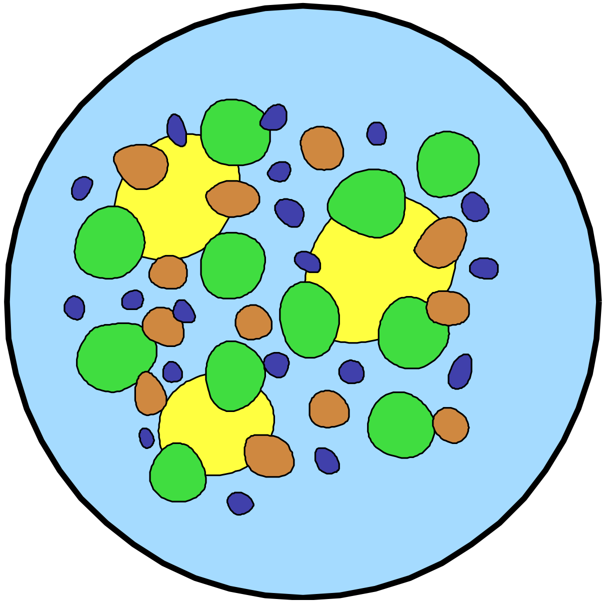


A. McLerran-Venugopalan Model

McLerran-Venugopalan Model

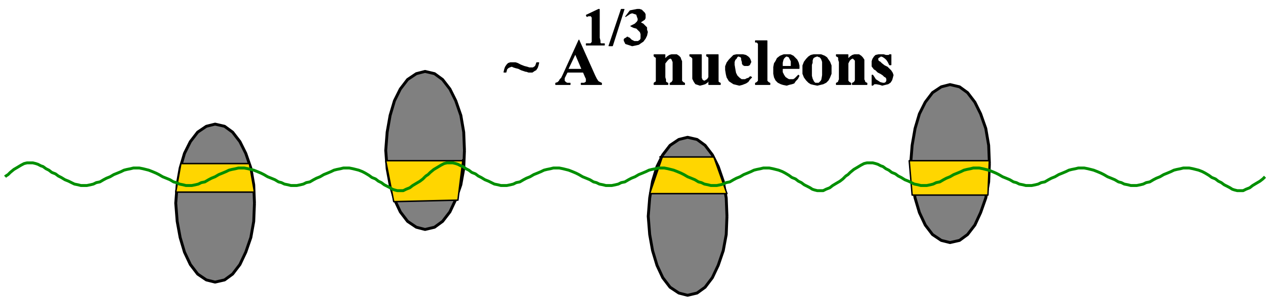
- The wave function of a single nucleus has many small-x quarks and gluons in it.
- In the transverse plane the nucleus is densely packed with gluons and quarks.

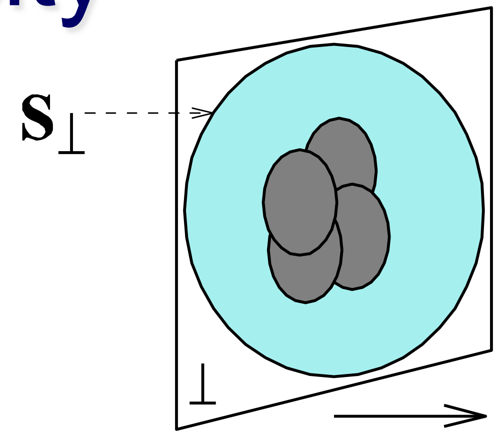




Large occupation number ⇨ Classical Field

Color Charge Density





Small-x gluon "sees" the whole nucleus coherently in the longitudinal direction! It "sees" many color charges which form a net effective color charge Q = g (# charges)$^{1/2}$, such that $Q^2 = g^2$ #charges (random walk).

Define color charge density

$$\mu^2 = \frac{Q^2}{S_\perp} = \frac{g^2 \ \# charges}{S_\perp} \propto g^2 \frac{A}{S_\perp} \propto A^{1/3}$$

McLerran
Venugopalan
'93-'94

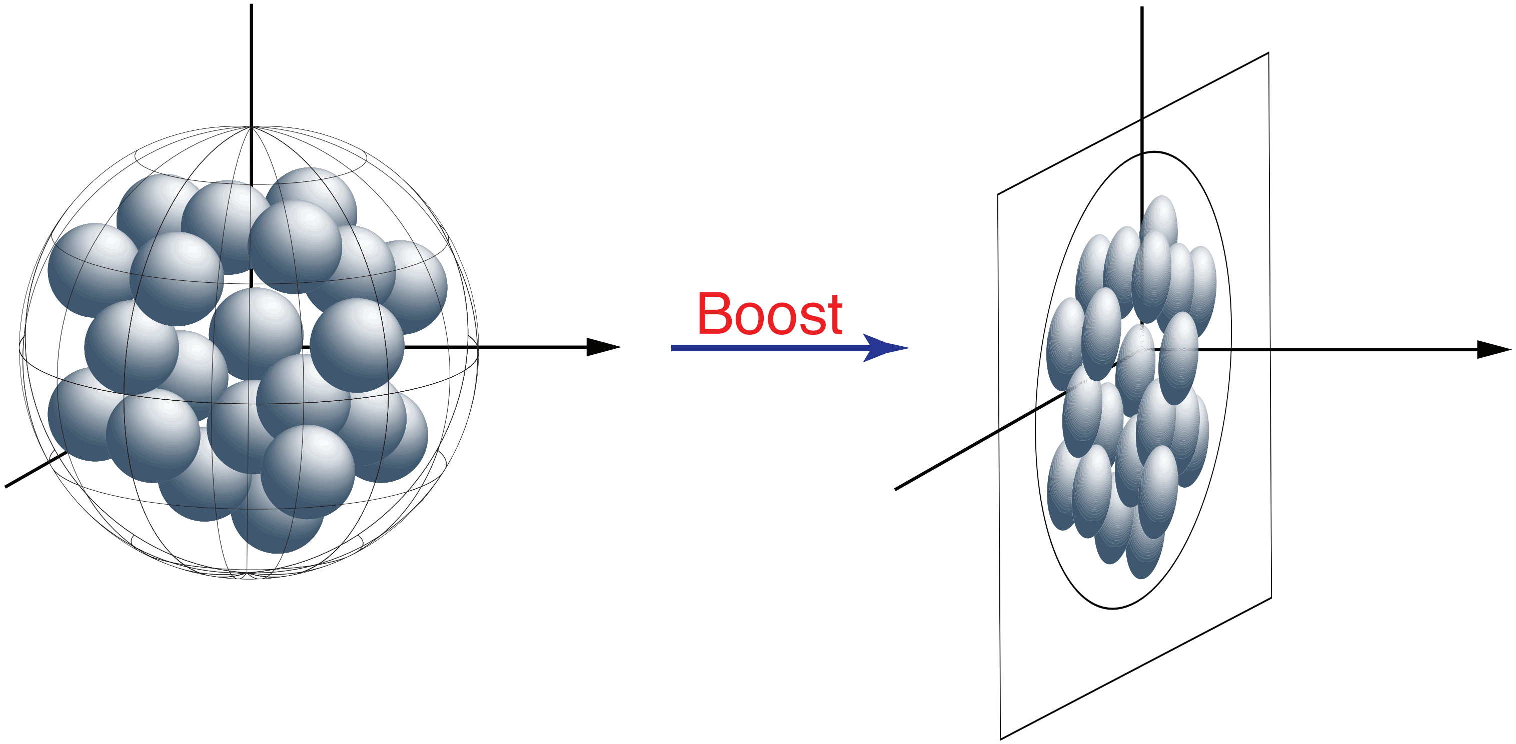
such that for a large nucleus (A>>1)

$$\mu^2 \propto \Lambda_{QCD}^2 A^{1/3} \gg \Lambda_{QCD}^2 \quad \Longrightarrow \quad \alpha_s(\mu^2) \ll 1$$

Nuclear small-x wave function is perturbative!!!

$$\mu = Q_s$$

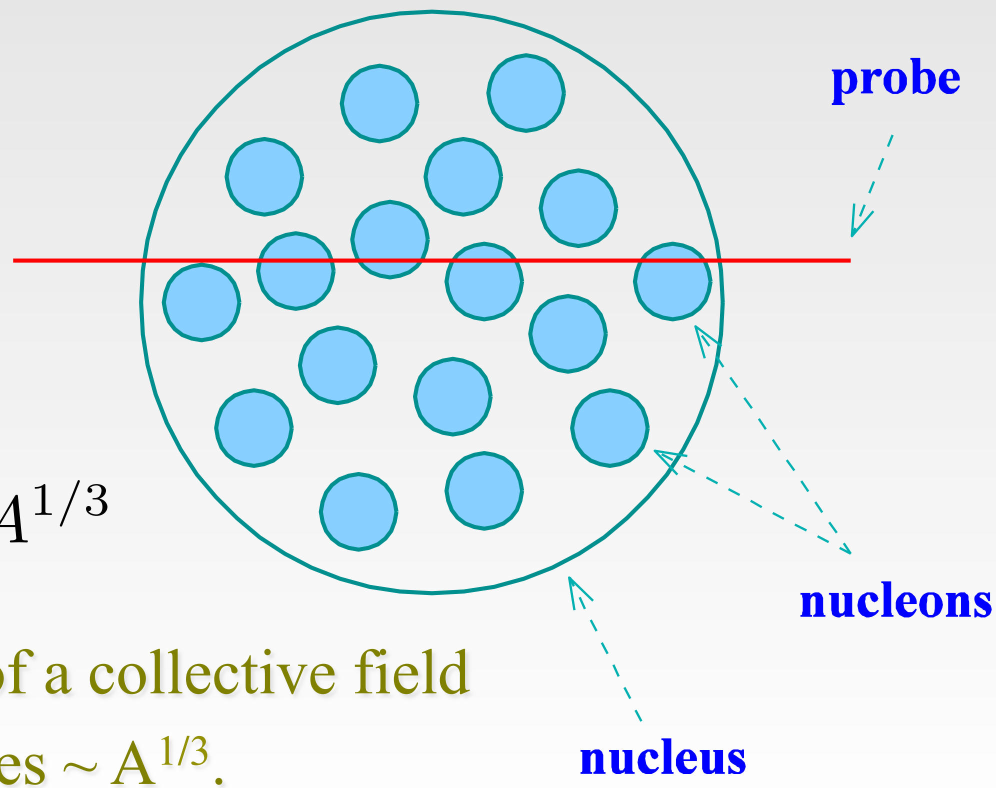
McLerran-Venugopalan Model



- Large parton density gives a large momentum scale Q_s (the saturation scale): Q_s^2 ~ # partons per unit transverse area.
- For $Q_s >> \Lambda_{QCD}$, get a theory at weak coupling $\alpha_s(Q_s^2) \ll 1$
- The leading gluon field is classical.

Saturation Scale

To argue that $Q_S^2 \sim A^{1/3}$ let us consider an example of a particle scattering on a nucleus. As it travels through the nucleus it bumps into nucleons. Along a straight line trajectory it encounters $\sim R \sim A^{1/3}$ nucleons, with R the nuclear radius and A the atomic number of the nucleus.



The particle receives $\sim A^{1/3}$ random kicks. Its momentum gets broadened by

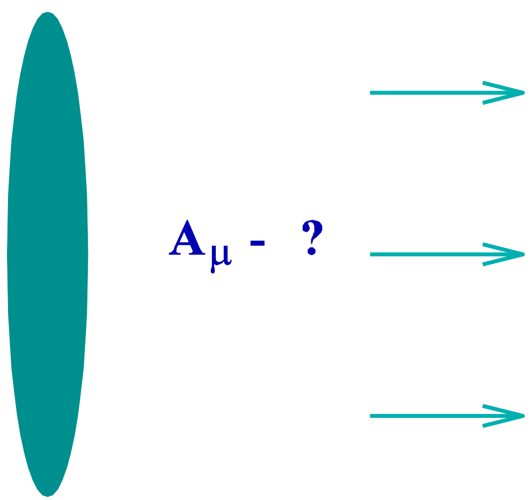
$$\Delta k \sim \sqrt{A^{1/3}} \Rightarrow (\Delta k)^2 \sim A^{1/3}$$

Saturation scale, as a feature of a collective field of the whole nucleus also scales $\sim A^{1/3}$.

McLerran-Venugopalan Model

- To find the classical gluon field A_μ of the nucleus one has to solve the non-linear analogue of Maxwell equations – the Yang-Mills equations, with the nucleus as a source of the color charge:

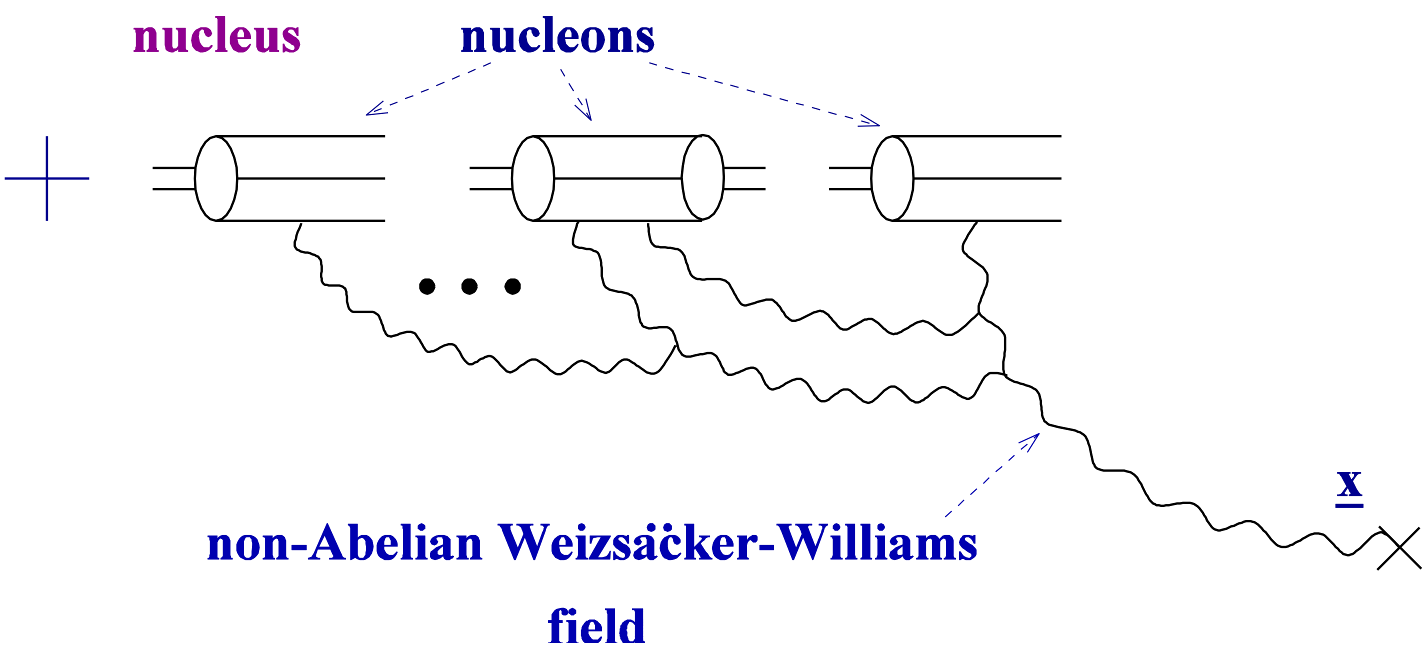
$$\mathcal{D}_\nu F^{\mu\nu} = J^\mu$$



nucleus is Lorentz contacted into a pancake

Yu. K. ’96; J. Jalilian-Marian et al, ‘96

Classical Field of a Nucleus

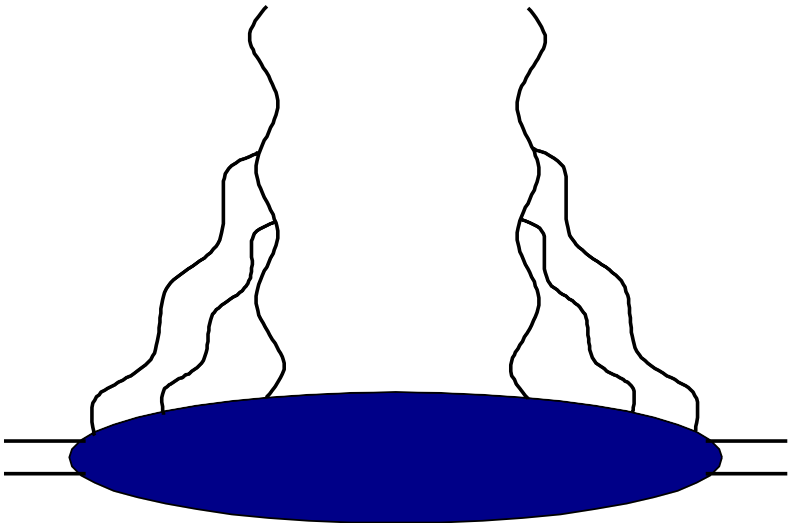


Here's one of the diagrams showing the non-Abelian gluon field of a large nucleus.

The resummation parameter is $\alpha_S^2 A^{1/3}$, corresponding to two gluons per nucleon approximation.

Classical Gluon Field of a Nucleus

Using the obtained classical gluon field one can construct corresponding gluon distribution function $\phi_A(x,k^2) \sim \langle \underline{A}(-k) \cdot \underline{A}(k) \rangle$



with the field in the $A^+=0$ gauge

$$\phi_A(x,k_T^2) = \frac{C_F}{\alpha_s\,\pi} \int \frac{d^2x_\perp}{x_\perp^2}\, e^{i\,\underline{k}\cdot\underline{x}} \left[1 - \exp\left(-\frac{x_\perp^2\,Q_s^2}{4}\ln\frac{1}{x_\perp\,\Lambda}\right)\right]$$

J. Jalilian-Marian et al, ' 97; Yu. K. and A. Mueller, '98

⇨ $Q_S=\mu$ is the saturation scale $Q_S^2 \sim A^{1/3}$

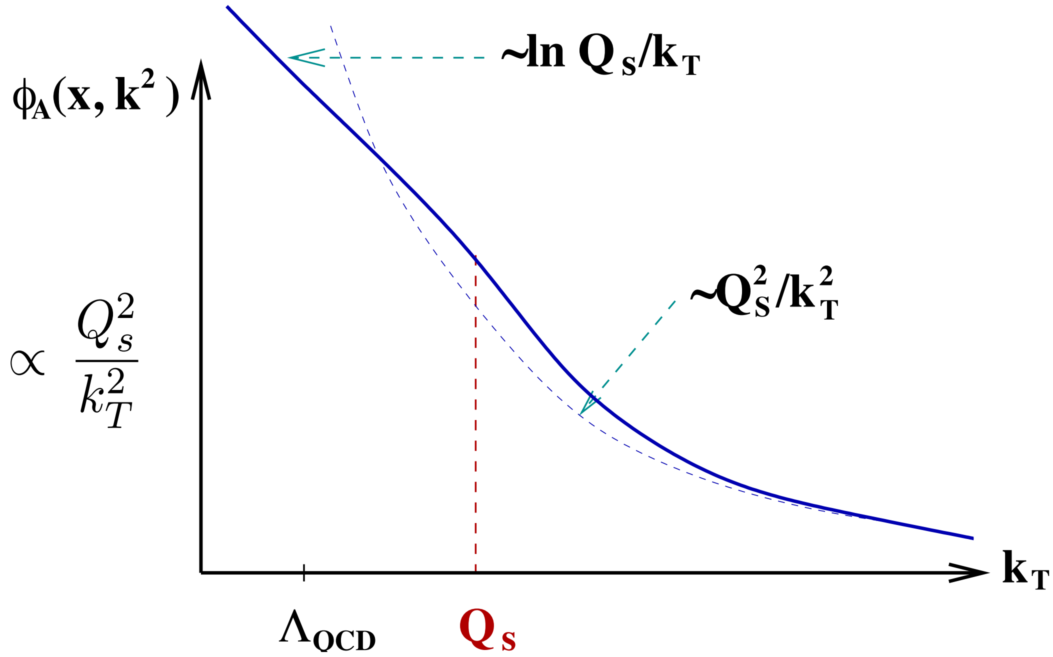
⇨ Note that $\phi \sim \langle A_\mu A_\mu \rangle \sim 1/\alpha$ such that $A_\mu \sim 1/g$, which is what one would expect for a classical field.

$$\phi_A(x,k_T^2) = \frac{C_F}{\alpha_s\,\pi}\int \frac{d^2x_\perp}{x_\perp^2}\, e^{i\,\underline{k}\cdot\underline{x}} \left[1-\exp\left(-\frac{x_\perp^2\,Q_s^2}{4}\,\ln\frac{1}{x_\perp\,\Lambda}\right)\right]$$

⇨ In the UV limit of k→∞, x_T is small and one obtains

$$\phi_A(x,k_T^2) \sim \int d^2x_\perp\, e^{i\,\underline{k}\cdot\underline{x}}\, Q_s^2\, \ln\frac{1}{x_\perp\,\Lambda} \propto \frac{Q_s^2}{k_T^2}$$

which is the usual LO result.



⇨ In the IR limit of small k_T, x_T is large and we get

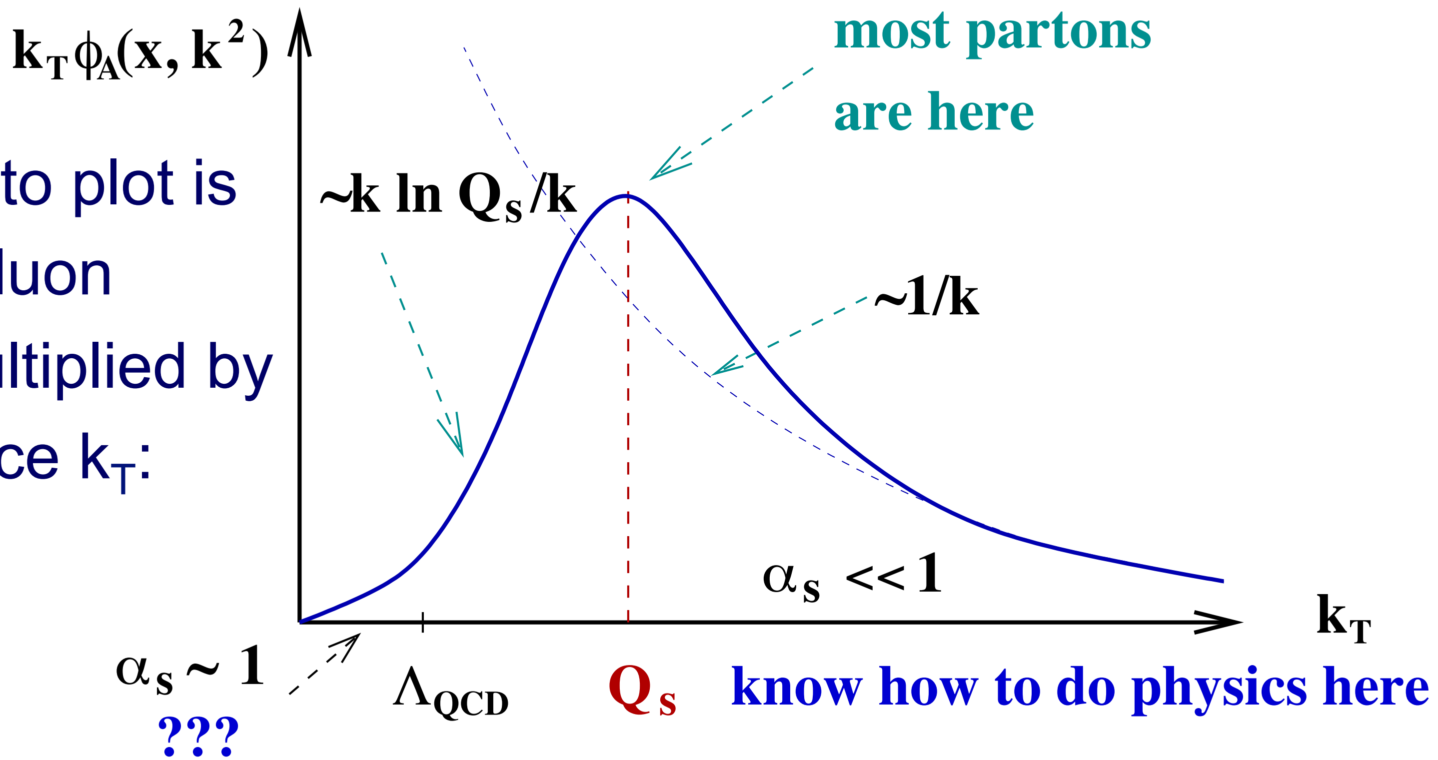
$$\phi_A(x,k_T^2) \approx \frac{C_F}{\alpha_s\,\pi}\int_{1/Q_s} \frac{d^2x_\perp}{x_\perp^2}\, e^{i\,\underline{k}\cdot\underline{x}} \propto \ln\frac{Q_s}{k_T}$$

SATURATION !

Divergence is regularized.

Classical Gluon Distribution

A good object to plot is the classical gluon distribution multiplied by the phase space k_T:



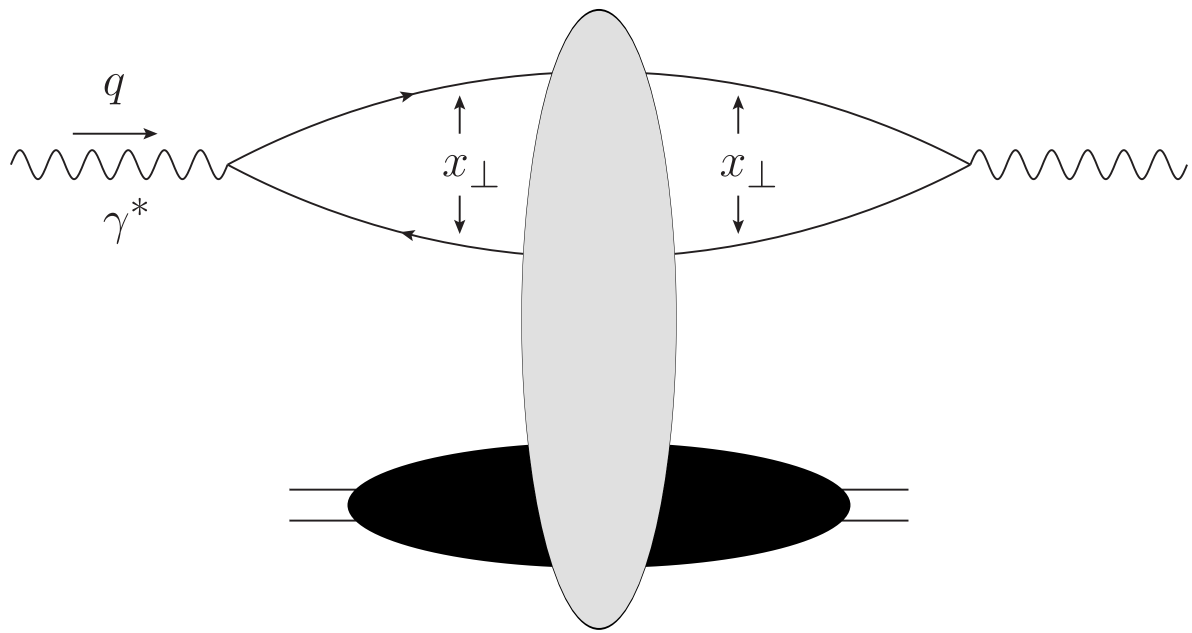
⇨ Most gluons in the nuclear wave function have transverse momentum of the order of $k_T \sim Q_S$ and $Q_S^2 \sim A^{1/3}$

⇨ We have a small coupling description of the whole wave function in the classical approximation.

