Determination of $K\pi$ scattering lengths at physical kinematics

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Scattering phase shifts

For a system with two (pseudo)scalar particles, the S-matrix in centre-of-mass frame can be written in partial wave basis as:

$$\langle E', p', l', m' \mid S \mid E, 0, l, m\rangle = \delta(E' - E)\delta(p)\delta_{ll'}\delta_{mm'}S_l(E)$$

which is required by Lorentz invariance of the S-matrix, specifically $[H, S] = 0$, $[P, S] = 0$, $[J^2, S] = 0$, $[J_3, S] = 0$ and $[J_\pm, S] = 0$. Furthermore, unitarity of the S-matrix implies $S^\dagger S = SS^\dagger = 1$ gives

$$S_l(E) = e^{2i\delta_l(E)}$$

This means that the two (pseudo)scalar particle scattering can be expressed in terms of a single real parameter $\delta_l(E)$ called the *phase shift*.

Scattering length

At low energies (or equivalently momenta, k), phase shifts have the following threshold behaviour:

$$\delta_l(k) \sim k^{l+1}$$

- The dominant contribution will come from the s-wave ($l = 0$).
- We can define a constant called the *scattering length*:

$$(\delta_0(k)/k)^{-1} = \frac{1}{a_0} + \frac{r_{eff}}{2}k^2 + \mathcal{O}(k^4)$$

$K\pi$ scattering

With $m_u = m_d \equiv m_{ud}$ and QCD interactions only, isospin becomes a good quantum number.
Pions have $I = 1$, kaons have $I = 1/2$, so $K\pi$ can be in $I = 3/2$ or $I = 1/2$ state. Specifically:

$$|I = 3/2; I_z = 3/2\rangle = |K^+\pi^+\rangle$$

$$|I = 3/2; I_z = 1/2\rangle = \frac{1}{\sqrt{3}}\,|K^0\pi^+\rangle + \sqrt{\frac{2}{3}}\,|K^+\pi^0\rangle$$

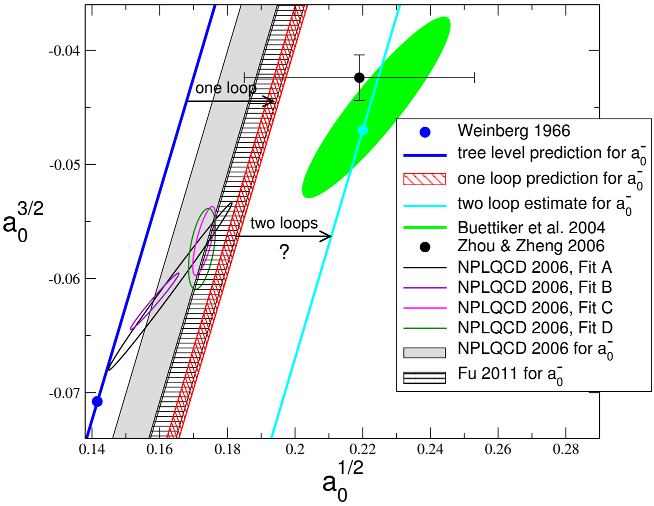
$$|I = 3/2; I_z = -1/2\rangle = \frac{1}{\sqrt{3}}\,|K^+\pi^-\rangle + \sqrt{\frac{2}{3}}\,|K^0\pi^0\rangle$$

$$|I = 3/2; I_z = -3/2\rangle = |K^0\pi^-\rangle$$

$$|I = 1/2; I_z = 1/2\rangle = \frac{1}{\sqrt{3}}\,|K^+\pi^0\rangle - \sqrt{\frac{2}{3}}\,|K^0\pi^+\rangle$$

$$|I = 1/2; I_z = -1/2\rangle = -\frac{1}{\sqrt{3}}\,|K^0\pi^0\rangle + \sqrt{\frac{2}{3}}\,|K^+\pi^-\rangle$$

Results so far



Plot courtesy of G. Colangelo.

Lattice results

	$a_0^{3/2} m_\pi$	$a_0^{1/2} m_\pi$
NPLQCD [1]	$-0.0574(16)\left(\begin{array}{c}+24 \\ -58\end{array}\right)$	$0.1725(13)\left(\begin{array}{c}+23 \\ -156\end{array}\right)$
Fu [2]	-0.0512(18)	0.1819(35)
PACS-CS[3]	-0.0602(31)(26)	0.183(18)(35)

Calculation also done by Lang et. al. [4], but without extrapolation to physical point.