B. Glauber-Mueller Rescatterings

Dipole picture of DIS

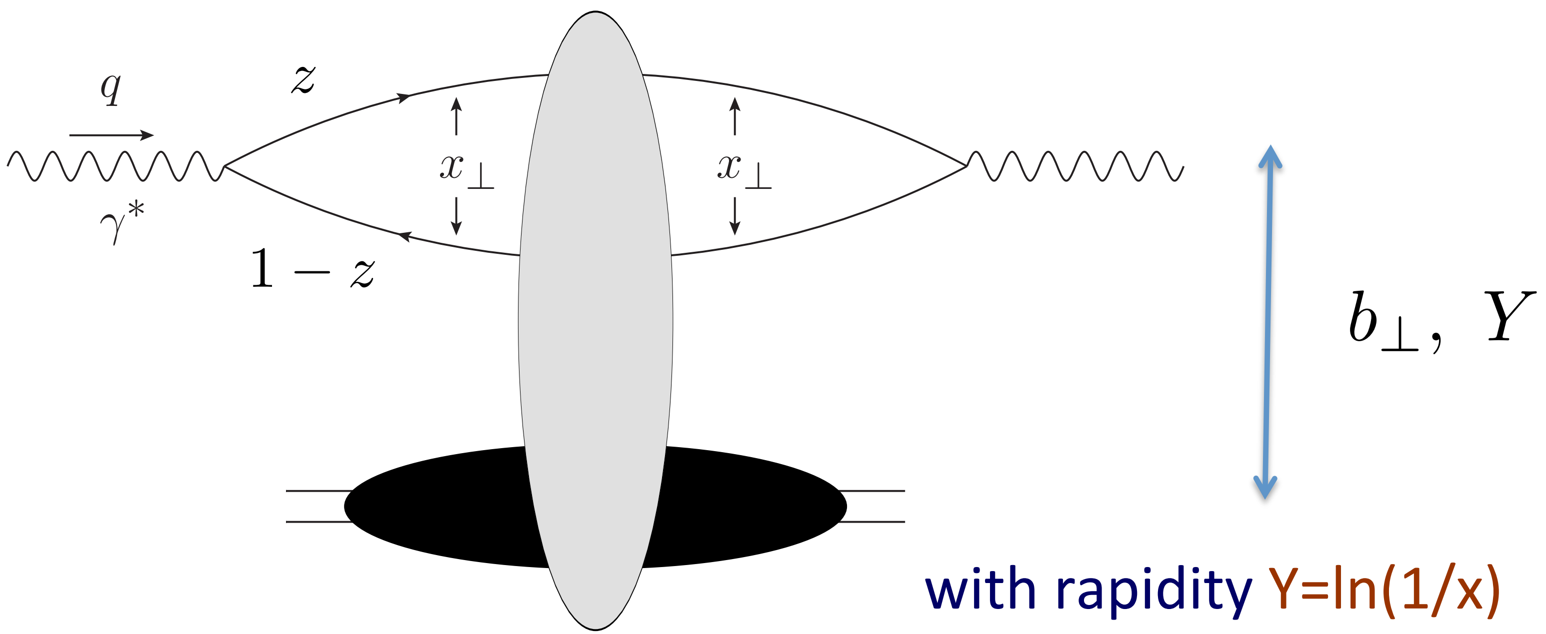
- In the dipole picture of DIS the virtual photon splits into a quark-antiquark pair, which then interacts with the target.
- The total DIS cross section and structure functions are calculated via:



Dipole Amplitude

- The total DIS cross section is expressed in terms of the (Im part of the) forward quark dipole amplitude N:

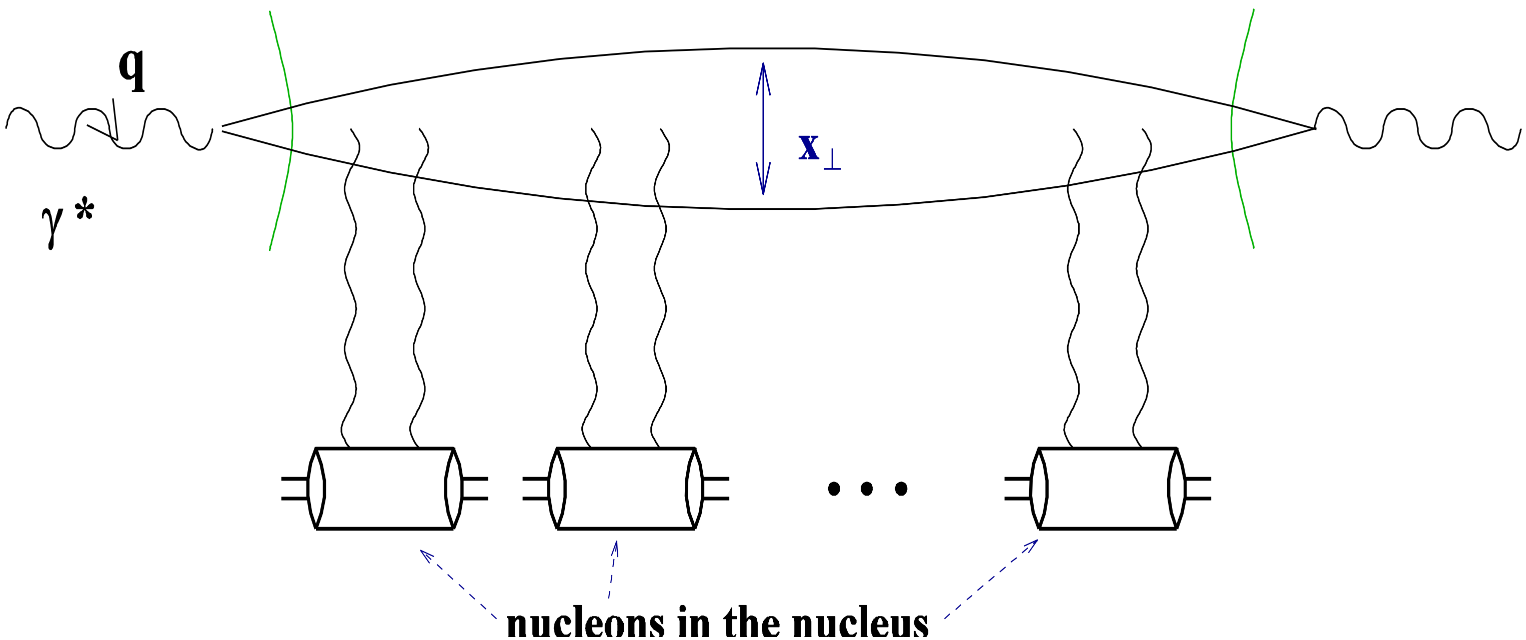
$$\sigma_{tot}^{\gamma^* A} = \int \frac{d^2 x_\perp}{2\pi} \, d^2 b_\perp \int_0^1 \frac{dz}{z\,(1-z)} \, |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 \; N(\vec{x}_\perp, \vec{b}_\perp, Y)$$



with rapidity Y=ln(1/x)

DIS in the Classical Approximation

The DIS process in the rest frame of the target is shown below. It factorizes into



$$\sigma_{tot}^{\gamma * A}(x_{Bj}, Q^2) = |\Psi^{\gamma * \to q\bar{q}}|^2 \otimes N(x_\perp, Y = \ln 1/x_{Bj})$$

with rapidity Y=ln(1/x)

Dipole Amplitude

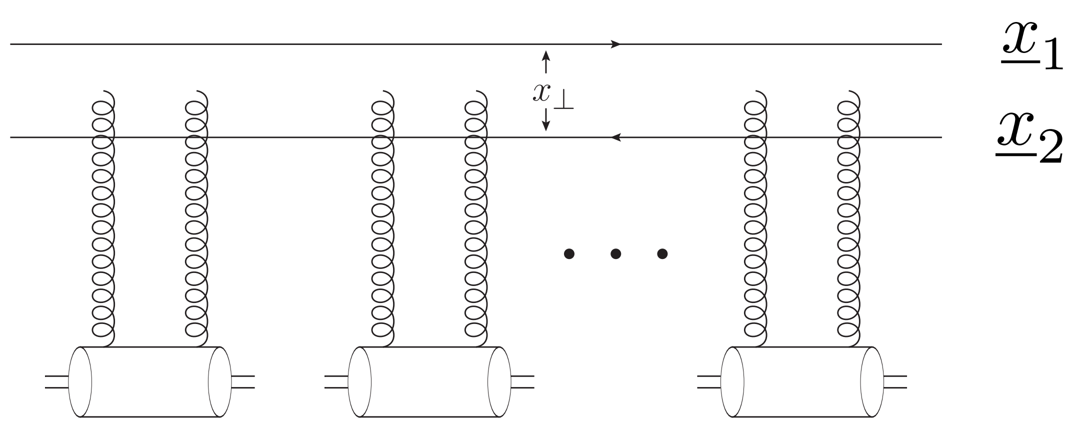
- The quark dipole amplitude is defined by

$$N(\underline{x}_1, \underline{x}_2) = 1 - \frac{1}{N_c} \left\langle \mathrm{tr} \left[V(\underline{x}_1) \, V^\dagger(\underline{x}_2) \right] \right\rangle$$

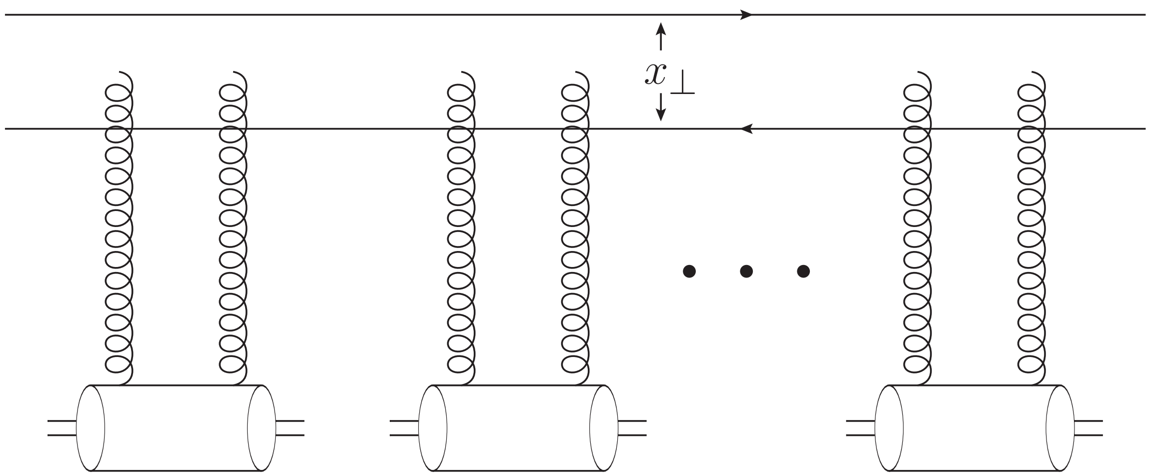
- Here we use the Wilson lines along the light-cone direction

$$V(\underline{x}) = \mathrm{P} \exp \left[i \, g \int\limits_{-\infty}^{\infty} dx^+ \, A^-(x^+, x^- = 0, \underline{x}) \right]$$

- In the classical Glauber-Mueller/McLerran-Venugopalan approach the dipole amplitude resums multiple rescatterings:

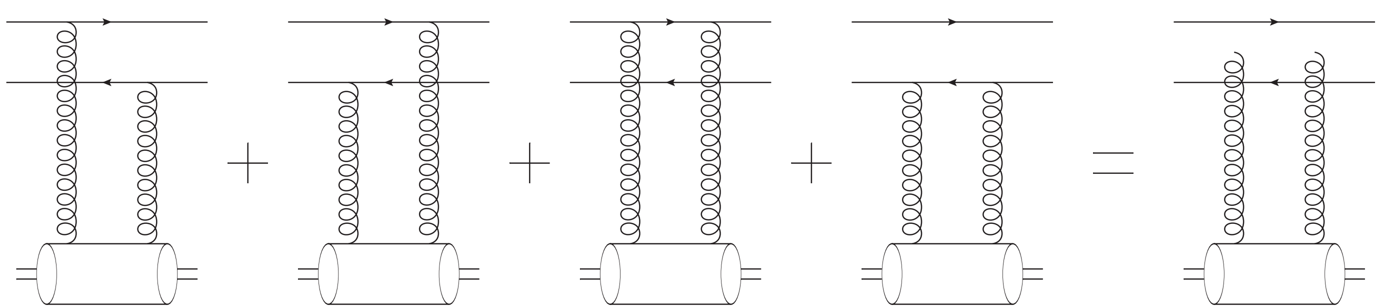


Quasi-classical dipole amplitude



A.H. Mueller, ‘90

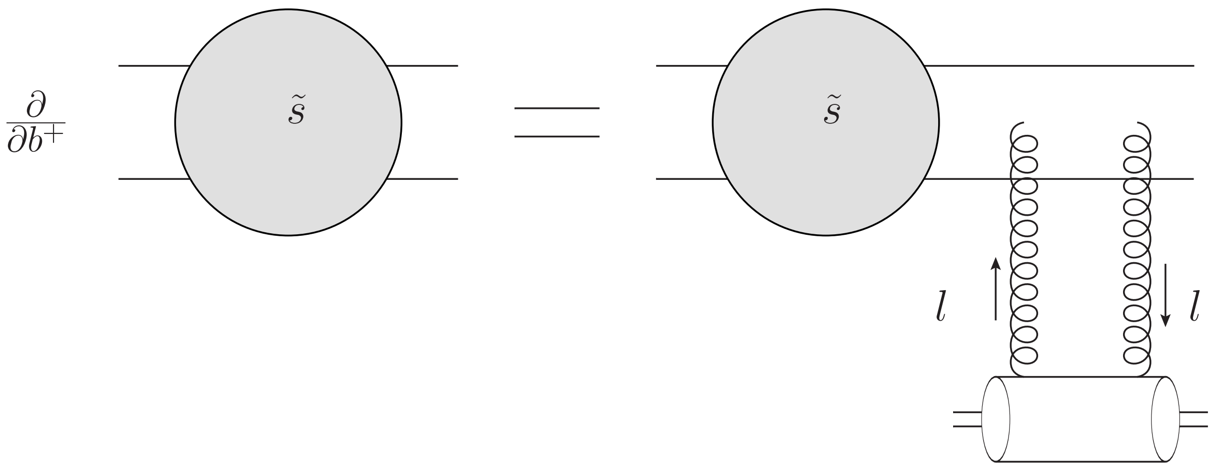
Lowest-order interaction with each nucleon – two gluon exchange – the same resummation parameter as in the MV model: $\alpha_s^2 \, A^{1/3}$



Quasi-classical dipole amplitude

- To resum multiple rescatterings, note that the nucleons are independent of each other and rescatterings on the nucleons are also independent.

- One then writes an equation (Mueller '90)



Each scattering!

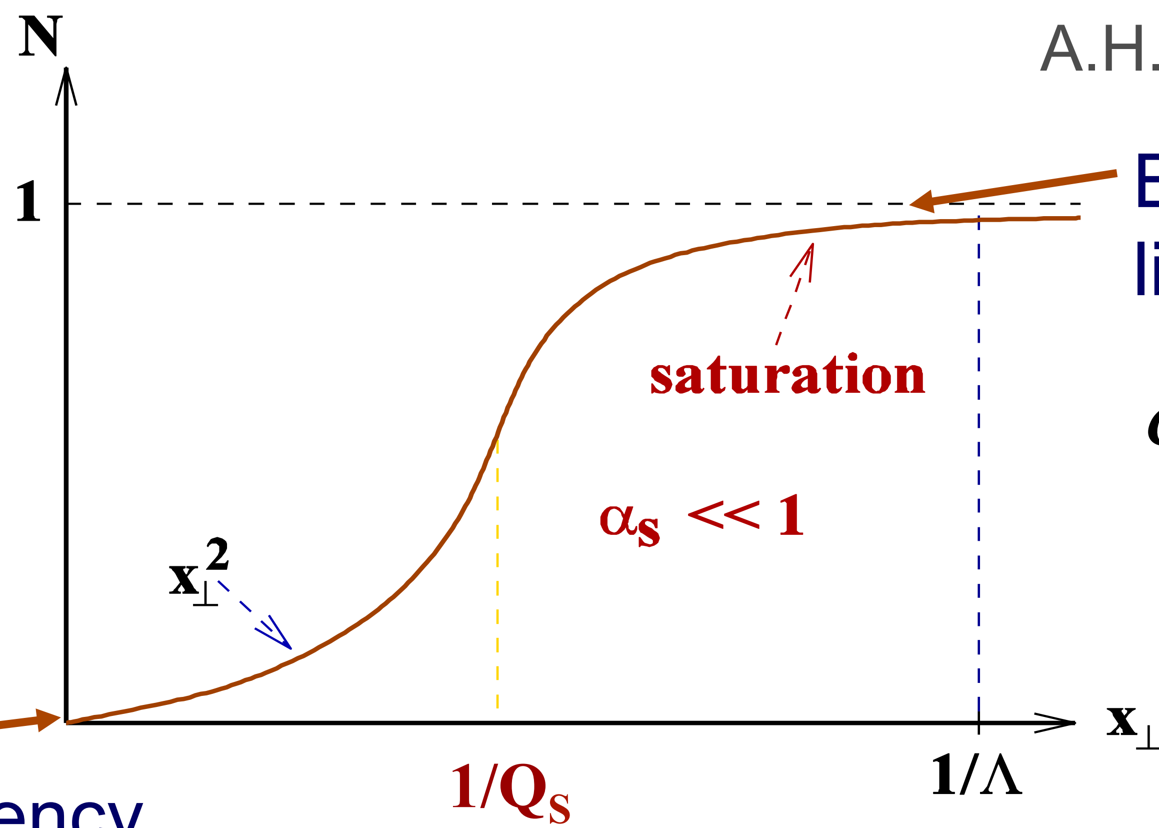
$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2\, Q_s^2}{4}\, \ln \frac{1}{x_\perp\, \Lambda}\right]$$

DIS in the Classical Approximation

The dipole-nucleus amplitude in the classical approximation is

$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2 \, Q_s^2}{4} \, \ln \frac{1}{x_\perp \Lambda}\right]$$

A.H. Mueller, '90



Black disk limit,

$$\sigma_{tot} < 2\pi R^2$$

Color transparency

Black Disk Limit

- Start with basic scattering theory: the final and initial states are related by the S-matrix operator,

$$|\psi_f\rangle = \hat{S}\,|\psi_i\rangle$$

- Write it as $$|\psi_f\rangle = |\psi_i\rangle + \left[\hat{S} - 1\right]\,|\psi_i\rangle$$

- The total cross section is

$$\sigma_{tot} \propto \left|\left[\hat{S} - 1\right]\,|\psi_i\rangle\right|^2 = 2 - S - S^*$$

where the forward matrix element of the S-matrix operator is

$$S = \langle\psi_i|\,\hat{S}\,|\psi_i\rangle$$

and we have used unitarity of the S-matrix

$$\hat{S}\,\hat{S}^\dagger = 1$$

Black Disk Limit

- Now, since $$|\psi_f\rangle = |\psi_i\rangle + \left[\hat{S} - 1\right] |\psi_i\rangle$$

  the elastic cross section is

$$\sigma_{el} \propto \left|\langle\psi_i| \left[\hat{S} - 1\right] |\psi_i\rangle\right|^2 = |1 - S|^2$$

- The inelastic cross section can be found via

$$\sigma_{tot} = \sigma_{inel} + \sigma_{el}$$

- In the end, for scattering with impact parameter b we write

$$\sigma_{tot} = 2 \int d^2b \, [1 - \mathrm{Re}\, S(b)]$$

$$\sigma_{el} = \int d^2b \, |1 - S(b)|^2$$

$$\sigma_{inel} = \int d^2b \left[1 - |S(b)|^2\right]$$

Unitarity Limit

- Unitarity implies that

$$1 = \langle\psi_i|\,\hat{S}\,\hat{S}^\dagger\,|\psi_i\rangle = \sum_X \langle\psi_i|\,\hat{S}|X\rangle\,\langle X|\hat{S}^\dagger\,|\psi_i\rangle \geq |S|^2$$

- Therefore

$$|S| \leq 1$$

leading to the unitarity bound on the total cross section

$$\sigma_{tot} = 2\int d^2b\,[1 - \mathrm{Re}\,S(b)] \leq 4\int d^2b = 4\pi R^2$$

- Notice that when S=-1 the inelastic cross section is zero and

$$\sigma_{tot} = 2\int d^2b\,[1 - \mathrm{Re}\,S(b)]$$

$$\sigma_{tot} = 4\pi R^2 = \sigma_{el}$$

$$\sigma_{el} = \int d^2b\,|1 - S(b)|^2$$

This limit is realized in low-energy scattering!

$$\sigma_{inel} = \int d^2b\left[1 - |S(b)|^2\right]$$

Black Disk Limit

- At high energy inelastic processes dominate over elastic. Imposing

$$\sigma_{inel} \geq \sigma_{el}$$

we get

$$\mathrm{Re}\, S \geq 0$$

- The bound on the total cross section is (aka the **black disk limit**)

$$\sigma_{tot} = 2 \int d^2b \, [1 - \mathrm{Re}\, S] \leq 2 \int d^2b = 2\pi R^2$$

- The inelastic and elastic cross sections at the black disk limit are

$$\sigma_{inel} = \sigma_{el} = \pi R^2$$

$$\sigma_{tot} = 2 \int d^2b \, [1 - \mathrm{Re}\, S(b)]$$

$$\sigma_{el} = \int d^2b \, |1 - S(b)|^2$$

$$\sigma_{inel} = \int d^2b \left[1 - |S(b)|^2\right]$$

Notation

- At high energies $$\mathrm{Im}\, S \approx 0$$

  while the dipole amplitude N is the imaginary part of the T-matrix (S=1+iT), such that
  $$\mathrm{Re}\, S = 1 - N$$

- The cross sections are
$$\sigma_{tot} = 2 \int d^2b\, N(x_\perp, b_\perp)$$
$$\sigma_{el} = \int d^2b\, N^2(x_\perp, b_\perp)$$
$$\sigma_{inel} = \int d^2b \left[2\, N(x_\perp, b_\perp) - N^2(x_\perp, b_\perp)\right]$$

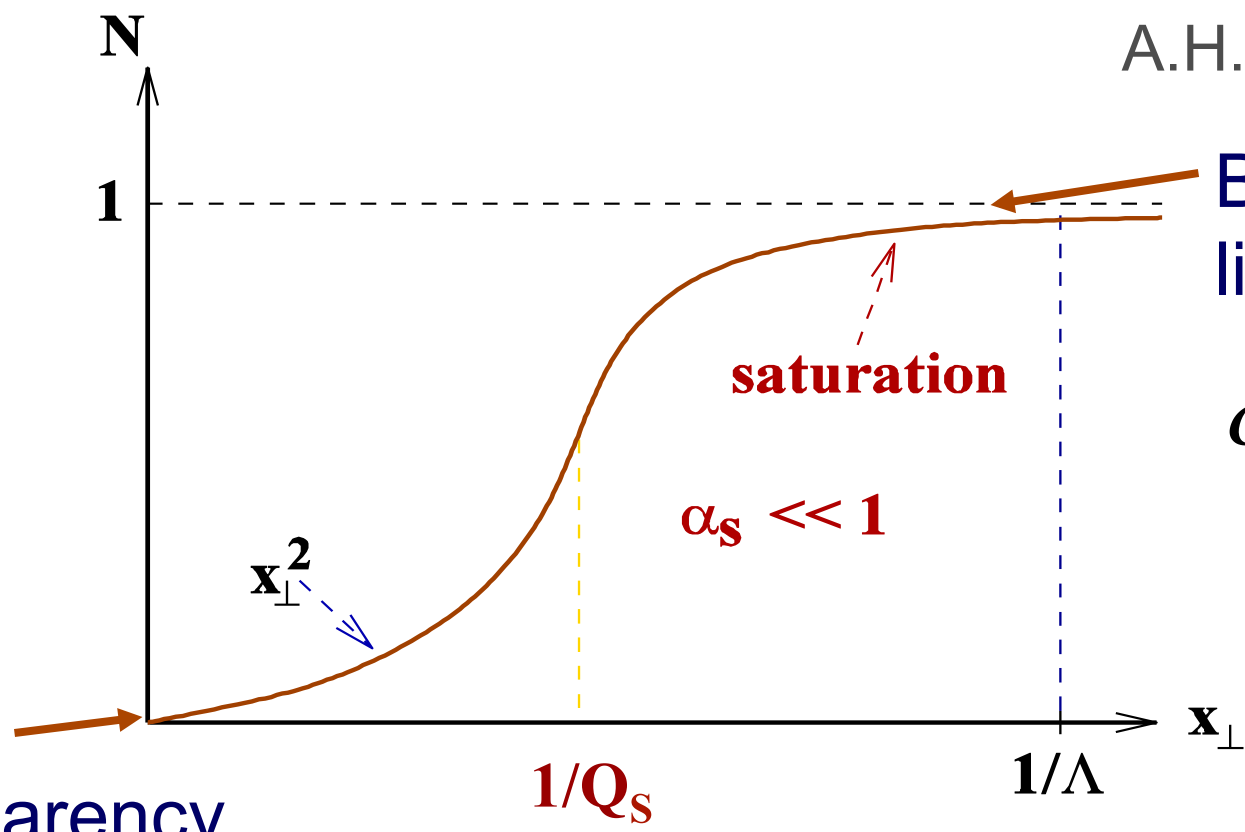
- We see that N=1 is the black disk limit. Hence $N \leq 1$ as we saw above.

DIS in the Classical Approximation

The dipole-nucleus amplitude in the classical approximation is

$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2\, Q_s^2}{4}\, \ln\frac{1}{x_\perp\, \Lambda}\right]$$

A.H. Mueller, '90



Black disk limit,

$$\sigma_{tot} < 2\pi R^2$$

Color transparency

Summary

- We have reviewed the McLerran-Venugopalan model for the small-x wave function of a large nucleus.
- We saw the onset of gluon saturation and the appearance of a large transverse momentum scale – the saturation scale:

$$Q_s^2 \sim A^{1/3}$$

- We applied the quasi-classical approach to DIS, obtaining Glauber-Mueller formula for multiple rescatterings of a dipole in a nucleus.
- We saw that onset of saturation insures that unitarity (the black disk limit) is not violated. Saturation is a consequence of unitarity!

Quantum Small-x Evolution

A. Birds-Eye View

Why Evolve?

- No energy or rapidity dependence in classical field and resulting cross sections.

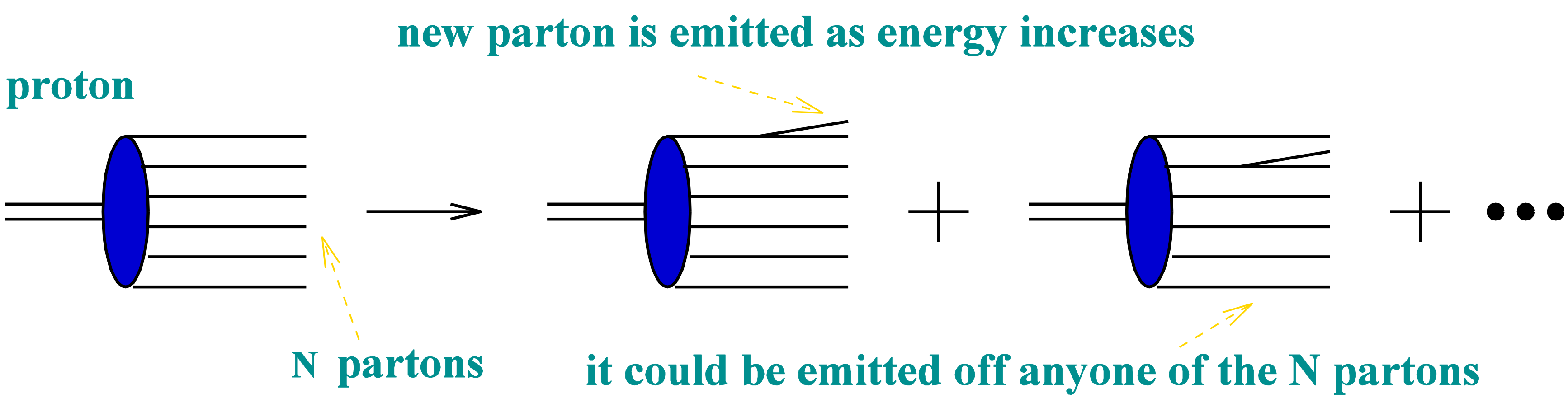
- Energy/rapidity-dependence comes in through quantum corrections.

- Quantum corrections are included through "evolution equations".

BFKL Equation

Balitsky, Fadin, Kuraev, Lipatov ‘78

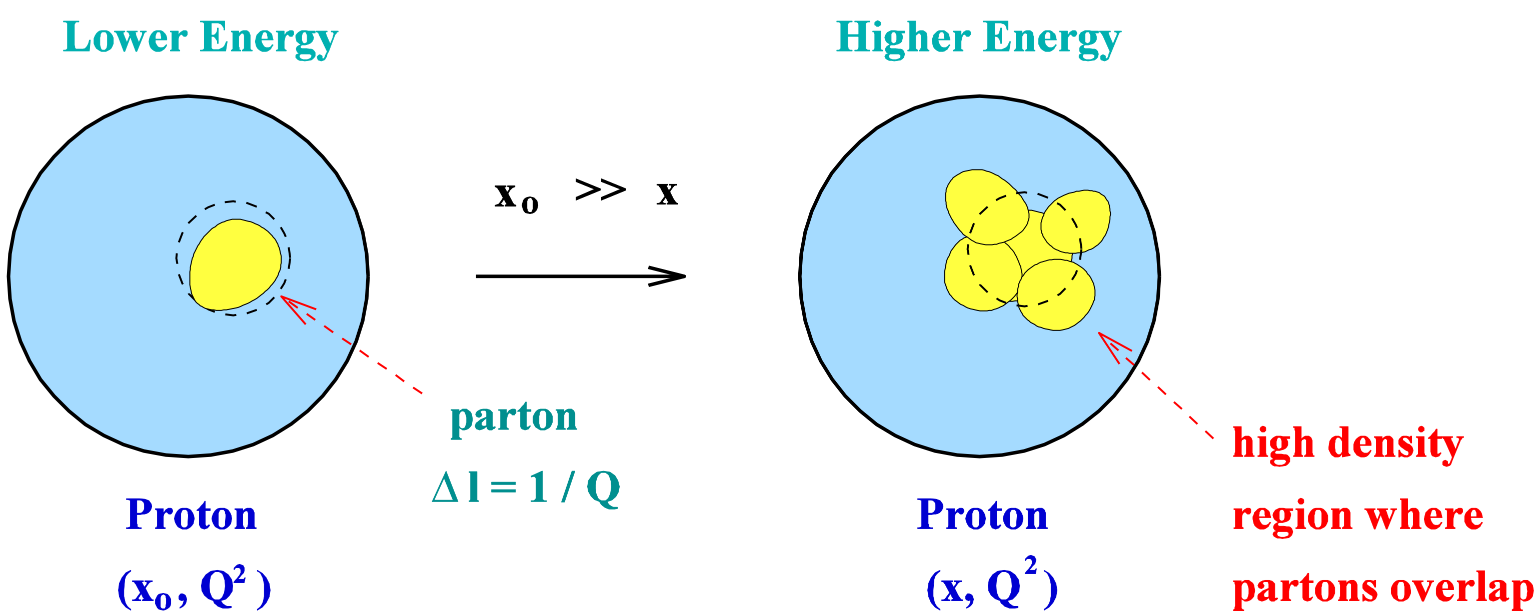
Start with N particles in the proton’s wave function. As we increase the energy a new particle can be emitted by either one of the N particles. The number of newly emitted particles is proportional to N.