This work: evaluation of scattering length **directly at physical point**.

[1] S. R. Beane et al. [NPLQCD Collaboration], Phys. Rev. D 74 (2006) 114503

[2] Z. Fu, Phys. Rev. D 85 (2012) 074501

[3] Kiyoshi Sasaki (Tokyo Inst. Tech.) et al. [PACS-CS Collaboration], Phys.Rev. D89 (2014) 054502

[4] C. B. Lang, L. Leskovec, D. Mohler and S. Prelovsek, Phys . Rev. D 86 (2012) 054508

S-wave phase shifts from a lattice - Lüscher's formula

In infinite volume, the two meson energies can be visualised on the complex energy plane as a branch cut starting at $m_1 + m_2$. In finite volume this is replaced by series of poles.

Position of these poles can are related to the s-wave phase shifts by Lüscher's condition [5]:

$$\delta(k) + \phi(q) = n\pi$$

$$\tan\phi(q) = \frac{q\pi^{3/2}}{Z_{00}(1;q)}$$

$$Z_{00}(1;q) = \frac{1}{\sqrt{4\pi}} \sum_{k\in\mathbb{Z}^3} \frac{1}{q^2 - k^2}$$

[5] M. Lüscher, Nucl. Phys. B354 (1991) 531-578

Two meson correlation function - choice of interpolators

$$C'^{ij}_{K\pi}(t) \equiv \langle 0 \mid O^{i\dagger}_{K_\pi}(t) O^{j}_{K\pi}(0) \mid 0 \rangle \tag{1}$$

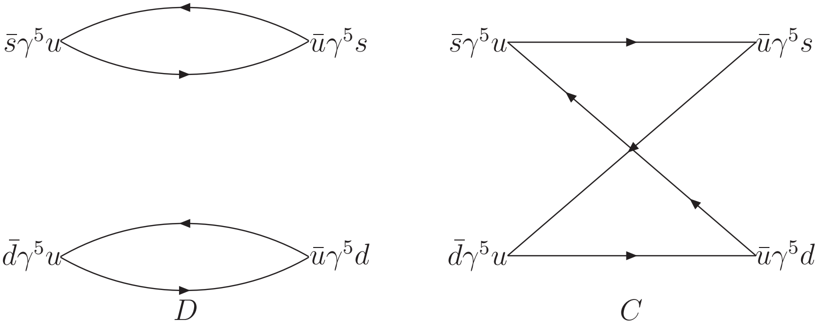
$$= \sum_n \langle 0 \mid O^{i\dagger}_{K\pi} \mid n \rangle \langle n \mid O^{j}_{K\pi} \mid 0 \rangle e^{-E_n t} \tag{2}$$

$$\xrightarrow{t \to \infty} \langle 0 \mid O^{i\dagger}_{K\pi} \mid K\pi \rangle \langle K\pi \mid O^{j}_{K\pi} \mid 0 \rangle e^{-E_{K\pi} t} \tag{3}$$

We use:

$$O^{\pm}_{K_\pi}(t) = \bar{s}(t \pm 2)\gamma^5 u(t \pm 2)\bar{d}(t)\gamma^5 u(t)$$

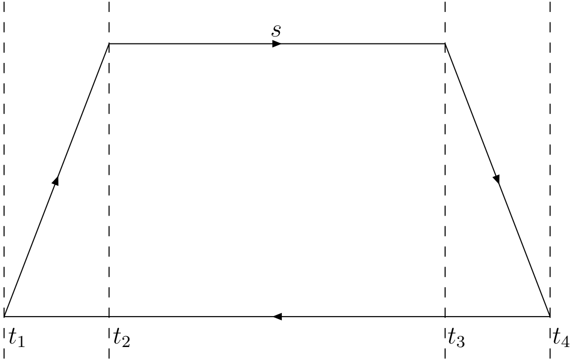
$K\pi$ I=3/2 contractions



$$D = \mathrm{Tr}\left(S^\dagger(t;2)L(t;2)\right)\mathrm{Tr}\left(L(t+2;0)^\dagger L(t+2;0)\right)$$
$$C = \mathrm{Tr}\left(S^\dagger(t;2)L(2;0)L^\dagger(t+2;0)L(t+2;2)\right)$$