The BFKL equation for the number of partons N reads:

$$\frac{\partial}{\partial \ln(1/x)} N(x,Q^2) = \alpha_S K_{BFKL} \otimes N(x,Q^2)$$

BFKL Equation as a High Density Machine



- As energy increases BFKL evolution produces more partons, roughly of the same size. The partons overlap each other creating areas of very high density.
- Number density of partons, along with corresponding cross sections grows as a power of energy

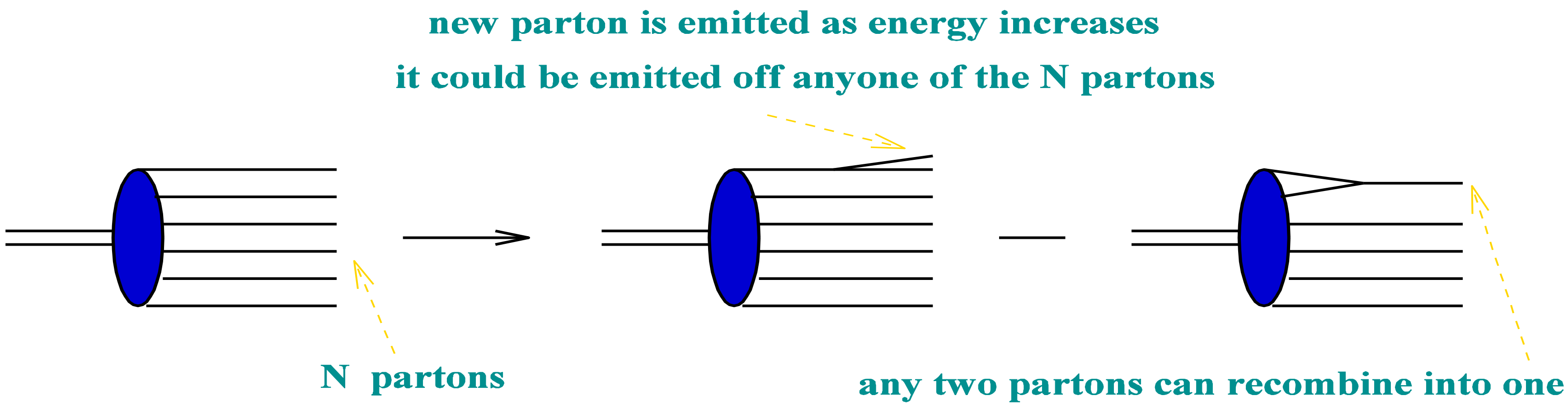
$$N \sim s^{\Delta}$$

- But can parton densities rise forever? Can gluon fields be infinitely strong? Can the cross sections rise forever?
- No! There exists a black disk limit for cross sections, which we know from Quantum Mechanics: for a scattering on a disk of radius R the total cross section is bounded by

$$\sigma_{tot} \leq 2\pi R^2$$

Nonlinear Equation

At very high energy parton recombination becomes important. Partons not only split into more partons, but also recombine. Recombination reduces the number of partons in the wave function.

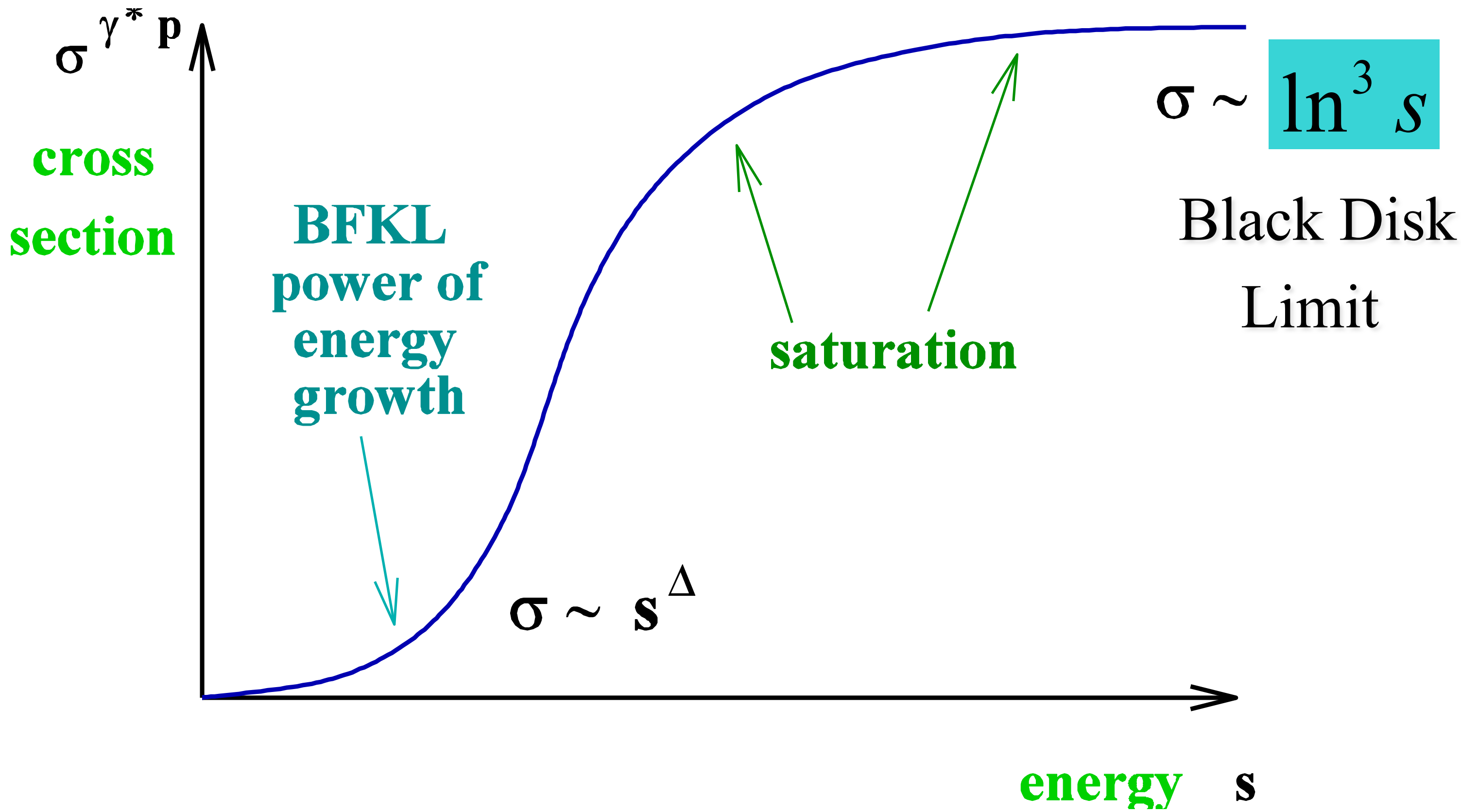


$$\frac{\partial}{\partial Y} N(x, k_T^2) = \alpha_s \, K_{BFKL} \otimes N(x, k_T^2) - \alpha_s \, [N(x, k_T^2)]^2$$

Number of parton pairs ~ N^2

I. Balitsky ’96 (effective Lagrangian)
Yu. K. ’99 (large N_C QCD)

Nonlinear Equation: Saturation

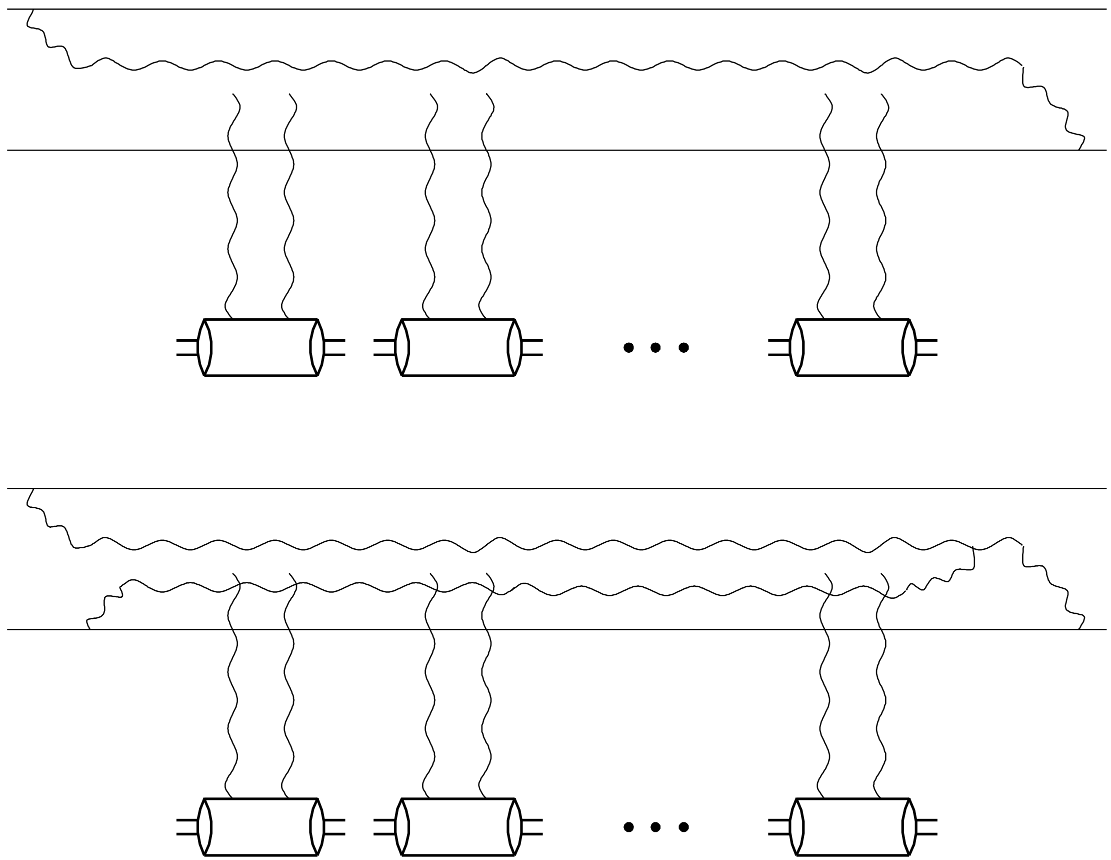


Gluon recombination tries to reduce the number of gluons in the wave function. At very high energy recombination begins to compensate gluon splitting. Gluon density reaches a limit and does not grow anymore. So do total DIS cross sections. Unitarity is restored!

B. In-Depth Discussion

Quantum Evolution

As energy increases the higher Fock states including gluons on top of the quark-antiquark pair become important. They generate a cascade of gluons.

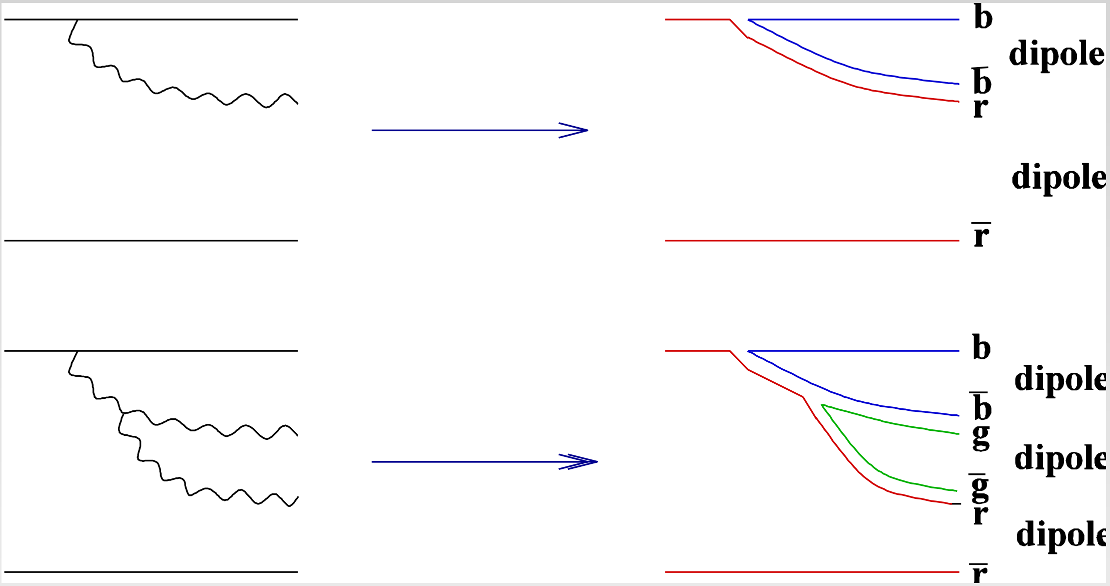


These extra gluons bring in powers of $\alpha_s \ln s$, such that when $\alpha_s \ll 1$ and $\ln s \gg 1$ this parameter is $\alpha_s \ln s \sim 1$ (leading logarithmic approximation, LLA).

Resumming Gluonic Cascade

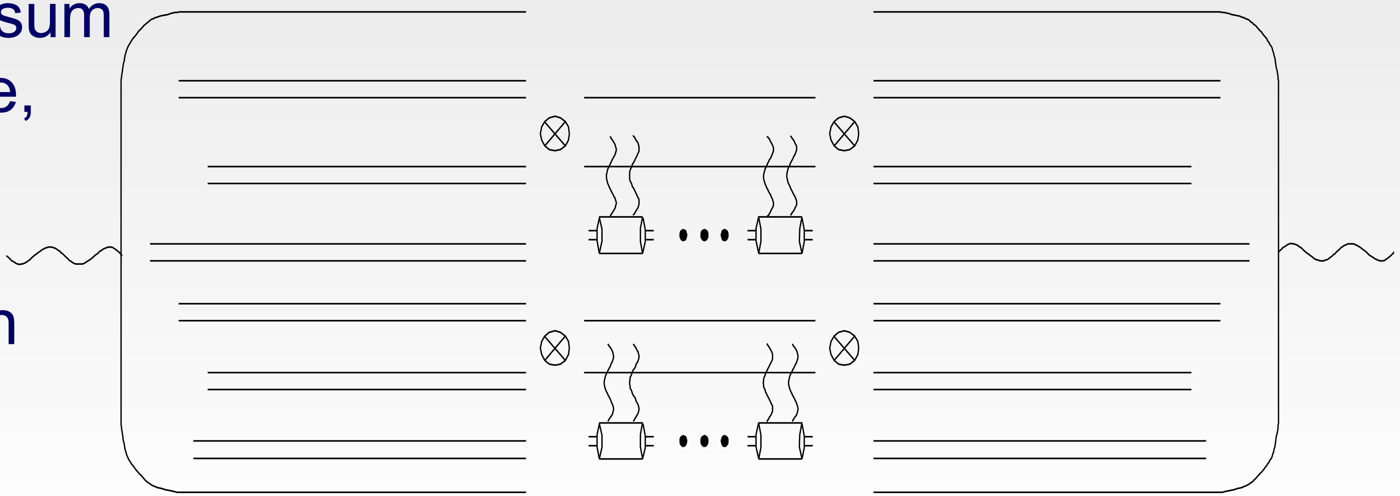
In the large-N_C limit of QCD the gluon corrections become color dipoles. Gluon cascade becomes a dipole cascade.

A. H. Mueller, ’93-’94

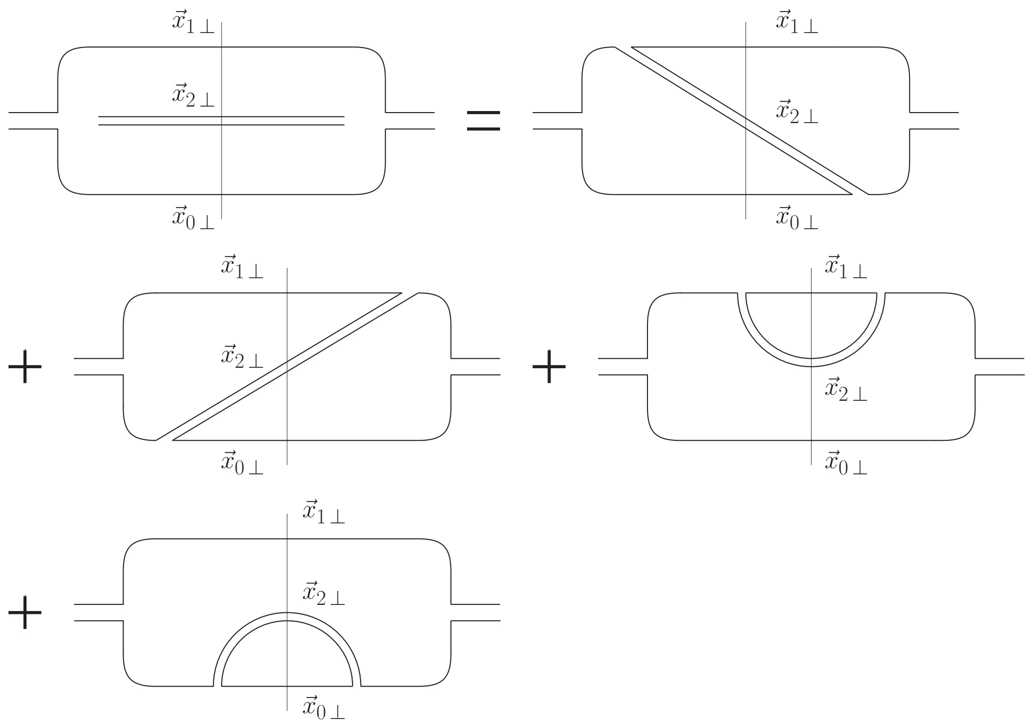


We need to resum dipole cascade, with each final state dipole interacting with the target.

Yu. K. ‘99



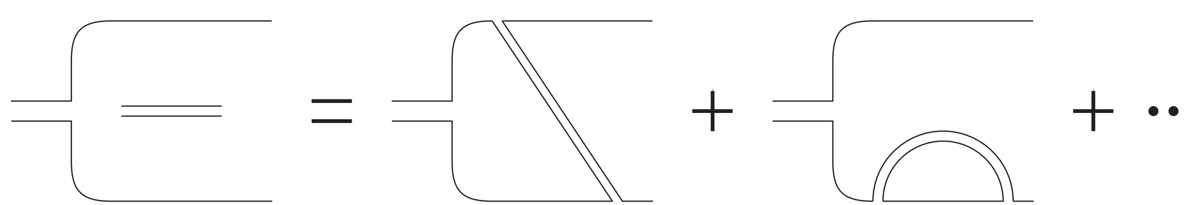
Notation (Large-N_c)



Real emissions in the amplitude squared

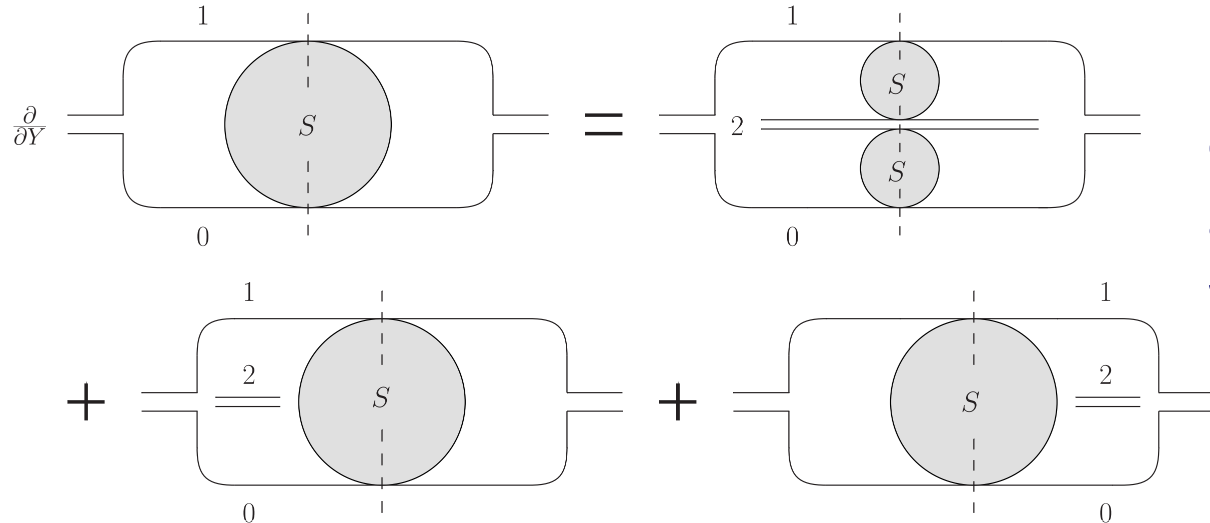
(dashed line – all Glauber-Mueller exchanges at light-cone time =0)

Virtual corrections in the amplitude (wave function)



Nonlinear Evolution

To sum up the gluon cascade at large-N_C we write the following equation for the dipole S-matrix:



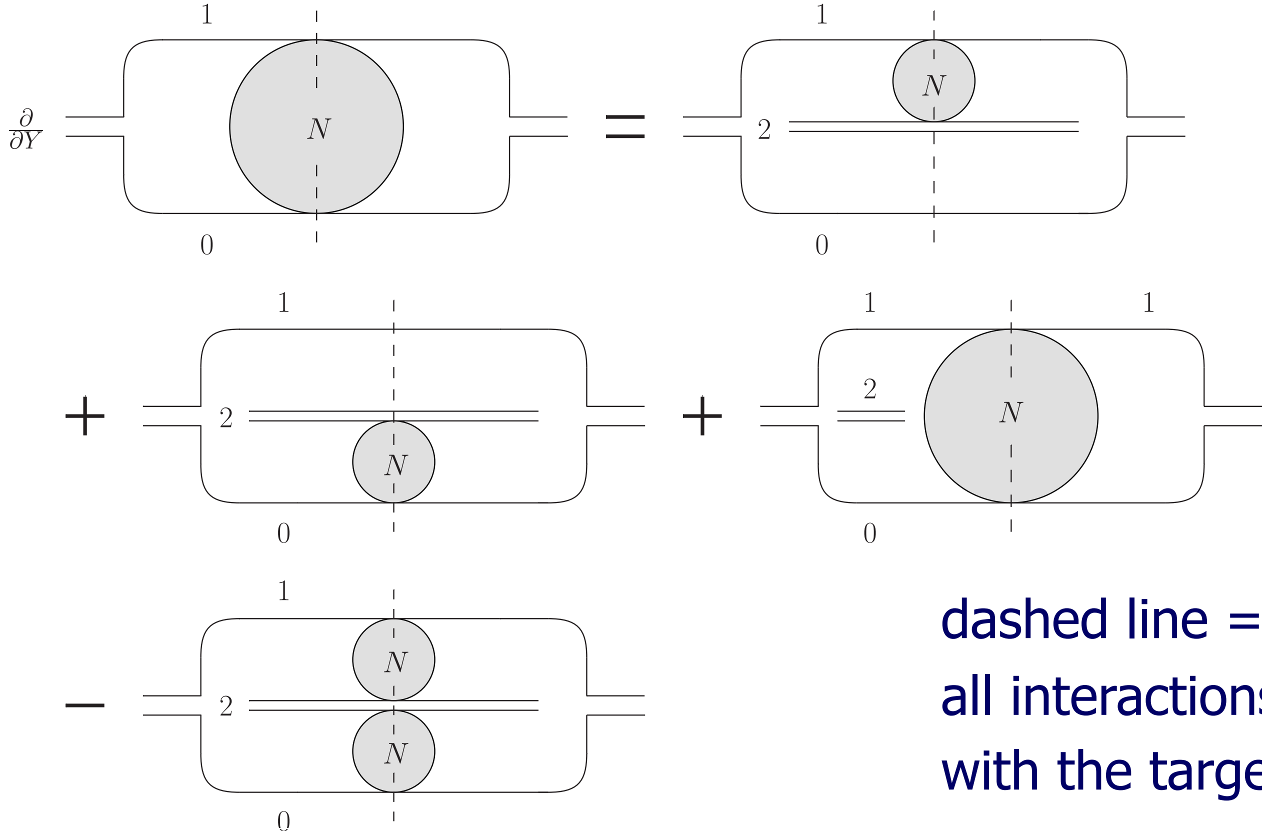
dashed line = all interactions with the target

$$\partial_Y S_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s \, N_c}{2\pi^2} \int d^2 x_2 \, \frac{x_{01}^2}{x_{02}^2 \, x_{21}^2} \, \left[S_{\mathbf{x}_0,\mathbf{x}_2}(Y) \, S_{\mathbf{x}_2,\mathbf{x}_1}(Y) - S_{\mathbf{x}_0,\mathbf{x}_1}(Y) \right]$$

Remembering that S= 1-N we can rewrite this equation in terms of the dipole scattering amplitude N.

Nonlinear evolution at large N_c

As N=1-S we write

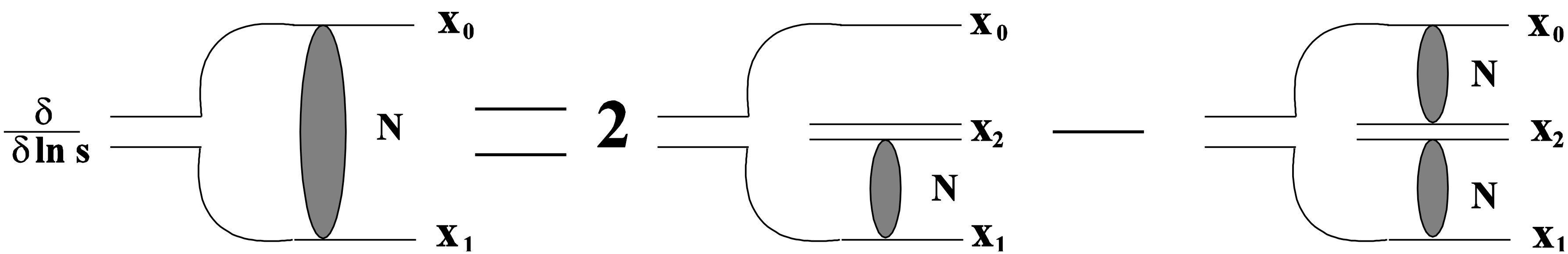


dashed line =
all interactions
with the target

$$\partial_Y N_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{21}^2} \left[N_{\mathbf{x}_0,\mathbf{x}_2}(Y) + N_{\mathbf{x}_2,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_2}(Y)\, N_{\mathbf{x}_2,\mathbf{x}_1}(Y) \right]$$

Balitsky ‘96, Yu.K. ‘99

Nonlinear Evolution Equation



We can resum the dipole cascade

$$\frac{\partial N(x_{01},Y)}{\partial Y} = \frac{\alpha_S N_C}{\pi^2}\int d^2x_2 \left[\frac{x_{01}^2}{x_{02}^2\, x_{12}^2} - 2\pi\,\delta^2(\underline{x}_{01} - \underline{x}_{02})\ln\left(\frac{x_{01}}{\rho}\right)\right] N(x_{02},Y)$$
$$- \frac{\alpha_S N_C}{2\pi^2}\int d^2x_2 \frac{x_{01}^2}{x_{02}^2\, x_{12}^2} N(x_{02},Y)\, N(x_{12},Y)$$

I. Balitsky, ' 96, HE effective lagrangian
Yu. K., ' 99, large N_C QCD

$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2 Q_s^2}{4}\ \ln\frac{1}{x_\perp \Lambda}\right]$$

← initial condition

⇨ Linear part is BFKL, quadratic term brings in damping

Resummation parameter

- BK equation resums powers of

$$\alpha_s \, N_c \, Y$$

- The Galuber-Mueller/McLerran-Venugopalan initial conditions for it resum powers of

$$\alpha_s^2 \, A^{1/3}$$

Going Beyond Large N_C: JIMWLK

To do calculations beyond the large-N_C limit on has to use a functional integro-differential equation written by Iancu, Jalilian-Marian, Kovner, Leonidov, McLerran and Weigert (JIMWLK):

$$\frac{\partial Z}{\partial Y} = \alpha_s \left\{ \frac{1}{2} \frac{\delta^2}{\delta\rho(u)\,\delta\rho(v)} [Z\,\chi(u,v)] - \frac{\delta}{\delta\rho(u)} [Z\,\sigma(u)] \right\}$$

where the functional $Z[\rho]$ can then be used for obtaining wave function-averaged observables (like Wilson loops for DIS):

$$\langle O \rangle = \int D\rho\, Z[\rho]\, O[\rho]$$

Going Beyond Large N_C: JIMWLK

- The JIMWLK equation has been solved on the lattice by Rummukainen and H. Weigert '04

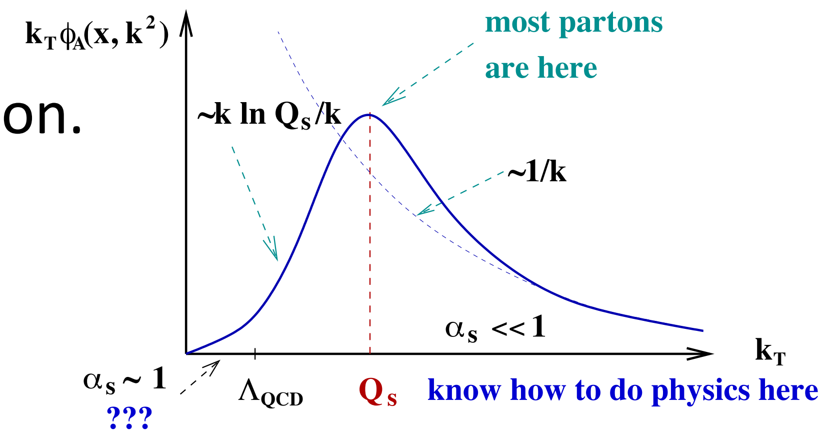
- For the dipole amplitude $N(x_0, x_1, Y)$, the **relative** corrections to the large-N_C limit BK equation are **< 0.001 !** Not the naïve $1/N_C^2 \sim 0.1$! (For realistic rapidities/ energies.)

- The reason for that is dynamical, and is largely due to saturation effects suppressing the bulk of the potential $1/N_C^2$ corrections (Yu.K., J. Kuokkanen, K. Rummukainen, H. Weigert, '08).

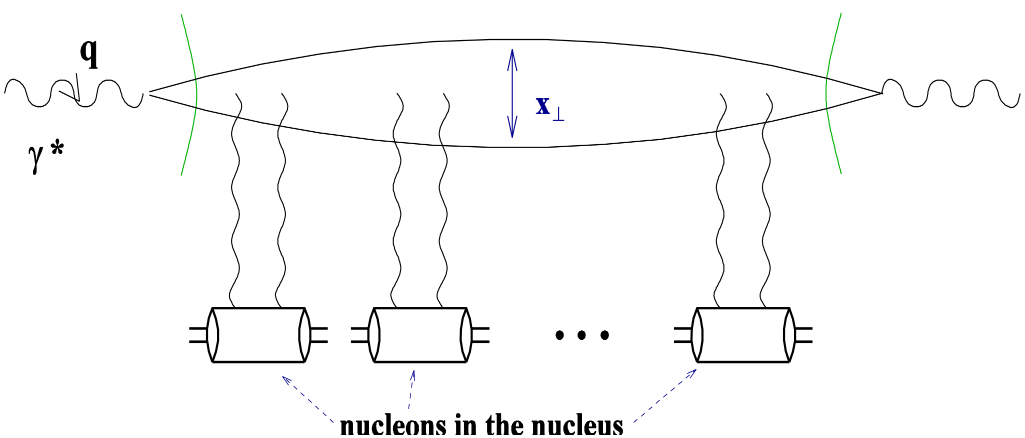
Last time

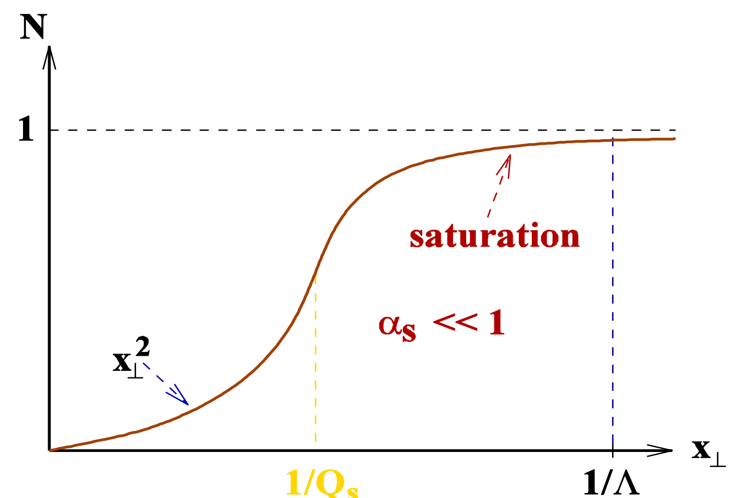
- We discussed the McLerran-Venugopalan (MV) model: classical gluon field of a nucleus.
- Found the classical gluon distribution.
- Argued that the saturation scale grows as

$$Q_s^2 \sim A^{1/3}$$



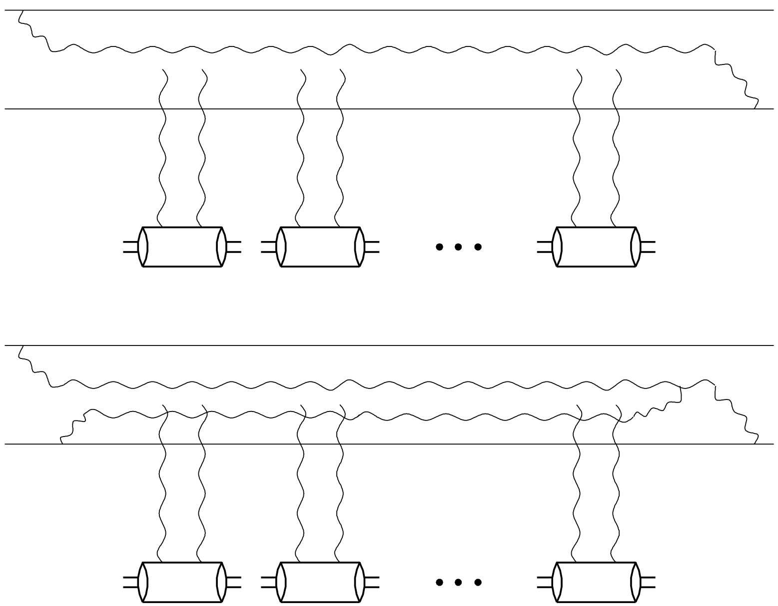
- Considered DIS in the quasi-classical picture:

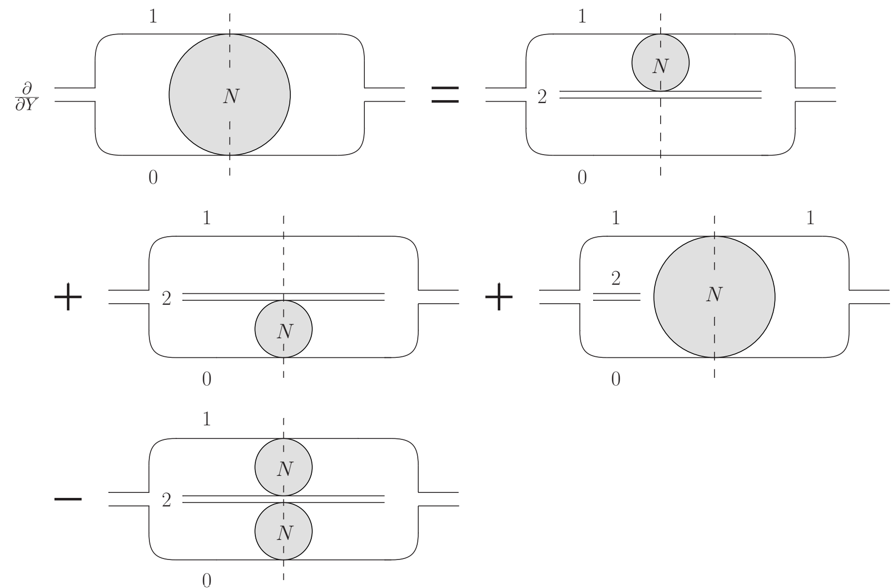




Last time

- Derived the nonlinear (BK) evolution equation:





- Resummation parameter is (leading log approximation):

$$\alpha_s \, Y = \alpha_s \, \ln \frac{1}{x} \sim \alpha_s \, \ln s$$

Last time

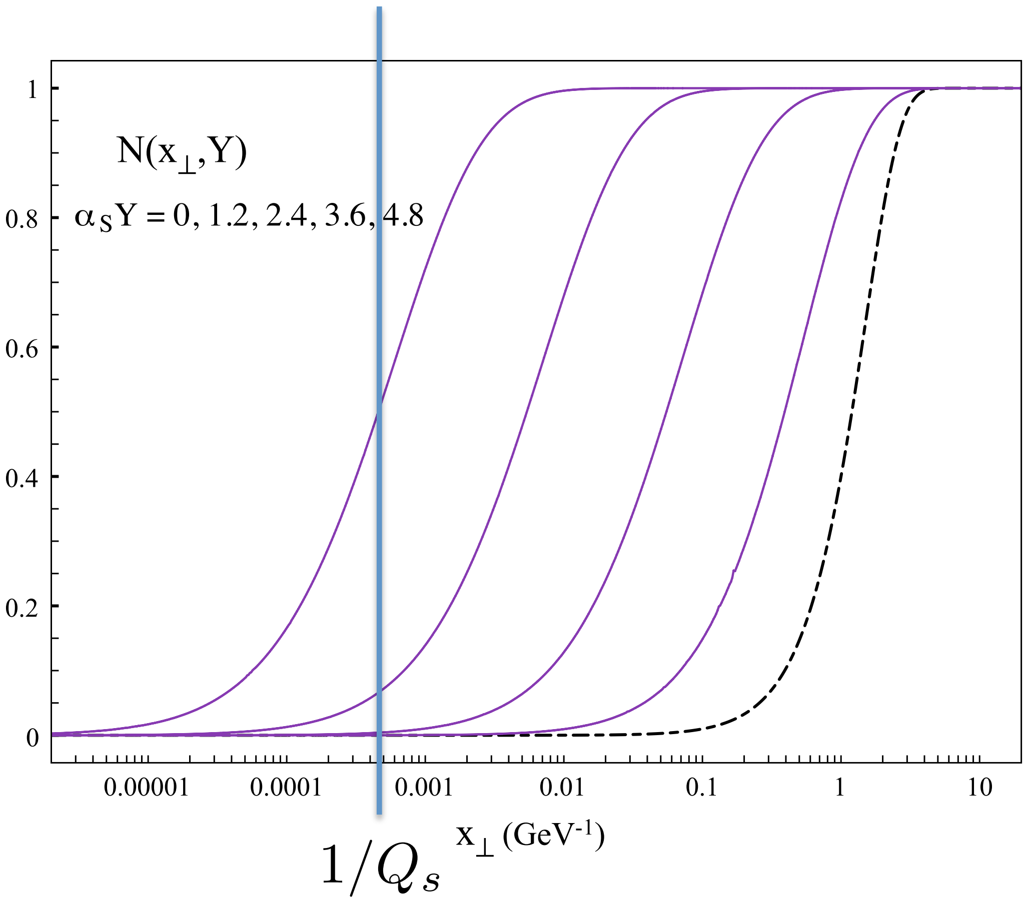
- The equation reads:

$$\partial_Y N_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \frac{x_{01}^2}{x_{02}^2 x_{21}^2} [N_{\mathbf{x}_0,\mathbf{x}_2}(Y) + N_{\mathbf{x}_2,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_2}(Y) N_{\mathbf{x}_2,\mathbf{x}_1}(Y)]$$

- It combines BFKL evolution (the linear part) and the quadratic damping correction.
- All-N_c evolution is JIMWLK.
- Gelis+Golec-Biernat: BK is the "Heisenberg representation", while JIMWLK is the "Schrödinger representation".
- Now let's discuss its solution.

C. Solution of BK Equation

Solution of BK equation

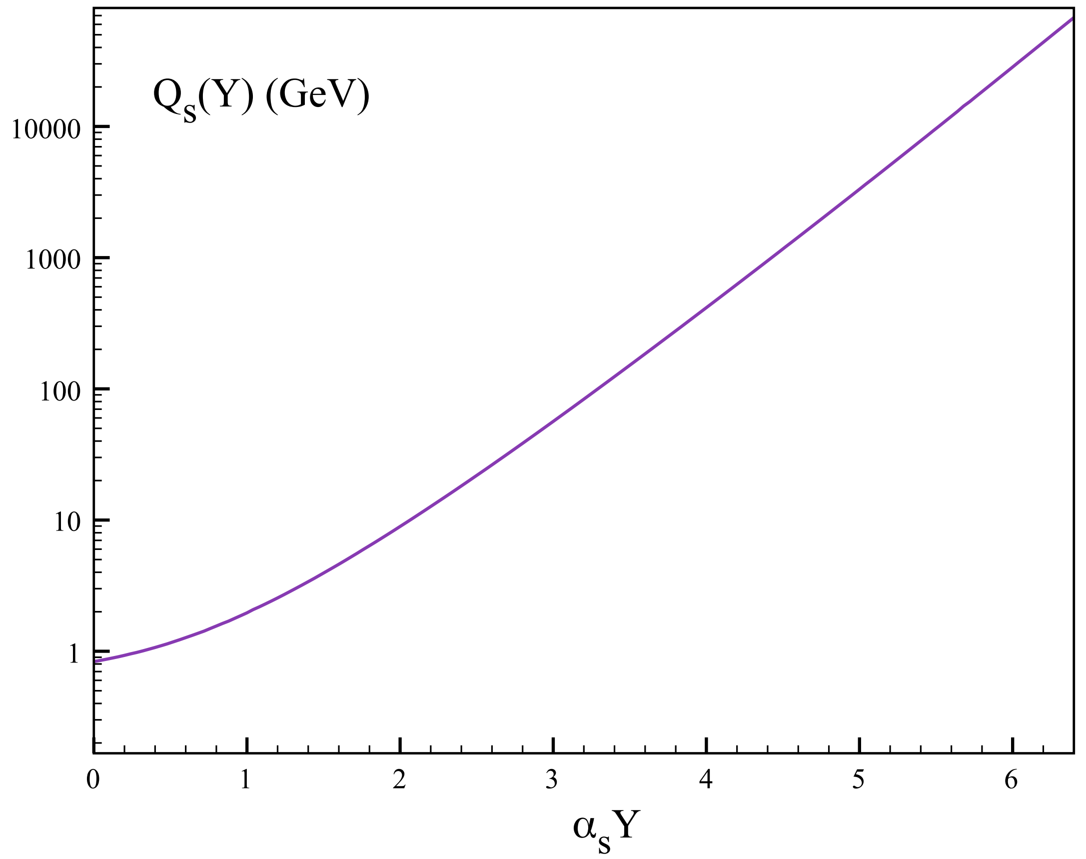


numerical solution by J. Albacete '03 (earlier solutions were found numerically by Golec-Biernat, Motyka, Stasto, by Braun and by Lublinsky et al in '01)

BK solution preserves the black disk limit, N<1 always (unlike the linear BFKL equation)

$$\sigma^{q\bar{q}A} = 2 \int d^2b \, N(x_\perp, b_\perp, Y)$$

Saturation scale



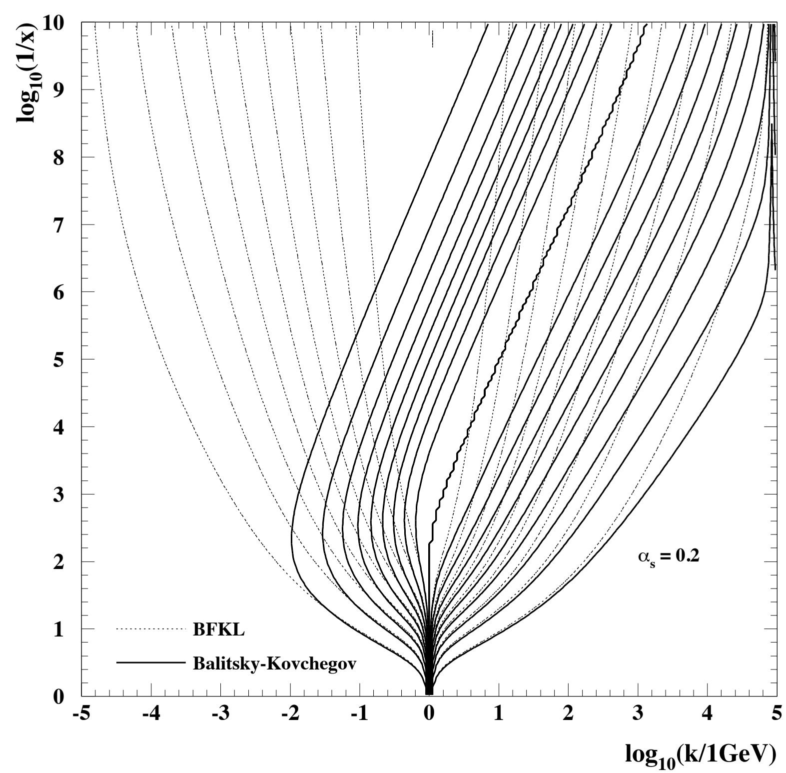
numerical solution by J. Albacete

BK Solution

- Preserves the black disk limit, N<1 always.

$$\sigma^{q\bar{q}A} = 2 \int d^2b\, N(x_\perp, b_\perp, Y)$$

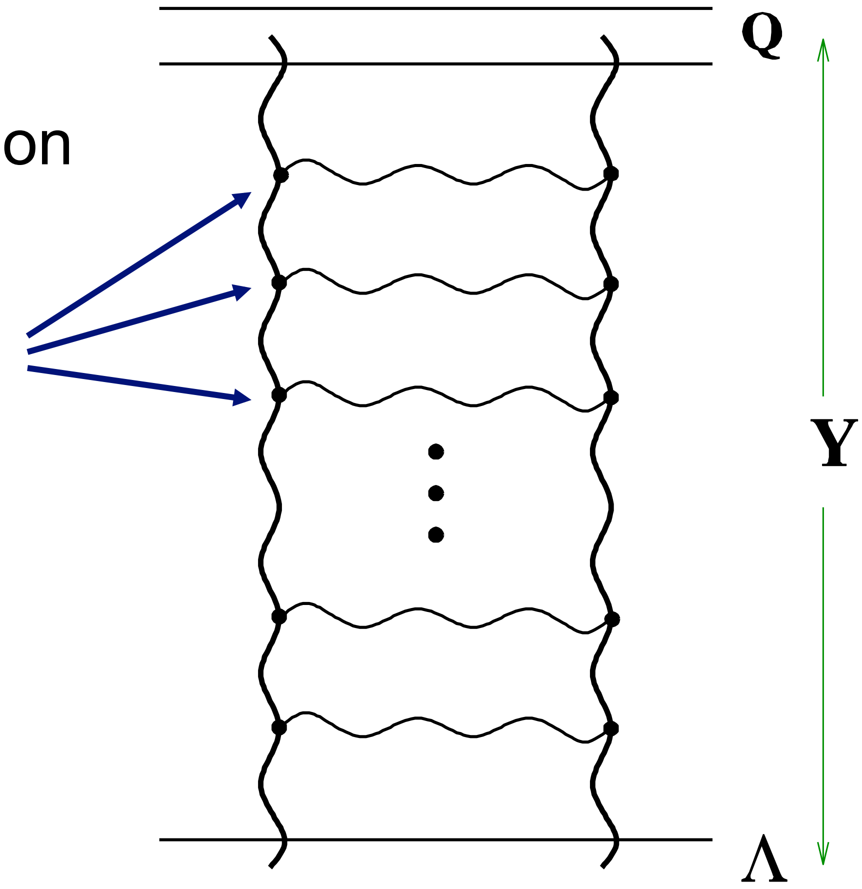
- Avoids the IR problem of BFKL evolution due to the saturation scale screening the IR:



Golec-Biernat, Motyka, Stasto '02

BFKL Equation

In the conventional Feynman diagram picture the BFKL equation can be represented by a ladder graph shown here. Each rung of the ladder brings in a power of α ln s.



The resulting dipole amplitude grows as a power of energy

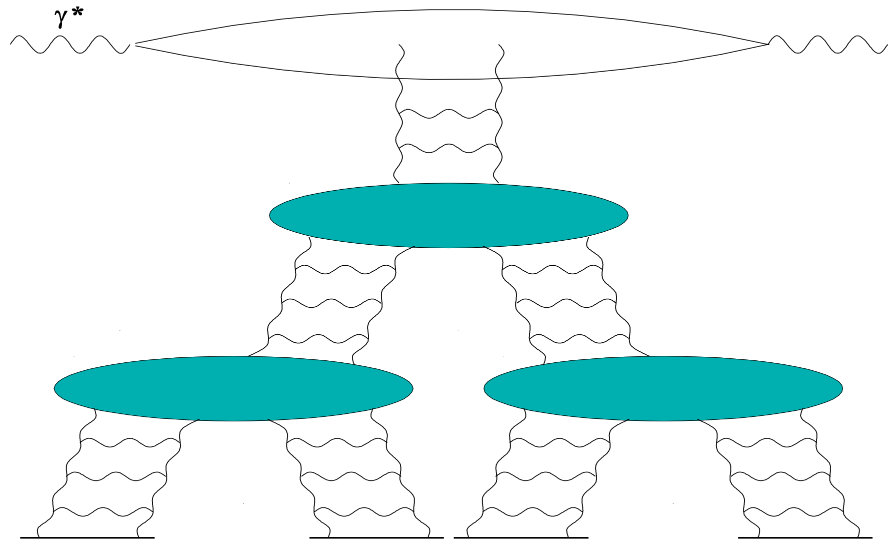
$$N \sim s^{\Delta}$$

violating Froissart unitarity bound

$$\sigma_{tot} \leq const \ln^2 s$$

GLR-MQ Equation

Gribov, Levin and Ryskin ('81)
proposed summing up "fan" diagrams:



Mueller and Qiu (' 85) summed
"fan" diagrams for large Q^2.

The GLR-MQ equation reads:

$$\frac{\partial}{\partial \ln 1/x} \phi(x, k_T^2) = \alpha_s K_{BFKL} \otimes \phi(x, k_T^2) - \alpha_s [\phi(x, k_T^2)]^2$$

GLR-MQ equation has the same principle of recombination as BK and JIMWLK. GLR-MQ equation was thought about as the first nonlinear correction to the linear BFKL evolution. An AGL (Ayala, Gay Ducati, Levin '96) equation was suggested to resum higher-order nonlinear corrections.

BK/JIMWLK derivation showed that for the dipole amplitude N (!) there are no more terms in the large-N_C limit and obtained the correct kernel for the non-linear term (compared to GLR suggestion).

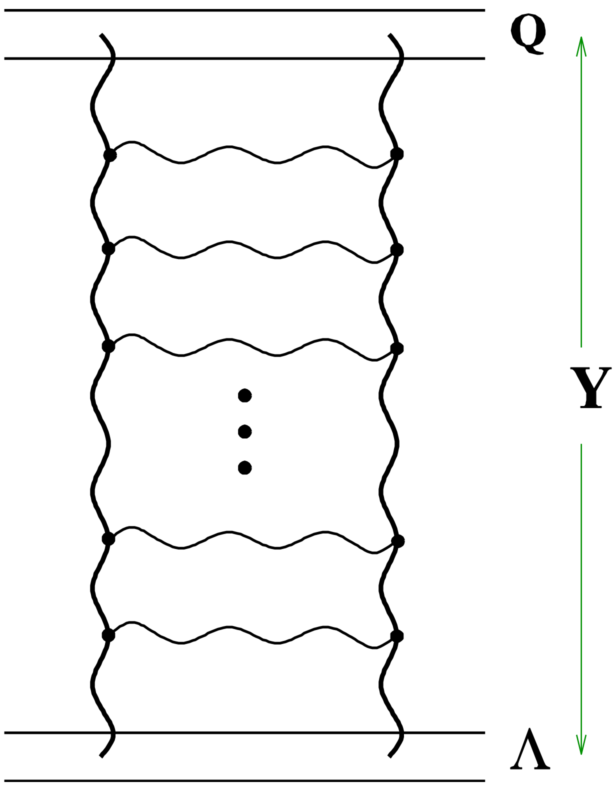
Energy Dependence of the Saturation Scale

Single BFKL ladder gives scattering amplitude of the order $N \sim \frac{\Lambda}{k_T}\, s^{\Delta}$

Nonlinear saturation effects become important when N ~ N^2 ⇨ N ~ 1. This happens at

$$k_T = Q_s \sim \Lambda\, s^{\Delta}$$

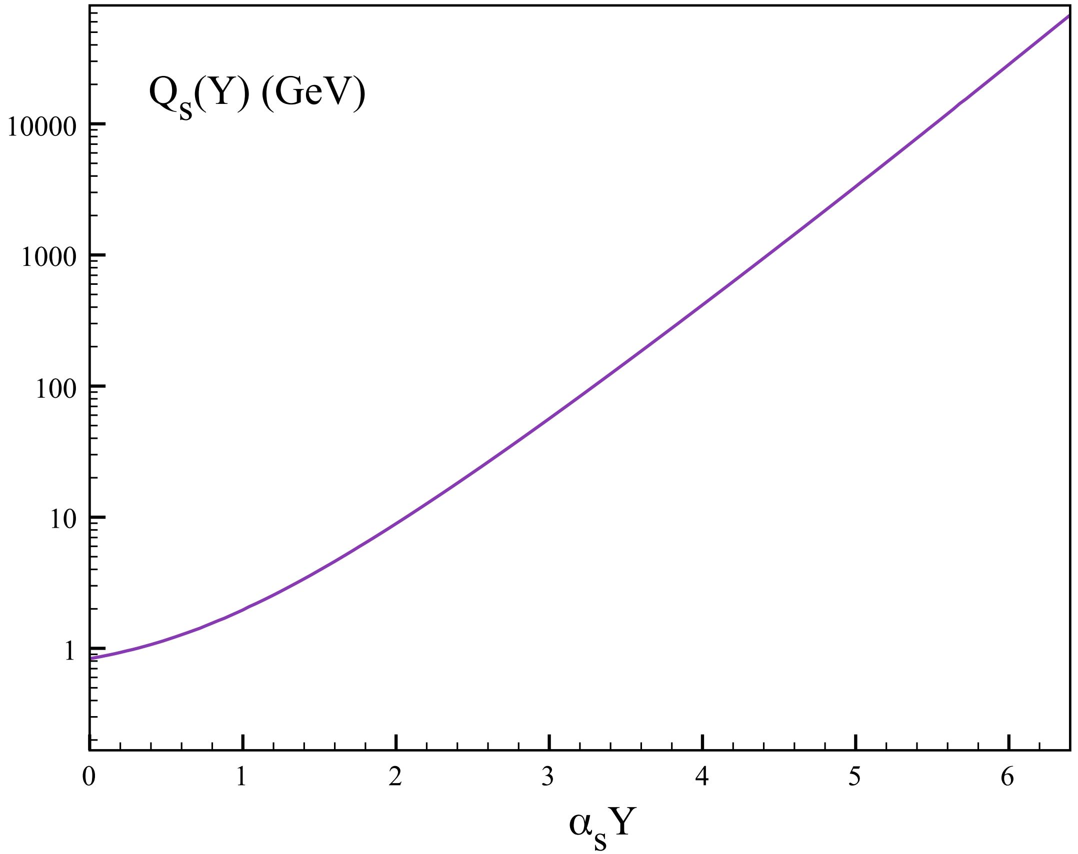
Saturation scale grows with energy!



Typical partons in the wave function have $k_T \sim Q_S$, so that their characteristic size is of the order $r \sim 1/k_T \sim 1/Q_S$.

⇨ Typical parton size decreases with energy!

Saturation scale

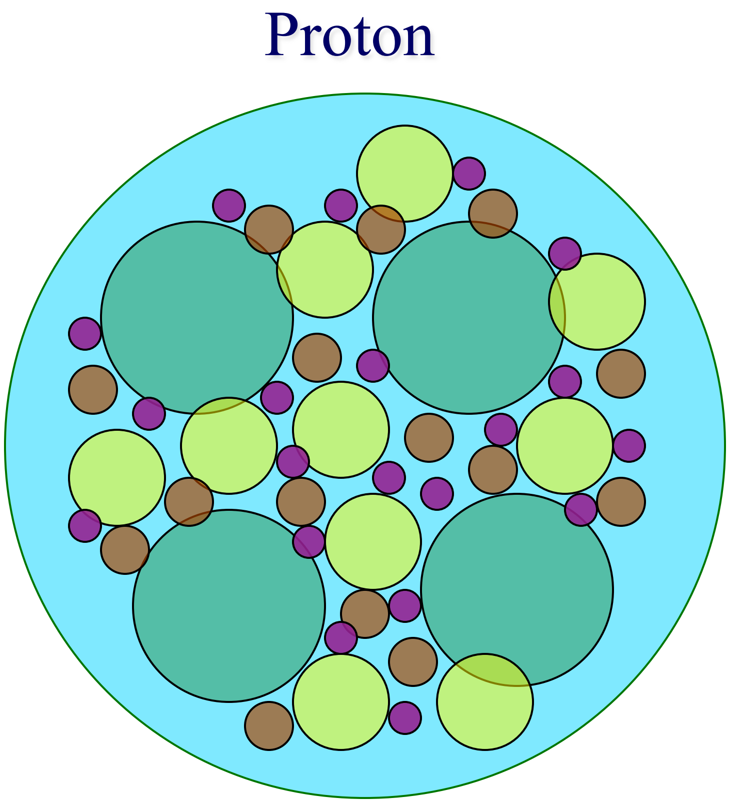


numerical solution by J. Albacete

Nonlinear Evolution at Work

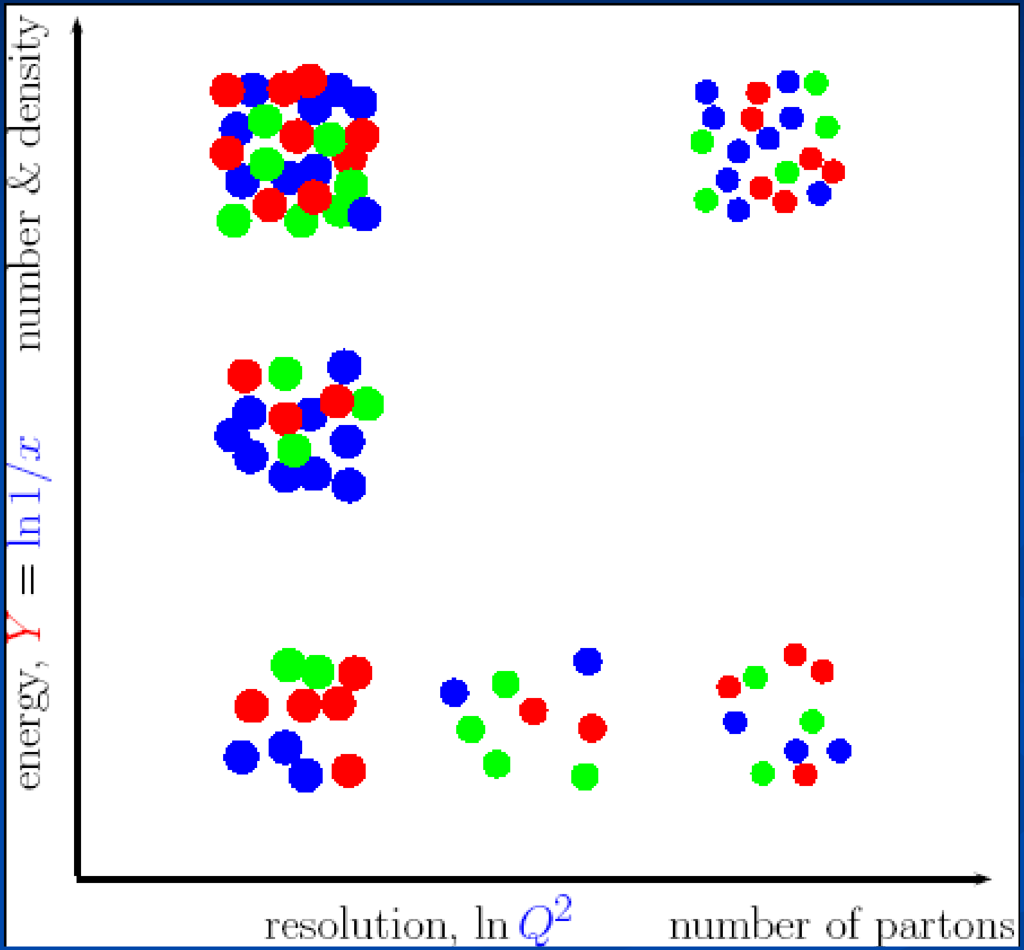
✓ First partons are produced overlapping each other, all of them about the same size.