$$C_{K\pi}^{I=3/2}(t) = D - C$$

Rectangle graph for I=1/2 correlator



$$C_{K\pi}^{I=1/2}(t) = D + 0.5C - 1.5R$$

κ resonance

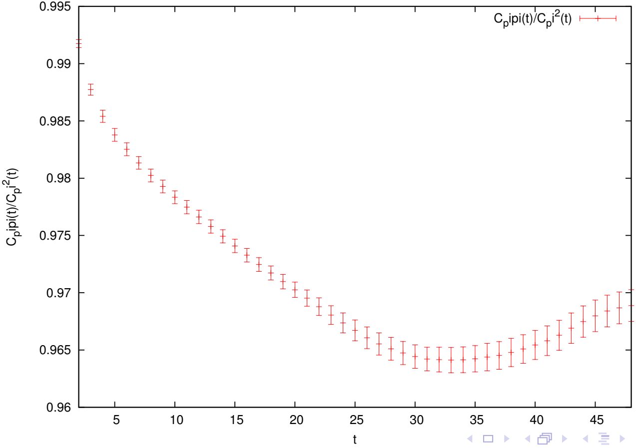
$K\pi$ at I=1/2 has a scalar resonance $K_0^*(800)$ (called κ). At physical kinematics, κ mass is larger than $E_{K\pi}^{I=1/2}$, so the ground state still corresponds to $E_{K\pi}^{I=1/2}$.
Previous studies have shown that the two-meson interpolators have a good overlap with $K\pi$ states for light pion masses. Another viable choice would be the scalar meson interpolator:

$$O(t) = \bar{s}(t)d(t). \tag{4}$$

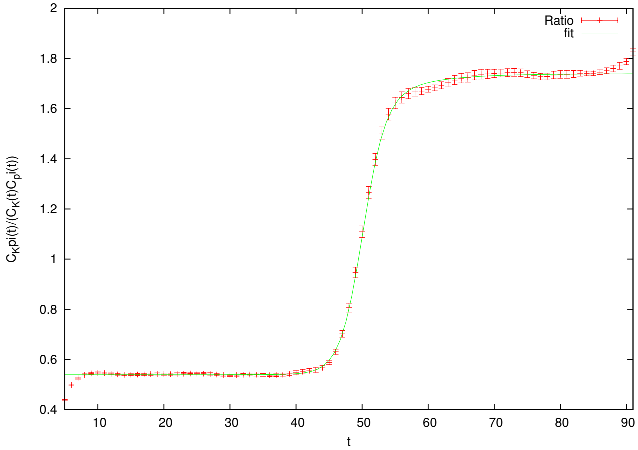
This operator however was shown to only have a good overlap with $K\pi$ state for heavy pion masses ($>$ 700 MeV) and was not considered in our test run.

For $\pi\pi$ scattering the energy difference can be accurately predicted from:

$$\frac{C_{\pi\pi}}{C_\pi^2} \approx N e^{-(E_{\pi\pi}-2m_\pi)t} \approx N(1-(E_{\pi\pi}-2m_\pi)t)$$



$K\pi$ I=3/2 scattering



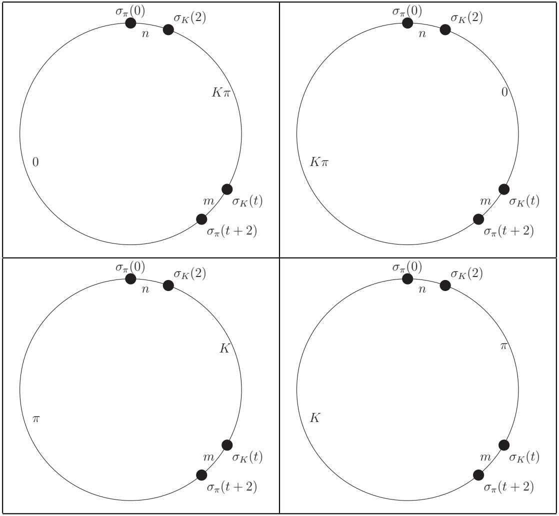
Around-the-world effects

In lattice simulations we often use (anti)periodic boundary conditions in the time direction. This means that, rather than e.g. $\langle 0 \mid O_1(t_1)O(t_2) \mid 0\rangle$ for some operators O_1, O_2, we're actually calculating:

$$\sum_n \langle n, t = T \sim 0 \mid e^{-H(T-t_1)} O_1 e^{-H(t_1-t_2)} O_2 e^{-Ht_2} \mid n, t = 0\rangle$$

This sum will contain not only the desired term, but also other contributions, referred to as 'around-the-world effects'.