✓ When some critical density is reached no more partons of given size can fit in the wave function. The proton starts producing smaller partons to fit them in.



Color Glass Condensate

Map of High Energy QCD

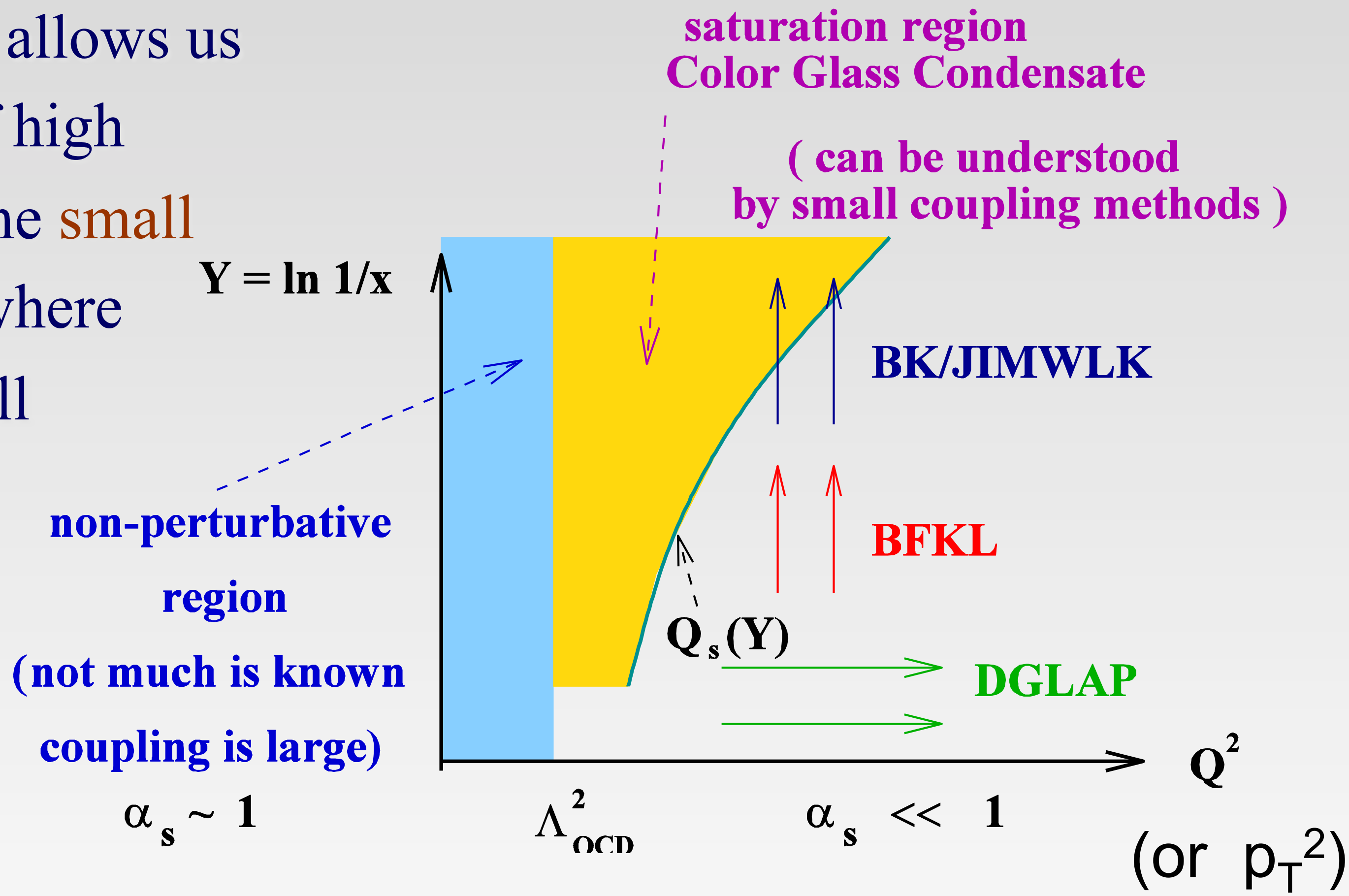


energy

size of gluons

Map of High Energy QCD

Saturation physics allows us to study regions of high parton density in the small coupling regime, where calculations are still under control!



Transition to saturation region is characterized by the saturation scale

$$Q_S^2 \sim A^{1/3} \left(\frac{1}{x} \right)^{\lambda}$$

Geometric Scaling

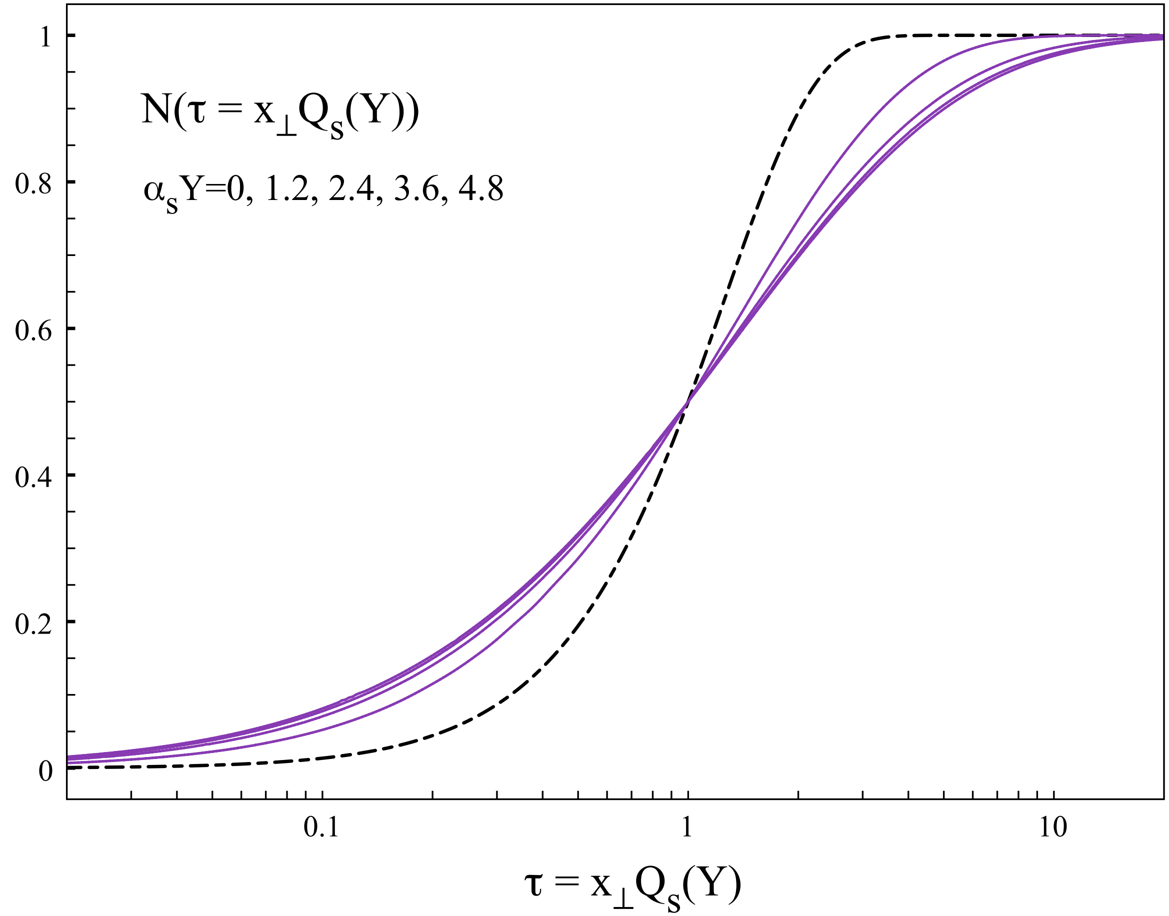
- One of the predictions of the JIMWLK/BK evolution equations is geometric scaling:

  DIS cross section should be a function of one parameter:

$$\sigma_{DIS}(x, Q^2) = \sigma_{DIS}(Q^2 / Q_S^2(x))$$

(Levin, Tuchin ' 99; Iancu, Itakura, McLerran ' 02)

Geometric Scaling



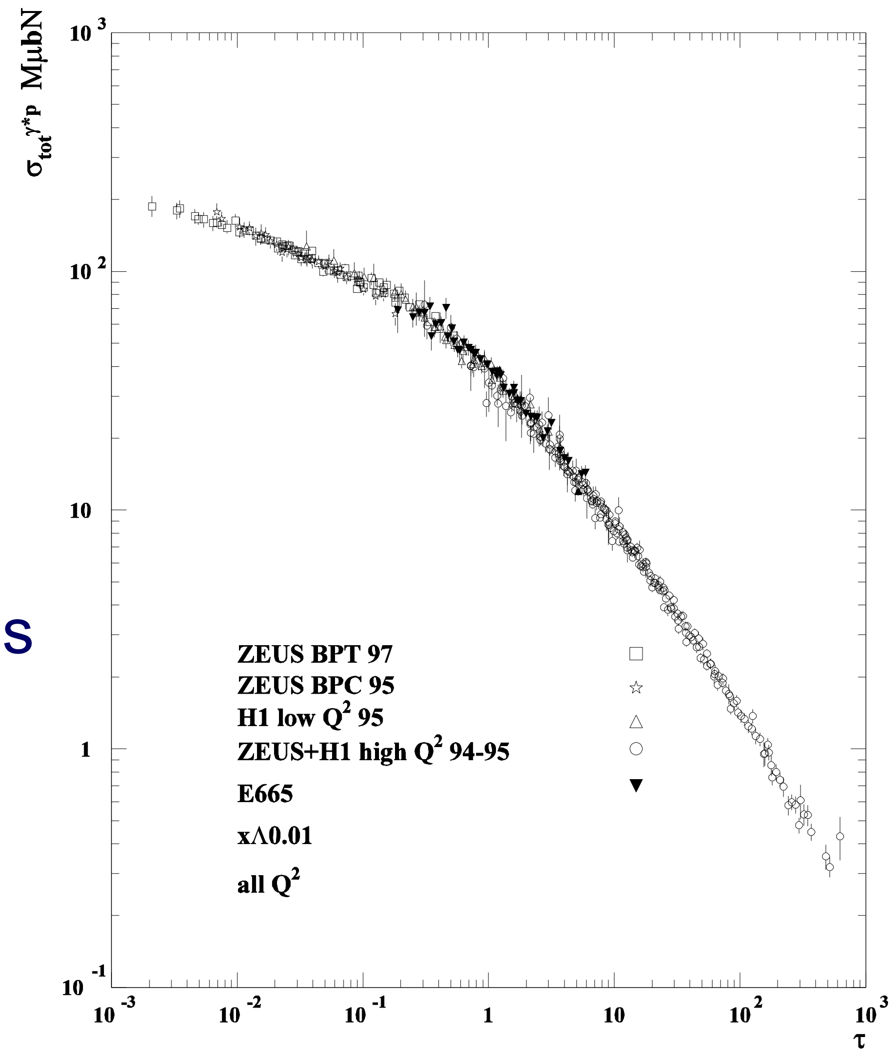
numerical solution by J. Albacete

Geometric Scaling in DIS

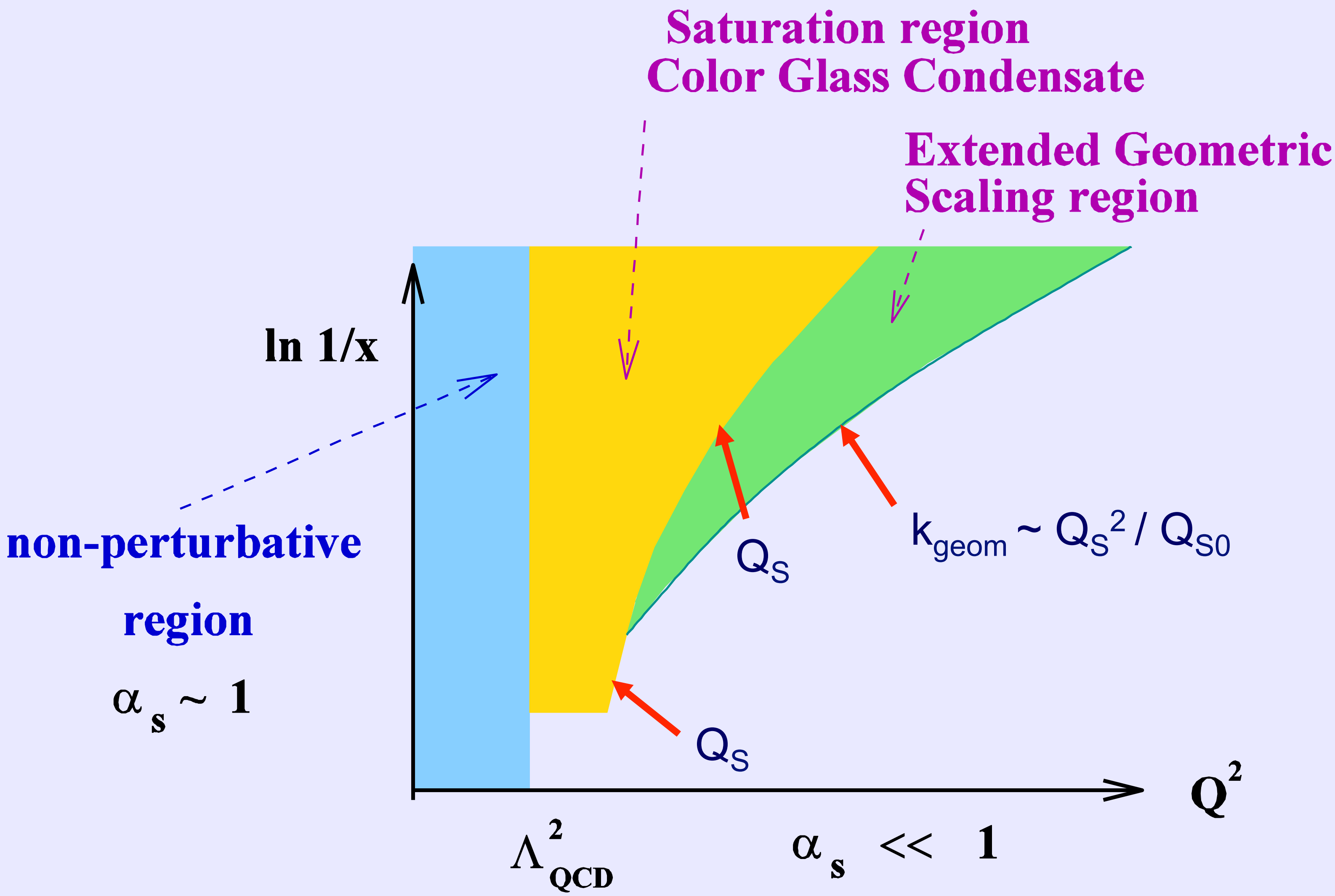
Geometric scaling has been observed in DIS data by Stasto, Golec-Biernat, Kwiecinski in ’00.

Here they plot the total DIS cross section, which is a function of 2 variables - Q^2 and x, as a function of just <u>one</u> variable:

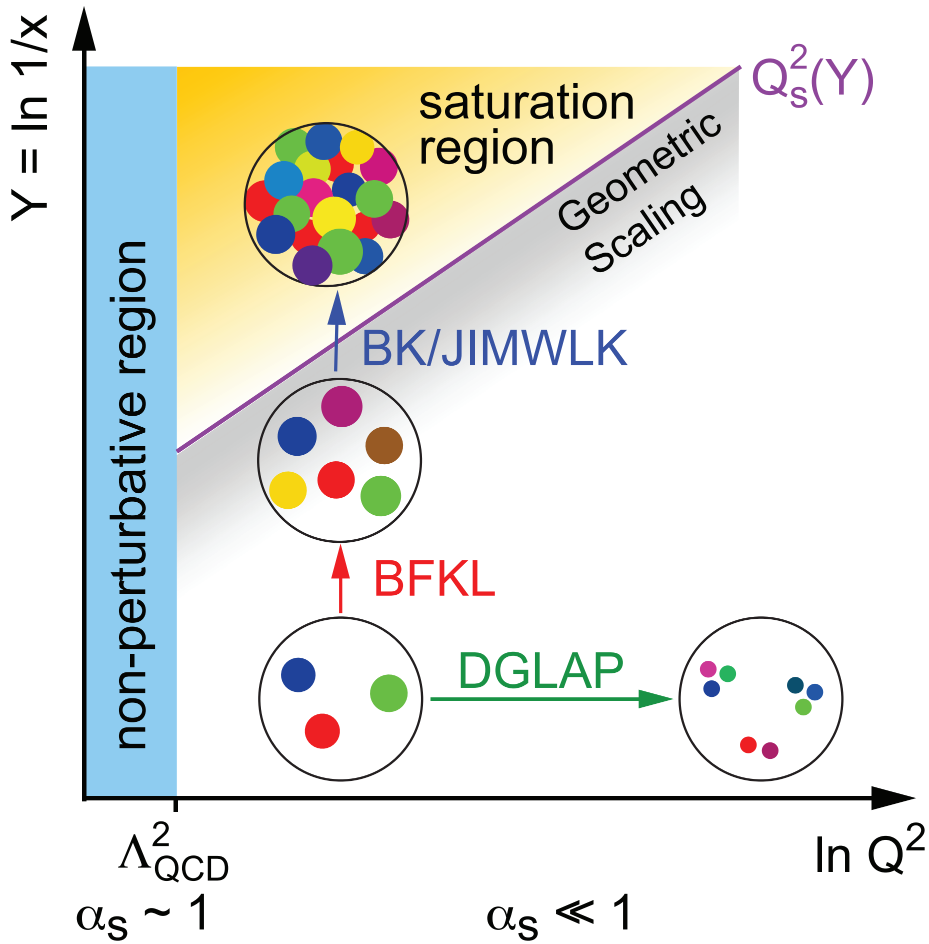
$$\tau = \frac{Q^2}{Q_s^2}$$



Map of High Energy QCD



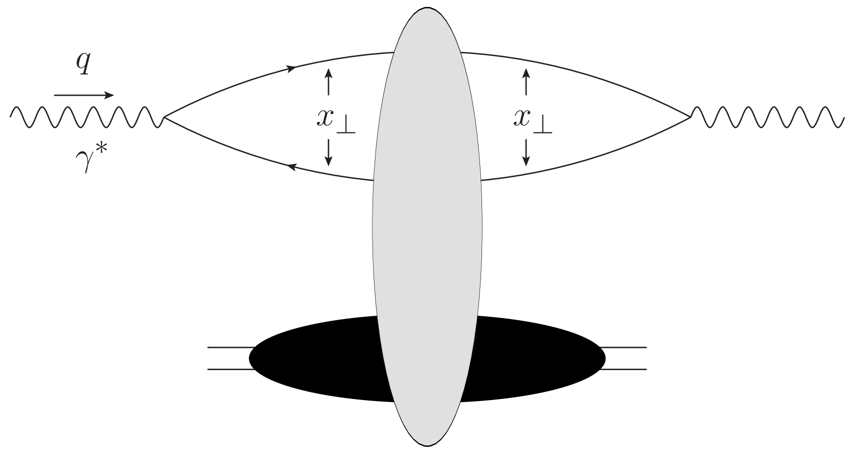
Map of High Energy QCD



D. Connection to Conventional Approaches

Dipole Amplitude and Other Operators

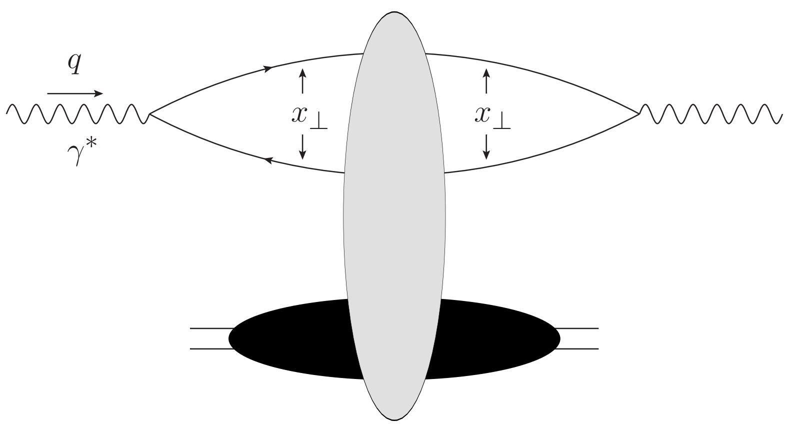
- Dipole scattering amplitude is a universal degree of freedom in saturation physics.
- It describes the total DIS cross section and structure functions:

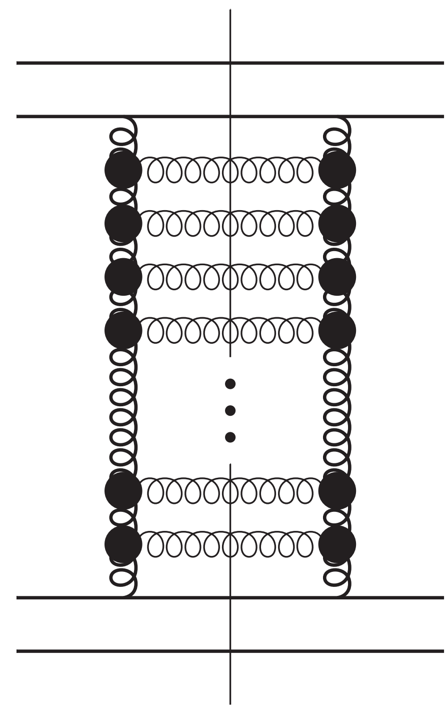


- It also describes single inclusive quark and gluon production cross section in DIS and in p+A collisions.
- Works for diffraction in DIS and p+A.
- For correlations need also quadrupoles (J.Jalilian-Marian, Yu.K. '04; Dominguez et al '11) and other Wilson line operators.

Dipole vs. BFKL Evolution

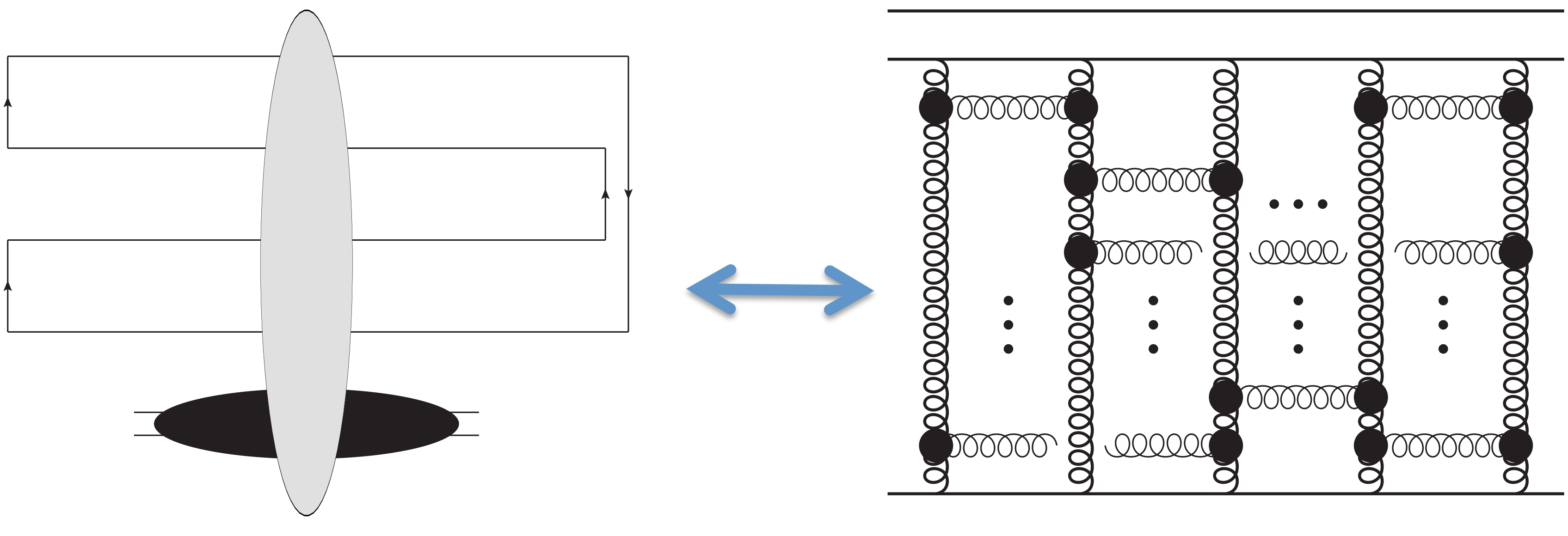
- In the linear regime, dipole evolution is BFKL:





BKP Evolution vs Quadrupoles, etc

- What is the analogue of the Bartels-Kwiecinski-Praszalowicz (BKP) equation?

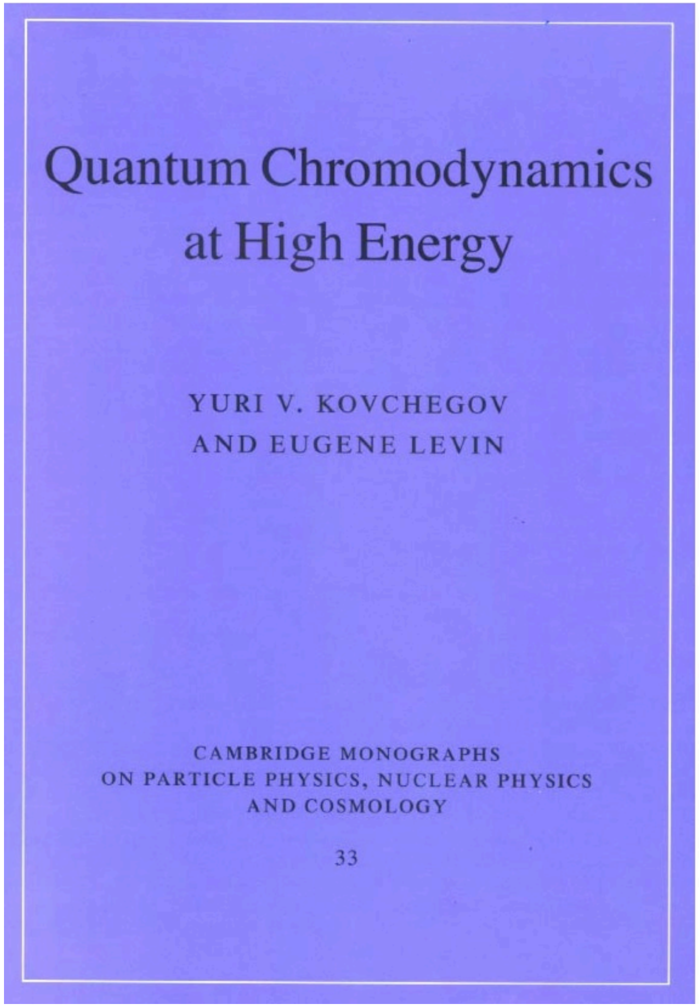


- It seems that color quadrupoles and higher multipoles are the answer. (Altinoluk et al, 2013)

References

- E.Iancu, R.Venugopalan, hep-ph/0303204.
- H.Weigert, hep-ph/0501087
- J.Jalilian-Marian, Yu.K., hep-ph/0505052
- F. Gelis et al, arXiv:1002.0333 [hep-ph]
- J.L. Albacete, C. Marquet, arXiv:1401.4866 [hep-ph]
- and…

References



Published in September 2012
by Cambridge U Press

Conclusions

- We have constructed nuclear/hadronic wave function in the quasi-classical approximation (MV model), and studied DIS in the same approximation
- We included small-x evolution corrections into the DIS process, obtaining nonlinear BK/JIMWLK evolution equations
- We found the saturation scale justifying the whole procedure.

$$Q_S^2 \sim A^{1/3} \left(\frac{1}{x}\right)^{\lambda}$$

- Saturation/CGC physics predicts geometric scaling observed experimentally at HERA.

More Recent Progress

A. Running Coupling

Non-linear evolution: fixed coupling

- Theoretically nothing is wrong with it: preserves unitarity (black disk limit), prevents the IR catastrophe.
- Phenomenologically there is a problem though: LO BFKL intercept is way too large (compared to 0.2-0.3 needed to describe experiment)

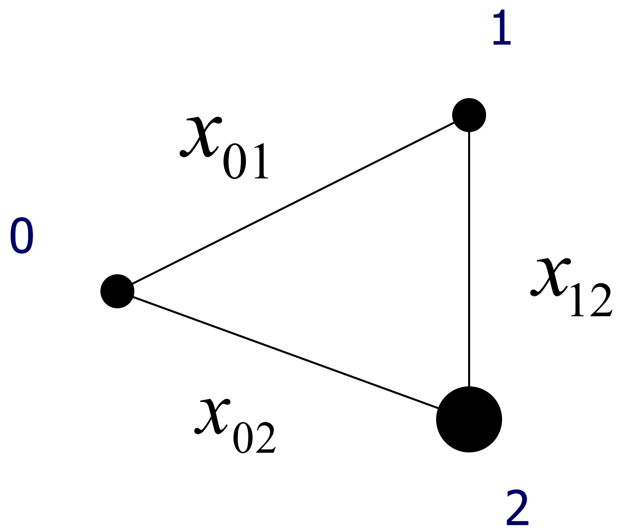
$$\alpha_P - 1 = 2.77 \frac{\alpha_s N_c}{\pi} \approx 0.79$$

- Full NLO calculation (order-α^2 kernel): tough, but done (see Balitsky and Chirilli ' 07).

- First let's try to determine the scale of the coupling.

What Sets the Scale for the Running Coupling?

$$\frac{\partial N(x_0, x_1, Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$
$$\times [N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y) N(x_2, x_1, Y)]$$



transverse plane

What Sets the Scale for the Running Coupling?

$$\frac{\partial N(x_0, x_1, Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2} \int d^2 x_2 \, \frac{x_{01}^2}{x_{02}^2 \, x_{12}^2}$$
$$\times [N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y) \, N(x_2, x_1, Y)]$$

$\alpha_S(???)$

In order to perform consistent calculations it is important to know the scale of the running coupling constant in the evolution equation.

There are three possible scales – the sizes of the "parent" dipole and "daughter" dipoles x_{01}, x_{21}, x_{20} . Which one is it?

Preview

- The answer is that the running coupling corrections come in as a **"triumvirate"** of couplings (H. Weigert, Yu. K. '06; I. Balitsky, '06):

$$\alpha_\mu \Rightarrow \frac{\alpha_s(\ldots)\,\alpha_s(\ldots)}{\alpha_s(\ldots)}$$

cf. Braun '94, Levin '94

- The scales of three couplings are somewhat involved.

Main Principle

To set the scale of the coupling constant we will first calculate the $\alpha_S N_f$ corrections to BK/JIMWLK evolution kernel to all orders.

We then would complete N_f to the QCD beta-function

$$\beta_2 = \frac{11 N_C - 2 N_f}{12 \pi}$$

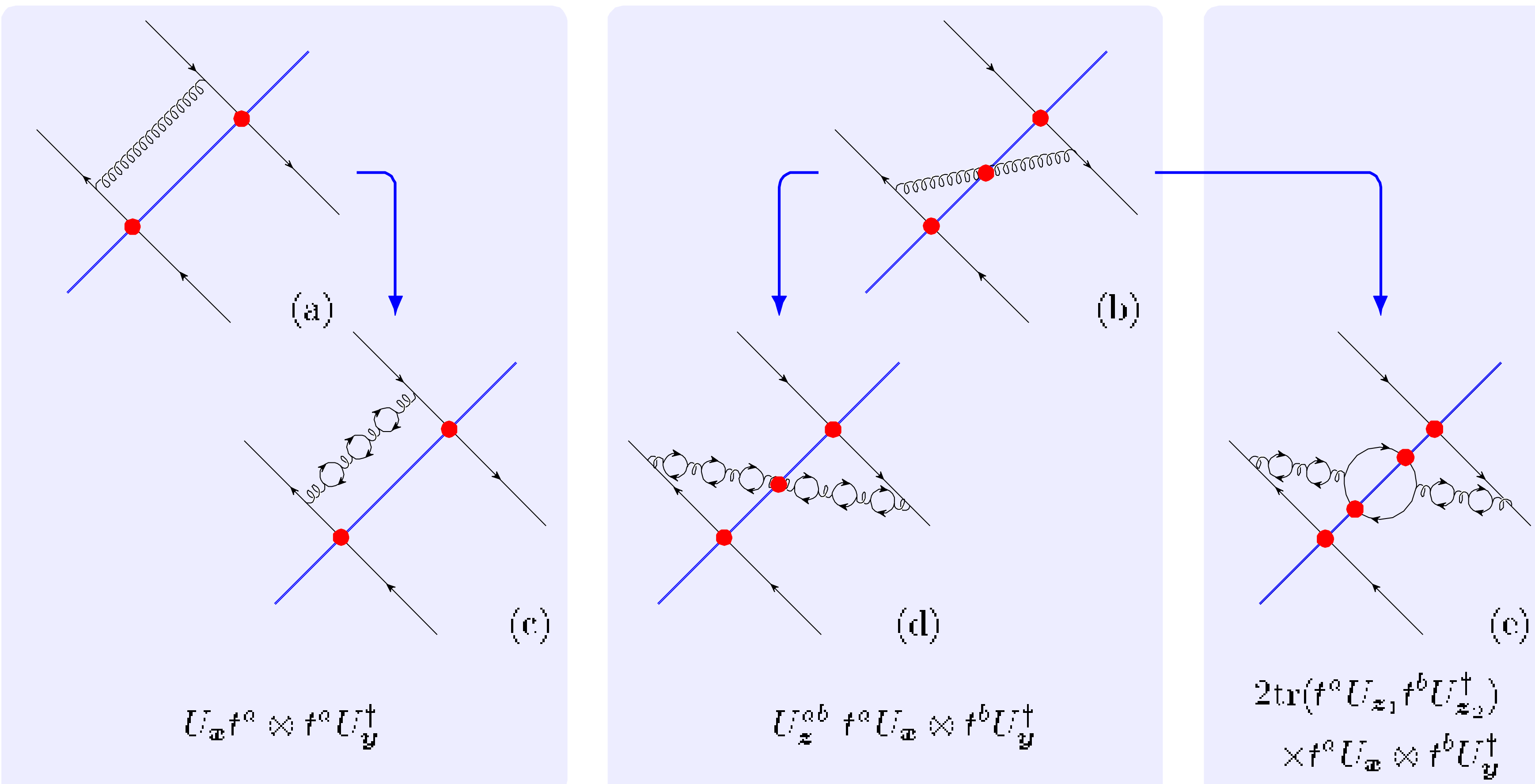
by replacing $N_f \to -6 \pi \beta_2$ to obtain the scale of the running coupling:

$$\alpha_s(Q^2) = \frac{\alpha_\mu}{1 + \alpha_\mu \beta_2 \ln(Q^2/\mu^2)}$$

BLM prescription (Brodsky, Lepage, Mackenzie '83)

Running Coupling Corrections to All Orders

One has to insert fermion bubbles to all orders:



Results: Transverse Momentum Space

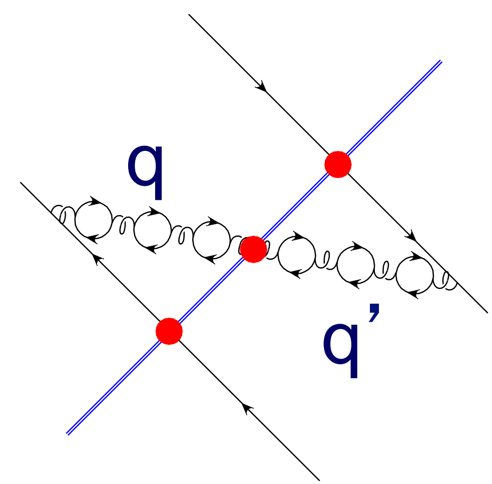
The resulting JIMWLK kernel with running coupling corrections is

$$\alpha_\mu K(\mathbf{x}_0,\mathbf{x}_1;\mathbf{z}) = 4\int \frac{d^2q\, d^2q'}{(2\pi)^4} e^{-i\mathbf{q}\cdot(\mathbf{z}-\mathbf{x}_0)+i\mathbf{q}'\cdot(\mathbf{z}-\mathbf{x}_1)} \frac{\mathbf{q}\cdot\mathbf{q}'}{\mathbf{q}^2\,\mathbf{q}'^2} \frac{\alpha_S(\mathbf{q}^2)\,\alpha_S(\mathbf{q}'^2)}{\alpha_S(Q^2)}$$

where

$$\ln\frac{Q^2}{\mu^2} = \frac{\mathbf{q}^2\ln(\mathbf{q}^2/\mu^2) - \mathbf{q}'^2\ln(\mathbf{q}'^2/\mu^2)}{\mathbf{q}^2-\mathbf{q}'^2} - \frac{\mathbf{q}^2\,\mathbf{q}'^2}{\mathbf{q}\cdot\mathbf{q}'}\frac{\ln(\mathbf{q}^2/\mathbf{q}'^2)}{\mathbf{q}^2-\mathbf{q}'^2}$$

The BK kernel is obtained from the above by summing over all possible emissions of the gluon off the quark and anti-quark lines.



Running Coupling BK

Here's the BK equation with the running coupling corrections
(H. Weigert, Yu. K. '06; I. Balitsky, '06):

$$\frac{\partial N(x_0, x_1, Y)}{\partial Y} = \frac{N_C}{2\pi^2} \int d^2 x_2$$
$$\times \left[\frac{\alpha_S(1/x_{02}^2)}{x_{02}^2} + \frac{\alpha_S(1/x_{12}^2)}{x_{12}^2} - 2\,\frac{\alpha_S(1/x_{02}^2)\,\alpha_S(1/x_{12}^2)}{\alpha_S(1/R^2)}\,\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{x_{02}^2\,x_{12}^2} \right]$$
$$\times [N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y)\,N(x_2, x_1, Y)]$$

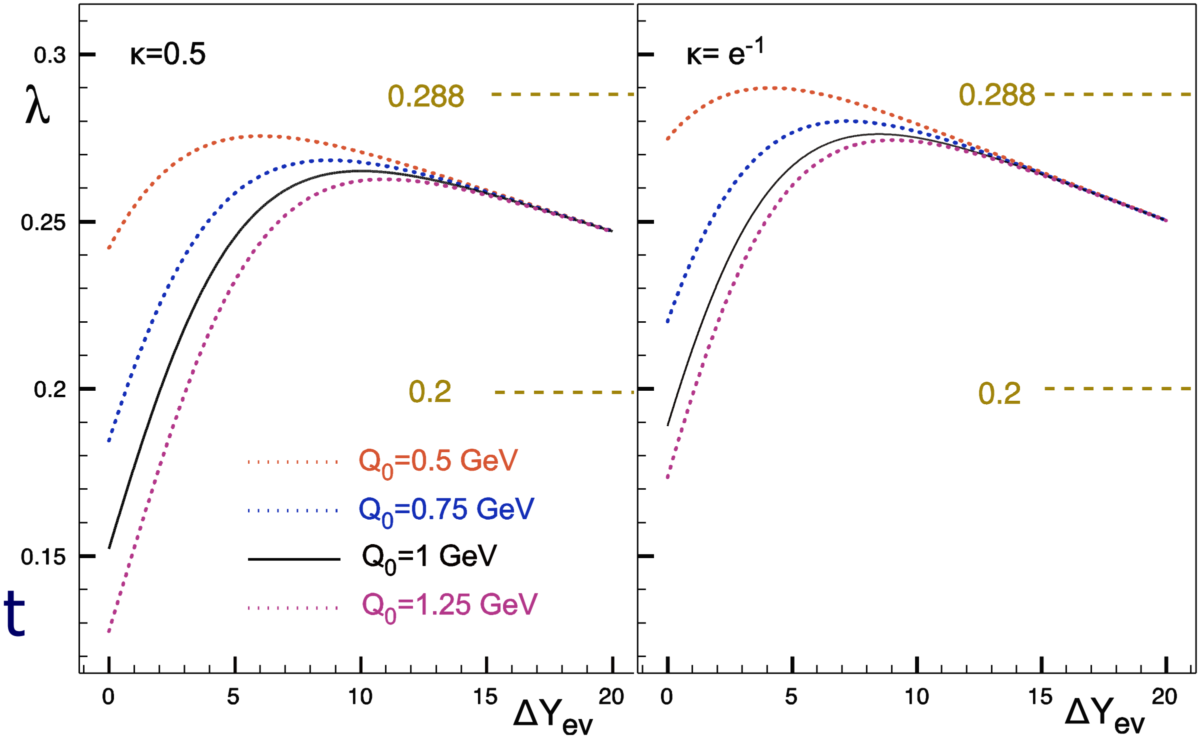
where

$$\ln R^2 \mu^2 = \frac{x_{20}^2 \ln(x_{21}^2 \mu^2) - x_{21}^2 \ln(x_{20}^2 \mu^2)}{x_{20}^2 - x_{21}^2} + \frac{x_{20}^2\, x_{21}^2}{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}\,\frac{\ln(x_{20}^2 / x_{21}^2)}{x_{20}^2 - x_{21}^2}$$

What does the running coupling do?

- Slows down the evolution with energy / rapidity.

$$\lambda = \frac{d\ln Q_s^2(Y)}{dY}$$



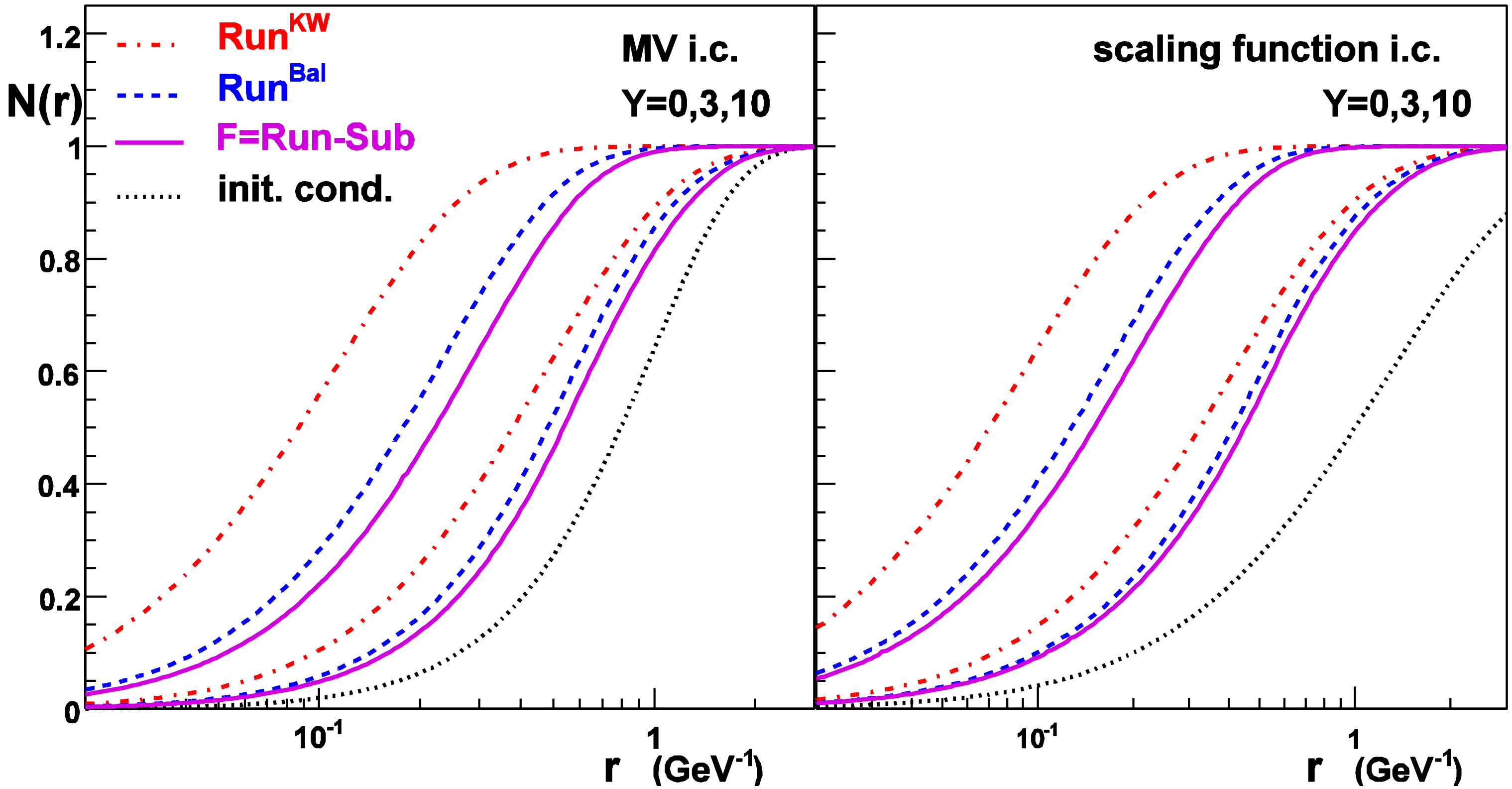
down from about

$\lambda \approx 0.7 \div 0.8$

at fixed coupling

Albacete '07

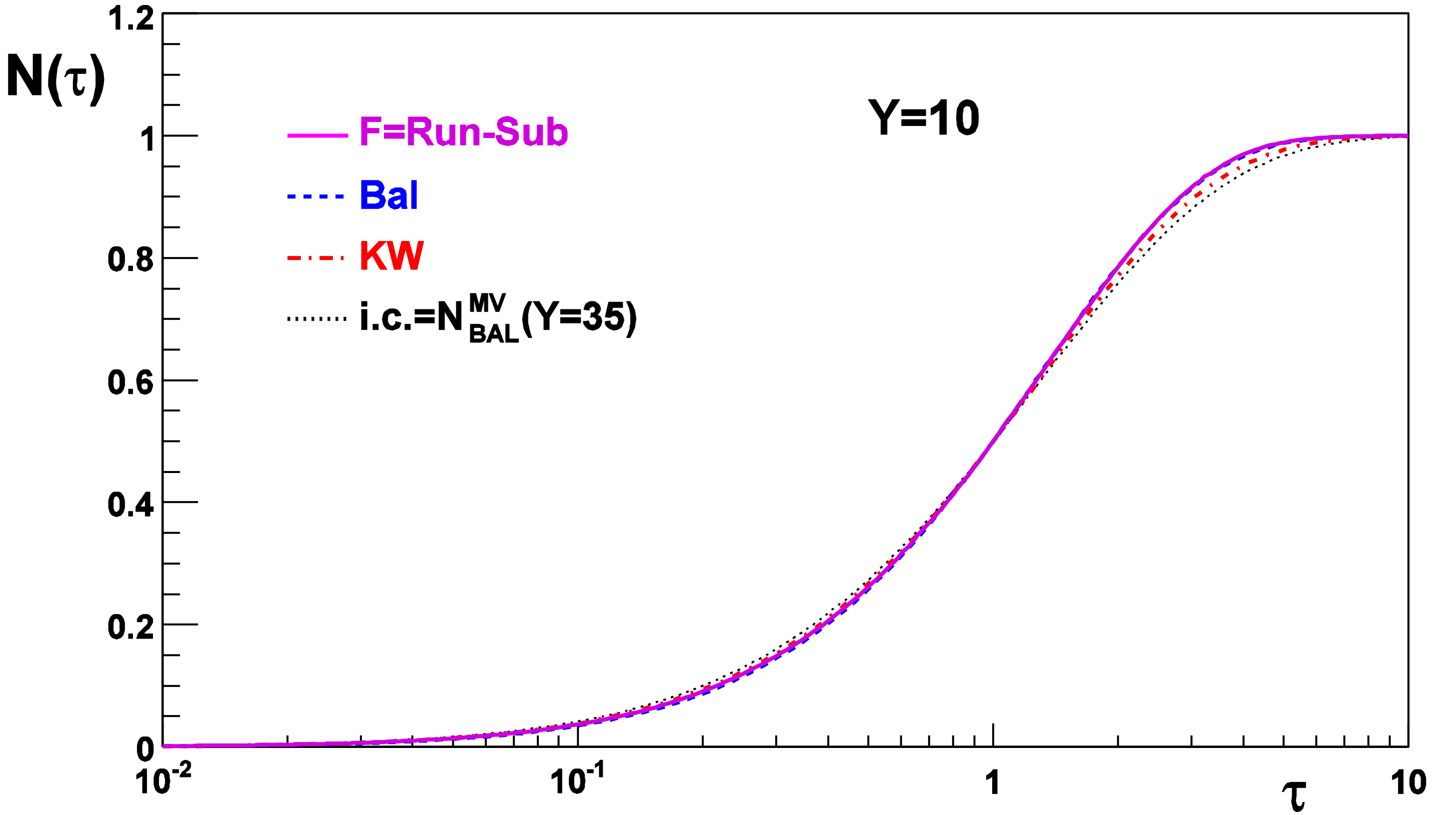
Solution of the Full Equation



Different curves – different ways of separating running coupling from NLO corrections. Solid curve includes all corrections.

J. Albacete, Yu.K. '07

Geometric Scaling



$$\tau = r\, Q_S(Y)$$

At high enough rapidity we recover geometric scaling, all solutions fall on the same curve. This has been known for fixed coupling: however, the shape of the scaling function is different in the running coupling case!

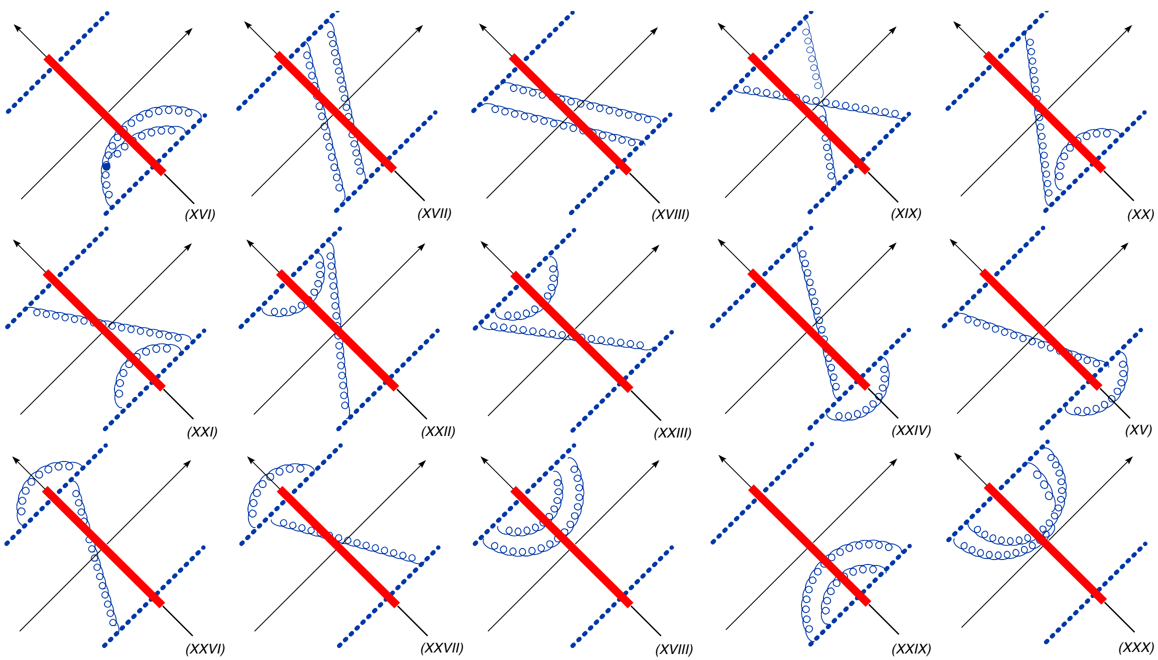
J. Albacete, Yu.K. '07

B. NLO BFKL/BK/JIMWLK

NLO BK

- NLO BK evolution was calculated by Balitsky and Chirilli in 2007.
- It resums powers of $\alpha_s^2 Y$ (NLO) in addition to powers of $\alpha_s Y$ (LO).
- Here's a sampler of relevant diagrams (need kernel to order-α^2):

Diagrams with 2 gluons interaction



NLO BK

- The large-N_C limit:

$$\frac{d}{d\eta}N(x,y) = \frac{\alpha_s N_c}{2\pi^2}\int d^2z \frac{(x-y)^2}{X^2Y^2}\left\{1 + \frac{\alpha_s N_c}{4\pi}\left[\frac{11}{3}\ln(x-y)^2\mu^2 - \frac{11}{3}\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - 2\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\right]\right\}$$
$$\times[N(x,z)+N(z,y)-N(x,y)-N(x,z)N(z,y)]$$
$$+\frac{\alpha_s^2 N_c^2}{8\pi^4}\int d^2z d^2z'\left\{-\frac{2}{(z-z')^4} + \left[\frac{X^2Y'^2+X'^2Y^2-4(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2-X'^2Y^2)} + \frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}\right.\right.$$
$$\left.\left.+\frac{(x-y)^2}{X^2Y'^2(z-z')^2}\right]\ln\frac{X^2Y'^2}{X'^2Y^2}\right\}[N(z,z')-N(x,z)N(z,z')-N(z,z')N(z',y)-N(x,z)N(z',y)+N(x,z)N(z,y)$$
$$+N(x,z)N(z,z')N(z',y)]. \qquad (136)$$

(yet to be solved numerically)

NLO JIMWLK

- Very recently NLO evolution has been calculated for other Wilson line operators (not just dipoles), most notably the 3-Wilson line operator (Grabovsky '13, Balitsky & Chirilli '13, Kovner, Lublinsky, Mulian '13, Balitsky and Grabovsky '14).

- The NLO JIMWLK Hamiltonian was constructed as well (Kovner, Lublinsky, Mulian '13, '14).

- However, the equations do not close, that is, the operators on the right hand side can not be expressed in terms of the operator on the left. Hence can't solve.

- To find the expectation values of the corresponding operators, one has to perform a lattice calculation with the NLO JIMWLK Hamiltonian, generating field configurations to be used for averaging the operators.

NLO Dipole Evolution at any N_C

- NLO BK equation is the large-N_C limit of (Balitsky and Chrilli ' 07)

$$
\begin{aligned}
&\frac{d}{d\eta}\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\} \qquad (5)\\
&= \frac{\alpha_s}{2\pi^2}\int d^2z\, \frac{(x-y)^2}{X^2Y^2}\Big\{1+\frac{\alpha_s}{4\pi}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2} + (\frac{67}{9}-\frac{\pi^2}{3})N_c - \frac{10}{9}n_f \\
&\qquad - 2N_c\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big]\Big\}\,[\mathrm{Tr}\{\hat U_x\hat U_z^\dagger\}\mathrm{Tr}\{\hat U_z\hat U_y^\dagger\} - N_c\mathrm{Tr}\{\hat U_x\hat U_y^\dagger\}] \\
&+ \frac{\alpha_s^2}{16\pi^4}\int d^2z d^2z' \Big[\Big(-\frac{4}{(z-z')^4} + \Big\{2\frac{X^2Y'^2+X'^2Y^2-4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2-X'^2Y^2]} \\
&+ \frac{(x-y)^4}{X^2Y'^2-X'^2Y^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big] + \frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}-\frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big) \\
&\qquad \times [\mathrm{Tr}\{\hat U_x\hat U_z^\dagger\}\mathrm{Tr}\{\hat U_z\hat U_{z'}^\dagger\}\mathrm{Tr}\{\hat U_{z'}\hat U_y^\dagger\} - \mathrm{Tr}\{\hat U_x\hat U_z^\dagger\hat U_{z'}U_y^\dagger\hat U_z\hat U_{z'}^\dagger\} - (z'\to z)] \\
&+ \Big\{\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big] - \frac{(x-y)^4}{X^2Y'^2X'^2Y^2}\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\,\mathrm{Tr}\{\hat U_x\hat U_z^\dagger\}\mathrm{Tr}\{\hat U_z\hat U_{z'}^\dagger\}\mathrm{Tr}\{\hat U_{z'}\hat U_y^\dagger\} \\
&+ 4n_f\Big\{\frac{4}{(z-z')^4} - 2\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2-X'^2Y^2-)}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\mathrm{Tr}\{t^a\hat U_x t^b\hat U_y^\dagger\}[\mathrm{Tr}\{t^a\hat U_z t^b\hat U_{z'}^\dagger\} - (z'\to z)]\Big]
\end{aligned}
$$

Summary

- Running coupling and NLO corrections have been calculated for BK and JIMWLK equations.

- rcBK and rcJIMWLK have been solved numerically and used in phenomenology (DIS, pA, AA) with reasonable success.

- NLO BK and NLO JIMWLK have not yet been solved.

C. DIS Phenomenology

Three-step prescription

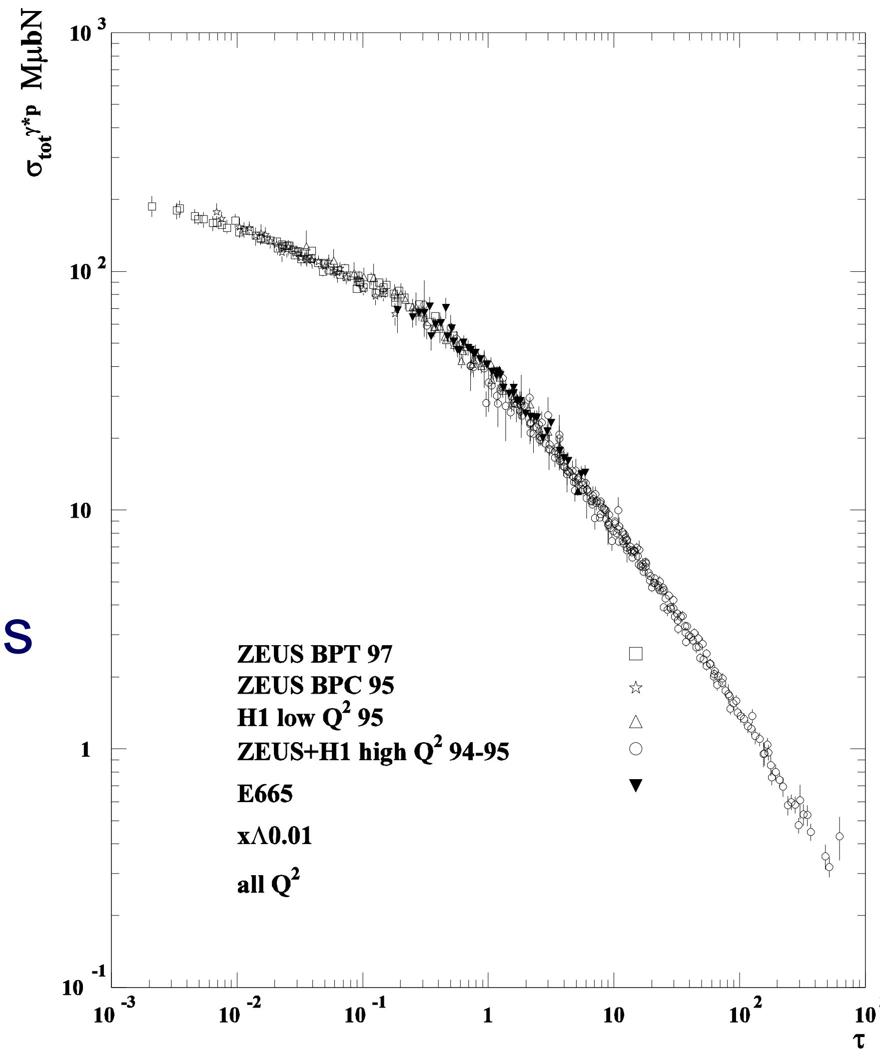
- Calculate the observable in the classical approximation.
- Include nonlinear small-x evolution corrections (BK/JIMWLK), introducing energy-dependence.
- To compare with experiment, need to fix the scale of the running coupling.
- NLO corrections to BK/JIMWLK need to be included as well. This has not been done yet.