Around-the-world effects



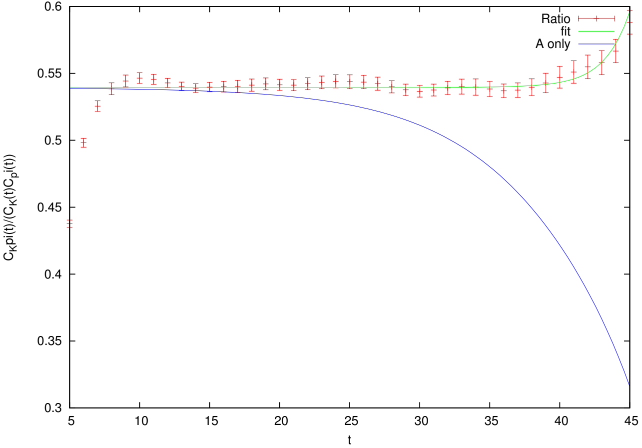
5 parameter fit

$$C_{K\pi}(t) = Ae^{-E_{K\pi}(t-2)} + Be^{-E_{K\pi}(T-t-2)} + Ce^{-m_K(t-2)}e^{-m_\pi(T-t-2)} + De^{-m_\pi(t-2)}e^{-m_K(T-t-2)}$$

with:

$$A = \langle 0 \mid O_{snk} \mid K\pi\rangle\langle K\pi \mid O_{src} \mid 0\rangle$$
$$B = \langle K\pi \mid O_{snk} \mid 0\rangle\langle 0 \mid O_{src} \mid K\pi\rangle$$
$$C = \langle \pi \mid O_{snk} \mid K\rangle\langle K \mid O_{src} \mid \pi\rangle$$
$$D = \langle K \mid O_{snk} \mid \pi\rangle\langle \pi \mid O_{src} \mid K\rangle$$

5-parameter fit - test results



Physical run parameters

Lattice size	$48^3 \times 96$
Gauge action	Iwasaki
Fermion acrion	Möbius DWF
L_s	24
M	1.8
β	2.13
am_s	0.0362
am_l	0.00078
a^{-1}	1.73(3) GeV
am_K	0.08079(24)
am_π	0.28886(35)

- quark sources every second time slice (48 per configuration)
- antiperiodic boundary conditions in time direction only

Preliminary results

All results shown are **PRELIMINARY**, based on 20 gauge configurations.
I=3/2

$$E_{K\pi} = 0.37007(35)$$
$$E_{K\pi} - m_K - m_\pi = 0.00041(29)$$
$$a_0^{3/2} m_\pi = -0.036(24)$$

I=1/2

$$E_{K\pi} = 0.36779(38)$$
$$E_{K\pi} - m_K - m_\pi = -0.00186(42)$$
$$a_0^{1/2} m_\pi = 0.190(50)$$

Comparison

	$a_0^{3/2} m_\pi$	$a_0^{1/2} m_\pi$
NPLQCD	$-0.0574(16)\left(\begin{array}{c}+24\\-58\end{array}\right)$	$0.1725(13)\left(\begin{array}{c}+23\\-156\end{array}\right)$
Fu	-0.0512(18)	0.1819(35)
PACS-CS	-0.0602(31)(26)	0.183(18)(35)
RBC-UKQCD **(preliminary)**	-0.036(24)	0.190(50)

Conclusions

- We are able to generate ensembles with physical pion and kaon masses.
- Calculation of $K\pi$ energies at low values of $m_\pi T$ and $m_K T$ suffers from significant around-the world effects.
- Around-the-world effects can be treated reliably using a 5-parameter fit.
- First calculation of scattering lengths that does not rely on chiral perturbation theory.
- Although low statistics prevent us from obtaining an accurate $I = 3/2$ result, we can get a good estimate for $I = 1/2$, which has been dominated by χPT errors in previous calculations.

Thank you for your attention!