Geometric Scaling in DIS

Geometric scaling has been observed in DIS data by Stasto, Golec-Biernat, Kwiecinski in ’00.

Here they plot the total DIS cross section, which is a function of 2 variables - Q^2 and x, as a function of just <u>one</u> variable:

$$\tau = \frac{Q^2}{Q_s^2}$$

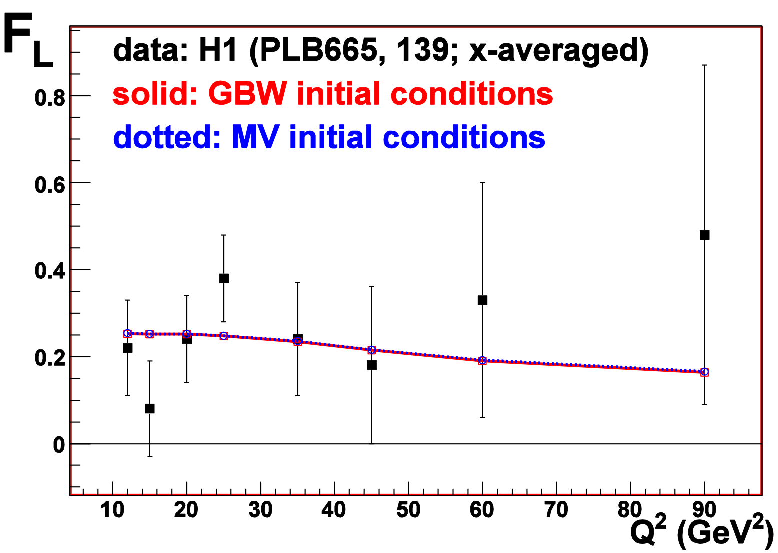


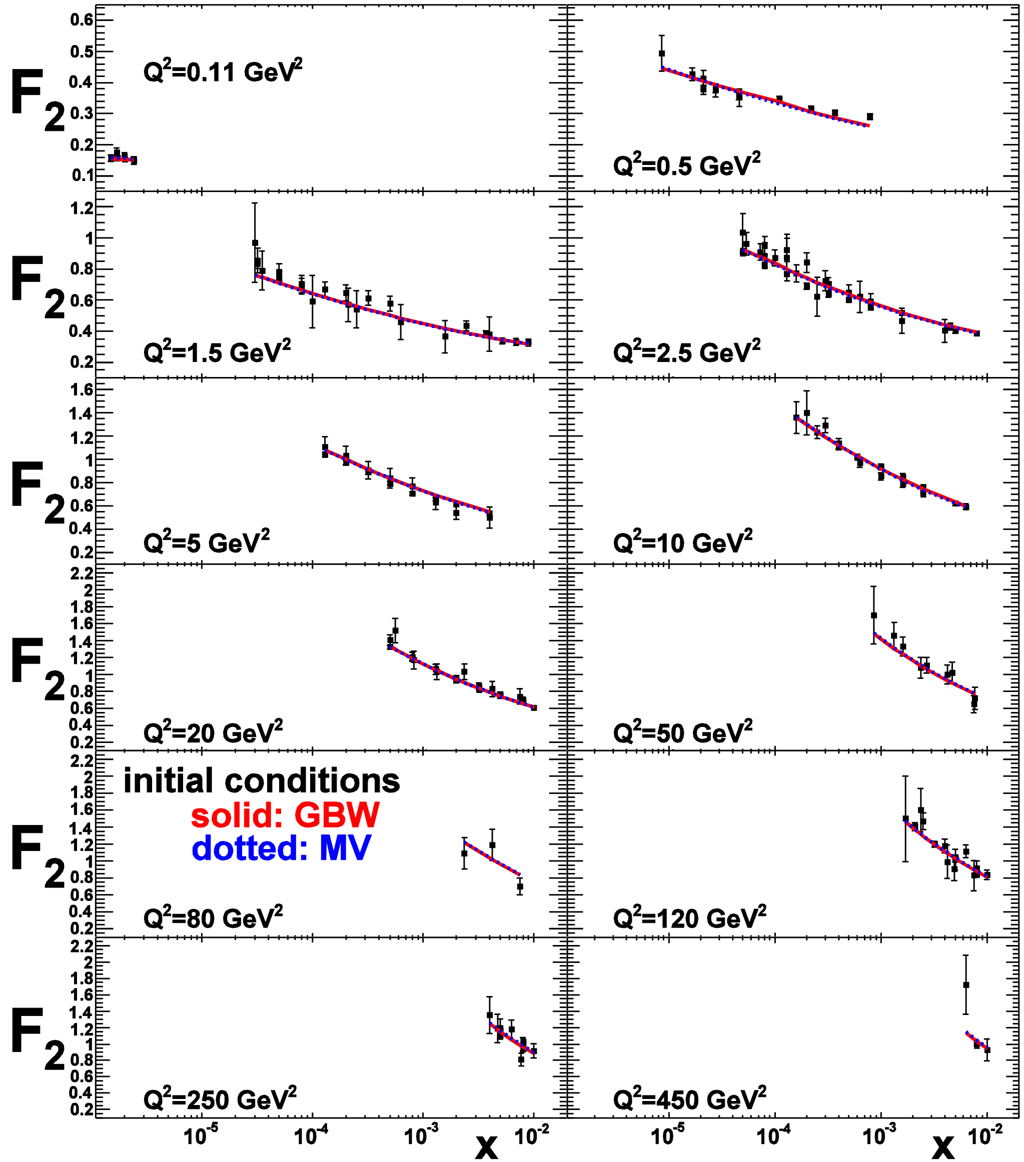
Comparison of rcBK with HERA F2 Data

DIS structure functions:

$$F_{2,\,L} = \frac{Q^2}{4\,\pi^2\,\alpha_{EM}}\,\sigma_{tot,\,L}^{\gamma * p}$$

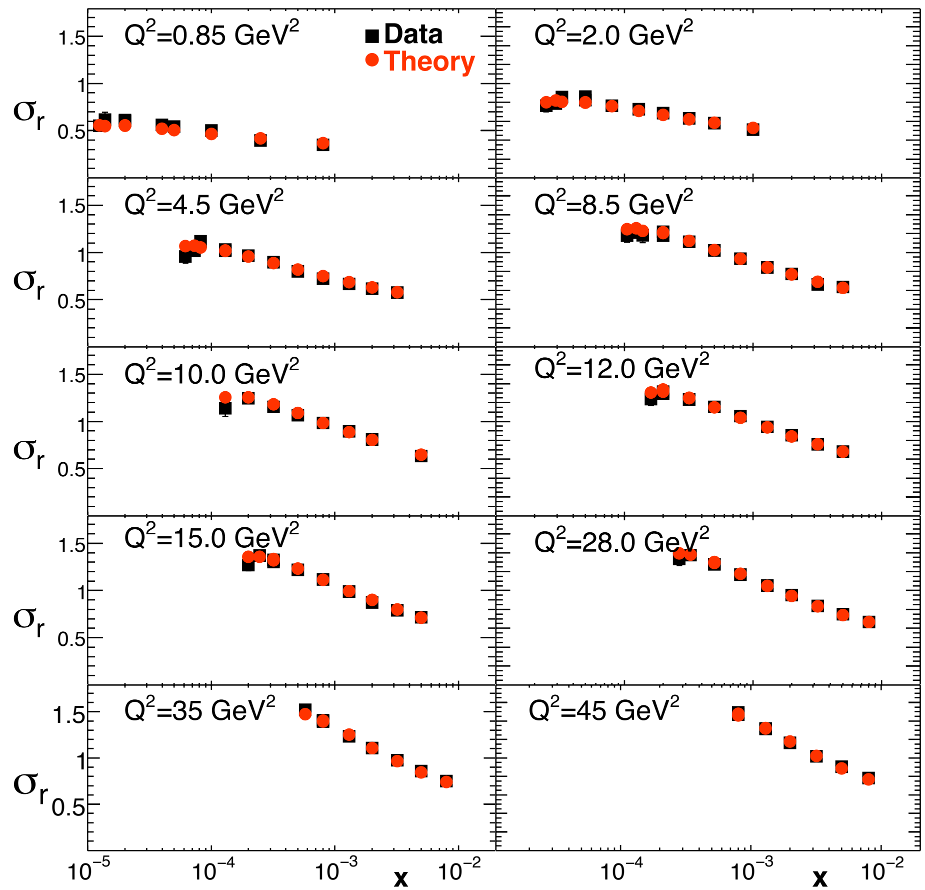
from Albacete, Armesto, Milhano, Salgado '09





Comparison with the combined H1 and ZEUS data

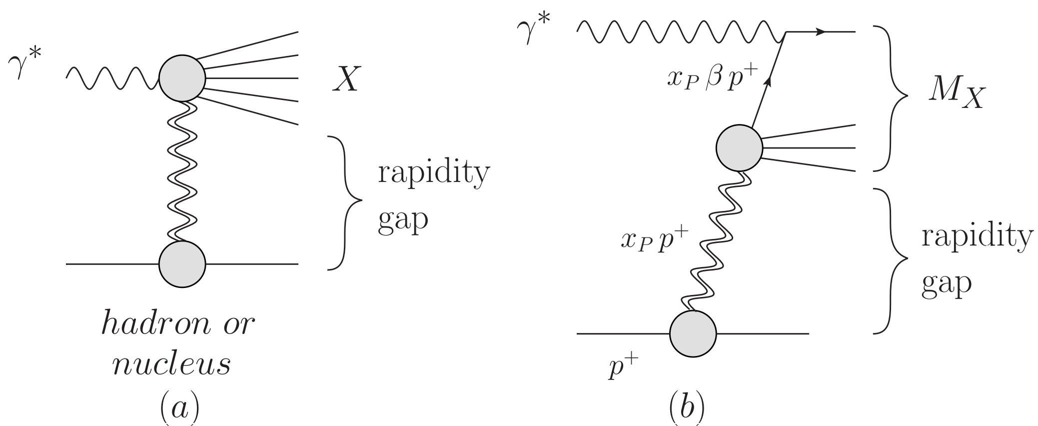
Albacete, Armesto, Milhano, Qiuroga Arias, and Salgado '11

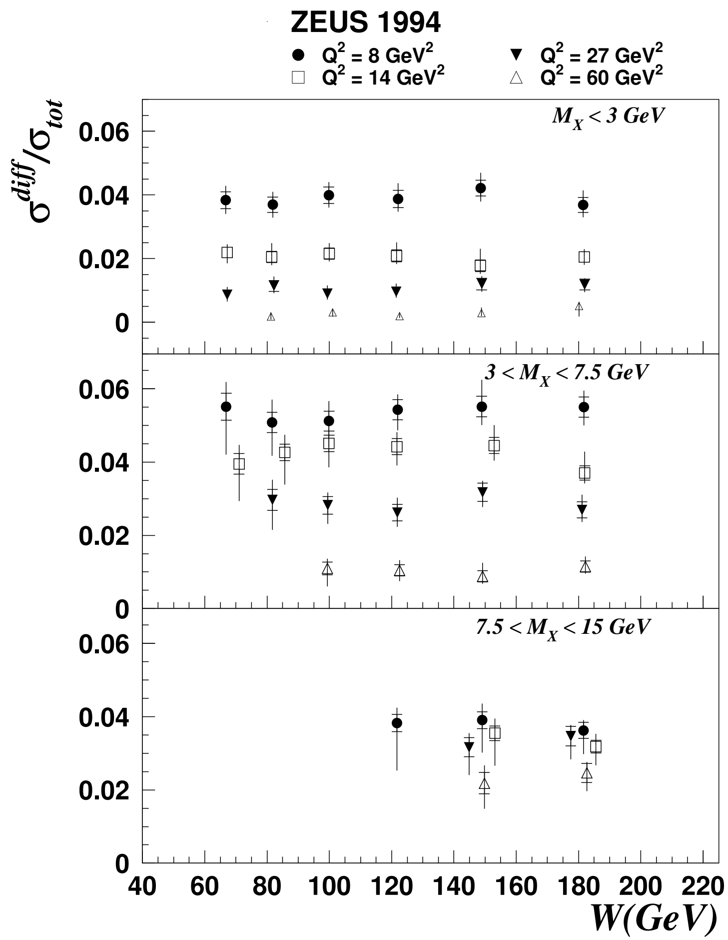


reduced cross section:

$$\sigma_r = F_2 - \frac{y^2}{1+(1-y)^2} F_L$$

Diffractive cross section

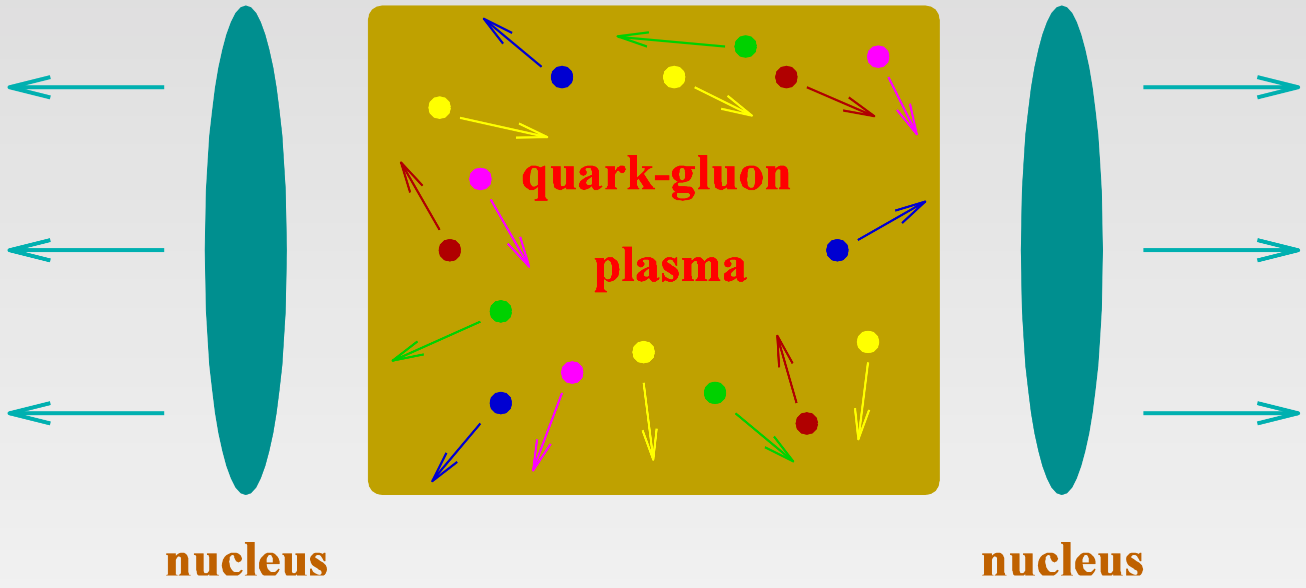




Also agrees with the saturation/CGC expectations.

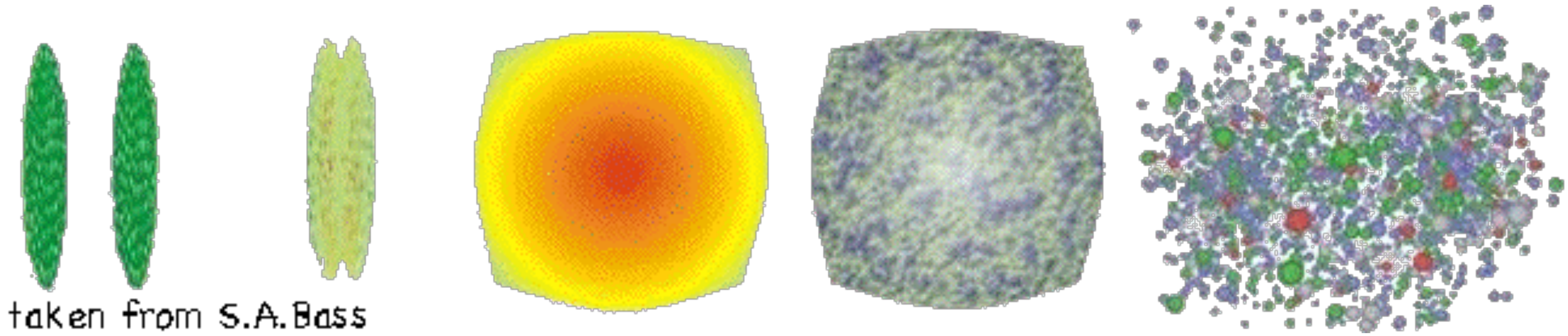
D. Heavy Ion Phenomenology

Heavy Ion Collisions



⇨ Quarks and gluons are confined inside hadrons. In heavy ion collisions people are trying to create a new state of matter called Quark-Gluon Plasma: a soup of de-confined quarks and gluons.

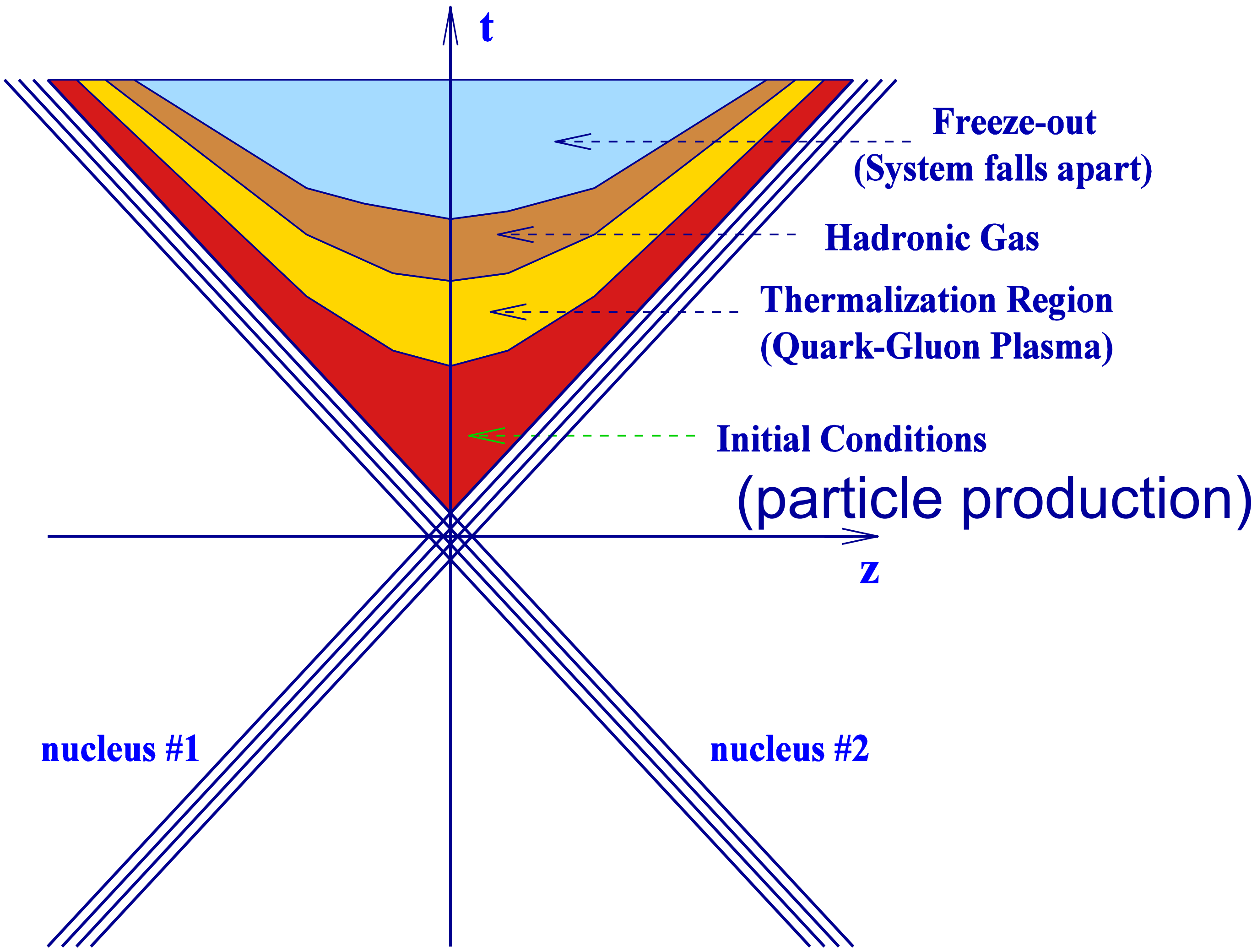
Heavy Ion Collisions



Time evolution of the collision:

- Initial collision and particle production
- Thermalization and formation of quark-gluon plasma (QGP)
- Hadronization: QGP becomes a hadron gas
- Decoupling followed by free-streaming

Timeline of a Heavy Ion Collision

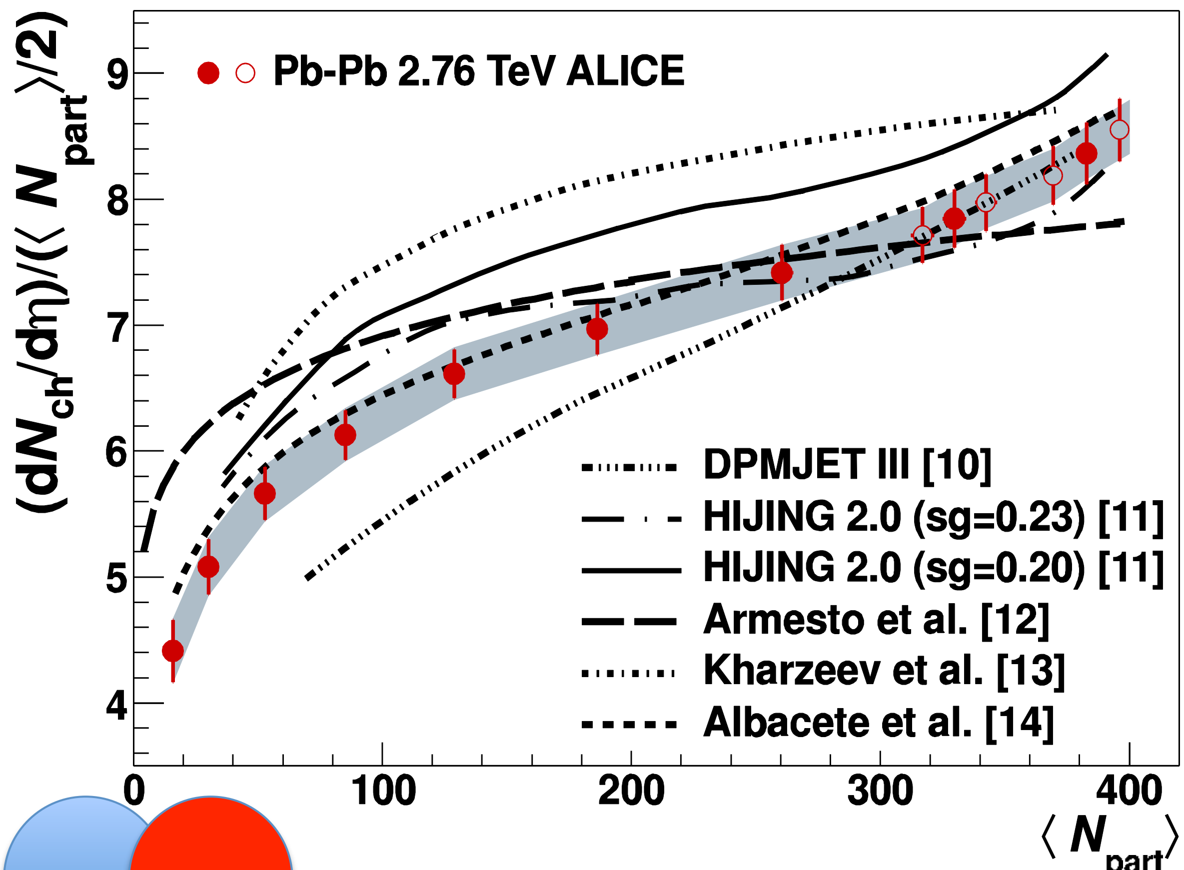


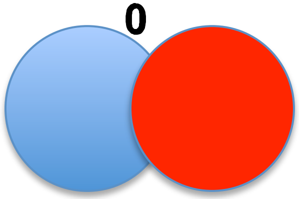
Three-step prescription

- Calculate the observable in the classical approximation.
- Include nonlinear small-x evolution corrections, introducing energy-dependence.
- To compare with experiment, need to find the scale of the running coupling.

CGC Multiplicity Prediction

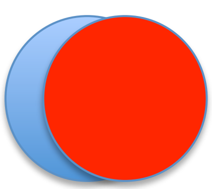
Number of hadrons
per nucleon-nucleon collision
at mid-rapidity.





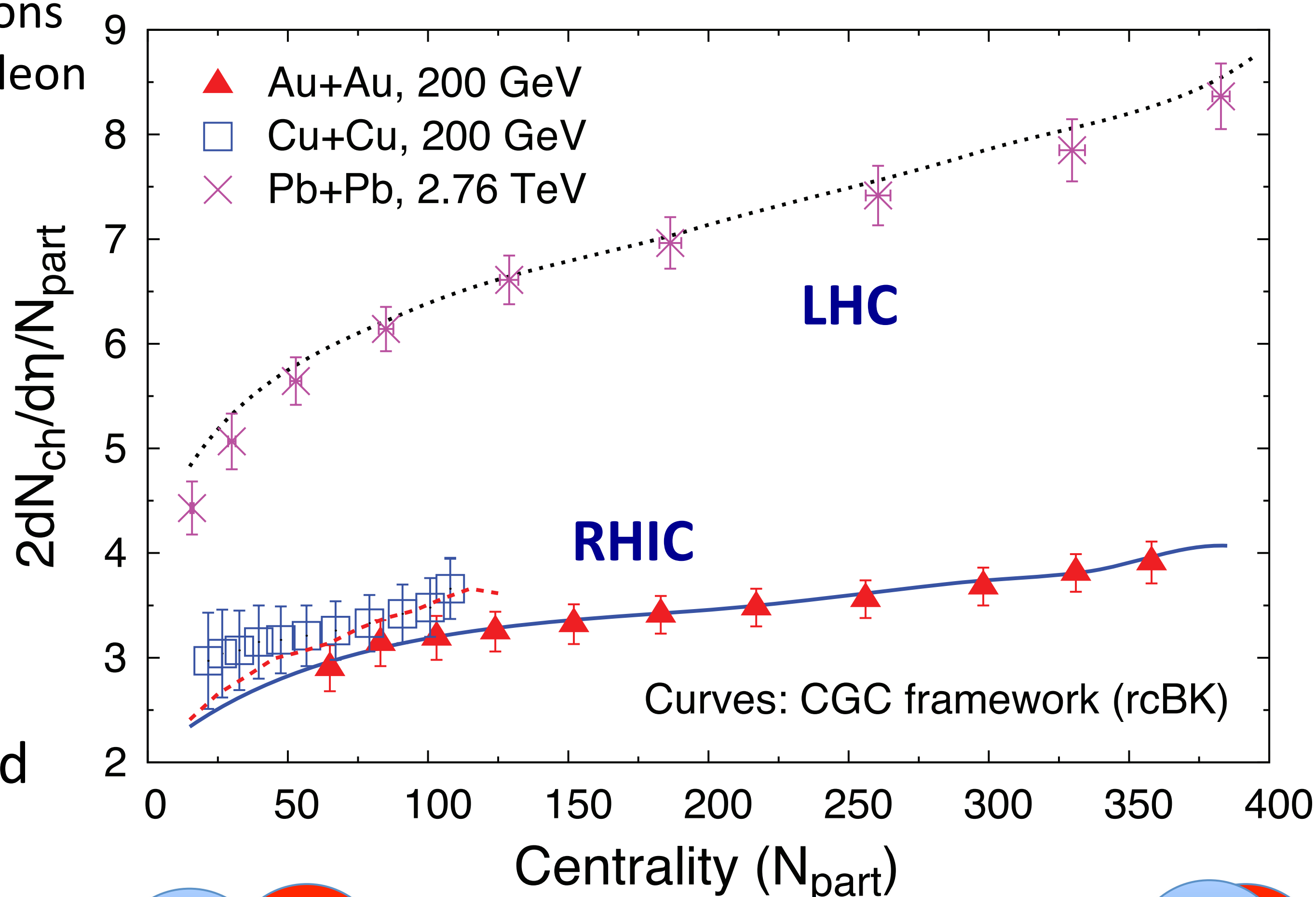
CGC prediction by
Albacete and Dumitru '10
for LHC multiplicity
and its centrality
dependence was
quite successful.

centrality

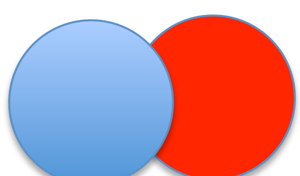


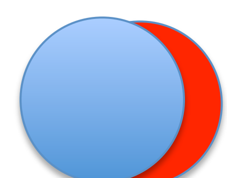
Multiplicity with centrality at RHIC and LHC

Number of hadrons per nucleon-nucleon collision at mid-rapidity.



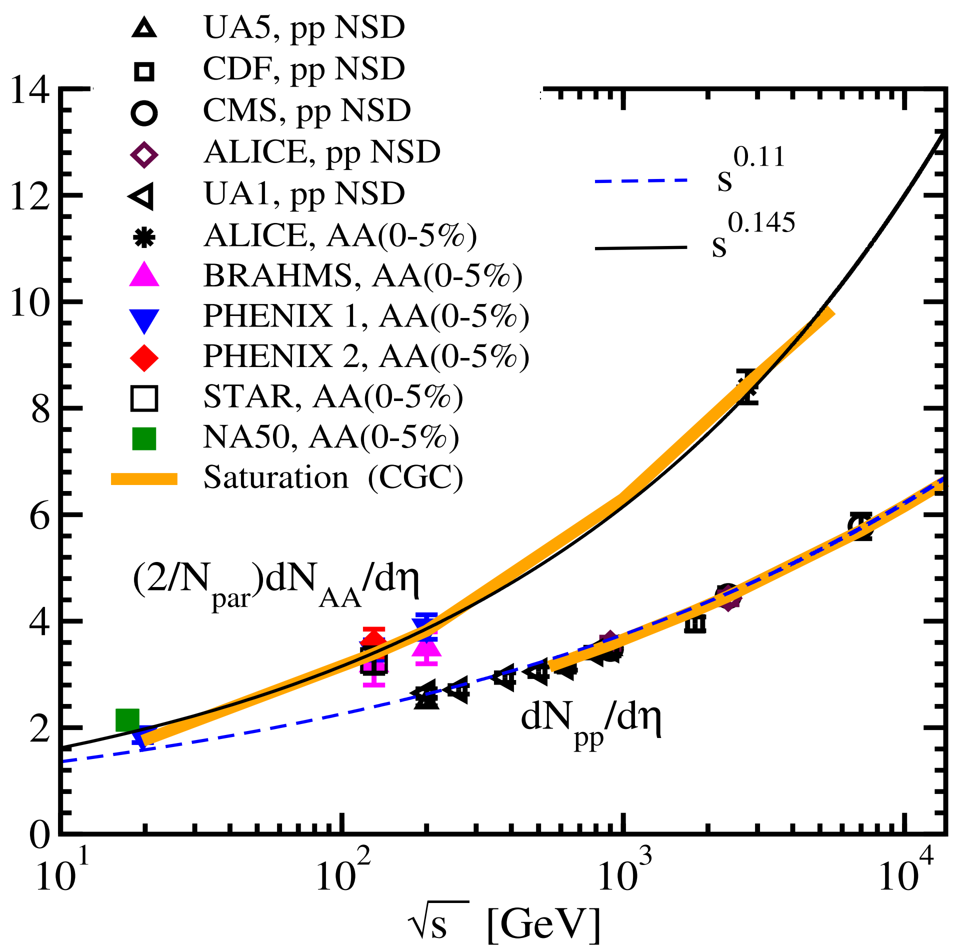
Albacete and Dumitru '10





Multiplicity vs. collision energy

Number of hadrons per unit rapidity at mid-rapidity

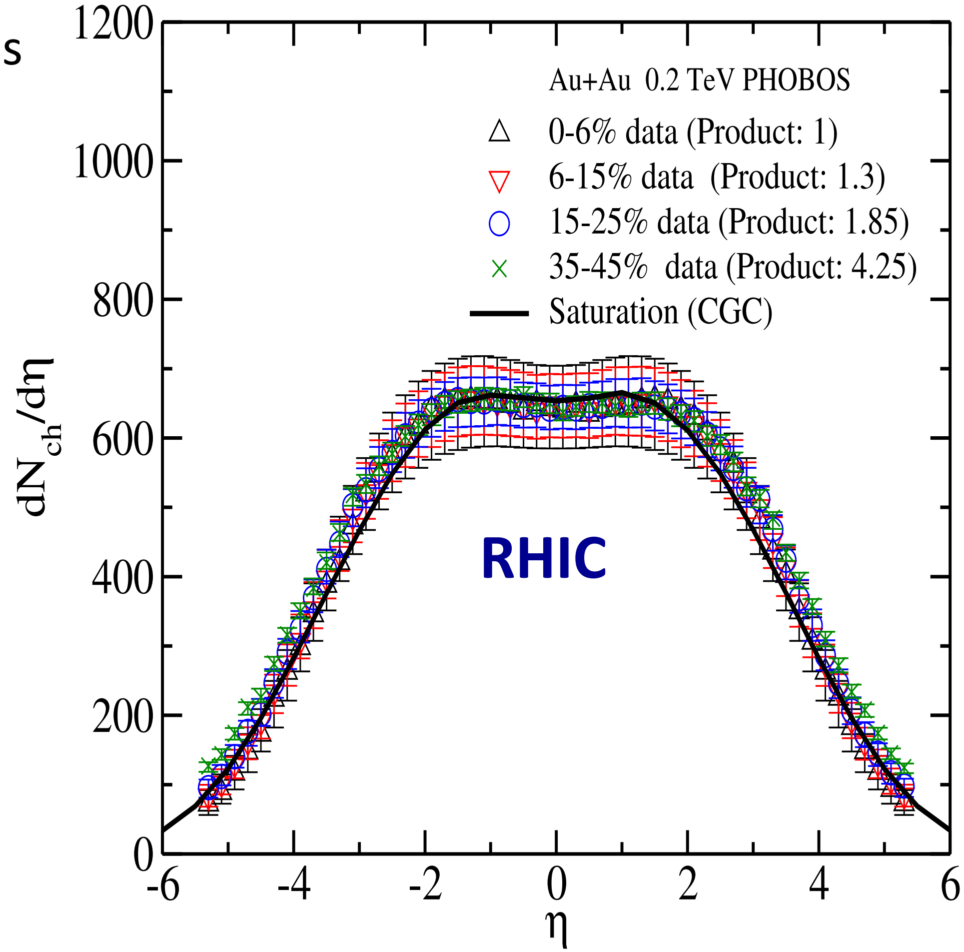


center-of-mass energy

Levin, Rezaeian '11

Multiplicity vs. rapidity

Number of hadrons per unit rapidity

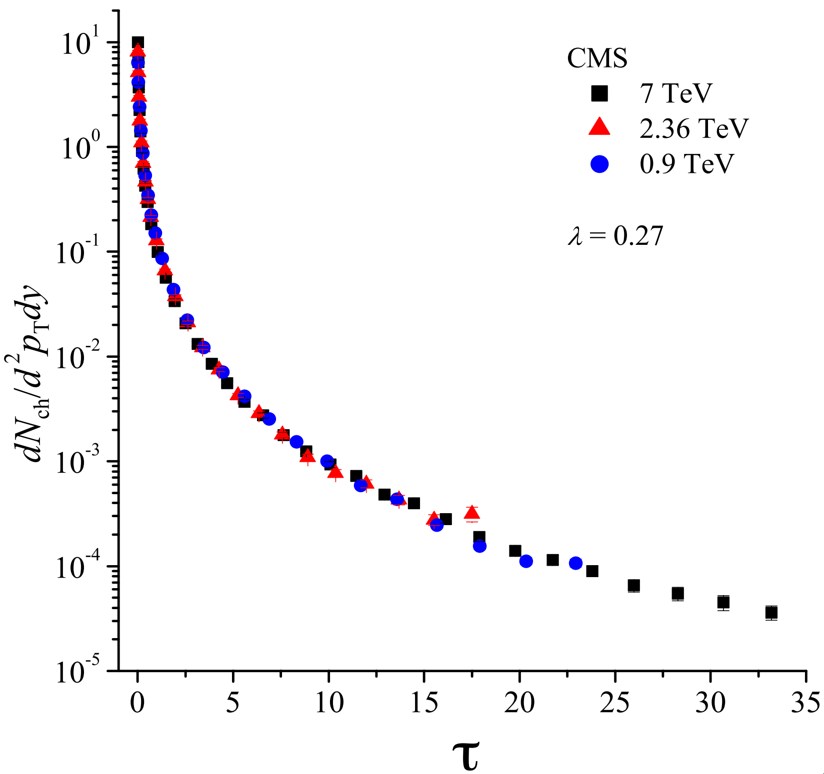


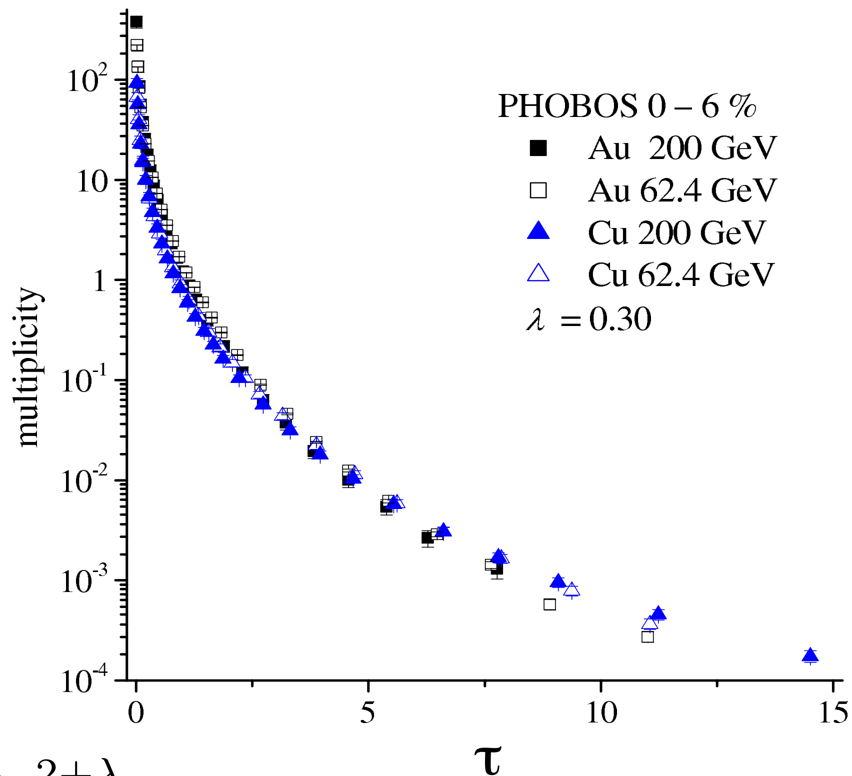
rapidity

Levin and Rezaeian, '11

Geometric Scaling

- Geometric scaling appears to be present in particle spectra both et RHIC and at LHC (McLerran&Praszalowicz (2011), Praszalowicz (2011)):



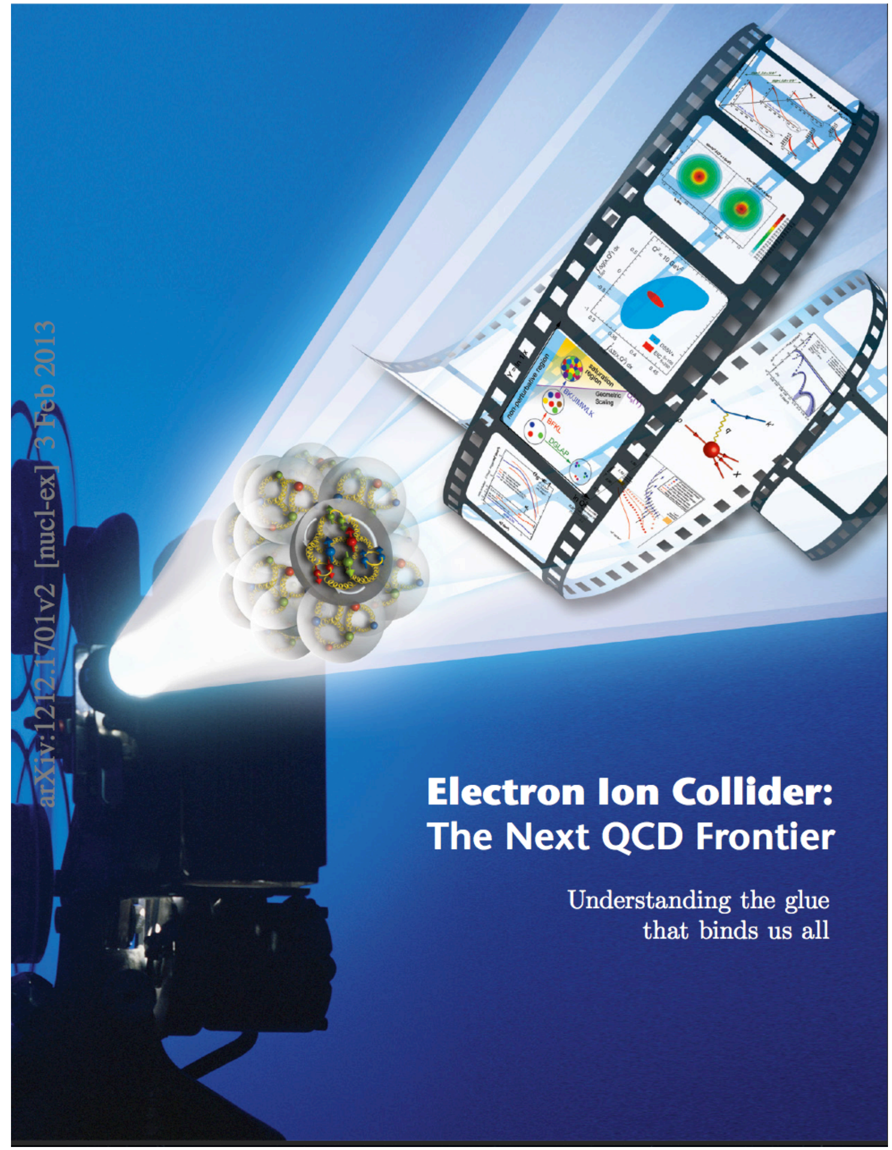


$$\tau = \left(\frac{p_T}{Q_s}\right)^{2+\lambda}$$

E. A case for EIC

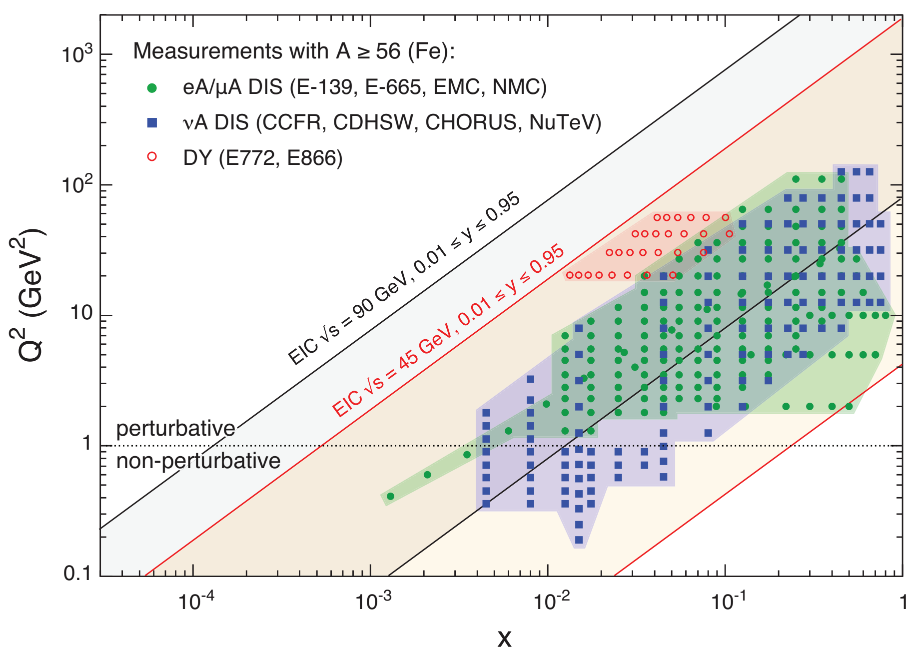
Electron-Ion Collider (EIC) White Paper

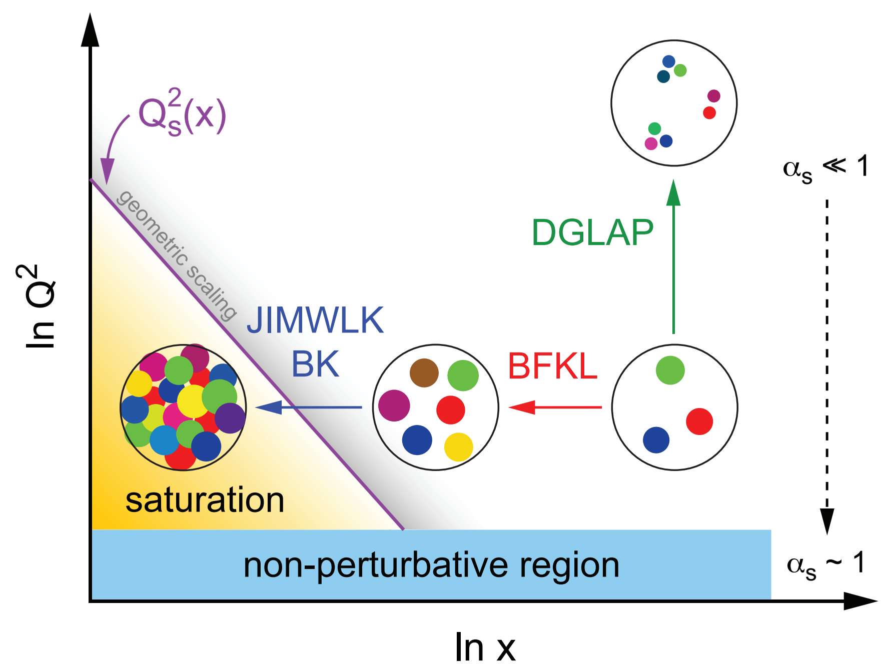
- EIC WP was finished in late 2012
- A several-year effort by a 19-member committee + 58 co-authors
- arXiv:1212.1701 [nucl-ex]
- EIC can be realized as eRHIC (BNL) or as ELIC (JLab)



Can Saturation Discovery be Completed at EIC?

EIC has an unprecedented small-x reach for DIS on large nuclear targets, allowing to seal the discovery of saturation physics and study of its properties:





Diffraction on a black disk

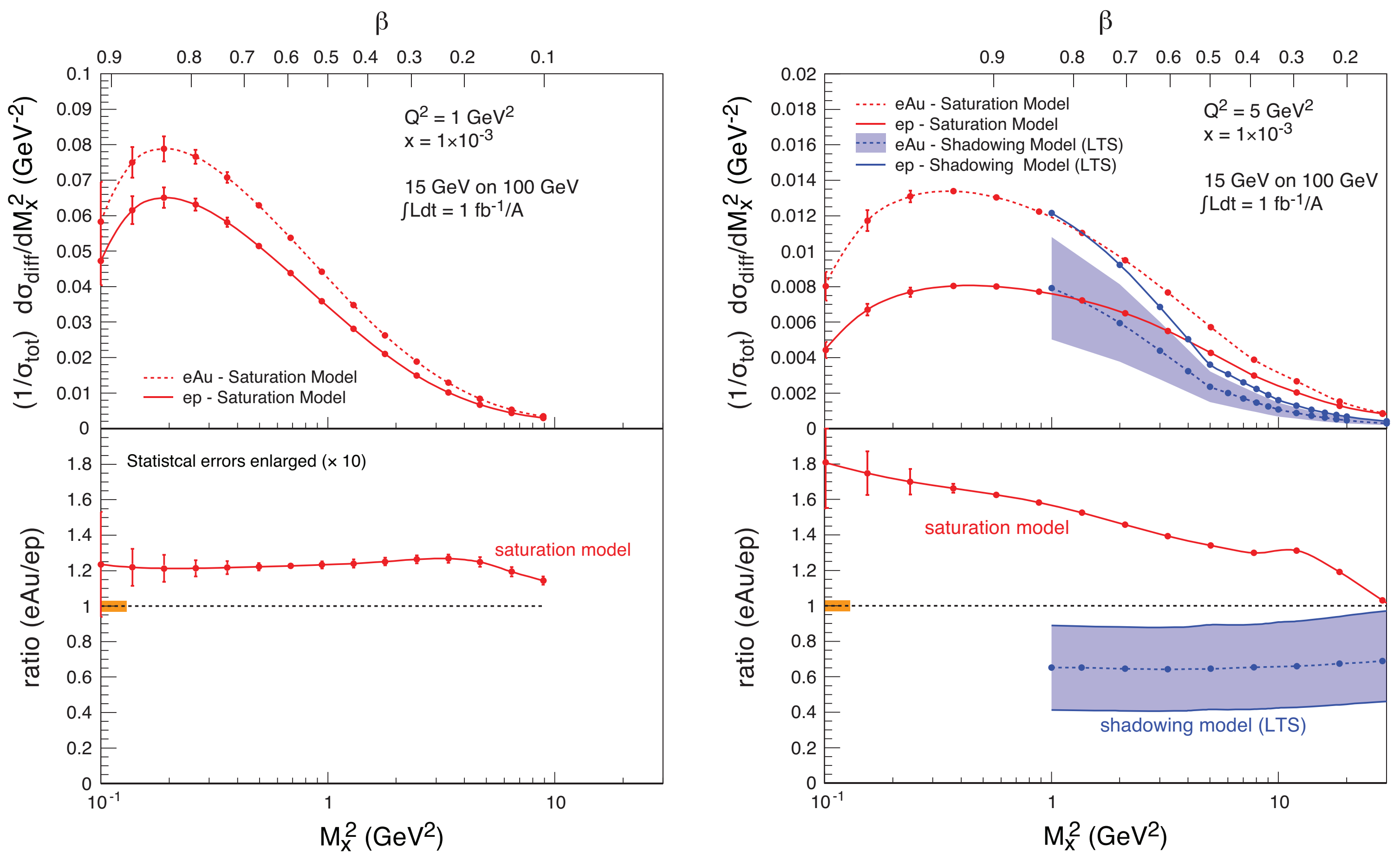
- For low Q^2 (large dipole sizes) the black disk limit is reached with N=1
- Diffraction (elastic scattering) becomes a half of the total cross section

$$\frac{\sigma_{el}^{q\bar{q}A}}{\sigma_{tot}^{q\bar{q}A}} = \frac{\int d^2b\, N^2}{2\int d^2b\, N} \longrightarrow \frac{1}{2}$$

- Large fraction of diffractive events in DIS is a signature of reaching the black disk limit!

Diffractive over total cross sections

- Here's an EIC stage-I measurement which may distinguish saturation from non-saturation approaches:



sat = Kowalski et al '08, plots generated by Marquet

no-sat = Leading Twist Shadowing (LTS), Frankfurt, Guzey, Strikman '04, plots by Guzey

Conclusions

- The field has evolved tremendously over recent two decades, with the community making real conceptual progress in understanding QCD in high energy hadronic and nuclear collisions.
- High energy collisions probe a dense system of gluons (Color Glass Condensate), described by nonlinear BK/JIMWLK evolution equations with highly non-trivial behavior.
- Calculation of higher-order corrections to the evolution equations is a rapidly developing field with many new results.
- Progress in understanding higher order corrections led to an amazingly good agreement of saturation physics fits and predictions (!) with many DIS, p+A, and A+A experiments at HERA, RHIC, and LHC.

Backup Slides

Conclusions

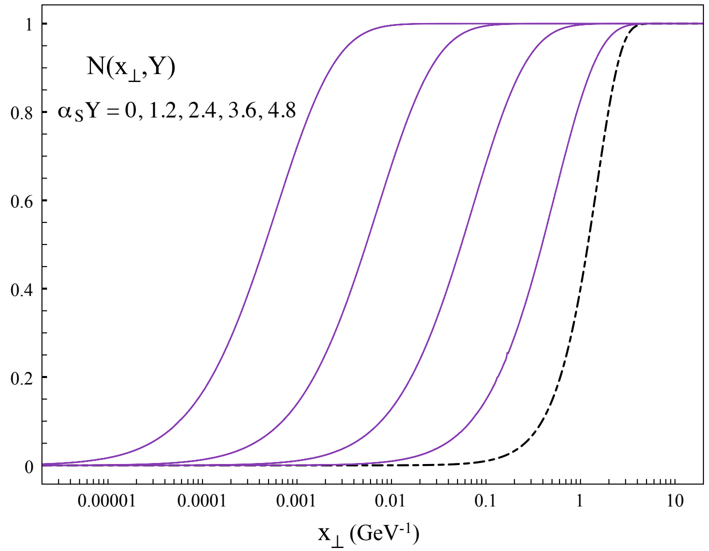
- In these lectures I introduced a 3-step approach to CGC: classical physics, small-x evolution, and running coupling corrections.
- This prescription appears to describe a wide range of small-x data on DIS, p(d)A, and AA collisions.

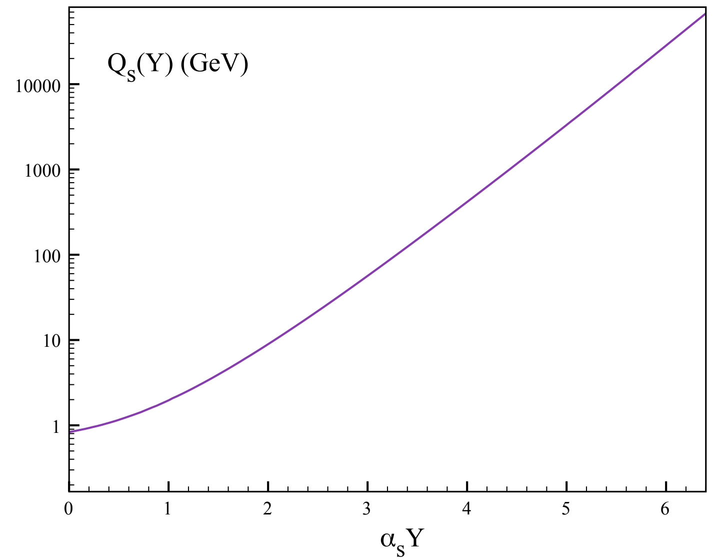
Last time

- The equation reads:

$$\partial_Y N_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{21}^2} \left[N_{\mathbf{x}_0,\mathbf{x}_2}(Y) + N_{\mathbf{x}_2,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_2}(Y)\, N_{\mathbf{x}_2,\mathbf{x}_1}(Y)\right]$$

- It combines BFKL evolution (the linear part) and the quadratic damping correction.
- We discussed its solution:





Dipole universality

- So far a wide range of observables, from total DIS cross section and structure functions, to the hadronic p_T spectra in pA are described in terms of a single quantity – dipole scattering amplitude.
- This is a new universal degree of freedom. (Gelis, Jalilian-Marian '02; Goncalves, Kugeratsky, Machado, Navarro '06; AGBS '12, etc.)
- However, there are observables, like two-particle correlations, which are described in terms of the higher-order correlators, like quadrupoles, etc.

NLO Corrections

- Note also that two iterations of NLO evolution kernel is parametrically of the same order as a combination of one LO and one NNLO kernels:

$$(\alpha_s^2\, Y)^2 \sim (\alpha_s\, Y)\, (\alpha_s^3\, Y)$$

- Does this mean that NLO kernel can only be inserted once into the LO evolution?

- Things simplify if you know the solution of the equation. For instance, in DGLAP case, perturbative expansion in the kernel naturally translates into the perturbative expansion in the anomalous dimensions.

- Nonlinear equations are hard. Let’s consider the linear BFKL evolution.

The Problem

- We want to find the BFKL Green function. It satisfies the BFKL equation

$$\partial_Y G(k,k',Y) = \int d^2q\, K(k,q)\, G(q,k',Y)$$

with the initial condition

$$G(k,k',Y=0) = \frac{1}{2\pi k}\delta(k-k')$$

- K(k,q) represents a BFKL kernel at an unspecified order in α_s.
- We need to find the eigenfunctions and eigenvalues for the kernel.

BFKL Equation in *N*=4 SYM Theory

- The form of the BFKL equation's solution is straightforward to determine in *N*=4 SYM theory: there the eigenfunctions are fixed by conformal symmetry and are simply $E^{n,\nu}$ (eigenfunctions of the Casimir operators of the Mobius group).

- In the angle-independent case at hand $E^{n,\nu}$'s reduce to simple powers of momentum k and we write the BFKL Green function in *N*=4 SYM theory as

$$G(k, k', Y) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi^2\, k\, k'}\, e^{\left[\alpha\, \chi_0(\nu) + \alpha^2\, \chi_1(\nu) + \ldots\right] Y} \left(\frac{k^2}{k'^2}\right)^{i\,\nu}$$

- Perturbative expansion takes place in the exponent (the eigenvalue).

Solving BFKL Equation in QCD

- QCD is not a conformal theory: we can not fix the all-order BFKL eigenfunctions by a symmetry argument.
- While simple powers are eigenfunctions for the LO kernel, they are not eigenfunctions for the NLO kernel due to the running coupling effects:

$$\int d^2q\, K^{\text{LO+NLO}}(k,q)\; q^{2\gamma-2} = \left[\bar{\alpha}_\mu\, \chi_0(\gamma) - \bar{\alpha}_\mu^2\, \beta_2\, \chi_0(\gamma)\, \ln\frac{k^2}{\mu^2} - \frac{1}{2}\, \bar{\alpha}_\mu^2\, \beta_2\, \chi_0'(\gamma) + \bar{\alpha}_\mu^2\, \chi_1(\gamma)\right] k^{2\gamma-2}.$$

LO BFKL eigenvalue

1-loop running coupling

Conformal NLO terms

NLO terms

$$\bar{\alpha}_\mu \equiv \frac{\alpha_\mu\, N_c}{\pi}$$

$$\chi_0(\gamma) = 2\,\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

$$\beta_2 = \frac{11\, N_c - 2\, N_f}{12\, N_c}$$

The Strategy

- Since the BFKL kernel is known perturbatively up to NLO

$$K(k,q) = \bar{\alpha}_\mu \, K^{\rm LO}(k,q) + \bar{\alpha}_\mu^2 \, K^{\rm NLO}(k,q) + \mathcal{O}(\bar{\alpha}_\mu^3)$$

it appears logical to construct the eigenfunctions order-by-order in the coupling as well. (Solving NLO BFKL equation exactly would exceed the precision of the approximation as NLO^2 = LO x NNLO.)

G. Chirilli, Yu.K. '13

- To find the eigenfunctions we thus write

$$H_\gamma(k) = k^{2\gamma-2} \, \left[1 + \bar{\alpha}_\mu \, f_\gamma(k) + \ldots \right]$$

and (perturbatively) impose the eigenfunction condition

$$\int d^2 q \, K^{\rm LO+NLO}(k,q) \, H_\gamma(q) = \Delta(\gamma) \, H_\gamma(k)$$

where the eigenvalue $\Delta(\gamma)$ is also an unknown.

NLO BFKL Solution

$$G(k, k', Y) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi^2} \, e^{\left[\bar{\alpha}_\mu \, \chi_0(\nu) + \bar{\alpha}_\mu^2 \, \chi_1(\nu)\right] Y} \, H_{\frac{1}{2}+i\,\nu}(k) \, \left[H_{\frac{1}{2}+i\,\nu}(k')\right]^*$$

- Note that the perturbative expansion is present both in the exponent and in the eigenfunctions (G. Chirilli, Yu.K. '13).
- The procedure can be repeated at higher orders in α_s and was implemented at NNLO already (G. Chirilli, Yu.K. '14).